

To the Graduate Council:

I am submitting herewith a dissertation written by Jenq-Der Tsou entitled "The Effect of Die Flow on the Dynamics of Isothermal Melt Spinning." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Polymer Engineering.

Donald C. Bogue
Donald C. Bogue, Major Professor

We have read this dissertation
and recommend its acceptance:

Edward S. Clark

J. E. Spruiell

John F. Fellus

John Ash

Accepted for the Council:

Lew Minkler
The Graduate School

THE EFFECT OF DIE FLOW ON THE DYNAMICS OF
ISOTHERMAL MELT SPINNING

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ABSTRACT

The dynamics of both steady and unsteady isothermal melt spinning have been studied, with emphasis on the effect of the die flow variables. Experimental results reported by earlier workers and supplemental data obtained in the present work are compared with theory and are in reasonable agreement.

An analysis of die flow effects must begin with consideration of the extrudate swelling (die swell) which occurs at the end of the die. This problem has been treated at various levels of rigor, ranging from finite-element solutions to one by Matsui and Bogue, which uses simplified kinematical assumptions. This analysis can be brought into agreement with some of the earlier theories and with experiment by the use of two adjustable parameters. This modified kinematical assumption also provides the initial stress conditions required in the subsequent analyses.

Next, one must consider the dynamics of steady spinning, especially as it is influenced by die flow (or die swell) effects. The experimental results show that the differential model used in the present work and an integral model can be used almost interchangeably to predict the dynamics of steady spinning, provided that one uses the same kinematics in the near die flow region.

Finally, the steady flow solution can be subjected to an infinitesimal (linear) disturbance and the stability of the spinline can be studied theoretically. This is the so-called "draw resonance" problem in which the spinline shows unbounded, periodic disturbances

above some critical draw ratio. Certain unusual effects associated with the thinning behavior of polymer melts and the die flow conditions are thus clarified. Considered in terms of the parameter "a" (having to do with the Bogue-White deformation rate dependent relaxation time) and the stress ratio (the ratio of the first normal stress difference to the shear stress in the die flow), one finds that the behavior is quite different depending on whether "a" is less or greater than $1/\sqrt{3}$.

In the existing analysis, there are two stable regions of spinning, one at low values of draw ratio and another at high values of draw ratio. Similar results come from the present analysis if $a < 1/\sqrt{3}$, although there is an effect of the stress ratio which cannot be predicted by the existing theories. Except in the second stable region at high draw ratios, it was shown that increasing the stress ratio makes the spinline less stable.

For $a > 1/\sqrt{3}$, however, the second stable region no longer exists; the critical draw ratio is determined by the maximum draw ratio attainable under any conditions (that is, from the steady spinning analysis). Beyond a certain maximum draw ratio, the velocity gradient goes to infinity and ductile failure occurs. Similarly, increasing the stress ratio results in lower critical draw ratios.

The agreement between theory and experiment is quite good for one of the two polypropylenes studied and is in at least qualitative agreement for the other, the first material having the narrower molecular weight distribution. A completely quantitative comparison is difficult to make because of the uncertainty in choosing the single relaxation time which is required by the theory.

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CHAPTER 1

INTRODUCTION

The formation of fibers of uniform diameter by the melt spinning process is an important industrial operation. The extruded filament is drawn down in cross-section by winding it up at a speed which is greater than the mean extrusion velocity. The draw ratio (the velocity ratio of the take-up velocity to the mean extrusion velocity) is the primary variable of interest. Above some critical value of draw ratio, the fiber spinline sometimes shows instabilities and exhibits periodic diameter fluctuations. Since this phenomenon, known as "draw resonance," sometimes limits productivity, it is important to define precisely the conditions under which it occurs.

The total spinline flow involves the near die (die swell) region and the downstream region. While the dynamics of melt spinning have been treated extensively, there have been few attempts to connect it to the flow prior to the spinline, that is, the flow in the die and near die regions. It is the main purpose of the present work to make this connection.

The problem of the flow in the die involves an entrance flow region, a developed tube flow region, and finally a post die (die swell) region. This problem has received considerable attention in the literature. The formation of irregular distorted extrudates is also well-known and is termed as "melt fracture." From flow visualization studies, it is known that the instabilities which give rise to these distorted extrudates develop in the die entrance and in

the die. While melt fracture would surely affect the downstream flow, it will not be considered further here. Thus, the present work considers only dies with large aspect ratio and die flow conditions where no melt fracture occurs. Even so, one can expect an effect of die flow on the downstream spinline dynamics because of the viscoelastic nature of polymeric materials.

The downstream spinline flow problem, which is assumed to be a purely elongational flow beginning at some point downstream of the die exit, has been treated by simply introducing some assumptions using different differential viscoelastic models. Analysis of both steady and unsteady spinning can be done with this differential treatment. However, no die flow effects can be predicted. By using an integral type viscoelastic model, one can treat the die flow and spinline flow together if one makes the assumption that the kinematics change in a prescribed (discontinuous) way at the die exit. While this leads to an analysis of steady spinning, it does not seem adaptable to a stability analysis. The present work thus tries to bring together the differential and integral approaches and to provide some understanding of the effects of die flow on the spinline stability.

CHAPTER 2

BACKGROUND

A. DIE SWELL

It has long been known that the jets of viscoelastic materials expand upon exiting from a capillary die, a phenomenon known as die swell (or extrudate swell). It is generally agreed that this swelling of the extrudate is typical of non-Newtonian, viscoelastic fluids and is related to their elastic properties. The many theoretical and experimental references concerning this phenomenon have been reviewed by Han (1), Vlachopoulos (2) and Huang (3). The following sections summarize the important literature references, with the additional comments relative to the present work.

A-1. Purely Viscous Fluids

For Newtonian fluids at high values of the Reynolds number, it is easy to show on the basis of mass and momentum balances that the diameter of the liquid jets exhibits a 13% contraction (4). On the other hand, at low Reynolds number, Middleman and Gavis (5) in measuring the die swell of viscoelastic jets discovered that a Newtonian liquid jet also swells at Reynolds number less than 16, the maximum being about 12%. The hydrodynamic problem of Newtonian swelling was solved by Tanner (6) using a finite-element program to solve the Navier-Stokes equation for creeping flow. He obtained a swelling of 13%. More details and further results on the finite-element method, applied to Newtonian and non-Newtonian

inelastic jets have been given in a series papers by Tanner and coworkers (7,8,9,10). They found that for power law fluids, the higher the flow index, the larger the die swell. The die swell values are in the range of 1 to 1.13 for pseudoplastic fluids with flow index ranging from 0 to 1 (8). Reddy and Tanner (10) showed that, for a slit die, the die swell of the Newtonian fluid is 1.19, they also studied the die swell of the second-order fluid.

The influence of take-up force on the die swell of Newtonian fluids was analyzed by Fisher, Denn and Tanner (11) using the finite-element method. They also studied the effects of gravity and surface tension. They found that the location of maximum die swell is closer to the spinneret as the force is increased, and the die swell is decreased as the force is increased. The uniform velocity (purely elongational flow) is found within one diameter of the spinneret. They also compared the experimental results obtained by Chang and Denn (12) on corn syrup. There were several experimental uncertainties having to do with establishing the local diameter accurately on the photographs and having to do with possible shearing action near the take-up device. Despite these uncertainties, the theoretical method predicts the die swell reasonably well. Keunings, Crochet and Denn (13) recently did studies of die swell and of the position of the developed flow (that is, where the assumption of purely elongational flow is applicable) for Newtonian fluids. They concluded that the developed position is about $0.9D$ or $0.6D$, depending on the different numerical techniques used in their finite-element method. Here D is the diameter of the spinneret. They also did similar work on viscoelastic fluids, as will be discussed in the next section.

A-2. Viscoelastic Fluids

The first attempt to derive an analytical expression for the die swell of polymer solutions assumed a high Reynolds number flow, where the response is largely determined by momentum fluxes and axial normal stresses, was done by Metzner, Sailor and White (14). However, for polymer melts because the Reynolds number is small and the momentum fluxes are negligible compared to the rheological force, their results cannot be used to relate normal stress to die swell, as shown by LaNieve and Bogue (15) and Graessley et al. (16).

Spencer and Dillon (17) assumed that extruded molten polymers behave as rubber-like solids and the extrudate swell is due to recovery as the material emerges from the die. Nakajima and Shida (18) evaluated the tensile stress required to pull the extrudate until its diameter becomes equal to the die diameter. Using an energy balance, Bagley and Duffey (19) obtained a die swell and a recoverable shear relationship. Grassley et al. (16) using Nakajima's (18) approach evaluated the energy required to deform the extrudate elastically to the diameter of the capillary. Later, Vlachopoulos et al. (20) obtained a result similar to that of Graessley et al. (16) based on the recoverable shear at the wall. It was shown that all of their expressions yield comparable values for the extrudate swell. For large values of die swell, they obtained

$$SR = \frac{P_{11} - P_{22}}{2P_{12}} \propto [B(0)]^2 \quad (2.1)$$

where SR is known as the recoverable shear or the stress ratio, P_{11}

and P_{22} are the normal stresses, and P_{12} is the shear stress. The die swell or extrudate swell is defined by

$$B = \frac{d}{D} \quad (2.2)$$

and $B(0)$ indicates the die swell without take-up force. In Eq. (2.2) D and d are the diameters of the capillary and the extrudate, respectively.

The modern view of the die swell of polymer melts--that die swell is the result of the unconstrained recovery of the stress developed in Poiseuille flow, in which inertial effects are neglected--has been reviewed by White (21). The first reasonably satisfactory treatment of this problem was done by Tanner (22). He used a constitutive equation of the BKZ (23) type and assumed N_2 , the second normal stress difference, is negligible. The sudden unconstrained die swell recovery expressed by Tanner (22) has the form

$$\begin{cases} X^1 = (x^1/\lambda_T^2) + g(x^2) \\ X^2 = \lambda_T x^2 \\ X^3 = \lambda_T x^3 \end{cases} \quad (2.3)$$

Tanner transformed the reference frame from the present time t (somewhere downstream of the swelled region with spatial coordinate x) to just before the stress is released where the coordinate is (X) and time is t' ($t' = 0^-$, where $t = 0$ indicates the position at sudden expansion) and arrived at the expression

$$B(0) = \frac{1}{\lambda_T} = \frac{\int_0^{d/2} (N_1 + G - \sigma_{12}^2/G) r \, dr}{\int_0^{d/2} G r \, dr} \quad (2.4)$$

where $N_1 = P_{11} - P_{22}$ is the first normal stress difference. G is the modulus, σ_{12} is the shear stress and r is the radial position. For a capillary die, assuming a single element Maxwellian fluid, the above equation yields

$$\begin{aligned} B(0) &= \left[1 + \frac{1}{2} \left(\frac{N_{1w}}{2\sigma_{12w}} \right)^2 \right]^{1/6} \\ &= \left[1 + \frac{1}{2} (\tau_o \dot{\gamma}_w)^2 \right]^{1/6} \end{aligned} \quad (2.5)$$

where the subscript "w" indicates values measured at the capillary wall. τ_o is the relaxation time and $\dot{\gamma}_w$ is the strain rate at the wall. Tanner found agreement between theory and existing experimental data of polystyrene if he added 0.1 to the right side of Eq.(2.5) to account for the viscous effect.

White (21) reproduced Tanner's result using a single element Maxwell model with the effective relaxation time defined as

$$\eta = \tau_{\text{eff}} G \quad (2.6)$$

where τ_{eff} is the effective relaxation time and η is the viscosity. He obtained results for both the Finger and Cauchy fluids, following different kinds of prior steady flow. White and Roman (24,25) developed a modified formulation of Tanner's theory where the melt

recovers from Poiseuille flow into a state of uniaxial extension. They found that the swell decreases rapidly with increasing applied take-up force according to

$$B^6 = [B(0)^6] - \frac{4F \tau_{\text{eff}} B^4}{\pi d^2 \eta} \quad (2.7)$$

$B(0)$ is the die swell without take-up force F and is represented by Tanner's swell equation, that is Eq.(2.5), but τ_{eff} is used instead of τ_0 . White and Roman also compared the unconstrained die swell data obtained in various ways (frozen, annealed into hot silicon oil, photographed emerging into air, photographed emerging through 180°C silicon oil) in the absence of take-up force for several polymer melts. They showed that data from the frozen extrudate method and the photographic method fit Eq. (2.5) reasonably well, but the die swell is much higher for the other two methods.

Another viscoelastic type of analysis, one using multiple time constants, was done by Matsui and Bogue (26) and Wust (27). Some of the basic ideas come from an analysis done by Lodge (28), who showed that multiple time constants give rise to both an instantaneous and delayed recoil. Matsui and Bogue (26) derived an equation for the rheological tensile stress, assuming that the velocity distribution in the die is a fully developed parabolic form and that this flow changes instantaneously to a uniform elongational flow at the die exit. They make the following kinematical assumptions:

$$\text{For } 0 < t' < t \quad \begin{cases} x^1 = \lambda X^1 \\ x^2 = \lambda^{-\frac{1}{2}} X^2 \\ x^3 = \lambda^{-\frac{1}{2}} X^3 \end{cases} \quad \text{where} \quad \frac{A(t')}{A(t)} = \frac{V(t)}{V(t')} \quad (2.8)$$

$$\text{For } t' < 0 \quad \begin{cases} x^1 = \lambda_o (X^1 + X^{2\dot{\gamma}t'}) \\ x^2 = \lambda_o^{-\frac{1}{2}} X^2 \\ x^3 = \lambda_o^{-\frac{1}{2}} X^3 \end{cases} \quad \text{where} \quad \lambda_o = \frac{A_{\text{die}}}{A(t)} = \frac{V(t)}{\bar{V}} \quad (2.9)$$

The corresponding Finger strain tensors are

$$C_{ij}^{-1} = \begin{vmatrix} \lambda^2 & 0 & 0 \\ 0 & \lambda^{-1} & 0 \\ 0 & 0 & \lambda^{-1} \end{vmatrix} \quad (0 < t' < t) \quad (2.10)$$

$$C_{ij}^{-1} = \begin{vmatrix} \lambda_o^{2(1+\dot{\gamma}^2 t'^2)} & \lambda_o^{1/2 \dot{\gamma} t'} & 0 \\ \lambda_o^{1/2 \dot{\gamma} t'} & \lambda_o^{-1} & 0 \\ 0 & 0 & \lambda_o^{-1} \end{vmatrix} \quad (t' < 0) \quad (2.11)$$

where A is the area, V is the velocity, $\bar{V}$ is the average velocity in the die, and $\dot{\gamma}$ is the shear rate defined as

$$\dot{\gamma}(r) = \frac{4\bar{V} r}{R^2} \quad (t' < 0) \quad (2.12)$$

where R is the radius of the die and r is the radial position.

Wust (27) showed that the result derived by Matsui and Bogue is able to predict die swell if the axial stress is known. The instantaneous die swell is obtained by setting $t = 0$;

$$\sigma_{11} = \sum_n \frac{G_n}{n} [(1 + F_n) B^{-4} - B^2] \quad (2.13)$$

where

$$\begin{cases} F_n = \int_0^1 \frac{\xi^2}{(K_n^{-1} + a\xi)^2} d\xi \\ K_n = \frac{8\bar{V}\tau_n}{D} = \gamma_w \tau_{n,die} \\ \xi = \left(\frac{r}{R_{die}}\right)^2 \end{cases} \quad (2.14)$$

and σ_{11} is the radius-averaged tensile stress.

For unconstrained recovery, that is $F = 0$, and assuming a Lodge fluid ($a = 0$), Eq. (2.13) reduces to

$$[B(0)]^6 = 1 + \left(\frac{\tau_0}{\gamma_w}\right)^2 \quad (2.15)$$

Eq. (2.15) is the result for an one element Maxwellian model. This result shows great similarity to the result of Tanner (22), and White and Roman (24,25), however, there is a $1/2$ which appears in Eq. (2.5), but not in Eq. (2.15) even though both consider the sudden expansion at $t = 0$.

There has been continuing theoretical analyses of the die swell of viscoelastic fluids under different die flow conditions using the finite-element method. Crochet and Keunings (29) studied the die swell of a single element Maxwell model at the exit of slit, circular and annular dies, assuming fully developed Poiseuille flow in the die. For $SR = 0$, the die swell for the Newtonian free jet was recovered. However, when SR is larger than zero, the die swell first decreases and then increases as SR is increased for the slit die. The same behavior was obtained by Caswell and Viriyayuthakron for the capillary die, using an integral type single element Maxwell model (30). Coleman (31,32) developed a finite-element method for analyzing the die swell of Dewitt fluids. He was able to calculate the die swell at the exit of a tapered die. In the special case of the Maxwell fluid, his method can be extended to recoverable shears of about 1.25 (32). Crochet and Keunings (33,34) pointed out that the range of recoverable shears and the values of the die swells and exit pressure losses obtained depend on the numerical algorithm used in the finite-element method. They showed that mesh effects also have a dominant influence on their results. Their numerical results lie above the theoretical result of Tanner (22), but the agreement of Tanner's analysis (that is, Eq. (2.5) with the addition of the zero shear die swell of Newtonian fluid) is still good up to $SR = 3$ (34). They indicated that the addition of a retardation time in the Maxwell model (that is, using an Oldroyd fluid B) can increase the recoverable shear value up to 4 without the occurrence of divergence in the numerical scheme (33). The influence of the take-up force on the die swell and also

the initial position where the velocity profile becomes uniform was studied by Keunings, Crochet and Denn (13) for the Maxwell fluid and Phan-Thien and Tanner fluid (35,36). At $SR = 1$, the maximum recoverable shear which allows convergence for the Maxwell fluid, they found that the location of the maximum die swell occurs further from the spinneret than in the Newtonian case. The position at which the velocity is uniform (that is, within 5% of the mean) is about $0.9D$ from the spinneret exit. The ratio P_{11}/P_{12} varies across the radius but radial uniformity (that is, variations less than 10% from the mean) occurs at the axial spinline position of $1.3D$. They concluded that the assumption of purely elongational flow is valid within two diameters of the spinneret exit, but downstream of the point of the maximum die swell.

There has been extensive work on the effects of molecular weight and molecular weight distribution on the die swell by Grasseley et al. (16), Vinogradov and coworkers (37), Han (1), Racin and Bogue (38), Huang and White (39), Minoshima, White and Spruiell (40,41) and Yamane (42). They concluded that the die swell properties are strongly dependent on the molecular weight distribution. Following Oda, White and Clark (43), the following rheological expression is used:

$$N_1 = A \sigma_{12}^b \quad (2.16)$$

where the parameters A and b depend on molecular weight distribution. Racin and Bogue (38) showed that the parameter b varies from 2.0 for narrowly distributed polystyrene samples to about 1.4-1.6 for broadly

distributed ones. Following Tanner's analysis (22), according to different integration across the cross section, Racin and Bogue (38), Huang and White (39) found

$$B(0) = \left[1 + \frac{4 - b}{b + 2} \left(\frac{N_{1w}}{2\sigma_{12w}} \right)^2 \right]^{1/6} + Co \quad (2.17)$$

$$B(0) = \left[1 + \frac{1}{b} \left(\frac{N_{1w}}{2\sigma_{12w}} \right)^2 \right]^{1/6} + Co \quad (2.18)$$

where Co is the extrudate swell correction as the shear stress or shear rate approaches zero. Racin and Bogue also showed that the Co values depend on the molecular weight distribution. For their polystyrene samples, the Co values range from 0.08 to 0.3 with the broader the molecular weight distribution, the higher the Co value. Racin and Bogue (38), Huang and White (39) found that Eqs. (2.16) to (2.18) correlate the unconstrained die swell of polystyrene reasonably well. Minoshima (40) and Yamane (42) used a special case of the above equation (namely $b = 2$, $Co = 0.1$) and in this special case, neither of the above equations can predict the die swell of the polypropylene they used.

Recently, the theory of elastic recovery following the flow of a plastic viscoelastic fluid model, as derived by White (44), has been further developed by White and Huang (45). This model responds as an elastic solid below the yield value and can be applied approximately to represent the flow of concentrated suspensions of small interacting particles in polymer fluids. Unconstrained recovery from both

uniaxial extension and shear flow are considered by them. The approach is similar to that of Tanner (22) and White and Roman (24,25).

B. DYNAMICS OF MELT SPINNING

Melt spinning is a fundamental process for making synthetic fibers. The process is shown schematically for a single filament in Figure (2-1). A filament is extruded from a spinneret into ambient air that is below the solidification temperature. The solidified filament is taken-up at a higher velocity than the mean extrusion velocity, causing a drawdown of the fiber. The location of the point of solidification can be controlled precisely by rapid quenching with an air or water bath, or solidification may be allowed to occur solely because of heat exchange with the surroundings all along the filament length. The spinline problem, beginning at some point downstream of the die swell, has been assumed to be a purely elongational flow. The review of this problem began with the work of Ziabicki in 1967 (46) and continued in 1976 (47). More detailed reviews have been given recently by Denn (48) and White (49).

B-1. Basic Balances and Boundary Conditions

Fluid mechanical analyses of melt spinning have all been based on balance equations derived by averaging the mass, momentum, and energy equations over a circular filament cross-section at each axial position. The most careful derivation is given by Matovich and Pearson (50). Beginning with the Cauchy equation of motion and

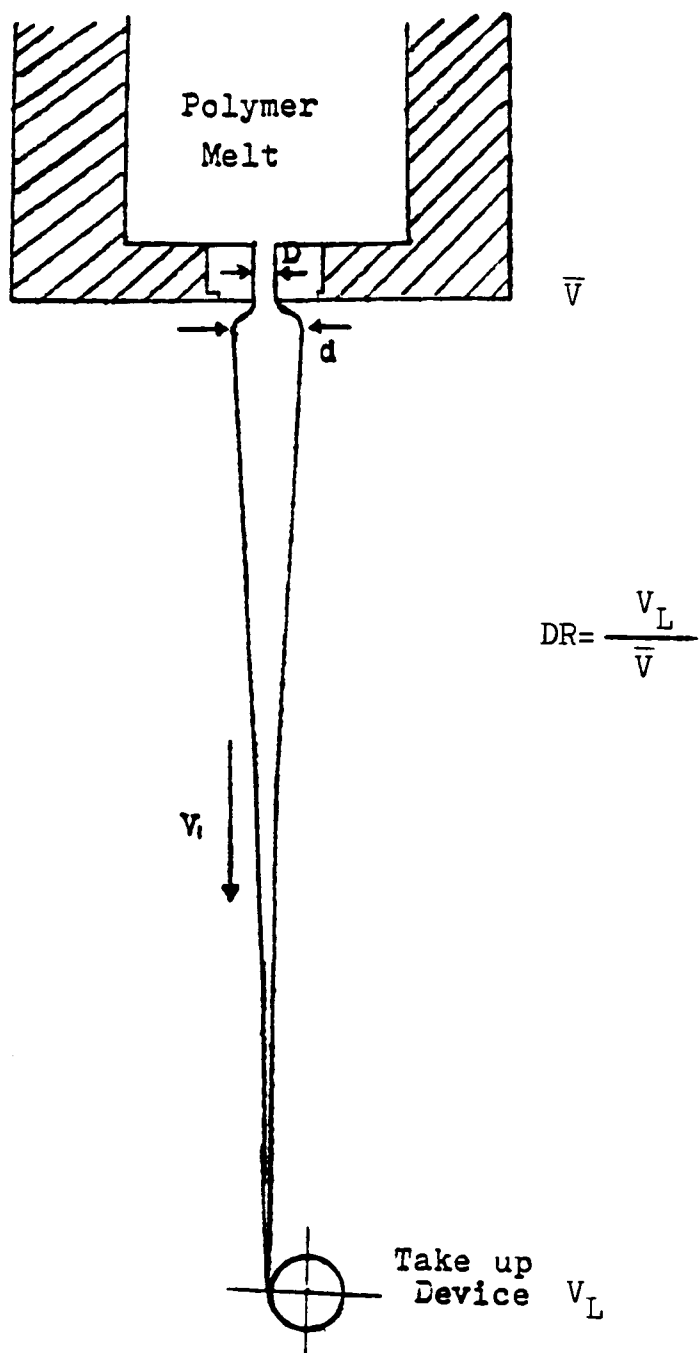


Figure 2-1. Schematic diagram of the spinline in melt spinning.

assuming that all derivatives with respect to the angular direction are zero, Ide and White (51) showed the resulting one dimensional asymptote momentum equation to be

$$\pi R^2 \rho \left(\frac{\partial U_1}{\partial t} + U_1 \frac{\partial U_1}{\partial x_1} \right) = \frac{\partial}{\partial x_1} (\pi R^2 \sigma_{11}) + 2 \pi S \frac{\partial R}{\partial x_1} + \pi R^2 \rho g \quad (2.19)$$

where the subscript 1 denotes the drawing direction and 2 is any direction in the orthogonal plane, R is the radius of fiber, ρ is the density of the fiber and t is time. σ_{11} is the total stress tensor, and S is the surface tension.

Another approach is to use a macroscopic force balance for a fiber spinline. As summarized by Bogue and White (52) and White (49), if one makes a balance between the take-up point and an arbitrary position in the spinline, we have

$$F_{\text{rheo}} = F + F_{\text{grav}} - F_{\text{drag}} - F_{\text{iner}} - F_s \quad (2.20)$$

where F_{rheo} is the rheological force, F is the applied tension, F_{grav} is the force of gravity, F_{drag} is the force due to frictional air drag, F_s is the force due to surface tension and F_{iner} is the inertia force. The above terms are defined as

$$F_{\text{rheo}} = \int_0^A \sigma_{11} da \quad (2.21)$$

$$F_{\text{grav}} = \int_x^L g A \rho dx_1 \quad (2.22)$$

$$F_{\text{drag}} = \int_x^L C_f \frac{\rho_a U_a^2}{2} \pi R dx_1 \quad (2.23)$$

$$F_s = C S \quad (2.24)$$

$$F_{\text{iner}} = \rho Q (U_L - U_x) \quad (2.25)$$

where A is the cross-sectional area, ρ_a is the density of the surrounding medium, Q is the volumetric flow rate, C is a constant determined by the filament shape and C_f is the friction factor.

The relative magnitudes of different forces on the spinline are discussed under varying conditions by Ziabicki (46, 47) and by Bankar, Spruiell and White (53). For moderate drawdowns, which is of interest in this work, only the rheological force is important, the Eq. (2.20) reduces to

$$F_{\text{rheo}} = \pi R \sigma_{11}^2 = \pi R (P_{11} - P_{22})^2 = F \quad (2.26)$$

Here σ_{22} is taken as zero. Also all the stresses are radius-averaged.

The continuity equation or mass balance equation can be stated as

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial x_1} (AU_1) = 0 \quad (2.27)$$

which for steady state spinning reduces to

$$\frac{\partial}{\partial x_1} (AU_1) = 0 \quad (2.28)$$

or

$$Q = U_1 A = U_1 \pi R^2 = \text{Constant} \quad (2.29)$$

The energy equation is started by neglecting axial conduction and viscous dissipation. Imaging a radius-averaged temperature (or assuming that the temperature is constant at each axial position), one has (1)

$$\rho C_p A \left(\frac{\partial T}{\partial t} + U_1 \frac{\partial T}{\partial x_1} \right) = -h \pi d (T - T_a) - \pi d e \sigma (T^4 - T_a^4) + \rho \Delta H A \frac{dY}{dt} \quad (2.30)$$

where d is the filament diameter, T_a is the surrounding temperature, σ is the Stefan-Boltzmann constant, e is the emissivity of the filament, H is the heat of crystallization per unit mass, Y is the degree of crystallization per unit mass, and C_p is the heat capacity. The heat transfer coefficient h depends on the local filament velocity, the radius and the cross flow air velocity. A widely accepted expression for h for a moving fiber spinline is that developed and used by Kase and Matsuo (54):

$$Nu = \frac{hd}{K_{air}} = 0.42 Re^{\frac{1}{3}} (1 + m) \quad (2.31)$$

where

$$Re = \frac{d U_1 \rho_{air}}{\eta_{air}} \quad (2.32)$$

m is a function of the ratio of the velocity of the transverse air flow to the filament velocity and subscript "air" implies the ambient air. Other correlations for h are summarized by Ziabicki (47) and White (49).

To carry forward with the analysis of melt spinning, one must combine the above conservation equations with a constitutive equation, relating the force to the deformation history. The area of rheological equation of state of viscoelastic fluids has been reviewed by Bogue and White (52), Han (1), Bird et al. (55) and White (56).

Two distinct types of constitutive equations are especially useful. These are the differential models deriving from Clerk Maxwell and Zaremba and the integral models deriving from Boltzmann. The generalized 3-dimensional Maxwell model has the form

$$\underline{\sigma} = -P\underline{I} + \underline{P} \quad (2.33)$$

$$\underline{P} + \tau \frac{\delta \underline{P}}{\delta t} = 2\tau G \underline{d} \quad (2.34)$$

In the White-Metzner (57) formulation, which has been successful in correlating experimental data in polymer solutions and melts, the derivative has the form

$$\frac{\delta \underline{P}}{\delta t} = \frac{\partial \underline{P}}{\partial t} + (\underline{U} \cdot \nabla) \underline{P} - (\nabla \underline{U} \cdot \underline{P}) - \underline{P} \cdot (\nabla \underline{U})^T \quad (2.35)$$

where $\underline{\sigma}$ is the total stress tensor, $\underline{I}$ the unit tensor, $\underline{d}$ the rate of deformation tensor and P is the isotropic pressure. $\underline{d}$ is defined as

$$\underline{d} = \frac{1}{2} (\nabla \underline{U} + (\nabla \underline{U})^T) \quad (2.36)$$

The relaxation time τ is taken to be a function of the trace of $\underline{d}^2$, according to Bogue (58), Bogue and White (52)

$$\tau = \frac{\tau_o}{1 + a \Pi_d^{1/2} \tau_o} \quad (2.37)$$

If the relaxation time spectrum is considered, then, according to Denn and Marrucci (59)

$$\underline{P} = \sum_n \underline{P}_n \quad (2.38)$$

was introduced in Eq. (2.33), and the resulting n coupling equations is obvious.

Another generalization of the Maxwell model was given by Phan-Thien and Tanner (35) and Phan-Thien (36). Lodge's (60) network theory is involved in deriving the constitutive equation. Their constitutive equations has the form:

$$\underline{P} = \sum_n \underline{P}_n$$

$$Y(q, \underline{P}_n) \underline{P}_n + \tau_n \left[\frac{\delta \underline{P}}{\delta t} + \zeta (\underline{d} \cdot \underline{P}_n + \underline{P}_n \cdot \underline{d}) \right] = 2 \tau_n G_n \underline{d} \quad (2.39)$$

$$Y(q, \underline{P}_n) = e^{\frac{q}{G_n} (\text{tr } \underline{P}_n)} \quad (2.40)$$

Y is related to the rate of destruction of network junctions. The relaxation time τ_n and moduli G_n are obtained from linear viscoelastic measurements. The adjustable parameter ζ is obtained from dynamic viscosity-shear viscosity shift data and is roughly a constant of order 0.1. The other adjustable parameter q must be determined by the extensional experiment. If one use the linear part of Eq. (2.40)

(35), the elongational viscosity levels off and tends to a constant at high elongation rate. For an exponential form of Y , that is Eq. (2.40), there is a maximum elongational viscosity, and the elongational viscosity decreases as the elongational rate is increased.

The integral models generally have the form

$$\underline{\sigma} = -P\underline{I} + \int_{-\infty}^t [M_1(t-t')\underline{C}^{-1} - M_2(t-t')\underline{C}] dt' \quad (2.41)$$

The memory function M_1 and M_2 may be reasonably well represented by the following forms, according to Bogue and White (52) and Chen and Bogue (61):

$$M_1(Z) = (1 + \frac{\epsilon}{2}) \bar{M}(Z) \quad (2.42)$$

$$M_2(Z) = \frac{-\epsilon}{2} \bar{M}(Z) \quad (2.43)$$

$$\bar{M}(Z) = \sum_n \frac{G_n}{\tau_n} e^{-\frac{Z}{\tau_n}} \quad (2.44)$$

where

$$\tau_n = \frac{\tau_{no}}{1 + a(\frac{1}{2}) \frac{\tau_{no}}{\underline{d}}} \quad (2.45)$$

with

$$\overline{\Pi_d^{1/2}}^2 = 2(\text{Tr } \underline{d})^2 = \frac{1}{t} \int_0^t \Pi_d^{1/2}(s) ds \quad (2.46)$$

where ϵ and "a" are constants. τ_{no} is the asymptote of τ_n at low deformation rates. If $\epsilon = 0$ we have Finger fluid, if $\epsilon = -2$ we have Cauchy fluid, according to White (21).

By generalizing concepts first suggested by Morland and Lee (62), Matsui and Bogue (26) and Matsumoto and Bogue (63) proposed the following form for the non-isothermal viscoelastic fluids of Finger fluids:

$$\underline{\sigma} = -P\underline{I} + \int_{-\infty}^t \underline{\Sigma G_n} \frac{e^{-\int_{t'}^t \frac{dt''}{\tau_n^*(t'')}}}{\tau_n^*(t')} \underline{C}^{-1} dt' \quad (2.47)$$

where

$$\tau_n^*(.) = \tau_{no} a_T[T(.)] / [1 + a \overline{\Pi_d^{1/2}} \tau_{no} a_T[T(.)]] \quad (2.48)$$

and a_T is the temperature shift factor obtainable from the Williams-Landel-Ferry equation (64) or a similar equation (65).

Another integral type constitutive equation based on "reptation" or "snakelike" movement of polymer molecular along its own contour was derived by Doi and Edwards (65-68). They assumed that a polymer chain is constrained to pass through a series of sliplinks imposed by entanglement and that a fluid deformation displaces the sliplinks affinely. Their constitutive equation has the form

$$\underline{P}(t) = \frac{60 \eta_0}{\pi^2 \tau_1} \int_{-\infty}^t u(t - t') \underline{Q} dt' \quad (2.49)$$

$$u(t) = \sum_{n, \text{odd}} \frac{8}{\pi^2 \tau_1} e^{\frac{-tn}{\tau_1}} \quad (2.50)$$

where $\underline{Q}$ is a "universal tensor" depends only upon the deformation history and not upon material properties, and η_0 is the shear viscosity at low shear rates. τ_1 is the longest relaxation time. Eqs. (2.49) to (2.50) can reduce to the corotational Maxwell model and convected Maxwell model. It is shown that use of Eqs. (2.49) to (2.50), the elongational viscosity has a slight maximum and then decreases as the elongational rate is increased.

In another non-isothermal formulation, Marrucci (69) used a simple molecular model based on a system of Hookean dumbbells to develop the expression

$$\underline{P} + \tau \left[\frac{\delta \underline{P}}{\delta t} - \frac{D \ln T}{Dt} \underline{P} \right] = 2 \tau G \underline{d} \quad (2.51)$$

which can be considered as generalization of Eq. (2.34). τ and G are functions of temperature. Gupta and Metzner (70) have followed the same arguments and developed the form

$$\underline{P} + \tau \left[\frac{\delta \underline{P}}{\delta t} + B \frac{D \ln T}{Dt} \underline{P} \right] = 2 \tau G \left[\underline{d} - (B + 1) \frac{D \ln T}{Dt} \underline{I} \right] \quad (2.52)$$

which reduces to Marrucci's results when B equals -1.

In the steady state, the above equations define a boundary value problem. In most situations the flow rate and temperature will be specified at the beginning of the spinline and the take-up velocity at the end is known; the take-up force can not be specified independently unless the take-up velocity or filament length is left unspecified. The proper initial conditions for the above equations have not been definitively established, since the asymptotic equations are not applicable at the spinneret. Use of finite-element method, Fisher, Denn and Tanner (11) show that the position where the asymptotic equations are applicable is within $1D$ from the spinneret for Newtonian fluid, and that the higher the take-up force, the closer the distance to the die exit. Recently, Keunings, Corchet and Denn (13) gave the same conclusion for Maxwell fluid and Phan-Thien and Tanner fluid except $2D$, instead of $1D$, is required. However, except in Matsui and Bogue (26), Wust (27), and Nam (71), who used the Matsui and Bogue's kinematical approximation and integral constitutive model, no literature connecting the die flow problem and the spinline problem based on a differential treatment is available.

B-2. Steady State Spinning

To obtain a description of the spinline dynamics, the balance equations just presented must be solved subject to various boundary conditions. Such solutions were first given by Kase and Matsuo (54) in 1965 and Matovich and Pearson (50) for purely viscous fluids. Since then, a number of modifications and generalizations have been made. This area has been reviewed by White (49) and Nam (71), from which the following discussion is summarized.

Zeichner (72) in 1972 gave an analytical solution of the isothermal melt spinning for a convected simple Maxwell model as a part of a larger study including spinline instabilities. His solution predicts a linear velocity profile at high spinline tension and high elasticity. He assumed that

$$P_{11} \gg P_{22} \quad (2.53)$$

that is, $P_{22} = 0$ along the spinline, and no initial stress condition is required because of Eq.(2.26). Similarly, White and Ide (73, 74) used the White-Metzner fluid model with Bogue-White deformation rate dependent relaxation time (Eq. (2.37)) and the same assumption to analyze spinnability and isothermal spinning of filament. They showed that the parameter "a" has a significant effect on the shape of spinline velocity profile. A more complete solution which includes the P_{22} component was given by Denn, Petrie and Avenas (75). However, the solution is of a high order differential equation and requires an additional initial stress condition. They claimed that the influence of the initial stress could be ignored for the case of a high Weissenberg number and high tensile tension in the spinline, and the behaviour of $F1(0) \neq 1$ is not very different from $F1(0) = 1$, except at small dimensionless spinline distance. Thus they assumed

$$F1(0) = \frac{Q}{FU_0} P_{11} = 1 \quad (2.54)$$

That is

$$P_{22}(0) = 0 \quad (2.55)$$

according to Eq. (2.26). They found that, when $\frac{\epsilon}{\alpha} \rightarrow 0$,

$$DR = 1 + \alpha^{-1} \quad (2.56)$$

where the Weissenberg number α is

$$\alpha = \frac{\tau U_0}{L} \quad (2.57)$$

and

$$\epsilon = \frac{Q \eta}{FL} \quad (2.58)$$

L is the length of the spinline, Q is the volume flow rate, η ($\eta = \tau G$) is the viscosity, U_0 is the velocity at the starting position presumably at the position of maximum die swell or somewhere downstream the maximum die swell where the elongational flow can be applied. Their results were in agreement with the experimental results of Kanel (76) and Spearot and Metzner (77).

Fisher and Denn (78) in 1976 extended the prior analysis of Denn et al. (75) for convected Maxwell fluid but with power law deformation rate dependence of relaxation time; that is

$$\tau = \frac{K}{G} \frac{(n-1)/2}{\dot{\gamma}} \quad (2.59)$$

Numerical results for steady state velocity profile are in good agreement for a shear thinning polystyrene. Denn and Marrucci (59) in 1977 applied a spectrum of relaxation times. They found that the use of a relaxation time spectrum leads to higher forces and a more linear

velocity profile than that would be obtained for a Maxwellian material of the same mean properties in shear.

Phan-Thien (36) studied the steady melt spinning of Phan-Thien and Tanner fluid, that is Eqs. (2.38) to (2.40). They used the relaxation spectrum of IUPAC LDPE with $\zeta = 0.1$ and $q = 0.015$ and the same initial stress assumption as that used by Denn et al. (75) and Fisher and Denn (78). The numerical results show good agreement with the experimental data of Spearot (77).

The analytical solution for the corotational Maxwell model, that is, one in which the Zaremba time derivative is used instead of the convected derivative in Eq. (2.34), was obtained by Petrie (79). He showed that this fluid gives a more rapid drawdown than a Newtonian fluid.

Recently, Gagon and Denn (80), and George (81) studied the computer simulation of steady polymer melt spinning including inertia and air drag effects. The former authors studied the viscoelastic fluids described by Phan-Thien and Tanner's constitutive equation (35) which can be reduced to the convected Maxwell model under certain conditions. Their results were compared with experiments for PET for high speed spinning.

Numerical solutions of the asymptotic equations for non-isothermal pure viscous Newtonian fluid were given by Kase and Matsuo (54) in 1965. In 1969, Hamana, Matsui and Kato (82) presented a more concise formulation of the non-isothermal melt spinning problem which includes air drag, gravitation and momentum flux forces for Newtonian polymer melt. Lamonte and Han (83) in 1972 made a

significant improvement in the model of Hamana, Matsui and Kato by including non-Newtonian effects on the spinline elongational viscosity. The use of differential model to analyze non-isothermal melt spinning was initiated by Fisher and Denn (84). They used a convected Maxwell model developed by Marrucci (69) as shown in Eq. (2.47) and assumed that viscosity has an Arrhenius form. The influence of increasing Stanton number was shown not only to make the temperature change more rapidly along the spinline but also to make the velocity vary linearly.

In addition to the differential model just discussed, the spinning problem has been also dealt with integral models. Early studies for the integral model were done by Acierno, Dalton, Rodriguez and White (85) and Chen, Hagler, Abbott, Bogue and White (86). These studies mostly involved an analysis of the spinline itself and comparisons of rheological modeling with experiment, and did not include the effect of die flow. The effect of die flow was explicitly included by Matsui and Bogue (26). The kinematical assumption used is shown in Eqs. (2.47) and (2.48). The numerical solutions given by Wust (27), and in more detail by Nam (71), predict both die swell and the velocity distribution along the spinline. Computer solutions were in good agreement for fitting the experimental velocity profile along the spinline for different molecular weight distribution polypropylenes, with the three element model fitting better than one element model. There exists a "blow-up" point at some specified spinline length, where a numerical solution cannot be obtained if the thinning parameter " a " of Eq. (2.37) is larger than $1/\sqrt{3}$.

Recently, Larson (87) studied the isothermal melt spinning for Doi-Edwards fluid (that is, Eqs. (2.49) to (2.50)) and found that the velocity profile grows supraexponentially and no solution can be obtained at some spinline length. The velocity profiles for the Doi-Edwards fluid and the corotational Maxwell model are coincident, including the value at which the solution ceases to exist. The analytical solution of the corotational Maxwell model was obtained by Petrie (79).

B-3. Instabilities in Melt Spinning

The spinning of fibers from polymer melts is sometimes limited at high take-up rates by the onset of a flow instability known as "draw resonance." The instability manifests itself at a critical draw ratio by the occurrence of sustained oscillations in the force required to maintain a constant take-up velocity. These force oscillations are accompanied by in-phase oscillations in the take-up diameter and the instability is always recorded as a systematic deviation from unity of the ratio of maximum to minimum diameter of the drawn filament. The theoretical and experimental aspects of this problem will now be reviewed.

The first reported observation appear to have been by Christensen (88) in 1962, in a discussion of coating wire with polypropylene. A review of the literature through 1976 is given by Petrie and Denn (89). More recently, White(49) gave a summary of the literature on draw resonance. He also discussed other flow instabilities in melt spinning.

The first theoretical analyses have been developed independently by Kase, Matsuo and Yoshimoto (90), Matovich and Pearson (91), and Gelder (92) for isothermal melt spinning of Newtonian fluids. They performed linear stability analyses and showed that the instability occurs when the draw ratio is greater than 20.21. The nonlinear stability analysis of a Newtonian spinline to finite amplitude disturbance was studied by Ishihara and Kase (93, 94) and by Fisher and Denn (95) using different approaches. It is found that the critical draw ratio for the growth of disturbances is the same as that obtained by linear stability theory, since all finite disturbances decay at a draw ratio lower than the critical draw ratio. At draw ratios just beyond the onset of draw resonance, the period of the finite-amplitude limit cycle is only slightly larger than that given by linear theory. Hyun (96) used a different approach based on the competition between a "throughput wave" velocity and the actual material velocity and predicted a critical draw ratio of 19.74. Hyun's work is disputed by Denn (97), who has been answered in turn by Hyun (98). Shah and Pearson (99) and Ishihara and Kase (100) considered power law fluids and showed that the spinline becomes more stable as the power law exponent is increased. For pseudoplastic fluids, the critical draw ratio is less than 20.21 and when the exponent is higher than 1.5, the critical draw ratio is very high.

Zeichner (72) in 1972 first considered the stability for a single element convected Maxwell fluid, and later Fisher and Denn (78) extended his analysis for a convected Maxwell model with a variable relaxation time based on a power law representation (that is,

Eq. (2.59)). They found that the critical draw ratio depends on the power law exponent n and a viscoelastic parameter (a kind of Weissenberg number) defined by

$$\alpha = \frac{(n-1)/2}{G} K \left(\frac{U_0}{L} \right)^n \quad (2.60)$$

where K is a constant defined in power law viscosity function. U_0 is the velocity at the initial position. The results are shown in Figure (2-2). The critical draw ratio (DRC) is relatively constant over a wide range of α , indicating that the onset of the instability is governed by the purely viscous behavior defined for $\alpha \rightarrow 0$. At higher values of α , the fluid elasticity causes stabilization. Finally, the stability envelop doubles back, indicating regions of stability at both large α and large draw ratio. A nonlinear stability analysis was also made by Fisher and Denn (78) for the Lodge fluid. The same conclusion as that for Newtonian fluids was drawn. This indicates that a linear stability analysis does define the complete stability boundary. Recently, Chang and Denn (101) studied the stability of Phan-Thien two parameters fluid (that is, Eqs. (2.38) to (2.40)). The results of their linear stability analysis are shown in Figure (2-3) for single element model with

$$\alpha = \frac{\tau_{\max} U_0}{L} \quad (2.61)$$

where $\tau_{\max}$ is the maximum zero shear relaxation time. The important difference between their results and those of Fisher and Denn (see

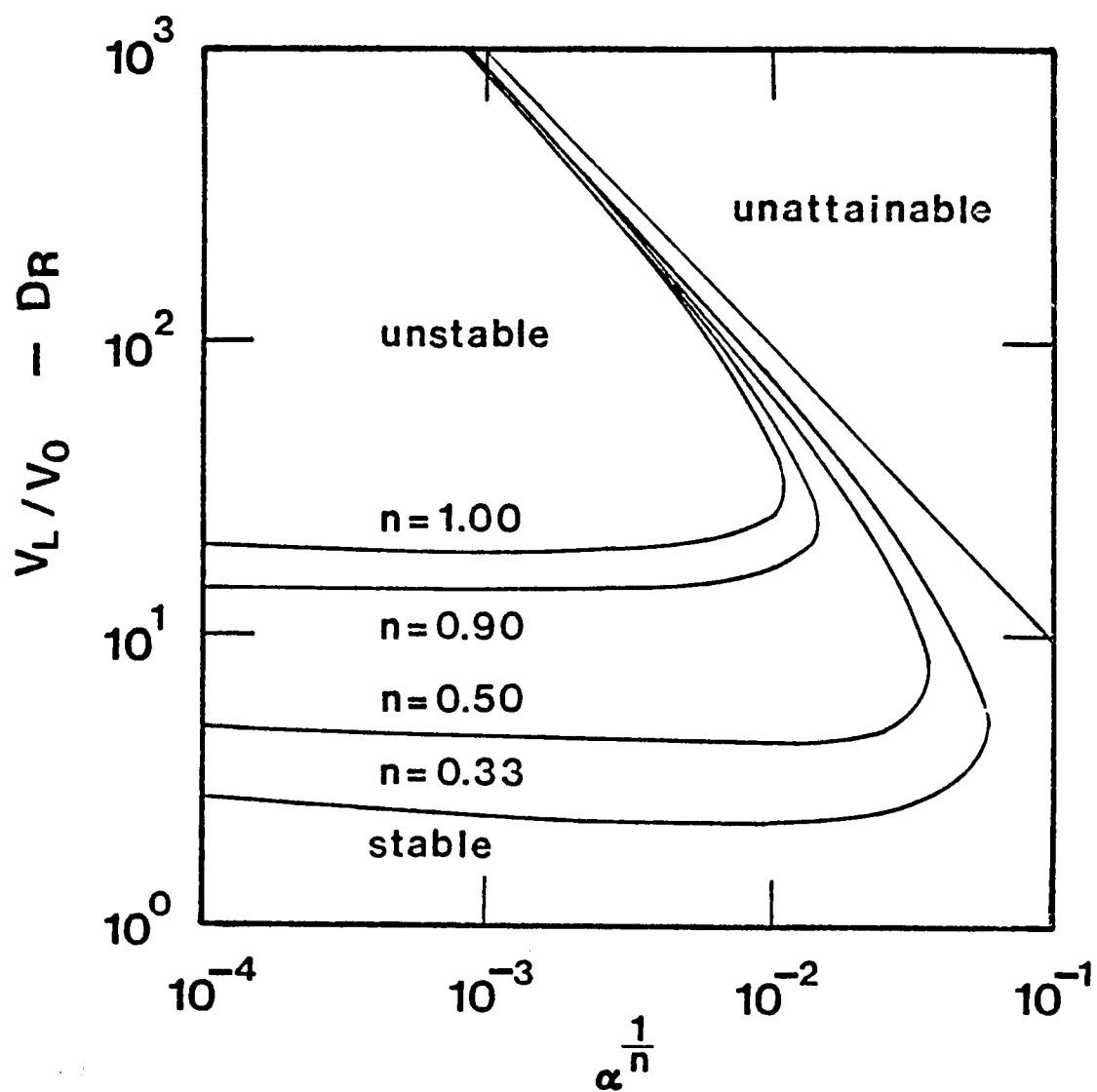


Figure 2-2. Critical draw ratio as a function of a viscoelastic parameter and the power law index (from Fisher and Denn).

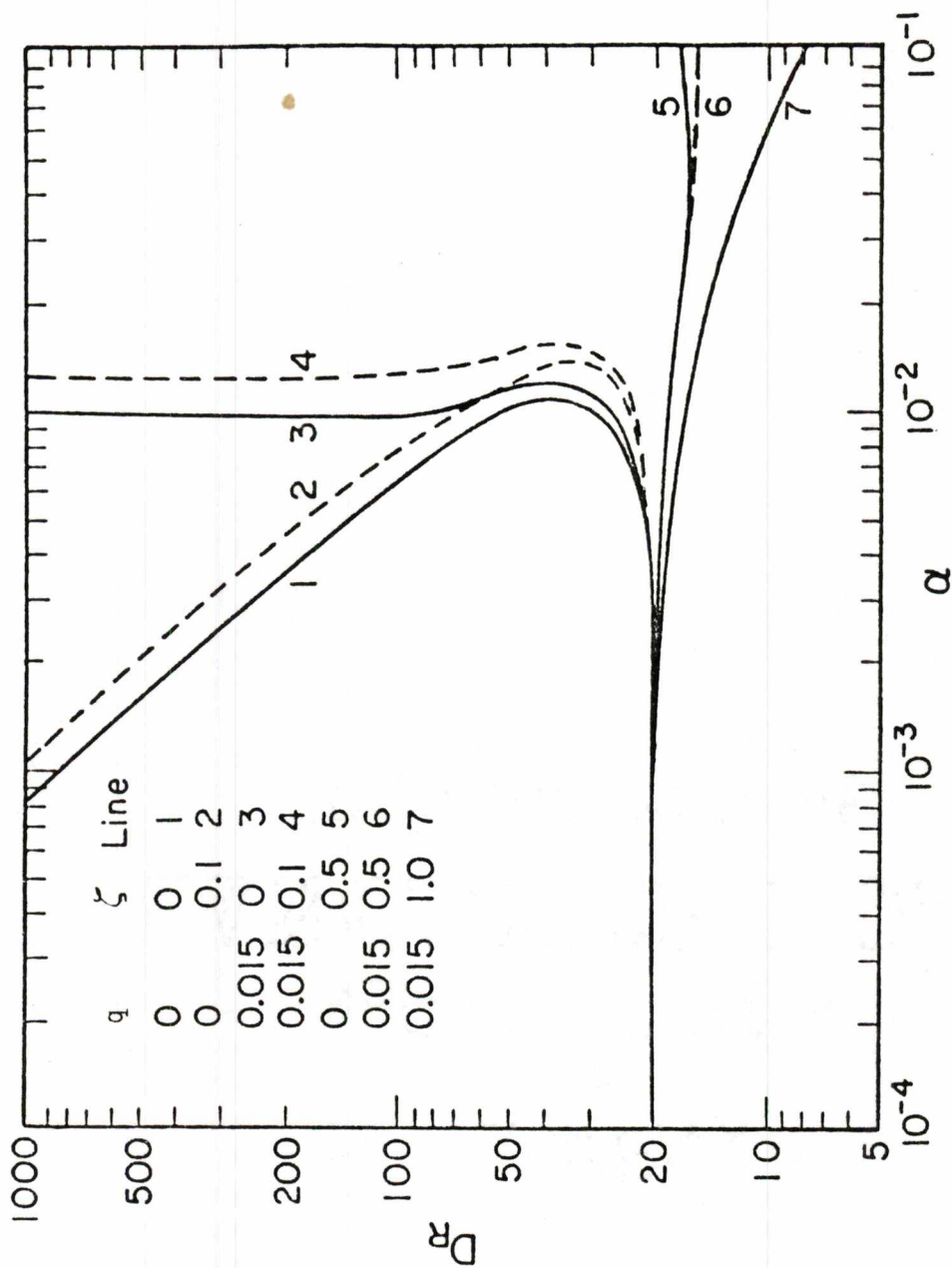


Figure 2-3. Critical draw ratio as a function of α for the single element Phan-Thien model (from Chang and Denn).

Figure (2-2)) is the absence of a second stable region at high draw ratios for $q > 0$. Note that the stresses are bounded for $q > 0$, as shown by Phan-Thien (36). They also showed that the stability behavior is the same for the single element and the two element model as long as α is based on $\tau_{\max}$ and not on a mean relaxation time, although there are quantitative differences near $\alpha_{\max}$ where the stability envelop doubles back.

The above theoretical analysis is carried out under the conditions in which gravity, surface tension, inertia, and air drag are unimportant, that is, only the rheological force is concerned. Also, only isothermal melt spinning is considered.

Experimental studies of isothermal draw resonance of polymer melt through 1976 are given by Petrie and Denn(102). Weinberger and coworkers (103, 104) studied polysiloxane, and Ishihara and Kase (93) studied the polyethylene terephthalate, both of which show nearly Newtonian behavior. The critical draw ratio is approximately 20. This value is relevant to the theory discussed above. The latter authors also found that the magnitude of the ratio of maximum-to-minimum diameter of draw resonance decreased with decreasing length of melt zone at constant draw ratio of about 50 in one series of experiments. This was interpreted in terms of the upper stable region of Figure (2-2), as shown by Fisher and Denn (78). The studies of polyolefins and polystyrene by Zeichner (72) shows that the critical draw ratio is less than 20. In some cases an order of magnitude less. A comparative studies over a range of different polymer melts are reported by Weinberger et al. (105) and by White and

Ide (74). The former authors found results that are qualitatively consistent with the linear stability theory of Fisher and Denn (78) if shear flow rheological properties are chosen. White and Ide (74) tried to show the similarity in response of polymer melts in elongational flow and melt spinning. A linear stability analysis of isothermal uniform elongational flow was done by Ide and White (51). They used the White-Metzner model with a Bogue-White deformation rate dependence of relaxation time (that is, Eqs. (2.33) to (2.37)) and used the assumption of Eq. (2.53). They predicted that polymer melts which exhibit strong thinning behavior (large "a") tend to show ductile failure in elongational flow experiments and draw resonance at low drawdown in isothermal melt spinning. However, the kinematics of the melt spinning is not the same as that of the uniform elongational flow. Therefore no quantitative results for the melt spinning problem have been obtained using the Bogue-White deformation rate dependence of relaxation time.

Chang and Denn (12) studied the onset of draw resonance of a Newtonian and a viscoelastic fluid. They showed that draw resonance occurs in accordance with the prediction of stability theory for Newtonian liquids and also for viscoelastic liquids provided the throughput is small. The theory of Fisher and Denn (78), however does not include a throughput effect, as was observed experimentally by Chang and Denn (12).

The influence of molecular weight distribution on the onset of draw resonance was studied by Minoshima et al. (40, 41) and Yamane (42). They concluded that broadening the molecular weight distribution destabilizes the spinline and is also associated with the Bogue-White thinning parameter "a."

Systematic experimental studies of the influence of die flow on the critical draw ratio were done recently by Nam (71). Various dies with different diameters and various plunger speeds were used in his studies. The influence of die flow on critical draw ratio for two polypropylenes used in his studies is obviously complicated as shown in Figures (2-4) to (2-7), and various empirical correlating groups were used. With them a rough correlation of the effects was obtained. The stability theory of Fisher and Denn (78) cannot reasonably explain his experimental results.

More general linear stability theory, including the effects of inertia, surface tension and gravitational force, was done by Pearson and Shah (106) and Chang, Denn and Geyline (107) for purely viscous fluids. Their results showed that gravity stabilizes the filament while surface tension destabilizes it. The latter authors also compared the experimental results done by Donnelly and Weinberger (103), D'Andrea and Weinberger (108) and Chan and Denn (12).

The influence of cooling on spinline stability is modeled in papers by Shah and Pearson (109, 110), and Kase (111) for Newtonian fluids. Han, Lamonte and Shah (112) make calculations for a viscous non-Newtonian fluids. Fisher and Denn (84) considered convected Maxwell model derived by Marrucci (69) and defined a Stanton number as

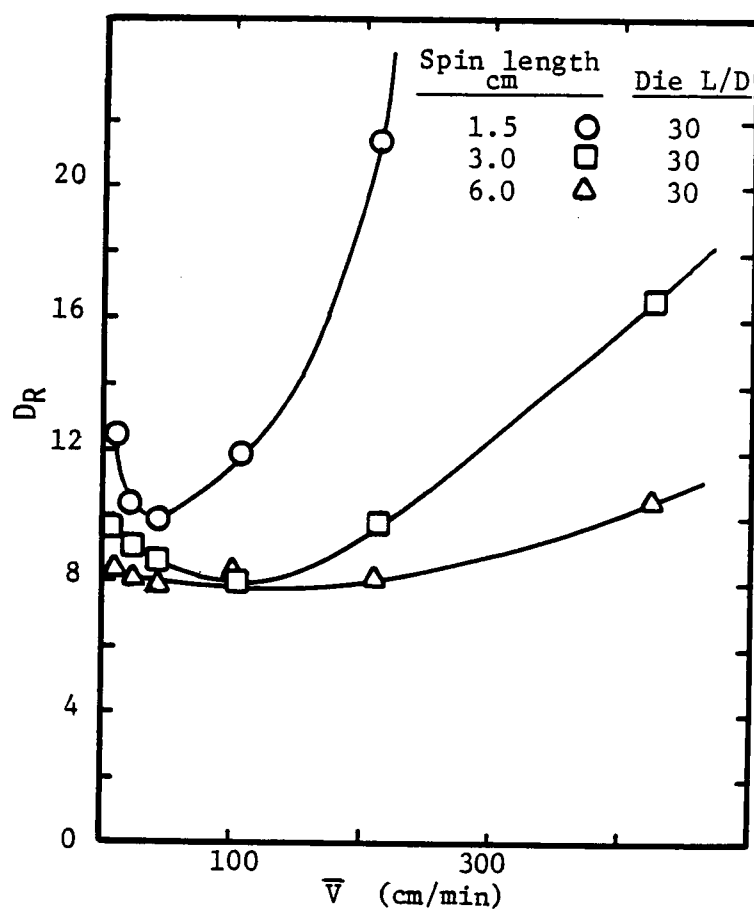


Figure 2-4. Experimental critical draw ratio (from Nam, PP 106-4, 180°C, $D = 0.041$ in.).

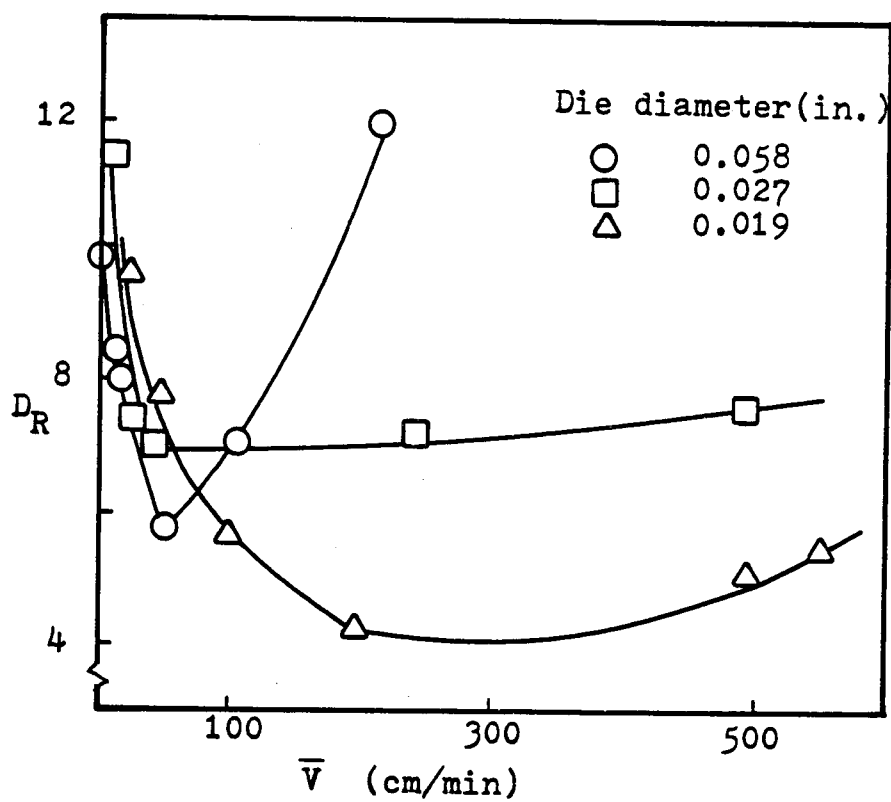


Figure 2-5. Experimental critical draw ratio (from Nam, PP 106-4, 180°C, $L = 3$ cm).

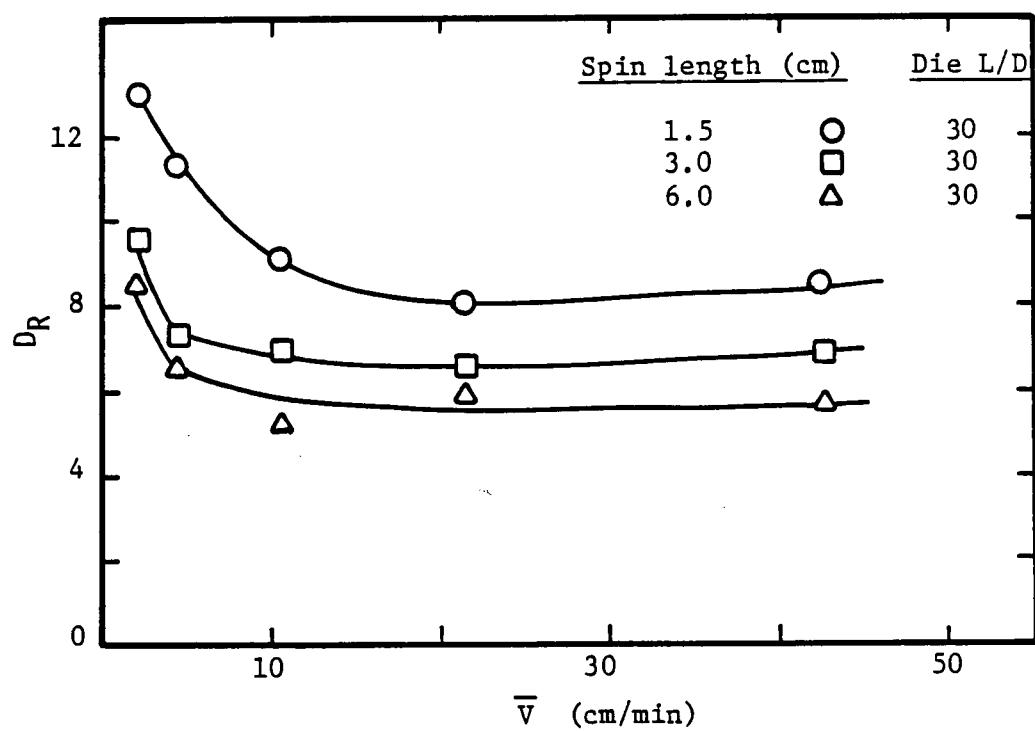


Figure 2-6. Experimental critical draw ratio (from Nam, PP 6823, 200°C, $D = 0.041$ in.).

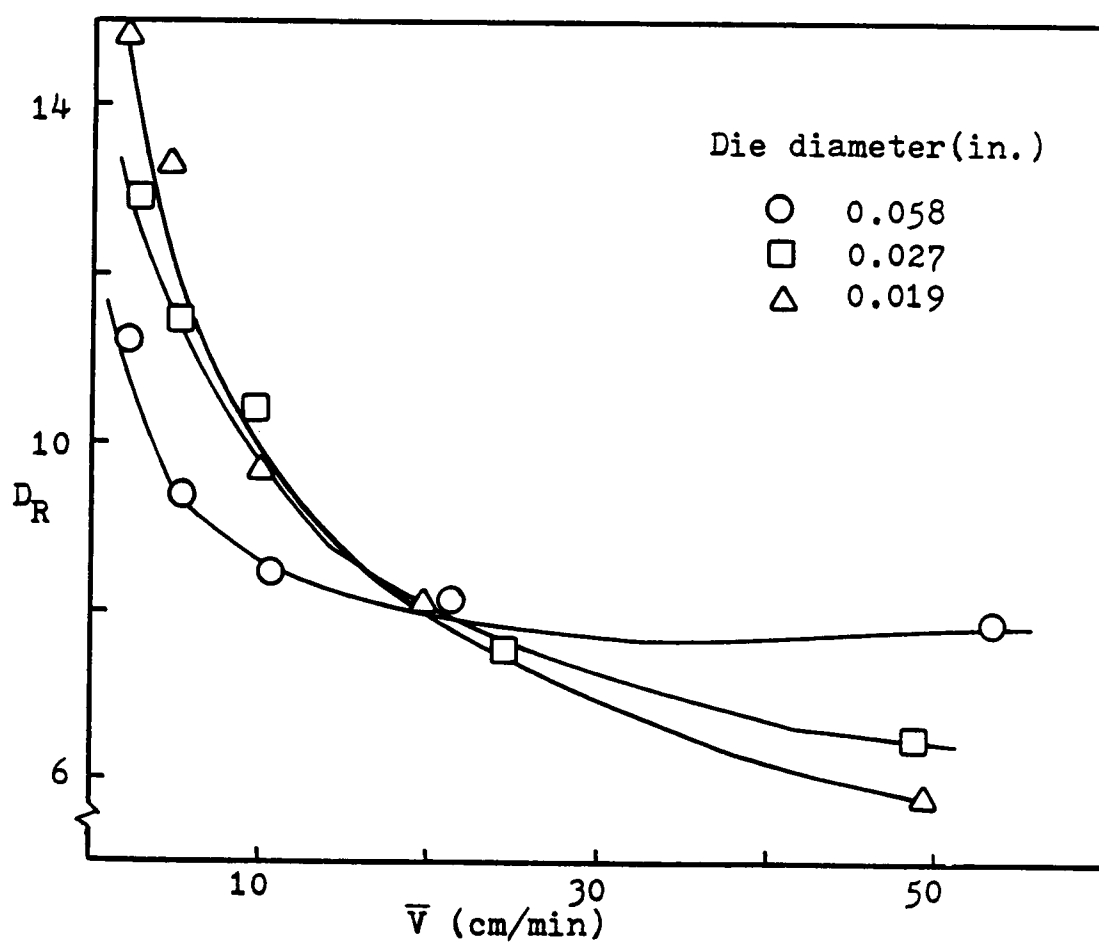


Figure 2-7. Experimental critical draw ratio (from Nam, PP 6823, 200°C, $L = 3$ cm).

$$St = 1.67 \times 10^{-4} (L/\rho C_p A^{5/6} U_o^{2/3}) \quad (2.62)$$

They found that cooling inhibits the onset of the draw resonance and the interaction of cooling and viscoelasticity results in a stabilization at high Stanton numbers which cannot occur for an inelastic fluids.

