A Comparative Study of the Gas Turbine Simulation Program (GSP) 11 and GasTurb 11 on Their Respective Simulations for a Single-Spool Turbojet

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Dedication

This thesis is, of course, dedicated to my parents. It is because of their love and support that I was able to find the strength to complete my studies. They are the reason why I have made it this far. They are the Atlas that holds up my world.

Acknowledgments

I would like to take this opportunity to acknowledge all my professors, past and present, who tried to instill and pass on their knowledge and wisdom to me. They are directly responsible for shaping my professional identity. I would also like to thank Dee Merriman and Scott VanZandbergen for taking care of me during my stay at UTSI. And a heartfelt thank you to Dr. Brian Canfield for proofreading my thesis, to ensure my English is coherent.

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Abstract

GasTurb 11 and the Gas Turbine Simulation Program (GSP) 11 are two commercially available gas turbine simulation programs used by industrial professionals and academic researchers throughout the world. The two programs use a pseudo-perfect gas assumption in their calculations, where the specific heat is taken as a function of temperature and gas composition but not pressure. This assumption allows the two programs to make more realistic calculations of gas turbine engine performance. This is in contrast to the ideal and perfect gas assumptions used in classroom calculations. In addition, GasTurb 11 and GSP 11 both utilize component maps, comprised from test data, to model off-design turbojet component behavior. This is different from the referencing technique where off-design performance is calculated based on the ratio of off-design to on-design conditions. This thesis presents a comparative study of the two simulation programs with the traditional ideal-perfect gas calculations and the referencing technique. The scope of the thesis is limited to the turbojet without afterburners.

The main focus is on comparing the simulated thrusts and the corresponding fuel flow rates needed to maintain full throttle for different flight conditions. The ratio of fuel flow rate to thrust is by definition the thrust-specific fuel consumption and it is also compared. The thrust-specific fuel consumption is a measurement of the fuel economy of the turbojet engine. The on-design comparisons show that the ideal-perfect gas calculations tend to underestimate the turbojet engine performance as compared to GasTurb 11 and GSP 11. It is also evident that the lower the efficiency of the turbojet components, the greater the discrepancy. With the off-design calculations at a fixed flight Mach number and varying altitudes, the ideal-perfect gas calculations report a more aggressive decrease for thrust and fuel flow rate. At a fixed altitude and varying Mach number, on the other hand, the ideal-perfect gas calculations report a more aggressive increase in thrust and fuel flow rate as compared to GasTurb 11 and GSP 11.

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Nomenclature

Symbol	*Definition*	*Symbol*	*Definition*
A	Area	$\hat{n}$	Normal vector
a	Speed of sound	P	Pressure
c_p	Specific heat at constant pressure	P_t	Total Pressure
c_v	Specific heat at constant volume	Q	Heat interaction
E_0	Total energy	q	Mass specific heat interaction
E	Internal energy	$\overline{R}$	Universal gas constant
e	Mass specific internal energy	R	Gas constant
e_c	Compressor polytropic efficiency	S	Entropy
e_t	Turbine polytropic efficiency	s	Mass specific entropy
F	Force	T	Temperature
F_T	Thrust	T_t	Total temperature
f	Fuel air ratio	u	Velocity
g	Gravitational acceleration	V	Volume
h	Mass specific enthalpy	W_T	Total work
h_{PR}	Low heating value of fuel	W_s	Shaft work
M	Mach Number	W_m	Mechanical work
m	Mass	W_f	Flow work
MFP	Mass flow parameter	w	Mass specific work
N	Spool speed	z	Altitude

Greek Letters

γ	Specific heat ratio	$\pi_{d\max}$	Pressure change due to wall friction
η	efficiency	ρ	Density
η_r	Nozzle ram recovery	τ	Temperature ratio
v	Mass specific Volume	τ_λ	$c_{pt}T_{t4}/c_{pc}T_{t0}$
π	Pressure ratio		

Subscripts

b	Burner, combustor	n	Nozzle
c	Compressor	r	Stagnation ratio
d	Diffusor, inlet	t	Turbine
f	Fuel	cv	Control volume
m	Mechanical	cs	Control surface

Chapter 1 Introduction

"My wish is that you will have as much fun in propulsion as I have."
Hans von Ohain
German Inventor of the Gas Turbine Engine [1]

The scope of this thesis involves the comparison of two commercially available gas turbine engine simulation programs, GasTurb 11[2] and the Gas Turbine Simulation Program (GSP) 11[3], to traditional thermodynamic analysis presented in standard university textbooks. In the classroom, it is common to assume ideal and perfect gas conditions to help simplify the calculations. With GasTurb 11 and GSP 11, however, the gas properties are assumed to be those of a pseudo-perfect gas and they are functions of temperature and gas composition but not pressure.[4] This allows GasTurb 11 and GSP 11 to achieve more realistic simulations.

The gas turbine engine is one of the fundamental technologies that has changed the world and helped to define the Twentieth Century. Most noticeably, it has changed the way we humans travel, as well as the way we fight wars. In terms of travel, we can now depart and arrive anywhere in the world within twenty-four hours. In terms of warfare, military supremacy is defined by the way in which a nation is able to rule and dominate the air space in a war zone. In both cases, airplanes are the technological marvel that makes it possible. And at the heart of the airplane is the gas turbine engine.

This thesis will serve as an homage to the gas turbine engine, specifically the turbojet. The first half of the thesis will serve as a summary of derivations of the gas turbine equations starting from the conservation principles. Once that is established, the thesis will then use the derived equations as a basis of comparison to GasTurb 11 and GSP 11 to see how the classroom assumptions compare to the real world and industry-proven simulations. Before that, however, it is prudent to present a brief historical description of the gas turbine engine, GasTurb 11, and GSP 11.