Most experiments, such as those reviewed by Petrie and Denn (102) showed that cooling increase the stability of Newtonian fluids. But there is some dispute in the case of viscoelastic fluids. Han and coworkers (113) reported that spinning into an isothermal chamber and also the length of the die have an effect on severity of draw resonance. Also, Matsumoto and Bogue (114) have shown that certain observations cannot be explained with the existing theoretical framework, specifically, the destabilizing effect of certain cooling histories in the case of polystyrene and the destabilizaing effect of short dies in the case of polypropylene. The reasons for the discrepancies in their observations are unclear.

CHAPTER 3

EXPERIMENT EQUIPMENT AND MATERIALS

Nam (71) has done a systematic studies of the effect of die flow on the critical draw ratio using different dies with sufficiently high of (L/D) ($L/D = 30$) for two different grades of polypropylene. The present author wished to use his experimental data, but certain experimental results are not presented in his dissertation, especially for the die flow conditions (data such as die swell values, the wall shear stress values etc.). In addition, some checks on the reproducibility of the data were felt to be desirable. All of the experimental work in the present research was done for PP 106-4. For PP 6823, results obtained by Yamane (42) for unconstrained die swell and those obtained by Nam (71) for the critical draw ratio values were used.

A. MATERIALS

The two materials investigated by the present author and by Nam are commercial polypropylenes with different molecular weight distributions, as shown in Table 3-1 (71). The rheological characterization of these two materials has been done by Minoshima (40, 41) for PP 106-4 and Yamane (42) for PP 6823; these are summarized by Nam, (71) as shown in Table 3-2.

TABLE 3-1. Polypropylene Used

Polymer	Company	Melt Index	Mw($\times 10^{-5}$)	Mw/Mn
PP106-4	Arco	11.6	2.32	4.7
PP 6823	Hercules	0.33	4.6	9

TABLE 3-2. Summary of Rheological Parameters Used

Polymer	T (°C)	η_0 (poise)	a	τ_0 (sec.)	G(dyne/cm ²)	A*	b*
PP 106-4	180	4.8×10^4	0.6	0.1	4.8×10^5	6.10×10^{-3}	1.48
PP 6823	200	8.9×10^5	0.9	1.25	7.1×10^5	1.81×10^{-4}	1.76

*See Eq. (2.16)

B. EQUIPMENT

A schematic diagram of the present apparatus is shown in Figure (3-1). Details of the description can be found elsewhere (71). The polymer melt was extruded at 180°C for PP 106-4 or at 200°C for PP 6823 through a die (diameter of 0.041 in., or 0.029 in. and die aspect ratios $L/D = 30$). The extrudate passed into a long heated chamber (of inside diameter of 1.0 in. and lengths of 3 in. or 1.5 in.). Then, the fiber passed into an ice-water quench bath. Two sets of heaters were used to heat the 3 in. long chamber and were controlled by a two temperature controller with the control thermocouples at 2 cm above and 1.5 cm below the die exit (for the $D = 0.041$ in. die) This die provided the enough room to accomodate the Reticon Scan Camera to measure the spinline profile from the die exit to about 1.5 cm below the die through the glass windows of the chamber. Only one heater was used for the 1.5 in. long isothermal chamber. A thermocouple at 0.75 cm below the die exit for this chamber was used to provide a spinline length of 1.5 cm for the $D = 0.027$ in. die. This chamber and die does not provide enough room for Reticon Scan Camera; thus, no data are available for this spinline diameter profile.

The Reticon Scan Camera (EG & G Reticon, Sunnyvale, Calif., Model LC 100V 1024-1 Line Scan Camera with lens viewer) was used to measure the diameter along the spinline. This was done by reading the diameter value through the digital readout or by measuring the width of the image of the fiber shown on the oscilloscope screen. A

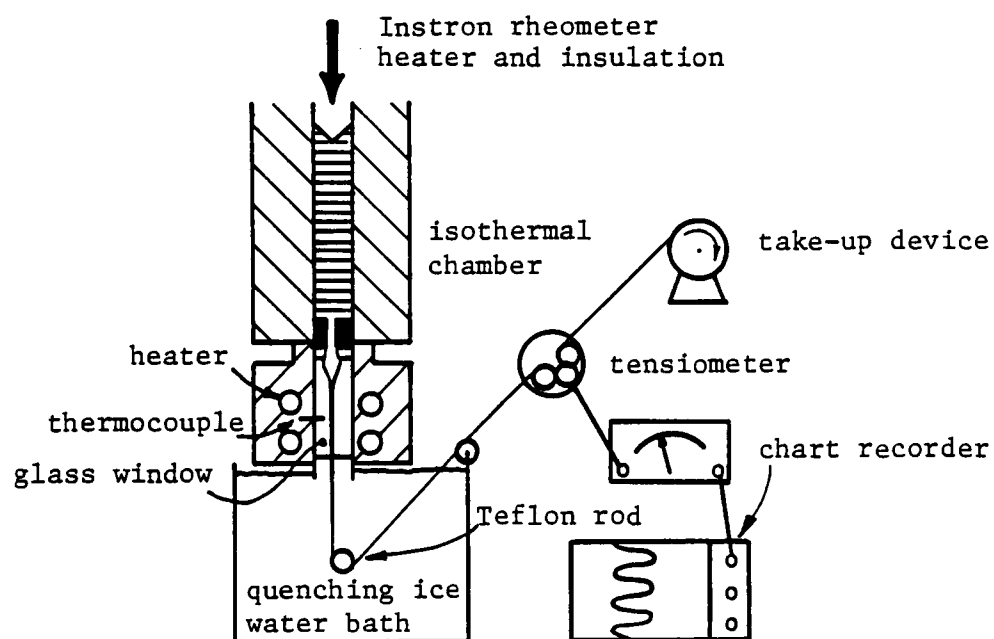


Figure 3-1. Schematic diagram of equipments.

bellows, whose length is adjustable, is extended from Reticon Scan Camera to magnify the image with a Tokina variable zoom lens (70 mm, 2.7 ft. focal length, and No. 11 lens opening), other details of Reticon Scan Camera are described by Nam (71).

Because of the transparency of the polymer melt, the image of the spun fiber shows two peaks with a light region between them. A modification of the circuit as suggested by Dr. K. Koyama, Yamagata University, Japan, is made so that the digital readout could give an accurate measurement of the diameter of the spinline. The reading speed from a digital readout can be controlled by a gain control. This gave an on-line measurement of the fluctuation of the diameter and its period (if any) during spinning.

Other equipment were a Rothschild Tensiometer, strip chart recorder (Model EV-205-11, Heath Co., Benton Harbor, Mich.), a potentiometric amplifier (Model EV-200-01, Heath Co., Benton Harbor, Mich.), and a take-up device with three different speed motors (one 1/15 HP, 1.6 RPM max., the second one 1/15 HP, 12 RPM max., the third one 1/8 HP, 345 RPM max., all B & B Motors with tachometers).

C. EXPERIMENTAL PROCEDURE

In studying the unconstrained die swell, the on-line diameter profile was measured by using the digital readout when the extrudate was less than 1 in. long after it is coming out from the capillary. The 3 in. long isothermal chamber and the die with 0.041 in. diameter and $(L/D) = 30$ were used. The position of the spinline was adjusted by turning the wheel of the camera holder. The edge of the wheel is

divided into 100 divisions so that turning one division (3.6 degrees) corresponds to changing the vertical position 0.024 mm. For each plunger speed, at least three measurements were made along the spinline, and the maximum value of the measurements was taken as the die swell value.

To study the dynamics of the melt spinning experimentally, the shear stress at the wall, the drawdown ratio, and the diameter profile along the spinline must be measured or controlled. The take-up force can also be measured by the tensiometer. However, the gravitational force, the frictional force between the thread and the Teflon rod, and the drag force in the water bath, etc. are also measured by the tensiometer. Several methods have been suggested by Minoshima (40, 41) to calibrate for these effects. However, to avoid the difficulties involved in correcting these factors, the take-up force was treated as a parameter in the present work; that is, it was calculated from the theoretical model for each set of operating conditions. The shear stress was calculated by the force indicated on the recorder chart of the Instron Capillary Rheometer without any correction for the entrance effect. An aspect ratio of 30 is believed to be large enough to ignore the entrance effect as shown by previous authors (1, 18, 71).

The periodic fluctuation of the force shown on the recorder chart of the tensiometer had the same period as that observed from the digital readout of the Reticon Scan Camera. Since no recorder is available for recording the diameter fluctuation using the scan camera, the period of fluctuation (if any) is calculated from the

recorder chart of the tensiometer. Other details of the experimental work are shown in the latter chapters, which take up the three parts of the present research in turn.

CHAPTER 4

DIE SWELL STUDIES

As shown in Chapter 2 (Section A), the analysis of die swell can be done either numerically, by using the finite-element method, or analytically, by making various kinematical assumptions near the die exit. In the former case, more accurate results can be obtained, but the numerical technique is complicated and troublesome because of the convergence problem. In addition, the conceptual framework for doing a stability analysis by the finite-element method has not been developed. Therefore, the emphasis in the present work was turned to the latter case, that is, to obtain an analytical expression of die swell (or the die flow effect) and finally to combine it to study the effect of die flow on the stability of melt spinning. To do so, one first notes that the analysis of unconstrained die swell of viscoelastic fluids done by Wust (27) and later by Nam (71), using the kinematics suggested by Matsui and Bogue (26), does not reduce to exactly the same results as that of Tanner (22), White (21), and White and Roman (24, 25). There is a difference of a factor of $1/2$ (compare Eqs. (2.5) and (2.15)), although both consider the instantaneous die swell. In order to provide the initial conditions for the dynamics studies of melt spinning (see next chapter), this discrepancy must be clarified. A detailed comparison between these two approaches is first made. A new kinematics is then suggested with an adjustable parameter k to account for the difference. The unconstrained die

swell and the die swell under applied take-up force are finally studied both theoretically and experimentally.

A. THEORETICAL ANALYSIS

Consider the kinematical assumptions Eqs. (2.3) and (2.9), used by Tanner (22) and Matsui and Bogue (26), respectively. If we transfer the Finger tensor for the die flow to the reference frame at $t = 0+$ (that is, after sudden expansion) from the reference frame at $t = 0-$ in Tanner's analysis (22), by the definition of the relative deformation gradient tensor and tensor analysis (115), we have

$$\frac{\partial \underline{x}_{t'}}{\partial \underline{x}_t} = \frac{\partial \underline{x}_{t'}}{\partial \underline{x}_{0-}} \frac{\partial \underline{x}_{0-}}{\partial \underline{x}_t} \quad (4.1)$$

or

$$\underline{F}_t(t') = \underline{F}_{0-}(t') \cdot \underline{F}_t(0-) \quad (4.2)$$

where $\underline{F}$ is the relative deformation gradient tensor and the subscript indicates the reference frame. $\underline{x}$ is the spatial coordinate of the material, t is the present time and t' is the past time. Therefore, the Cauchy tensor is

$$\begin{aligned} \underline{C}_t(t') &= \overset{T}{\underline{F}_t(t')} \cdot \underline{F}_t(t') \\ &= \overset{T}{\underline{F}_t(0-)} \cdot \overset{T}{\underline{F}_{0-}(t')} \cdot \underline{F}_{0-}(t') \cdot \underline{F}_t(0-) \\ &= \overset{T}{\underline{F}_t(0-)} \cdot \underline{C}_{0-}(t') \cdot \underline{F}_t(0-) \end{aligned} \quad (4.3)$$

and the $\underline{C}_{0-}(t')$ is clearly the Cauchy tensor for the die flow. Thus,

the Finger tensor is

$$\underline{C}_t^{-1}(t') = \underline{F}_t^{-1}(0-) \cdot \underline{C}_{0-}^{-1}(t') \cdot \left(\underline{F}_t^T(0-) \right)^{-1} \quad (4.4)$$

and the $\underline{C}_{0-}(t')$ is the Finger tensor of the die flow and can be represented as

$$\underline{C}_{0-}^{-1}(t') = \begin{vmatrix} 1 + \dot{\gamma}^2 t'^2 & \dot{\gamma} t' & 0 \\ \dot{\gamma} t' & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad (4.5)$$

Use the kinematical equations (2.3) and (4.5), one obtains

$$\underline{C}_{0-}^{-1}(t') = \begin{vmatrix} \lambda_o^2 & \dot{\gamma} t' - g' & 0 \\ \lambda_o^{1/2} (\dot{\gamma} t' - g') & \lambda_o^{-1} & 0 \\ 0 & 0 & \lambda_o^{-1} \end{vmatrix} \quad (4.6)$$

In Eq. (4.6) the following relationship is used, according to Eqs. (2.3) and (2.9),

$$\lambda_T^2 = \lambda_o \quad (4.7)$$

Comparing Eq. (4.6) to Eq. (2.11), it is clear that in the Matsui and Bogues' approximation, the lateral shear recovery denoted by function g is not included (compare White (21)). Consequently, instead of Eq. (2.9), the following kinematics is suggested:

$$\text{For } t' < 0 \quad \begin{cases} x^1 = \lambda_o (X^1 + kX^2 \dot{\gamma} t') \\ x^2 = \lambda_o^{-\frac{1}{2}} X^2 \\ x^3 = \lambda_o^{-\frac{1}{2}} X^3 \end{cases} \quad \text{where} \quad \lambda_o = \frac{A_{die}}{A(t)} = \frac{V(t)}{\bar{V}} \quad (2.9')$$

Now, one uses Eqs. (2.8), (2.9') and the Bogue-White model (Eqs. (2.39) to (2.43)) but a local rather than an average deformation rate dependence of relaxation time is used. Following some manipulation (see Appendix A), one obtains, for a single element Maxwellian model,

$$P_{11}(0)/G = (1 + F_s^2 k^2)/B^2 \quad (4.8)$$

$$P_{22}(0)/G = B^2 \quad (4.9)$$

$$B^6 = (1 + F_s^2 k^2) - \frac{4F_s^2}{G\pi D^2} B^2 = [B(0)]^6 - \frac{4F_s^2}{G\pi d^2} B^4 \quad (4.10)$$

The stress F_s is, (1) for $a \neq 0$,

$$F_s^2 = (4/a^2) [(-1/(c+1)) + 1.5 - 3c + 3c^2 \ln(1 + 1/c)] \quad (4.11)$$

and

$$c = D/(8\bar{V}a \dot{\gamma}_o \tau_o) = 1/(a \dot{\gamma}_w \tau_o) \quad (4.12)$$

or (2) for $a = 0$ (that is, the Lodge fluid), one obtains

$$F_s^2 = (\tau_o \dot{\gamma}_w)^2 = (N_{1w}/2\sigma_{12w})^2 \quad (4.13)$$

and the unconstrained die swell is

$$\begin{aligned}
 B(0)^6 &= 1 + k^2 (\tau_{oy_w})^2 \\
 &= 1 + k^2 (N_{1w}/2\sigma_{12w})^2
 \end{aligned}
 \tag{4.14}$$

Following White (21) and White and Roman (24, 25), if one defines

$$\eta = \tau_{eff} G \tag{4.15}$$

Then Eq. (4.10) becomes

$$\begin{aligned}
 B^6 &= [B(0)]^6 - \frac{4F}{G\pi d^2} B^4 \\
 &= [B(0)]^6 - \frac{4F\tau_{eff}}{\pi d^2 \eta} B^4
 \end{aligned}
 \tag{4.16}$$

Some interesting values of k^2 are as follows: (1) for $k^2 = 1/2$, we obtain Tanner's result, that is, Eq. (2.5), when $a = 0$. White and Roman's result can also be obtained, if we further use Eq. (4.15); (2) $k^2 = 1$, we obtain Matsui and Bogues' result, that is Eq. (2.15); (3) for $k^2 = 1/b$ or $k^2 = (4-b)/(b+2)$, we obtain results which have to do with the molecular weight distribution effect, that is, Eqs. (2.17) and (2.18).

The present author notes that Eqs. (4.8) to (4.10) can be used to provide the initial stress conditions. This is important in the studies of the dynamics of the melt spinning since these conditions connectd the die flow to the spinline flow.

Because of the rearrangement of the velocity profile, we also know that there is a die swell at low shear rates, the correction factor C_0 such as shown in Eq. (2.17) or Eq. (2.18) is retained in correlating the experimental data. From the above analysis, it is known that the adjustable parameter k has several advantages. In addition, it is clear that the Matsui-Bogues' approximation can reduce to Tanner's result when $a = 0$ and $k^2 = 1/2$ under the circumstance of single element Maxwellian model, that is, material with one constant relaxation time and one constant modulus.

The solutions of Eqs. (4.8) to (4.14) are shown in Figures (4-1) to (4-3). As shown in Figure (4-1) for rubberlike fluids (that is, $a = 0$), the unconstrained die swell is increased as the values of k^2 and $\dot{\gamma}^*$ are increased. Here, the $\dot{\gamma}^*$ is defined by

$$\dot{\gamma}^* = \tau_0 \dot{\gamma}_w = (N_{1w}/2\sigma_{12w}) \quad (4.17)$$

As $\dot{\gamma}^*$ is small, the unconstrained die swell approaches to C_0 . In above figures, C_0 is chosen as 0.1 (Tanner's correction factor for polystyrene).

When " a " is different from zero the unconstrained die swell increases sharply first and then levels off as $\dot{\gamma}^*$ is increased. In addition, as " a " is increased, the $B(0)$ is decreased at the same $\dot{\gamma}^*$ as shown in Figure (4-2). Although $\dot{\gamma}^*$ is affected by the τ_0 value chosen, which in turn depends on the rheological properties of the polymer, the experimental work done by earlier workers (38, 40, 42) never showed this behavior. Below the shear rate where melt fracture

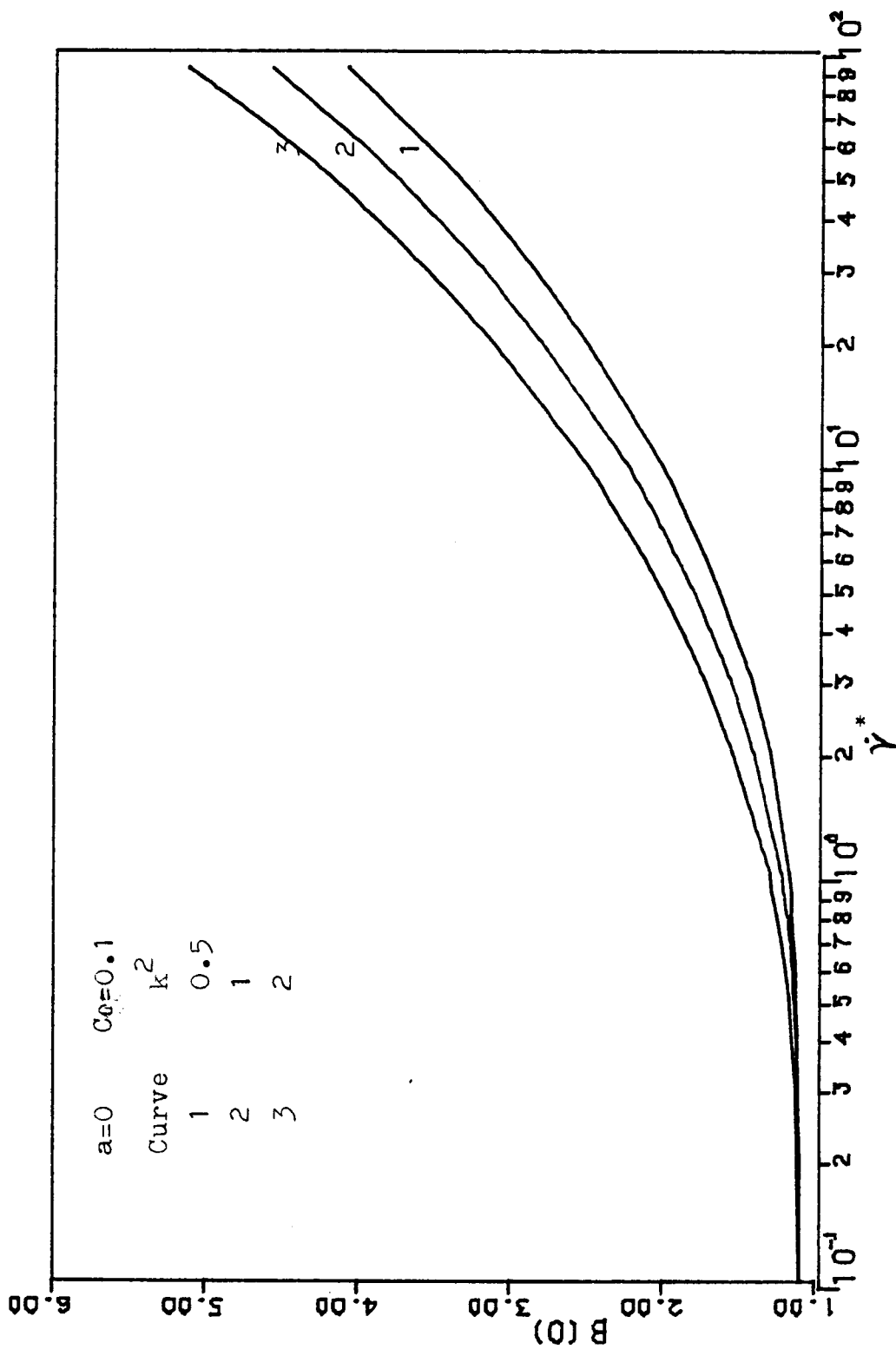


Figure 4-1. Unconstrained die swell of a Lodge fluid as a function of $\dot{\gamma}^*$, for various values of k .

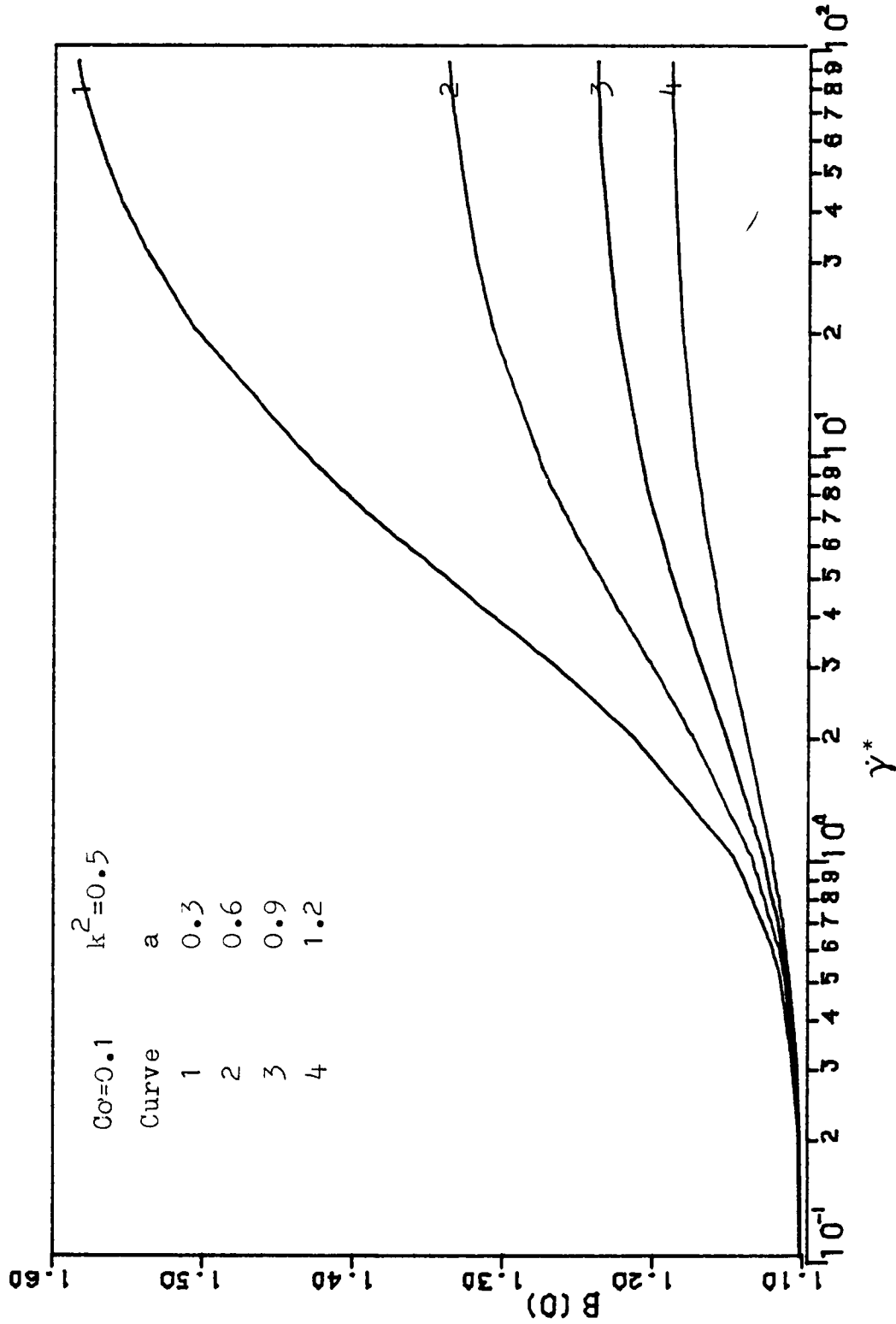


Figure 4-2. Unconstrained die swell of the Bogue-White model as a function of $\dot{\gamma}^*$, for various values of "a."

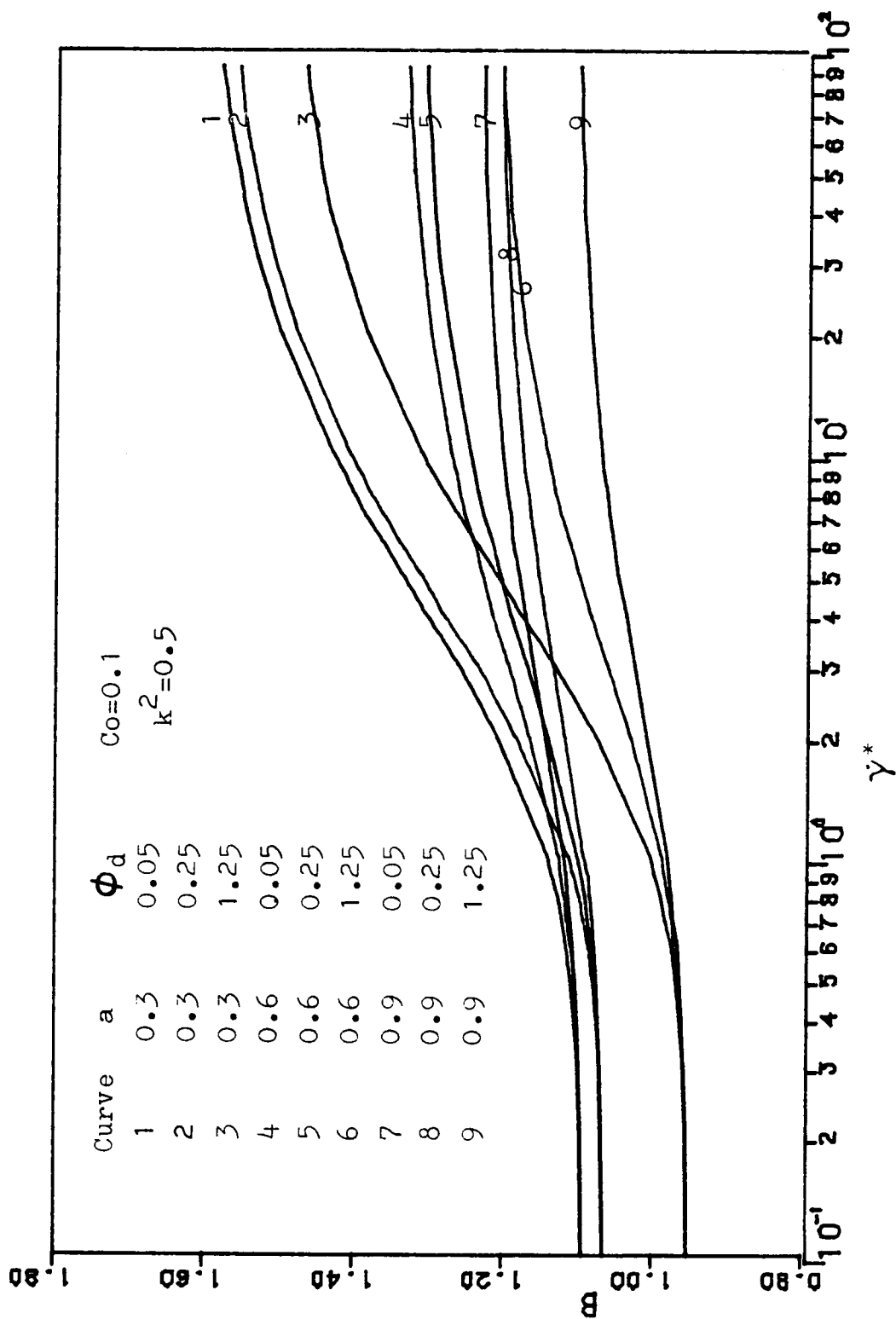


Figure 4-3. Die swell of the Bogue-White model as a function of $\dot{\gamma}^*$, for various values of "a" and ϕ_d .

is observed, it is always found that $B(0)$ increases rapidly with increasing shear rate. Also, as molecular weight distribution is increased, which corresponds to a higher value of "a," the die swell is also increased (40-42). The influence of take-up force on the die swell, under different values of the shear thinning parameter "a," is shown in Figure (4-3). Increasing the dimensionless take-up force ϕ_d , which is defined as

$$\phi_d = \frac{4F}{\pi d^2 G} \quad (4.18)$$

the die swell is decreased. However, the same difficulties (that is, the die swell levels off as $\dot{\gamma}^*$ is increased and the die swell gets smaller as "a" is increased at constant $\dot{\gamma}^*$) still remain, no improvement of these phenomena can be expected, even one uses the multiple elements model, as can be concluded by looking at Eqs. (4.10) to (4.12).

Attention is now turned to equations of the type used by Tanner, White and White and Roman; that is, Eqs. (2.5) and (2.7), together with the use of an effective relaxation time as suggested by White (21) and White and Roman (24, 25). As is clearly seen in Appendix B, results similar to Eqs. (4.8) to Eq. (4.10) can be derived by using the kinematics suggested by Tanner and a single element Maxwell model with an effective relaxation time:

$$\frac{P_{11}(0)}{G} = \frac{1 + \frac{1}{2}(\tau_{eff} \dot{\gamma}_w)^2}{B^4} = \frac{1 + \frac{1}{2}(N_{1w}/2^{\sigma} \dot{\gamma}_w)^2}{B^4} \quad (4.19)$$

$$\frac{P_{22}(0)}{G} = B^2 \quad (4.20)$$

$$\begin{aligned} B^6 &= 1 + \frac{1}{2} (N_{1w} / 2\sigma_{12w})^2 - \frac{4F}{G \pi D^2} B^4 \\ &= [B(0)]^6 - \phi_d B^4 \end{aligned} \quad (4.21)$$

That is, the relationship containing the ratio of first normal stress difference to shear stress is derived; this stress ratio depends on the shear flow condition in the die. Consequently, it is possible to improve the prediction of die swell if one uses the experimental (empirical) relationship between N_{1w} and σ_{12w} . The power law type empirical relationship and the parameters for PP 106-4 and PP 6823 used in this work have been obtained by Nam (71) and are shown in the following equations and in Table 3-2 (page 44):

$$\text{PP 106-4} \quad N_{1w} = 6.1 \times 10^{-3} (\sigma_{12w})^{1.48} \quad (4.22)$$

$$\text{PP 6823} \quad N_{1w} = 1.8 \times 10^{-4} (\sigma_{12w})^{1.76} \quad (4.23)$$

In other words, the ratio $N_{1w} / 2\sigma_{12w}$ is taken directly from extrapolated data rather than by using predictions based on the Bogue-White model.

Experimental work was therefore done for both unconstrained die swell and die swell under an applied take-up force. Details of the experimental procedure have been shown in Chapter 3. To correlate the experimental results, the adjustable parameters k and C_0 are retained.

B. COMPARISON WITH EXPERIMENTAL DATA

Experimentally, one can measure the shear stress at the wall, the drawdown ratio, and the diameter profile along the spinline. The take-up force is not easily measured and will here be taken as a calculated quantity. The first normal stress difference and the ratio of it to the shear stress is thus calculated from Eqs. (4.22) to (4.23). For unconstrained die swell, Figures (4-4) to (4-7) show the experimental results obtained by Minoshima (40) together with curves obtained from computer for PP 106-4. The experimental data from the scan camera measurements are also shown. It can be seen clearly that Minoshima's data is much higher than that obtained from scan camera. In Minoshima's experiment, the extrudate was cut at the die ($L/D = 40$, $D = 0.058$ in.) exit at the point where the samples became 2 in. long and were annealed in a silicon oil bath at 180°C for 5 min and finally put into polyethylene glycol at 110°C for 30 min. After this annealing process, the diameter of the extrudate was then measured by a micrometer and a density correction was applied. Thus, his data have to do with the condition of total relaxation.

From the eyepiece of the Reticon Scan Camera, the position where the maximum die swell occurs can be viewed clearly. It is found that the maximum die swell is located within about $1.5D$. With a higher plunger speed, the larger is the distance from the die exit. At the very low speeds, it is very difficult to determine either the value of the die swell or the position where it occurs, because of the relaxation of the polymer melt. At high plunger speeds, the flow rate

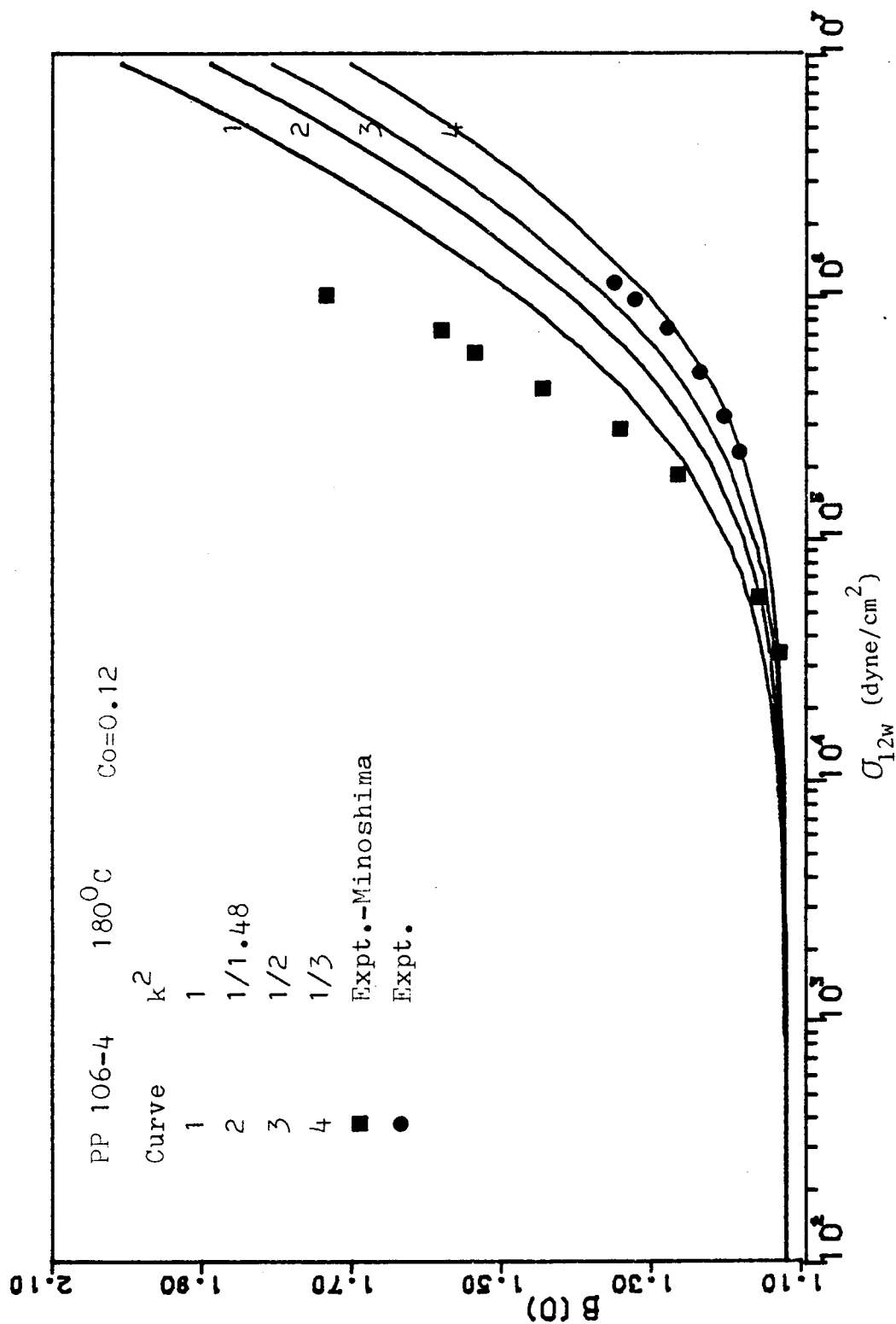


Figure 4-4. Unconstrained die swell of PP 106-4 as a function of σ_{12w} , for $Co = 0.12$ and various k values.

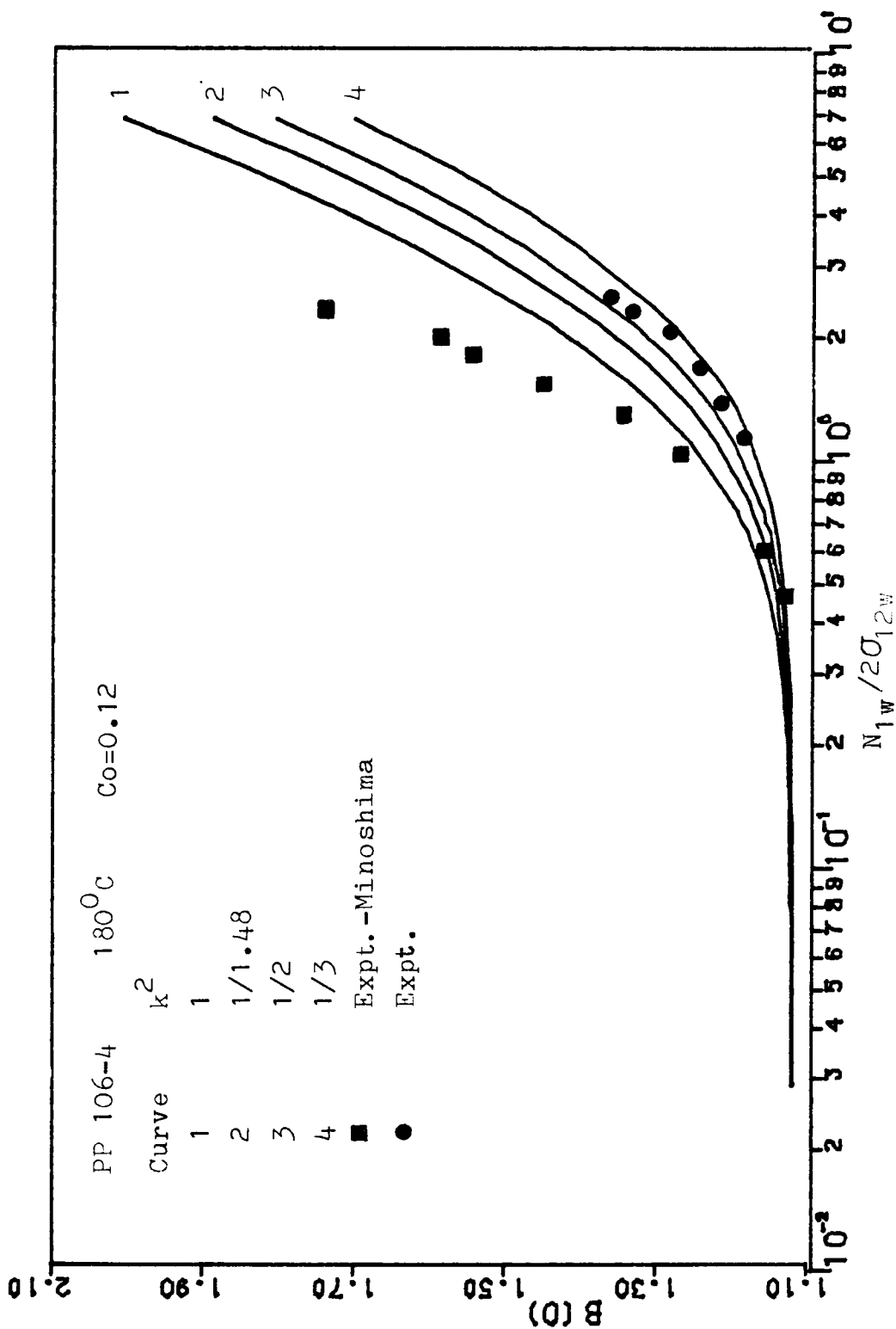


Figure 4-5. Unconstrained die swell of PP 106-4 as a function of $N_{1w}/2\sigma_{12w}$ for $Co = 0.12$ and various k values.

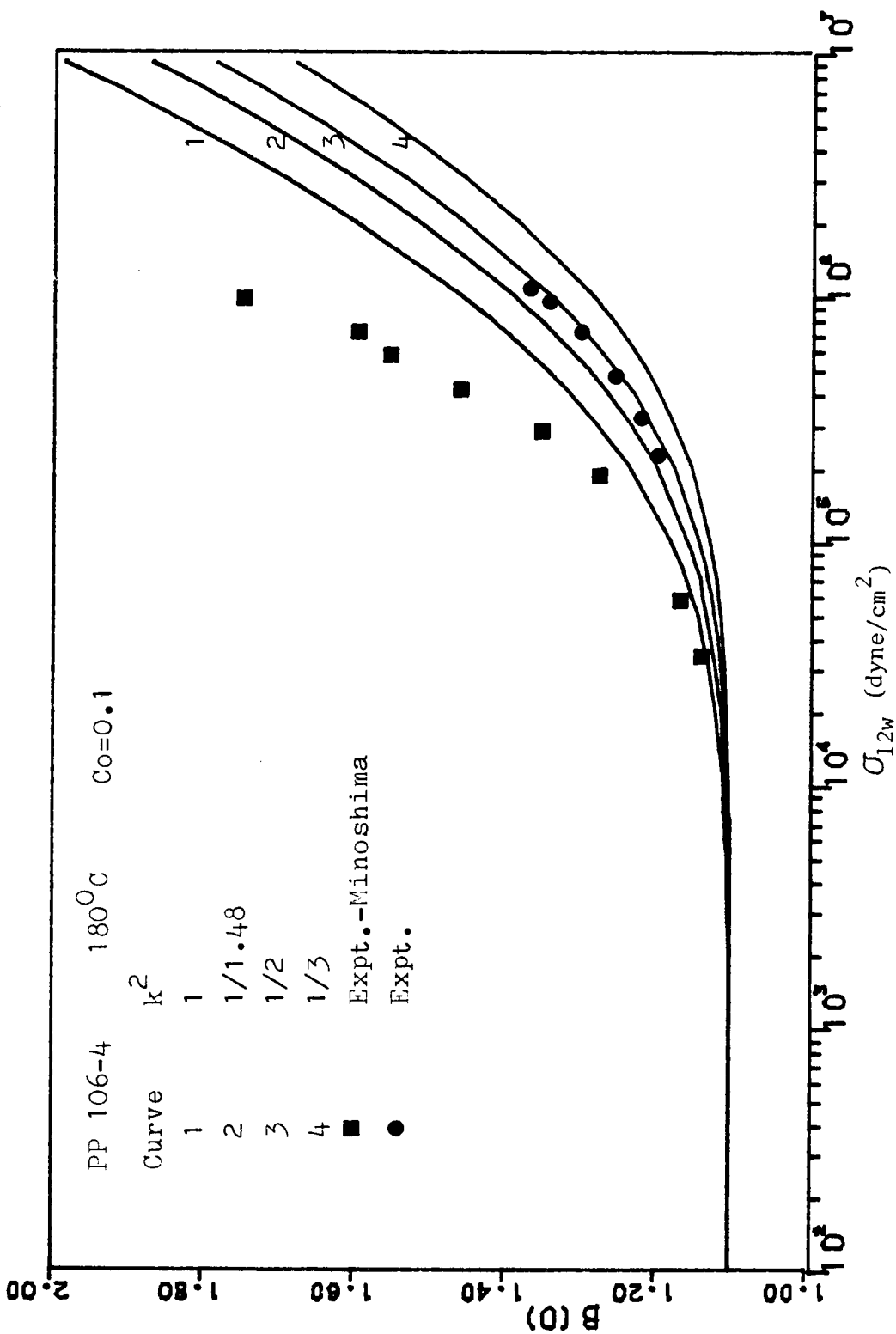


Figure 4-6. Unconstrained die swell of PP 106-4 as a function of σ_{12w} , for $Co = 0.1$ and various k values.

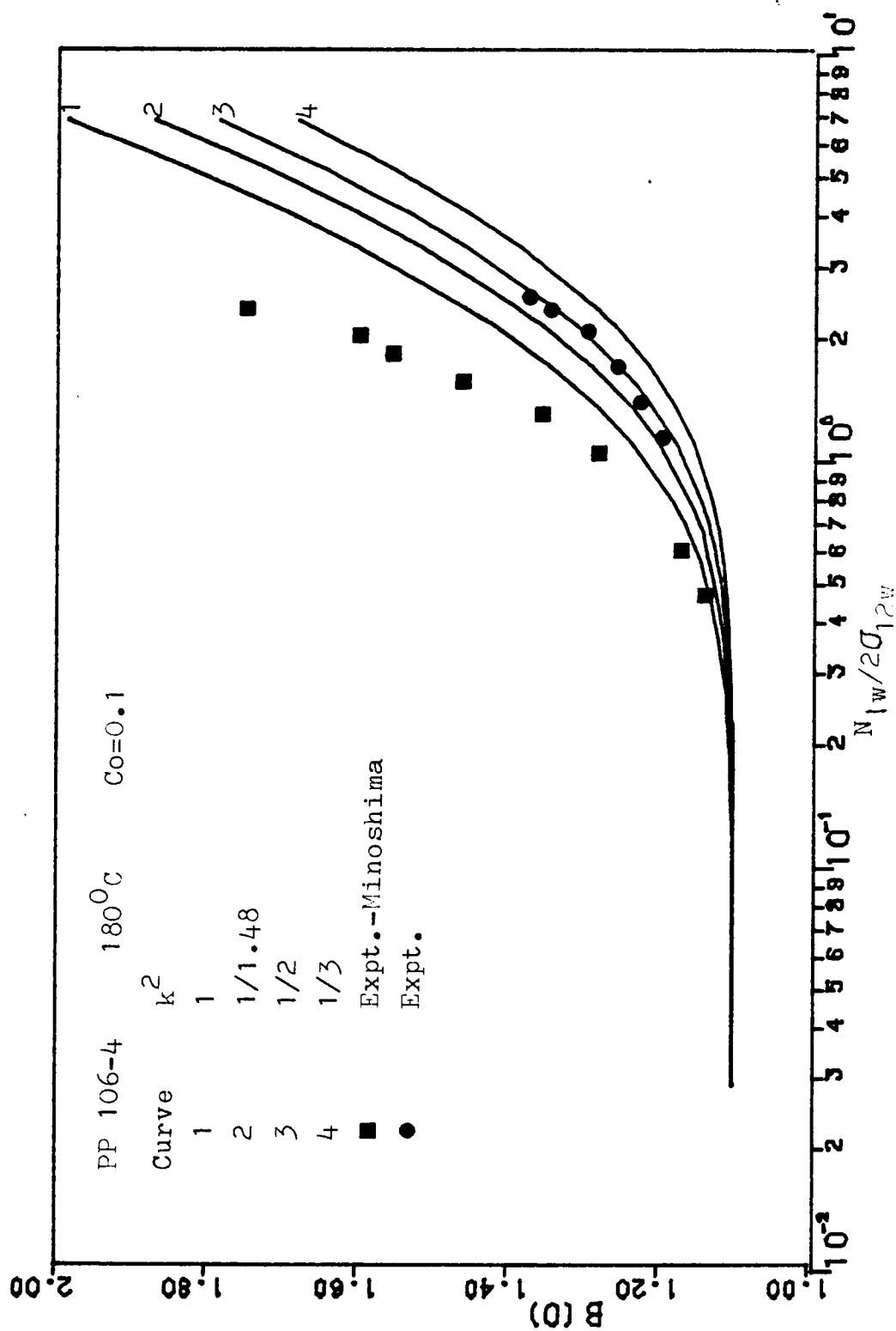


Figure 4-7. Unconstrained die swell of PP 106-4 as a function of $N_{1w}/2\sigma_{12w}$, for $Co = 0.1$ and various k values.

of the polymer melt is too high and gravity influences the reproducibility of the measurements. In Figures (4-4) to (4-7), the solid circles are obtained by taking the largest reading in the digital read out from the scan camera when the extrudate is less than 2.5 cm long; these data are from the middle range of the Instron plunger speed available, that is, from 0.05 in./min to 2 in./min. The capillary die used had a diameter of 0.041 in. with $L/D = 30$, which is believed long enough to neglect the entrance effect (1,18,71). The isothermal chamber used by Yamane (42) in his die swell measurements was used.

In order to fit the experimental results obtained from scan camera for PP 106-4, which, in reality, is closer to the spinning experiment, either of the following equations can be used as shown in Figures (4-4) to (4-7):

$$B(0) = \left(1 + \frac{1}{2} (N_{1w}/2\sigma_{12w})^2 \right)^{1/6} + 0.1 \quad (4.24)$$

or

$$B(0) = \left(1 + \frac{1}{3} (N_{1w}/2\sigma_{12w})^2 \right)^{1/6} + 0.12 \quad (4.25)$$

Clearly, Eq. (4.24) is Tanner's Equation.

That the values of the die swell vary considerably depending on the method of measurement has been observed by White and Roman (24), but the differences are not clearly understood. White and Roman showed that the frozen extrudates and the photographed melts fit Eq. (4.24) reasonably well, and the die swell is much higher if one

measures the die swell of the annealed extrudates. It seems consistent that the annealing method, as was done by Minoshima, gives higher values than the direct on-line methods. Apparently, some residual stresses must remain in the extrudate when one is observing die swell by a direct on-line measurement. Thus, a discrepancy between the values obtained by the scan camera method and the values obtained by Minoshima's annealing method is expected. Since the scan camera method closely represents the situation of melt spinning, these results are chosen to study the effects of die flow.

For PP 6823, Yamane (42) has carried out his die swell measurements by taking a picture of the extrudate in the isothermal chamber when the extrudate became 1 in. long. His experiment is exactly the same as that obtained using the scan camera, except that one can read the diameter profile directly in the latter case from the digital readout. For PP 6823, the experimental results can be fit reasonably well by either of the following equations as shown in Figures (4-8) to (4-11):

$$B(0) = \left[1 + \frac{1}{2} (N_{1w}/2\sigma_{12w})^2 \right]^{1/6} + 0.3 \quad (4.26)$$

or

$$B(0) = \left[1 + \frac{1}{3} (N_{1w}/2\sigma_{12w})^2 \right]^{1/6} + 0.33 \quad (4.27)$$

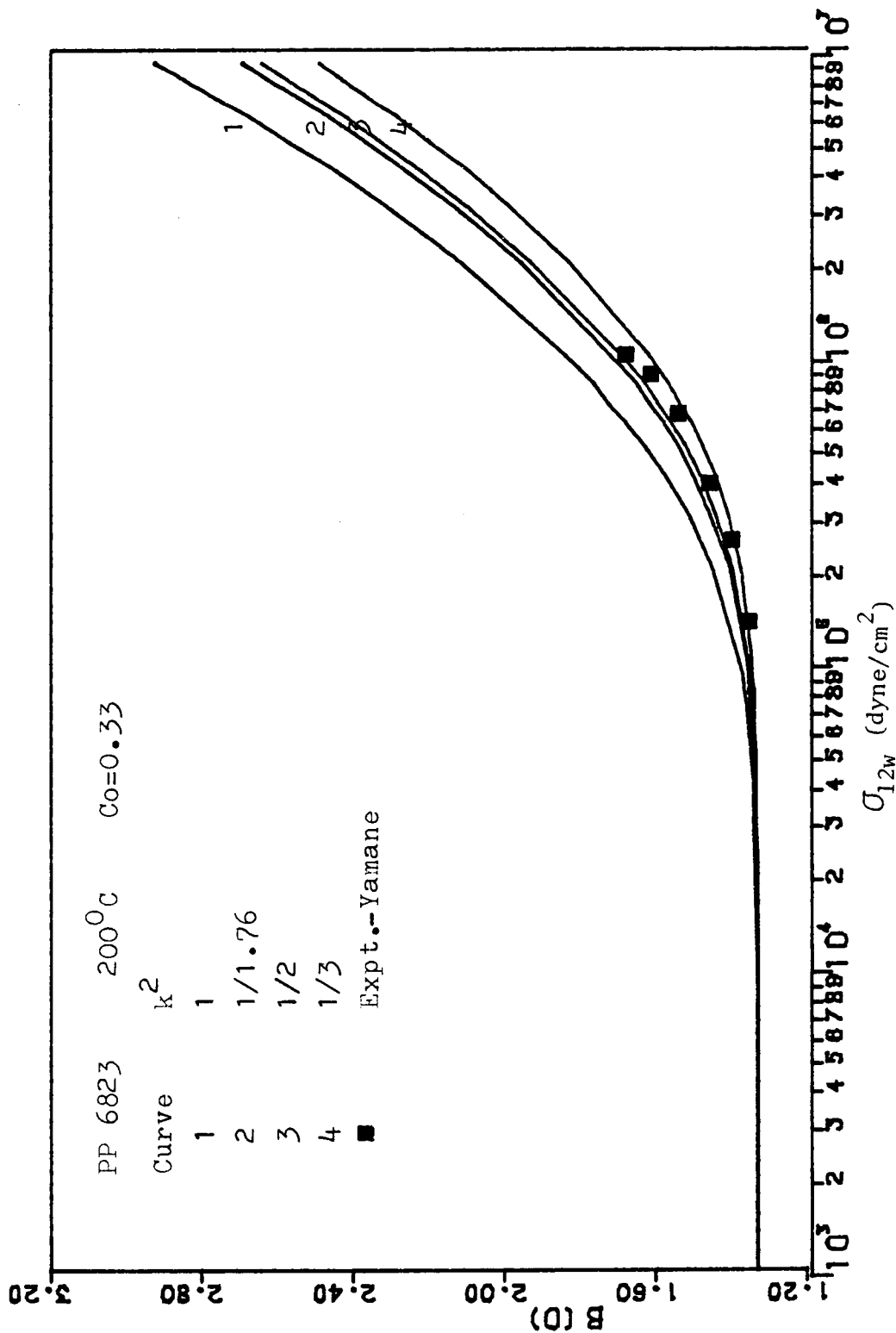


Figure 4-8. Unconstrained die swell of PP 6823 as a function of σ_{12w} , for $Co = 0.33$ and various k values.

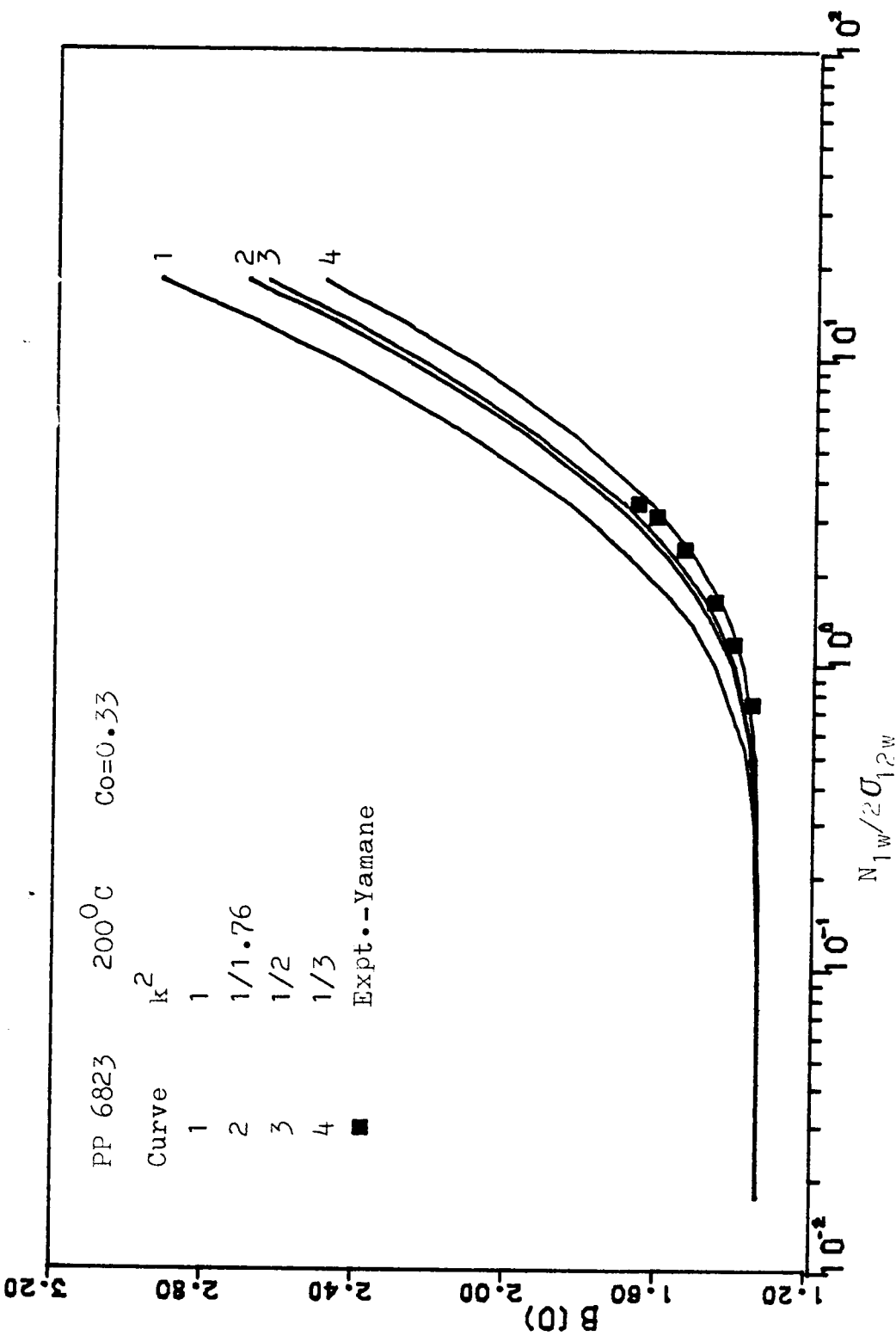


Figure 4-9. Unconstrained die swell of PP 6823 as a function of $N_{1w}/2\sigma_{12w}$, for $Co = 0.33$ and various k values.

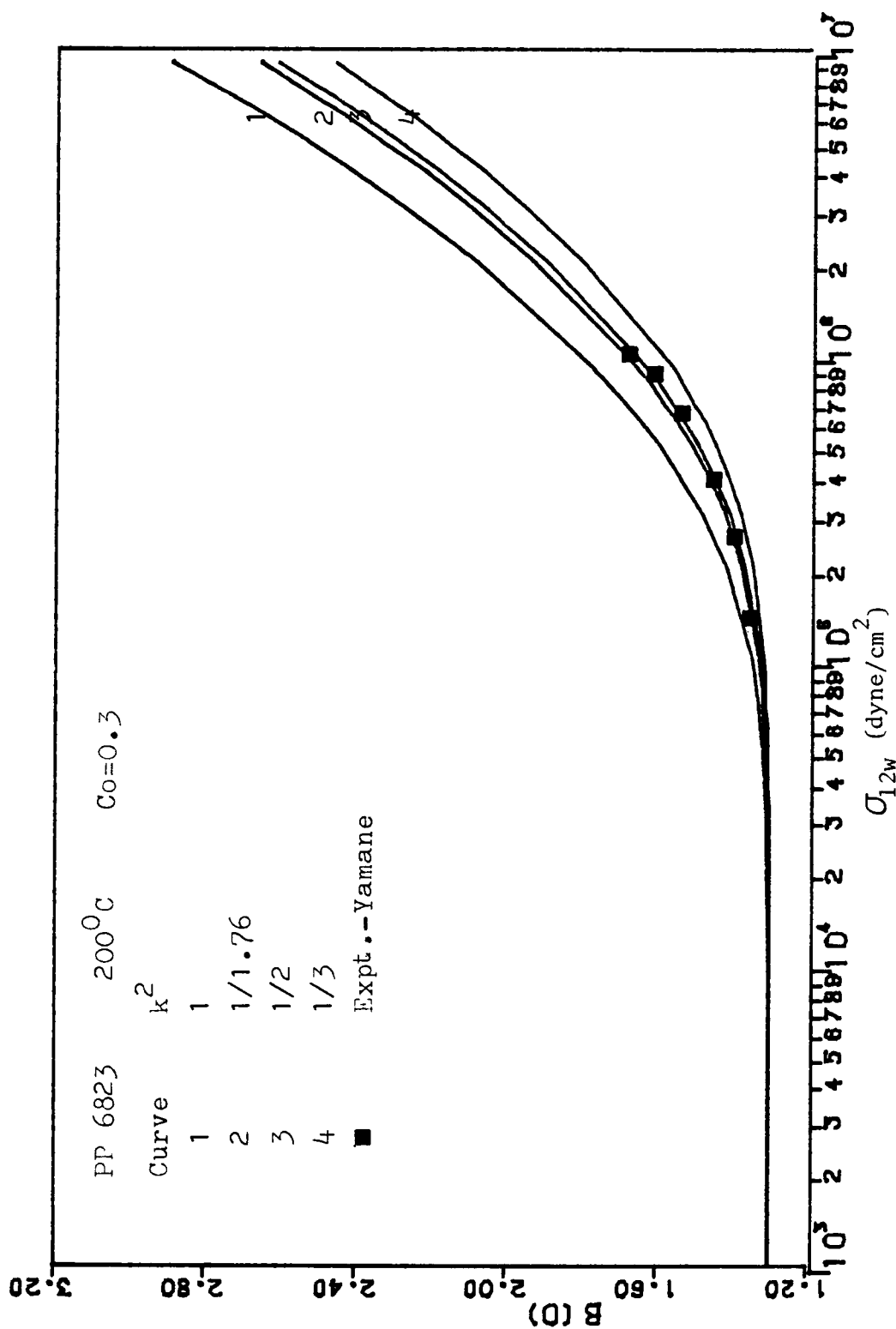


Figure 4-10. Unconstrained die swell of PP 6823 as a function of σ_{12w} , for $C_0 = 0.3$ and various k values.