1.1. History of the Gas Turbine Engine

The birth of the gas turbine engine can be attributed to two extraordinary gentlemen, Frank Whittle of Britain and Hans von Ohain of Germany. They began working on the concept of a gas turbine engine at around the same time, but they worked independently of each other. More amazingly, they were each unaware of the other's work. Nevertheless, they both came to the same conclusion and were directly responsible for shaping the design of the gas turbine engine we know today.

As a cadet in the Royal Air Force, Frank Whittle began visualizing the concept of the modern gas turbine engine in 1926 at the tender age of only nineteen.[5] He envisioned a propulsive device containing intake fans driven by a turbine, where the turbine is in turn driven by hot gases produced by a combustion device. At a time when aircrafts were dependent upon propellers and were limited to a dismal 150mph, Whittle's superiors at the Royal Air Force were skeptical of the idea and they lacked the imagination and faith to believe in the potential of Whittle's design. Refusing to be discouraged, Whittle filed to patent his engine idea in 1930 and was granted a patent in 1933. Whittle formed his company, Power Jets, in March 1936 to devote himself in the development of the gas turbine engine. His efforts bore fruit on June 30, 1939 when he held a demonstration for authorities where his gas turbine engine operated successfully. It was at this demonstration that the authorities were finally convinced of the potential of the gas turbine engine and began to provide support. This led to the development of the Gloster Meteor, the first British jet aircraft, which had its maiden flight on May 15, 1941. At the heart of the Gloster Meteor, of course, was the gas turbine engine designed by Whittle. For more details on Whittle's development of the jet engine, see reference [6].

It is noteworthy to mention that in the same year, General Henry H. Arnold saw the demonstration of the Gloster Meteor. He was instrumental in bring the technology to the United States. Arnold Air Force Base in Tullahoma, Tennessee is named after General Henry H. Arnold, who many considered to be the father of the United States Air Force.[7] The Arnold Engineering Development Center, an organization within Arnold Air Force Base, in cooperation with the University of Tennessee, created the University of Tennessee Space Institute in 1964. The author of this thesis is a student at the University of Tennessee Space Institute, where this thesis was written.

While Whittle was making history in Britain, Hans von Ohain began making his design of his gas turbine engine in 1933. Von Ohain was pursuing a Ph.D. in physics at the Georgia Augusta University of Göttingen specializing in aerodynamics. Incredibly, von Ohain began his research into the development of the gas turbine engine because he found the aircraft engines at the time were insufficient.[5] He simply wanted to gift the world with a better aircraft engine. Putting his physics degree to good use, von Ohain drafted his concept of the gas turbine engine, a compressor driven by the turbine. With the help of his car mechanic, Max Hahn, von Ohain was able to produce prototypes that attracted the attention of Ernst Heinkel, an aircraft manufacturer. In April 1936, Heinkel invited von Ohain and Hahn to work for him and they were able to produce a working engine in September 1937. Just a year later, von Ohain's engine was at the heart of the Heinkel He 178 test aircraft and it successfully completed its first test flight on August 27, 1938. For more details on von Ohain's development of the jet engine, see reference [8].

1.2. History of GasTurb

GasTurb was developed by Dr. Joachim Kurzke of Dachau, Germany, who began developing the software in 1991.[9] Kurzke was an engineer at MTU Aero Engines for 28 years, where his focus was engine test analysis and performance prediction. His experience gained from MTU Aero Engines has made Kurzke an expert in gas turbine engine performance and a well published author.[10-12] Retired from MTU Aero Engines in October 2004, Kurzke now divides his time on the development of GasTurb along with consulting and teaching gas turbine performance.

1.3. History of the Gas Turbine Simulation Program (GSP)

In 1975, NASA developed the DYNGEN program to simulate turbojet and turbofan engine performance.[13] However, researchers at the aerospace department of the Delft University of Technology (TU Delft), located in the Netherlands, found DYNGEN to be problematic to use and unstable. As a result, the researchers at TU Delft decided to make improvements to

DYNGEN. In 1986, TU Delft, in partnership with the National Aerospace Laboratory of the Netherlands (NLR), completed improvements to DYNGEN's numerical iteration process and user interface. Thus, GSP was born. With continual development at NLR, GSP was adapted for Windows in 1996. GSP is now currently still being used by NLR, and it is the main simulation tool for gas turbine engine analysis and data gathering for technological papers.[14-16] In addition to NLR, notable GSP users include, but are not limited to, the following: BAE Systems of Britain, the Royal Military Academy of Belgium, and the Hellenic Air Force of Greece.

Chapter 2 Fundamental Equations for Turbojet Engine Analysis

> *"The technology at the leading edge changes so rapidly that you have to keep current after you get out of school. I think probably the most important thing is having good fundamentals"*
>
> Gordon Earle Moore
> **Co-founder and Chairman Emeritus of Intel Corporation [17]**

In this section, the author will present the fundamental equations and concepts that are used to derive the gas turbine equations necessary for evaluating turbojet performance. The following is a summary of textbook knowledge, taken from various sources.[1, 5, 18, 19] It is presented here for the reader's convenience, as it demonstrates a clear path to how the gas turbine equations are derived. It also serves the purpose of demonstrating the author's understanding of the derivations.

2.1. Conservation of Mass and Momentum

The concepts of conservation of mass and momentum are the most basic principles that govern the fluid interaction within the turbojet engine. Equation (2.1) is the standard integral form of the conservation of mass. The first term on the left-hand side represents the change of mass within the control volume with respect to time and the second term represents the flux of mass through the control surface:

$$\frac{d}{dt}\int_{cv} \rho dV + \int_{cs} \rho\left(u \cdot \hat{n}\right) dA = 0 . \tag{2.1}$$

In the performance analysis of the turbojet, the flow is assumed to be a one-dimensional steady state. This assumption helps to reduce equation (2.1) to equation (2.2):

$$\rho_2 u_2 A_2 - \rho_1 u_1 A_1 = 0 \tag{2.2}$$

The assumption of a one-dimensional, steady-state flow allows the advantage of calculating turbojet performance by only accounting for the conditions at the beginning and the end of the control volume. The advantage of such analysis will become clear later.

The same assumption is applied to the conservation of momentum, where the traditional integral form is presented below:

$$\frac{d}{dt}\int_{cv} u\rho dV + \int_{cs} u\rho\left(u \cdot \hat{n}\right) dA = \Sigma F \tag{2.3}$$

Similarly, equation (2.3) is transformed into equation (2.4) by again assuming a one-dimensional steady state. The sum of the forces is simply the difference between the pressure forces at the opposite ends of the control volume and any additional body forces:

$$\rho_2 u_2^2 A_2 - \rho_1 u_1^2 A_1 = P_1 A_1 - P_2 A_2 + F_{Body} \tag{2.4}$$

2.2. First Law of Thermodynamics

The First Law concerns the principle of conservation of energy. The most useful form of the First Law for gas turbine engine analysis is equation (2.5). The equation states that heat, Q, is the sum of the change in total energy, ΔE_0, and the total work, W_T:

$$Q = \Delta E_0 + W_T \tag{2.5}$$

The total energy in equation (2.5) is defined as the sum of the internal energy, E, kinetic energy, $m\frac{u^2}{2}$, and potential energy, mgz, as shown below:

$$E_0 = E + m\frac{u^2}{2} + mgz$$

The total work is defined as the sum of shaft work, W, mechanical work, W_m, and flow work, W_f, as shown below:

$$W_T = \left(W_s + W_m + W_f\right)$$

The purpose of the above definitions is to distinguish between the two different working environments, a closed system and an open system.

2.2.1. Closed System

A closed system is defined as an isolated environment with no mass flow across the control surface. In this case, the total work of a closed system, W_{csys}, is simply the sum of shaft and mechanical works:

$$W_{csys} = W_s + W_m$$

Using lower case to denote mass specific values, equation (2.5) is transformed into equation (2.6) for a closed system:

$$q - w_{csys} = \left(e + \frac{u^2}{2} + gz\right)_2 - \left(e + \frac{u^2}{2} + gz\right)_1 \tag{2.6}$$

A closed system is not of interest in the analysis of gas turbine engines. However, the author believes that it is important to illustrate the difference.

2.2.2. Open System

The open system environment is where the gas turbine engine operates. Therefore, the open system is how we will analyze the performance of the turbojet. Firstly, work is defined as the dot product of force and displacement. Moreover, force is also defined in terms of pressure and area. Below, work is related in terms of pressure and specific volume, ν, as demonstrated by the following relationship:

$$dW = Fdx = PAdx = PdV = Pd\left(m\nu\right) = mPd\nu$$

Assuming mass to be constant, the above equation can defined in terms of specific work as shown below. The following relation is how we will define the additional flow work associated with the open system:

$$dw = Pd\nu \rightarrow w = P\nu$$

At this point, equation (2.7) introduces enthalpy. Enthalpy is defined as the sum of the specific internal energy and flow work:

$$h \equiv e + P\nu \tag{2.7}$$

As a result, the First Law equation for an open system can be defined in terms of enthalpy, where flow work is introduced into equation (2.6), resulting in a new total work, w. The derivation is shown below:

$$\begin{aligned} q - w &= \left(e + \frac{u^2}{2} + gz\right)_2 - \left(e + \frac{u^2}{2} + gz\right)_1 + \Delta w_f \\ &= \left(e + \frac{u^2}{2} + gz\right)_2 - \left(e + \frac{u^2}{2} + gz\right)_1 + \Delta P\nu \\ &= \left(e + P\nu + \frac{u^2}{2} + gz\right)_2 - \left(e + P\nu + \frac{u^2}{2} + gz\right)_1 \end{aligned}$$

Equation (2.8) shows the final form of the First Law used in gas turbine performance analysis:

$$q - w = \left(h + \frac{u^2}{2} + gz\right)_2 - \left(h + \frac{u^2}{2} + gz\right)_1 \tag{2.8}$$

2.3. Ideal Gas

In order to close the system of governing equations required to calculate the performance of gas turbine engines, we need additional definitions that describe the atmospheric and flight conditions. Equation (2.9) is the ideal gas law, where $\overline{R}$ is the universal gas constant and n is the number of moles of gas:

$$PV = n\overline{R}T \tag{2.9}$$

However, in gas turbine engine calculations, it is preferred to use the specific gas constant, R, and the specific volume relevant to the flight condition. Equation (2.9) is, therefore, transformed into equation (2.10) through the derivation below, where M_w is the molecular weight of the gas:

$$\frac{PV}{nM_wT}=\frac{\bar{R}}{M_w}\rightarrow\frac{PV}{mT}=R$$

$$Pv=RT \tag{2.10}$$

We define any gas that obeys equation (2.10) as an ideal gas.

2.4. Specific Heats and Perfect Gas

Specific heat refers to the amount of energy needed to increase the temperature of a unit mass of a substance by one degree of temperature.[20] There are two types of specific heat, the specific heat at constant volume and the specific heat at constant pressure. The specific heat at constant volume is defined as the change in internal energy relative to the change in temperature:

$$c_v=\frac{de}{dT} \tag{2.11}$$

The specific heat at constant pressure is defined as the change in enthalpy relative to the change in temperature:

$$c_p=\frac{dh}{dT} \tag{2.12}$$

Any gas with constant specific heats is referred to as a perfect gas.