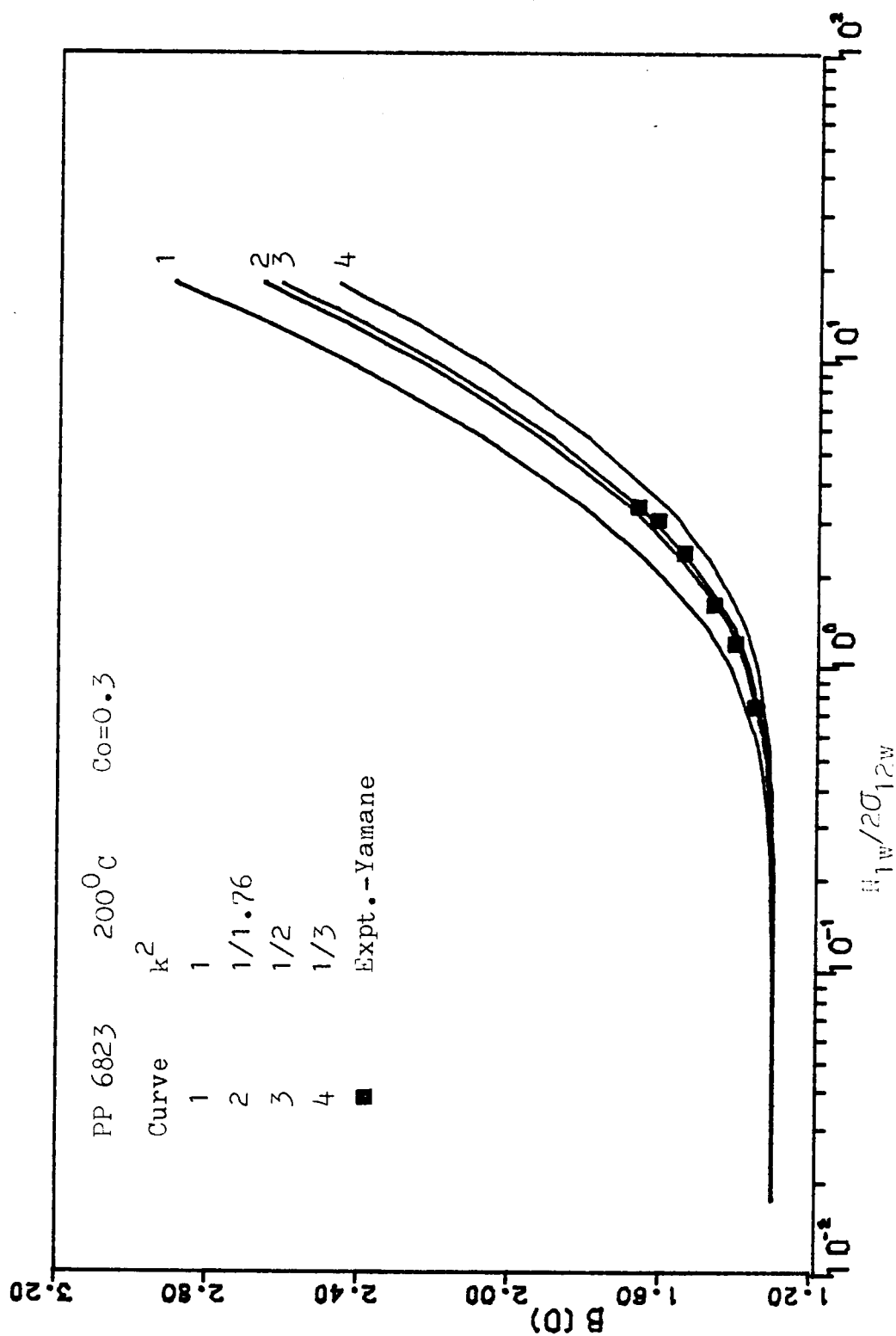


Figure 4-11. Unstrained die swell of PP 6823 as a function of $N_{1w}/2\sigma_{12w}$, for $Co = 0.3$ and various k values.

It is difficult to obtain a good fit if we choose a Co value close to 0.1 such as in the case of PP 106-4. The value of 0.13 for Co has been obtained for Newtonian fluids by Tanner (6) using finite element method. Although the flow of the PP 6823 melt, as well as the PP 106-4 melt, behave as Newtonian fluids as the shear stress approaches zero, the deviation of Co from 0.13 for PP 6823 is not clear. The breadth of the molecular weight distribution, the use of the single effective relaxation time or the averaging method used to determine the normal stress and the shear stress relationship will all affect the shape of the theoretical curve. Racin and Bogue (38) has found that the Co values are strongly affected by the molecular weight distribution. The Co values can varied from less than 0.1 to about 0.3 if one increases the molecular weight distribution. Measurement of the die swell at much lower stresses may give some help to determine the Co volue, however, the Reticon Scan Camera and photographic method cannot give an accurate measurement of die swell at low plunger speeds because of the relaxation of the polymer melt.

We turn now to the problem of die swell under an applied take-up force. Such data for PP 106-4 were measured by using the scan camera. The experiment was carried out using a capillary die with $D = 0.041$ in. and $L/D = 30$ and with plunger speeds ranging from 0.05 in./min to 2 in./min. The spinline length is 3 cm long. The results are shown in Figures (4-12) to (4-17), together with the theoretical curves obtained from Eq. (4.21), that is,

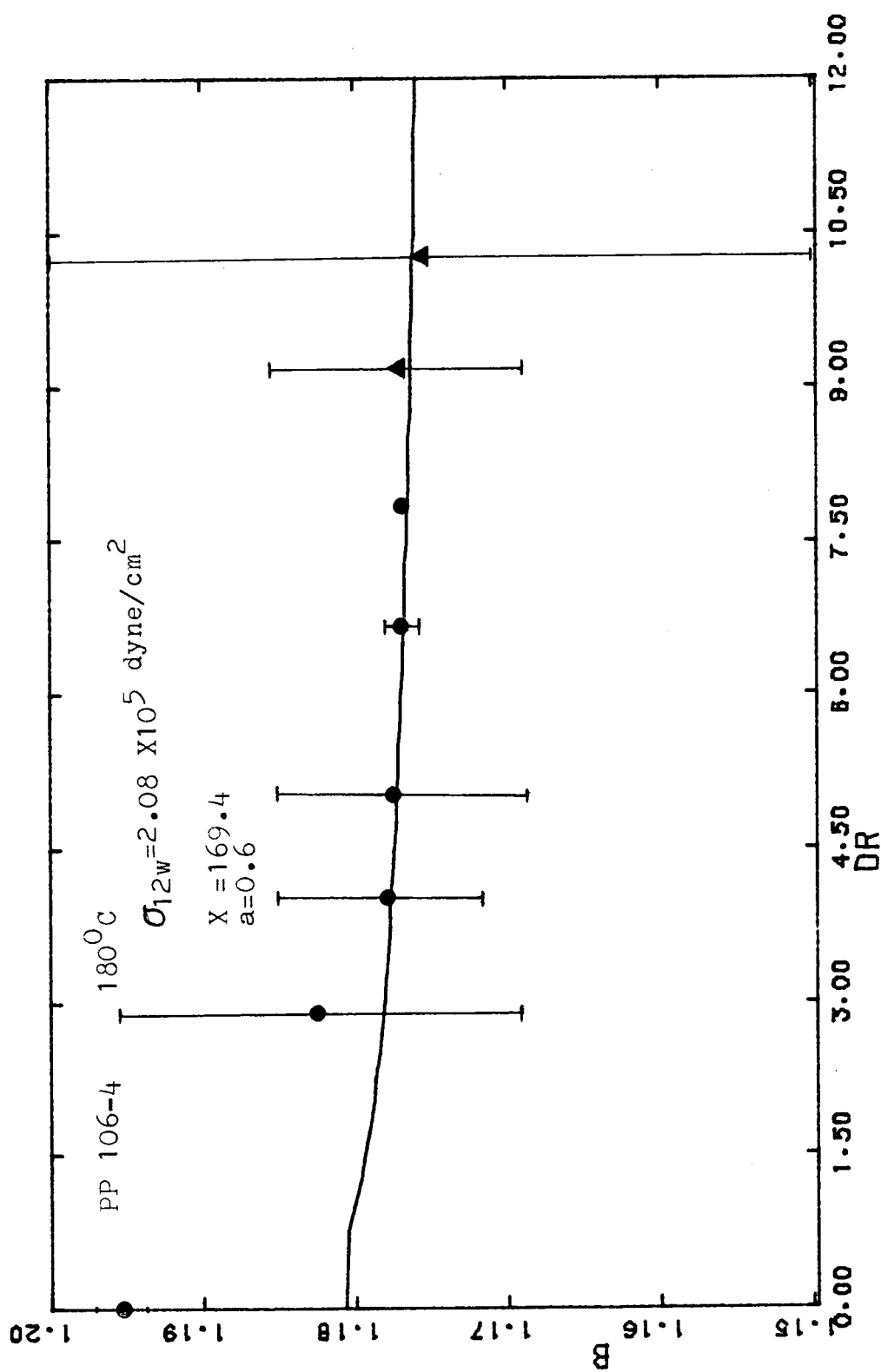


Figure 4-12. Die swell of PP 106-4 as a function of DR ($V_p = 0.05 \text{ in./min}$, $L = 3 \text{ cm}$).

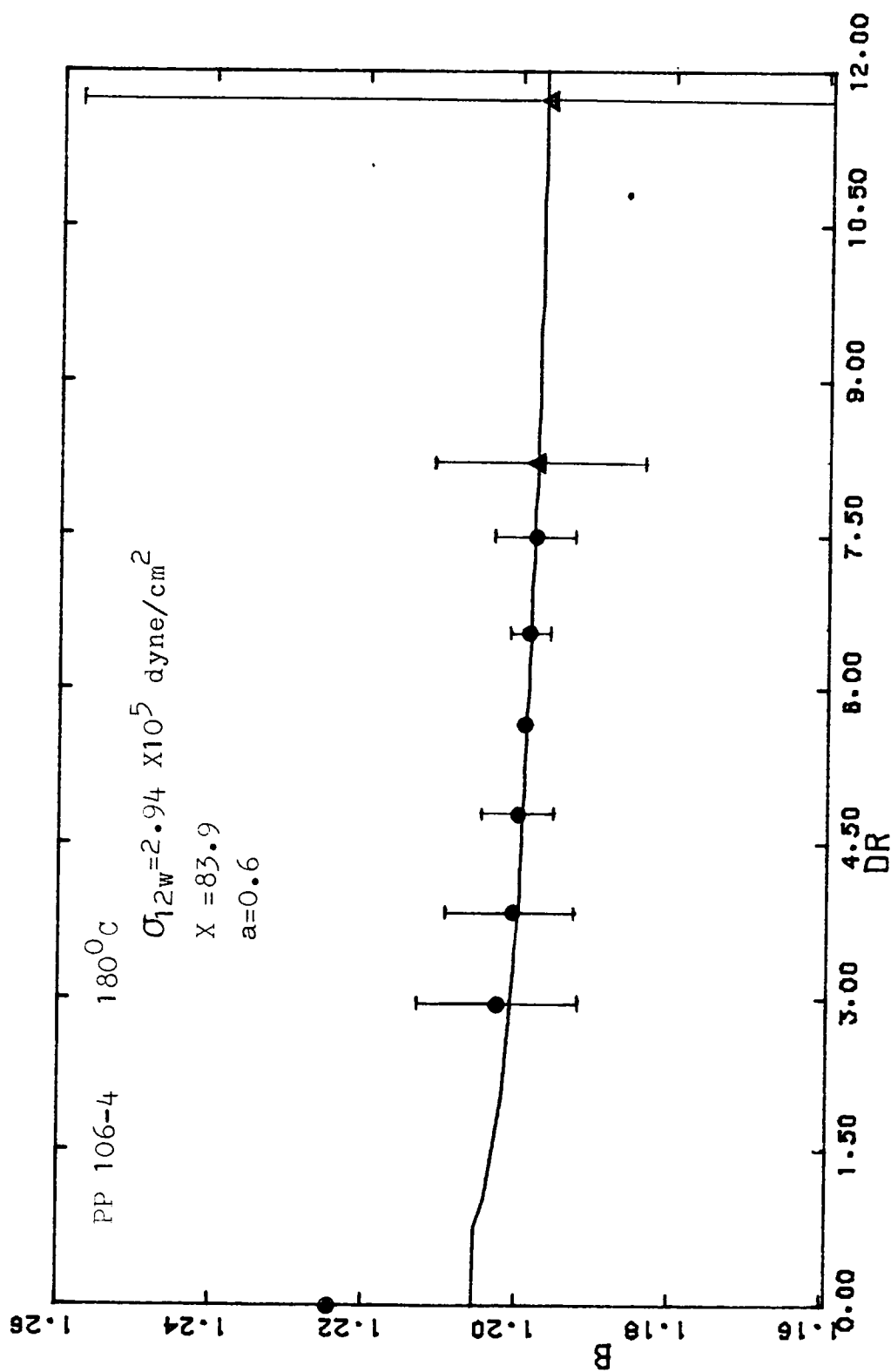


Figure 4-13. Die swell of PP 106-4 as a function of DR ($V_p = 0.1 \text{ in./min}$, $L = 3 \text{ cm}$).

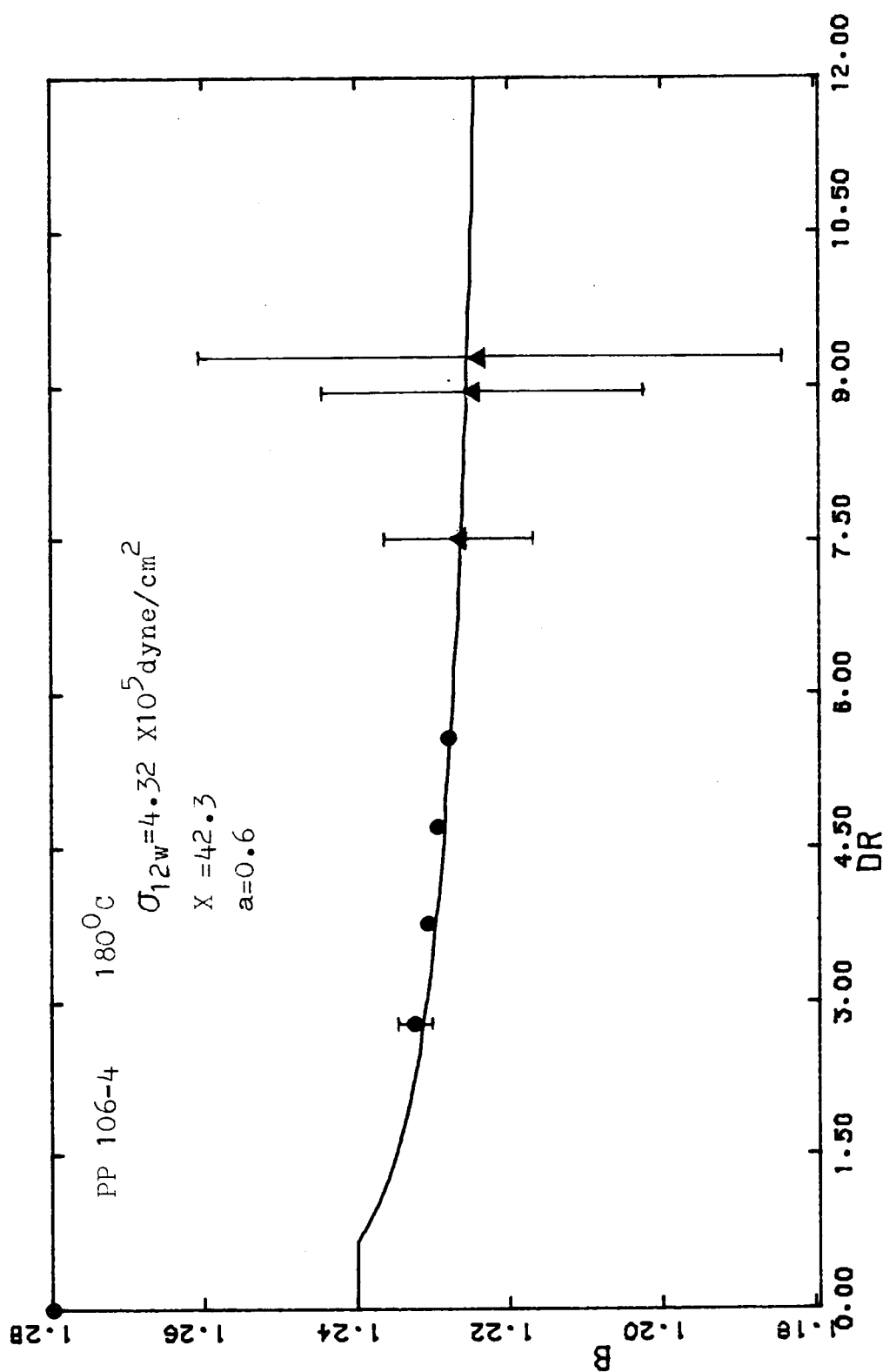


Figure 4-14. Die swell of PP 106-4 as a function of DR ($V_p = 0.2 \text{ in./min}$, $L = 3 \text{ cm}$).

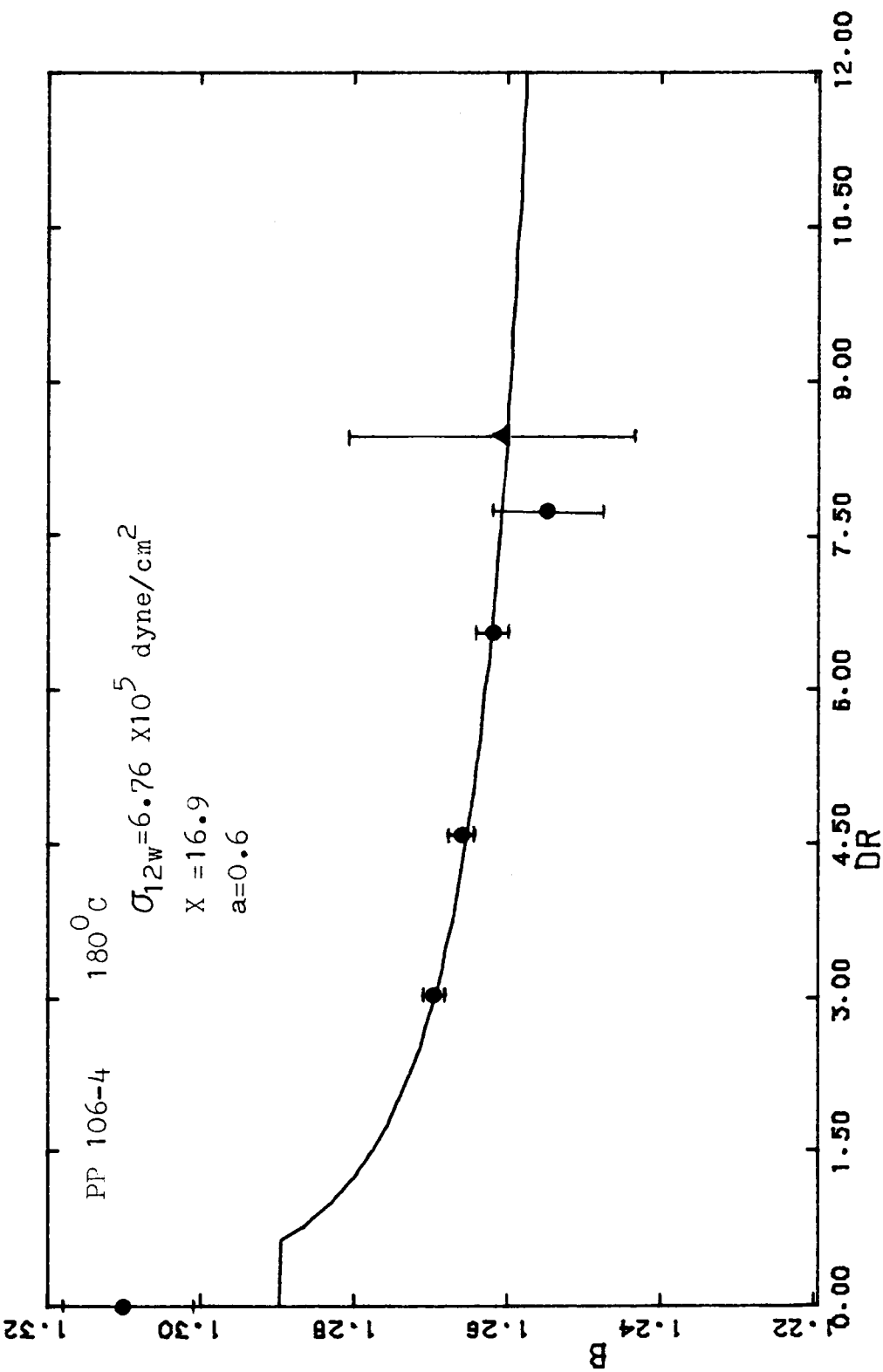


Figure 4-15. Die swell of PP 106-4 as a function of DR ($V_p = 0.5 \text{ in./min}$, $L = 3 \text{ cm}$).

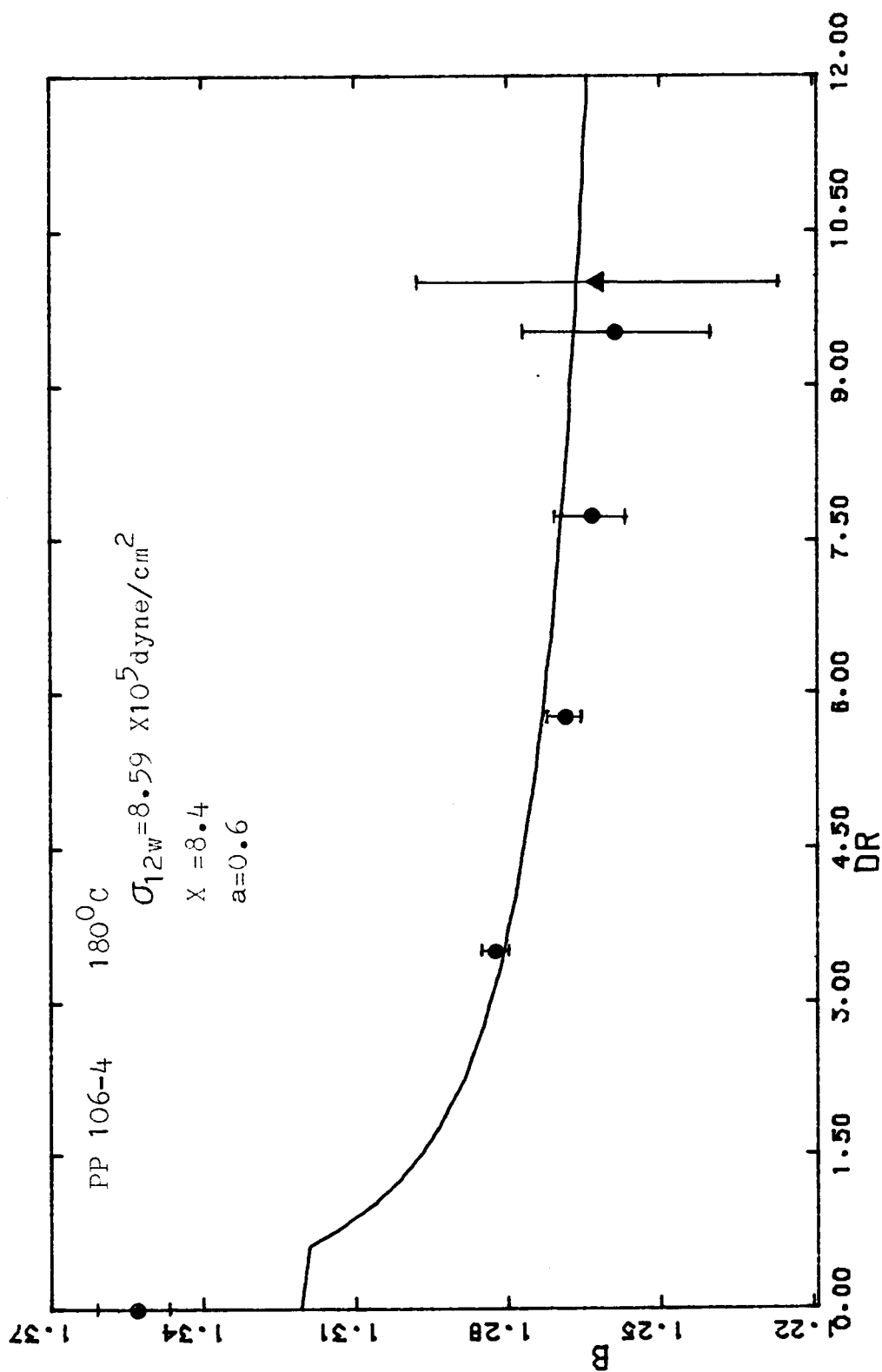


Figure 4-16. Die swell of PP 106-4 as a function of DR ($V_p = 1 \text{ in./min}$, $L = 3 \text{ cm}$).

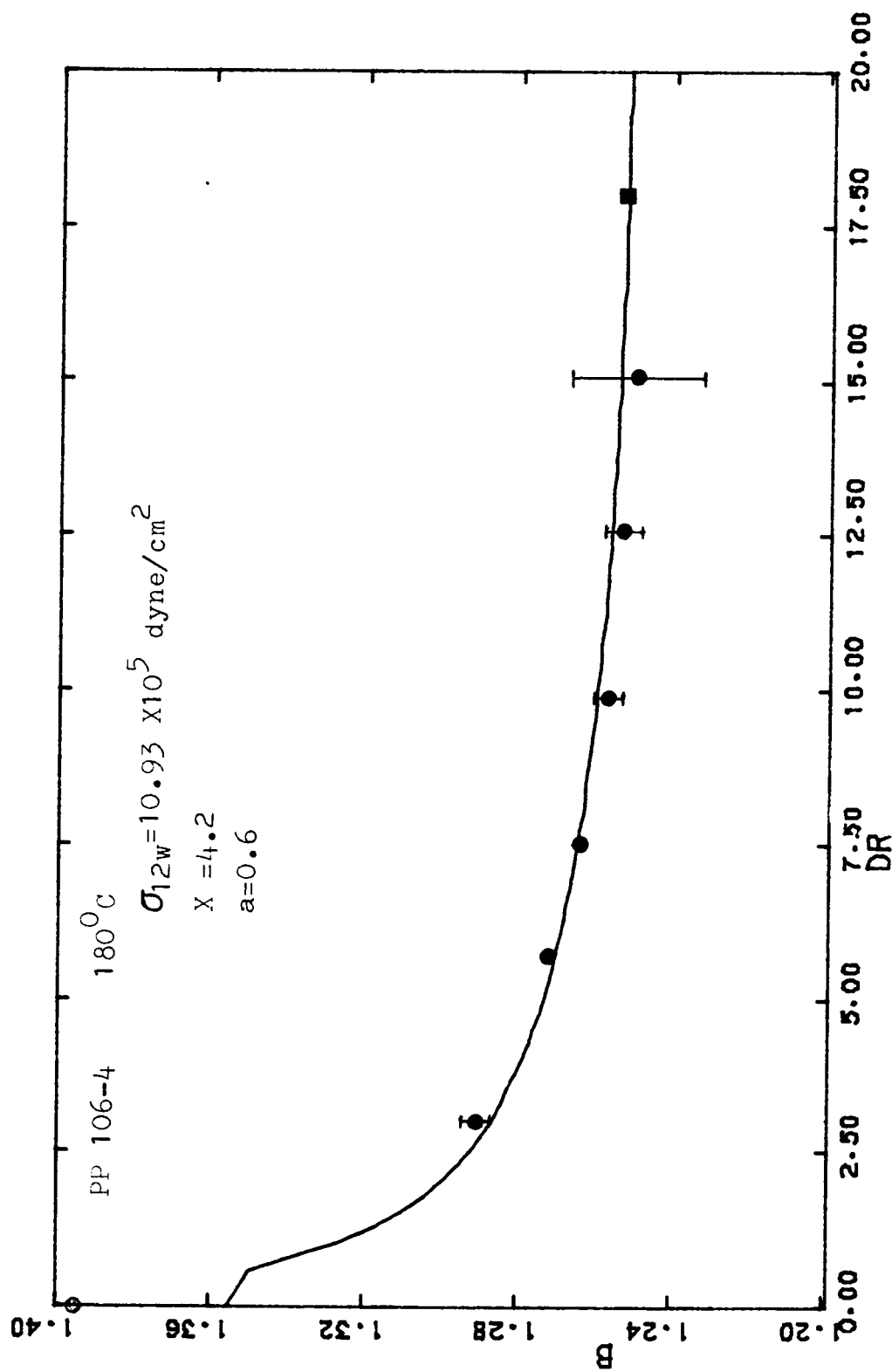


Figure 4-17. Die swell of PP 106-4 as a function of DR ($V_p = 2 \text{ in./min}$, $L = 3 \text{ cm}$).

$$B^6 = [B(0)]^6 - \phi_d^4 B^4 \quad (4.28)$$

with

$$B(0) = \left(1 + k^2 (N_{1w}/2\sigma_{12w})^2 \right)^{1/6} + Co \quad (4.29)$$

The computer scheme to calculate B can be easily generated from the computer program included in Appendix D. One introduces values for the shear stress, dimensionless spinline length, deformation thinning parameter "a" (for PP 106-4, $a = 0.6$ as determined by Nam (71)), the draw ratio, and the parameters k and Co . The unknown parameters are B or ϕ_d . To obtain them an iteration method is provided until the draw ratio at that specified spinline length is obtained. Empirically, we have assumed that the starting position, where the assumption of purely elongational flow is applicable, is at the location of maximum die swell. This assumption originally came from the study of diameter profile along the spinline for Newtonian fluids done by Fisher, Denn and Tanner (11) using the finite-element method. They concluded that the assumption of purely elongational flow is valid no more than 1D from the exit of the spinneret but varies with the take-up forces and the stress ratio, and that the position of purely elongational flow occurs downstream of the point of maximum die swell. Recently, similar conclusions have been drawn by Keunings, Crochet and Denn (13) for viscoelastic fluids, except that 2D, instead of 1D, is required to use the asymptotic equation for the spinline flow. This distance is small compared to the spinline length. Since we can obtain the initial conditions for the spinline

flow (Eqs. (4-19) to (4-21)) at the location of the maximum die swell and its position is somewhere between the die exit and the position where the purely elongational flow is occurred, it is reasonable to choose the location of maximum die swell as the starting position. We hope by this idea, or assumption, that we can predict, at least qualitatively, the effect of die flow on the dynamics of melt spinning, especially on the onset of the draw resonance.

In Figures (4-12) to (4-17), the solid circles indicate the average of the random die swell values obtained experimentally; the solid triangles indicate not only the average die swell value but also the position where there is a periodic fluctuation (so-called draw resonance) of the die swell value, as seen from both the digital readout of the scan camera and the tensiometer; and the solid squares indicate the value of the draw ratio where the spinline breaks as one increases the draw ratio slowly as the experiment progresses.

From the force recorder of the Instron Capillary Rheometer, a negligible decrease of force is observed when we increase the draw ratio or change the spinline length. Thus the shear stress is assumed to be unchanged for the same plunger speed in the computer calculation. From Figures (4-12) to (4-17), it is clear that the die swell decreases as the draw ratio is increased. The location of the maximum die swell is always less than 1.5 mm from the spinneret exit, with the higher the draw ratio, the lesser the distance from the spinneret. For example, when the plunger speed is 0.5 in./min the location of the maximum die swell is about 1.3 mm for $DR = 3.1$ and is about 1.1 mm for DR larger than 6.8. However, because of the random

fluctuation of the die swell values, it is not easy to determine the position accurately. A recorder rather than a digital readout for determining the diameter profile along each position of the spinline would be a great help to determine the maximum die swell position.

As shown in Figures (4-12) to (4-13), at the lower plunger speeds, when the draw ratio is low, there is an instability in which one sees a growing "bulge" as the material comes out of the die. This "bulge" occasionally breaks away but in an erratic way. No periodic fluctuation can be seen either in the digital readout or in the recorder chart of the tensiometer. A similar phenomenon was observed by Matsumoto and Bogue (114).

When the draw ratio is sufficiently high, the fluctuation is almost surely a result of the draw resonance phenomenon, because the fluctuations are periodic and are accompanied by periodic fluctuations in the tensiometer readings. Finally at high plunger speeds, there is an interesting phenomenon, that is, the spinline eventually breaks without showing any periodic fluctuation when one increases the draw ratio slowly.

To correlate the die swells under different draw ratios with Eqs. (4.28) and (4.29), we found that if one chooses $k^2 = 1/2$ and $Co = 0.1$ we have reasonably good fit as shown in the solid line of Figures (4-12) to (4-17). It seems that although there are two pairs of k^2 and Co that can be chosen to correlate the unconstrained die swell of PP 106-4 (because we are dealing with a limited set of data), only one pair of these two gives a reasonably good correlation of the die swell under different draw ratios. However, correlating the die

swell for PP 6823 is not so simple, because of the experimental and theoretical difficulties already discussed. While other pairs of values of k and Co might be possible, Eqs. (4.28) and (4.26), that is, $k^2 = 1/2$ and $Co = 0.3$ are chosen to study the effect of die flow on the onset of draw resonance for pp 6823. These values are reasonable in view of the prior literature and the theoretical analysis done in previous section.

CHAPTER 5

COMBINED DIE FLOW AND SPINLINE FLOW STUDIES:

STEADY SPINNING

As shown in Chapter 2 (Section B), isothermal melt spinning of viscoelastic polymer melt has been solved by using different assumptions for the initial conditions in various differential type constitutive equations. This problem has also been solved by assuming the kinematics near the die exit for an integral type constitutive equation. In the latter case, one can combine the die flow and the spinline flow together to study the effect of the die flow on the dynamics of steady melt spinning. However, it seems impossible to do a stability analysis using an integral model. By using a differential model, it is possible to deal with the stability analysis, although this approach suffers from the disadvantage of not knowing how to set the initial condition. If, however, one is willing to start the problem at the point of maximum die swell (which occurs immediately after the exit for a one time constant formations) and makes the assumption that the flow is purely elongational there, we can proceed with the differential stability analysis. The reasonableness of this assumption, and the details of carrying it out, will now be discussed.

The studies of the velocity profile along the spinline have been done by Fisher, Denn and Tanner (11) for Newtonian fluids, and by Keunings, Crochet and Denn (13) for viscoelastic fluids by using a finite-element method. That purely elongational flow occurs downstream of the point of maximum die swell was found, but all of

these processes take place within a few diameter of the exit ($1D$ in the case of Newtonian fluids and about $2D$ in the case of viscoelastic fluids). This result together with the kinematics suggested by Matsui and Bogue (26), and later used by Wust (27) and Nam (71) in their steady melt spinning, leads the present author to choose the maximum die swell position as the starting point for the differential treatment. Moreover, finite-element solutions show that there is some effect of the take-up force or the draw ratio on the positions where the purely elongational flow is occurred. The positions are closer to the die exit when the take-up force is increased. This makes the approximation of the present work even more reasonable. In this manner, the influence of the die flow on both steady and unsteady melt spinning can be thoroughly studied.

In the present work, the White-Metzner model, with a Bogue-White deformation rate dependence of relaxation time, is used. The governing equations and the computer program for the steady melt spinning are shown in Appendixes C and D, respectively. In the first part of this chapter, a theoretical analysis, together with some assumptions used by previous authors, will be shown. Some comparisons between integral treatment and the differential treatment in steady spinning will also be provided. Finally, some experimental results of PP 106-4 will be compared to the presented theories. The unsteady melt spinning will be discussed in next chapter.

A. THEORETICAL ANALYSIS

A-1. Influence of Die Flow on the Initial Conditions

As we have shown in Chapter 4, Eqs. (4.19) to (4.22) together with Eq. (4.29) cannot only give a reasonable good fit to the experimental die swell data but also provide the initial stress conditions required in the differential treatment. However, there is the problem, that is, there is discrepancy between the analyses of Matsui and Bogue (26), and Tanner (22) for unconstrained die swell. This can be resolved by introducing an adjustable parameter k to account for the lateral shear recovery. But the Bogue-White model does not predict die swell very well. The present author then first showed that the use of a single element model with effective relaxation time can predict the on-line maximum die swell reasonably well. Finally the present author showed that if one specified the take-up force (or the corresponding draw ratio) at some specified spinline length and shear flow parameters (such as wall shear stress or the ratio of the first normal stress difference to the shear stress) one can obtain the die swell and the initial stresses. In summary, then, the governing equation are

$$B^6 = [B(0)]^6 - \phi_d^4 B^4 \quad (5.1)$$

$$B(0) = \left[1 + k \left(N_{1w}/2\sigma_{12w} \right)^2 \right]^{1/6} + C_0 \quad (5.2)$$

$$P_{11}(0)/G = [B(0)]^6 / B^4 \quad (5.3)$$

$$P_{22}(0)/G = B^2 \quad (5.4)$$

where

$$\phi_d = \sigma_{11}(0)/G = 4F/(G\pi d^2) = \phi_I B^{-2} \quad (5.5)$$

$$\phi_I = 4F/(G\pi D^2) \quad (5.6)$$

and d is the diameter at maximum die swell position.

For further use, the following dimensionless groups of initial stresses are more useful:

$$F1(0) = P_{11}(0)/\sigma_{11}(0) = QP_{11}(0)/FU_0 \quad (5.7)$$

$$F2(0) = P_{22}(0)/\sigma_{11}(0) \quad (5.8)$$

Figures (5-1) to (5-3) show the effect of die flow on the die swell and initial stress conditions at three different ϕ_I values for polymer melts whose k^2 and Co are $1/2$ and 0.13 respectively. The choice of these values is based on the theoretical analysis done by Crochet and Keunings (34). They showed that these values give agreement with that obtained from the finite-element method up to $N_{1w}/2\sigma_{12w}$ value of 3 for the Maxwell fluid. Although an effective relaxation time (or the empirical relationship between first normal stress difference and shear stress) has been used in the above equations, we expect these values are still useful for polymer melts. It is seen in Figure (5-1) that the die swell increases as $N_{1w}/2\sigma_{12w}$

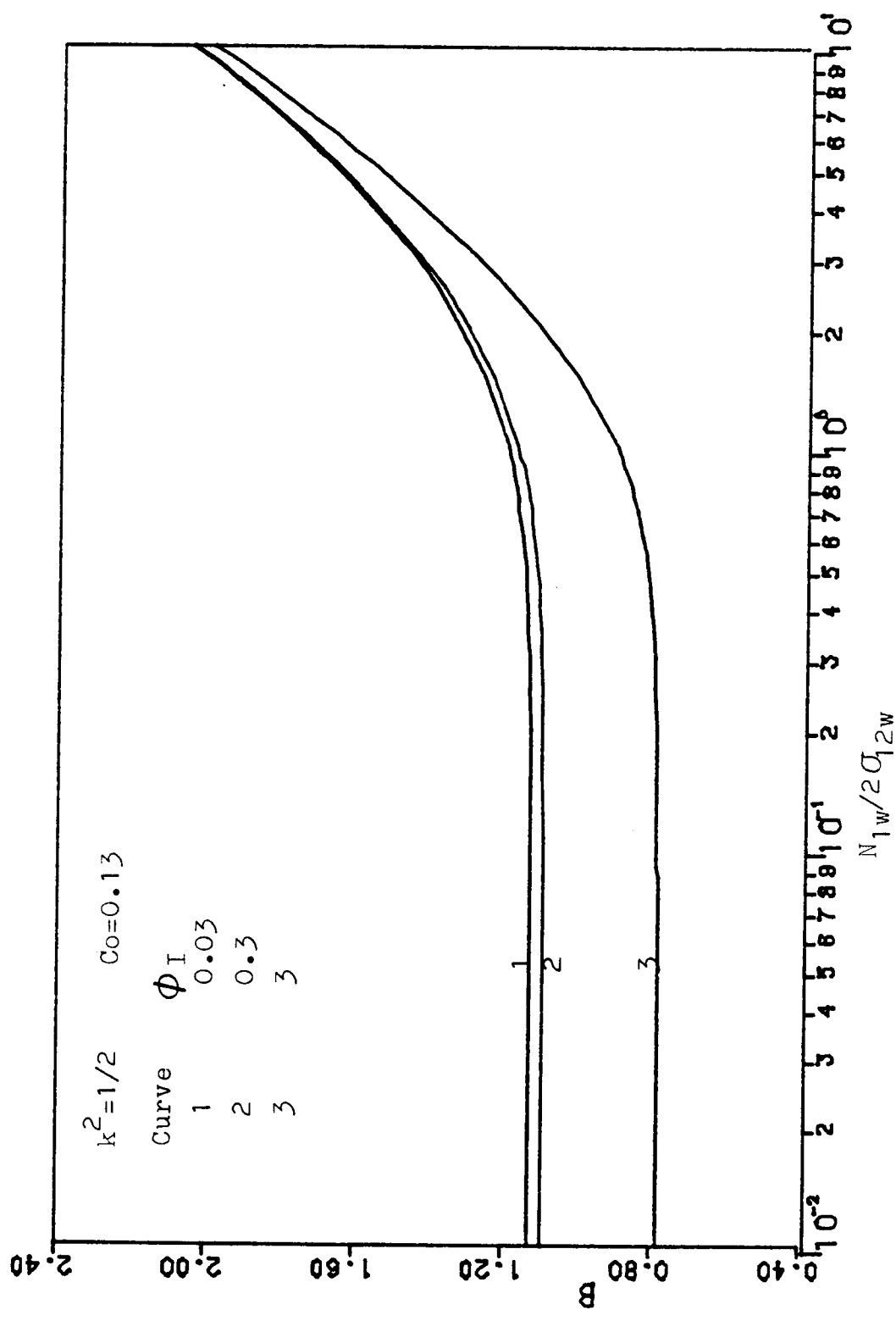


Figure 5-1. Die swell as a function of $N_{1w}/2\sigma_{12w}$, for various ϕ_I values.

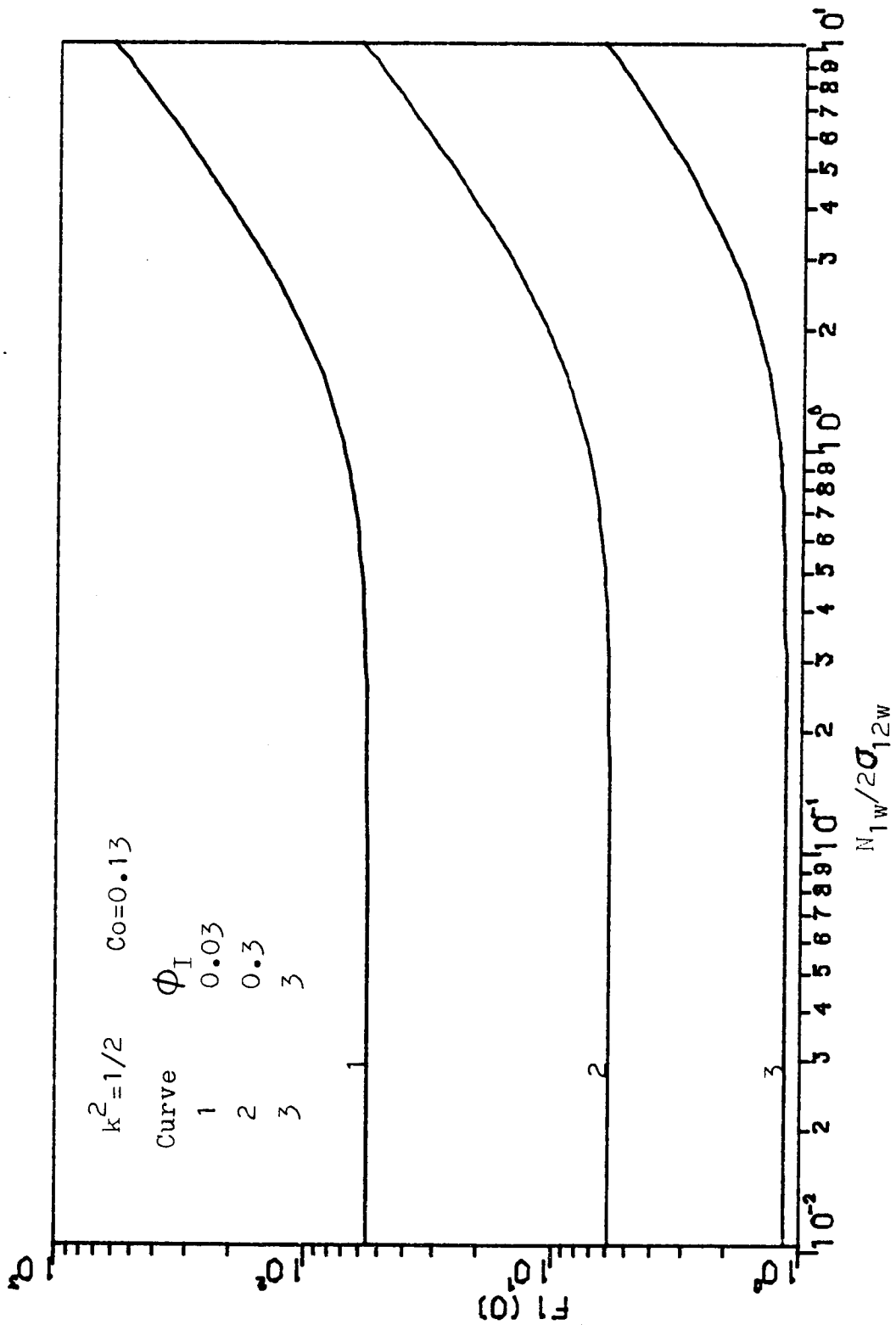


Figure 5-2. $F_1(0)$ as a function of $N_{1w}/2\sigma_{12w}$, for various ϕ_I values.

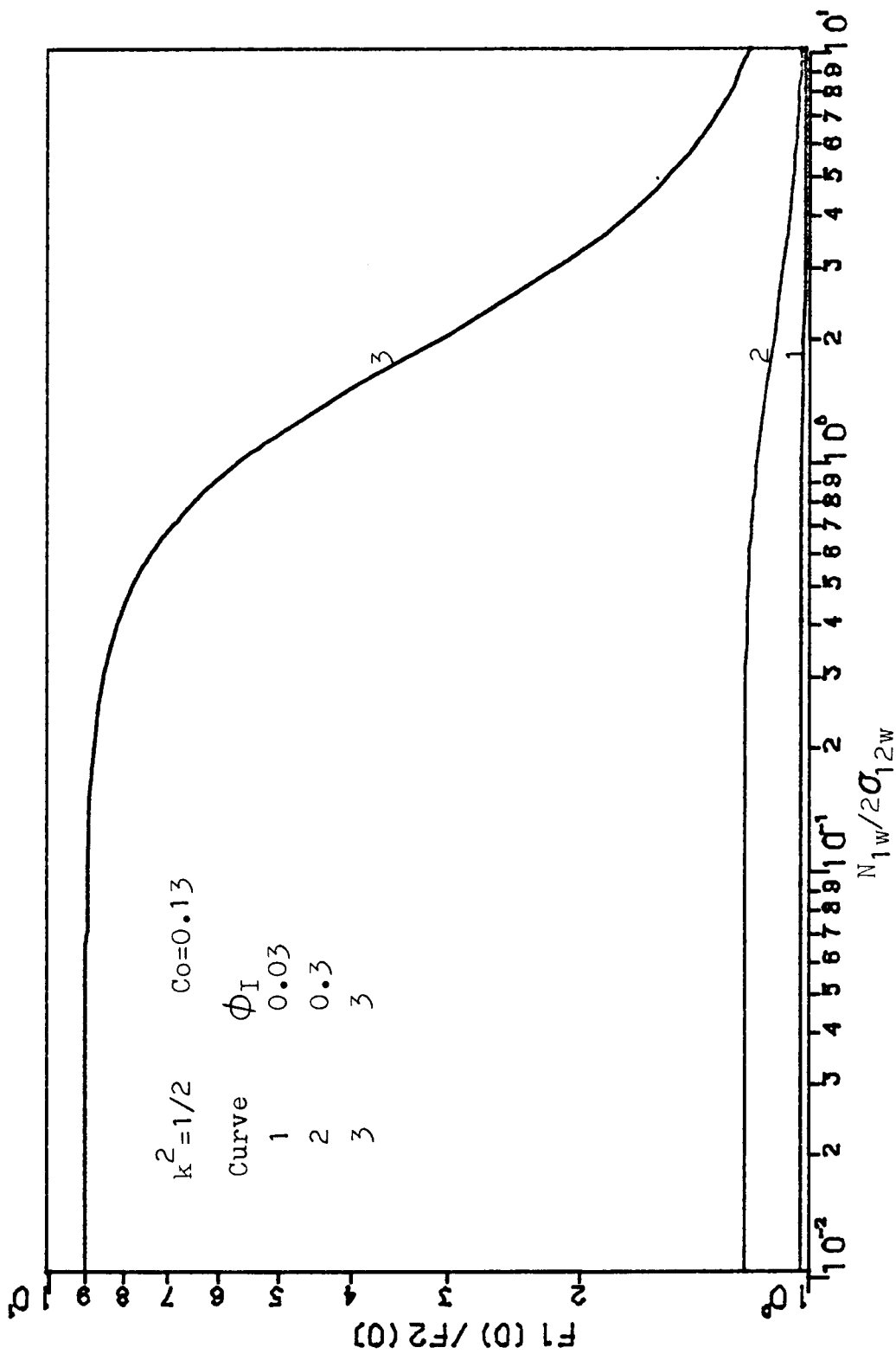


Figure 5-3. $F1(0)/F2(0)$ as a function of $N_{1w}/2\sigma_{12w}$, for various ϕ_I values.

is increased and decreases as the take-up force is increased; these are expected results, as shown in Chapter 4 and by the finite-element methods done by other authors. Figure (5-2) shows clearly that $F1(0)$ is different from 1, which is the value assumed by previous authors (36, 59, 72-75, 78, 80, 84). The value of $F1(0)$ is much higher than 1 as $N_{1w}/2\sigma_{12w}$ is increased or as the take-up force is decreased. Only when the take-up force is sufficiently high does $F1(0)$ become close to 1. Also, $F2(0)$ has a similar magnitude as $F1(0)$ when the take-up force is lower or the elasticity of the polymer melt (denoted by the value of $N_{1w}/2\sigma_{12w}$) is larger, as shown in Figure (5-3). Physically, these results seem reasonable. At a low take-up force or for highly elastic materials, the die swell is large. This gives a high value for $F2(0)$, which indicates a value for the radial normal stress.

The influence of take-up force on the die swell and initial stress can be clearly seen in Figures (5-4) to (5-6). In this figures, the $N_{1w}/2\sigma_{12w}$ are taken as parameters. Each line in Figures (5-4) to (5-6) corresponds to the situation of increasing the take-up force (or the draw ratio) at a specified die flow condition. In Figure (5-4), we see that the die swell can be less than 1 if the recoverable shear is low and the take-up force is high. Also $F1(0)$ is less than 10 only when ϕ_I is larger than 1 for $SR = 5$, as shown in Figure (5-5). Referring to Figures (5-5) and (5-6), we conclude that the assumption of $F1(0) \gg F2(0)$ used by the previous authors is applicable when the take-up force is sufficiently high, and when the elasticity is low. Note that $N_{1w}/2\sigma_{12w}$ is ranging from 0.5 to 5 for PP 106-4 and PP 6823 in this work as shown in the previous chapter.

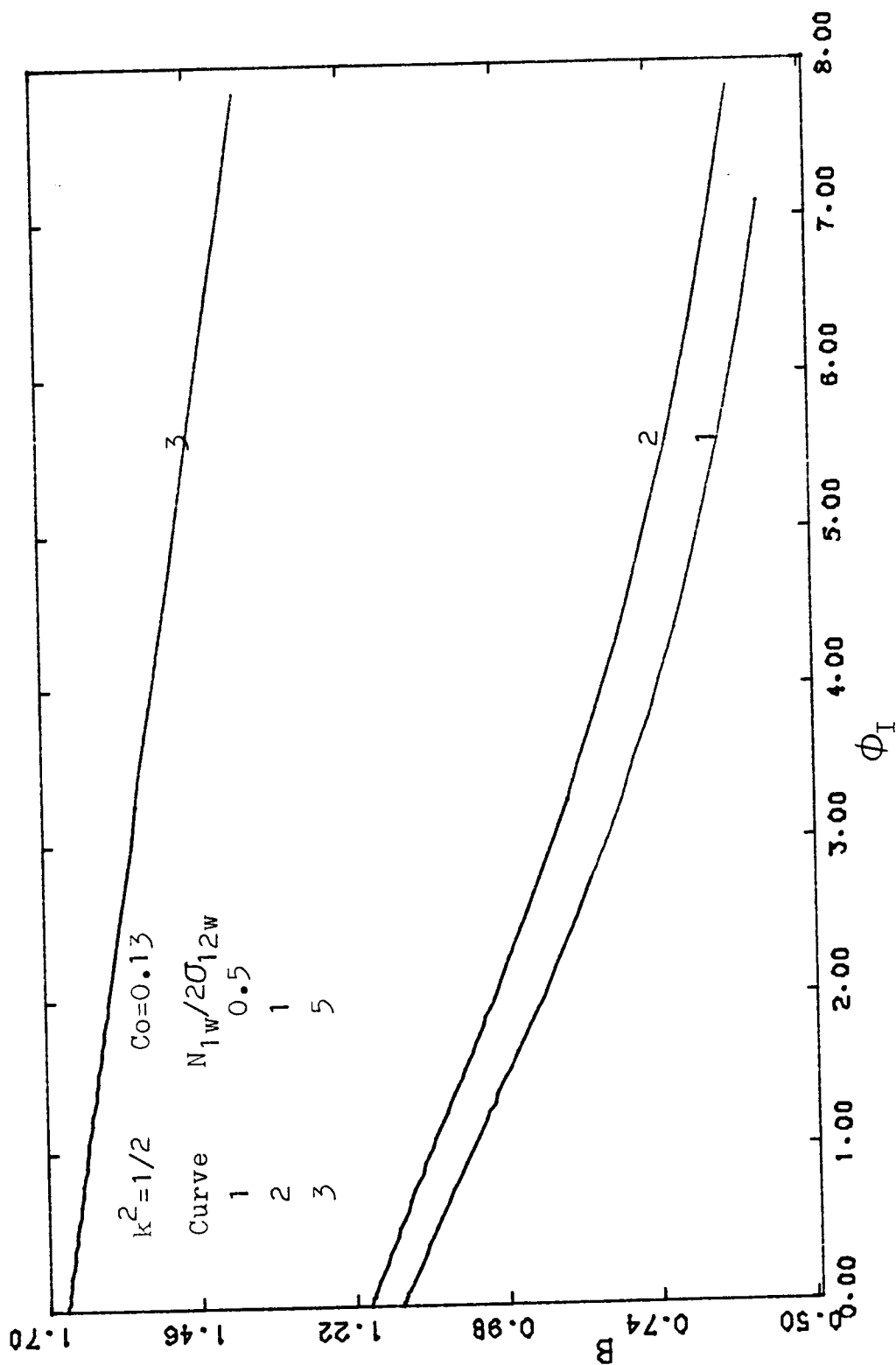


Figure 5-4. Die swell as a function of ϕ_I , for various $N_{1w}/2\sigma_{12w}$ values.

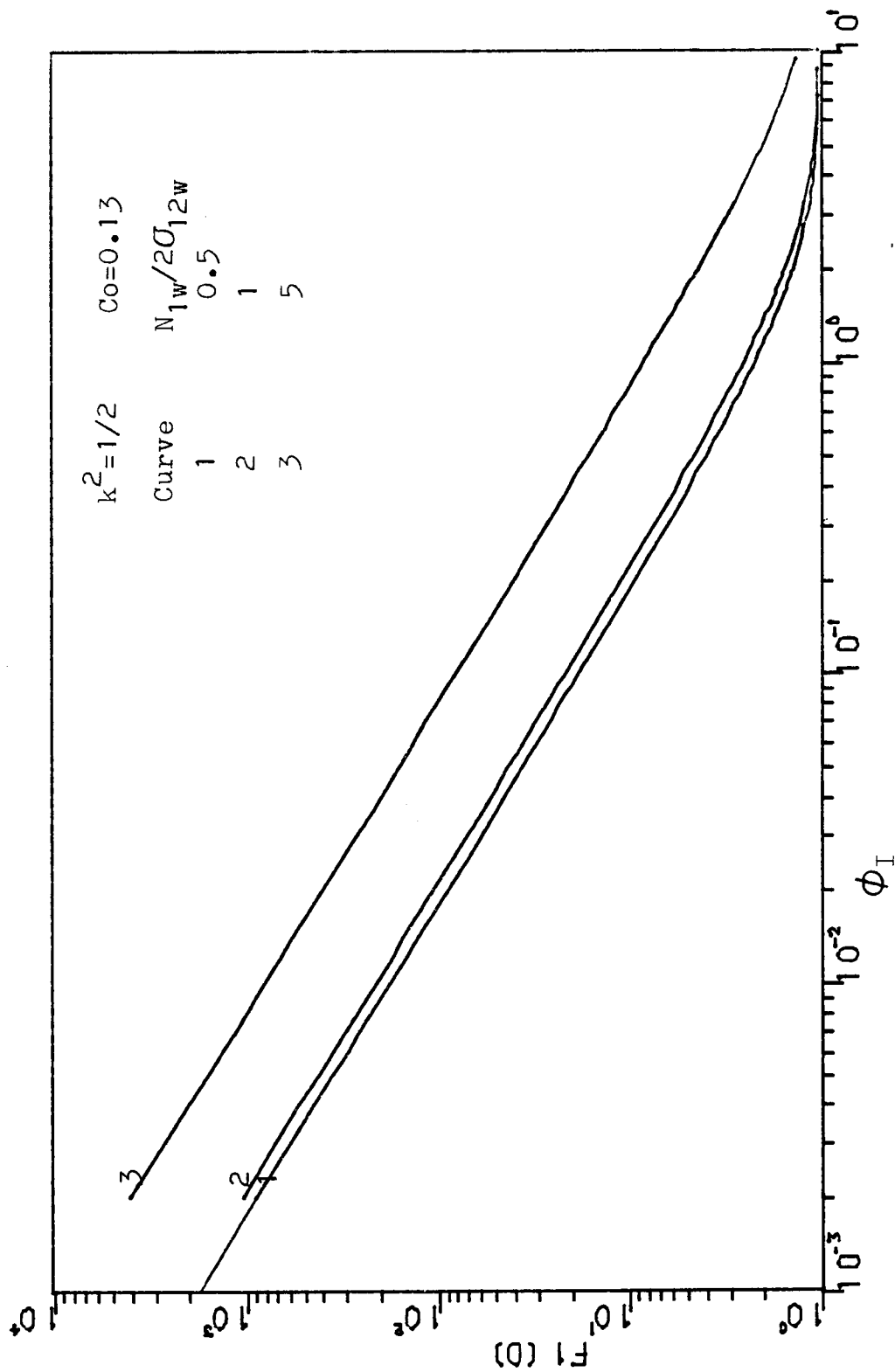


Figure 5-5. $F_1(0)$ as a function of ϕ_I , for various $N_{1w}/2\sigma_{12w}$ values.

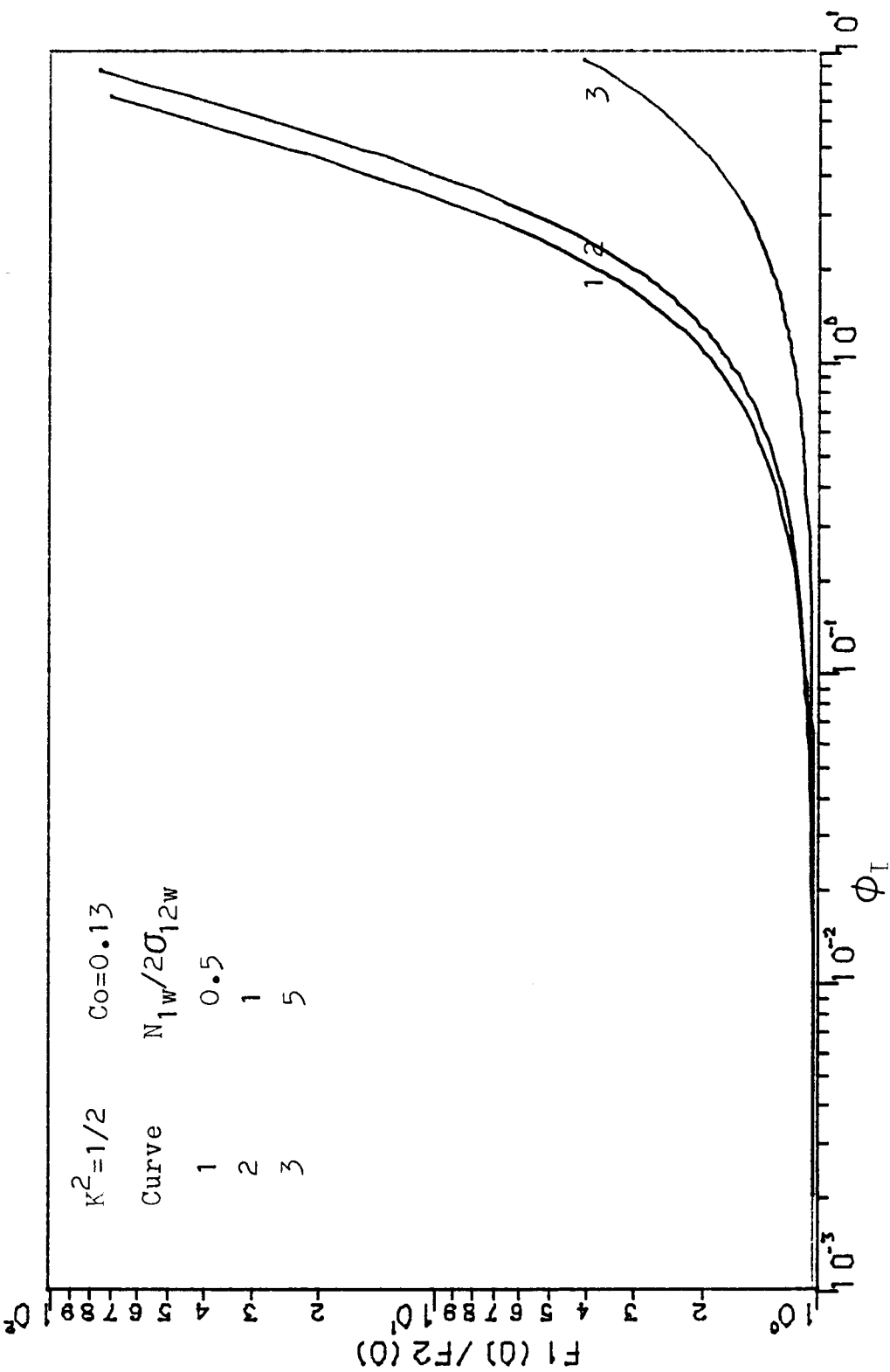


Figure 5-6. $F1(0)/F2(0)$ as a function of $N_{1w}/2\sigma_{12w}$, for various ϕ_I values.

A-2. Governing Equations in Melt Spinning Studies

The differential type constitutive equation of White-Metzner type (57) is used in the present work. The deformation rate dependence of the relaxation time is that first assumed by Bogue (58) and Bogue and White (52). That is, the constitutive equations are (see Appendix C):

$$\begin{cases} \underline{\sigma} = - \underline{P} \underline{I} + \underline{P} \\ P_{11} + \tau(\underline{\Pi}) \left[\frac{\partial P_{11}}{\partial t} + U_1 \frac{\partial P_{11}}{\partial x_1} - 2P_{11} \frac{\partial U_1}{\partial x_1} \right] = 2 \tau(\underline{\Pi}) G \frac{\partial U}{\partial x_1} \\ P_{22} + \tau(\underline{\Pi}) \left[\frac{\partial P_{22}}{\partial t} + U_1 \frac{\partial P_{22}}{\partial x_1} + P_{22} \frac{\partial U_1}{\partial x_1} \right] = - \tau(\underline{\Pi}) G \frac{\partial U_1}{\partial x_1} \end{cases} \quad (5.9)$$

and

$$\tau(\underline{\Pi}_d) = \tau_o / (1 + \sqrt{3} a \tau_o (\partial U_1 / \partial x_1)) \quad (5.10)$$

The force balance equation for moderate drawdown, where only rheological force is important, according to Eq. (2.26), is

$$F_{\text{rheo}} = \frac{1}{4} \pi d^2 \sigma_{11} = F \quad (5.11)$$

The mass balance equation is

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial x} (A U_1) = 0 \quad (5.12)$$

The above governing equations are valid for both steady and unsteady isothermal spinning starting from the point where the purely elongational flow is applicable. The starting position is chosen at the location of the maximum die swell, which is reasonable, as shown at the beginning of this chapter. Since we are dealing with the instantaneous expansion, the maximum die swell is, of course, located at the die exit. The boundary conditions are at $x = 0$:

$$\begin{cases} A(x_1, t) = A_0 \quad (\text{or } R(x_1, t) = R_0) \\ U(x_1, t) = U_0 \\ P_{11}(x_1, t) = P_{11}(0) \end{cases} \quad (5.13)$$

and at $x = L$, where L is the spinline length,

$$U(x_1, t) = U(L, t) \quad (\text{or } F \text{ is known}) \quad (5.14)$$

Note that if we specified F (or later ϕ_I or ϕ_d according to Eq. (5.5) or Eq. (5.6)), the $U(L, t)$ must be remained unknown. In other words, for each take-up force F , we have a corresponding draw ratio at $x = L$.

To normalize the above equations, the following dimensionless variables are introduced:

$$\begin{cases} A^* = \frac{A}{A_0} \quad (\text{or } R^* = \frac{R}{R_0}) \\ x^* = \frac{x}{\tau_o U_0}, \quad U^* = \frac{U}{U_0} \end{cases} \quad (5.15)$$

$$\begin{cases} \theta = \frac{t}{\tau_0} \\ F1 = P_{11}/\sigma_{11}(0) \\ F2 = P_{22}/\sigma_{11}(0) \end{cases} \quad (5.15)$$

where $\sigma_{11}(0)$ is the stress value at x^* equals zero. Note that the above dimensionless groups are all based on the values at the initial position. Substituting Eq. (5.15) into the above equations, one can obtain after some manipulation,

Rheological equations:

$$F1 + \frac{1}{1 + \sqrt{3}a \frac{\partial U}{\partial x}} \left[\frac{\partial F1}{\partial \theta} + U \frac{\partial F1}{\partial x} - 2F1 \frac{\partial U}{\partial x} \right] = \frac{2}{1 + \sqrt{3}a \frac{\partial U}{\partial x}} \left(\frac{1}{\phi_d} \right) \frac{\partial U}{\partial x} \quad (5.16)$$

$$F2 + \frac{1}{1 + \sqrt{3}a \frac{\partial U}{\partial x}} \left[\frac{\partial F2}{\partial \theta} + U \frac{\partial F2}{\partial x} + F2 \frac{\partial U}{\partial x} \right] = \frac{-1}{1 + \sqrt{3}a \frac{\partial U}{\partial x}} \left(\frac{1}{\phi_d} \right) \frac{\partial U}{\partial x} \quad (5.17)$$

Force balance:

$$F1 - F2 = U \quad \text{or} \quad \frac{\partial}{\partial x} [A(F1 - F2)] = 0 \quad (5.18)$$

Mass balance :

$$\frac{\partial A}{\partial \theta} + \frac{\partial}{\partial x} (AU) = 0 \quad (5.19)$$

and the boundary conditions are, at $x = 0$;

$$A(x, \theta) = 1 \quad (\text{or} \quad R(x, \theta) = 1)$$

$$U(x, \theta) = 1 \quad (5.20)$$

$$F1(x, \theta) = P_{11}(0)/\sigma_{11}(0)$$

and at take-up position with $x = L / \tau_o U_o$;

$$U(x, \theta) = U(L, \theta) / U_o \quad (\text{or } \phi_I = \text{Constant}) \quad (5.21)$$

Note that in the above normalized equations the superscript * is omitted for simplicity, and $F_1(0, \theta)$ is determined automatically when one considers Eqs. (5.1) to (5.7) together.

In order to calculate the dimensionless velocity (or radius) based on the average velocity in the die (or die radius), the dimensionless spinline distance is defined as

$$X = x_1 / \tau_o \bar{V} = (x_1 / \tau_o U_o) (U_o / \bar{V}) = x / B^2 \quad (5.22)$$

and the dimensionless velocity is defined by

$$V^*(X, \theta) = \frac{V(X, \theta)}{\bar{V}} = \left(\frac{V(X, \theta)}{U_o} \right) \left(\frac{U_o}{\bar{V}} \right) = \frac{U(X, \theta)}{B^2} \quad (5.23)$$

Thus, if one solves Eqs. (5.16) to (5.19) together with Eqs. (5.1) to (5.8) to obtain $U(x, \theta)$, one can transfer the coordinate x to the coordinate X by using Eq. (5.22) and finally calculate the dimensionless velocity (or radius) based on the die flow condition. In this manner, the effect of die flow on the dynamics of spinning can be thoroughly studied.

A-3. Steady Melt Spinning Studies

In steady state, for which the variables are time independent, the above normalized equations reduce to

$$F1_s + \frac{1}{1 + \sqrt{3}a \frac{dU_s}{dx}} \left[U_s \frac{dF1_s}{dx} - 2 F1_s \frac{dU_s}{dx} \right] = \frac{2}{1 + \sqrt{3}a \frac{dU_s}{dx}} \left(\frac{1}{\phi_d} \right) \frac{dU_s}{dx} \quad (5.24)$$

$$F2_s + \frac{1}{1 + \sqrt{3}a \frac{dU_s}{dx}} \left[U_s \frac{dF2_s}{dx} + F2_s \frac{dU_s}{dx} \right] = \frac{-1}{1 + \sqrt{3}a \frac{dU_s}{dx}} \left(\frac{1}{\phi_d} \right) \frac{dU_s}{dx} \quad (5.25)$$

$$F1_s - F2_s = U_s \quad (5.26)$$

$$R_s^2 U_s = 1 \quad (5.27)$$

and the boundary conditions are

$$R_s(0) = 1 \quad (5.28)$$

$$U_s(0) = 1 \quad (5.29)$$

and

$$U(x_1/\tau_0 U_0) \Big|_{x_1=L} = U_L/U_0 \quad (\text{or } V(x_1/\bar{V}\tau_0) \Big|_{x_1=L} = DR) \quad (5.30)$$

or

$$\phi_I^2 = 4F/(\pi D G) = \phi_d^2 = \text{Constant} \quad (5.31)$$

The subscript "s" which indicates the steady state value are omitted in the following treatment for simplicity.

Elimination of F2 from Eqs. (5.24) to (5.26), F1 can be expressed by

$$F1 = \frac{U}{\frac{dU}{dx}} + \frac{\sqrt{3}a}{3} U + \frac{2}{3} U - \frac{1}{\phi_d} \quad (5.32)$$

Taking the derivative of F1 with respect to x and substituting this result together with Eq. (5.32) into Eq.(5.24), after some manipulation, one obtains

$$\begin{aligned} U \frac{d^2 U}{dx^2} = & (3a^2 + \sqrt{3}a - 2)U \left(\frac{dU}{dx} \right)^3 - \left(\frac{3\sqrt{3}a}{\phi_d} \right) \left(\frac{dU}{dx} \right)^3 \\ & + (2\sqrt{3}a + 1)U \left(\frac{dU}{dx} \right)^2 - \left(\frac{3}{\phi_d} \right) \left(\frac{dU}{dx} \right)^2 + U \frac{dU}{dx} \end{aligned} \quad (5.33)$$

Eq. (5.33) is the final form of the governing equation to solve $U(x)$. It is a second-order nonlinear ordinary differential equation. The boundary conditions, according to Eqs. (5.28) to (5.32), are

$$U(0) = 1 \quad (5.34)$$

$$\frac{dU}{dx}(0) = \frac{1}{3F1(0) - (2 + \sqrt{3}a) + (3/\phi_d)} \quad (5.35)$$

and the parameters $F1(0)$ and ϕ_d (or ϕ_I) are obtainable from Eqs. (5.1) to (5.8), which connect the die flow to the spinline flow.

Figure (5-7) shows the dimensionless velocity profile $V(X)$ under different die flow conditions and various Bogue-White thinning parameters "a," for $\phi_I = 0.5$. The computer program for obtaining these results is shown in Appendix E, as a part of combined die flow and spinline flow studies, including both steady melt spinning and instabilities studies. The values of k^2 and Co in Eq. (5.2) are

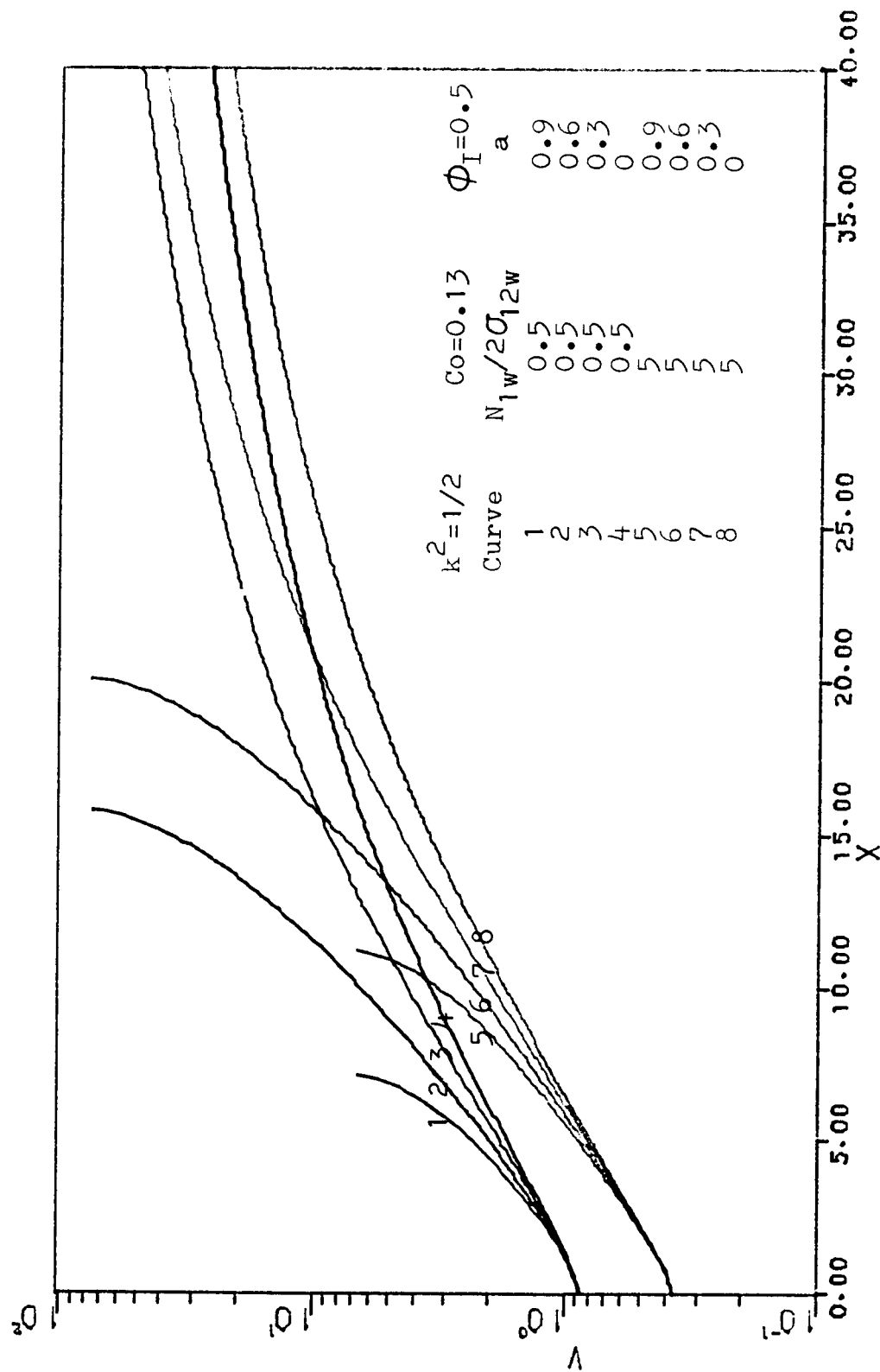


Figure 5-7. Velocity Profile for various values of "a" and $N_{1w}/2\sigma_{12w}$ ($\phi_I = 0.5$).

chosen as $1/2$ and 0.13 , respectively. For any ϕ_I value, a Newton-Raphson method is used which iterates the die swell value until the corresponding ϕ_I value equals to the given ϕ_I value. The initial stress conditions ϕ_d and $F1(0)$ are then calculated by using Eqs. (5.5) and (5.7). These values are used to calculate the initial velocity gradient, that is, Eq. (5.35). The initial conditions (Eqs. (5.34) and (5.35)) and the governing equations constitute an initial value problem. This initial value problem is solved by a fifth or sixth order Runge-Kutta method (the DVERK package in UTCC). The step size of this subroutine is selfadjusted. The minimum step size, for example, of x is about 0.000002 for the chosen step size of 0.5 as checked by CSMP (Continuous System Modeling Program) method. This subroutine take the final x value in the previous step as the starting x value for the next increment of x . Consequently, it does not take much CPU (Central Processing Unit) time. The increment of x is always chosen as $0.04/\phi_I$.

An important feature in Figure (5-7) is that at some specific value of spinline length X or x , depending on ϕ_I and SR values, the computer does not yield a solution. This is true for $a > 1/\sqrt{3}$, as concluded from several trials. To understand this "blow-up" phenomenon, the velocity gradient along the spinline is plotted, as shown in Figure (5-8). Clearly, the "blow-up" point is the position where the velocity gradient approaches infinity. Physically, it is the position where the ductile necking behavior is occurred. Figure (5-7) also shows that as the thinning parameter " a " is increased, the draw ratio or the velocity is increased at the same die flow

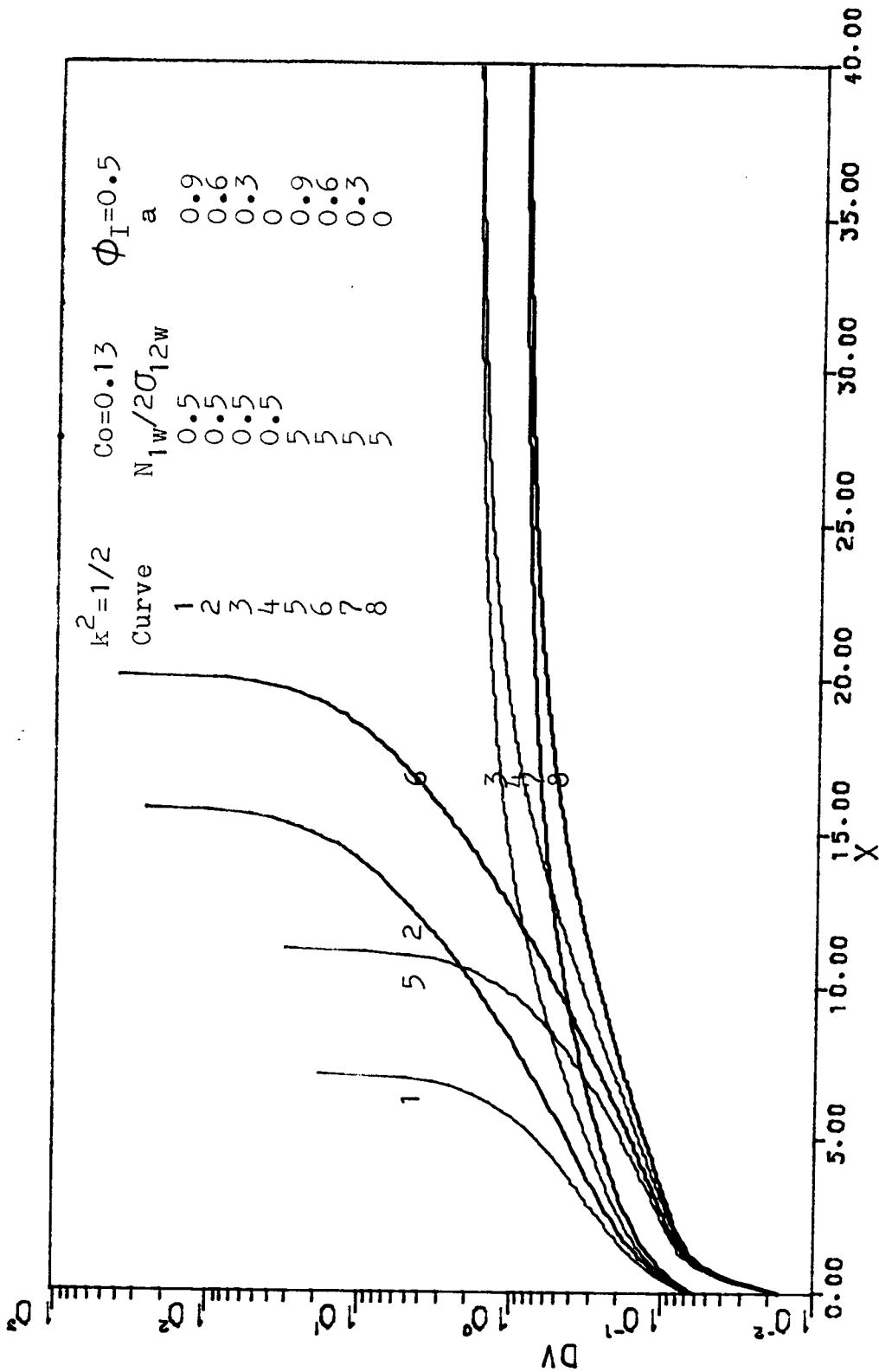


Figure 5-8. Velocity gradient profile for various values of "a" and $N_{1w}/2\sigma_{12w}$ ($\phi_I = 0.5$).

condition. It is also shown that the higher the stress ratio the lower the draw ratio for the same "a" value at the same take-up force and constant X. However, we must note that the values of $N_{1w}/2\sigma_{12w}$, a, ϕ_I , and X are all dependent on rheological properties, and X and V are normalized based on the velocity in the die, which also affects the value of $N_{1w}/2\sigma_{12w}$. That is, all variables are interrelated. In reality, one can obtain "a," τ_o , and G values by using the shear flow properties or uniaxial elongational flow properties of polymer melt as shown by several authors (33, 114, 7); also the $N_{1w}/2\sigma_{12w}$ can be obtained by the cone and plate rheometer if the data are extrapolated to the high shear rate region. Then, given a specified die diameter with sufficiently high aspect ratio (L/D) and plunger speed, one can determine the shear stress and the mean velocity in the die.

In the spinning experiment, it is easier to determine draw ratio at the specified spinline length rather than the take-up force. The take-up force measured by the tensiometer also includes the gravitational force, the frictional force between thread and Teflon rod, the drag force in the water bath, etc. Several methods have been suggested by Minoshima (40, 41) to correlate these effects. However, in the present work, the take-up force is treated as a parameter, that is, it is theoretically related to the draw ratio value at the specified spinline length.

Figures (5-9) to (5-11) show the velocity profiles under different die flow conditions for $a = 0, 0.3$, and 0.6 , when $DR = 10$ at $X = 40$ is specified. Each line corresponds to different take-up forces as shown in Table 5-1. It is clear that, for constant DR at

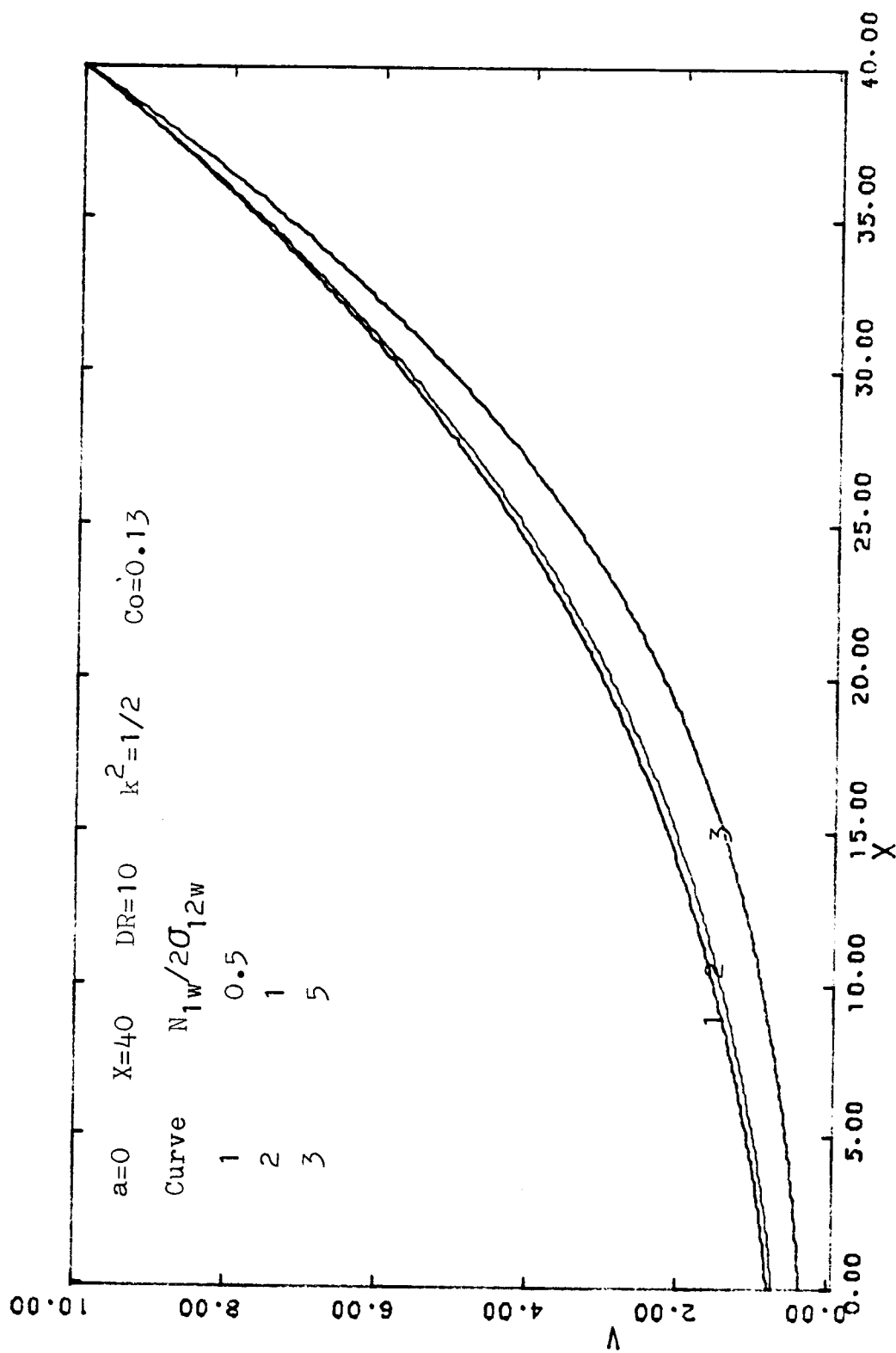


Figure 5-9. Velocity profile for various values of $N_{1w}/2\sigma_{12w}$ ($DR = 10$, $X = 40$, $a = 0$).

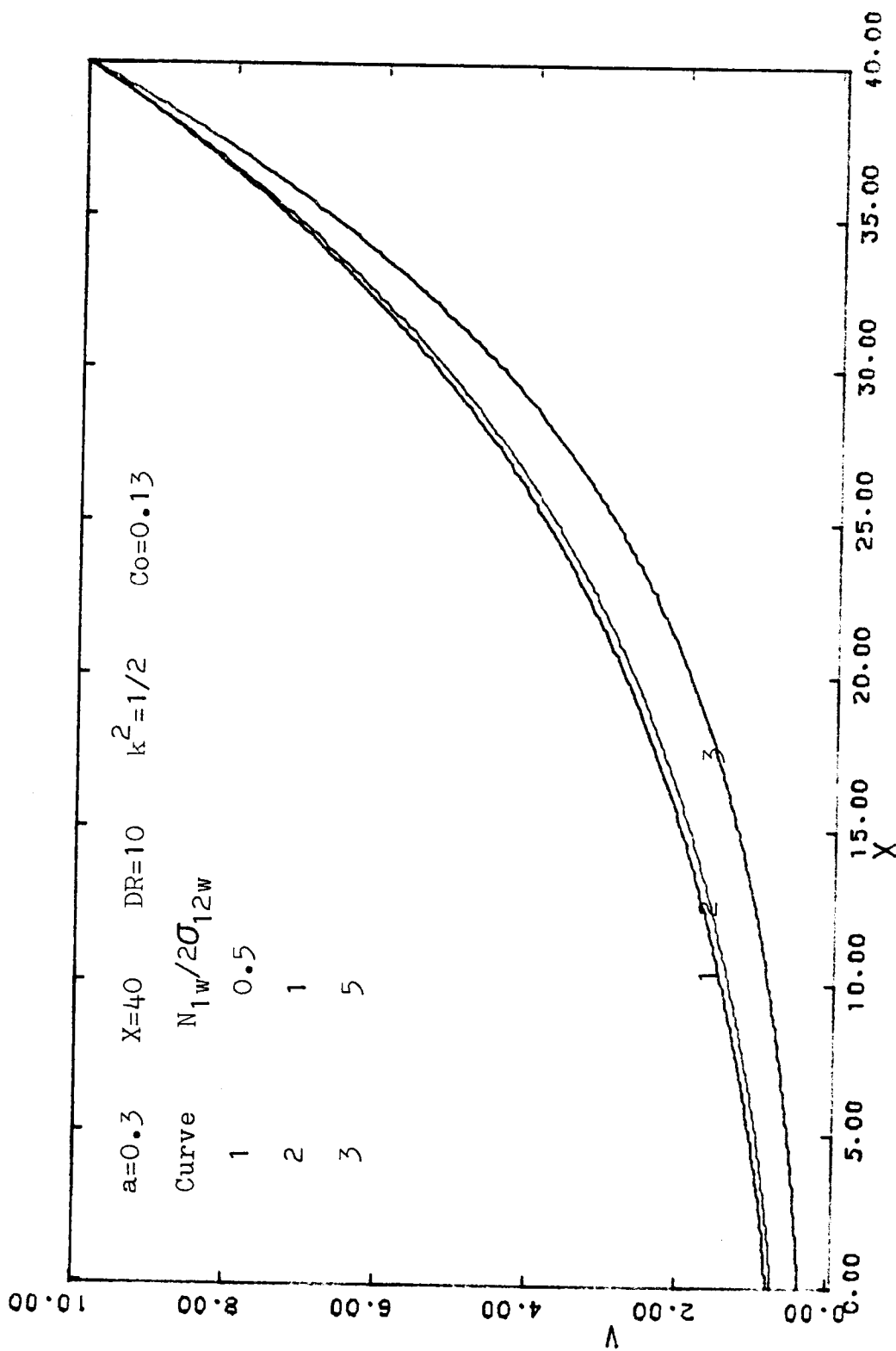


Figure 5-10. Velocity profile for various values of $N_{1w}/2\sigma_{12w}$ ($DR = 10$, $X = 40$, $a = 0.3$).

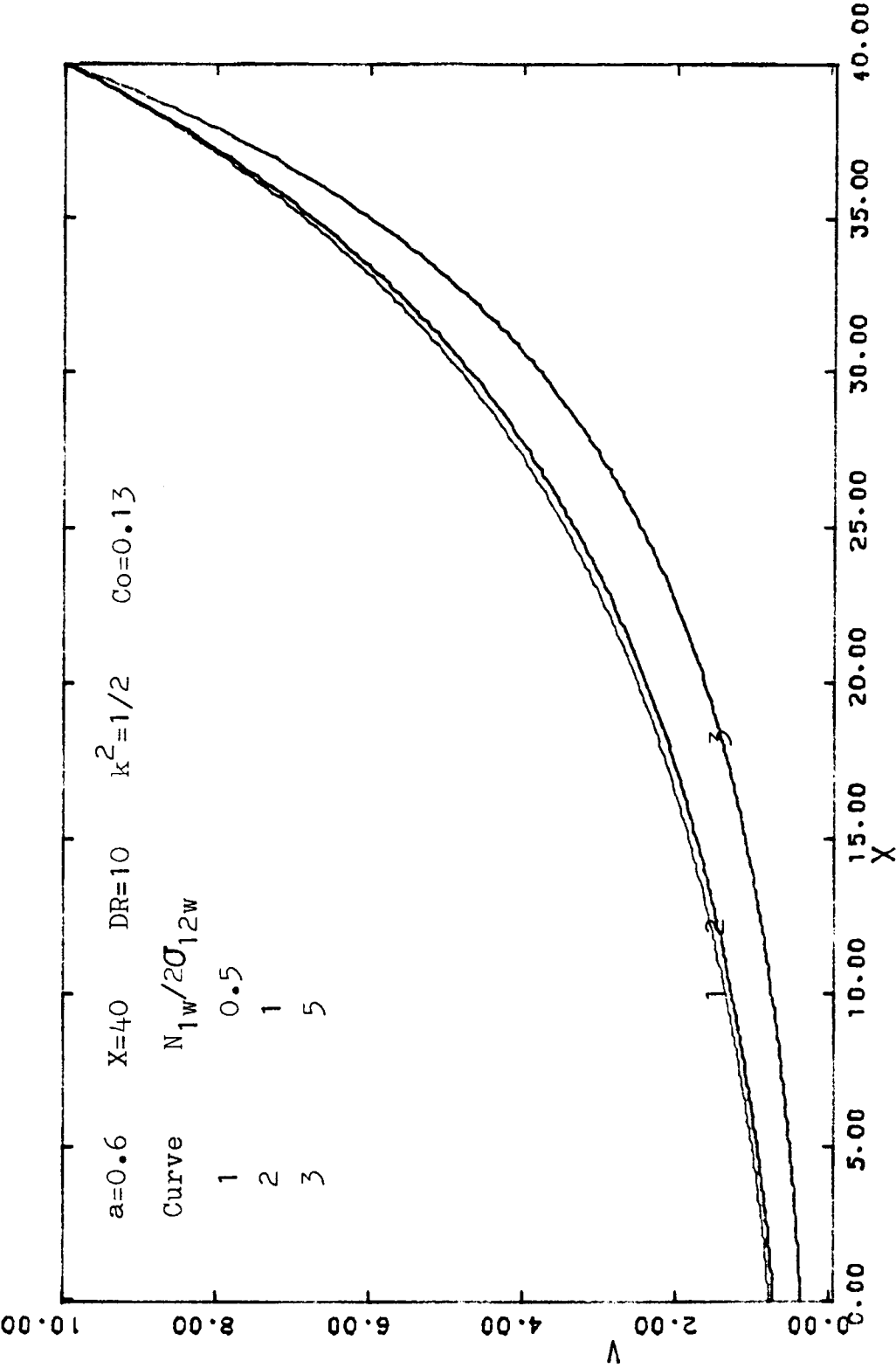


Figure 5-11. Velocity profile for various values of $N_{1w}/2\sigma_{12w}$ ($DR = 10$, $X = 40$, $a = 0.6$).

Table 5-1. Calculated Die Swell and Force at $X = 40$, $DR = 10$.

a	SR*	B	ϕ_I
0	0.5	1.1261	0.2143
0.3	0.5	1.1290	0.1880
0.6	0.5	1.1313	0.1675
0.9	0.5	1.1331	0.1512
0	1	1.1783	0.2223
0.3	1	1.1810	0.1947
0.6	1	1.1831	0.1733
0.9	1	1.1847	0.1563
0	5	1.6630	0.2835
0.3	5	1.6643	0.2464
0.6	5	1.6653	0.2181
0.9	5	1.6661	0.1957

*SR: Stress Ratio = $N_{1w}/2\sigma_{12w}$.

specified X , the velocity is higher along X when the stress ratio is lower, for the same "a" value. The effect of "a" values on the velocity profile is shown in Figures (5-12) to (5-14). The velocity is higher for a lower "a" value. To obtain these figures, an iteration method to obtain the value of B or take-up force is required. This computer program has been explained in Chapter 4 when the comparison between the die swell done under different draw ratio and sinline length X is made. The computer program is shown in Appendix D.

It is interesting to know the relationship between the maximum draw ratio obtainable (corresponding to the "blow-up" point) and X . This is shown in Figure (5-15) for $a = 0.6, 0.9$ and $N_{lw}/2\sigma_{12w} = 0.5, 1, 5$. It is shown that at the same X , as "a" is increased, the maximum draw ratio (DRMAX) is decreased, and the higher the $N_{lw}/2\sigma_{12w}$, the lower the DRMAX. Note that each point in Figure (5-15) corresponds to different values of ϕ_I ; these ϕ_I values are shown in Figure (5-16) for comparison. These figures are obtained by slowly increasing the ϕ_I value. There is a small fluctuation of DRMAX as X is made small or ϕ_I is made large; this is because of the difficulty in determining accurately the position where the velocity gradient approaches infinity. The computer "overflows" when the velocity gradient is sufficiently large. It is also known that the DRMAX decreases rapidly and that after some sufficiently high take-up force (or low X), the DRMAX levels off to a constant value.

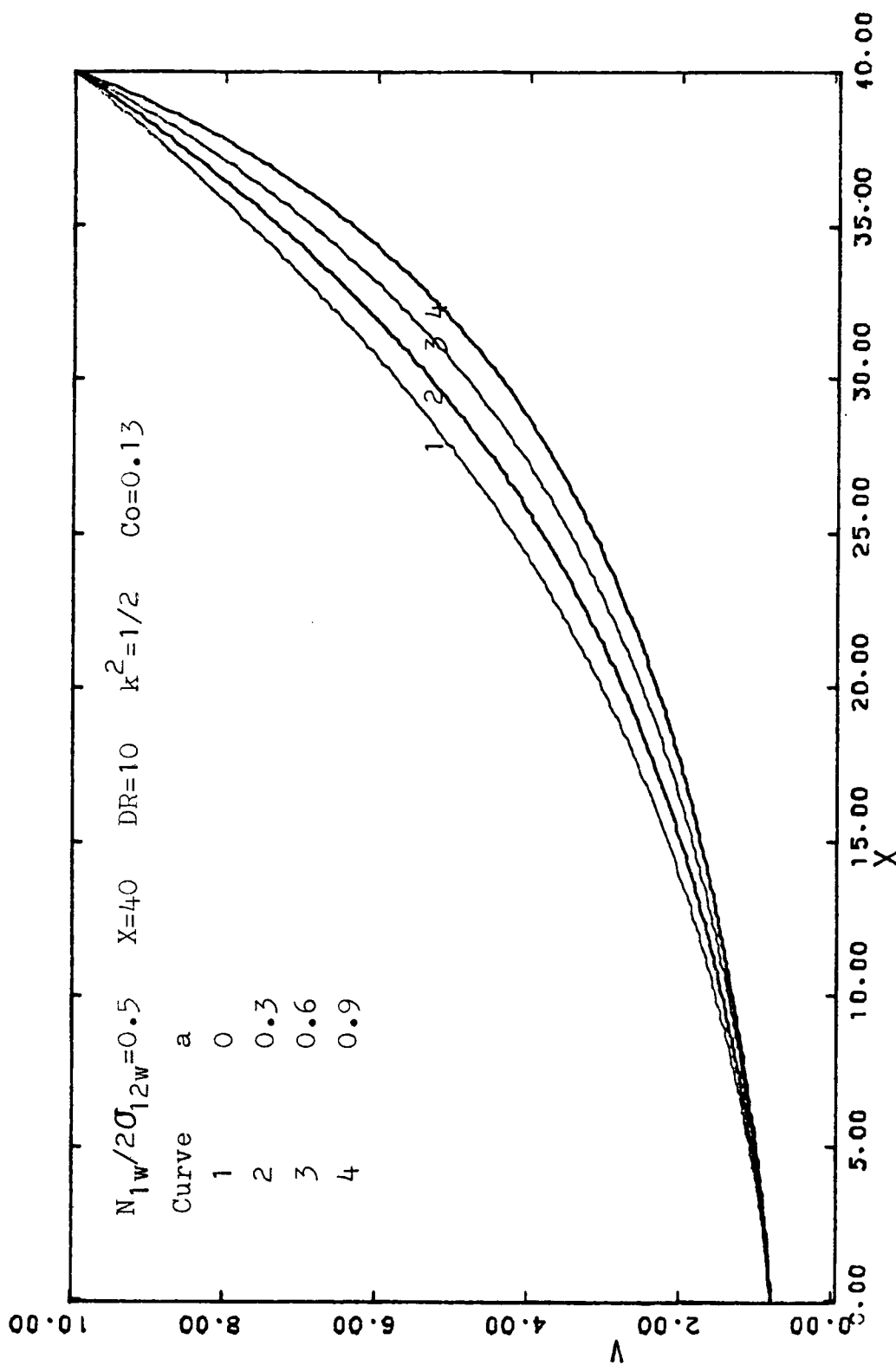


Figure 5-12. Velocity profile for various values of " a " ($DR = 10$, $X = 40$, $N_{1w}/2\sigma_{12w} = 0.5$).

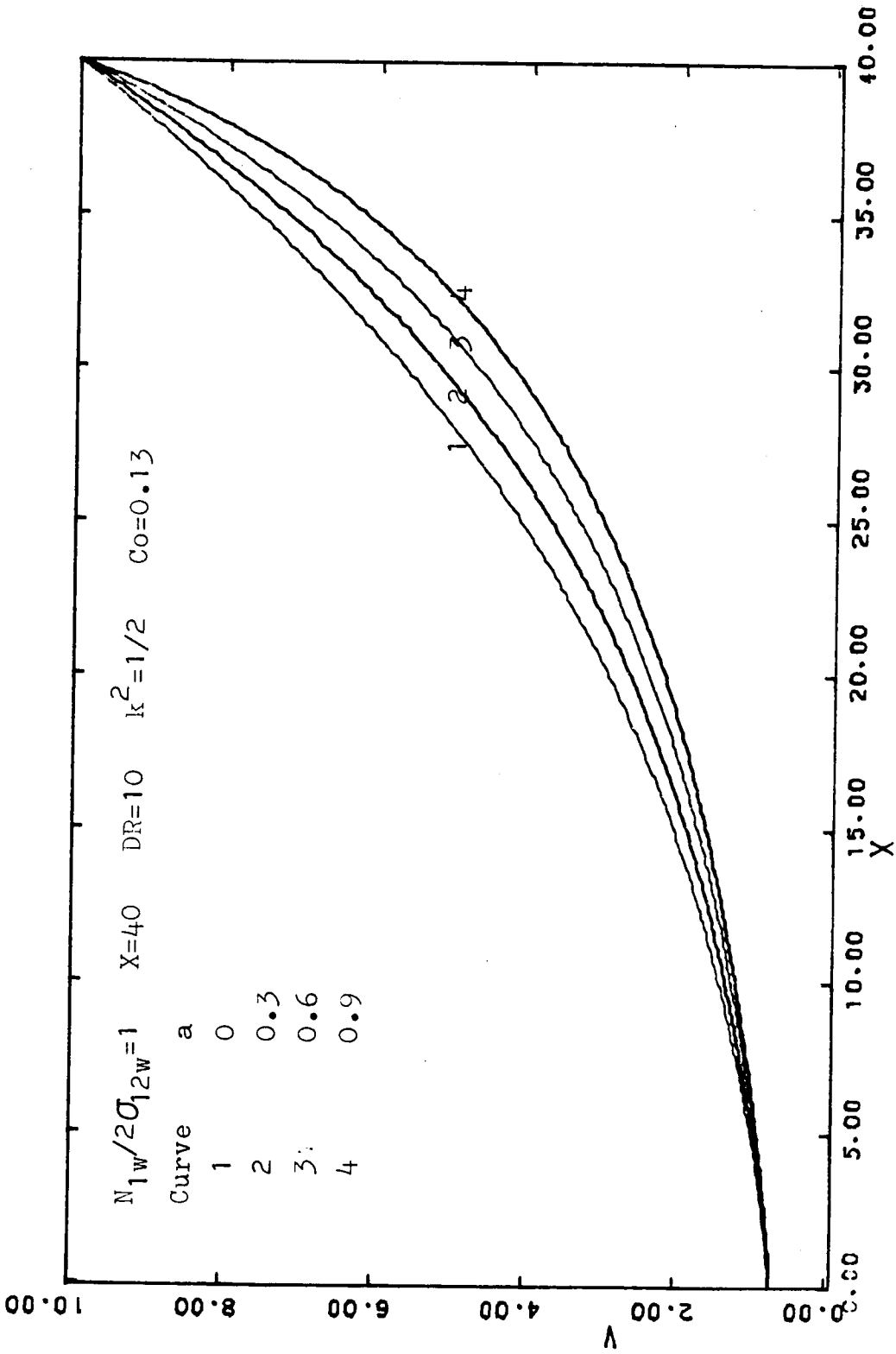


Figure 5-13. Velocity profile for various values of "a" ($DR = 10$, $X = 40$, $N_{1w}/2\sigma_{12w} = 1$).

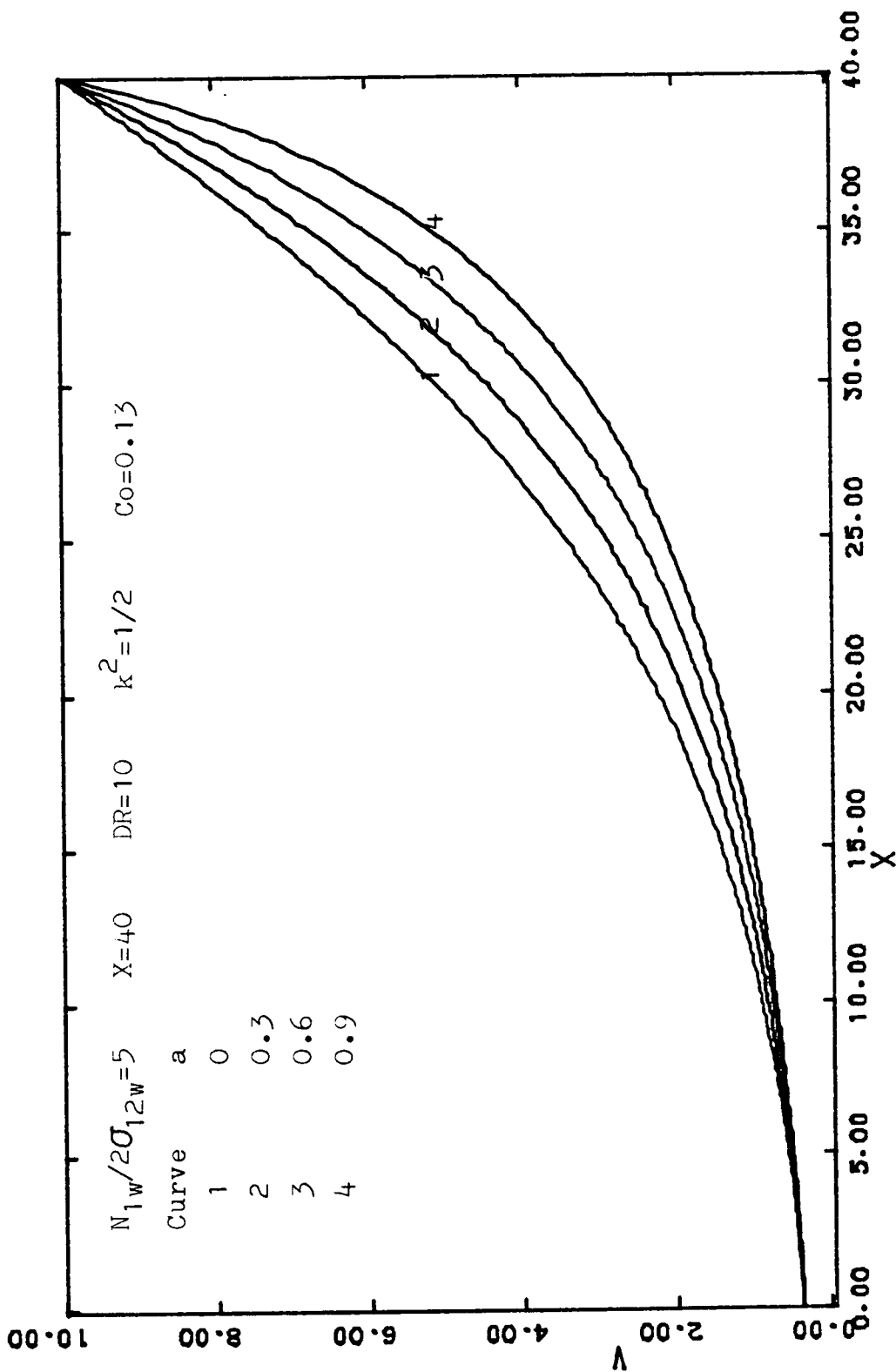


Figure 5-14. Velocity profile for various values of "a" ($DR = 10$, $X = 40$, $N_{1w}/2\sigma_{12w} = 5$).

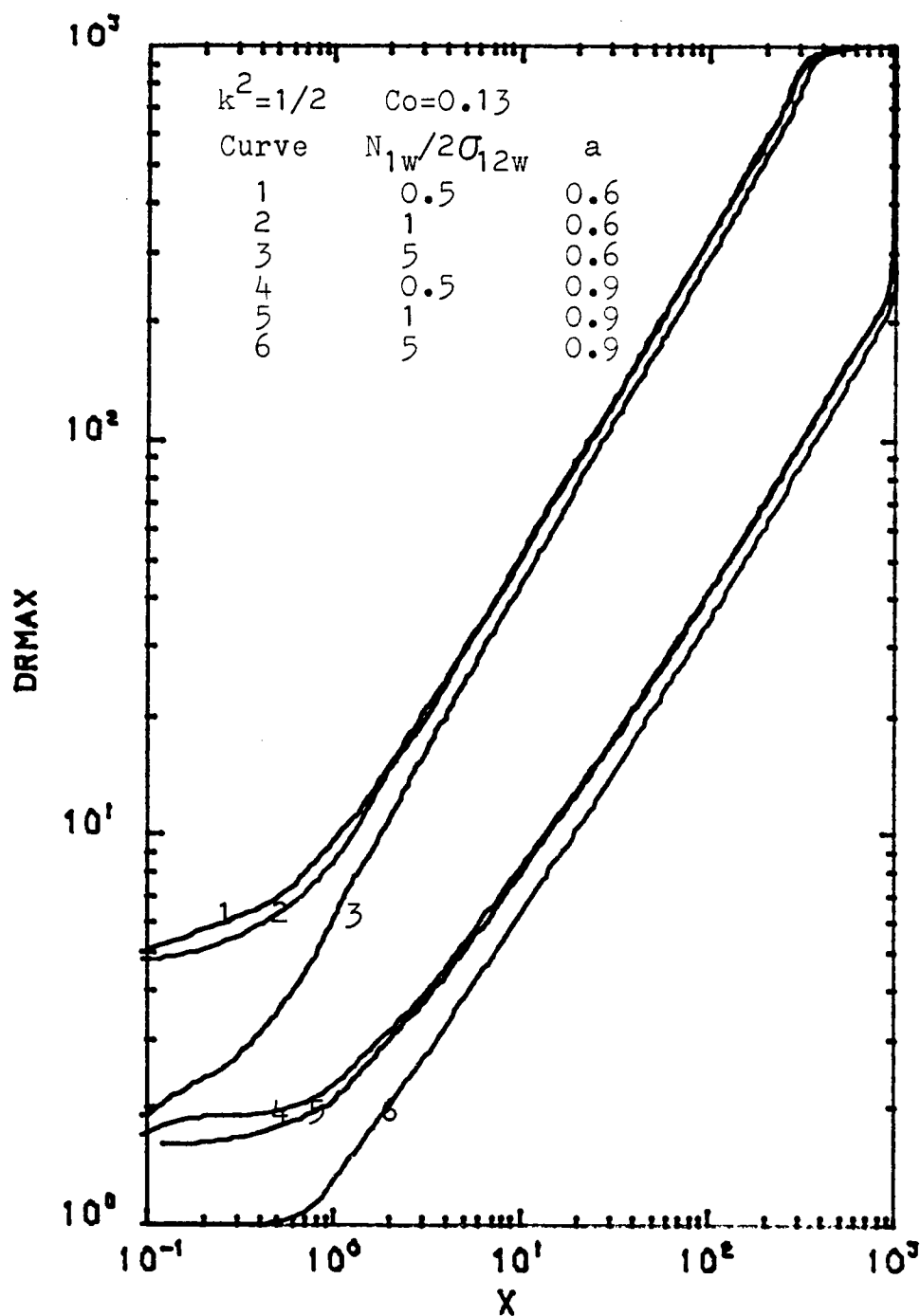


Figure 5-15. Maximum draw ratio obtainable as a function of X for various $N_{1w}/2\sigma_{12w}$ ($a = 0.6, 0.9$).

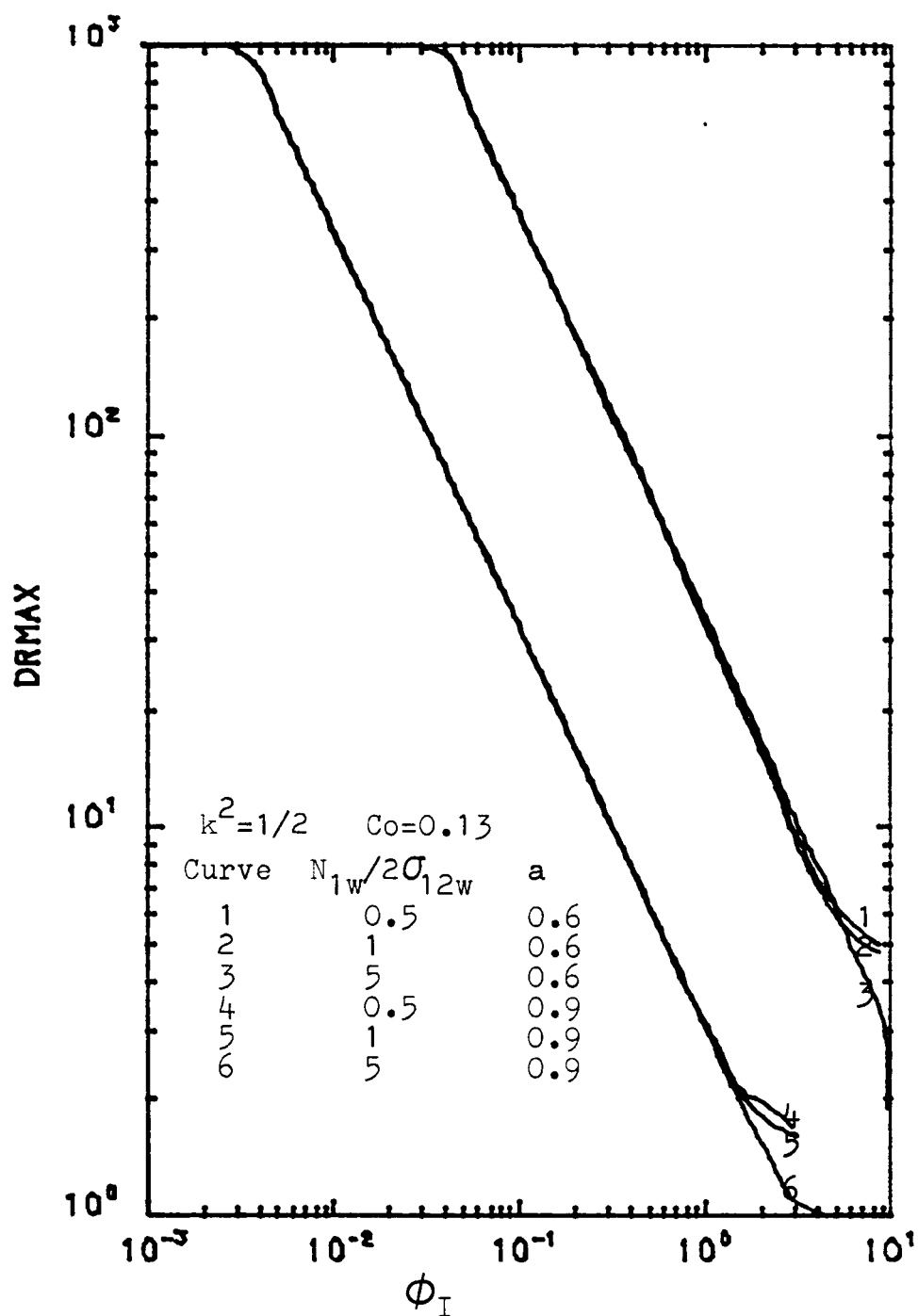


Figure 5-16. Maximum draw ratio obtainable as a function of ϕ_I for various $N_{1w}/2\sigma_{12w}$ ($a = 0.6, 0.9$).

There are some important features concerning Eqs. (5.32) and (5.33). First of all, consider the assumption used by Zeichner (72) and White and Ide (74), that is,

$$F_2 = 0 \quad (5.36)$$

along the entire spinline. Then, from Eqs. (5.26) and (5.35), one shows that

$$F_1 = U \quad (5.37)$$

$$\frac{dU}{dx}(0) = \frac{1}{(1 - \sqrt{3}a) + (3/\phi_d)} \quad (5.38)$$

Therefore, the initial velocity gradient is obtained naturally if one knows the take-up force ϕ_d and the Bogue-White parameter "a." No influence of $F_1(0)$ can be expected. In addition, if one substitutes Eq. (5.36) into Eq. (5.32), one obtains

$$1 + (\sqrt{3}a - 1) \frac{dU}{dx} = \frac{3}{\phi_d} \frac{1}{U} \frac{dU}{dx} \quad (5.39)$$

which, upon integration, gives the analytical result:

$$x + (\sqrt{3}a - 1) (U - 1) = (3/\phi_d) \ln U \quad (5.40)$$

Compare Eq. (5.40) to White-Ides' result, that is,

$$x + (\sqrt{3}a - 1) (U - 1) = (2/\phi_d) \ln U \quad (5.41)$$

One concludes that White and Ide do not consider Eq. (5.25). Eq. (5.40) is now termed as modified White-Ide (mod. W-I) solution.

There are two ways to obtain the Newtonian solution using Eq. (5.40). Mathematically, one sets $a = 1/\sqrt{3}$ in Eq. (5.40) and obtains

$$U = e^{(\phi_d x/3)} \quad (5.42)$$

Since the velocity profile for Newtonian fluids has the form

$$U = e^{(Fx/3\eta Q)} \quad (5.43)$$

where Q is the volume flow rate, η is the Newtonian viscosity. Comparing Eqs. (5.42) and (5.43), one knows that the Newtonian solution can be obtained by assuming:

$$a = 1/\sqrt{3}, \quad \phi_d = FL/Q\eta, \quad x = x_1/L \quad (5.44)$$

Physically, for purely viscous fluids, there is no time effect involved. The relaxation time is zero for purely viscous fluids. Therefore τ_o should equal zero, and an asymptotic Newtonian solution can be obtained when

$$x \gg (U - 1) \quad (5.45)$$

Figure (5-17) shows the velocity profile of Eqs. (5.40) and (5.41) for $a = 0.6$ at $\phi_d = 0.4$. The velocity profile doubles back at some x values. Physically, this double back region is impossible. For a polymer melt (τ_o , G , and " a " are specified) we can have one draw ratio at specified spinline length (x or ϕ_d is specified). In other words, for a given set of τ_o , a , G , and x values, only one draw ratio value can be imagined under a specified take-up force. In addition, it is clear that White-Ide solution (W-I solution) gives a higher draw

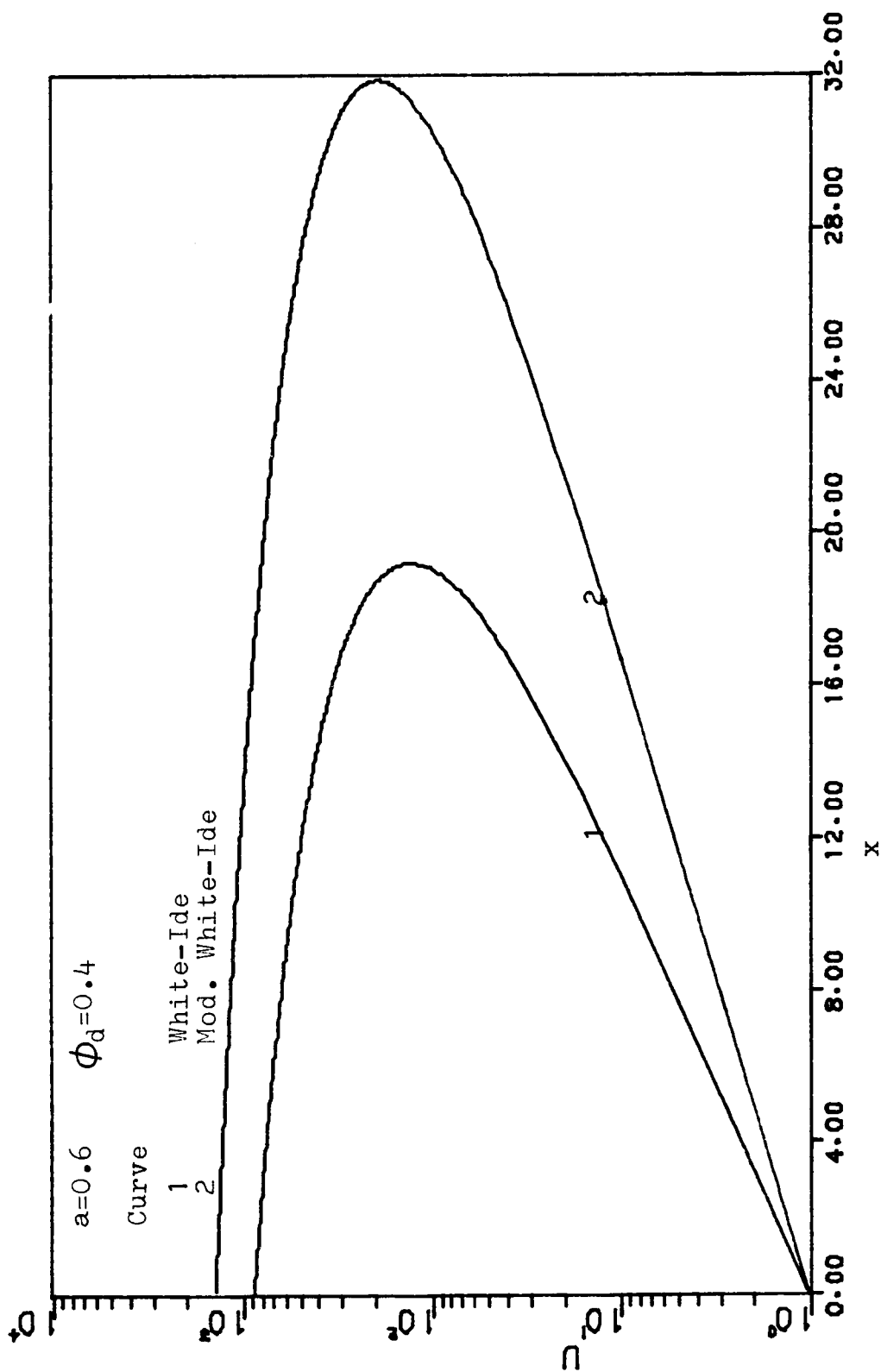


Figure 5-17. Comparison of velocity profile for the White-Ide solution and modified White-Ide solution ($a = 0.6$, $\phi_d = 0.4$)

ratio than that of mod. W-I solution. This also indicates that to obtain the same draw ratio, the mod. W-I solution needs a higher ϕ_d value than the unmodified form.

Secondly, consider the assumption used by Denn et al. (59, 75) and Fisher and Denn (78, 84), Phan-Thien (36), and Chang-Denn (101), that is,

$$F1(0) = 1 \quad (5.46)$$

which is corresponding to

$$F2(0) = 0 \quad (5.47)$$

Note that they make the assumption $F2 = 0$ only at $x = 0$. From Eq. (5.35), one obtains

$$\frac{dU}{dx}(0) = \frac{1}{(1 - \sqrt{3}a) + (3/\phi_d)} \quad (5.38)$$

Consequently, the initial velocity gradient is the same as that of the modified White-Ide solution, and no effect of $F1(0)$ can be expected. Note that, to have an elongational flow in melt spinning, the initial velocity gradient shown in Eqs. (5.35) and (5.38) should be positive. Comparing Eqs. (5.35) and (5.38) and using Figures (5-2) and (5-5) (page 88 and 92) we conclude that the actual initial velocity gradient is less than that predicted by the assumption used by those authors.

Figure (5-18) shows the numerical solutions of Eq. (5.33) with the boundary conditions (5.34) and (5.38) for $a = 0.6$ and $\phi_d = 0.4$. This solution is termed the modified Fisher-Denn solution (mod. F-D solution) because the assumption of Fisher and Denn is involved.

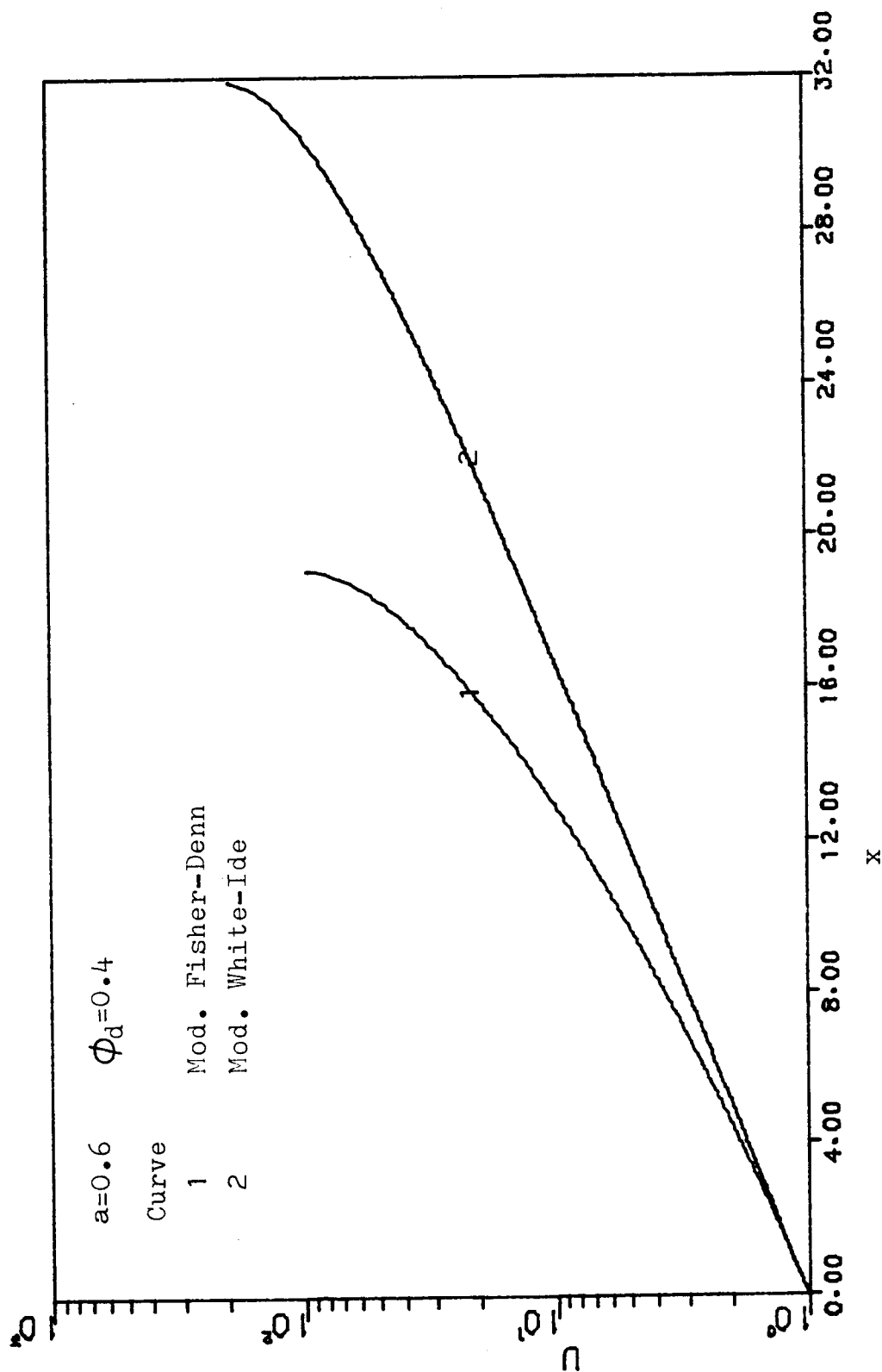


Figure 5-18. Comparison of velocity profile for the modified White-Ide solution and modified Fisher-Denn solution ($a = 0.6$, $\phi_d = 0.4$).

However the solution is not the same as that done by Fisher and Denn (78) except for the case of $a = 0$, which is the same as $n = 1$ in Fisher and Denn's treatment. Also shown in Figure (5-18) is the numerical solution of the modified White-Ide solution, that is Eq. (5.39) with the boundary condition (5.34). Clearly the mod. F-D solution gives a higher velocity than mod. W-I solution. As $a > 1/\sqrt{3}$; there exist the "blow-up" point where the velocity gradient approaches infinity, as shown in Figure (5-19) for the velocity gradient profile of Figure (5-18). To ensure that the DVERK package used in the numerical method is useful, Figures (5-17) and (5-18) are plotted in Figure (5-20). It is clear that the "blow-up" point in the numerical solution is corresponding to the inflection point in the analytical solution. That the numerical solution gives the same results as obtained from the analytical solution gives confidence in the computer scheme.

In the following discussion, we compare the results obtained by several different assumptions. As shown above, mod. W-I solution and mod. F-D solution suffer from the disadvantage of leaving an undefined starting position. In the present work, we have shown that choosing the maximum die swell position as the starting position for the differential treatment is reasonable and that it connects the die flow to the spinline flow. This kind of treatment is termed as the "general differential treatment." In Figures (5-21) and (5-22), PP 106-4, for which k^2 and Co are $1/2$ and 0.1 (as shown in Chapter 4) is chosen as an example. Knowing the shear stress at the wall, the draw ratio at some specified spinline length, the velocity profile can

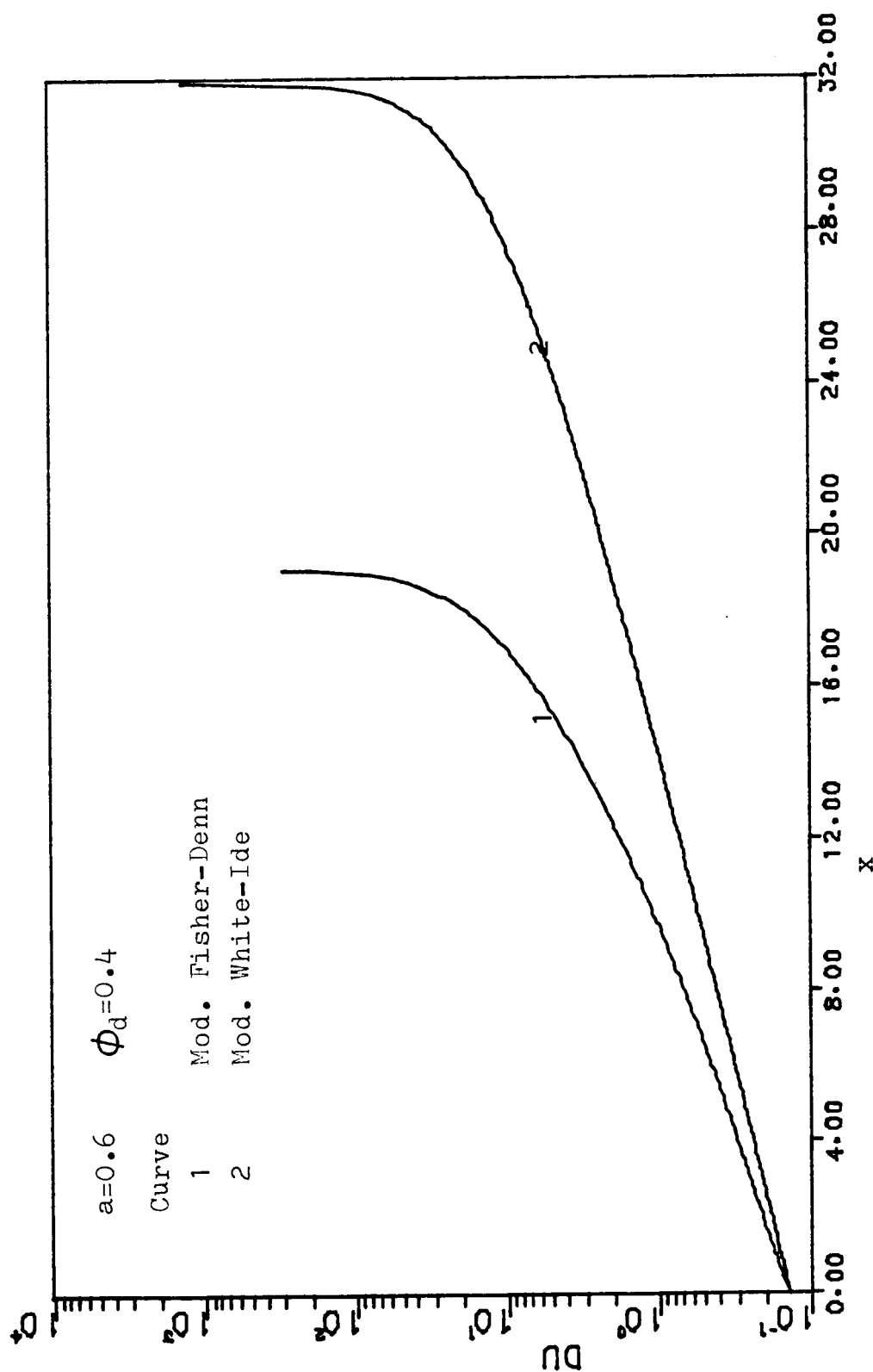


Figure 5-19. Velocity gradient profile for the modified White-Ide solution and modified Fisher-Denn solution ($a = 0.6$, $\phi_d = 0.4$).