2.5. Relationship Between Specific Heats and Gas Constant

The relationship below is derived using the definition of enthalpy and the ideal gas relationship:

$$h=e+Pv=e+RT\rightarrow dh=de+RdT$$

Additionally, the above relationship is further transformed using the definitions of the specific heats. Equation (2.13) is the derived result that relates the specific heats and the gas constant:

$$\begin{gathered} dh=de+RdT \\ c_pdT=c_vdT+RdT \\ c_p=c_v+R \end{gathered} \tag{2.13}$$

2.6. Entropy

Entropy introduces the concept of irreversibility and the direction of thermodynamic processes. Equation (2.14) defines entropy as the change in heat relative to temperature for a reversible process:

$$dS\equiv\left(\frac{dQ}{T}\right)_{rev} \tag{2.14}$$

According to equation (2.14), in a reversible and adiabatic process, where $dQ = 0$, the entropy must be zero. We define this particular process as an isentropic process.

2.7. Relationship Between Entropy, Specific Heat and Gas Constant

Equations (2.15) and (2.16) demonstrate how entropy is related to various gas state properties. They are derived from the conservation of energy:

$$dq = de + dw$$
$$Tds = de + Pdv$$

The chain rule is applied on Pdv to arrive at the following identity:

$$d\left(Pv\right) = dPv + Pdv$$

The above identity is substituted into the previous relationship to arrive at equation (2.15):

$$Tds = de + d\left(Pv\right) - vdP$$
$$ds = \frac{d\left(e + Pv\right)}{T} - \frac{dP}{\rho T}$$
$$ds = c_p \frac{dT}{T} - R\frac{dP}{P} \tag{2.15}$$

Equation (2.16) is derived similarly, in terms of the specific heat at constant volume:

$$ds = \frac{de}{T} + \frac{Pdv}{T}$$
$$ds = c_v \frac{dT}{T} - R\frac{d\left(1/\rho\right)}{1/\rho} \tag{2.16}$$

2.8. Isentropic Relations

Equation (2.17) introduces the definition of the specific heat ratio. Assuming the fluid to be ideal and perfect, the specific heat ratio will therefore be constant:

$$\gamma = \frac{c_p}{c_v} \tag{2.17}$$

In addition to the specific heat ratio, the isentropic assumption allows the derivation of useful relationships between pressure, temperature, and density in terms of the specific heat ratio. Starting with equation (2.15), the entropy is set to zero for the isentropic condition:

$$0 = c_p \frac{dT}{T} - R\frac{dP}{P}$$

$$R\frac{dP}{P} = c_p \frac{dT}{T}$$

Taking the proper integral of both sides and utilizing the properties of natural logarithms, the following relationship between pressure and temperature is achieved:

$$R\int_{P_1}^{P_2} \frac{dP}{P} = c_p \int_{T_1}^{T_2} \frac{dT}{T}$$

$$\left(\ln P_2 - \ln P_1\right) = \frac{c_p}{R}\left(\ln T_2 - \ln T_1\right)$$

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{c_p}{R}}$$

Equation (2.18) is derived from the above relationship using equation (2.13), and the specific heat ratio:

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \tag{2.18}$$

Similarly, equation (2.19) is derived by following the same procedure as above. It shows the relationship between density and temperature:

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}} \tag{2.19}$$

Equation (2.20) shows the relationship between pressure and density:

$$\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} \tag{2.20}$$

2.9. Speed of Sound

Turbojet engine performance analysis relies heavily on the value of the speed of sound. Therefore it is important to understand how the speed of sound is derived. Imagine a moving gas travelling through a stationary sound wave, as illustrated by Figure 1.

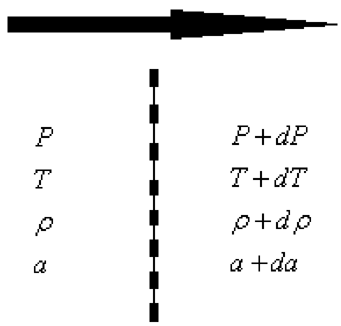

Figure 1. Gas (arrow) moving through a stationary sound wave (dashed line).

Figure 1 shows the initial conditions of pressure, temperature, density, and the speed of sound to the left of stationary sound wave. After crossing the stationary sound wave, the initial conditions are changed by the addition of small perturbations. Equation (2.21) is achieved by equating the mass fluxes before and after the stationary sound wave. Applying some algebra and neglecting higher order terms, an expression for the speed of sound, a, is obtained:

$$\begin{aligned}\rho a &= (\rho + d\rho)(a + da) \\ &= \rho a + \rho da + a d\rho + d\rho da\end{aligned}$$

$$a = -\rho \frac{da}{d\rho} \tag{2.21}$$

Equation (2.22) is obtained by equating the momentum fluxes across the stationary sound wave, where once again, higher order terms are neglected:

$$\begin{aligned}P + \rho a^2 &= (P + dP) + (\rho + d\rho)(a + da)^2 \\ dP &= -2a\rho da - a^2 d\rho\end{aligned}$$

$$da = \frac{dP + a^2 d\rho}{-2a\rho} \tag{2.22}$$

Combining equations (2.21) and (2.22) together we arrive at a relationship that relates the speed of sound to pressure and density:

$$a = -\rho \frac{\frac{dP}{d\rho} + a^2}{-2a\rho}$$

$$a^2 = \frac{dP}{d\rho} \tag{2.23}$$

Using the isentropic relations between pressure and density, equation (2.24) is obtained using the following through the following derivation:

$$\frac{P_1}{P_2}=\left(\frac{\rho_1}{\rho_2}\right)^{\gamma}\rightarrow\frac{P_1}{\rho_1^{\gamma}}=\frac{P_2}{\rho_2^{\gamma}}=\text{constant}=c$$

$$P=c\rho^{\gamma}\rightarrow c=\frac{P}{\rho^{\gamma}} \tag{2.24}$$

Equation (2.24) is substituted into equation (2.23), where the appropriate derivative is taken with respect to density. Using the ideal gas law, Equation (2.25) is obtained. Equation (2.25) is the speed of sound for a perfect gas:

$$a^2=\frac{dP}{d\rho}=\frac{d\left(c\rho^{\gamma}\right)}{d\rho}=c\gamma\rho^{\gamma-1}=\frac{P}{\rho^{\gamma}}\gamma\rho^{\gamma-1}=\frac{\gamma P}{\rho}$$