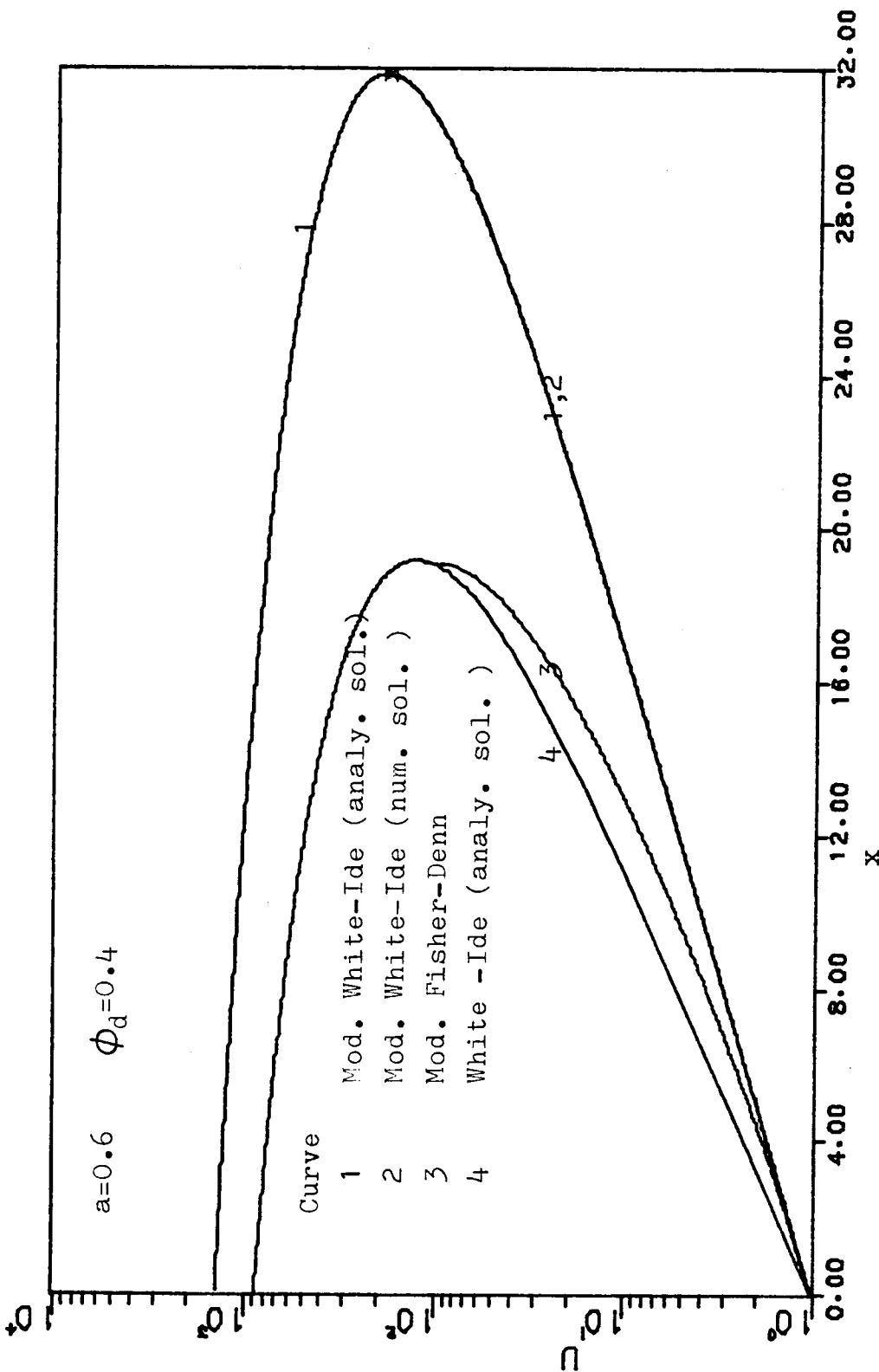


Figure 5-20. Comparison of numerical and analytical solutions ($a = 0.6$, $\phi_d = 0.4$).

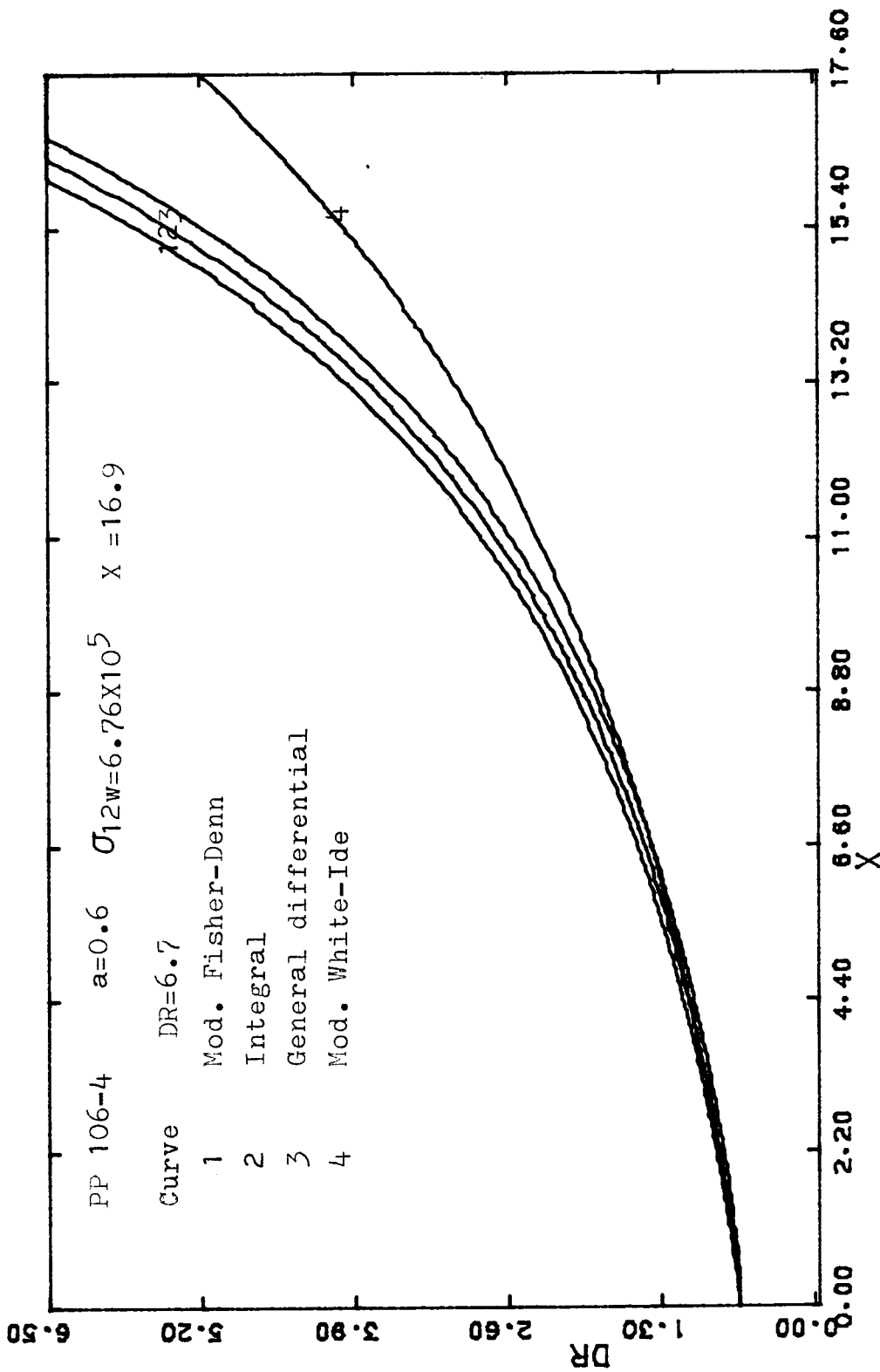


Figure 5-21. Velocity profile of PP 106-4 for various models ($DR = 6.78$, $X = 16.94$, $\sigma_{12w} = 6.76 \times 10^5$ dyne/cm²).

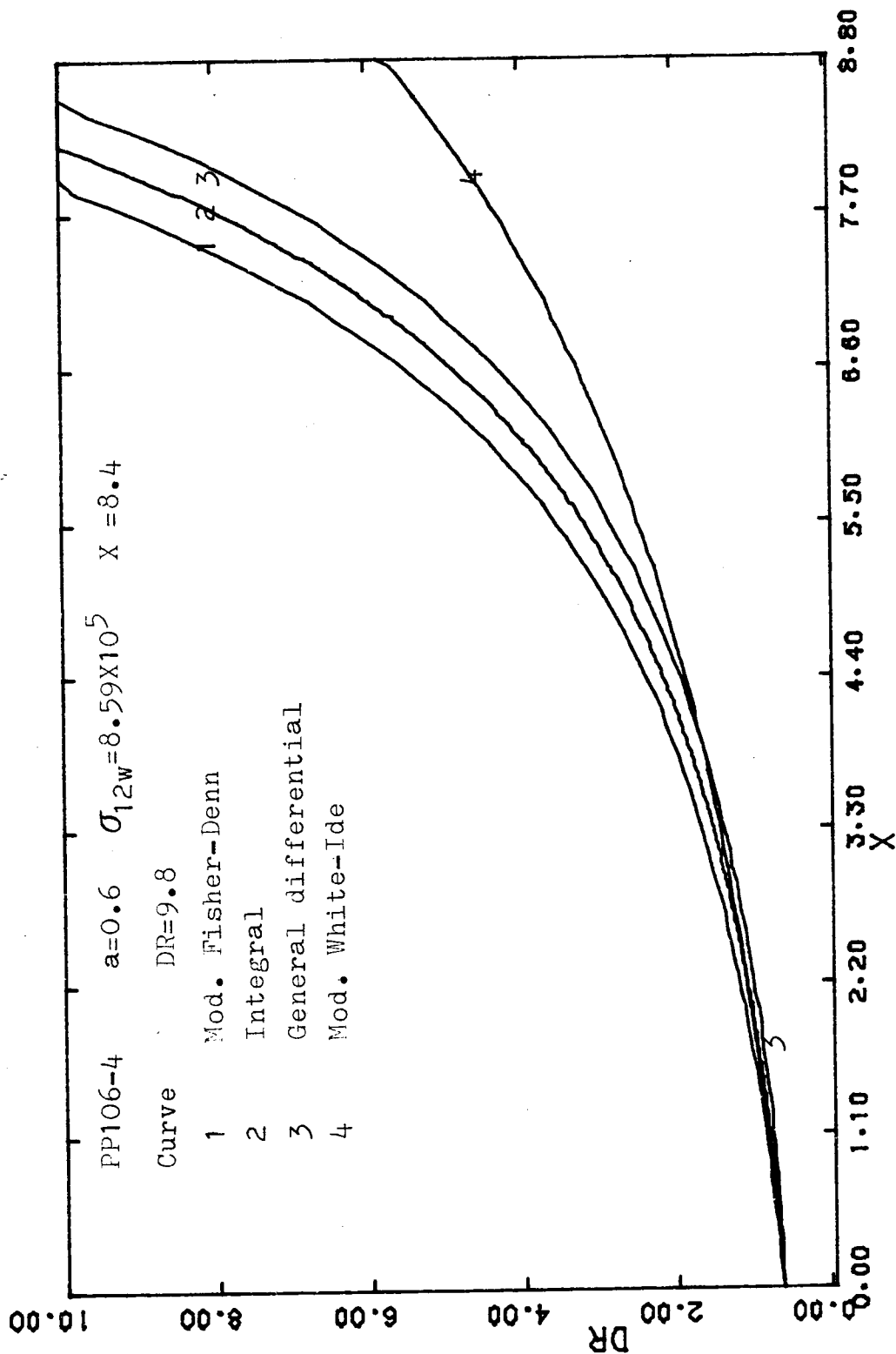


Figure 5-22. Velocity profile of PP 106-4 for various models (DR = 9.86, $X = 8.47$, $\sigma_{12w} = 8.59 \times 10^5$ dyne/cm²).

be obtained as shown in Curve 3 in these figures. The corresponding take-up force is then used in mod. W-I solution, mod. F-D solution and the integral treatment used by Nam (71). The velocity profile of the mod. W-I solution and the mod. F-D solution are transferred to the dimensionless velocity based on the mean die flow velocity by using Eqs. (5.22) and (5.23) to obtain the same initial velocity. Thus all the curves in Figure (5-21) have the same initial position, initial velocity and the same ϕ_d (or ϕ_I according to Eq. (5.5)). Clearly, it is shown that the assumption used by Zeichner (72) and White-Ide (74) gives the lowest velocity and that the less restrictive assumption used by Denn et al. (75) and Fisher and Denn (78) gives the highest velocity profile. The integral treatment and the general differential treatment give a velocity profile between the above two methods. In other words, to obtain DR at a specified X , a higher force is required for the mod. W-I method, and a lower force is required for the mod. F-D method compared to the general differential treatment as used in the present work.

Another way to compare the results obtained by these four different methods is shown in Figures (5-23) to (5-25) for different values of ϕ_I . These figures show that the "blow-up" point occurs at higher value of X if one makes the assumption used by Zeichner and White and Ide, and occurs at lower X value if the more rigorous analysis of Denn et al. and Fisher and Denn is used. The velocity increases less rapidly in the near die region but more rapidly than mod. W-I solution as X proceeds as shown in Figure (5-26).

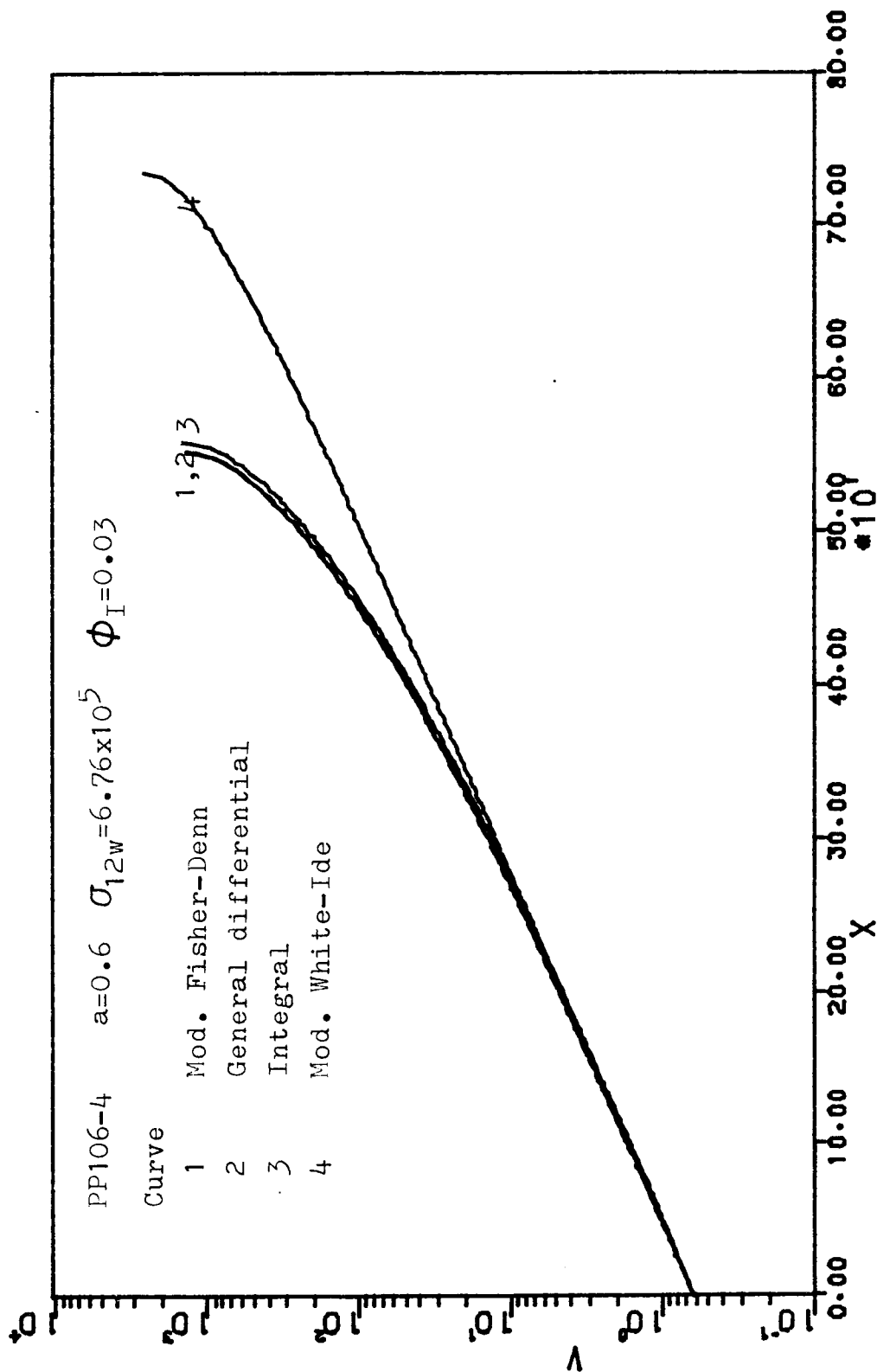


Figure 5-23. Velocity profile of PP 106-4 for various models ($\phi_I = 0.03$, $\sigma_{12w} = 6.76 \times 10^5$ dyne/cm²).

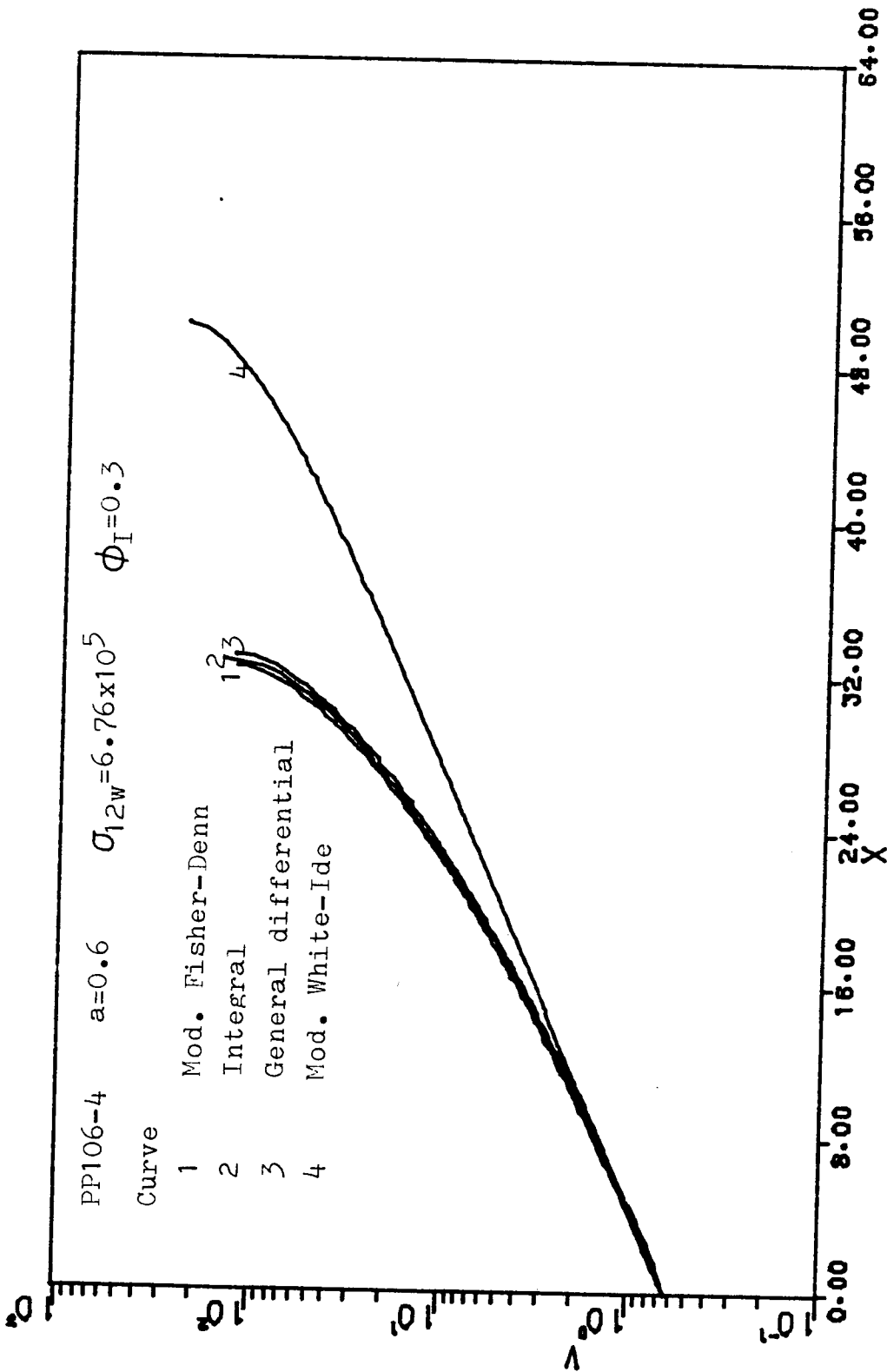


Figure 5-24. Velocity profile of PP 106-4 for various models ($\phi_I = 0.3$, $\sigma_{12w} = 6.76 \times 10^5$ dyne/cm²).

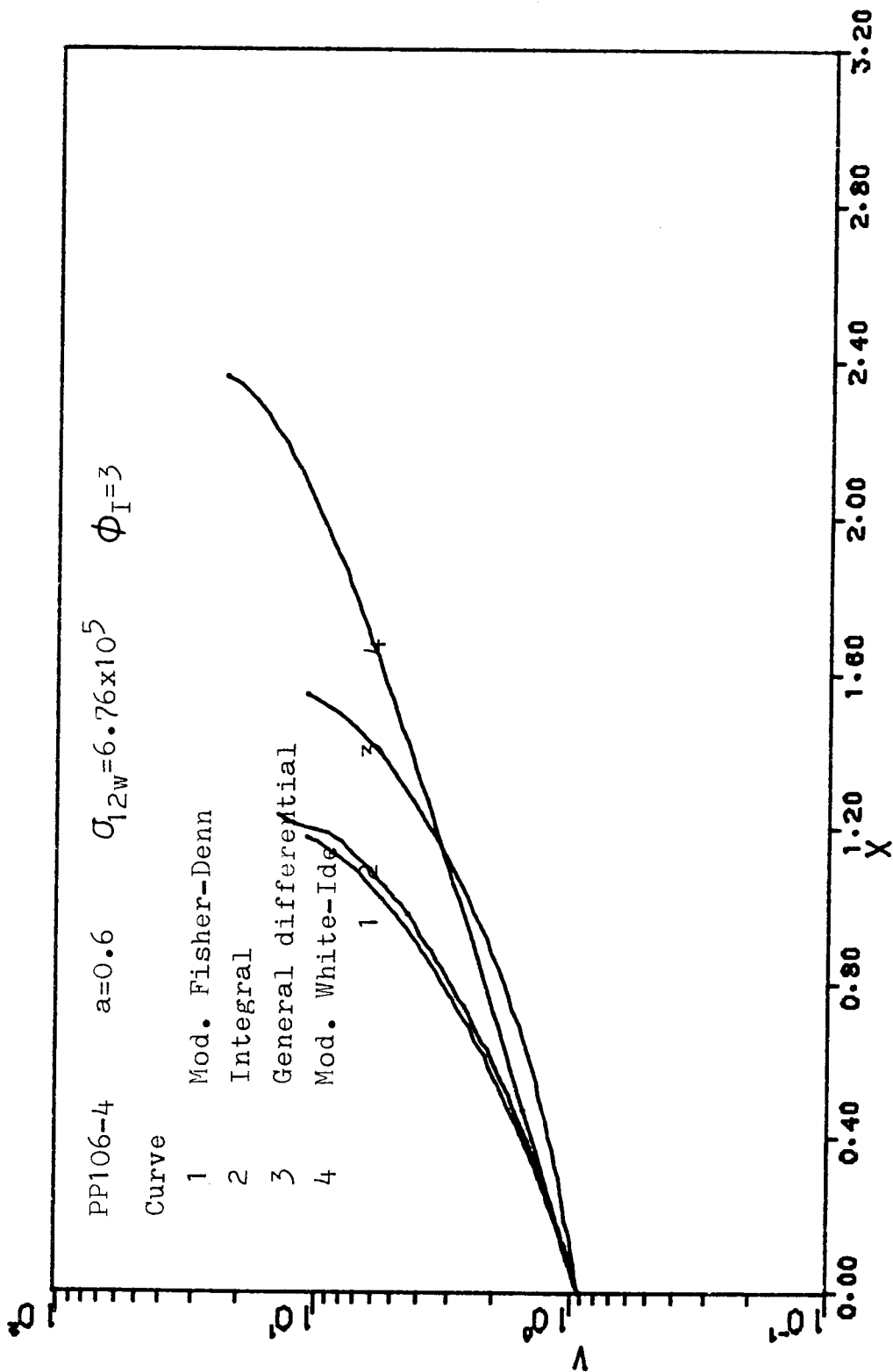


Figure 5-25. Velocity profile of PP 106-4 for various models ($\phi_I = 3$, $\sigma_{12w} = 6.76 \times 10^5$ dyne/cm²).

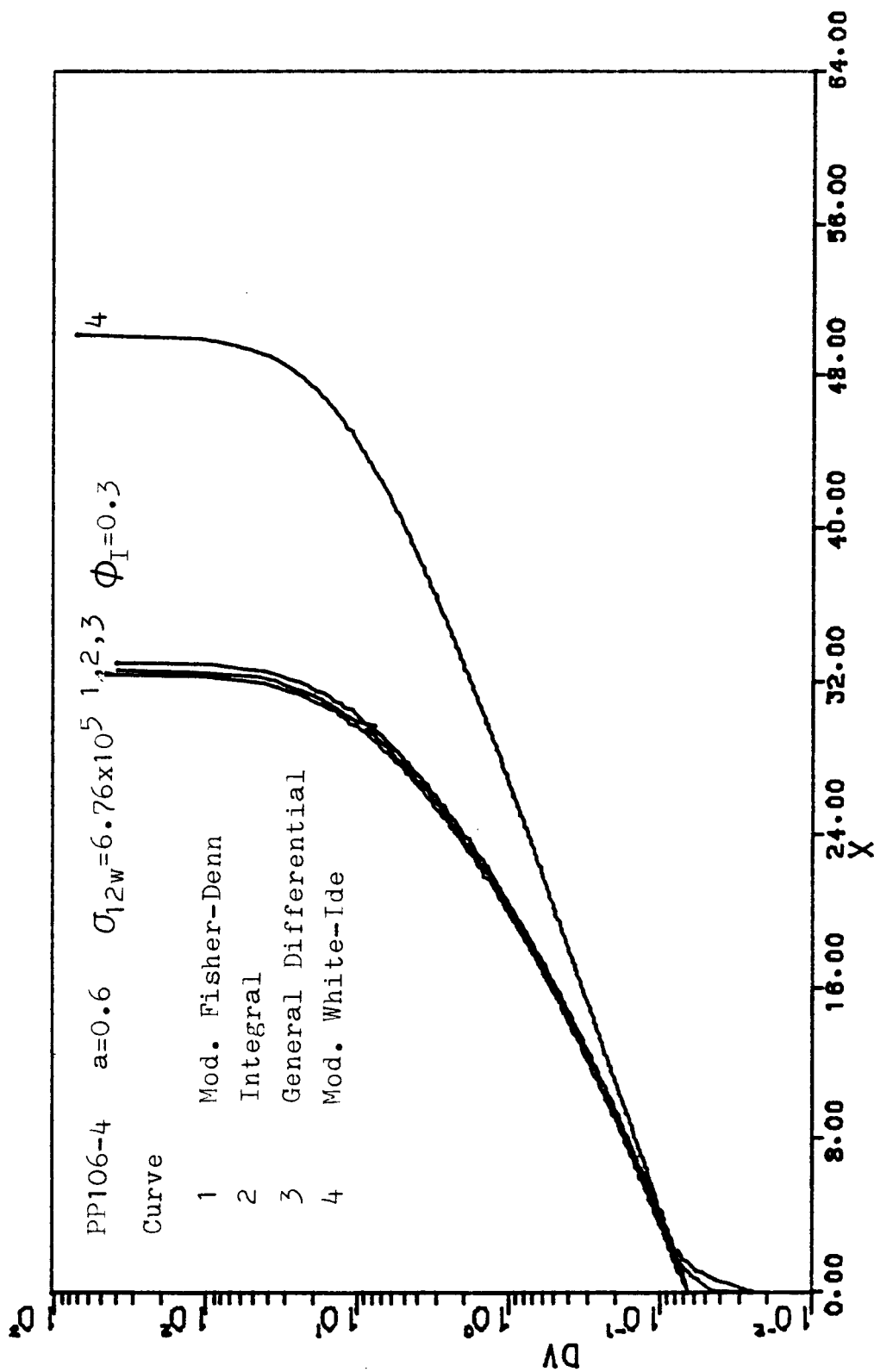


Figure 5-26. Velocity gradient profile of PP 106-4 for various models ($\phi_I = 0.3$, $\sigma_{12w} = 6.76 \times 10^5$ dyne/cm²).

One curious result is that the general treatment used in the present work and the integral treatment used by the previous authors (27, 71) do not give exactly the same result. As shown in Figures (5-21) to (5-25), except at a low take-up force (Figure (5-23)), the integral method always give a little higher value of the velocity than that of the general differential treatment. We have changed the kinematics (see Appendix A) and used the Eqs. (5.1) to (5.8) to connect the die flow to the spinline flow in the integral method, since the Bogue-White model cannot predict the die swell behavior well as shown in Chapter 4. However, the numerical scheme used by the previous authors (71) is retained. To further check this difference, the velocity profile (as shown in Figure (5-27) obtained for different values of "a" and $\phi_I = 0.1$) is introduced into the integral numerical scheme and the ϕ_I value is calculated at each spinline position. The results are shown in Figure (5-28). Clearly, ϕ_I is not a constant (equal to 0.1) along the spinline, as it should be, even when $a = 0$. Since when $a = 0$ the integral model and the White-Metzner model are the same, this discrepancy, especially far from the die exit, must be due to a difficulty in the numerical treatment in the integral method. The "dip" near the die exit is not clearly understood and is perhaps a difficulty in the hypothesis of the integration. The numerical scheme used in the present work is checked by CSMP method, and is compared with the analytical solution of Eq. (5.40) for the special case of the modified White-Ide solution, and is also checked with the stability analysis obtained by Fisher and Denn for $a = 0$. We therefore have considerable confidence in our numerical scheme. Since we are not

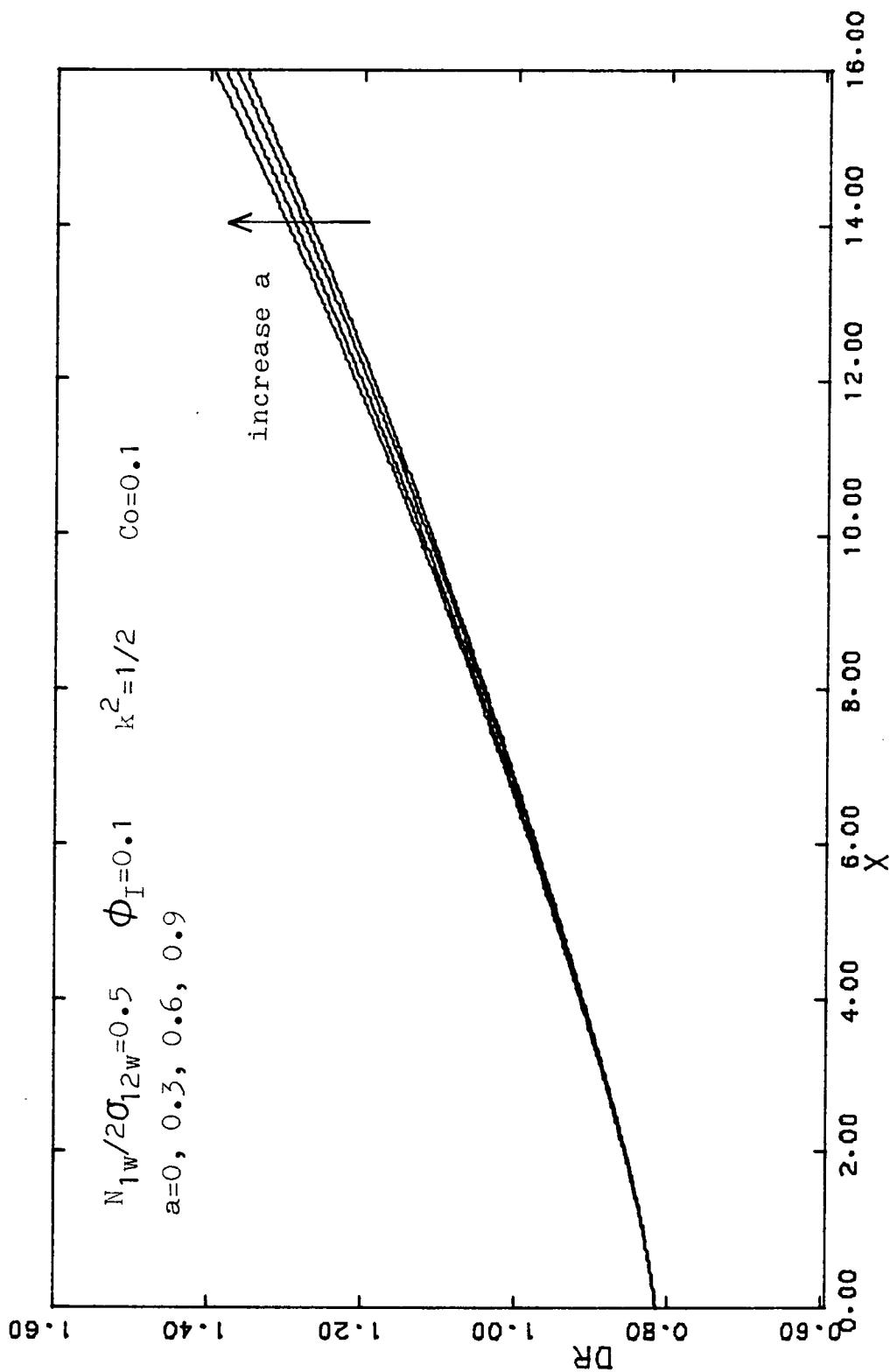


Figure 5-27. Velocity profile obtained from the general differential treatment, for $N_{1w}/2\sigma_{12w} = 0.5$, $\phi_I = 0.1$, and various values of "a."

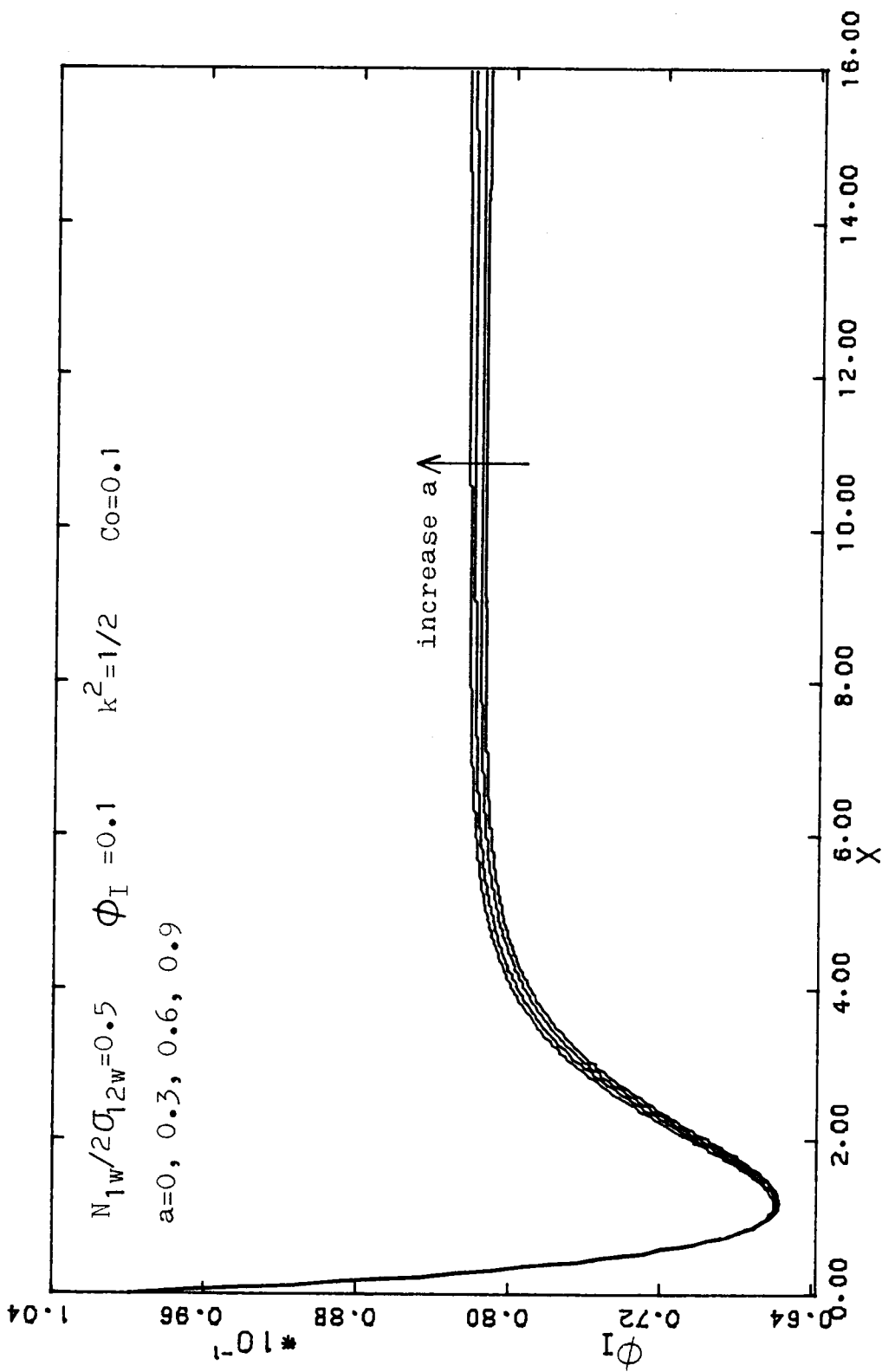


Figure 5-28. Values of ϕ_I along the spinline obtained from the integral treatment for the velocity profile of Figure 5-27.

interested in using the integral method further, no additional research concerning this discrepancy was made in the present work.

B. COMPARISON WITH EXPERIMENTAL DATA

Experimentally, the diameter of the spinline can be read by using the digital readout of the Reticon Scan Camera. The position where the reading is taken can be adjusted by turning the wheel of the camera holder. The edge of the wheel is divided into 100 divisions, that is, each division equals 3.6 degrees. Move of one division of the wheel corresponds to changing the vertical distance 0.024 mm. The capillary die used in this experiment has the dimensions $D = 0.041$ in., $(L/D) = 30$, and has a long tapered entrance. It is the same as that used to study the die swell in Chapter 4. The long tapered entrance is required to provide the necessary room for inserting the lense of the scan camera under the barrel of the Instron Capillary Rheometer so that the image of the spinline can be viewed starting from the die exit. The total length of the spinline profile which can be studied by adjusting the vertical distance is about 13 mm, for the particular die and isothermal chamber used by Yamane (114) and Nam (7). The material studied is PP 106-4, for which k^2 and Co values are 1/2 and 0.1, respectively.

Because of the transparency of the amorphous polymer melt, there are two peaks of the image as seen on the oscilloscope. The digital readout can only give the width value of the first peak. Some modification of the electronic circuit was made, as suggested by Dr. K. Koyama of Yamagata university, Japan. Using his modification the

Reticon Scan Camera ignores the lighted portion between these two peaks and the digital readout shows the diameter of the spinline. However, the small change of the diameter will need a highly sensitive recorder, which was not available, the diameter was read directly from the digital readout. Thus, measuring the diameter profile is time consuming work; one changes the vertical position of the scan camera, adjusts the focus, and writes down the random fluctuating values of diameter, and so on. But using the scan camera provides an accurate on-line technique.

Figures (5-29) to (5-32) show the experimental velocity profiles (indicated by the solid circles) for plunger speeds ranging from 0.1 in./min to 1 in./min and a spinline length of 3 cm. The X value is calculated from the mean velocity in the die and $\tau_o = 0.1$. The shear stress is obtained from the force recorder of the Instron Capillary Rheometer without correction for the entrance effect (since $(L/D) = 30$ which is large enough to ignore the entrance effect, as shown by (1, 18, 71)). The curves in Figures (5-29) to (5-32) are obtained theoretically using several different methods. As mentioned before, each curve in each figure is calculated from the same take-up force ϕ_I obtained from the general differential treatment for that specified DR and X values. The experimental results show that there is a delayed die swell region within 1.5 mm of the die exit, that is about $1.5D$. This value is almost the same as (although slightly higher than) that obtained by Crochet, Keunings and Denn (13) using the finite-element method. They showed that the position of the maximum die swell is located within $1D$. Because of the delayed die

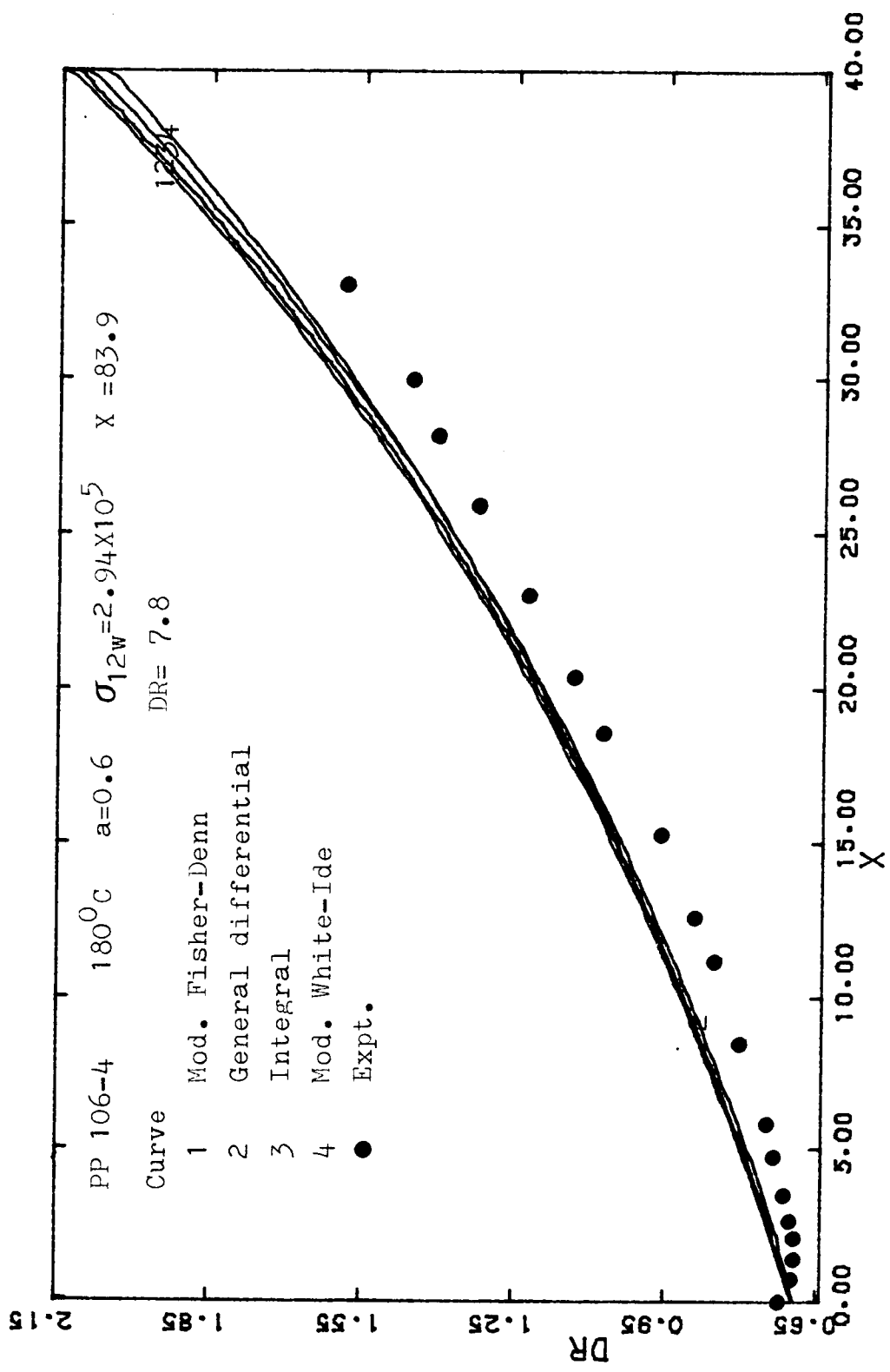


Figure 5-29. Comparison of theoretical and experimental velocity profiles ($V_p = 0.1$ in./min, $L = 3$ cm, $DR = 7.86$).

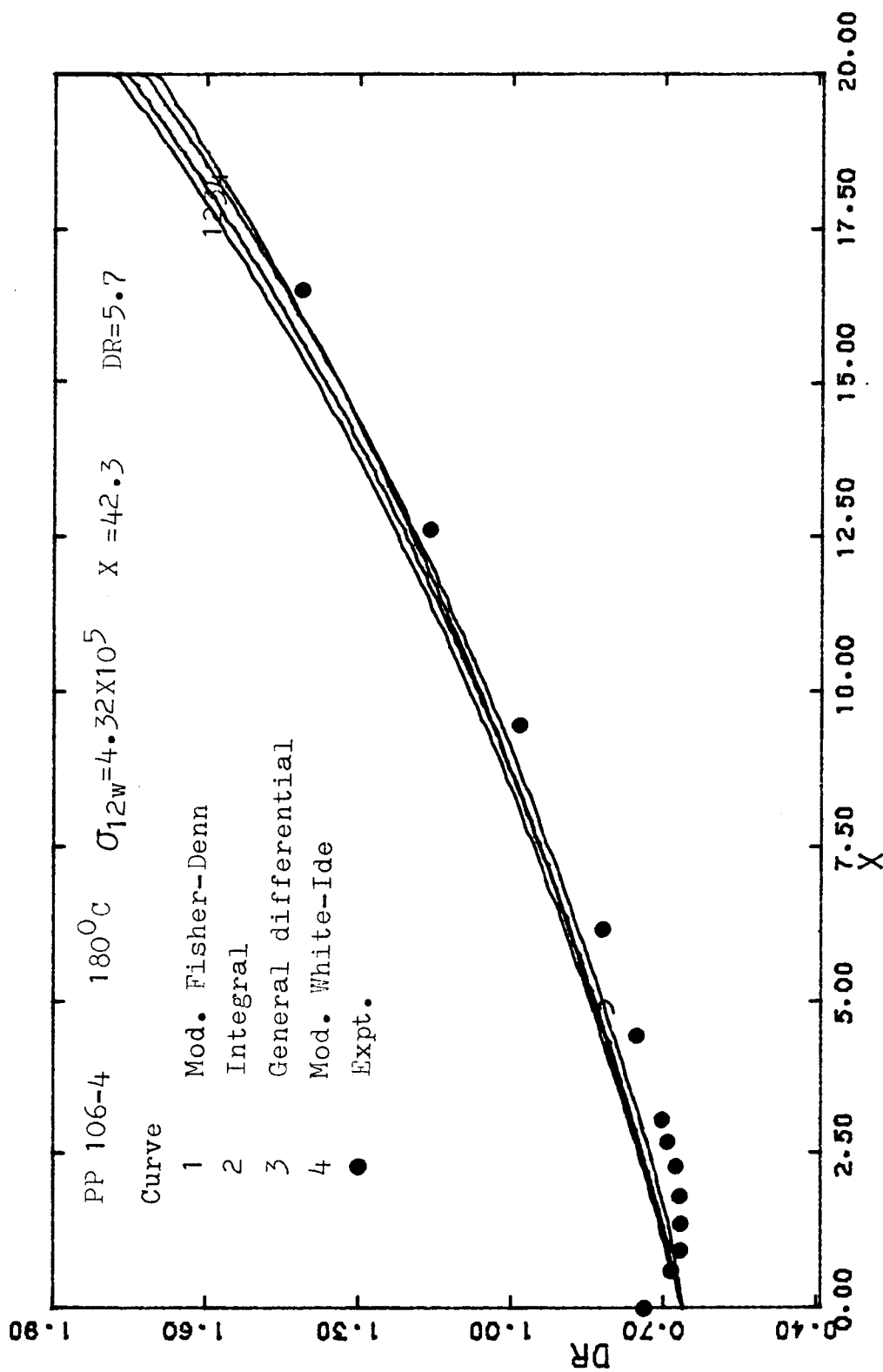


Figure 5-30. Comparison of theoretical and experimental velocity profiles ($V_p = 0.2$ in./min, $L = 3$ cm, $DR = 5.73$).

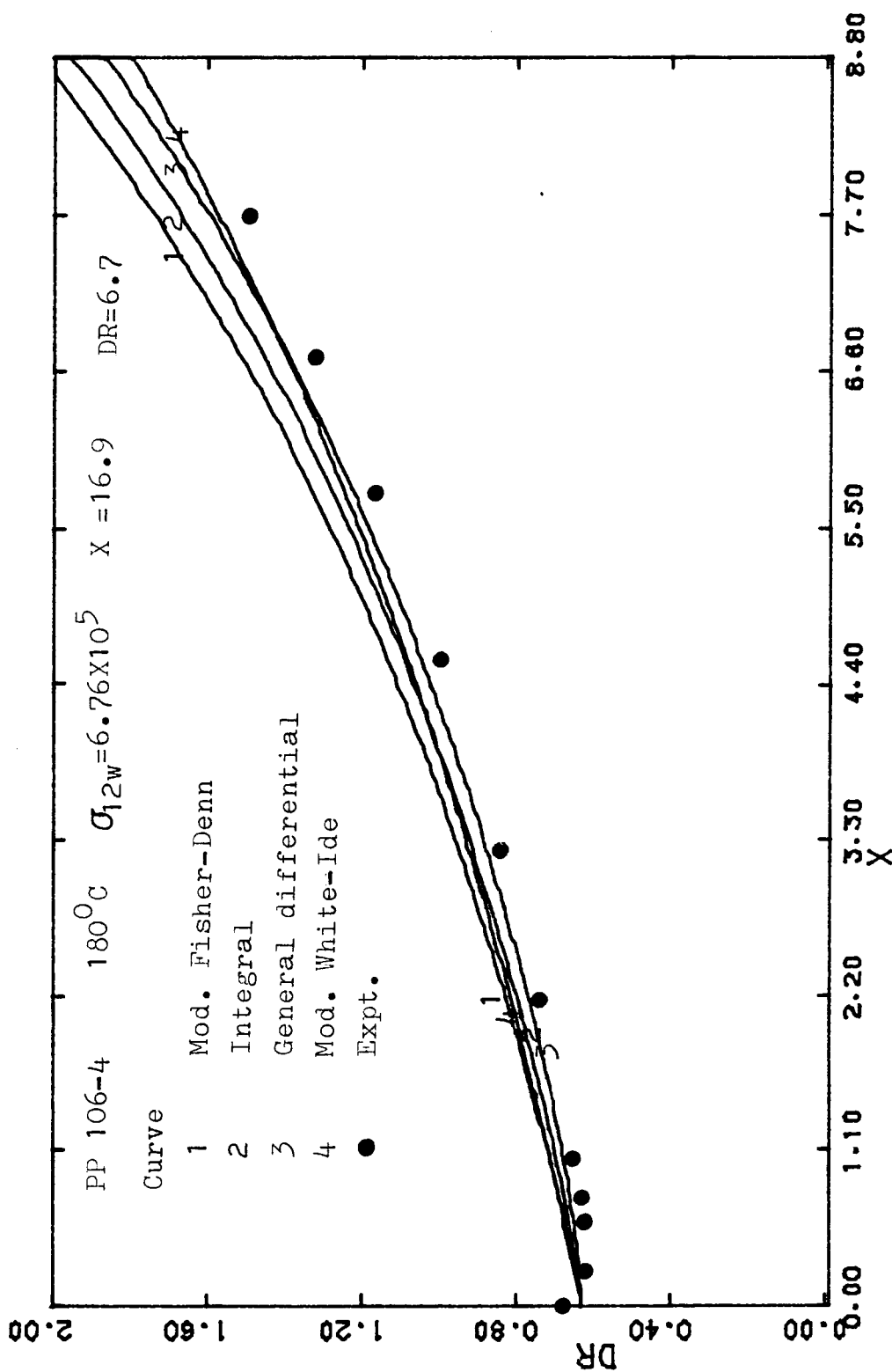


Figure 5-31. Comparison of theoretical and experimental velocity profiles ($V_p = 0.5$ in./min, $L = 3$ cm, $DR = 6.78$).

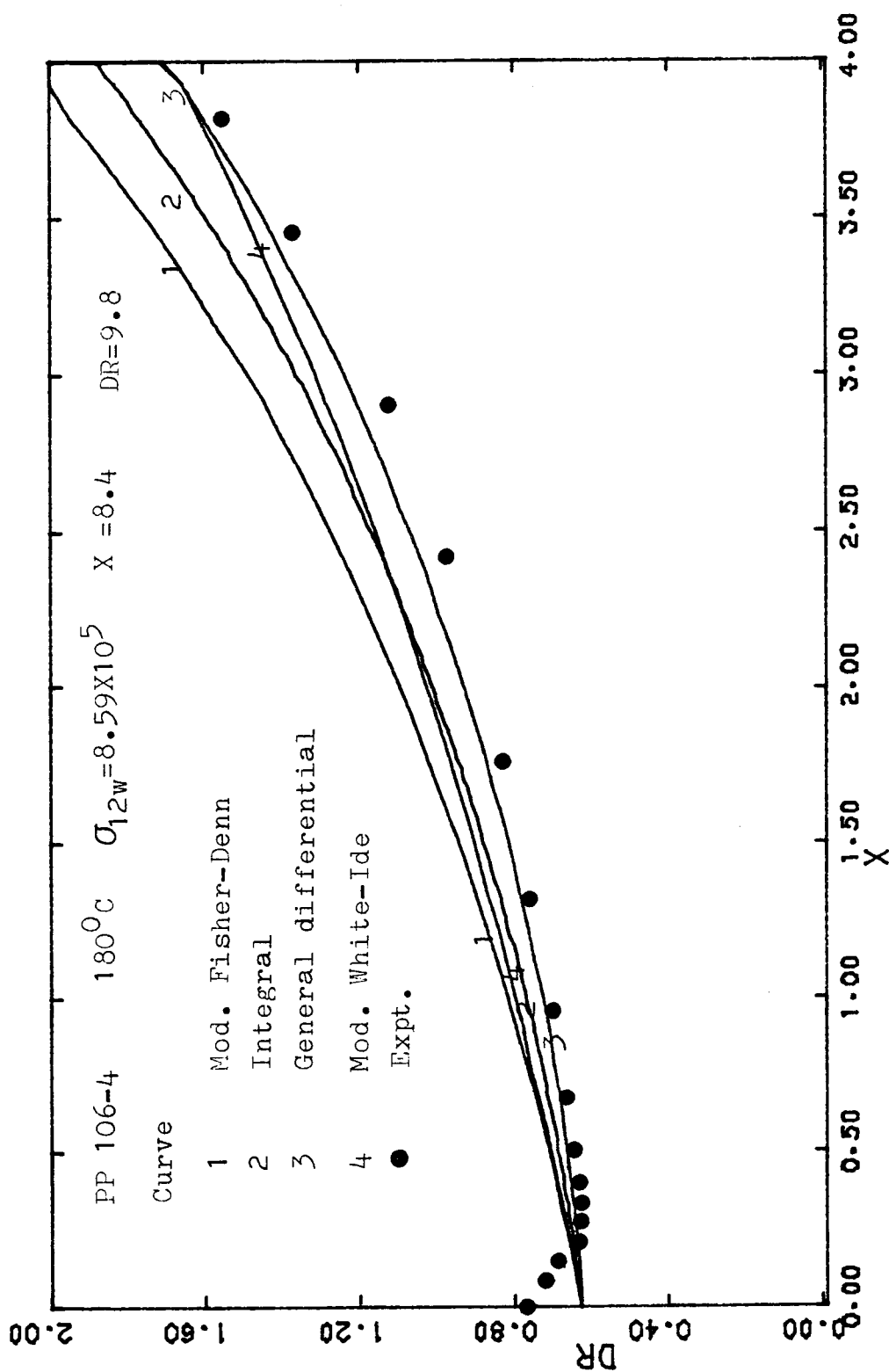


Figure 5-32. Comparison of theoretical and experimental velocity profiles ($V_p = 1$ in./min, $L = 3$ cm, $DR = 9.8$).

swell, the experimental results become lower than that predicted by any of the theories.

The theoretical curves in Figures (5-29) to (5-32) deal with the kinematical assumption of sudden expansion at the die exit as shown in the previous chapter. Thus, no delayed die swell can be predicted. In order to compare the experimental results, it is better to move the origin of the experimental data to the left until the position of the maximum die swell coincides with the origin of the theoretical curves. In this manner, one should see that the general differential treatment and the integral method would give a better prediction of the velocity profile along the spinline.

It should be mentioned again that each theoretical curve in Figure (5-29) to (5-32) has the same values. That is, except for the general differential treatment, the other methods may not give the same DR at the specified X, as shown in Figures (5-21) to (5-22) (page 122-123).

CHAPTER 6

COMBINED DIE FLOW AND SPINLINE FLOW STUDIES:

UNSTEADY SPINNING

As shown in Chapter 2 (Section B), an isothermal stability analysis of the spinline has been done for both purely viscous fluids (90, 91, 92, 93, 95, 99, 100) and viscoelastic fluids; these include the convected Maxwell model (72) with power law type deformation rate dependence of relaxation time (78). The results of nonlinear analyses show that the critical draw ratio for the growth of the disturbances is the same as that obtained by the linear theory and that the period of the finite-amplitude limit cycle is only slightly larger than that given by the linear theory. Thus, if one is not interested in the amplitude and wave form of the draw resonance, the linear stability analysis provides the desired information about the critical draw ratio and the resonance period at that draw ratio. However, none of the above analyses has been related to the die flow effect. The experimental results done by Nam (7), and some supplemental data shown in Chapter 4, do show that there is a die flow effect on the value of the critical draw ratio. The main purpose of the present work is to interpret this phenomenon by making a linear stability analysis which combines the die flow and the spinline flow together.

The White-Metzner model (with Bogue-White type deformation rate dependence of relaxation time (that is, Eqs. (2.33) to (2.37)) is used in the present work. Although we can deal with a multielement Maxwell model, only one Maxwell element is considered in the present work.

Using a two Maxwell elements, Chang and Denn (101) showed that the stability behavior is the same as for a single element, as long as the highest relaxation time is used. The Bogue-White type deformation rate dependence of relaxation time allows a zero shear viscosity and both unbounded and bounded stress growth. This nonlinear parameter "a" has proved to be useful in interpreting the failure mechanisms in elongated fibers (40, 41, 42, 51, 73, 74).

As shown in Chapter 5, the initial stresses are calculated from die flow and thus, no assumption, as was used by previous authors, is needed in the present work. In the first part of this chapter, a linear stability analysis will be made for different values of the thinning parameter "a" under three different $N_{1w}/2\sigma_{12w}$ (or the so-called stress ratio (SR)) values. The k^2 and Co in Eq. (5.2) are chosen as 1/2 and 0.13, respectively. The choice of these values is based on the finite-element results done by Crochet and Keunings (34). They showed that these values provide a good prediction of die swell value up to $N_{1w}/2\sigma_{12w} = 3$, as compared to the finite-element results for convected single element Maxwell fluid. In the second part of this chapter, the experimental results done by Nam (71) and in the present work will be compared with the theory.

A. THEORETICAL ANALYSIS

The linear stability analysis begins by introducing the perturbation variables as follows:

$$\begin{cases}
 \phi = A(x, \theta) - A_s(x) \\
 \gamma = F1(x, \theta) - F1_s(x) \\
 \delta = F2(x, \theta) - F2_s(x) \\
 \psi = U(x, \theta) - U_s(x)
 \end{cases} \quad (6.1)$$

and substituting Eq. (6.1) into the governing equations (5.16) to (5.19). After linearization one obtains

$$\begin{aligned}
 \gamma + \sqrt{3}aF1_s \frac{\partial \psi}{\partial x} + \sqrt{3}a \gamma \frac{dU_s}{dx} + \frac{\partial \gamma}{\partial \theta} + U_s \frac{\partial \gamma}{\partial x} + \psi \frac{dF1_s}{dx} - 2F1_s \frac{\partial \psi}{\partial x} - 2\gamma \frac{dU_s}{dx} \\
 = \frac{\partial \psi}{\partial x} \frac{2}{\phi_d}
 \end{aligned} \quad (6.2)$$

$$\begin{aligned}
 \delta + \sqrt{3}aF2_s \frac{\partial \psi}{\partial x} + \sqrt{3}a \delta \frac{dU_s}{dx} + \frac{\partial \delta}{\partial \theta} + U_s \frac{\partial \delta}{\partial x} + \psi \frac{dF2_s}{dx} + F2_s \frac{\partial \psi}{\partial x} + \delta \frac{dU_s}{dx} \\
 = \frac{\partial \psi}{\partial x} \frac{-1}{\phi_d}
 \end{aligned} \quad (6.3)$$

$$\frac{\partial}{\partial x}(F1_s \phi) - \frac{\partial}{\partial x}(F2_s \phi) + \frac{\partial}{\partial x}(A_s \gamma) - \frac{\partial}{\partial x}(A_s \delta) = 0 \quad (6.4)$$

$$\frac{\partial \phi}{\partial \theta} + \frac{\partial}{\partial x}(U_s \phi) + \frac{\partial}{\partial x}(A_s \psi) = 0 \quad (6.5)$$

The subscript "s" indicates the steady state values obtainable by solving the steady state governing equations shown in Chapter 5. The above coupled partial differential equations, together with different kinds of disturbances for the perturbation variables at the

initial position or at the take-up position, can be solved by a finite-difference method, as has been discussed by several authors (90, 92, 93, 99, 100, 106, 109, 110, 111). Another method, that is, the separation variables method, which reduces the problem to one of solving the eigenvalues of a set of coupled ordinary differential equations was used by Zeichner (72), Fisher and Denn (78, 84) and Chang and Denn (101). Since a linear stability analysis can provide the desired information about the critical draw ratio and the resonance period at that draw ratio, no nonlinear stability analysis will be done in the present work. In addition, to avoid solving the coupled partial differential equations, the latter method is used.

Following Fisher and Denn (78), separation-of-variables solutions are sought in the form:

$$\phi = \sum_{n=1}^{\infty} e^{\lambda_n \theta} \phi_n(x) + \sum_{n=1}^{\infty} e^{\lambda_n^* \theta} \phi_n^*(x) \quad (6.6)$$

$$\gamma = \sum_{n=1}^{\infty} e^{\lambda_n \theta} \Gamma_n(x) + \sum_{n=1}^{\infty} e^{\lambda_n^* \theta} \Gamma_n^*(x) \quad (6.7)$$

$$\delta = \sum_{n=1}^{\infty} e^{\lambda_n \theta} \Delta_n(x) + \sum_{n=1}^{\infty} e^{\lambda_n^* \theta} \Delta_n^*(x) \quad (6.8)$$

$$\psi = \sum_{n=1}^{\infty} e^{\lambda_n \theta} \Psi_n(x) + \sum_{n=1}^{\infty} e^{\lambda_n^* \theta} \Psi_n^*(x) \quad (6.9)$$

and the boundary conditions are, at $x = 0$,

$$\phi(0, \theta) = \psi(0, \theta) = \gamma(0, \theta) = 0 \quad (6.10)$$

and, at $x = L/\tau_o U_o$

$$\psi(x, \theta) = 0 \quad (6.11)$$

In Eqs. (6.6) to (6.9), the superscript * denotes the complex conjugate of the complex quantity. Substituting Eqs. (6.6) to (6.9) into Eqs. (6.3) to (6.5) and defining

$$\Sigma_n = \Gamma_n - \Delta_n \quad (6.12)$$

leads to the eigenvalue problem.

$$\begin{aligned} \frac{d\psi_n}{dx} = & \frac{1}{(3F1_s - 2U_s - \sqrt{3}aU_s + 3/\phi_d)} [\lambda_n(\Sigma_n + U_s^2 \phi_n) + \Sigma_n \\ & + \frac{dU_s}{dx} (2\Sigma_n - 3\Gamma_n + \sqrt{3}a\Sigma_n)] \end{aligned} \quad (6.13)$$

$$\frac{d\phi_n}{dx} = \left(\frac{1}{U_s}\right) \left[\frac{\psi_n}{U_s^2} \frac{dU_s}{dx} - \frac{1}{U_s} \frac{d\psi_n}{dx} - \left(\lambda_n + \frac{dU_s}{dx}\right) \phi_n \right] \quad (6.14)$$

$$\begin{aligned} \frac{d\Gamma_n}{dx} = & \left(\frac{1}{U_s}\right) \left[\frac{d\psi_n}{dx} \left(\frac{2}{\phi_d} + 2F1_s - \sqrt{3}aF1_s \right) + \frac{dU_s}{dx} (2\Gamma_n - \sqrt{3}a\Gamma_n) \right. \\ & \left. - \psi_n \frac{dF1_s}{dx} - \Gamma_n - \lambda_n \Gamma_n \right] \end{aligned} \quad (6.15)$$

$$\begin{aligned} \frac{d\Sigma_n}{dx} = & \left(\frac{-1}{U_s}\right) \left[\Sigma_n - \frac{3}{\phi_d} \frac{d\psi_n}{dx} + \Sigma_n \left(\lambda_n + \sqrt{3}a \frac{dU_s}{dx} + \frac{dU_s}{dx} \right) + \frac{dU_s}{dx} \right. \\ & \left. \left(\psi_n - 3\Gamma_n \right) + \frac{d\psi_n}{dx} (U_s - 3F1_s + \sqrt{3}aU_s) \right] \end{aligned} \quad (6.16)$$

The boundary conditions are

$$\phi_n(0) = \psi_n(0) = \Gamma_n(0) = 0 \quad (6.17)$$

$$\psi_n\left(\frac{L}{\tau_o U_o}\right) = 0 \quad (6.18)$$

ϕ_n , ψ_n , Γ_n and Σ_n are complex functions, and the eigenvalue, λ_n , are, in general, complex numbers. Solutions of Eqs. (6.13) to (6.18) are obtained by a direct integration using a Runge-Kutta fifth or sixth order integration subroutine with automatic step size adjustment (University of Tennessee Computer Center library subroutine known as "DVERK,") normalizing $\Sigma_n(0)$ to unity and varying λ_n until the boundary condition Eq. (6.18) is satisfied. Since the system is unstable to infinitesimal disturbances if the real part of any λ_n is positive, the critical draw ratio occurs when the largest real part of the eigenvalue goes to zero. The computer program is given in Appendixes D and E. In Appendix D, the computer program is written for the specified draw ratio at some specified spinline length X , and the computer program shown in Appendix E is written to study the eigenvalues at each position x along the velocity profile for constant dimensionless take-up force ϕ_d or ϕ_I . Although both programs can deal with the instability analysis using the assumption of Denn and his coworkers concerning the initial condition (78, 84, 101), all the results shown in this chapter are based on the general differential treatment, except in the following paragraph.