$$a=\sqrt{\frac{\gamma P}{\rho}}\rightarrow a=\sqrt{\gamma RT} \tag{2.25}$$

2.10. Stagnation Properties

Stagnation values are obtained by bringing the flow to rest isentropically. Therefore, for a perfect gas, equation (2.8) is simply reduced to the following equations:

$$h_1+\frac{u_1^2}{2}=h_2+\frac{u_2^2}{2}$$

$$c_pT_1+\frac{u_1^2}{2}=c_pT_2+\frac{u_2^2}{2}$$

Since stagnation represents the flow at rest, $u_2=0$, and T_2 becomes the stagnation, or total, temperature, T_t:

$$c_pT+\frac{u^2}{2}=c_pT_t$$

After isolating the temperature to one side and utilizing the relationships $c_p=c_v+R$, and $\gamma=\frac{c_p}{c_v}$ as well as equation (2.25), the stagnation relationship in terms of the Mach number is obtained:

$$\frac{T_t}{T}=1+\frac{u^2}{2c_pT}=1+\frac{u^2}{\frac{2\gamma RT}{\gamma-1}}=1+\frac{(\gamma-1)u^2}{2a^2}$$

$$\frac{T_t}{T}=1+\frac{\gamma-1}{2}M^2 \tag{2.26}$$

Since the stagnation temperature ratio is achieved isentropically, the isentropic relations are used to derive the relation for stagnation pressure:

$$\frac{P_t}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} \tag{2.27}$$

Similarly, equation (2.28) is the relationship for stagnation density:

$$\frac{\rho_t}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}} \tag{2.28}$$

Chapter 3 Turbojet Performance Analysis

The equations presented in this chapter for analyzing turbojet performance are typical of what is taught at universities. The author will now summarize these equations using Mattingly's text[1] as the main source of reference. The main focus will be to derive an expression to calculate turbojet engine thrust based on the user-defined component efficiencies. Ideal and perfect gas assumptions are applied throughout the derivation.

3.1. Anatomy of a Turbojet

"The whole is [greater] than the sum of its parts"
Aristotle
Greek Philosopher [21]

A turbojet engine is made up of several components. The typical station numbering corresponding to each component is show in Figure 2. Numbers 6 through 8 are reserved for stations in the afterburner, which is located between the turbine and nozzle. They are omitted here.

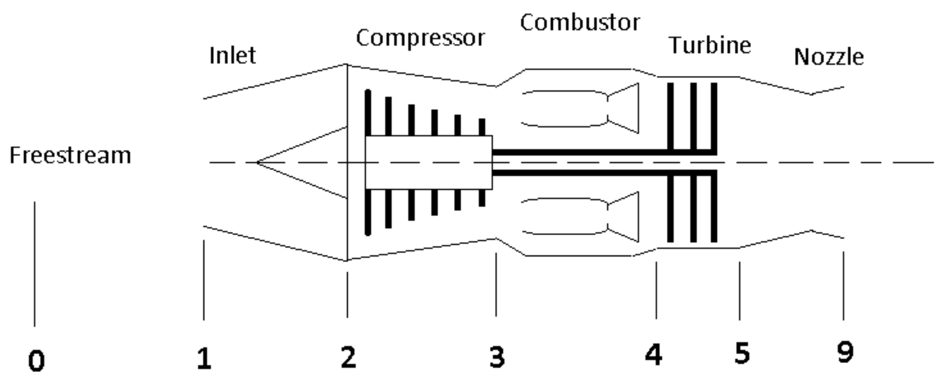

Figure 2. Anatomy of a Turbojet.

During flight operation, the inlet acts as a diffuser in order to decelerate the flow to a subsonic condition before it enters the compressor. The compressor, driven by the turbine, injects work into the flow, thereby increasing the pressure. The compressed flow enters the combustor, where the combustion process injects chemical energy to greatly heat and accelerate the flow. The turbine takes some of the energy from the flow, in order to drive the compressor, before the flow leaves the engine through the nozzle. For the turbojet engine analysis in this thesis, the nozzle is designed to be optimally expanded at sea level static.

3.2. Ratio Definitions

Using the station numberings defined above, we will now define useful temperature and pressure ratios based on the station numberings. These ratios will help to simplify the expressions. The ratios are summarized in Table 1.

Table 1. Temperature and Pressure Ratios.

Temperature Ratio		Pressure Ratio	
$\tau_r = 1 + \frac{\gamma - 1}{2} M_0^2$	$\tau_b = \frac{T_{t4}}{T_{t3}}$	$\pi_r = \left(1 + \frac{\gamma - 1}{2} M_0^2\right)^{\frac{\gamma}{\gamma - 1}}$	$\pi_b = \frac{P_{t4}}{P_{t3}}$
$\tau_\lambda = \frac{c_{pt} T_{t4}}{c_{pc} T_{t0}}$	$\tau_t = \frac{T_{t5}}{T_{t4}}$	$\pi_d = \frac{P_{t2}}{P_{t0}}$	$\pi_t = \frac{P_{t5}}{P_{t4}}$
$\tau_d = \frac{T_{t2}}{T_{t0}}$	$\tau_n = \frac{T_{t9}}{T_{t5}}$	$\pi_c = \frac{P_{t3}}{P_{t2}}$	$\pi_n = \frac{P_{t9}}{P_{t7}}$
$\tau_c = \frac{T_{t3}}{T_{t2}}$			

The ratios use the same subscript identifiers as those found in Mattingly's textbook.[1]

3.3. Component-Specific Definitions

In order to properly calculate the performance of a turbojet, certain component properties must be defined.