To insure that the numerical scheme in the computer program is correct, the first and second eigenvalues shown by Fisher and Denn (78) for the Lodge fluids at $\alpha = 0.006$ were introduced into the computer program (Appendix D). Since different dimensionless groups are used, the eigenvalues obtained by Fisher and Denn should be multiplied by α and the initial dimensionless velocity and $F_1(0)$ should set to equal 1. That is,

$$\begin{cases} x = X = (1/\alpha) = (1/0.006) \\ B = 1 \\ F_1(0) = 1 \end{cases} \quad (6.19)$$

should be used in Appendix D. In this manner, we found that the desired boundary conditions shown in Eq. (6.18) is obtained for the thinning parameter "a" equals zero. Thus confidence in the numerical scheme of the computer program was established.

Using the dimensionless groups defined in the present work, that is, Eqs. (5.15) and (5.22) to (5.23), it is found easier to trace the change of the first eigenvalue along the velocity profile at different specified dimensionless take-up forces ϕ_d or ϕ_I . Figures (6-1) and (6-2) show the change of the real part and imaginary part of the first eigenvalue along the spinline for the typical case of $\phi_d = 0.17$, $a = 0$, and for different values of $N_{1w}/2\sigma_{12w}$ (0.5, 1, 5). The real part of the second eigenvalue is found to be negative. Since we are interested in finding the position where the real part of the first eigenvalue changes its sign, it is not necessary to find all eigenvalues. Thus it makes this method much simpler than the other

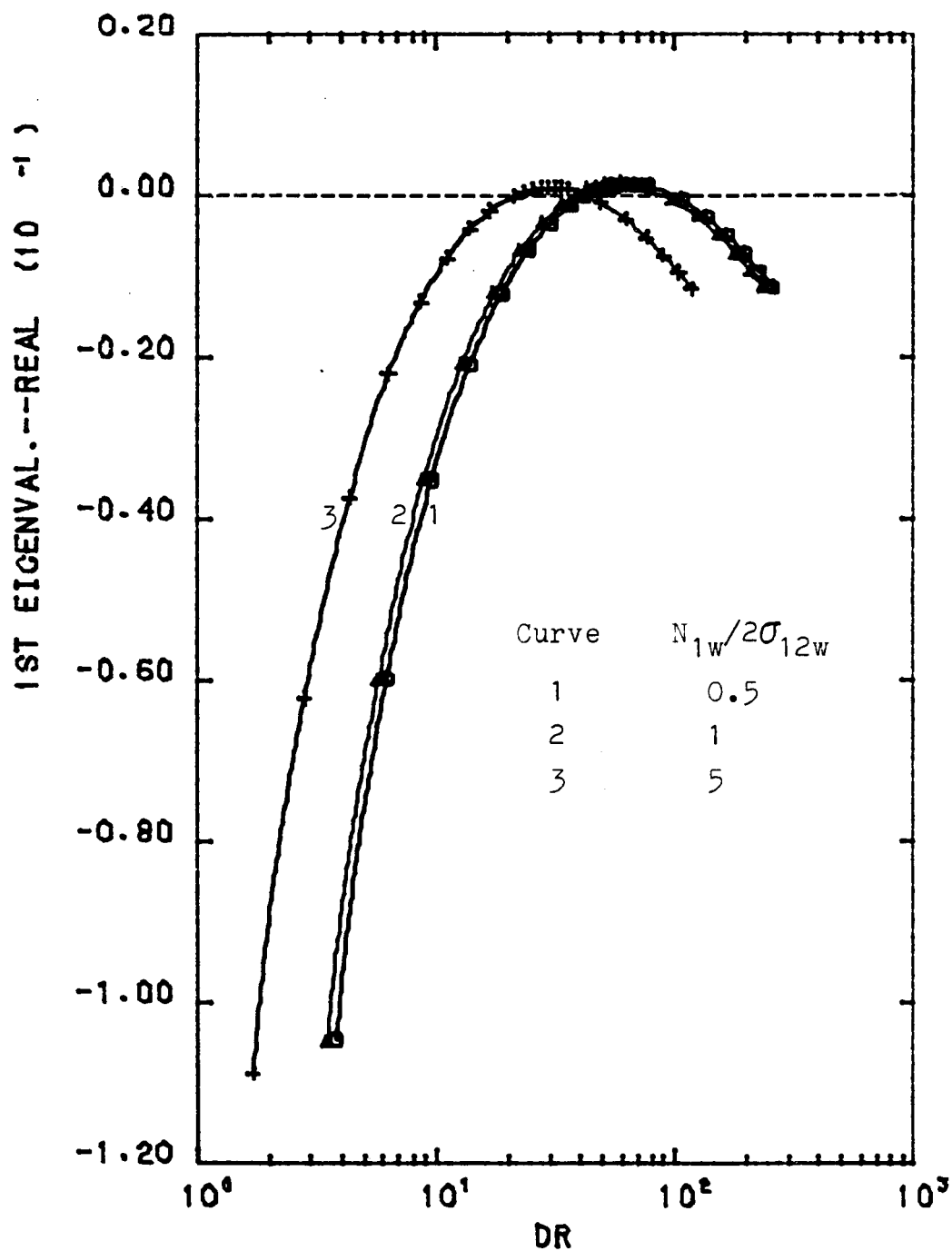


Figure 6-1. The real part of the first eigenvalue as a function of DR.

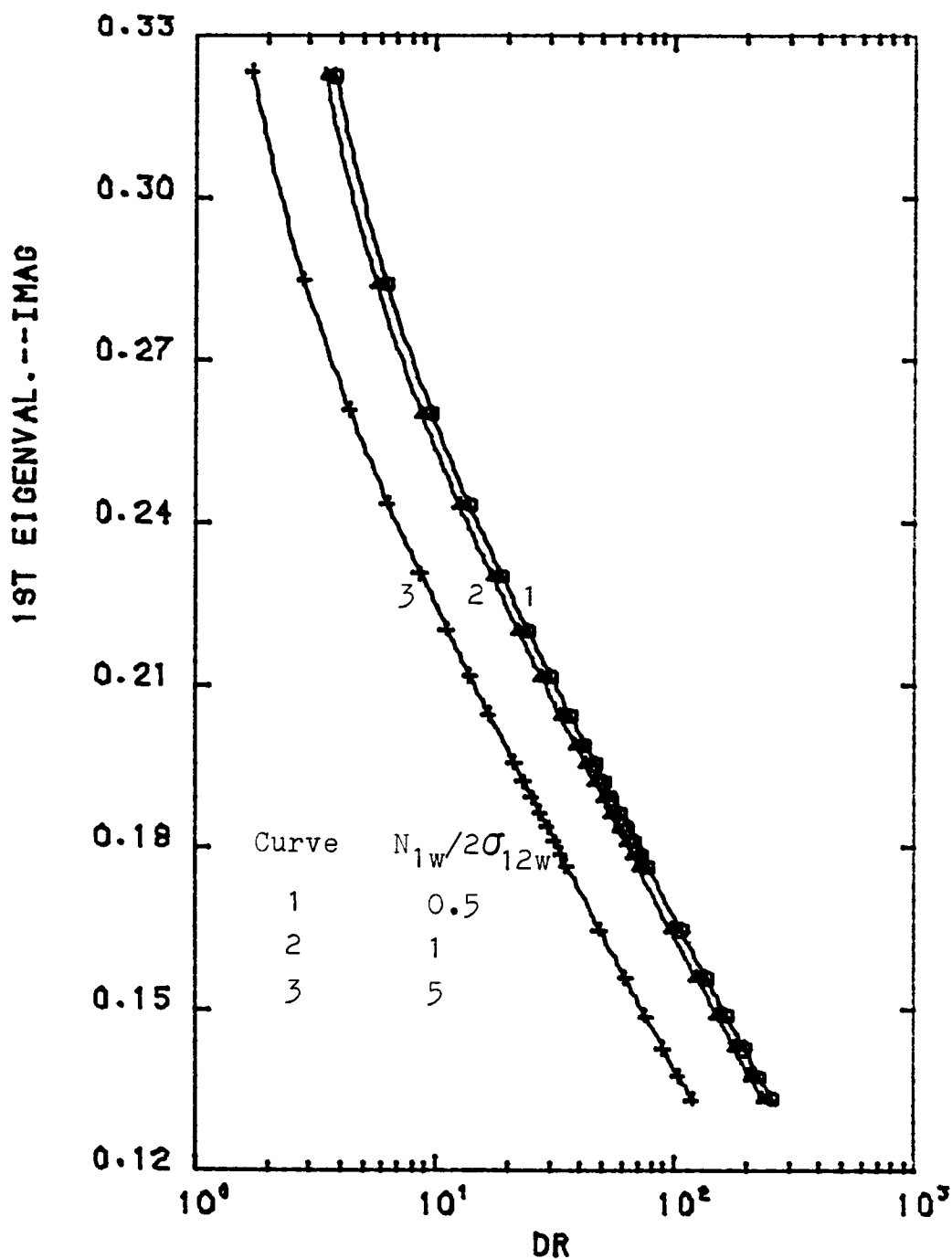


Figure 6-2. The imaginary part of the first eigenvalue as a function of DR.

methods. However, a great deal of computer work is needed, since two parameters (that is, $N_{1w}/2\sigma_{12w}$ and "a") are involved.

The important feature in Figure (6-1) is that there is a second stable region at sufficiently high draw ratios since the real part of the first eigenvalue goes from positive to negative again. The existence of the second stable region is found for all case in which $a < 1/\sqrt{3}$. These are indicated in Figures (6-3) to (6-6) for $a = 0, 0.2, 0.4, 1/\sqrt{3}$ and three different values of $N_{1w}/2\sigma_{12w}$. Note that all the dimensionless variables are based on die flow. These are

$$\begin{cases} \alpha = \tau_o \bar{V}/L \\ DR = V/\bar{V} \end{cases} \quad (6.20)$$

and DRC indicates the critical draw ratio where the real part of the first eigenvalue goes to zero.

The second stable region is no longer existed as $a > 1/\sqrt{3}$, as shown in Figures (6-6) to (6-8). As shown in Figure (6-6) for $a = 1/\sqrt{3}$, the critical draw ratio increased rapidly to infinity after some specified α values, depending on the die flow parameter $N_{1w}/2\sigma_{12w}$. When the material shows more thinning behavior, say $a > 1/\sqrt{3}$, the maximum draw ratio available dominates the critical draw ratio value, as shown in Figures (6-7) and (6-8) for $a = 0.6$ and 0.9 , respectively. The maximum draw ratio is the steady state value after which the velocity gradient approaches infinity, as has been shown in the previous chapter. The real part of the first eigenvalue at the maximum draw ratio available changes from positive to negative, as shown in Figure (6-9) for $a = 0.9$. This means that we have three

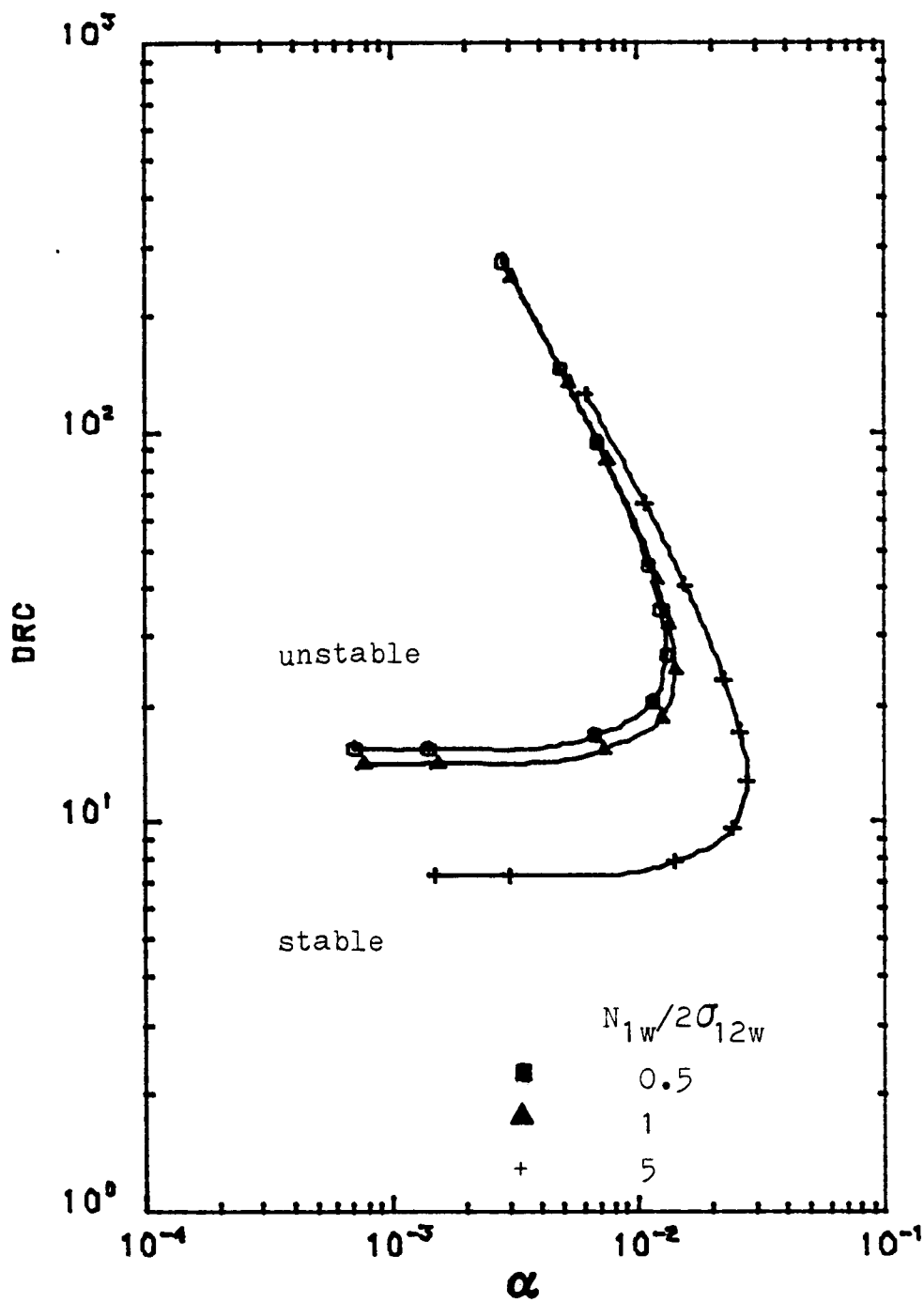


Figure 6-3. Critical draw ratio as a function of α for $a = 0$.

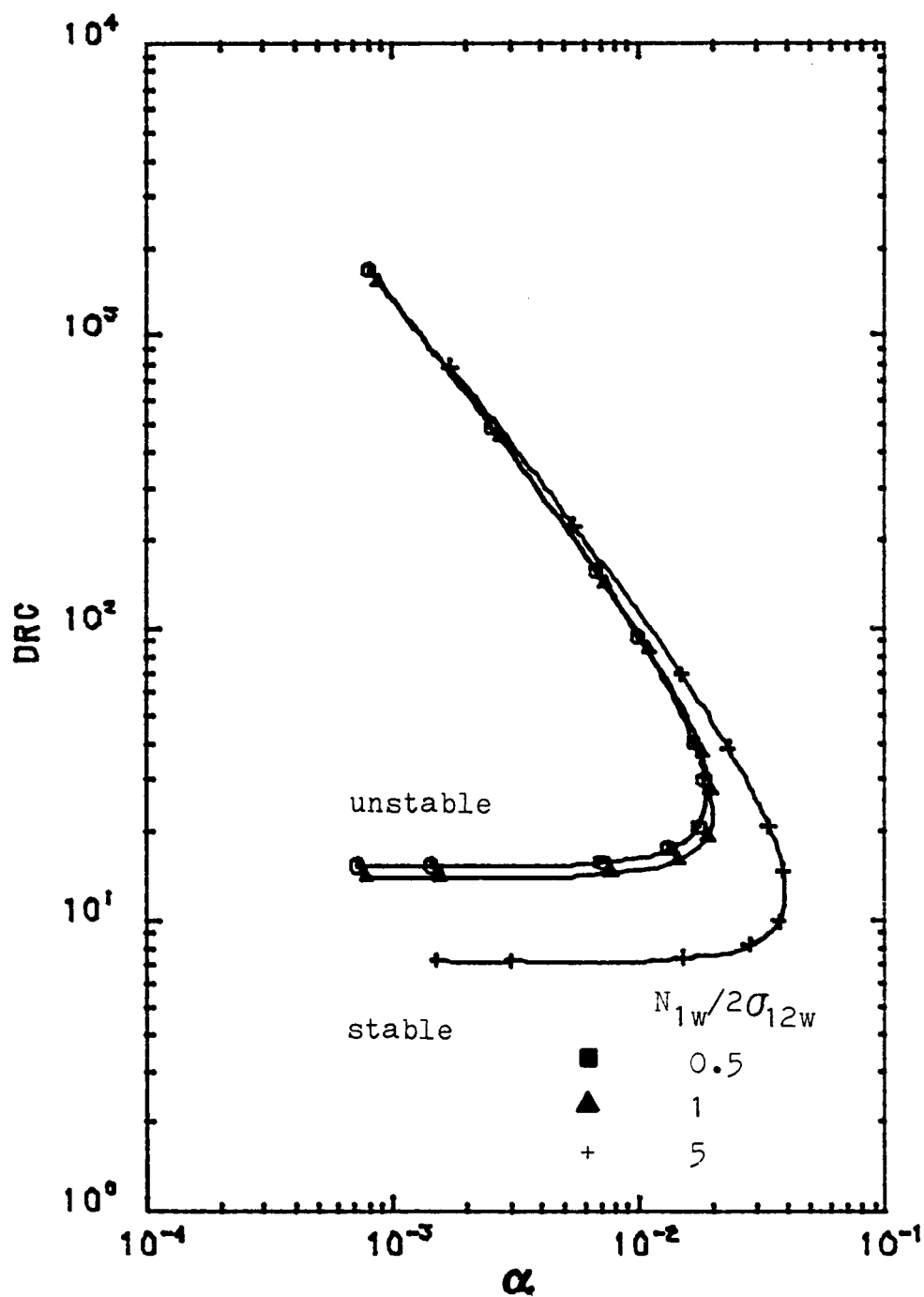


Figure 6-4. Critical draw ratio as a function of α for $a = 0.2$.

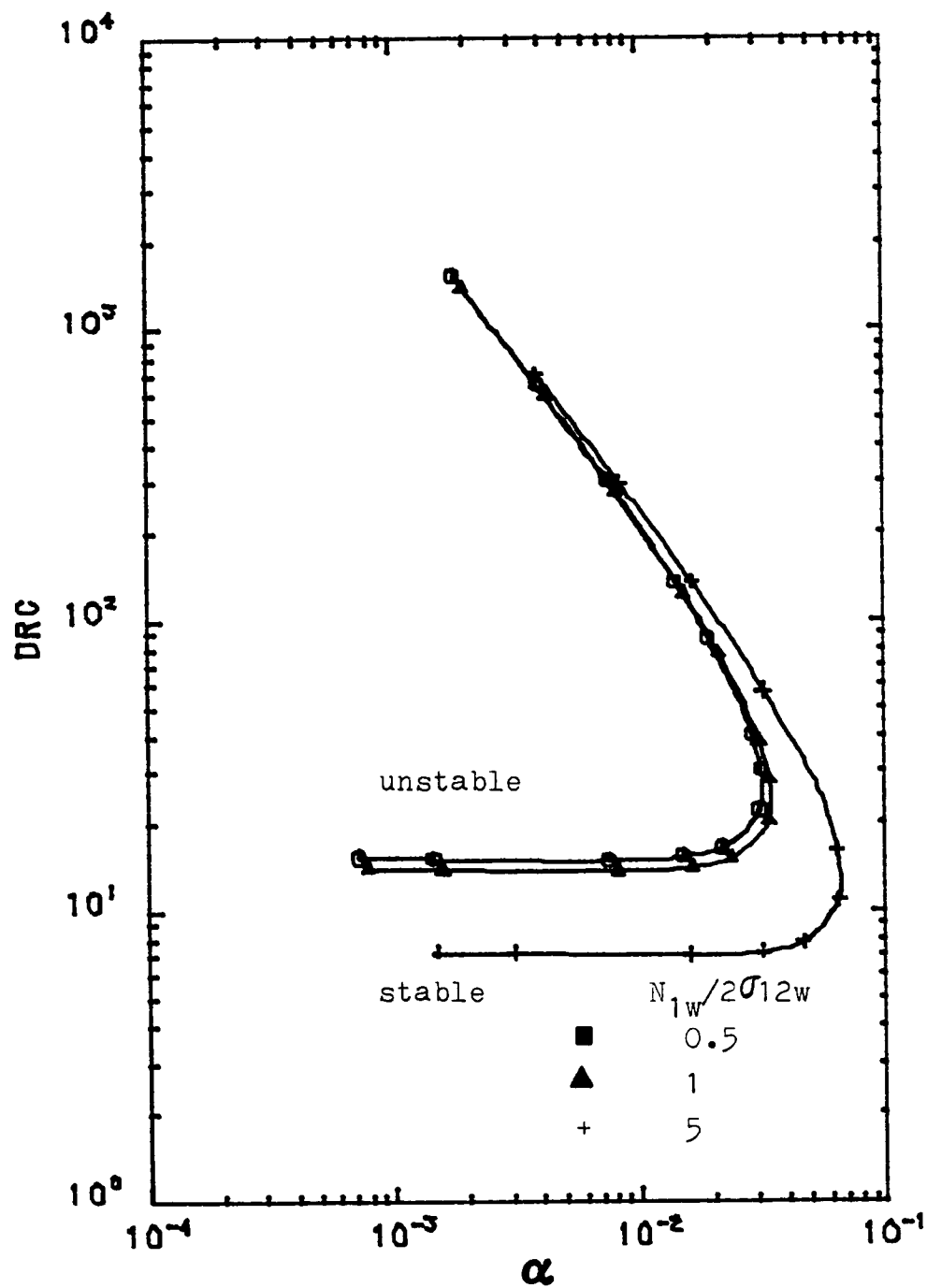


Figure 6-5. Critical draw ratio as a function of α for $a = 0.4$.

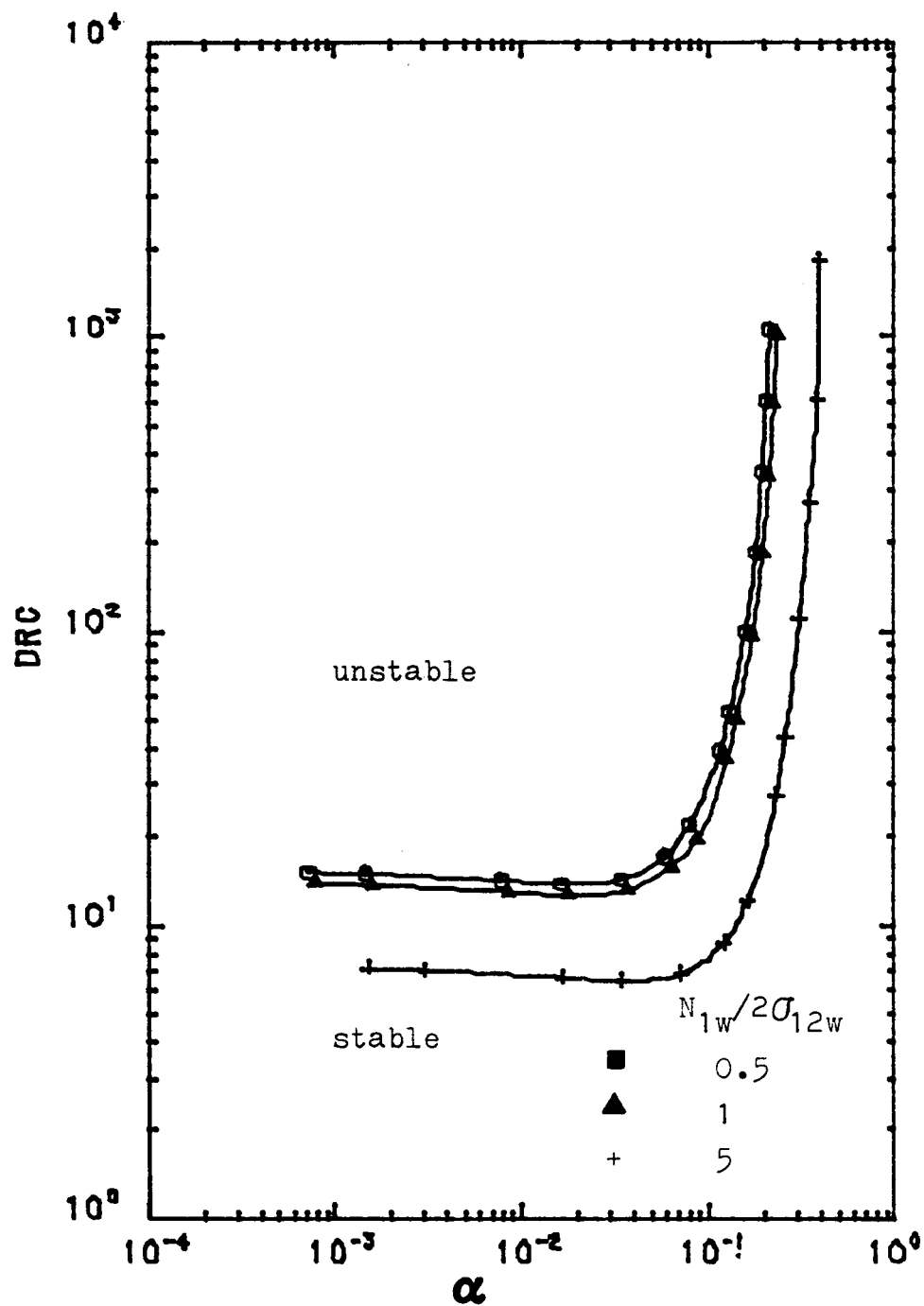


Figure 6-6. Critical draw ratio as a function of α for $a = 1/\sqrt{3}$.

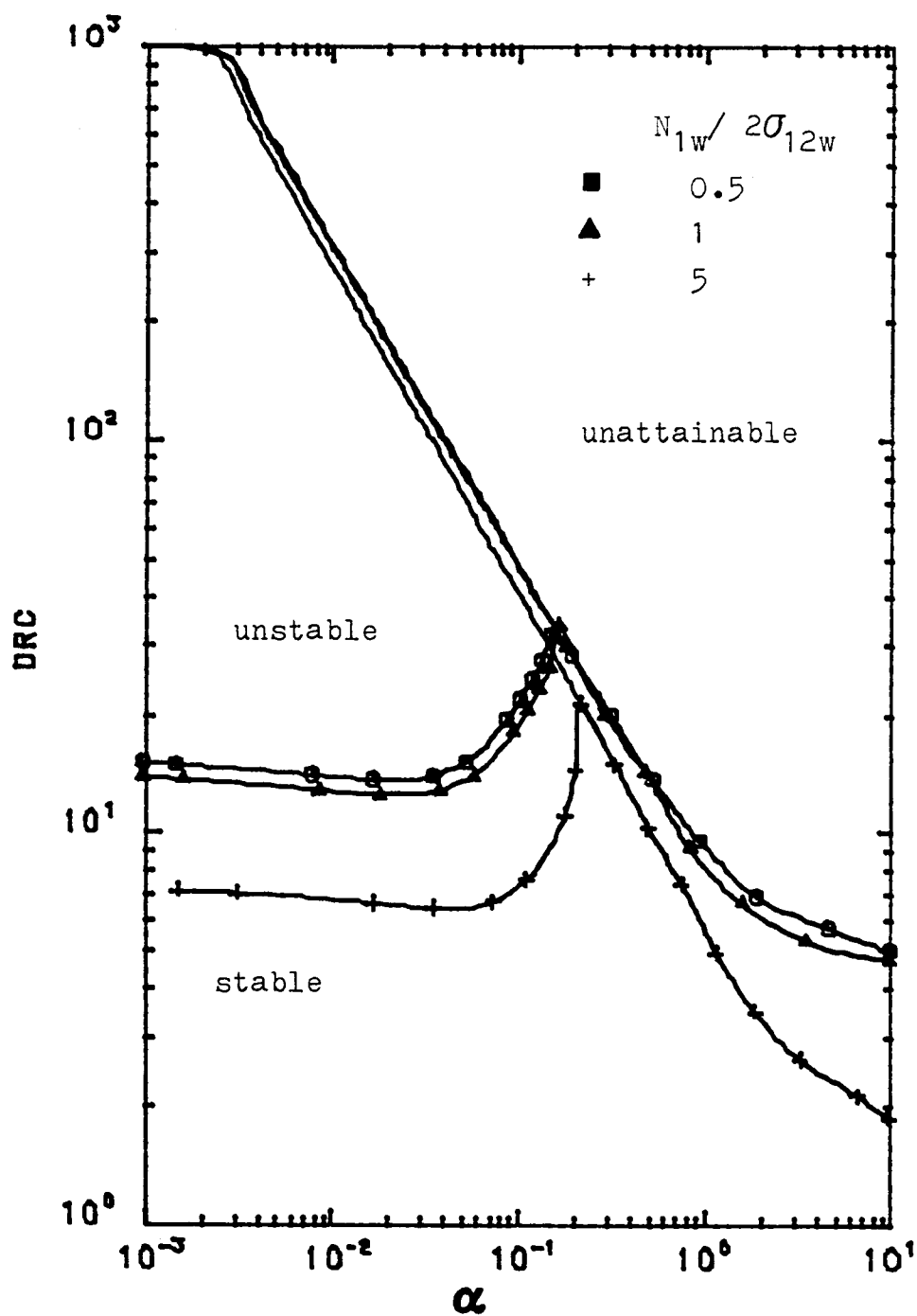


Figure 6-7. Critical draw ratio as a function of α for $a = 0.6$.

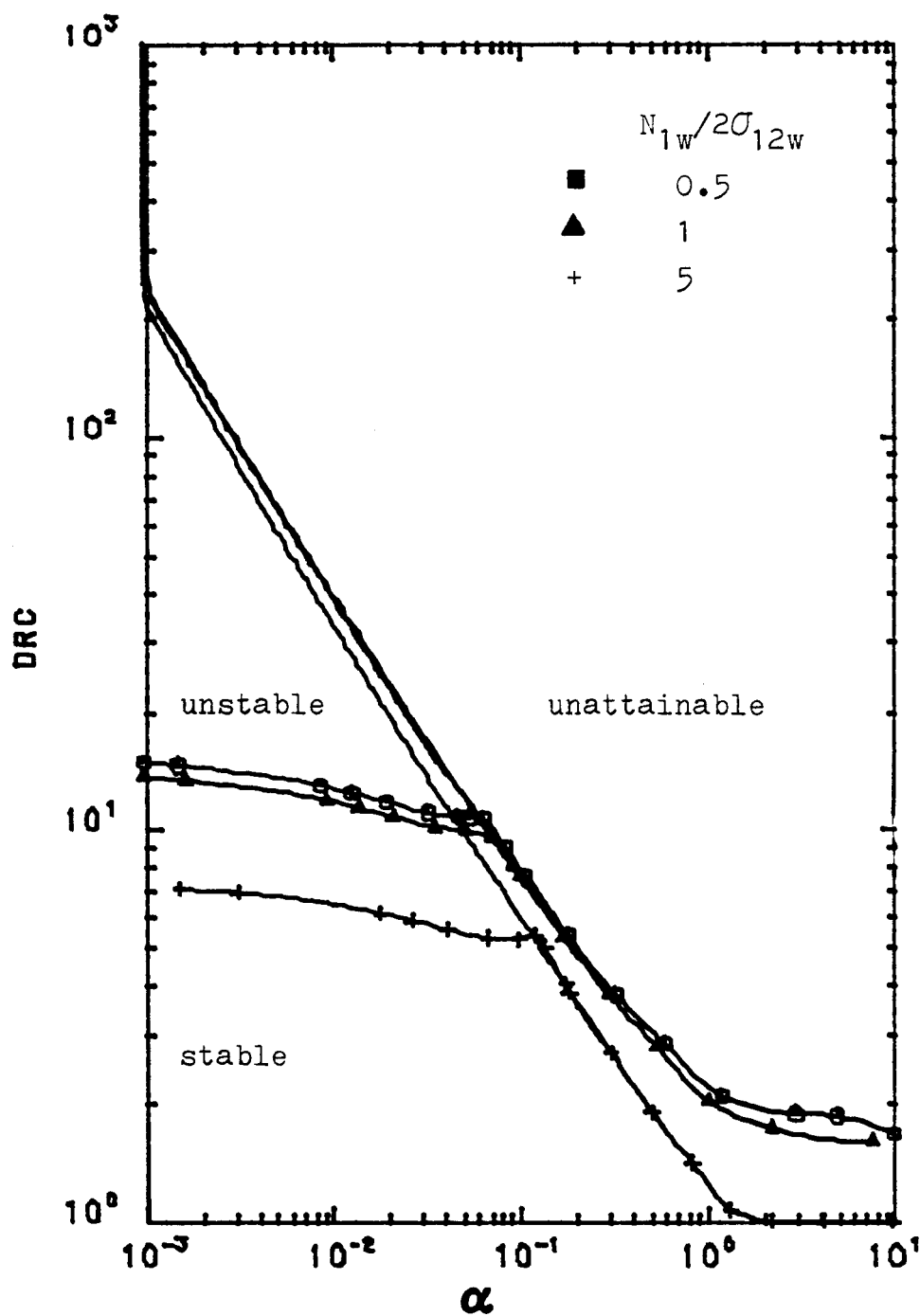


Figure 6-8. Critical draw ratio as a function of α for $a = 0.9$.

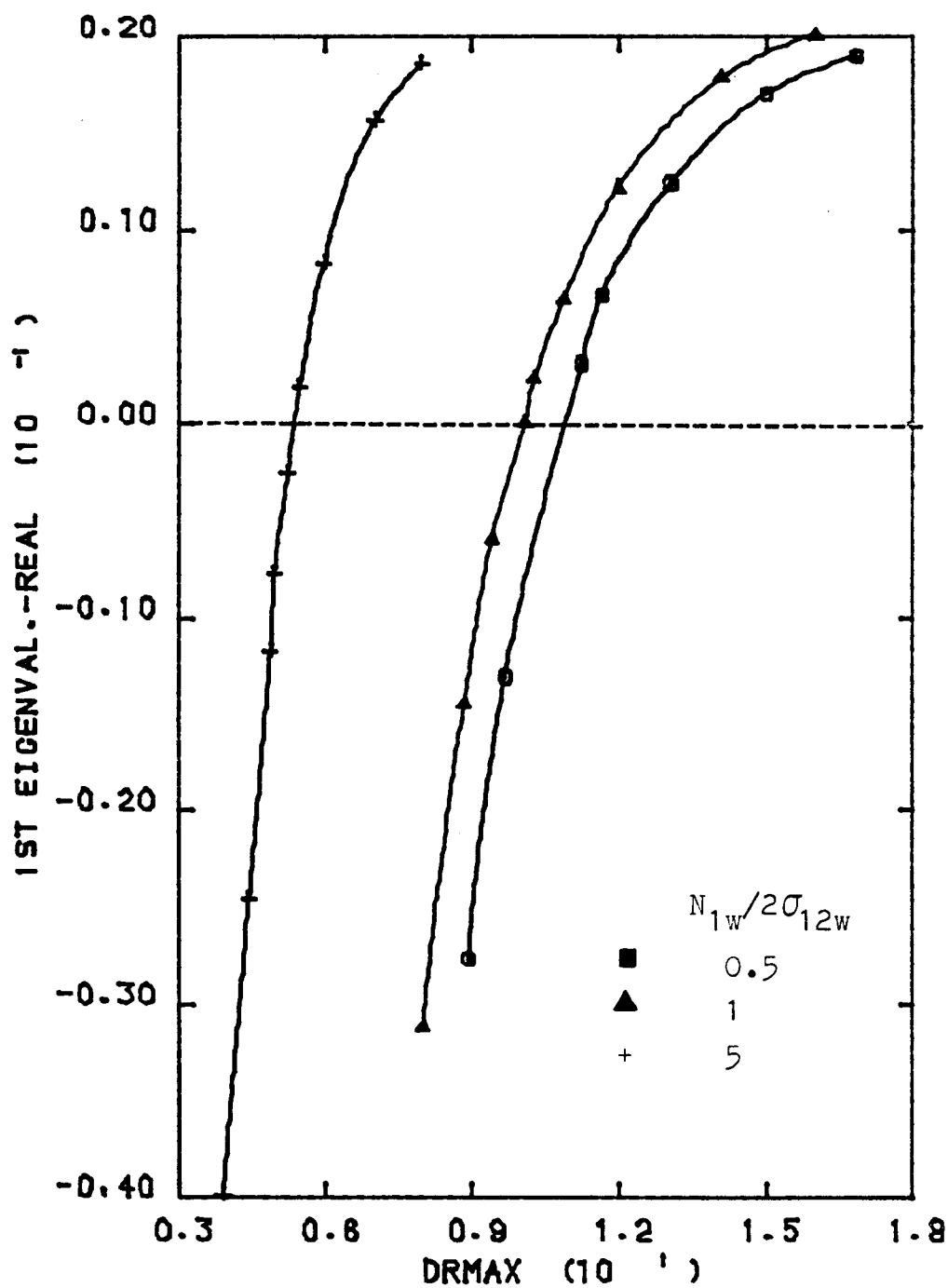


Figure 6-9. The real part of the first eigenvalue at maximum draw ratio for $a = 0.9$.

regions which correspond to the unattainable region (or ductile failure region), the unstable region, and the stable region, as $a > 1/\sqrt{3}$. For $a < 1/\sqrt{3}$, only two regions, that is, the stable region and unstable region, are obtained. (If force is made sufficiently high, the spinline is always stable, as indicated in the following paragraph, but the die swell can be less than 1, as shown in Figure (5-4) (page 91), which makes the critical draw ratios go to infinity).

Another important feature is that as $a < 1/\sqrt{3}$, there is a take-up force value (depending on die flow parameter $N_{lw}/2\sigma_{12w}$) beyond which the spinline is always stable. These are shown in Figures (6-10) to (6-12). As $a = 1/\sqrt{3}$, as shown in Figure (6-11), the critical draw ratio is increased rapidly when the take-up force is sufficiently high. However, as $a > 1/\sqrt{3}$, again the ductile failure dominates the critical draw ratio value (or the maximum draw ratio value) as the take-up force is increased, as shown in Figure (6-12) for $a = 0.6$. If $a < 1/\sqrt{3}$, the critical take-up force, beyond which the spinline is stable, is increased as the deformation thinning parameter "a" is increased or as the stress ratio $N_{lw}/2\sigma_{12w}$ is increased. However, as $a > 1/\sqrt{3}$, the critical take-up force (beyond which the critical draw ratio is controlled by the maximum draw ratio available, that is, ductile failure) decreases as "a" is increased, or as $N_{lw}/2\sigma_{12w}$ is decreased, as shown in Figure (6-13).

The influence of die flow is somewhat complicated because of the interrelationship among the dimensionless variables ϕ_I , α and $N_{lw}/2\sigma_{12w}$. If one uses different dies with sufficiently high aspect ratio but the same material, each figure from Figures (6-3) to (6-8)

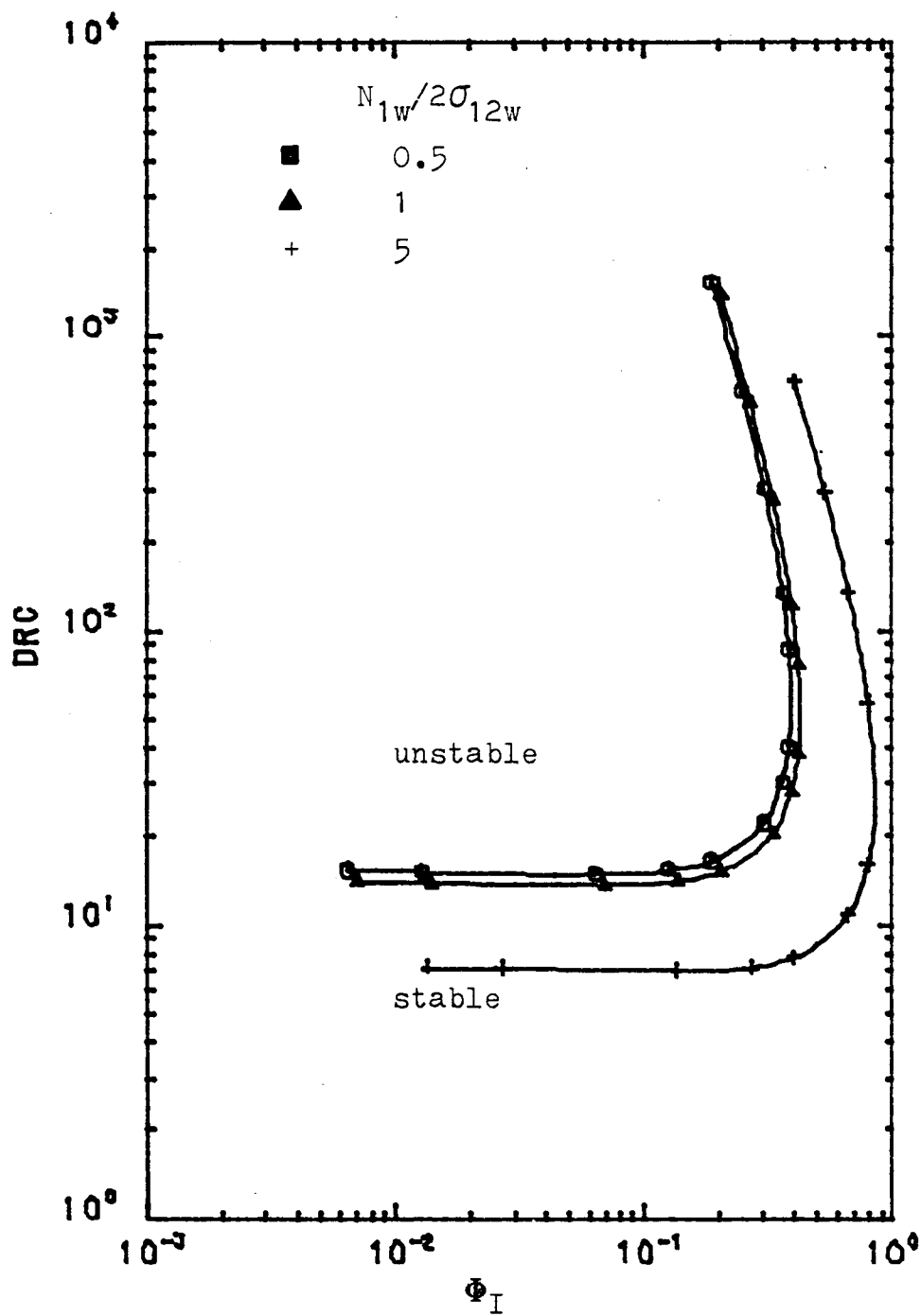


Figure 6-10. Critical draw ratio as a function of ϕ_I for $a = 0.4$.

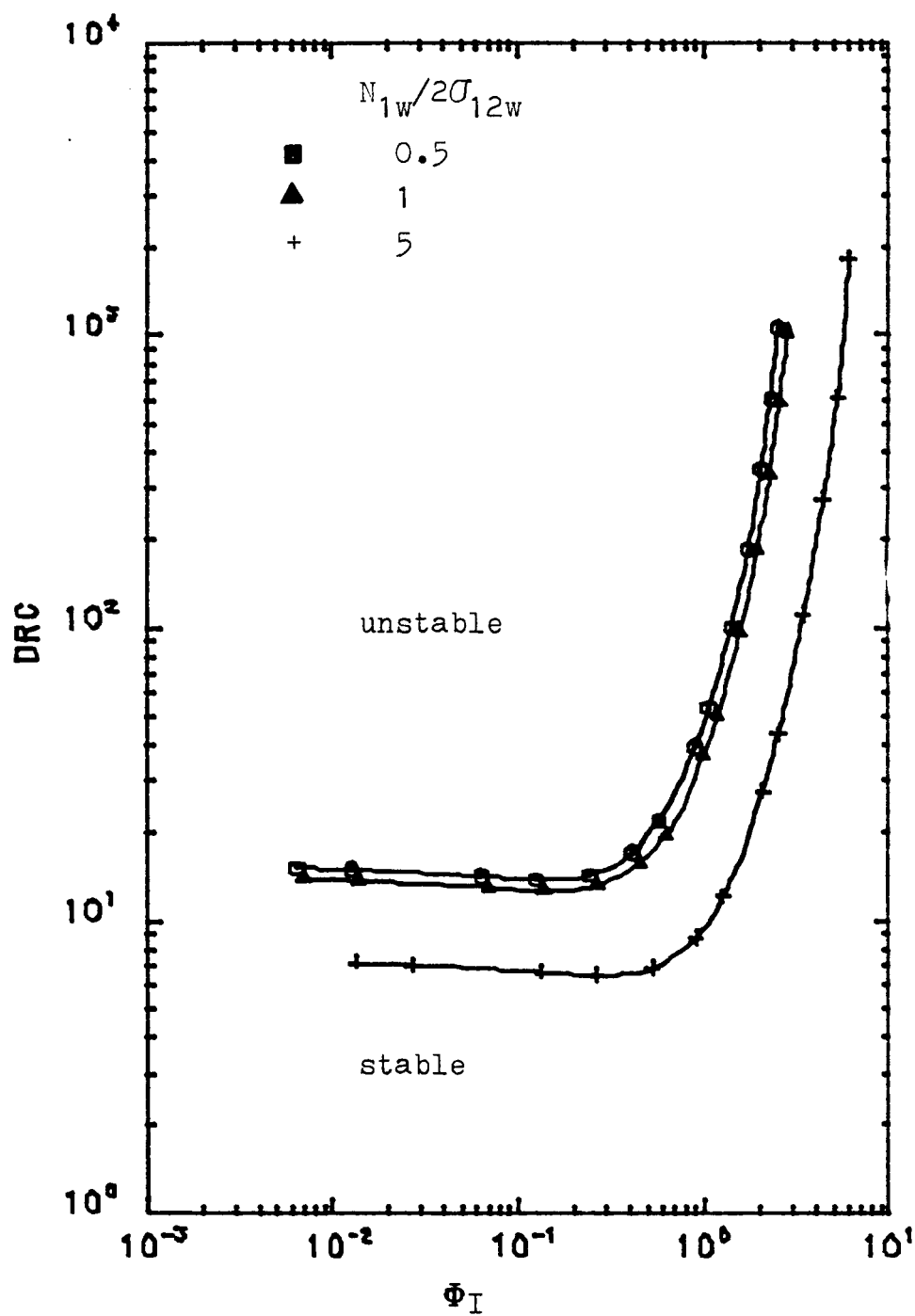


Figure 6-11. Critical draw ratio as a function of ϕ_I for $a = 1/\sqrt{3}$.

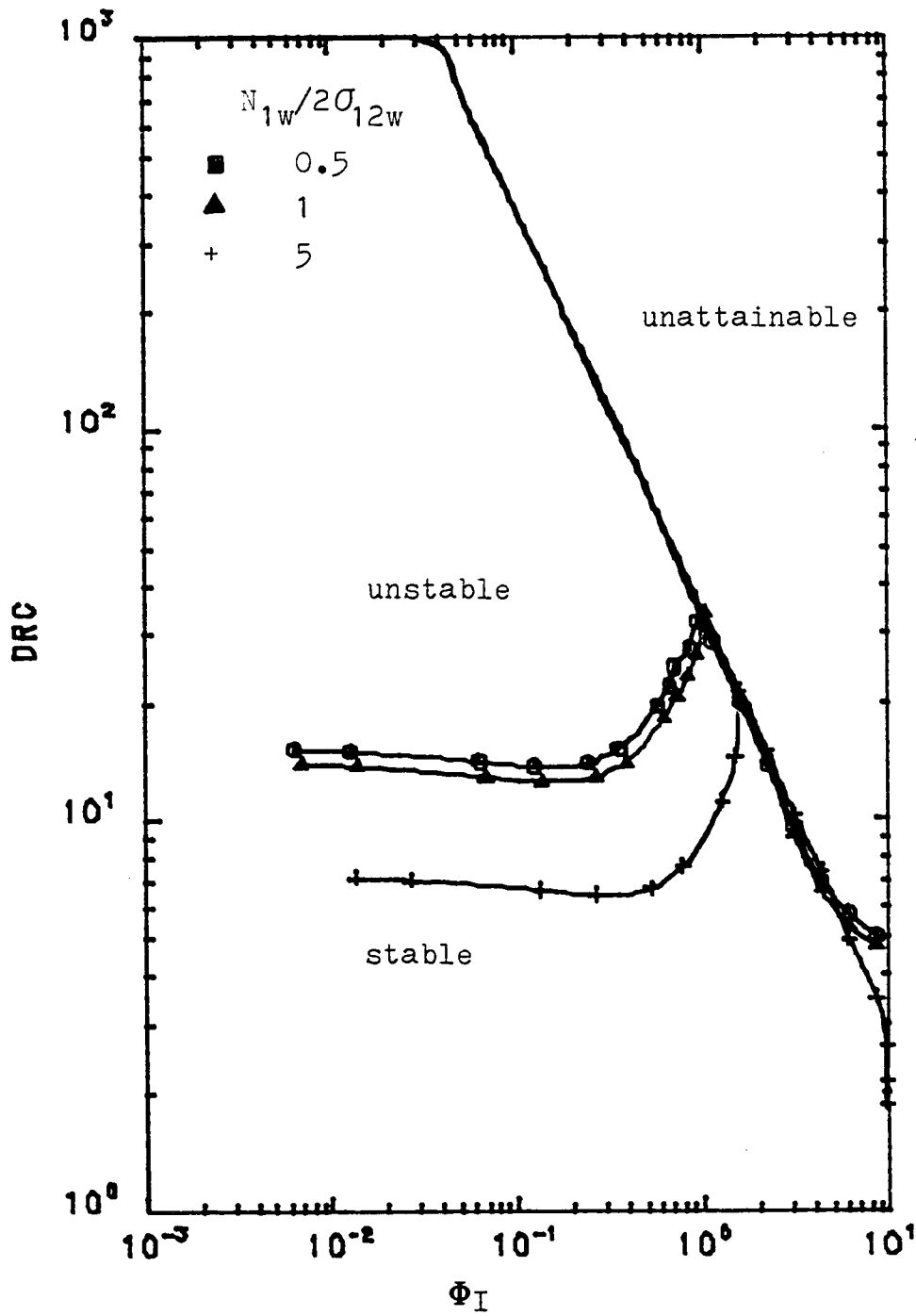


Figure 6-12. Critical draw ratio as a function of ϕ_I for $a = 0.6$.

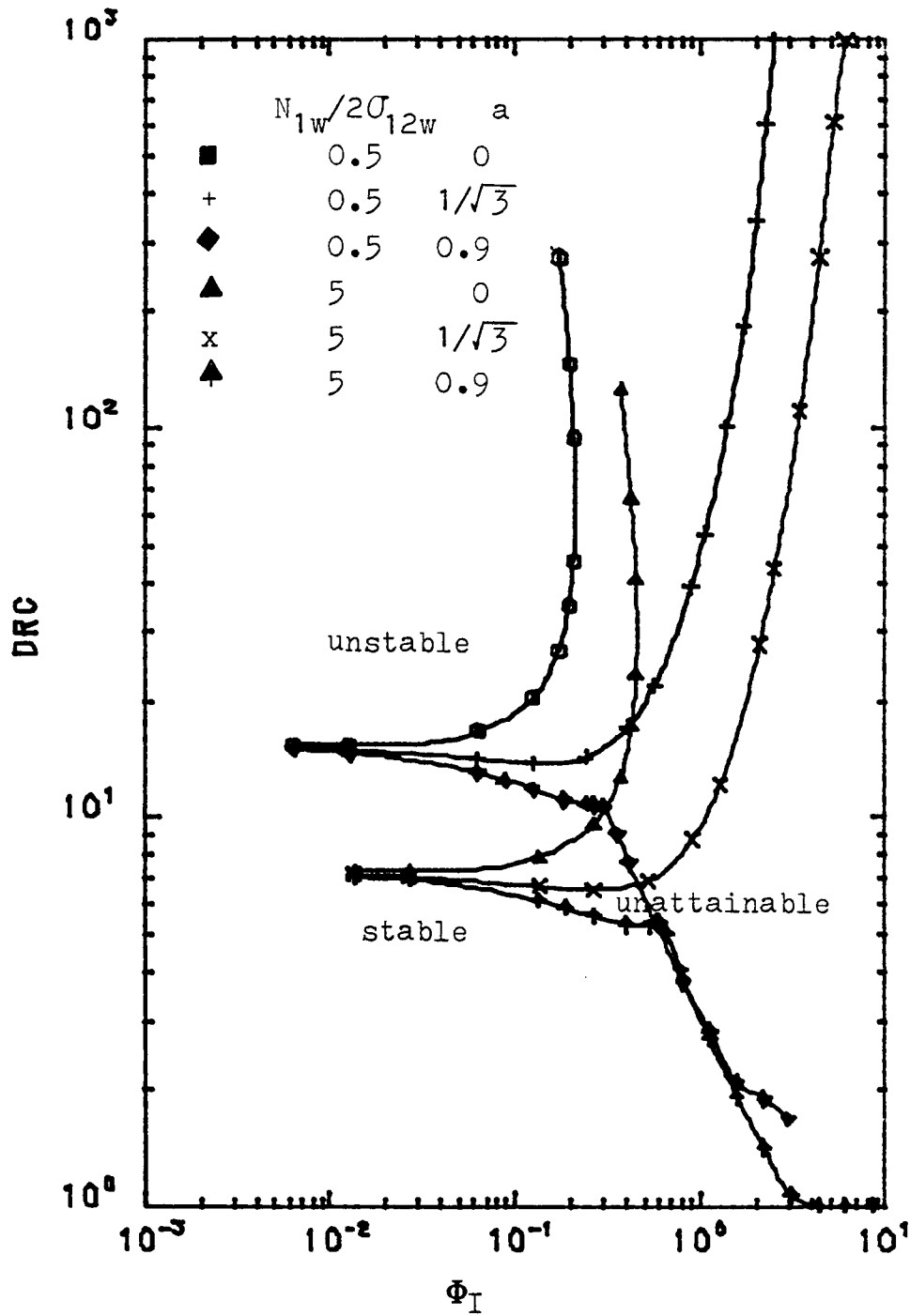


Figure 6-13. Critical draw ratio as a function of Φ_I for different values of "a" and $N_{1w}/2\sigma_{12w}$.

can be used to predict the influence of the die flow. Increasing the shear stress (by increasing the extrusion speed or decreasing the die diameter), which corresponds to increasing $N_{1w}/2\sigma_{12w}$, can increase or decrease the critical draw ratio, depending on the α value. That is, the curves slope slightly downward for small values of α and steeply upward for large values α (at small values of "a").

The influence of the materials used and the die flow on the critical draw ratio can also be viewed clearly in the master plot shown in Figure (6-14). Generally, increasing the thinning parameter "a," decreases the critical draw ratio value at the same α and stress ratio value, if one does not consider the second stable region when $a < 1/\sqrt{3}$. In addition, as the stress ratio is increased, the critical draw ratio is decreased if one keeps α and "a" constant. Finally, the critical draw ratio is not affected by the "a" value but depends on the stress ratio, when the α is sufficiently small, as seen in Figure (6-14). The critical draw ratio is slightly decreased as α is increased, and is increased to the second stable region as $a < 1/\sqrt{3}$ or to infinity as $a = 1/\sqrt{3}$, or to some steady maximum draw ratio value as $a > 1/\sqrt{3}$.

The linear stability analysis results of Fisher and Denn (78) and Chang and Denn (101) as shown in Figures (2-2) (page 32) and (2-3) (page 33) for two models do not predict a die flow effect. In their analyses, no die flow expression is involved. The initial position of their studies is chosen somewhere downstream of the spinline, consequently, both initial velocity and $F1(0)$ value, which, in turn, are dependent on the die flow properties, are unknown. If one chooses

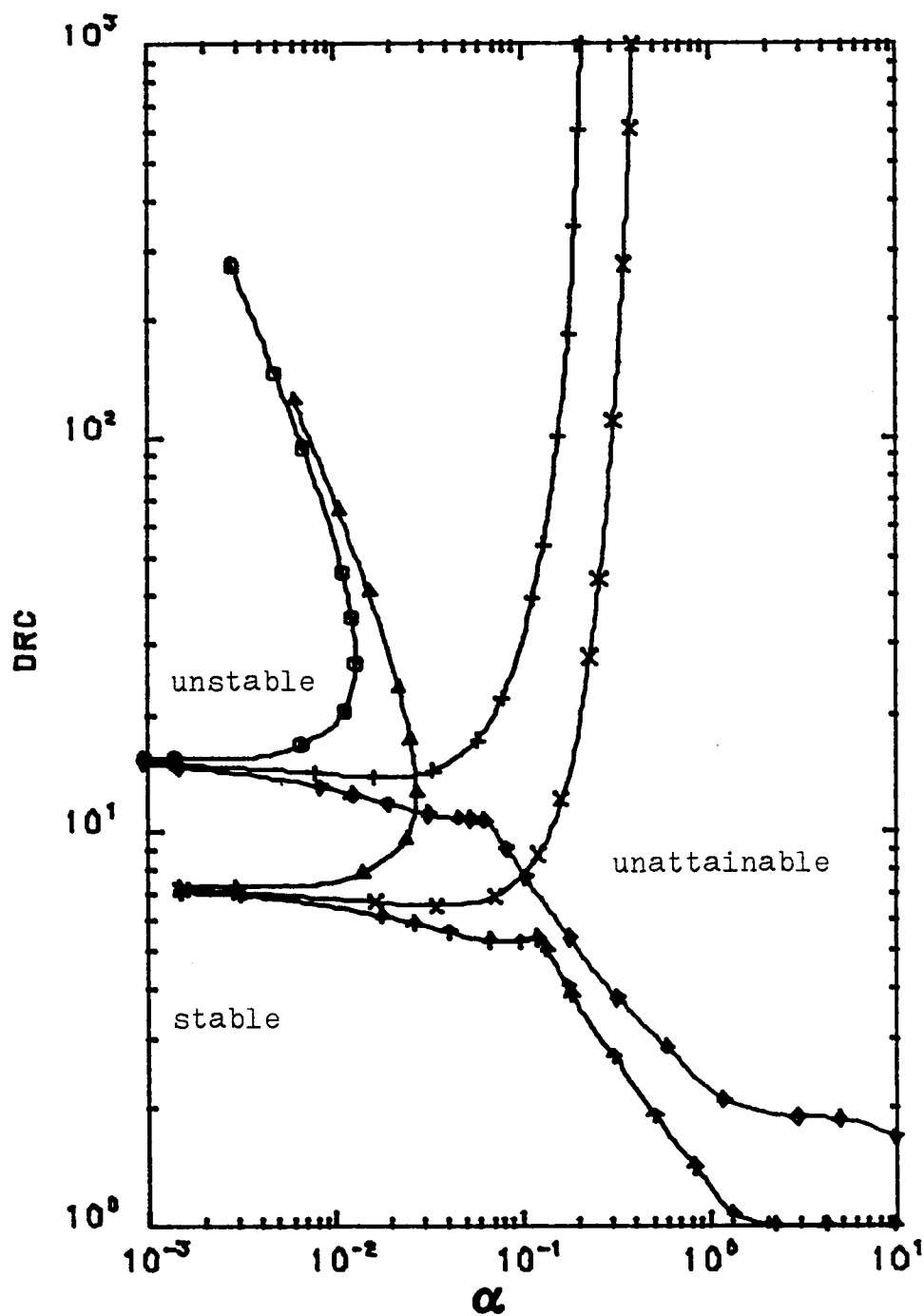


Figure 6-14. Critical draw ratio as a function of α for different values of "a" and $N_{1w}/2\sigma_{12w}$.

the expression such as Eqs. (5.1) to (5.4), the influence of die flow on the critical draw ratio value using these or other different models could also be obtained.

Although it is difficult to compare the results obtained from different viscoelastic models, some comments can still be made. We have shown in Chapter 5 that only when the stress ratio is low and the take-up force is sufficiently high, does the assumption that $F_1(0) \gg F_2(0)$ become valid. Consider Figures (5-4) (page 91) and (5-6) (page 93), we may say that the assumption used by Fisher and Denn (78) and Chang and Denn (101) is useful for small stress ratios, if one wishes to consider the die flow effect. Consequently, from our stability results, an increase in the stress ratio would move their stability envelopes downward and to the right.

Another difficulty in comparing the critical draw ratio obtained from these three different models is the shape of the stability envelope. Although these three models are degenerated to the Maxwell model, different nonlinearity parameters are involved. For the Maxwell model, the shape is the same (although the values may not be because of the die flow effect). The results of Fisher and Denn always have a second stable region, which is the same as in the present work for $a < 1/\sqrt{3}$. In contrast the results of the present work are rather similar to those of Chang and Denn (101); in both of these cases the second stable region or the absence of it depend on the "a" value or "q" value. The absence of the second stable region for $a > 1/\sqrt{3}$ is the result of ductile failure, whereas the absence of the second stable region in the Phan-Thien model is the result of the

usual draw resonance instability. The q value in the Phan-Thien model must be obtained from extensional experiments and is currently available only for low density polyethylene (where $q = 0.015$).

The only systematic experimental studies of the influence of die flow on the critical draw ratio were done by Nam (71) and in the present work. In the following, the experimental data obtained by Nam and the present author are compared with the theoretical results shown above.

B. COMPARISON WITH EXPERIMENTAL DATA

Experimentally, one should decide which fluctuations are merely "noise" and which are due to draw resonance. The experimental work done by previous authors (40, 42, 63, 71, 112) has shown that some irregular fluctuation always exists over a wide range of spinning conditions, especially in the case of polypropylene. This was also shown by the present author in Figures (4-12) to (4-17) (pages 73 to 78). The amplitude of the fluctuation, although no periodicity is observed, becomes much more severe as the draw ratio is made sufficiently low. This unusual effect will be discussed later. The onset of draw resonance can be seen either in the digital readout or on the recorder chart of the tensiometer. The period of the periodic fluctuation can be obtained from either measurement. These two methods give almost the same results. Table 6-1 shows various period measurements obtained by these two methods. In the digital readout method, a stop watch was used to measure the time required to complete a cyclic change at maximum die swell position. It would be more

Table 6-1. Comparison of Period Obtained by Two Different Methods.

DR	$\frac{B_{\max}}{B_{\min}}$	Period (sec)*	Period (sec)**
2.92	1.018	-----	-----
4.01	1.012	-----	-----
5.13	1.013	-----	-----
6.60	1.004	-----	-----
7.71	1.002	-----	-----
8.82	1.000	-----	-----
9.54	1.013	12.37	12.50
10.30	1.049	12.85	12.50
11.51	1.168	12.90	12.50
12.42	1.230	12.45	12.50
13.36	1.427	12.64	12.50

Material: PP 106-4 at 180°C.

Experimental conditions: plunger speed = 0.05 in./min, D = 0.041 in. L=3 cm.

*Data from digital readout.

**Data from tensiometer.

accurate to determine both the diameter profile and the period if a recorder rather than a digital readout had been available. It was more convenient to use the recorder of the tensiometer to determine both the period and the onset of the draw resonance. The critical draw ratio was determined by slightly increasing or decreasing the draw ratio below or above the value where the periodicity is found on the recorder chart of the tensiometer.

The random, erratic fluctuations cannot be explained within the framework of the stability theory, which always predicts periodic disturbances. Random fluctuations could be induced by changes in a large number of system variables, such as temperature homogeneity of polymer melt and surroundings, the polydispersity of polymeric material, and random fluctuation of output and spinline tension as deduced from the steady spinning analysis. However, the present author (and also the previous authors at the University of Tennessee (71, 114)) have an intuitive feeling this "instability" is somehow due to the inability of the spinline to pull away the material that is being continuously supplied by the plunger. In some physical way there is insufficient force in the spinline to overcome some "adhesion" effect near the die, perhaps due to surface tension.

One of the major purposes in the present work is to try to interpret the experimental data shown by Nam (71). The experimental data of Nam have been shown in Figures (2-4) to (2-7) (pages 37 to 40) for two different grades of polypropylene. To insure that Nam's data are reproducible, a set of experiments was first done by the present author using a die with a diameter of 0.041 in., $(L/D) = 30$ and a

spinline length of 3 cm (for PP 106-4). The experimental results are shown in Table 6-2. It is found that there is quantitatively some difference but the tendency of the effect of the die flow on the critical draw ratio is exactly the same. Thus, it is believed that Nam's data can be used to compare the effects of die flow on the critical draw ratio both experimentally and theoretically.