3.3.1. Inlet Definitions

The inlets are assumed to be adiabatic, meaning $\tau_d = 1$. For calculating the pressure across the inlet we first define the property $\pi_{d\max}$ as the pressure change due to wall friction and η_r as the ram recovery. The product of these two will result in the pressure ratio across the inlet:

$$\pi_d = \pi_{d\max} \eta_r \tag{3.1}$$

The value of η_r is defined by the Military Specification 5008B.[1] Equation (3.2) shows how η_r changes with the Mach number:

$$\eta_r = \begin{cases} 1 & ; M_0 \le 1 \\ 1 - 0.075(M_0 - 1)^{1.35} & ; 1 < M_0 < 5 \\ \dfrac{800}{M_0^4 + 935} & ; 5 < M_0 \end{cases} \tag{3.2}$$

Military Specification 5008B is derived from empirical data of various different nozzles. Equation (3.2) serves as an empirical average of the tested nozzles, which provides a good estimation of nozzle behavior in general.

3.3.2. Compressor Polytropic Efficiency

The polytropic efficiency is the ratio between ideal work and actual work per unit mass with respect to differential pressure change. It is used to calculate turbojet performance because the polytropic efficiency is essentially constant for highly efficient turbojets. It is, therefore, an accurate estimation for modern engines. Using the definition of enthalpy and assuming isentropic flow, the compressor polytropic efficiency is derived as

$$e_c = \frac{dh_{ti}}{dh_t} = \frac{RT\,\frac{dP}{P}}{C_p dT}$$

Equation (3.3) is obtained by taking the proper integrals and following the derivations below:

$$\int_{T_1}^{T_2} \frac{dT}{T} = \frac{R}{e_c C_p} \int_{P_1}^{P_2} \frac{dP}{P} \rightarrow \frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{e_c \gamma}}$$

$$\tau_c = \pi_c^{\frac{\gamma-1}{e_c \gamma}} \tag{3.3}$$

3.3.1. Compressor Isentropic Efficiency

The isentropic efficiency is defined as the ratio between the ideal work and the actual work as shown below. The compressor isentropic efficiency is needed during off-design analysis for relating off-design compressor pressure and temperature ratios.

$$\eta_c = \frac{h_{t3i} - h_{t2}}{h_{t3} - h_{t2}} = \frac{c_{pc}\left(T_{t3i} - T_{t2}\right)}{c_{pc}\left(T_{t3} - T_{t2}\right)} = \frac{\tau_{ci} - 1}{\tau_c - 1} = \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\tau_c - 1} \tag{3.4}$$

3.3.2. Turbine Polytropic Efficiency

Similar to the compressor polytropic efficiency, the turbine polytropic efficiency can be considered as constant for a modern engine. The derivation for the turbine polytropic efficiency is the same as the compressor polytropic efficiency.

$$e_t = \frac{dh_t}{dh_{ti}} = \frac{C_p dT}{RT\,\frac{dP}{P}}$$

Equation (3.5) is the result after following the proper derivation procedures.

$$\pi_t = \tau_t^{\frac{\gamma}{e_t(\gamma-1)}} \tag{3.5}$$

3.4. On-Design Engine Performance Defined by Mattingly[1]

On-design is defined as the atmospheric and flight condition for which the engine is sized. The turbojet analysis calculated in this thesis is taken at sea level static, meaning the on-design point has both the flight Mach number and altitude equal to zero. The performance of the turbojet engine at the on-design point is also referred to as the reference point performance. This is due to the fact that when conducting calculations with different atmospheric and flight conditions, it becomes necessary to refer back to the on-design conditions.

On-design calculations will assume ideal and perfect gas conditions with two separate but constant specific heats before and after the combustor, meaning: $R_c = R_{0,1,2,3}$, $R_t = R_{4,5,9}$, $\gamma_c = \gamma_{0,1,2,3}$, $\gamma_t = \gamma_{4,5,9}$, $c_{pc} = c_{p0,1,2,3}$, and $c_{pt} = c_{p4,5,9}$.

3.4.1. Thrust Equation

To derive the thrust equation, a control volume analysis is applied to an engine as shown in Figure 3. The outward normal vector is positive and F_T is the reaction force to the thrust.

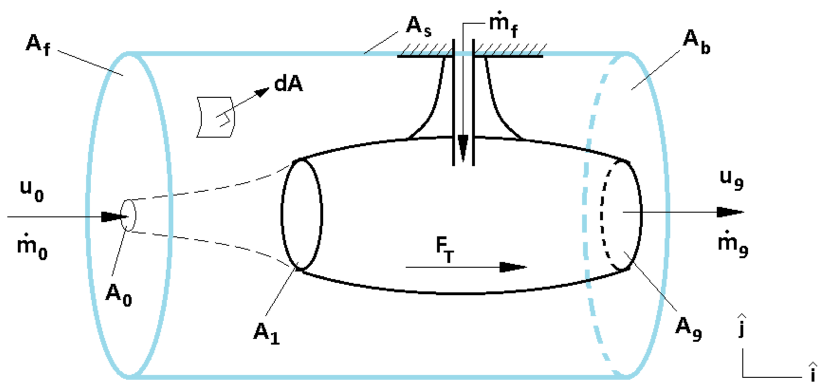

Figure 3. Control Volume Analysis of a Stationary Turbojet.

Air flow is assumed to be one-dimensional and steady-state and force is considered to be positive in the $\hat{i}$ direction. The resulting thrust force is calculated by applying the principles of conservation of mass and momentum to the control volume shown in Figure 3. The results of mass and momentum rates and the pressure forces are summarized in Table 2.

Table 2. Summary of Mass and Momentum Flow and Pressure Forces.

Control Surface	A_f	A_0	A_b	A_9	A_s
Mass Flow	$-\rho_0 u_0 A_f$	$-\rho_0 u_0 A_0$	$\rho_0 u_0 A_b$	$\rho_9 u_9 A_9$	$-\dot{m}_f$
Axial Momentum Flow	$-\rho_0 u_0^2 A_f$	$-\rho_0 u_0^2 A_0$	$\rho_0 u_0^2 A_b$	$\rho_9 u_9^2 A_9$	0
Axial Pressure Force	$P_0 A_f$	$P_0 A_0$	$-P_0 A_b$	$-P_9 A_9$	0

Using the conservation of mass, the mass flow terms from Table 2 are summed in equation (3.6):

$$0 = -\rho_0 u_0 A_f - \rho_0 u_0 A_0 - \dot{m}_f + \rho_0 u_0 A_b + \rho_9 u_9 A_9 \tag{3.6}$$

Factoring out $-\rho_0 u_0$, equation (3.6) is reduced into equation (3.7)

$$\dot{m}_f = -\rho_0 u_0 \left(A_f + A_0 - A_b \right) + \rho_9 u_9 A_9 \tag{3.7}$$

At this juncture the mass flow identities are introduced, where f is the fuel-air ratio:

$$\dot{m}_f + \dot{m}_0 = \dot{m}_9 \quad ; \quad \frac{\dot{m}_f}{\dot{m}_0} = f \quad ; \quad \frac{\dot{m}_9}{\dot{m}_0} = 1 + f$$

Using the identities $\dot{m}_f + \dot{m}_0 = \dot{m}_9$ and $A_f + A_0 = A_b + A_9$, equation (3.7) is transformed into equation (3.8):

$$\dot{m}_9 - \dot{m}_0 = -\rho_0 u_0 A_9 + \rho_9 u_9 A_9 \tag{3.8}$$

The mass flows in equation (3.8) is transformed into their respective density, velocity and area to form equation (3.9):

$$\rho_9 u_9 A_9 - \rho_0 u_0 A_0 = -\rho_0 u_0 A_9 + \rho_9 u_9 A_9 \tag{3.9}$$

Simplifying equation (3.9) resulted in the relationship $A_9 = A_0$:

$$\begin{aligned} \rho_0 u_0 A_9 - \rho_0 u_0 A_0 &= \rho_9 u_9 A_9 - \rho_9 u_9 A_9 \\ \rho_0 u_0 \left(A_9 - A_0 \right) &= 0 \\ A_9 - A_0 &= 0 \end{aligned}$$

Using the conservation of momentum, the axial momentum flow and axial pressure force terms from Table 2 and the thrust reaction force are summed:

$$-\rho_0 u_0^2 A_f - \rho_0 u_0^2 A_0 + \rho_0 u_0^2 A_b + \rho_9 u_9^2 A_9 = P_0 A_f + P_0 A_0 - P_0 A_b - P_9 A_9 + F_T \tag{3.10}$$

Using the identities $\rho_0 u_0^2 A_0 = \dot{m}_0 u_0$, $A_b = A_f + A_0 - A_9$ and $\rho_9 u_9^2 A_9 = \dot{m}_9 u_9$ to transform the left hand side of equation (3.10), while factoring out P_0 on the right hand side, equation (3.10) is turned into equation

$$-\rho_0 u_0^2 A_f - \dot{m}_0 u_0 + \rho_0 u_0^2 \left(A_f + A_0 - A_9 \right) + \dot{m}_9 u_9 = P_0 \left(A_f + A_0 - A_b \right) - P_9 A_9 + F_T \tag{3.11}$$

By applying the relationship $A_0 = A_9$ to the left hand side of equation (3.11) and knowing that $A_f + A_0 - A_b = A_9$ on the right hand side, equation (3.11) is reduced into equation (3.12):

$$-\rho_0 u_0^2 A_f - \dot{m}_0 u_0 + \rho_0 u_0^2 A_f + \dot{m}_9 u_9 = P_0 A_9 - P_9 A_9 + F_T \tag{3.12}$$

Simplifying and rearranging equation (3.12), equation (3.13) is obtained as the thrust equation:

$$-\dot{m}_0 u_0 + \dot{m}_9 u_9 = P_0 A_9 - P_9 A_9 + F_T$$

$$F_T = \dot{m}_9 u_9 - \dot{m}_0 u_0 + \left(P_9 - P_0\right) A_9 \tag{3.13}$$

3.4.2. Specific Thrust

Equation (3.13), however, is insufficiently defined for gas turbine analysis. The thrust equation must be represented in terms of variables we can calculate from the user-defined turbojet engine parameters. These parameters are shown in Table 3.

Table 3. Turbojet User-Defined Design Parameters

Flight Conditions		Design Limits	Component Performance Constants		Design Choices
P_0	γ_c	T_{t4}	π_b	η_c	π_c
T_0	c_{pc}	$\pi_{d\max}$	π_n	η_b	P_0/P_9
M_0	γ_t	h_{PR}	e_c	η_m	
$\dot{m}_0$	c_{pt}		e_t		

To conduct the calculations in terms of the user-defined design parameters, Equation (3.13) is transformed into the specific thrust by dividing by $\dot{m}_0$ and the speed of sound and Mach number are introduced:

$$\frac{F_T}{\dot{m}_0} = \left(\frac{\dot{m}_9}{\dot{m}_0} u_9 - u_0\right) + \frac{A_9}{\dot{m}_0}\left(P_9 - P_0\right) = a_0\left(\frac{\dot{m}_9}{\dot{m}_0}\frac{u_9}{a_0} - M_0\right) + \frac{A_9 P_9}{\dot{m}_0}\left(1 - \frac{P_0}{P_9}\right)$$