The results in Table 6-2 are plotted in Figure (6-15), together with the curves obtained from theory. The two curves in Figure (6-15) are corresponding to Co values of 0.1 and 0.13, the former giving a somewhat better experimental fit to the data and the latter coming from the finite-element result. The k^2 value is chosen as 1/2. From Figure (6-15), however, it is found that there is not much difference between these two Co values, and therefore the choice of the parameter Co is not critical. While the trends are generally in agreement, some special comments are needed concerning the data at very low X . For this point (that is, at plunger speed of 2 in./min) no periodic fluctuation is seen on the recorder chart of the tensiometer, but is seen at the as-spun fiber, the fiber already shows a periodic change of its diameter. Because of high take-up velocity, about a half of the fiber is in the "necking" region during one cycle. The measured period is about 0.40 second. The theoretical stability analysis at that processing conditions shows that the maximum draw ratio is stable. The maximum draw ratio, if it is stable, is also termed as critical draw ratio in the present work.

Table 6-2. Critical Draw Ratios and Periods for PP 106-4 (L = 3 cm).

Vp (in./min)	SR	α	Nam		Present Work	
			DRC	P (sec)	DRC	P (sec)
0.05	1.089	0.0059	9.8	----	9.54	12.5
0.1	1.286	0.0118	9.0	7.40	8.23	7.14
0.2	1.546	0.0238	8.5	----	7.52	4.00
0.5	1.918	0.0590	8.2	1.72	8.45	1.67
1.0	2.150	0.1180	9.8	0.94	10.08	0.86
2.0	2.414	0.2390	16.8	0.42	18--21*	0.40**

Experimental conditions: T = 180°C, L = 3 cm, D = 0.041 in.

*Spinline is broken without a periodic fluctuation occurring on the recorder chart of the tensiometer.

**Period measured by using the fiber just before the fiber is broken.

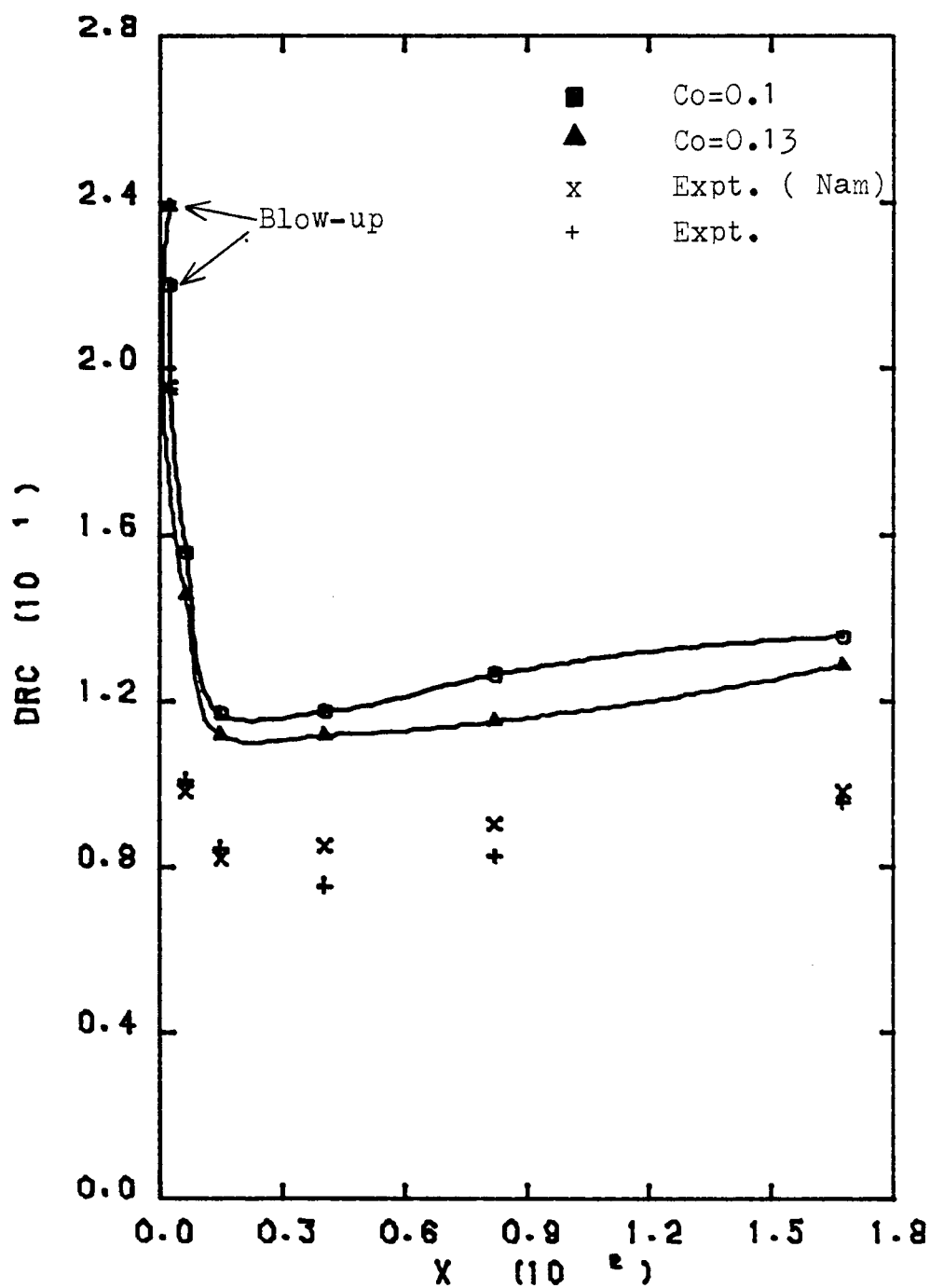


Figure 6-15. Comparison of theoretical and experimental critical draw ratios for PP 106-4 ($D = 0.041$ in., $L = 3$ cm).

As shown in Chapter 2, it is found that the period of the limit cycle from the nonlinear stability theory is the same as that obtained from linear stability theory at the critical draw ratio. From our linear stability analysis, it is known that, at the onset of draw resonance, the real part of the first eigenvalue is zero, and the velocity is perturbed starting from the initial position and arrives at the take-up velocity in one cycle. This velocity perturbation is due to the imaginary part of the first eigenvalue. Although it seems that there is no relationship between the period of oscillation and the imaginary part of the first eigenvalue, because of many eigenvalues involved, it has been shown that the imaginary part of the first eigenvalue at critical draw ratio can be used to predict the period of the fluctuation (107). Figure (6-16) indicates the period calculated by the theory, that is

$$\text{Period} = \frac{2\pi \tau_o}{\lambda_I} \quad (6.21)$$

where λ_I is the imaginary part of the first eigenvalue at the critical draw ratio. The theoretical period at the maximum draw ratio, which is stable, is also shown in Figure (6-16), for comparison. Surprisingly, the theoretical work can predict the period of the fluctuation very well, although the theoretical value is slightly lower than the experimental value. Here, the theoretical curve is based on $Co = 0.1$ (if one use $Co = 0.13$, only a little higher period than that obtained by $Co = 0.1$ is obtained).

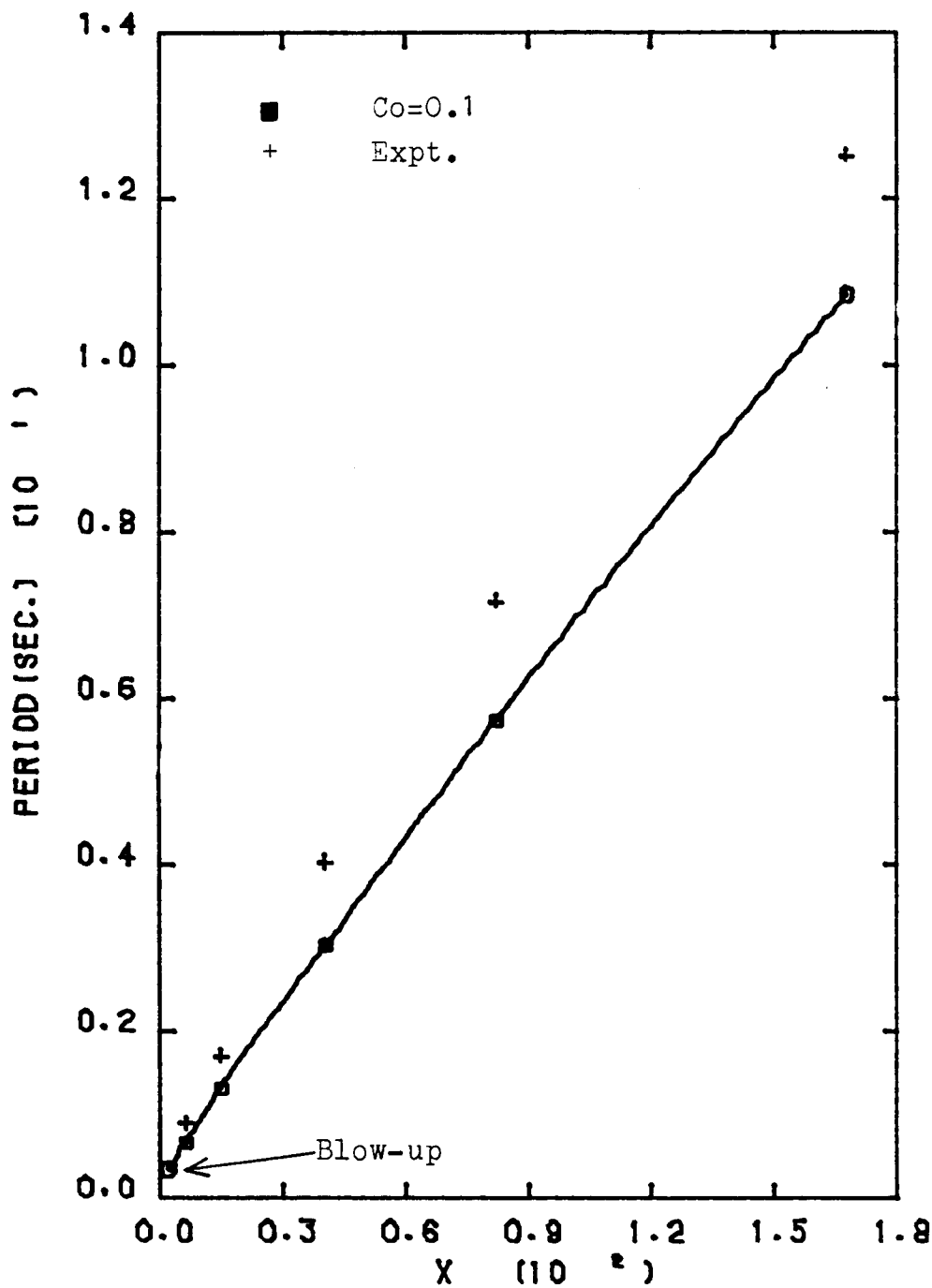


Figure 6-16. Comparison of theoretical and experimental periods for PP 106-4 ($D = 0.041$ in., $L = 3$ cm).

In the following we try to qualitatively discuss whether the present theory can predict the effect of die flow on the critical draw ratio as observed by Nam (71). We have shown in Figure (6-15) that the present analysis can predict the correct tendency for the curve at 3 cm spinline length in Figure (2-4) (page 37). First, we know that the data points in Figure (2-4) from left to right are corresponding to the plunger speed ranging from 0.05 in./min to 2 in./min as deduced from Table 6-2. The corresponding stress ratio values are the same as that in Table 6-2 but the α value is double or half of the value shown in Table 6-2 (for spinline lengths of 1.5 cm and 6 cm, respectively). Second, looking at Figure (6-7) (page 153), we find that we can predict the tendency of the effect of the die flow on the critical draw ratio, but not quantitatively. The theory always gives a higher critical draw ratio than that obtained from experiment. This result can be expected. In our theory, the maximum die swell position is chosen as the starting position for the analysis. Studies of velocity profile using the finite-element method as reviewed in Chapter 2 and discussed in Chapter 4 showed that the development of uniaxial flow occurs somewhere downstream of the maximum die swell position. Thus, choosing the maximum die swell position should give a higher draw ratio because of the lower initial velocity at that position.

The effect of spinline length (L), however, is not always correctly predicted. As shown in Figure (6-7), the theory predicts the following: (1) at low α values ($\alpha < 2.0 \times 10^{-2}$), the critical draw ratio decreases slightly with increasing α (or decreasing L);

and (2) at high α values, the critical draw ratio increases steeply with increasing α (or decreasing L). Thus, one can qualitatively explain the right-hand portion of the experimental results of Figure (2-4), but not the left-hand side (for example, $L = 1.5$ cm, $\bar{V} = 10.6$ cm/min, corresponds to $\alpha = 1.18 \times 10^{-2}$). Because the theoretical results are so strongly dependent on the particular value of τ_0 that is used, it is not possible to say whether there is quantitative agreement or not. The spinline length effects shown in the left-hand portion of Figure (2-4) cannot be explained by any of the existing theories. If we make very large changes in L , perhaps the theoretical effects might be seen. It is encouraging that the theory does work in the portion where there are large effects of L , that is, the right hand portion of Figure (2-4).

Now let us consider die diameter effects. In Figure (6-7), it is found that the minimum critical draw ratio (at $\alpha < 1 \times 10^{-1}$) occurs at α values ranging from 2.0×10^{-2} for stress ratio of 0.5 to 6×10^{-2} for stress ratio of 5. One should keep in mind that the stress ratio increases as the shear stress or shear rate is increased. (see Eq. (4.22)). In this manner, we find that the curve of $D = 0.058$ in. in Figure (2-5) (page 38) is easily interpreted because the α value is calculated as ranging from 1.18×10^{-3} to 5.9×10^{-2} and the minimum critical draw ratio of that curve ($\bar{V} = 42.5$ cm/min) is at $\alpha = 2.95 \times 10^{-2}$. This is a reasonable value compared with the theory. However, the curve of $D = 0.027$ in. which has a minimum at $\alpha = 2.72 \times 10^{-2}$ is not easy to interpret. According to the theory the minimum should occur at a high α value because of the increasing

of stress ratio due to decrease of the die diameter. Experimental data for a slightly higher plunger speed would be helpful. The critical draw ratio at $\alpha = 1.36 \times 10^{-1}$ and 2.72×10^{-1} are almost the same because of the maximum "bump" in the Figure (6-7), but the mechanisms on either side are different: at low α is due to true usual instability, at high α it is due to a ductile failure mechanism. The curve for $D = 0.019$ in. in Figure (2-5) has a minimum critical draw ratio at $\alpha = 1.10 \times 10^{-1}$; that is due to the higher stress ratio (the apparent shear rates are 546.7 sec., 95.26 sec., and 48.05 sec., at the minimum for $D = 0.019$ in., 0.027 in. and $D = 0.058$ in. as shown in Figure (2-5)). However, this value is slightly higher than that predicted by the theory, since the stress ratio is about 2.5. The critical draw ratio at $\alpha = 2.75 \times 10^{-1}$ for $D = 0.019$ in., where the stress ratio is about 2.8, is due to the stable maximum draw ratio under the conditions shown in Figure (6-7).

Thus far, we find that the theoretical framework presented can predict, at least qualitatively, the experimental data given by Nam (71) for PP 106-4. In the following, the experimental data given by Nam for PP 6823, as shown in Figures (2-6) (page 39) and (2-7) (page 40) are interpreted by Figure (6-8).

In general for PP 6823, one sees from Figures (2-6) and (2-7) that there is a continuously decreasing or decreasing then flatting of the critical draw ratio curve, in contrast to the upswing in the case of PP 106-4. These results are qualitatively in agreement with theory (Figure (6-8) (page 154)) provided that one is in the left-hand portion of the figure (the right-hand or steeply decreasing portion

corresponds to ductile failure). In order to be in the correct portion of that figure, τ_o must be adjusted empirically. These general observations will now be discussed in detail.

Consider first Figure (6-8), where it is found that the maximum draw ratio is stable at α ranging from 6×10^{-2} to 1.1×10^{-1} , for stress ratios from 0.5 to 5. Also, the maximum draw ratio decreases rapidly as α is increased. The highest α values in Figure (2-6) are 5.90×10^{-1} , 2.95×10^{-1} and 1.48×10^{-1} for spinline lengths of 1.5 cm, 3 cm, and 6 cm, respectively. These values are so high that according to Figure (6-8) an apparent decrease of the critical draw ratio should be expected. Consequently, if the value of "a" chosen is correct ($a = 0.9$), the τ_o value should be about 1/3 of that used to calculate the α values. That is, instead of $\tau_o = 1.25$ sec, $\tau_o = 0.4$ sec should be used. Otherwise, the "a" value should be smaller than 0.9 but larger than 0.6. In addition, we have shown that $Co = 0.3$ and $Co = 0.33$ are required for $k^2 = 1/2$ and $1/3$, respectively, to fit the experimental unconstrained die swell of PP 6823. It would be helpful to consider the effects of these values on the stability envelope of PP 6823. However, looking at Figures (6-8) and (2-6) to (2-7), we may say that the theory can predict the tendency of the effect of die flow on the critical draw ratio of PP 6823, if $\tau_o = 0.4$ sec is chosen. The slightly higher critical draw ratio is due to the initial position chosen in the present work, and the increasing of critical draw ratio for both PP 106-4 and PP 6823 as the spinline length is decreased for sufficiently small α value, at the same stress ratio, cannot be explained by any of the theories.

Let us now return to PP 106-4 ($a = 0.6$) for consideration of the steeply decreasing right-hand portion of Figure (6-7). The critical draw ratio decreases sharply as the α value is further increased, because of the decrease of the maximum draw ratio. It is interesting to see whether this behavior occurs or not. New experimental work was thus done for PP 106-4 using $D = 0.027$ in. and a spinline length of 1.5 cm. The experimental results are shown in Table 6-3 and Figure (6-17). Also shown in Figure (6-17) is the theoretical results with $Co = 0.1$ and $Co = 0.13$. Clearly the theory can predict the tendency very well. Note that the critical draw ratios at $\alpha = 2.72 \times 10^{-1}$ ($X = 3.67$) and 5.44×10^{-1} ($X = 1.84$) are values where the spinline broke and no periodic fluctuation was observed during the increase of the draw ratio. The theoretical results at these two points all show that at these specified α values and stress ratios (as shown in Table 6-3) the maximum draw ratio available is stable. Thus, we believe the present stability analysis can, at least qualitatively, predict the tendency of the effect of die flow on the critical draw ratio.

Figure (6-18) shows the period of the fluctuation measured by the tensiometer and calculated by the spun fiber just before breaking, as indicated in Table 6-3. Theoretical results, according to Eq. (6-21), are also shown in Figure (6-18), using $Co = 0.1$. Good agreement is clearly seen. This result not only gives further confidence in the present theory, but also shows the validity of using a value of $\tau_o = 0.1$ sec for PP 106-4.

Table 6-3. Critical Draw Ratios and Periods for PP 106-4 (L = 1.5 cm)

Vp (in./min)	SR	α	DR	Period (sec)
0.05	1.573	0.0272	11.18	2.5
0.1	1.755	0.0544	8.79	1.44
0.2	1.967	0.1090	9.93	0.81
0.5	2.359	0.2720	17.8--18.8*	0.26**
1.0	2.625	0.5440	6.35-7.24*	0.15**

Experimental conditions: D = 0.027 in., L = 1.5 cm, T = 180°C.

*Spinline is broken without a periodic fluctuation occurring on the recorder chart of the tensiometer.

**Period measured by using the fiber just before the fiber is broken.

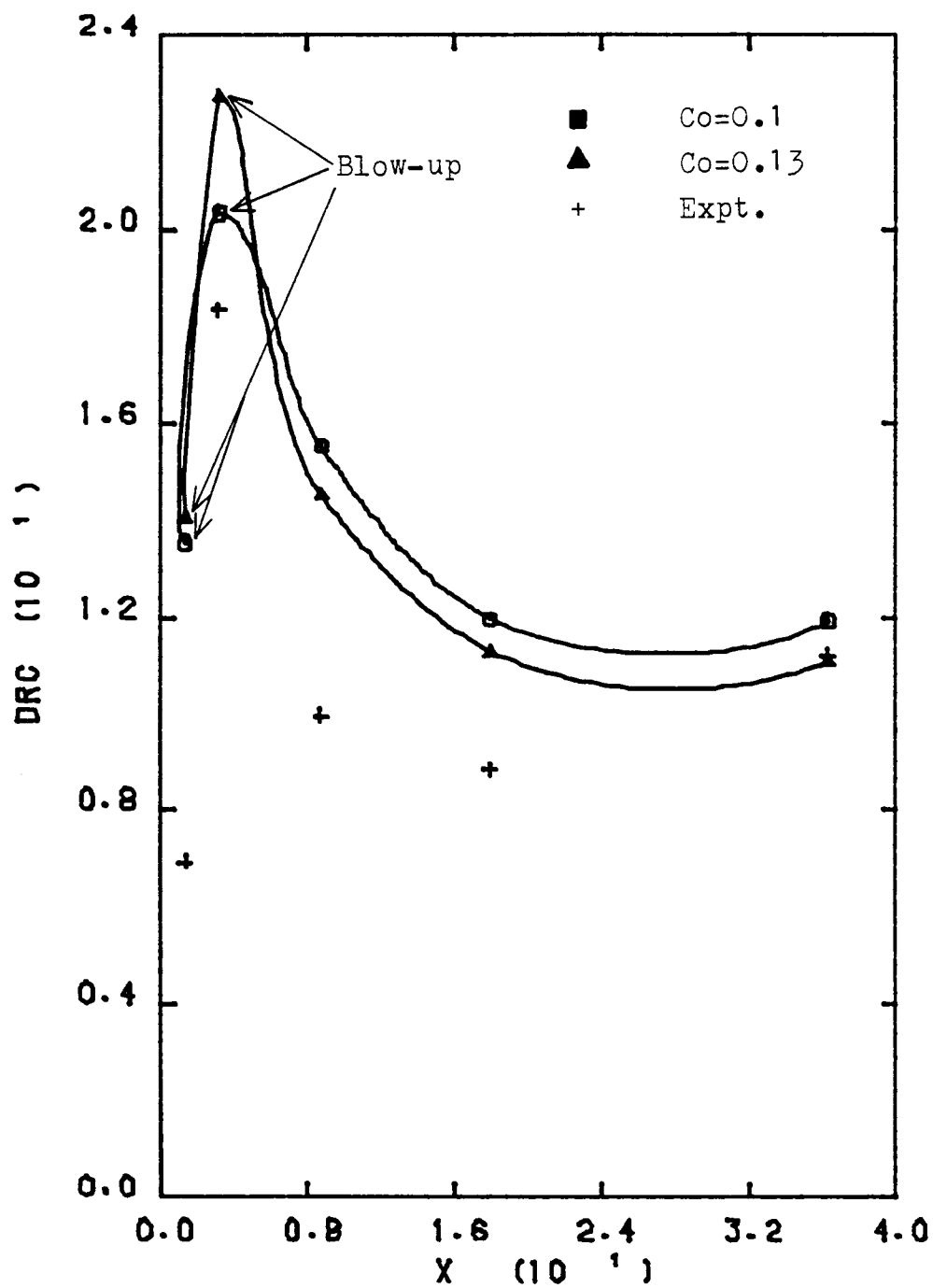


Figure 6-17. Comparison of theoretical and experimental critical draw ratios for PP 106-4 ($D = 0.027$ in., $L = 1.5$ cm).

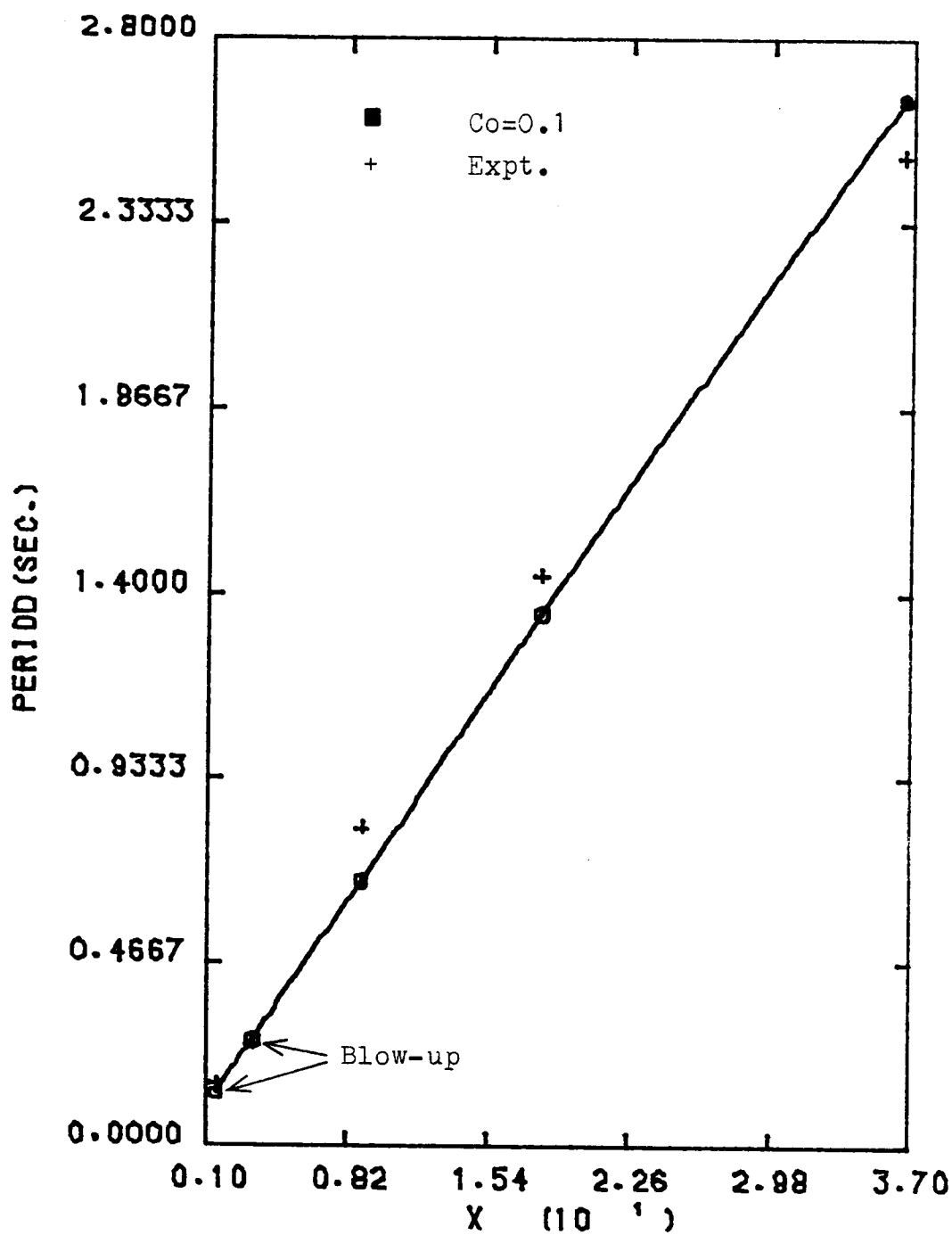


Figure 6-18. Comparison of theoretical and experimental periods for PP 106-4 ($D = 0.027$ in., $L = 1.5$ cm).

CHAPTER 7

CONCLUSIONS

The first theoretical analysis of the effects of die flow on the dynamics of isothermal melt spinning is provided by the present work. The analysis was based on kinematical assumptions involving instantaneous expansion (die swell) at the exit of the die, followed by purely elongational flow immediately thereafter. First, a comparison of various steady state solutions was made; second, a linear stability analysis using a differential treatment of the spinline was made; finally, comparisons with experimental results were presented. In this chapter, an attempt is made to pull out the important conclusions from these three areas of work.

A. DIE SWELL

The die swell problem has been dealt with in a variety ways. The most rigorous method is a finite-element analysis, while two less complicated analyses are those of Tanner, and Matsui and Bogue. The simplified kinematical assumption used in the Matsui and Bogue treatment makes their final answer somewhat different from the more rigorous analysis of Tanner. However, by introducing an adjustable parameter (k) these two analyses can be brought in agreement. In both of these theories, an additional parameter (C_0), which has to do with the limiting die swell in the absence of elastic stress, must be introduced. By a suitable adjustment of the parameters k and C_0 , a reasonable fit of die swell data can be obtained. Furthermore, it was

shown that the kinematics in the near die flow region (with the two adjustable parameters k and C_0) also can provide the initial stress conditions required in the subsequent work.

B. EFFECTS OF DIE FLOW ON THE STEADY SPINNING

A comparison of the predictions of four rheological models in steady spinning was made. One of these was the integral model of Matsui and Bogue and the other three were various differential treatments. The differential treatments vary in the kinds of initial boundary conditions that are used. That due to White and Ide permits one to obtain a simple analytical form. Fisher and Denn did a more rigorous analysis but made an arbitrary assumption of the initial stress condition. In the present work, the initial stress condition was obtained from an independent analysis of the die flow. All of the differential treatments give similar results, except in the case of a very strongly nonlinear rheological model, as will be discussed below. Generally, Fisher and Denn's treatment gives a slightly higher velocity profile at constant take-up force, or gives a lower take-up force at a constant draw ratio. In general, the integral treatment is in reasonable agreement with the results of the present differential treatment. However, there are differences very near the die exit, apparently due to the assumed kinematics that Matsui and Bogue introduced.

All of the treatments, except that of Fisher and Denn, use the Bogue-White rheological model. Using this model, one finds that the velocity gradient goes to infinity under certain circumstances (that is, $a > 1/\sqrt{3}$). This infinite velocity gradient is associated with the

incipient appearance of ductile failure. The various differential treatments show significant differences among themselves near this ductile failure point.

A principle objective of the present work was to study the effect of die flow on the subsequent spinline velocity profile. An important effect has to do with the fact that the die flow causes die swell at the exit of the die. For small values of "a," the various treatments can be brought nearly into agreement by merely adjusting the initial velocity, that is, the velocity in the die swell region. For large values of "a" (that is, "a" near or above $1/\sqrt{3}$), the effect of the die flow is felt much more strongly in the downstream flow.

Comparisons of spinline velocities with experimental data were made. In making those comparisons, one must first deal with the die swell region. The one element theories that were used predict sudden expansion, while the experimental data show delayed die swell. Except for this difference, reasonable agreement is obtained between theory and experiment, although the check is not a definitive one because the velocity profile is forced to fit at the take-up point.

C. EFFECTS OF DIE FLOW ON THE UNSTEADY SPINNING

A linear stability analysis was applied to a formulation based on a differential description of the spinline, in a manner analogous to that of Fisher and Denn. Where comparisons could be made, the present stability analysis is in good agreement with that of Fisher and Denn.

The results of the stability analysis depend strongly on the parameter "a." For small values of "a" ($a < 1/\sqrt{3}$), the stability envelopes are not qualitatively different from those presented by Fisher and Denn, although there is an effect of the stress ratio, with larger stress ratio resulting in less stable spinning conditions (that is, lower critical draw ratio).

For large "a" ($a > 1/\sqrt{3}$), the results are significantly different. At small values of the elasticity parameter α , the stability envelopes are qualitatively similar to those previously obtained. However, at some critical point, a ductile failure mechanism begins; at this point the critical draw ratio drops very sharply with increasing α .

The question of deciding on the onset of draw resonance is one of inspecting the amplitudes and the periods of the fluctuations. At very low forces, which corresponds to very low take-up velocities, there is an irregular, nonperiodic disturbance which is not well-understood. However, at draw ratios in the range of about 5 to 20, the onset of draw resonance is rather clearly observed and is characterized by a distinct periodicity in the disturbance.

The observed periods of fluctuations were in good agreement with theory based on using the imaginary part of the first eigenvalue. Comparisons of the results of the stability analysis with the experimental data for two polypropylenes (PP 106-4 and PP 6823) were made. For the first material, which has an "a" value of about 0.6, the agreement is quite satisfactory, although the predicted critical draw ratio are somewhat higher than the observed values. The results

for the second material, which is a highly thinning polypropylene ($a = 0.9$), show only qualitative agreement. To obtain quantitative agreement it would be necessary to adjust the time constant which appears in the theory by a factor of about $1/3$.

The earlier data of Nam, some of which were duplicated in the present work for PP 106-4, show complicated effects of die velocity and the spinline length on the critical draw ratio. For large die velocities, which correspond to high elasticity numbers α , there is a large effect of the die flow on the critical draw ratio and this was correctly predicted, at least in an qualitative way, by the stability analysis. At small die flow velocities (small values of α) the effect of die flow on the critical draw ratio is smaller but nonetheless consistently observed. This latter effect cannot be explained by the present analysis.

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APPENDIXES

APPENDIX A

GOVERNING EQUATIONS FOR INTEGRAL TREATMENT OF STEADY SPINNING

A-1. DERIVATION OF GOVERNING EQUATIONS

The integral model used has the form:

$$\begin{aligned}\underline{\sigma} &= - P \underline{I} + \underline{P} = - p \underline{I} + \int_{-\infty}^t M_1(t - t') \underline{C}^{-1} dt' \\ &= - P \underline{I} + \int_{-\infty}^0 M_1(t - t') \underline{C}^{-1} dt + \int_0^t M_1(t - t') \underline{C}^{-1} dt'\end{aligned}\quad (A.1)$$

and

$$\begin{aligned}M_1(t - t') &= \sum_n (G_n / \tau_n) e^{-(t-t')/\tau_n} \\ &= \sum_n (G_n / \tau_n) e^{-\int_{t'}^t \frac{dt''}{\tau_n}}\end{aligned}\quad (A.2)$$

In the above equations, only Finger fluid is considered and the general type of memory function M_1 is involved to include the multielement Maxwellian model under non-isothermal condition.

The kinematics used are Eqs. (2.8) and (2.9'). After calculating Finger tensors and substituting them into the above equations, assuming $\sigma_{22} = 0$, and making radius-averaged, one has

$$P_{11} = \sum_n G_n \left[\int_0^1 \int_{-\infty}^0 \frac{\lambda_o^2}{\tau_n} e^{-\int_{t'}^t \frac{dt''}{\tau_n}} \left(1 + \frac{16}{R^2} k \bar{V} t' \xi^2 \right) \xi dt' d\xi \right]$$

$$+ \int_0^t \frac{\lambda}{\tau_n} e^{-\int_{t'}^t \frac{dt''}{\tau_n}} dt' \right) \quad (A.3)$$

$$P_{22} = \sum_n G_n \left(\int_0^1 \int_{-\infty}^0 \frac{\lambda_o^{-1}}{\tau_n} e^{-\int_{t'}^t \frac{dt''}{\tau_n}} \xi dt' d\xi + \int_0^t \frac{\lambda^{-1}}{\tau_n} e^{-\int_{t'}^t \frac{dt''}{\tau_n}} dt' \right) \quad (A.4)$$

and

$$\sigma_{11} = P_{11} - P_{22} \quad (A.5)$$

Introducing Bogue-White deformation rate dependence of relaxation time, that is,

$$\tau_n = \left(\tau_o / (1 + a \tau_{no} \Pi_d^{1/2}) \right) \quad (A.6)$$

and noting that

$$\Pi_d^{1/2} = \begin{cases} \dot{\gamma} = 4\bar{V} r/R^2 = 4\bar{V} \xi/R \\ \sqrt{3} |dV/dX| \end{cases} \quad (A.7)$$

$$dt'' = dX''/V(X'') \quad (A.8)$$

Substituting Eqs. (A.6) and (A.8) into Eqs. (A.3) to (A.5), after some lengthy manipulation, one can obtain

$$P_{11} = \sum_n G_n \left(e^{-\int_0^X \frac{dX''}{\tau_{no}(X'')V(X'')}} \left(\frac{V(X)}{V(0+)} - \sqrt{3}a \left[\left(\frac{V(X)}{V} \right)^2 + F_n^k \left(\frac{V(X)}{V} \right)^2 \right] \right) \right. \\ \left. + \int_0^X \left(\frac{1}{\tau_{no}(X')V(X')} + \frac{\sqrt{3}a}{V(X')} \left| \frac{dV(X')}{dX'} \right| \right) \left(e^{-\int_{X'}^X \frac{dX''}{\tau_{no}(X'')V(X'')}} \left(\frac{V(X)}{V(X')} \right)^{2-\sqrt{3}a} dX' \right) \right) \quad (A.9)$$

$$P_{22} = \sum_n G_n \left(e^{-\int_0^X \frac{dX''}{\tau_{no}(X'')V(X'')}} \left(\frac{V(X)}{V(0+)} - \sqrt{3}a \frac{V(X)}{V} - 1 \right) \right. \\ \left. + \int_0^X \left(\frac{1}{\tau_{no}(X')V(X')} + \frac{\sqrt{3}a}{V(X')} \left| \frac{dV(X')}{dX'} \right| \right) \left(e^{-\int_{X'}^X \frac{dX''}{\tau_{no}(X'')V(X'')}} \left(\frac{V(X)}{V(X')} \right)^{-1-\sqrt{3}a} dX' \right) \right) \quad (A.10)$$

and

$$\sigma_{11} = P_{11} - P_{22} \quad (A.11)$$

where

$$\left[F_n = \frac{4}{a^2} \left(\frac{-1}{(c+1)} + 1.5 - 3c + 3c^2 \ln\left(1 + \frac{1}{c}\right) \right) \quad \text{for } a \neq 0 \right. \\ \left. F_n = \frac{4\bar{V}}{\tau_{no} R} \quad \text{for } a = 0 \right] \quad (A.12)$$

$$c = \frac{R}{4\bar{V} \tau_{no} a} \quad (A.13)$$

Following Nam (71), the following dimensionless groups are introduced:

$$X^* = \frac{X}{\bar{V} \tau_{no}} \quad (\text{A.14})$$

$$V^* = \frac{V}{\bar{V}} \quad (\text{A.15})$$

Consequently, we have the following working equations (neglecting *)

$$P_{11} = \sum_n G_n \left[\left(e^{-\int_0^{X''} \frac{dX''}{V(X'')}} \right) \left(\frac{V(X)}{V(0+)} - \sqrt{3}a \left[(1 + F_n^2 k^2) (V(X))^2 \right] \right) \right. \\ \left. + \int_0^X \left(e^{-\int_{X''}^X \frac{dX''}{V(X'')}} \right) \left(\frac{1}{V(X')} + \frac{\sqrt{3}a}{V(X')} \left| \frac{dV(X')}{dX'} \right| \right) \left(\frac{V(X)}{V(X')} \right)^{2-\sqrt{3}a} dX' \right] \quad (\text{A.16})$$

$$P_{22} = \sum_n G_n \left[\left(e^{-\int_0^{X''} \frac{dX''}{V(X'')}} \right) \left(\frac{V(X)}{V(0+)} - \sqrt{3}a \left(\frac{1}{V(X)} \right) \right) \right. \\ \left. + \int_0^X \left(e^{-\int_{X''}^X \frac{dX''}{V(X'')}} \right) \left(\frac{1}{V(X')} + \frac{\sqrt{3}a}{V(X')} \left| \frac{dV(X')}{dX'} \right| \right) \left(\frac{V(X)}{V(X')} \right)^{-1-\sqrt{3}a} dX' \right] \quad (\text{A.17})$$

and

$$\sigma_{11} = \sum_n G_n \left[\left(e^{-\int_0^{X''} \frac{dX''}{V(X'')}} \right) \left(\frac{V(X)}{V(0+)} - \sqrt{3}a \left[(1 + F_n^2 k^2) (V(X))^2 - \frac{1}{V(X)} \right] \right) \right. \\ \left. + \int_0^X \left(e^{-\int_{X''}^X \frac{dX''}{V(X'')}} \right) \left(\frac{1}{V(X')} + \frac{\sqrt{3}a}{V(X')} \left| \frac{dV(X')}{dX'} \right| \right) \left(\frac{V(X)}{V(X')} \right)^{-1-\sqrt{3}a} dX' \right] +$$

$$\int_0^X (e^{-\int_{X''}^X \frac{dX''}{V(X'')}}) \left[\frac{1}{V(X')} + \frac{\sqrt{3}a}{V(X')} \left| \frac{dV(X')}{dX'} \right| \left(\frac{V(X)}{V(X')} \right)^{2-\sqrt{3}a} - \left(\frac{V(X)}{V(X')} \right)^{-1-\sqrt{3}a} \right] dX' \quad (A.18)$$

Eq.(A.18) reduces to Nam's result when k^2 equals 1.

For one element Maxwellian model, the following dimensionless groups are defined:

$$\phi_I = 4F/(G\pi D^2) = \sigma_{11}(0-)/G = B^2 \sigma_{11}(0+)/G = \phi_d B^2 \quad (A.19)$$

$$\phi_d = \sigma_{11}(0+)/G = 4F/(G\pi d^2) = \phi_I V(0+) \quad (A.20)$$

$$F1 = P_{11}/\sigma_{11}(0+) \quad (A.21)$$

$$F2 = P_{22}/\sigma_{11}(0+) \quad (A.22)$$

Note that ϕ_I is a constant for a given die diameter D and take up force. In the following, the "+" is omitted for simplicity. Substituting Eqs. (A.19) to (A.22) into Eqs. (A.16) to (A.18), we have the working equations for single element model:

$$P_{11}/G = (e^{-\int_0^{X''} \frac{dX''}{V(X'')}}) \left(\frac{V(X)}{V(0)} \right)^{-\sqrt{3}a} \left[(1 + F_s^2 k^2) (V(X))^2 \right] + \int_0^X (e^{-\int_{X''}^X \frac{dX''}{V(X'')}}) \left[\frac{1}{V(X')} + \frac{\sqrt{3}a}{V(X')} \left| \frac{dV(X')}{dX'} \right| \right] \left(\frac{V(X)}{V(X')} \right)^{2-\sqrt{3}a} dX' \quad (A.23)$$

$$P_{22}/G = \left(e^{-\int_0^{X''} \frac{dX''}{V(X'')}} \right) \left(\frac{V(X)}{V(0)} \right)^{-\sqrt{3}a} \left(\frac{1}{V(X)} \right) + \int_0^X \left(e^{-\int_{X''}^X \frac{dX''}{V(X'')}} \right) \left(\frac{1}{V(X')} + \frac{\sqrt{3}a}{V(X')} \left| \frac{dV(X')}{dX'} \right| \right) \left(\frac{V(X)}{V(X')} \right)^{-1-\sqrt{3}a} dX' \quad (\text{A.24})$$

and

$$\phi_I = \frac{1}{V(X)} \left(e^{-\int_0^{X''} \frac{dX''}{V(X'')}} \right) \left(\frac{V(X)}{V(0)} \right)^{-\sqrt{3}a} \left[\left(1 + F_s^2 \right) (V(X))^2 - \frac{1}{V(X)} \right] + \int_0^X \left(e^{-\int_{X''}^X \frac{dX''}{V(X'')}} \right) \left(\frac{1}{V(X')} + \frac{\sqrt{3}a}{V(X')} \left| \frac{dV(X')}{dX'} \right| \right) \left(\frac{V(X)}{V(X')} \right)^{2-\sqrt{3}a} - \left(\frac{V(X)}{V(X')} \right)^{-1-\sqrt{3}a} dX' \quad (\text{A.25})$$

and

$$\begin{cases} F_s = \frac{4}{a^2} \left(\frac{-1}{c+1} + 1.5 - 3c + 3c \ln \left(1 + \frac{1}{c} \right) \right) & \text{for } a \neq 0 \\ F_s = 4\bar{V} \tau_o/R & \text{for } a = 0 \end{cases} \quad (\text{A.26})$$

$$c = \frac{R}{4\bar{V}a \tau_o} \quad (\text{A.27})$$

Eqs. (A.23) to (A.27) are the governing equations for determining the velocity profile or the radius profile in the present work. It can also provide the initial stress conditions for the differential treatment. The computer program used to obtain the

velocity profile is almost the same as that used by Nam, except that the adjustable parameter k and the calculation of $F1$ and $F2$ are introduced.

A-2. INITIAL CONDITIONS FOR DIFFERENTIAL TREATMENT

Setting X and ϕ_I equal zero in Eq. (A.25), one obtains the unconstrained die swell, that is,

$$[B(0)]^6 = 1 + F_s^2 k^2 \quad (A.28)$$

Setting X equals zero only, we can obtain the initial conditions for the differential treatment from Eqs. (A.23) to (A.25):

$$\begin{aligned} \phi_I &= B^2 [(1 + F_s^2 k^2) B^{-4} - B^2] \\ &= B^2 [B(0)]^6 B^{-4} - B^2 \end{aligned} \quad (A.29)$$

or

$$\begin{aligned} \phi_d &= [(1 + F_s^2 k^2) B^2 - B^2] \\ &= [B(0)]^6 B^{-4} - B^2 \end{aligned} \quad (A.30)$$

$$P_{11}(0)/G = (1 + F_s^2 k^2) B^2 = [B(0)]^6 / B^4 \quad (A.31)$$

$$P_{22}(0)/G = B^2 \quad (A.32)$$

or

$$F1(0) = P_{11}(0)/\sigma_{11}(0) \quad (A.33)$$

$$F2(0) = P_{22}(0)/\sigma_{11}(0) \quad (A.34)$$

APPENDIX B

DERIVATION OF INITIAL STRESS CONDITIONS FROM THE KINEMATICS SUGGESTED BY TANNER

The kinematics suggested by Tanner has the form as shown in Eq. (2.3). The integral model has the form:

$$\begin{aligned}\underline{\sigma} &= - \underline{P}\underline{I} + \underline{P} = - p\underline{I} + \int_{-\infty}^t M_1(t - t') \underline{C}^{-1} dt' \\ &= - \underline{P}\underline{I} + \int_0^{\infty} M_1(z) \underline{C}_t^{-1}(z) dz\end{aligned}\quad (B.1)$$

Then, according to tensor operation,

$$\begin{aligned}\underline{F}_t(0-) \cdot (\underline{\sigma} + \underline{P}\underline{I}) \cdot \underline{F}_t^T(0-) &= \int_0^{\infty} M_1(z) \underline{F}_t(0-) \cdot \underline{C}_t^{-1}(z) \cdot \underline{F}_t^T(0-) dz \\ &= \int_0^{\infty} M_1(z) \underline{C}_{0-}^{-1}(z) dz\end{aligned}\quad (B.2)$$

In the above equations, $\underline{F}$ is the relative deformation gradient tensor and the subscripts indicate the reference frame. Other definition of variables can be seen from Chapter 4. From Eq. (2.3), one obtains

$$\underline{F}_t(0-) = \begin{vmatrix} 1/\lambda_T^2 & g' & 0 \\ 0 & \lambda_T & 0 \\ 0 & 0 & \lambda_T \end{vmatrix}\quad (B.3)$$

If one further uses

$$M_1(z) = Ge^{(-z/\tau_{eff})} / \tau_{eff}\quad (B.4)$$

for single element model with effective relaxation time, one can

obtain, after performing the calculation using Eq. (4.5) and assume

$$\sigma_{22} = \sigma_{33} = 0,$$

$$(\sigma_{11} + P)/\lambda_T^4 + Pg' = G(1 + 2\gamma^2 \tau_{eff}^2) \quad (B.5)$$

$$\lambda_T^2 P = G \quad (B.6)$$

$$g' \lambda_T P = \tau_{eff} G \gamma \quad (B.7)$$

Furthermore, from Eqs. (B.5) to (B.7), one obtains

$$P_{11} = \lambda_T^4 G(1 + \tau_{eff}^2 \gamma^2) \quad (B.8)$$

$$P_{22} = B G \quad (B.9)$$

$$\sigma_{11}/\lambda_T^4 + G/\lambda_T^6 = G(1 + \tau_{eff}^2 \gamma^2) \quad (B.10)$$

Substituting $B = 1/\lambda_T$ into the above equations, and taking the radius-averaging on both sides, for capillary die, it is easily to obtain

$$P_{11}(0) = B^{-4} \left[1 + \frac{1}{2} (\tau_{eff}^2 \gamma_w^2) \right] \quad (B.11)$$

$$P_{22}(0)/G = B^2 \quad (B.12)$$

$$\phi_d = \sigma_{11}(0)/G = \left[1 + \frac{1}{2} (\tau_{eff}^2 \gamma_w^2) \right] B^{-4} - B^2 \quad (B.13)$$

For unconstrained die swell, that is, $\phi_d = 0$, we have from Eq. (B.13),

$$[B(0)]^6 = 1 + \frac{1}{2} (\tau_{\text{eff}} \dot{\gamma}_w)^2 \quad (\text{B.14})$$

which is Tanner's result when τ_{eff} equals τ_o .

Note that for single element Maxwellian model with effective relaxation time, we have

$$\tau_{\text{eff}} \dot{\gamma}_w = N_{1w}/2\sigma_{12w} \quad (\text{B.15})$$

Since τ_{eff} changes with the die flow conditions, the empirical relationship between N_{1w} and σ_{12w} is used in the present work, and the $1/2$ in Eqs. (B.11), (B.13) and (B.15) is replaced by k^2 . Also, note that if we use $(N_{1w}/2\sigma_{12w})$ instead of F_s in Eqs. (A.29) to (A.34), we have the same results for the initial conditions of both kinematical assumptions.

The present author also notes that there is a die swell at low shear conditions. To correct this purely viscous die swell, the correction factor Co such as shown in Eq. (2.17) or Eq. (2.18) is retained in correlating the experimental data. Consequently, the following equations are used to correlate the die flow effect:

$$B(0) = \left[1 + k (N_{1w}/2\sigma_{12w})^2 \right]^{1/6} + Co \quad (\text{B.16})$$

$$B^6 = [B(0)]^6 - \phi_d B^4 = [B(0)]^6 - \phi_I B^2 \quad (\text{B.17})$$

$$P_{11}(0)/G = [B(0)]^6 B^{-4} \quad (B.18)$$

$$P_{22}(0)/G = B^2 \quad (B.19)$$

$$F1(0) = P_{11}(0)/(P_{11}(0) - P_{22}(0)) = P_{11}(0)/\sigma_{11}(0) \quad (B.20)$$

$$F2(0) = P_{22}(0)/\sigma_{11}(0) \quad (B.21)$$

$$\phi_I = B^2 \phi_d = [B(0)]^6 B^{-2} - B^4 \quad (B.22)$$

In the present work, only rheological force is important. According to Eq. (2.26) and Eqs. (A.19) and (A.20), it is known that to compare the different approaches, it is better to use ϕ_I .

APPENDIX C

DERIVATION OF THE GOVERNING RHEOLOGICAL EQUATIONS IN MELT SPINNING USING WHITE-METZNER MODEL

The White-Metzner constitutive equation has the form:

$$\begin{cases} \underline{\sigma} = -p\underline{I} + \underline{P} \\ \underline{P} + \tau(\underline{\Pi}) \frac{\delta \underline{P}}{\delta t} = 2 \tau(\underline{\Pi}) G \underline{d} \\ \frac{\delta \underline{P}}{\delta t} = \frac{\partial \underline{P}}{\partial t} + (\underline{U} \cdot \nabla) \underline{P} - (\nabla \underline{U} \cdot \underline{P}) - \underline{P} \cdot (\nabla \underline{U})^T \end{cases} \quad (C.1)$$

and

$$\tau = \frac{\tau_o}{1 + a \frac{\underline{\Pi}^{1/2} \tau_o}{\underline{d}}} \quad (C.2)$$

For Purely elongational flow, which has been assumed valid beginning at some point downstream the die exit (in the present work, it is the maximum die swell position), the kinematics are

$$\begin{cases} \frac{\partial U_2}{\partial x_2} = \frac{\partial U_3}{\partial x_3} \\ U_1 = U_1(x_1) \end{cases} \quad (C.3)$$

and consequently, the rate of deformation tensor is

$$\underline{d} = \frac{1}{2} \begin{vmatrix} 2 \partial U_1 / \partial x_1 & \partial U_2 / \partial x_1 & \partial U_3 / \partial x_1 \\ \partial U_2 / \partial x_1 & 2 \partial U_2 / \partial x_2 & \partial U_3 / \partial x_2 + \partial U_2 / \partial x_3 \\ \partial U_3 / \partial x_1 & \partial U_3 / \partial x_2 + \partial U_2 / \partial x_3 & 2 \partial U_3 / \partial x_3 \end{vmatrix} \quad (C.4)$$

From the equation of continuity, one has, because of the symmetry about x_1 axis,

$$2(\partial U_2 / \partial x_2) + \partial U_1 / \partial x_1 = 0 \quad (C.5)$$

Integration of Eq. (C.4), one obtains

$$U_2 = -(1/2)(\partial U_1 / \partial x_1)x_2 \quad (C.6)$$

$$U_3 = -(1/2)(\partial U_1 / \partial x_1)x_3 \quad (C.7)$$

and finally the rate of deformation tensor Eq. (C.3) reduces to

$$\underline{d} = \frac{1}{2} \begin{vmatrix} 2\partial U_1 / \partial x_1 & -(1/2)(\partial^2 U_1 / \partial x_1^2)x_2 & -(1/2)(\partial^2 U_1 / \partial x_1^2)x_3 \\ -(1/2)(\partial^2 U_1 / \partial x_1^2)x_2 & -\partial U_1 / \partial x_1 & 0 \\ -(1/2)(\partial^2 U_1 / \partial x_1^2)x_3 & 0 & -\partial U_1 / \partial x_1 \end{vmatrix} \quad (C.8)$$

The off-diagonal terms in Eq.(C.8) are negligible compared to $\partial U_1 / \partial x_1$, so that Eq. (C.8) reduces to

$$\underline{d} = \frac{1}{2} \begin{vmatrix} 2\partial U_1 / \partial x_1 & 0 & 0 \\ 0 & -\partial U_1 / \partial x_1 & 0 \\ 0 & 0 & -\partial U_1 / \partial x_1 \end{vmatrix} \quad (C.9)$$

and

$$\Pi_{\underline{d}}^{1/2} = \left((2 \text{Tr } \underline{d}^2) \right)^{1/2} = \sqrt{3} \left| \partial U_1 / \partial x_1 \right| = \sqrt{3} \partial U_1 / \partial x_1 \quad (C.10)$$

Substituting Eqs. (C.9) to (C.10) into Eqs. (C.1) to (C.2) and further assuming that P_{11} are function of x_1 only, one can obtain the governing rheological equations:

$$\left\{ \begin{array}{l} \underline{\sigma} = - \underline{P} \underline{I} + \underline{P} \\ P_{11} + \tau(\underline{\Pi}) \frac{\partial P_{11}}{\partial t} + U_1 \frac{\partial P_{11}}{\partial x_1} - 2P_{11} \frac{\partial U_1}{\partial x_1} = 2 \tau(\underline{\Pi}) G \frac{\partial U_1}{\partial x_1} \\ P_{22} + \tau(\underline{\Pi}) \frac{\partial P_{22}}{\partial t} + U_1 \frac{\partial P_{22}}{\partial x_1} + P_{22} \frac{\partial U_1}{\partial x_1} = - \tau(\underline{\Pi}) G \frac{\partial U_1}{\partial x_1} \end{array} \right. \quad (C.11)$$

and the Bogue-White deformation rate dependence of relaxation time is

$$\tau = \tau_o / (1 + \sqrt{3} a \tau_o (\partial U_1 / \partial x_1)) \quad (C.12)$$

APPENDIX D

COMPUTER PROGRAM FOR TWO-POINTS BOUNDARY PROBLEM OF SPINNING

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C*****
C      THIS PROGRAM WORKS FOR DIE SWELL,MAX DRAW RATIO,VELOCITY AND
C      RADIUS PROFILE OF WHITE-METZNER MODEL FOR BOTH STABLE
C      AND STABILITY ANALYSIS UNDER A GIVEN DRAW RATIO
C      AND DIE FLOW CONDITIONS
C*****
C
C*****U--VELOCITY, Z--DISTANCE, Y,W,Q,C,--WORKING VARIABLES
C*****R--RADIUS, F1--STRESS IN 11 DIRECTION, DF1-DERIVATIVE OF F1
C*****YI--INITIAL VALUES OF PERTUBATIONS, CP--WORKING VARIABLES
C*****DU--DERIVATIVE OF U, DDU-DERIVATIVE OF DU, RTES-RESIDENCE TIME
C*****XAK--WORKING VARIABLES FOR EIGENVALUES
C*****CF--TOTAL VELOCITY PERTUBATION, CFX-WORKING VARIABLES
C*****YUR--REAL PART OF VELOCITY PERTUBATION
C*****YUI--IMAGINARY PART OF VELOCITY PERTUBATION
C*****XFT--DISTANCE TRIERS BASED ON DIE SWELL POSITION
C*****DRT--DRAW RATIO TRIERS BASED ON DIE SWELL POSITION
C*****BOT--DIE SWELL TRIERS
C*****PHI--DIMENSIONLESS TAKE-UP FORCE BASED ON DIE SWELL POSITION
C*****PHI--DIMENSIONLESS TAKE-UP FORCE BASED ON DIE FLOW
C*****AB--PARAMETER OF BOGUE-WHITE DEFORMATION RATE EQUATION
C*****SQR--SQUARE ROOT OF 3
C
      DIMENSION U(501),Z(501),Y(12),W(12),Q(12,9),C(24),DU(501)
      DIMENSION R(501),F1(501),DF1(501),YI(12),CP(501)
      DIMENSION DDU(501),CF(501),RTES(501)
      DIMENSION XAK(20),CFX(20)
      DIMENSION YUR(20),YUI(20)
      DIMENSION XFT(2),BOT(2),DRT(2)
      EXTERNAL DZ
      COMMON U,DU,F1,DF1,PHI,AB,AKR,AKI,SQR,DDU
      COMMON AA,I
C
C*****READ IN OR INPUT THE VALUES OF AB , DRF, XF,AND DELZ
C*****DRF--GIVEN DRAW RATIO BASED ON DIE FLOW VELOCITY
C*****XF--DIMENSIONLESS SPINLINE LENGTH BASED ON DIE FLOW VELOCITY
C*****DELZ--WALL SHEAR STRESS OR STRESS RATIO DEPENDING ON NA
C
10    CONTINUE
      AB=0.6
      DRF=14.930
20    CONTINUE
      XF=9.18420
30    CONTINUE
      DELZ=7.129E5
C

```

```

C*****GUESS A REASONABLE VALUE OF DIE SWELL
C*****I222--NUMBER OF ITERATION (IF > 25, STOP ITERATION)
C*****I111--CONTROL VARIABLE FOR DETERMINATION BOT FOR ITERATION
      BOT(2)=1.238744
      I222=0
      I111=1
      WRITE(5,270)AB,DELZ,DRF,XF
40    CONTINUE
      I222=I222+1
C*****B0--DIE SWELL VALUE
      B0=BOT(2)
50    CONTINUE
C*****XFT--SPINLINE LENGTH BASED ON VELOCITY AT DIE SWELL POSITION
C*****V0--VELOCITY AT DIE SWELL POSITION BASED ON DIE FLOW VELOCITY
      XFT(2)=XF*B0**2
      V0=(1./B0)**2
C*****MI--DETERMINING THE NUMBER OF INCREMENT FOR A GIVEN XFT
      MI=501
      DELX=XFT(2)/(MI-1.)
C*****
C*****READ IN OR SET NA:
C*****NA=1---PP1064 OR PP6823 WITH WALL SHEAR STRESS AS PRIMARY VAR.
C*****NA=2---PP1064 OR PP6823 WITH FAG1 AS PRIMARY VARIABLE
C*****NA=3---WHITE-METZNER-TANNER, IE.XK1=2,ZS=0.13
C*****
C      READ(5,140)NA
      NA=1
60    CONTINUE
C*****FAG1--STRESS RATIO
      IF(NA.NE.1)FAG1=DELZ
      IF(NA.EQ.3)GO TO 100
C
C*****INPUT THE RHEOLOGICAL PROPERTIES OF PP 106-4 AND PP 6823*****
C*****PP106-4,AB=0.6,T=0.1,G=4.8E5,A=6.1E-3,B=1.48,ZS=0.12,XK1=3*****
C*****PP 6823,AB=0.9,T=1.25,G=7.1E5,A=1.81E-4,B=1.76,ZS=0.33,XK1=3**
C
      IF(AB.EQ.0.6)GO TO 70
      T0=1.25
      G0=710000.
      A0=0.000181
      B00=1.76
      ZS=0.3
      XK1=2.
      GO TO 80
70    CONTINUE
      T0=0.1
      G0=480000.
      A0=0.0061
      B00=1.48
      ZS=0.1
      XK1=2.
80    CONTINUE

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      IF(NA.EQ.2)GO TO 90
C*****ZGMW--SHEAR STRESS, XN1W--FIRST NORMAL STRESS DIFFERENCE
C*****GAM--DIMENSIONLESS VARIABLE
      ZGMW=DELZ
      GAM=ZGMW/GO
      XN1W=A0*ZGMW**B00
C*****ZM IS CALCULATED FROM AB,ZGMW,AND LOW SHEAR RATE DIE SWELL**
C*****ZM--CALCULATED VARIABLE CORRESPONDING TO ZS
      FAG1=XN1W/(2.*ZGMW)
      GO TO 110
90    CONTINUE
      ZGMW=(2.*FAG1/A0)**(1./(B00-1.))
      XN1W=2.*ZGMW*FAG1
      GAM=ZGMW/GO
      GO TO 110
100   CONTINUE
      XK1=2.
      ZS=0.13
      ZGMW=8888.
      GAM=8888.
110   CONTINUE
      FAG=FAG1**2
      XK=1./XK1
      ZM=1./XK*(((1.+FAG*XK)**(1./6.))+ZS)**6-(1+FAG*XK))
      PAR=1.+(FAG+ZM)*XK
      PHI=(1.+(FAG+ZM)*XK)/B0**4-B0**2
      IF(PHI.GT.0.)GO TO 120
      B0=B0-0.0005
      GO TO 50
120   CONTINUE
C*****PHII--DIMENSIONLESS TAKE-UP FORCE BASE ON DIE FLOW
C*****F1(1)--INITIAL STRESS IN 11 DIRECTION
C*****F20--INITIAL STRESS IN 22 DIRECTION
C*****F120--F1(1)/F2
      PHII=PHI*B0**2
      F1(1)=(1.+(FAG+ZM)*XK)/B0**4/PHI
      F20=B0**2/PHI
      F120=PAR/B0**6
      WRITE(5,130)
130   FORMAT(1X,'NA=1--PP AND ZGMW,NA=2--PP AND FAG1,NA=3--W-M-T')
      WRITE(5,140)NA
140   FORMAT(1X,I1)
      WRITE(5,280)
      WRITE(5,290)AB,ZGMW,FAG1,GAM,B0,PHI,PHII,F1(1),F120
C
C*****DETERMINING THE REQUIRED SOLUTION:
C*****NC=1---MODIFIED WHITE-IDE SOLUTION (STEADY SOLUTION ONLY)
C*****NC=2---MODIFIED FISHER-DENN SOLUTION
C*****NC=3---GENERAL DIFFERENTIAL TREATMENT SOLUTION
C
      WRITE(5,150)
150   FORMAT(1X,'NC=1---MOD. W-I, NC=2---MOD. F-D, NC=3---GENERAL')

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```

C      READ(5,140)NC
      NC=3
      N=NC
      WRITE(5,140)NC
      SQR=SQRT(3.)*AB
C*****U--VELOCITY BASED ON DIE SWELL POSITION
C*****R--RADIUS BASED ON DIE SWELL POSITION
      U(1)=1.
      R(1)=SQRT(1./U(1))
      T=0.
      RTES(1)=T
      IF(N.NE.3)F1(1)=U(1)
      BX=3.*F1(1)+3./PHI-(2.+SQR)
      DU(1)=1./BX
      CP(1)=1./3.*(SQR+2.)*U(1)-1./PHI-F1(1)
      DDU(1)=(1./(U(1)**2))*(U(1)*DU(1)**3*(SQR-2.+3.*AB**2)-3./PHI
1*SQR*DU(1)**3+(2.*SQR+1.)*U(1)*DU(1)**2-3./PHI*DU(1)**2+U(1)
2*DU(1))
      IF(N.EQ.1)GO TO 160
      DF1(1)=1./3.+1./3.*DU(1)*(SQR+2.)-1./3.*U(1)*DDU(1)/(DU(1)**2)
      GO TO 170
160    CONTINUE
      DF1(1)=DU(1)
170    CONTINUE
      WRITE(5,180)AB,PHI,PHII,F1(1),VO,BO,ZS
180    FORMAT(/,2X,'AB=',G15.7,/,1X,'PHI=',G15.7,/,1X,'PHII=',G15.7,
1/,1X,'F1(1)=',G15.7,/,2X,'VO,BO=',2G15.7,/,2X,'ZS=',G15.7)
      IND=1
      NQ=12
      TOL=0.00001
      X1=0.
      Z(1)=X1
C*****Y(1)--VELOCITY, Y(2)--VELOCITY GRADIENT,-----STEADY SPINNING
      Y(1)=U(1)
      Y(2)=DU(1)
      WRITE(5,300)
      WRITE(5,310)Z(1),R(1),U(1),DU(1),IND,IER,F1(1),RTES(1),CP(1)
      DO 190 I=2,MI
      X=(I-1)*DELX
      CALL DVERK(N,DZ,X1,Y,X,TOL,IND,C,NQ,Q,IER)
      Z(I)=X1
      R(I)=SQRT(1./U(I))
      CP(I)=1./3.*(SQR+2.)*U(I)-1./PHI-F1(I)
      RTES(I)=T+(2.*DELX)/(U(I)+U(I-1))
      T=RTES(I)
      IF(IND.LT.0) GO TO 200
      IF(IER.GT.0) GO TO 200
      IF(DU(I).GT.1.E20)GO TO 200
      IM=I
190    CONTINUE
      GO TO 210
200    CONTINUE

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      IM=I
210  CONTINUE
C*****RTF--RESIDENCE TIME (SEC.)
      RTF=T*T0
      WRITE(5,220)IM,MI
220  FORMAT(/,1X,'MAX. I OR DU>E20(I < MI), AS I=',I4,2X,'MI=',I4)
      WRITE(5,310) Z(IM),R(IM),U(IM),DU(IM),IND,IER,F1(IM),RTES(IM),
        1CP(IM)
C*****UI--FINAL VELOCITY BASED ON DIE FLOW VELOCITY
C*****RI--FINAL RADIUS BASED ON DIE RADIUS
C*****ZI--FINAL SPILLINE LENGTH BASED ON DIE FLOW VELOCITY
      UI=V0*U(IM)
      RI=SQRT(1./UI)
      ZI=V0*Z(IM)
      WRITE(5,360)ZI,B0,UI,PHII
      IF(IM.EQ.MI)GO TO 230
      B0=B0+0.001
      GO TO 50
230  CONTINUE
      DRT(2)=UI
      IF(ABS(DRT(2)-DRF).LE.0.005)GO TO 260
      IF(I222.GE.25)GO TO 850
      IF(I111.NE.1)GO TO 240
      DRT(1)=DRT(2)
      XFT(1)=XFT(2)
      BOT(1)=B0
      BOT(2)=B0-0.0001
      I111=2
      GO TO 40
240  CONTINUE
      BOT(2)=(DRF-DRT(1))/(DRT(2)-DRT(1))*(BOT(2)-BOT(1))+BOT(1)
      DRT(1)=DRT(2)
      XFT(1)=XFT(2)
      BOT(1)=B0
      WRITE(5,250)BOT(2)
250  FORMAT(/,1X,G15.7,/)
      GO TO 40
260  CONTINUE
      WRITE(5,270)AB,DELZ,DRF,XF
      WRITE(1,270)AB,DELZ,DRF,XF
270  FORMAT(1X,'AB  =' ,G15.7,/,1X,'DELZ=' ,G15.7,/,1X,'DRF =' ,G15.7,
        1/,1X,'XF  =' ,G15.7)
      WRITE(1,130)
      WRITE(1,140)NA
      WRITE(1,150)
      WRITE(1,140)NC
      WRITE(5,280)
      WRITE(1,280)
280  FORMAT(/,12X,'AB  =' ,10X,'ZGMW=' ,10X,'FAG1=' ,10X,'GAM  =' ,
        1/,5X,'B0  =' ,10X,'PHI  =' ,10X,'PHII=' ,10X,'F10 =' ,10X,
        2'F120=')
      WRITE(1,290)AB,ZGMW,FAG1,GAM,B0,PHI,PHII,F1(1),F120

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WRITE(5,290)AB,ZGMW,FAG1,GAM,B0,PHI,PHII,F1(1),F120
290  FORMAT(7X,4G15.7,/,1X,5G15.7)
WRITE(1,220)IM,MI
WRITE(5,300)
WRITE(1,300)
300  FORMAT(5X,'Z(I)=' ,10X,'R(I)=' ,10X,'U(I)=' ,10X,'DU(I)='
      1,2X,'IND=' ,2X,'IER=' ,/,30X,'F1(I)=' ,9X,'RTES(I)=' ,8X,'CP(I)=' )
WRITE(1,310) Z(IM),R(IM),U(IM),DU(IM),IND,IER,F1(IM),RTES(IM),
      1CP(IM)
WRITE(5,310) Z(IM),R(IM),U(IM),DU(IM),IND,IER,F1(IM),RTES(IM),
      1CP(IM)
310  FORMAT(1X,4G15.7,2I4,/,30X,3G15.7)
WRITE(5,360)ZI,RI,UI,RTES(IM)
WRITE(1,360)ZI,RI,UI,RTES(IM)
WRITE(1,180)AB,PHI,PHII,F1(1),V0,B0,ZS
C*****DO STABILITY ANALYSIS OR NOT--BY CHOSING IN VALUE:*****
C*****READ IN OR SET VALUE OF "IN":
C*****IN<3---WRITE VEL. PRO. AND CHANGE FAG1 OR ZGMW ETC. IF DESIRED
C*****IN=4---DO STABLITIES ANALYSIS
C
      WRITE(1,320)
      WRITE(5,320)
320  FORMAT(1X,'IN<3--STEADY,IN=4--STAB. AN. ')
C  READ(5,140)IN
      IN=4
      WRITE(1,330)IN
      WRITE(5,330)IN
330  FORMAT(1X,I1)
      IF(IN.LE.3)GO TO 340
      IF(IN.EQ.4) GO TO 450
340  CONTINUE
      XJ=IM
      WRITE(20,350)XJ
      WRITE(1,350)XJ
350  FORMAT('DATA',T15,G15.6)
      DO 370 J=1,IM
C*****CALCULATING VELOCITY (OR RADIUS) PROFILES BASED ON DIE FLOW
      ZI=V0*Z(J)
      UI=V0*U(J)
      RI=SQRT(1./UI)
      WRITE(20,360)ZI,RI,UI,RTES(J)
      WRITE(1,360)ZI,UI,RI,RTES(J)
360  FORMAT(4G15.7)
370  CONTINUE
C*****CHANGE DELZ VALUE AND DO AGAIN IF NECESSARY
      WRITE(5,380)
      WRITE(1,380)
380  FORMAT(1X,'CHANG DELZ?Y/N')
      READ(5,390)AN
      WRITE(1,390)AN
390  FORMAT(A1)
      IF(AN.EQ.1HN)GO TO 850

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      DELZ=DELZ*10.
      IF(NA.EQ.1.AND.DELZ.LE.1.E7)GO TO 60
      IF(NA.NE.1.AND.DELZ.LE.5.E0)GO TO 60
      WRITE(5,400)
      WRITE(1,400)
400   FORMAT(1X,'CHANG DRF?Y/N')
      READ(5,390)AN
      WRITE(1,390)AN
      IF(AN.EQ.1HN)GO TO 410
      DRF=DRF*5.
      IF(DRF-100.)20,20,410
410   CONTINUE
      WRITE(5,420)
      WRITE(1,420)
420   FORMAT(1X,'CHANG XF?Y/N')
      READ(5,390)AN
      WRITE(1,390)AN
      IF(AN.EQ.1HN)GO TO 430
      XF=XF*5.
      IF(XF-1000.)30,30,850
430   CONTINUE
      WRITE(1,440)
      WRITE(5,440)
440   FORMAT(1X,'CHANGE NA OR NC?Y/N')
      READ(5,390)AN
      WRITE(1,390)AN
      IF(AN.EQ.1HN)GO TO 850
      GO TO 10
450   CONTINUE
C
C*****DO STABILITY ANALYSIS*****
C*****Y(1)--REAL PART OF VELOCITY PERTUBATION
C*****Y(2)--IMAGINARY PART OY VELOCITY PERTUBATION
C*****Y(3)--REAL PART OF AREA PERTUBATION
C*****Y(4)--IMAGINARY PART OF AREA PERTUBATION
C*****Y(5)--REAL PART OF F1 (STRESS IN 11 DIRECTION) PERTUBATION
C*****Y(6)--IMAGINARY PART OF F1 PERTUBATION
C*****Y(7)--REAL PART PERTUBATION OF F1-F2
C*****Y(8)--IMAGINARY PART PERTUBATION OF F1-F2
C*****Y(9)--STEADY STATE VELOCITY
C*****Y(10)--STEADY STATE VELOCITY GRADIENT
C*****
C
C*****DR--DRAW RATIO BASED ON DIE SWELL VELOCITY
C*****AL--ELASTICITY VALUE BASED ON DIE SWELL VELOCITY
      DR=U(IM)
      AL=1./Z(IM)
460   CONTINUE
      IF(NC.EQ.1)N=9
      IF(NC.GT.1)N=10
      DO 470 L=1,N
      YI(L)=0.0

```

```

470  CONTINUE
C*****YI(7)--INITIAL PERTUBATION OF Y(7)
C*****CF--TOTAL PERTUBATION OF VELOCITY
C*****IALER--CONTROL VARIABLE (NO. OF ITERATION)
C*****I1,I3,IAX,IAY--CONTROL VARIABLES FOR ITERATION OF EIGENVALUE
      YI(7)=1.0
      CF(1)=0.
      IALER=1
      II=1
      I3=1
      IAX=0
      IAY=0
C*****AKRMI CAN BE SET AS + OR - OR 0.0 *****
C*****READ IN OR SET AKIMD AND AKRMI
C*****AKIMD--REASONABLE GUESS OF IMAGINARY PART OF EIGENVALUE
C*****AKI--IMAGINARY PART OF EIGENVALUE
C*****AKR--REAL PART OF EIGENVALUE
C*****AKRMI--REASONABLE GUESS OF REAL PART OF EIGENVALUE
      AKIMD=0.9479058
      IF(IEIG.EQ.1)AKIMD=AKI
      AKI=AKIMD
C*****FOLLOWING IS THE ITERATION METHOD TO GET EIGENVALUE SOME
C*****MODIFICATIONS ARE REQUIRED DEPENDING ON THE EIGENVALUE
C*****GUESSED,SEVERAL TRIERS BEFORE SEEKING THE EXACT
C*****VALUES OF AKR AND AKI IS NECESSARY
C*****FIRST TRY AKR THEN AKI,AKI,AKR,AKR,AKI,AKI,AKR,AKR...
      AKRMI=0.001
      IF(IEIG.EQ.1)AKRMI=AKR/1.2
      IF(AKRMI.EQ.0.0)GO TO 810
      XAK(2)=AKRMI
      XAK(13)=XAK(2)*1.5
      DAK=(XAK(13)-XAK(2))/5.
      ICOL=1
480  CONTINUE
      IALX=0
      DO 610 IAL=8,13
      XAK(IAL)=XAK(2)+(IAL-8.)*DAK
      IF(ICOL.EQ.1)AKR=XAK(IAL)
      IF(ICOL.EQ.2)AKI=XAK(IAL)
      DO 490 L=1,N
      Y(L)=YI(L)
      Y(9)=U(1)
      Y(10)=DU(1)
490  CONTINUE
      IND=1
      NQ=12
      TOL=0.00001
      X1=0.
      Z(1)=X1
C*****THE EIGENFUNCTION Y(I) CAN BE WRITTEN IN DATA FILE IF REQUIRED
C      WRITE(5,500)
C      WRITE(1,500)

```

```

500  FORMAT(5X,'Z(I)=' ,10X,'R(I)=' ,10X,'U(I)=' ,10X,'CF(I)='
      1,2X,'IND=' ,2X,'IER=' ,/,30X,'Y(1)=' ,9X,'Y(2)=' ,8X,'Y(7)=' )
C    WRITE(1,310)Z(1),R(1),U(1),CF(1),IND,IER,Y(1),Y(2),Y(7)
C    WRITE(5,310)Z(1),R(1),U(1),CF(1),IND,IER,Y(1),Y(2),Y(7)
C*****
      DO 510 I=2,IM
      X=(I-1)*DELX
      CALL DVERK(N,DZ,X1,Y,X,TOL,IND,C,NQ,Q,IER)
      Z(I)=X1
      U(I)=Y(9)
      ZI=Z(I)*V0
      UI=V0*U(I)
      RI=SQRT(1./UI)
      CF(I)=ABS(Y(1))+ABS(Y(2))
C*****EIGENFUNCTION CAN BE WRITTEN IN DATA FILE, IF DESIRED *****
C    WRITE(5,310)Z(I),R(I),U(I),CF(I),IND,IER,Y(1),Y(2),Y(7)
C    WRITE(1,310)Z(I),R(I),U(I),CF(I),IND,IER,Y(1),Y(2),Y(7)
      IF(IND.LT.0)GO TO 520
      IF(IER.GT.0)GO TO 520
      ITM=I
510   CONTINUE
      GO TO 530
520   CONTINUE
      ITM=I
530   CONTINUE
      IF(ITM.NE.IM)GO TO 610
      WRITE(5,540)AKR,Y(1),AKI,Y(2),CF(ITM),Y(9),X1
      WRITE(1,540)AKR,Y(1),AKI,Y(2),CF(ITM),Y(9),X1
540   FORMAT(7G15.7)
      IF(CF(ITM).LE.5.E-2.AND.ITM.EQ.IM)GO TO 580
      IALX=IALX+1
      IF(ICOL.NE.1)GO TO 570
      AKRMA=XAK(IAL)
      AKRMD=XAK(IAL-1)
      AKRMI=XAK(IAL-2)
      CFX(IAL)=CF(ITM)
      CFR1=CFX(IAL-1)
      CFR2=CFX(IAL-2)
      CFRO=CFX(IAL)
      YUR(IAL)=Y(1)
      YUI(IAL)=Y(2)
      IF(IALX.EQ.2.AND.YUR(9).GT.0..AND.YUR(8).GT.0..AND.YUR(9)
1.GT.YUR(8))GO TO 550
      IF(IALX.EQ.2.AND.YUR(9).LT.0..AND.YUR(8).LE.0..AND.YUR(9)
1.LT.YUR(8))GO TO 550
      IF(IALX.GE.2.AND.YUR(IAL).LE.0.0.AND.YUR(IAL-1).GE.0.0)GO TO
1550
      IF(IALX.GE.2.AND.YUR(IAL).GE.0.0.AND.YUR(IAL-1).LE.0.0)GO TO
1550
      IF(IALX.GE.3.AND.ABS(YUR(IAL)).LE.1.E-1.AND.ABS(YUR(IAL-1)).LE.
11.E-1.AND.ABS(YUI(IAL)).GT.1..AND.ABS(YUI(IAL-1)).GT.1.)
2GO TO 740

```

```

IF(IALX.GE.3.AND.ABS(YUR(IAL)).LE.1.E-2.AND.ABS(YUR(IAL-1)).LE.
11.E-2.AND.ABS(YUI(IAL)).GT.0.1.AND.ABS(YUI(IAL-1)).GT.0.1)
2GO TO 740
IF(IAL.EQ.13.AND.YUR(11).GT.YUR(12).AND.
1YUR(12).GT.YUR(13))GO TO 620
IF(IAL.EQ.13.AND.YUR(11).LT.YUR(12).AND.
2YUR(12).LT.YUR(13))GO TO 620
IF(IALX.GE.2)CFER=ABS(YUR(IAL)-YUR(IAL-1))
IF(IALX.GT.2.AND.CFER.LE.5.E-4.AND.AKRMI.GT.0.)GO TO 630
IF(IALX.GE.2.AND.CFER.LE.5.E-4.AND.AKRMI.LT.0.)GO TO 650
GO TO 610
570 AKIMA=XAK(IAL)
AKIMD=XAK(IAL-1)
AKIMI=XAK(IAL-2)
YUI(IAL)=Y(2)
IF(AKR.EQ.0.0.AND.ABS(YUI(IAL)).LT.5.E-3.AND.ITM.EQ.IM)GO TO
1580
YUR(IAL)=Y(1)
CFX(IAL)=CF(ITM)
CFIO=CFX(IAL)
CFI1=CFX(IAL-1)
CFI2=CFX(IAL-2)
IF(IALX.EQ.2.AND.YUI(9).GT.0..AND.YUI(8).GT.0..AND.YUI(9)
1.GT.YUI(8))GO TO 590
IF(IALX.EQ.2.AND.YUI(9).LT.0..AND.YUI(8).LE.0..AND.YUI(9)
1.LT.YUI(8))GO TO 590
IF(IALX.GE.2.AND.YUI(IAL).LE.0.0.AND.YUI(IAL-1).GE.0.0)GO TO
1590
IF(IALX.GE.2.AND.YUI(IAL).GE.0.0.AND.YUI(IAL-1).LE.0.0)GO TO
1590
IF(IALX.GE.3.AND.ABS(YUI(IAL)).LE.1.E-1.AND.ABS(YUI(IAL-1)).LE.
11.E-1.AND.ABS(YUR(IAL)).GT.1..AND.ABS(YUR(IAL-1)).GT.1.)
2GO TO 800
IF(IALX.GE.3.AND.ABS(YUI(IAL)).LE.1.E-2.AND.ABS(YUI(IAL-1)).LE.
11.E-2.AND.ABS(YUR(IAL)).GT.0.1.AND.ABS(YUR(IAL-1)).GT.0.1)
2GO TO 800
IF(IAL.EQ.13.AND.YUI(11).GT.YUI(12).AND.
1YUI(12).GT.YUI(13))GO TO 620
IF(IAL.EQ.13.AND.YUI(11).LT.YUI(12).AND.
2YUI(12).LT.YUI(13))GO TO 620
610 CONTINUE
C*****NO AKR OR AKI EXIST IN THIS RANGE
620 CONTINUE
IF(ICOL.EQ.1)AKRMD=-(XAK(11)-XAK(13))/(YUR(11)-YUR(13))*YUR(13)
1+XAK(13)
IF(ICOL.EQ.2)AKIMD=-(XAK(11)-XAK(13))/(YUI(11)-YUI(13))*YUI(13)
2+XAK(13)
IF(ICOL.EQ.1.AND.AKRMI.GT.0.0)GO TO 630
IF(ICOL.EQ.1.AND.AKRMI.LE.0.0)GO TO 650
IF(ICOL.EQ.2)GO TO 670
630 CONTINUE
WRITE(5,640)AKRMA

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```

640  FORMAT(1X,'NO +AKR AS AKR< ',G15.7)
      IF(AKRMA.GT.5.)GO TO 580
      IF(AKRMA.LT.0.00010)GO TO 700
      IAX=IAX+1
      IF(IAX.EQ.3)GO TO 740
      ICOL=1
      AKI=AKIMD
      XAK(2)=AKRMD/1.1
      XAK(13)=AKRMD*1.1
      DAK=(XAK(13)-XAK(2))/5.
      GO TO 480
650  CONTINUE
      WRITE(5,660)AKRMA
660  FORMAT(1X,'NO -AKR AS AKR> ',G15.7)
      IF(AKRMA.LE.-10.)GO TO 580
      IF(AKRMA.GT.-0.00010)GO TO 580
      IAX=IAX+1
      IF(IAX.EQ.3)GO TO 740
      AKI=AKIMD
      ICOL=1
      XAK(2)=AKRMD/1.1
      XAK(13)=AKRMD*1.1
      DAK=(XAK(13)-XAK(2))/5.
      GO TO 480
670  CONTINUE
      WRITE(5,680)AKIMA
680  FORMAT(1X,'NO AKI AS AKI< ',G15.7)
      IF(AKIMA.GT.50.)GO TO 580
      IF(AKIMA.LT.0.0001)GO TO 580
      IF(AKR.EQ.0.)GO TO 810
      IAY=IAY+1
      IF(IAY.EQ.3)GO TO 800
      AKR=AKRMD
690  CONTINUE
      XAK(2)=AKIMD/1.01
      XAK(13)=AKIMD*1.01
      DAK=(XAK(13)-XAK(2))/5.
      ICOL=2
      GO TO 480
700  CONTINUE
      IALER=1
      IAX=0
      IAY=0
      AKI=AKIMD
      XAK(2)=-0.001
      XAK(13)=XAK(2)*1.5
      DAK=(XAK(13)-XAK(2))/5.
      ICOL=1
      GO TO 480
550  IF(IAL.GT.9)GO TO 720
      IF(IALER.GT.30)GO TO 580
      AKRMI=AKRMD/1.1

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```

720  CONTINUE
      WRITE(5,730)AKRMI,AKRMA
730  FORMAT(1X,'EXIST AKR BETW. <AKR> ',2G15.7)
      IF(AKRMA.GT.0.0.AND.AKRMA.LT.0.00010)GO TO 700
      IF(AKRMA.LT.0.0.AND.AKRMA.GT.-0.00010)GO TO 580
      IF(II.NE.1)GO TO 740
      AKI=AKIMD
      XAK(2)=AKRMI
      XAK(13)=AKRMA
      DAK=(XAK(13)-XAK(2))/5.
      ICOL=1
      II=2
      GO TO 480
740  CONTINUE
      IAX=0
      AKR=AKRMD
      IF(IALER.NE.1)GO TO 750
      XAK(2)=AKIMD/1.01
      XAK(13)=AKIMD*1.01
      GO TO 760
750  CONTINUE
      XAK(2)=AKIMI
      XAK(13)=AKIMA
760  ICOL=2
      DAK=(XAK(13)-XAK(2))/5.
      II=1
      GO TO 480
590  CONTINUE
      IF(AKR.EQ.0.)GO TO 810
      IF(IAL.GT.9)GO TO 780
      AKIMI=AKIMD/1.01
780  CONTINUE
      WRITE(5,790)AKIMI,AKIMA
790  FORMAT(1X,'EXIST AKI BETW. <AKI> ',2G15.7)
      IF(AKIMA.GT.50..OR.AKIMA.LT.0.0001)GO TO 580
      IF(I3.NE.1)GO TO 800
      AKR=AKRMD
      XAK(2)=AKIMI
      XAK(13)=AKIMA
      ICOL=2
      DAK=(XAK(13)-XAK(2))/5.
      I3=2
      IF(AKR.EQ.0.0)I3=1
      IF(AKR.EQ.0.0)IALER=IALER+1
      GO TO 480
800  CONTINUE
      IF(AKR.EQ.0.0)GO TO 810
      IAY=0
      AKI=AKIMD
      XAK(2)=AKRMI
      XAK(13)=AKRMA
      DAK=(XAK(13)-XAK(2))/5.

```

```

      ICOL=1
      I3=1
      IALER=IALER+1
      IF(IALER.GT.30)GO TO 580
      GO TO 480
810   CONTINUE
      AKRMD=0.
      AKR=AKRMD
      IF(IALER.NE.1)GO TO 820
      XAK(2)=AKIMD
      ICOL=2
      XAK(13)=AKIMD*1.01
      DAK=(XAK(13)-XAK(2))/5.
      GO TO 480
820   CONTINUE
      IF(IAL.GT.9)GO TO 830
      AKIMI=AKIMD/1.01
830   CONTINUE
      WRITE(5,790)AKIMI,AKIMA
      XAK(2)=AKIMI
      XAK(13)=AKIMA
      ICOL=2
      DAK=(XAK(13)-XAK(2))/5.
      IALER=IALER+1
      IF(IALER.GT.30)GO TO 580
      GO TO 480
580   CONTINUE
      IEIG=0
      IF(CF(ITM).LE.5.E-2)IEIG=1
      IF(AKR.EQ.0.0.AND.CF(ITM).LE.5.E-2)GO TO 850
      IF(AKR.LT.0)GO TO 850
850   CONTINUE
      STOP
      END
C*****
C*****
      SUBROUTINE DZ(N,X,Y,W)
      DIMENSION Y(N),W(N)
      DIMENSION U(501),DU(501),F1(501),DF1(501),DDU(501)
      REAL Y,W
      COMMON U,DU,F1,DF1,PHI,AB,AKR,AKI,SQR,DDU
      COMMON AA,I
      IF(N.GE.9)GO TO 45
      IF(N.LT.2)GO TO 40
      W(1)=Y(2)
      W(2)=(1./(Y(1)**2))*(Y(1)*Y(2)**3*(SQR-2.+3.*AB**2)-3./PHI
      1*SQR*Y(2)**3+(2.*SQR+1.)*Y(1)*Y(2)**2-3./PHI*Y(2)**2+Y(1)
      2*Y(2))
      U(I)=Y(1)
      DU(I)=W(1)
      F1(I)=(U(I)/(3.*DU(I)))+1./3.*U(I)*(2.+SQR)-1./PHI
      DDU(I)=(1./(U(I)**2))*(U(I)*DU(I)**3*(SQR-2.+3.*AB**2)-3./PHI

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```

1*SQR*DU(I)**3+(2.*SQR+1.)*U(I)*DU(I)**2-3./PHI*DU(I)**2+U(I)
2*DU(I))
DF1(I)=1./3.+1./3.*DU(I)*(SQR+2.)-1./3.*U(I)*DDU(I)/(DU(I)**2)
GO TO 60
40 W(1)=1./(3./(PHI*Y(1))+1.-SQR)
   U(I)=Y(1)
   DU(I)=W(1)
   F1(I)=U(I)
   DF1(I)=DU(I)
   DDU(I)=(1./(U(I)**2))*(U(I)*DU(I)**3*(SQR-2.+3.*AB**2)-3./PHI
1*SQR*DU(I)**3+(2.*SQR+1.)*U(I)*DU(I)**2-3./PHI*DU(I)**2+U(I)
2*DU(I))
   GO TO 60
C*****
45 CONTINUE
   IF(N.EQ.9)GO TO 50
   W(9)=Y(10)
   W(10)=(1./(Y(9)**2))*(Y(9)*Y(10)**3*(SQR-2.+3.*AB**2)-3./PHI
1*SQR*Y(10)**3+(2.*SQR+1.)*Y(9)*Y(10)**2-3./PHI*Y(10)**2+Y(9)
2*Y(10))
   U(I)=Y(9)
   DU(I)=W(9)
   F1(I)=(U(I)/(3.*DU(I)))+1./3.*U(I)*(2.+SQR)-1./PHI
   DDU(I)=(1./(U(I)**2))*(U(I)*DU(I)**3*(SQR-2.+3.*AB**2)-3./PHI
1*SQR*DU(I)**3+(2.*SQR+1.)*U(I)*DU(I)**2-3./PHI*DU(I)**2+U(I)
2*DU(I))
   DF1(I)=1./3.+1./3.*DU(I)*(SQR+2.)-1./3.*U(I)*DDU(I)/(DU(I)**2)
   GO TO 55
50 CONTINUE
   W(9)=1./(3./(PHI*Y(9))+1.-SQR)
   U(I)=Y(9)
   DU(I)=W(9)
   F1(I)=U(I)
   DF1(I)=DU(I)
   DDU(I)=(1./(U(I)**2))*(U(I)*DU(I)**3*(SQR-2.+3.*AB**2)-3./PHI
1*SQR*DU(I)**3+(2.*SQR+1.)*U(I)*DU(I)**2-3./PHI*DU(I)**2+U(I)
2*DU(I))
55 CONTINUE
   AA=1./(3./PHI+3.*F1(I)-2.*U(I)-SQR*U(I))
   W(1)=AA*(AKR*(Y(7)+U(I)**2*Y(3))+Y(7)+DU(I)*(2.*Y(7)-3.*Y(5)
1+SQR*Y(7))-AKI*(Y(8)+U(I)**2*Y(4)))
   W(2)=AA*(AKI*(Y(7)+U(I)**2*Y(3))+Y(8)+DU(I)*(2.*Y(8)-3.*Y(6)
1+SQR*Y(8))+AKR*(Y(8)+U(I)**2*Y(4)))
   W(3)=(1./U(I))*(DU(I)*Y(1)/U(I)**2-W(1)/U(I)-(AKR+DU(I))*Y(3)
1+AKI*Y(4))
   W(4)=(1./U(I))*(DU(I)*Y(2)/U(I)**2-W(2)/U(I)-(AKR+DU(I))*Y(4)
1-AKI*Y(3))
   W(5)=(1./U(I))*(W(1)*(2./PHI+(2.-SQR)*F1(I))+DU(I)*(2.-SQR)*Y(5)
1-Y(1)*DF1(I)-(1.+AKR)*Y(5)+AKI*Y(6))
   W(6)=(1./U(I))*(W(2)*(2./PHI+(2.-SQR)*F1(I))+DU(I)*(2.-SQR)*Y(6)
1-Y(2)*DF1(I)-(1.+AKR)*Y(6)-AKI*Y(5))
   W(7)=(-1./U(I))*(Y(7)-3./PHI*W(1)+Y(7)*(AKR+(SQR+1.)*DU(I))-AKI

```

```
1*Y(8)+DU(I)*(Y(1)-3.*Y(5))+W(1)*(U(I)-3.*F1(I)+SQR*U(I))
W(8)=(-1./U(I))*(Y(8)-3./PHI*W(2)+Y(8)*(AKR+(SQR+1.)*DU(I))+AKI
1*Y(7)+DU(I)*(Y(2)-3.*Y(6))+W(2)*(U(I)-3.*F1(I)+SQR*U(I))
60  CONTINUE
    RETURN
    END
```

APPENDIX E

COMPUTER PROGRAM FOR INITIAL PROBLEM OF SPINNING

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C*****
C      THIS PROGRAM WORKS FOR DIE SWELL,MAX DRAW RATIO,VELOCITY AND
C      RADIUS PROFILE OF WHITE-METZNER MODEL FOR BOTH STABLE AND
C      STABILITY ANALYSIS UNDER A CONSTANT TAKE-UP FORCE
C      AND DIE FLOW CONDITIONS
C*****
C
C*****U--VELOCITY, Z--DISTANCE, Y,W,Q,C,--WORKING VARIABLES
C*****R--RADIUS, F1--STRESS IN 11 DIRECTION, DF1-DERIVATIVE OF F1
C*****YI--INITIAL VALUES OF PERTUBATIONS, CP--WORKING VARIABLES
C*****DU--DERIVATIVE OF U, DDU-DERIVATIVE OF DU, RTES-RESIDENCE TIME
C*****XAK--WORKING VARIABLES FOR EIGENVALUES
C*****CF--TOTAL VELOCITY PERTUBATION, CFX-WORKING VARIABLES
C*****YUR--REAL PART OF VELOCITY PERTUBATION
C*****YUI--IMAGINARY PART OF VELOCITY PERTUBATION
C*****PHII--DIMENSIONLESS TAKE-UP FORCE BASED ON DIE FLOW
C*****PHI--DIMENSIONLESS TAKE-UP FORCE BASED ON DIE SWELL POSITION
C*****AB--PARAMETER OF BOGUE-WHITE DEFORMATION RATE EQUATION
C*****SQR--SQUARE ROOT OF 3
C
      DIMENSION U(1001),Z(1001),Y(12),W(12),Q(12,9),C(24),DU(1001)
      DIMENSION R(1001),F1(1001),DF1(1001),YI(12),CP(1001)
      DIMENSION DDU(1001),CF(1001),RTES(1001)
      DIMENSION XAK(20),CFX(20)
      DIMENSION YUR(20),YUI(20)
      EXTERNAL DZ
      COMMON U,DU,F1,DF1,PHI,AB,AKR,AKI,SQR,DDU
      COMMON AA,I
C
C*****READ IN OR INPUT THE VALUES OF AB , DELP,AND DELZ
C*****DELP--DIMENSIONLESS TAKE-UP FORCE (PHI OR PHII)
C*****DELZ--WALL SHEAR STRESS OR STRESS RATIO DEPENDING ON NA
C
      10    CONTINUE
           AB=0.9
           DELP=0.27
      20    CONTINUE
C*****PHID CAN BE PHII DEPENDING ON NEWTON-RAPHSON METHOD USED
           PHID=DELP
      30    CONTINUE
           DELZ=0.5
C
      40    CONTINUE
C*****MI--DETERMINING THE TOTAL SPINLINE LENGTH (DELX*MI)
           MI=1001
C*****SET DELX (ALWAYS 0.4/PHID)

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```

      DELX=0.0175
      WRITE(5,270)AB,DELZ,PHID,DELX
C*****
C*****READ IN OR SET NA:
C*****NA=1---PP1064 OR PP6823 WITH WALL SHEAR STRESS AS PRIMARY VAR.
C*****NA=2---PP1064 OR PP6823 WITH FAG1 AS PRIMARY VARIABLE
C*****NA=3---WHITE-METZNER-TANNER, IE.XK1=2,ZS=0.13
C*****
C      READ(5,140)NA
      NA=3
      60  CONTINUE
C*****FAG1--STRESS RATIO
      IF(NA.NE.1)FAG1=DELZ
      IF(NA.EQ.3)GO TO 100
C
C*****INPUT THE RHEOLOGICAL PROPERTIES OF PP 106-4 AND PP 6823*****
C*****PP106-4,AB=0.6,T=0.1,G=4.8E5,A=6.1E-3,B=1.48,ZS=0.12,XK1=3*****
C*****PP 6823,AB=0.9,T=1.25,G=7.1E5,A=1.81E-4,B=1.76,ZS=0.33,XK1=3**
C
      IF(AB.EQ.0.6)GO TO 70
      T0=1.25
      G0=710000.
      A0=0.000181
      B00=1.76
      ZS=0.3
      XK1=2.
      GO TO 80
      70  CONTINUE
      T0=0.1
      G0=480000.
      A0=0.0061
      B00=1.48
      ZS=0.1
      XK1=2.
      80  CONTINUE
      IF(NA.EQ.2)GO TO 90
C*****ZGMW--SHEAR STRESS, XN1W--FIRST NORMAL STRESS DIFFERENCE
C*****GAM--DIMENSIONLESS VARIABLE
      ZGMW=DELZ
      GAM=ZGMW/G0
      XN1W=A0*ZGMW**B00
C*****ZM IS CALCULATED FROM AB,ZGMW,AND LOW SHEAR RATE DIE SWELL**
C*****ZM--CALCULATED VARIABLE CORRESPONDING TO ZS
      FAG1=XN1W/(2.*ZGMW)
      GO TO 110
      90  CONTINUE
      ZGMW=(2.*FAG1/A0)**(1./(B00-1.))
      XN1W=2.*ZGMW*FAG1
      GAM=ZGMW/G0
      GO TO 110
      100 CONTINUE
      XK1=2.

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```

      ZS=0.13
      ZGMW=8888.
      GAM=8888.
110  CONTINUE
      FAG=FAG1**2
      XK=1./XK1
      ZM=1./XK*((1.+FAG*XK)**(1./6.)+ZS)**6-(1+FAG*XK)
      PAR=1.+(FAG+ZM)*XK
C*****BO--DIE SWELL VALUE
C*****VO--VELOCITY AT MAXIMUM DIE SWELL POSITION
C*****PHII--DIMENSIONLESS TAKE-UP FORCE BASE ON DIE FLOW
C*****F1(1)--INITIAL STRESS IN 11 DIRECTION
C*****F20--INITIAL STRESS IN 22 DIRECTION
C*****F120--F1(1)/F2
C
C*****NEWTON-RAPHSON METHOD IS USED TO CALCULATE BO *****
C*****B1 (BO GUSSED ) IS CHANGED TO OBTAIN CONSTANT PHID INPUT VALUE
      B1=0.5
1     B0=B1
C*****THE FOLLOWING EQUATION DEPEND PHI OR PHII IS KNOWN*****
C*****HERE,THE INPUT DELP IS PHI=PHID
2     ER=(B0**6+PHID*B0**4-PAR)/(6.*B0**5+4.*PHID*B0**3)
C*****IF PHID=PHII THE NEXT LINE MUST BE USED INSTEAD OF ABOVE LINE
C2    ER=(B0**6+PHID*B0**2-PAR)/(6.*B0**5+2.*PHID*B0)
      B0=B0-ER
      IF(ABS(ER)-1.E-6)3,2,2
3     PHI=(1.+(FAG+ZM)*XK)/B0**4-B0**2
      PHII=PHI*B0**2
      IF(ABS(PHI-PHID)-1.E-4)4,4,5
5     B1=B1+0.5
      IF(B1.GT.100.) GO TO 850
      GO TO 1
4     CONTINUE
C     PHII=PHI*B0**2
C     PHI=PHII/B0**2
      VO=1./B0**2
      IF(PHID.NE.0.)GO TO 6
      F1(1)=8888.
      F20=8888.
      F120=8888.
      GO TO 7
6     CONTINUE
      F1(1)=(1.+(FAG+ZM)*XK)/B0**4/PHI
      F20=B0**2/PHI
      F120=PAR/B0**6
7     CONTINUE
      WRITE(5,130)
130   FORMAT(1X,'NA=1--PP AND ZGMW,NA=2--PP AND FAG1,NA=3--W-M-T')
      WRITE(5,140)NA
140   FORMAT(1X,I1)
      WRITE(5,280)
      WRITE(5,290)AB,ZGMW,FAG1,GAM,B0,PHI,PHII,F1(1),F120

```

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C
C*****DETERMINING THE REQUIRED SOLUTION:
C*****NC=1---MODIFIED WHITE-IDE SOLUTION (STEADY SOLUTION ONLY)
C*****NC=2---MODIFIED FISHER-DENN SOLUTION
C*****NC=3---GENERAL DIFFERENTIAL TREATMENT SOLUTION
C
      WRITE(5,150)
150   FORMAT(1X,'NC=1---MOD. W-I, NC=2---MOD. F-D, NC=3---GENERAL')
C     READ(5,140)NC
      NC=3
      N=NC
      WRITE(5,140)NC
      SQR=SQRT(3.)*AB
C*****U--VELOCITY BASED ON DIE SWELL POSITION
C*****R--RADIUS BASED ON DIE SWELL POSITION
      U(1)=1.
      R(1)=SQRT(1./U(1))
      T=0.
      RTES(1)=T
      IF(N.NE.3)F1(1)=U(1)
      BX=3.*F1(1)+3./PHI-(2.+SQR)
      DU(1)=1./BX
      CP(1)=1./3.*(SQR+2.)*U(1)-1./PHI-F1(1)
      DDU(1)=(1./(U(1)**2))*(U(1)*DU(1)**3*(SQR-2.+3.*AB**2)-3./PHI
1      *SQR*DU(1)**3+(2.*SQR+1.)*U(1)*DU(1)**2-3./PHI*DU(1)**2+U(1)
2      *DU(1))
      IF(N.EQ.1)GO TO 160
      DF1(1)=1./3.+1./3.*DU(1)*(SQR+2.)-1./3.*U(1)*DDU(1)/(DU(1)**2)
      GO TO 170
160   CONTINUE
      DF1(1)=DU(1)
170   CONTINUE
      WRITE(5,180)AB,PHI,PHII,F1(1),V0,B0,ZS
180   FORMAT(/,2X,'AB=',G15.7,/,1X,'PHI=',G15.7,/,1X,'PHII=',G15.7,
1/,1X,'F1(1)=',G15.7,/,2X,'V0,B0=',2G15.7,/,2X,'ZS=',G15.7)
      IND=1
      NQ=12
      TOL=0.0001
      X1=0.
      Z(1)=X1
C*****Y(1)--VELOCITY, Y(2)--VELOCITY GRADIENT,-----STEADY SPINNING
      Y(1)=U(1)
      Y(2)=DU(1)
      WRITE(5,300)
      WRITE(5,310)Z(1),R(1),U(1),DU(1),IND,IER,F1(1),RTES(1),CP(1)
      DO 190 I=2,MI
      X=(I-1)*DELX
      CALL DVERK(N,DZ,X1,Y,X,TOL,IND,C,NQ,Q,IER)
      Z(I)=X1
      R(I)=SQRT(1./U(I))
      CP(I)=1./3.*(SQR+2.)*U(I)-1./PHI-F1(I)
      RTES(I)=T+(2.*DELX)/(U(I)+U(I-1))

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      T=RTES(I)
      IF(IND.LT.0) GO TO 200
      IF(IER.GT.0) GO TO 200
      IF(DU(I).GT.1.E20)GO TO 200
      IM=I
190   CONTINUE
      GO TO 210
200   CONTINUE
      IM=I-1
210   CONTINUE
C*****RTF--RESIDENCE TIME (SEC.)
      RTF=T*T0
      WRITE(5,220)IM,MI
220   FORMAT(/,1X,'MAX. I OR DU>E20(I < MI), AS I=',I4,2X,'MI=',I4)
      WRITE(5,310) Z(IM),R(IM),U(IM),DU(IM),IND,IER,F1(IM),RTES(IM),
        1CP(IM)
C*****UI--FINAL VELOCITY BASED ON DIE FLOW VELOCITY
C*****RI--FINAL RADIUS BASED ON DIE RADIUS
C*****ZI--FINAL SPILLINE LENGTH BASED ON DIE FLOW VELOCITY
      UI=V0*U(IM)
      RI=SQRT(1./UI)
      ZI=V0*Z(IM)
      WRITE(5,360)ZI,B0,UI,PHII
      WRITE(5,270)AB,DELZ,PHID,DELX
      WRITE(1,270)AB,DELZ,PHID,DELX
270   FORMAT(1X,'AB  =' ,G15.7,/,1X,'DELZ=' ,G15.7,/,1X,'PHID=' ,G15.7,
        1/,1X,'DELX=' ,G15.7)
      WRITE(1,130)
      WRITE(1,140)NA
      WRITE(1,150)
      WRITE(1,140)NC
      WRITE(5,280)
      WRITE(1,280)
280   FORMAT(/,12X,'AB  =' ,10X,'ZGMW=' ,10X,'FAG1=' ,10X,'GAM  =' ,
        1/,5X,'B0  =' ,10X,'PHI  =' ,10X,'PHII=' ,10X,'F10  =' ,10X,
        2'F120=' )
      WRITE(1,290)AB,ZGMW,FAG1,GAM,B0,PHI,PHII,F1(1),F120
      WRITE(5,290)AB,ZGMW,FAG1,GAM,B0,PHI,PHII,F1(1),F120
290   FORMAT(7X,4G15.7,/,1X,5G15.7)
      WRITE(1,220)IM,MI
      WRITE(5,300)
      WRITE(1,300)
300   FORMAT(5X,'Z(I)=' ,10X,'R(I)=' ,10X,'U(I)=' ,10X,'DU(I)='
        1,2X,'IND=' ,2X,'IER=' ,/,30X,'F1(I)=' ,9X,'RTES(I)=' ,8X,'CP(I)=' )
      WRITE(1,310) Z(IM),R(IM),U(IM),DU(IM),IND,IER,F1(IM),RTES(IM),
        1CP(IM)
      WRITE(5,310) Z(IM),R(IM),U(IM),DU(IM),IND,IER,F1(IM),RTES(IM),
        1CP(IM)
310   FORMAT(1X,4G15.7,2I4,/,30X,3G15.7)
      WRITE(5,360)ZI,RI,UI,RTES(IM)
      WRITE(1,360)ZI,RI,UI,RTES(IM)
      WRITE(1,180)AB,PHI,PHII,F1(1),V0,B0,ZS

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C*****DO STABILITY ANALYSIS OR NOT--BY CHOSING IN VALUE:*****
C*****READ IN OR SET VALUE OF "IN":
C*****IN<3---WRITE VEL. PRO. AND CHANGE FAG1 OR ZGMW ETC.
C*****IN=4---DO STABLITIES ANALYSIS
C
      WRITE(1,320)
      WRITE(5,320)
320   FORMAT(1X,'IN<3--STEADY,IN=4--STAB. AN. ')
C     READ(5,140)IN
      IN=4
      WRITE(1,330)IN
      WRITE(5,330)IN
330   FORMAT(1X,I1)
      IF(IN.LE.3)GO TO 340
      IF(IN.EQ.4) GO TO 450
340   CONTINUE
      XJ=IM
      WRITE(20,350)XJ
      WRITE(1,350)XJ
350   FORMAT('DATA',T15,G15.6)
      DO 370 J=1,IM
C*****CALCULATING VELOCITY (OR RADIUS) PROFILES BASED ON DIE FLOW
      ZI=V0*Z(J)
      UI=V0*U(J)
      RI=SQRT(1./UI)
      WRITE(20,360)ZI,RI,UI,RTES(J)
      WRITE(1,360)ZI,UI,RI,RTES(J)
360   FORMAT(4G15.7)
370   CONTINUE
C*****CHANGE DELZ VALUE AND DO AGAIN IF NECESSARY
      WRITE(5,380)
      WRITE(1,380)
380   FORMAT(1X,'CHANG DELZ?Y/N')
      READ(5,390)AN
      WRITE(1,390)AN
390   FORMAT(A1)
      IF(AN.EQ.1HN)GO TO 850
      DELZ=DELZ*10.
      IF(NA.EQ.1.AND.DELZ.LE.1.E7)GO TO 60
      IF(NA.NE.1.AND.DELZ.LE.5.E0)GO TO 60
      WRITE(5,400)
      WRITE(1,400)
400   FORMAT(1X,'CHANG DELP?Y/N')
      READ(5,390)AN
      WRITE(1,390)AN
      IF(AN.EQ.1HN)GO TO 410
      DELP=DELP*5.
      IF(DELP-5.)20,20,410
410   CONTINUE
      WRITE(1,440)
      WRITE(5,440)
440   FORMAT(1X,'CHANGE NA OR NC?Y/N')

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      READ(5,390)AN
      WRITE(1,390)AN
      IF(AN.EQ.1HN)GO TO 850
      GO TO 10
450   CONTINUE
C
C*****DO STABILITY ANALYSIS*****
C*****Y(1)--REAL PART OF VELOCITY PERTUBATION
C*****Y(2)--IMAGINARY PART OY VELOCITY PERTUBATION
C*****Y(3)--REAL PART OF AREA PERTUBATION
C*****Y(4)--IMAGINARY PART OF AREA PERTUBATION
C*****Y(5)--REAL PART OF F1 (STRESS IN 11 DIRECTION) PERTUBATION
C*****Y(6)--IMAGINARY PART OF F1 PERTUBATION
C*****Y(7)--REAL PART PERTUBATION OF F1-F2
C*****Y(8)--IMAGINARY PART PERTUBATION OF F1-F2
C*****Y(9)--STEADY STATE VELOCITY
C*****Y(10)--STEADY STATE VELOCITY GRADIENT
C*****
C
C*****SELECT THE POSITION OF SPINLINE AND THE CORRESPONDING*****
C*****VELOCITY BY CHOOSING THE IM VALUE*****
C*****DELX IS CHOSEN AS 1% OF THE SPINLINE LENGTH USED
C*****DR--DRAW RATIO BASED ON DIE SWELL VELOCITY
C*****AL--ELASTICITY VALUE BASED ON DIE SWELL VELOCITY
      IM=IM-0
      DELX=Z(IM)/100.
      DR=U(IM)
      AL=1./Z(IM)
      IM=101
460   CONTINUE
      IF(NC.EQ.1)N=9
      IF(NC.GT.1)N=10
      DO 470 L=1,N
      YI(L)=0.0
470   CONTINUE
C*****YI(7)--INITIAL PERTUBATION OF Y(7)
C*****CF--TOTAL PERTUBATION OF VELOCITY
C*****IALER--CONTROL VARIABLE (NO. OF ITERATION)
C*****I1,I3,IAX,IAY--CONTROL VARIABLES FOR ITERATION OF EIGENVALUE
      YI(7)=1.0
      CF(1)=0.
      IALER=1
      I1=1
      I3=1
      IAX=0
      IAY=0
C*****AKRMI CAN BE SET AS + OR - OR 0.0 *****
C*****READIN OR SET AKIMD AND AKRMI
C*****AKIMD--REASONABLE GUESS OF IMAGINARY PART OF EIGENVALUE
C*****AKI--IMAGINARY PART OF EIGENVALUE
C*****AKR--REAL PART OF EIGENVALUE
C*****AKRMI--REASONABLE GUESS OF REAL PART OF EIGENVALUE

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      AKIMD=0.69
      IF(IEIG.EQ.1)AKIMD=AKI
      AKI=AKIMD
C*****FOLLOWING IS THE ITERATION METHOD TO GET EIGENVALUE SOME
C*****MODIFICATIONS ARE REQUIRED DEPENDING ON THE EIGENVALUE
C*****GUESSED,SEVERAL TRIERS BEFORE SEEKING THE EXACT
C*****VALUES OF AKR AND AKI IS NECESSARY
C*****FIRST TRY AKR THEN AKI,AKI,AKR,AKR,AKI,AKI,AKR,AKR...
      AKRMI=-0.0001
      IF(IEIG.EQ.1)AKRMI=AKR/1.2
      IF(AKRMI.EQ.0.0)GO TO 810
      XAK(2)=AKRMI
      XAK(13)=XAK(2)*1.5
      DAK=(XAK(13)-XAK(2))/5.
      ICOL=1
480    CONTINUE
      IALX=0
      DO 610 IAL=8,13
      XAK(IAL)=XAK(2)+(IAL-8.)*DAK
      IF(ICOL.EQ.1)AKR=XAK(IAL)
      IF(ICOL.EQ.2)AKI=XAK(IAL)
      DO 490 L=1,N
      Y(L)=YI(L)
      Y(9)=U(1)
      Y(10)=DU(1)
490    CONTINUE
      IND=1
      NQ=12
      TOL=0.0001
      X1=0.
      Z(1)=X1
C*****THE EIGENFUNCTION Y(I) CAN BE WRITTEN IN DATA FILE IF REQUIRED
C      WRITE(5,500)
C      WRITE(1,500)
500    FORMAT(5X,'Z(I)=' ,10X,'R(I)=' ,10X,'U(I)=' ,10X,'CF(I)='
      1,2X,'IND=' ,2X,'IER=' ,/,30X,'Y(1)=' ,9X,'Y(2)=' ,8X,'Y(7)=' )
C      WRITE(1,310)Z(1),R(1),U(1),CF(1),IND,IER,Y(1),Y(2),Y(7)
C      WRITE(5,310)Z(1),R(1),U(1),CF(1),IND,IER,Y(1),Y(2),Y(7)
C*****
      DO 510 I=2,IM
      X=(I-1)*DELX
      CALL DVERK(N,DZ,X1,Y,X,TOL,IND,C,NQ,Q,IER)
      Z(I)=X1
      U(I)=Y(9)
      ZI=Z(I)*V0
      UI=V0*U(I)
      RI=SQRT(1./UI)
      CF(I)=ABS(Y(1))+ABS(Y(2))
C*****EIGENFUNCTION CAN BE WRITTEN IN DATA FILE, IF DESIRED *****
C      WRITE(5,310)Z(I),R(I),U(I),CF(I),IND,IER,Y(1),Y(2),Y(7)
C      WRITE(1,310)Z(I),R(I),U(I),CF(I),IND,IER,Y(1),Y(2),Y(7)
      IF(IND.LT.0)GO TO 520

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IF(IER.GT.0)GO TO 520
ITM=I
510 CONTINUE
GO TO 530
520 CONTINUE
ITM=I
530 CONTINUE
IF(ITM.NE.IM)GO TO 610
WRITE(5,540)AKR,Y(1),AKI,Y(2),CF(ITM),Y(9),X1
WRITE(1,540)AKR,Y(1),AKI,Y(2),CF(ITM),Y(9),X1
540 FORMAT(7G15.7)
IF(CF(ITM).LE.5.E-2.AND.ITM.EQ.IM)GO TO 580
IALX=IALX+1
IF(ICOL.NE.1)GO TO 570
AKRMA=XAK(IAL)
AKRMD=XAK(IAL-1)
AKRMI=XAK(IAL-2)
CFX(IAL)=CF(ITM)
CFR1=CFX(IAL-1)
CFR2=CFX(IAL-2)
CFRO=CFX(IAL)
YUR(IAL)=Y(1)
YUI(IAL)=Y(2)
IF(IALX.EQ.2.AND.YUR(9).GT.0..AND.YUR(8).GT.0..AND.YUR(9)
1.GT.YUR(8))GO TO 550
IF(IALX.EQ.2.AND.YUR(9).LT.0..AND.YUR(8).LE.0..AND.YUR(9)
1.LT.YUR(8))GO TO 550
IF(IALX.GE.2.AND.YUR(IAL).LE.0.0.AND.YUR(IAL-1).GE.0.0)GO TO
1550
IF(IALX.GE.2.AND.YUR(IAL).GE.0.0.AND.YUR(IAL-1).LE.0.0)GO TO
1550
IF(IALX.GE.3.AND.ABS(YUR(IAL)).LE.1.E-1.AND.ABS(YUR(IAL-1)).LE.
11.E-1.AND.ABS(YUI(IAL)).GT.1..AND.ABS(YUI(IAL-1)).GT.1.)
2GO TO 740
IF(IALX.GE.3.AND.ABS(YUR(IAL)).LE.1.E-2.AND.ABS(YUR(IAL-1)).LE.
11.E-2.AND.ABS(YUI(IAL)).GT.0.1.AND.ABS(YUI(IAL-1)).GT.0.1)
2GO TO 740
IF(IAL.EQ.13.AND.YUR(11).GT.YUR(12).AND.
1YUR(12).GT.YUR(13))GO TO 620
IF(IAL.EQ.13.AND.YUR(11).LT.YUR(12).AND.
2YUR(12).LT.YUR(13))GO TO 620
IF(IALX.GE.2)CFER=ABS(YUR(IAL)-YUR(IAL-1))
IF(IALX.GT.2.AND.CFER.LE.5.E-4.AND.AKRMI.GT.0.)GO TO 630
IF(IALX.GE.2.AND.CFER.LE.5.E-4.AND.AKRMI.LT.0.)GO TO 650
GO TO 610
570 AKIMA=XAK(IAL)
AKIMD=XAK(IAL-1)
AKIMI=XAK(IAL-2)
YUI(IAL)=Y(2)
IF(AKR.EQ.0.0.AND.ABS(YUI(IAL)).LT.5.E-3.AND.ITM.EQ.IM)GO TO
1580
YUR(IAL)=Y(1)

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CFX(IAL)=CF(ITM)
CFIO=CFX(IAL)
CFI1=CFX(IAL-1)
CFI2=CFX(IAL-2)
IF(IALX.EQ.2.AND.YUI(9).GT.0..AND.YUI(8).GT.0..AND.YUI(9)
1.GT.YUI(8))GO TO 590
IF(IALX.EQ.2.AND.YUI(9).LT.0..AND.YUI(8).LE.0..AND.YUI(9)
1.LT.YUI(8))GO TO 590
IF(IALX.GE.2.AND.YUI(IAL).LE.0.0.AND.YUI(IAL-1).GE.0.0)GO TO
1590
IF(IALX.GE.2.AND.YUI(IAL).GE.0.0.AND.YUI(IAL-1).LE.0.0)GO TO
1590
IF(IALX.GE.3.AND.ABS(YUI(IAL)).LE.1.E-1.AND.ABS(YUI(IAL-1)).LE.
11.E-1.AND.ABS(YUR(IAL)).GT.1..AND.ABS(YUR(IAL-1)).GT.1.)
2GO TO 800
IF(IALX.GE.3.AND.ABS(YUI(IAL)).LE.1.E-2.AND.ABS(YUI(IAL-1)).LE.
11.E-2.AND.ABS(YUR(IAL)).GT.0.1.AND.ABS(YUR(IAL-1)).GT.0.1)
2GO TO 800
IF(IAL.EQ.13.AND.YUI(11).GT.YUI(12).AND.
1YUI(12).GT.YUI(13))GO TO 620
IF(IAL.EQ.13.AND.YUI(11).LT.YUI(12).AND.
2YUI(12).LT.YUI(13))GO TO 620
610  CONTINUE
C*****NO AKR OR AKI EXIST IN THIS RANGE
620  CONTINUE
IF(ICOL.EQ.1)AKRMD=-(XAK(11)-XAK(13))/(YUR(11)-YUR(13))*YUR(13)
1+XAK(13)
IF(ICOL.EQ.2)AKIMD=-(XAK(11)-XAK(13))/(YUI(11)-YUI(13))*YUI(13)
2+XAK(13)
IF(ICOL.EQ.1.AND.AKRMI.GT.0.0)GO TO 630
IF(ICOL.EQ.1.AND.AKRMI.LE.0.0)GO TO 650
IF(ICOL.EQ.2)GO TO 670
630  CONTINUE
WRITE(5,640)AKRMA
640  FORMAT(1X,'NO +AKR AS AKR< ',G15.7)
IF(AKRMA.GT.5.)GO TO 580
IF(AKRMA.LT.0.00010)GO TO 700
IAX=IAX+1
IF(IAX.EQ.3)GO TO 740
ICOL=1
AKI=AKIMD
XAK(2)=AKRMD/1.05
XAK(13)=AKRMD*1.05
DAK=(XAK(13)-XAK(2))/5.
GO TO 480
650  CONTINUE
WRITE(5,660)AKRMA
660  FORMAT(1X,'NO -AKR AS AKR> ',G15.7)
IF(AKRMA.LE.-10.)GO TO 580
IF(AKRMA.GT.-0.00010)GO TO 580
IAX=IAX+1
IF(IAX.EQ.3)GO TO 740

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      AKI=AKIMD
      ICOL=1
      XAK(2)=AKRMD/1.05
      XAK(13)=AKRMD*1.05
      DAK=(XAK(13)-XAK(2))/5.
      GO TO 480
670   CONTINUE
      WRITE(5,680)AKIMA
680   FORMAT(1X,'NO AKI  AS AKI< ',G15.7)
      IF(AKIMA.GT.50.)GO TO 580
      IF(AKIMA.LT.0.0001)GO TO 580
      IF(AKR.EQ.0.)GO TO 810
      IAY=IAY+1
      IF(IAY.EQ.3)GO TO 800
      AKR=AKRMD
690   CONTINUE
      XAK(2)=AKIMD/1.001
      XAK(13)=AKIMD*1.001
      DAK=(XAK(13)-XAK(2))/5.
      ICOL=2
      GO TO 480
700   CONTINUE
      IALER=1
      IAX=0
      IAY=0
      AKI=AKIMD
      XAK(2)=-0.001
      XAK(13)=XAK(2)*1.5
      DAK=(XAK(13)-XAK(2))/5.
      ICOL=1
      GO TO 480
550   IF(IAL.GT.9)GO TO 720
      IF(IALER.GT.30)GO TO 580
      AKRMI=AKRMD/1.05
720   CONTINUE
      WRITE(5,730)AKRMI,AKRMA
730   FORMAT(1X,'EXIST AKR BETW. <AKR> ',2G15.7)
      IF(AKRMA.GT.0.0.AND.AKRMA.LT.0.00010)GO TO 700
      IF(AKRMA.LT.0.0.AND.AKRMA.GT.-0.00010)GO TO 580
      IF(II.NE.1)GO TO 740
      AKI=AKIMD
      XAK(2)=AKRMI
      XAK(13)=AKRMA
      DAK=(XAK(13)-XAK(2))/5.
      ICOL=1
      II=2
      GO TO 480
740   CONTINUE
      IAX=0
      AKR=AKRMD
      IF(IALER.NE.1)GO TO 750
      XAK(2)=AKIMD/1.001

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XAK(13)=AKIMD*1.001
GO TO 760
750 CONTINUE
XAK(2)=AKIMI
XAK(13)=AKIMA
760 ICOL=2
DAK=(XAK(13)-XAK(2))/5.
II=1
GO TO 480
590 CONTINUE
IF(AKR.EQ.0.)GO TO 810
IF(IAL.GT.9)GO TO 780
AKIMI=AKIMD/1.001
780 CONTINUE
WRITE(5,790)AKIMI,AKIMA
790 FORMAT(1X,'EXIST AKI BETW.<AKI> ',2G15.7)
IF(AKIMA.GT.50..OR.AKIMA.LT.0.0001)GO TO 580
IF(I3.NE.1)GO TO 800
AKR=AKRMD
XAK(2)=AKIMI
XAK(13)=AKIMA
ICOL=2
DAK=(XAK(13)-XAK(2))/5.
I3=2
IF(AKR.EQ.0.0)I3=1
IF(AKR.EQ.0.0)IALER=IALER+1
GO TO 480
800 CONTINUE
IF(AKR.EQ.0.0)GO TO 810
IAY=0
AKI=AKIMD
XAK(2)=AKRMI
XAK(13)=AKRMA
DAK=(XAK(13)-XAK(2))/5.
ICOL=1
I3=1
IALER=IALER+1
IF(IALER.GT.30)GO TO 580
GO TO 480
810 CONTINUE
AKRMD=0.
AKR=AKRMD
IF(IALER.NE.1)GO TO 820
XAK(2)=AKIMD
ICOL=2
XAK(13)=AKIMD*1.01
DAK=(XAK(13)-XAK(2))/5.
GO TO 480
820 CONTINUE
IF(IAL.GT.9)GO TO 830
AKIMI=AKIMD/1.01
830 CONTINUE

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WRITE(5,790)AKIMI,AKIMA
XAK(2)=AKIMI
XAK(13)=AKIMA
ICOL=2
DAK=(XAK(13)-XAK(2))/5.
IALER=IALER+1
IF(IALER.GT.30)GO TO 580
GO TO 480
580  CONTINUE
      IEIG=0
      IF(CF(ITM).LE.5.E-2)IEIG=1
      IF(AKR.EQ.0.0.AND.CF(ITM).LE.5.E-2)GO TO 850
C
C*****SOME MODIFICATIONS ARE REQUIRED IF ONE WISHES TO INCREASE*****
C*****THE SPINLINE LENGTH FOR STABILITY ANALYSIS*****
C
C*****DECREASE THE SPINLINE LENGTH UNTIL AKR < 0
      IM=IM-2
      DR=U(IM)
      AL=1./Z(IM)
      IF(AKR.LT.0)GO TO 850
      IF(IM.LE.80)850,460,460
850  CONTINUE
      STOP
      END
C*****
C*****
      SUBROUTINE DZ(N,X,Y,W)
      DIMENSION Y(N),W(N)
      DIMENSION U(1001),DU(1001),F1(1001),DF1(1001),DDU(1001)
      REAL Y,W
      COMMON U,DU,F1,DF1,PHI,AB,AKR,AKI,SQR,DDU
      COMMON AA,I
      IF(N.GE.9)GO TO 45
      IF(N.LT.2)GO TO 40
      W(1)=Y(2)
      W(2)=(1./(Y(1)**2))*(Y(1)*Y(2)**3*(SQR-2.+3.*AB**2)-3./PHI
1*SQR*Y(2)**3+(2.*SQR+1.)*Y(1)*Y(2)**2-3./PHI*Y(2)**2+Y(1)
2*Y(2))
      U(I)=Y(1)
      DU(I)=W(1)
      F1(I)=(U(I)/(3.*DU(I)))+1./3.*U(I)*(2.+SQR)-1./PHI
      DDU(I)=(1./(U(I)**2))*(U(I)*DU(I)**3*(SQR-2.+3.*AB**2)-3./PHI
1*SQR*DU(I)**3+(2.*SQR+1.)*U(I)*DU(I)**2-3./PHI*DU(I)**2+U(I)
2*DU(I))
      DF1(I)=1./3.+1./3.*DU(I)*(SQR+2.)-1./3.*U(I)*DDU(I)/(DU(I)**2)
      GO TO 60
40  W(1)=1./(3./(PHI*Y(1))+1.-SQR)
      U(I)=Y(1)
      DU(I)=W(1)
      F1(I)=U(I)
      DF1(I)=DU(I)

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DDU(I)=(1./(U(I)**2))* (U(I)*DU(I)**3*(SQR-2.+3.*AB**2)-3./PHI
1*SQR*DU(I)**3+(2.*SQR+1.)*U(I)*DU(I)**2-3./PHI*DU(I)**2+U(I)
2*DU(I))
GO TO 60

C*****
45  CONTINUE
    IF(N.EQ.9)GO TO 50
    W(9)=Y(10)
    W(10)=(1./(Y(9)**2))* (Y(9)*Y(10)**3*(SQR-2.+3.*AB**2)-3./PHI
1*SQR*Y(10)**3+(2.*SQR+1.)*Y(9)*Y(10)**2-3./PHI*Y(10)**2+Y(9)
2*Y(10))
    U(I)=Y(9)
    DU(I)=W(9)
    F1(I)=(U(I)/(3.*DU(I)))+1./3.*U(I)*(2.+SQR)-1./PHI
    DDU(I)=(1./(U(I)**2))* (U(I)*DU(I)**3*(SQR-2.+3.*AB**2)-3./PHI
1*SQR*DU(I)**3+(2.*SQR+1.)*U(I)*DU(I)**2-3./PHI*DU(I)**2+U(I)
2*DU(I))
    DF1(I)=1./3.+1./3.*DU(I)*(SQR+2.)-1./3.*U(I)*DDU(I)/(DU(I)**2)
GO TO 55
50  CONTINUE
    W(9)=1./(3./(PHI*Y(9))+1.-SQR)
    U(I)=Y(9)
    DU(I)=W(9)
    F1(I)=U(I)
    DF1(I)=DU(I)
    DDU(I)=(1./(U(I)**2))* (U(I)*DU(I)**3*(SQR-2.+3.*AB**2)-3./PHI
1*SQR*DU(I)**3+(2.*SQR+1.)*U(I)*DU(I)**2-3./PHI*DU(I)**2+U(I)
2*DU(I))
55  CONTINUE
    AA=1./(3./PHI+3.*F1(I)-2.*U(I)-SQR*U(I))
    W(1)=AA*(AKR*(Y(7)+U(I)**2*Y(3))+Y(7)+DU(I)*(2.*Y(7)-3.*Y(5)
1+SQR*Y(7))-AKI*(Y(8)+U(I)**2*Y(4)))
    W(2)=AA*(AKI*(Y(7)+U(I)**2*Y(3))+Y(8)+DU(I)*(2.*Y(8)-3.*Y(6)
1+SQR*Y(8))+AKR*(Y(8)+U(I)**2*Y(4)))
    W(3)=(1./U(I))* (DU(I)*Y(1)/U(I)**2-W(1)/U(I)-(AKR+DU(I))*Y(3)
1+AKI*Y(4))
    W(4)=(1./U(I))* (DU(I)*Y(2)/U(I)**2-W(2)/U(I)-(AKR+DU(I))*Y(4)
1-AKI*Y(3))
    W(5)=(1./U(I))* (W(1)*(2./PHI+(2.-SQR)*F1(I))+DU(I)*(2.-SQR)*Y(5)
1-Y(1)*DF1(I)-(1.+AKR)*Y(5)+AKI*Y(6))
    W(6)=(1./U(I))* (W(2)*(2./PHI+(2.-SQR)*F1(I))+DU(I)*(2.-SQR)*Y(6)
1-Y(2)*DF1(I)-(1.+AKR)*Y(6)-AKI*Y(5))
    W(7)=(-1./U(I))* (Y(7)-3./PHI*W(1)+Y(7)*(AKR+(SQR+1.)*DU(I))-AKI
1*Y(8)+DU(I)*(Y(1)-3.*Y(5))+W(1)*(U(I)-3.*F1(I)+SQR*U(I)))
    W(8)=(-1./U(I))* (Y(8)-3./PHI*W(2)+Y(8)*(AKR+(SQR+1.)*DU(I))+AKI
1*Y(7)+DU(I)*(Y(2)-3.*Y(6))+W(2)*(U(I)-3.*F1(I)+SQR*U(I)))
60  CONTINUE
    RETURN
    END

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VITA

The author, Jenq-Der Tsou (鄭正德), was born in Hsin Chu, Taiwan, on August 2, 1949, as the eldest son of Mr. and Mrs. Kuo-Pon Tsou. After graduating from Hsin Chu High School, he entered National Cheng Kung University in 1968 and received a Bachelor of Science degree in Chemical Engineering in 1972. After completing two years of military service, he worked for the Industrial Technology Research Institute for about one and a half years. In September 1975, he continued his education in graduate school at the same university. He obtained a Master of Science degree in Chemical Engineering under Professor Yen-Ping Shih in June 1977. Since then, he has been working for the Chung Shan Institute of Science and Technology, in Taiwan, Republic of China.

He married Lee-Yuan Kuo in January 1978. They have two children, Shi-Chin (5 years old) and Yun-Fu (4 years old).

In June 1981, he entered The University of Tennessee, Knoxville. After receiving a Ph.D in June 1984, he will rejoin the Chung Shan Institute of Science and Technology to continue his work.