The specific thrust equation is then expressed in terms of the gas constant and specific heat ratio:

$$\begin{aligned}\frac{F_T}{\dot{m}_0} &= a_0\left(\frac{\dot{m}_9}{\dot{m}_0}\frac{u_9}{a_0} - M_0\right) + \frac{\dot{m}_9}{\rho_9 A_9 u_9}\frac{A_9 P_9}{\dot{m}_0}\left(1 - \frac{P_0}{P_9}\right) \\ &= a_0\left(\frac{\dot{m}_9}{\dot{m}_0}\frac{u_9}{a_0} - M_0\right) + \frac{\dot{m}_9}{\dot{m}_0}\frac{P_9}{\dfrac{P_9}{R_9 T_9} u_9}\left(1 - \frac{P_0}{P_9}\right) \\ &= a_0\left(\frac{\dot{m}_9}{\dot{m}_0}\frac{u_9}{a_0} - M_0\right) + \frac{\dot{m}_9}{\dot{m}_0}\frac{R_9 T_9}{u_9}\frac{a_0^2}{\gamma_0 R_0 T_0}\left(1 - \frac{P_0}{P_9}\right)\end{aligned}$$

Equation (3.14) is obtained by factoring out the speed of sound and representing the mass flows in terms of the fuel-air ratio:

$$\frac{F_T}{\dot{m}_0} = a_0 \left(\left(1+f\right)\frac{u_9}{a_0} - M_0 + \left(1+f\right)\frac{a_0}{u_9}\frac{R_t T_9}{R_c T_0}\frac{1-\frac{P_0}{P_9}}{\gamma_c} \right) \tag{3.14}$$

Within Equation (3.14), the $\frac{u_9}{a_0}$ term can be express in terms of gas properties and pressure:

$$\left(\frac{u_9}{a_0}\right)^2 = \frac{a_9^2 M_9^2}{a_0^2} = \frac{\gamma_9 R_9 T_9}{\gamma_0 R_0 T_0} M_9^2 = \frac{\gamma_9 R_9 T_9}{\gamma_0 R_0 T_0}\frac{2}{\gamma_9 - 1}\left(\left(\frac{P_{t9}}{P_9}\right)^{\frac{\gamma_9 - 1}{\gamma_9}} - 1 \right) \tag{3.15}$$

In Equation (3.15), the $\frac{P_{t9}}{P_9}$ term can be described in terms of the user-defined design parameters and the pressure ratios from Table 1:

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9}\pi_r \pi_d \pi_c \pi_b \pi_t \pi_n \tag{3.16}$$

The only unknown value in Equation (3.16) is π_t, and it can be calculated using equation (3.5), which requires knowing the value of τ_t. τ_t can be calculated from the power balance between the compressor and turbine. The power balance is an application of the First Law:

$$\eta_m \dot{m}_4 c_{pt}\left(T_{t4} - T_{t5}\right) = \dot{m}_0 c_{pc}\left(T_{t3} - T_{t2}\right)$$

To obtain the desired ratios, the power balance is divided by $\dot{m}_0 c_{pc} T_{t2}$, and rearranged:

$$\frac{\eta_m \dot{m}_4 c_{pt}\left(T_{t4} - T_{t5}\right)}{\dot{m}_0 c_{pc} T_{t2}} = \frac{\dot{m}_0 c_{pc}\left(T_{t3} - T_{t2}\right)}{\dot{m}_0 c_{pc} T_{t2}}$$

$$\frac{\eta_m \left(\dot{m}_f + \dot{m}_0\right) c_{pt}\left(T_{t4} - T_{t5}\right)}{\dot{m}_0 c_{pc} T_{t2}} = \tau_c - 1$$

Using the temperature ratio definitions from Table 1, and the fuel-air ratio, the expression above is further transformed below:

$$\eta_m\left(f+1\right)\left(\frac{c_{pt}}{c_{pc}}\frac{T_{t4}}{T_0}\frac{T_0}{T_{t0}}\frac{T_{t0}}{T_{t2}}-\frac{c_{pt}}{c_{pc}}\frac{T_{t5}}{T_{t4}}\frac{T_{t4}}{T_0}\frac{T_0}{T_{t0}}\frac{T_{t0}}{T_{t2}}\right)=\tau_c-1$$

$$\eta_m\left(f+1\right)\left(\frac{\tau_\lambda}{\tau_r\tau_d}-\frac{\tau_\lambda\tau_t}{\tau_r\tau_d}\right)=\tau_c-1$$

$$\eta_m\left(f+1\right)\frac{\tau_\lambda}{\tau_r}\left(1-\tau_t\right)=\tau_c-1$$

Equation (3.17) is obtained by isolating for τ_t:

$$\left(1-\tau_t\right)=\frac{\tau_r\left(\tau_c-1\right)}{\eta_m\tau_\lambda\left(f+1\right)}$$

$$\tau_t=1-\frac{\tau_r\left(1-\tau_c\right)}{\eta_m\tau_\lambda\left(f+1\right)} \tag{3.17}$$

The fuel-air ratio in both Equations (3.17) and (3.14) can be calculated by applying the First Law to the combustor. The derivations are shown below where h_{PR} is the low heating value of fuel:

$$q-w=\left(h+\frac{u^2}{2}+gz\right)_{out}-\left(h+\frac{u^2}{2}+gz\right)_{in}$$

$$q+\left(h+\frac{u^2}{2}+gz\right)_{in}=\left(h+\frac{u^2}{2}+gz\right)_{out}$$

$$q+\left(h_3+\frac{u_3^2}{2}\right)=\left(h_4+\frac{u_4^2}{2}\right)$$

$$\dot{m}_f\eta_b h_{PR}+\dot{m}_0 c_{pc}T_{t3}=\dot{m}_4 c_{pt}T_{t4} \tag{3.18}$$

To express Equation (3.18) in terms of the desired ratios, the following derivation is applied, where the inlet is assumed to be adiabatic. Therefore $\tau_d=1$:

$$\frac{\dot{m}_f\eta_b h_{PR}}{\dot{m}_0 c_{pc}T_0}+\frac{\dot{m}_0 c_{pc}T_{t3}}{\dot{m}_0 c_{pc}T_0}=\frac{\dot{m}_4 c_{pt}T_{t4}}{\dot{m}_0 c_{pc}T_0}$$

$$f\frac{\eta_b h_{PR}}{c_{pc}T_0}+\tau_r\tau_d\tau_c=\frac{\left(\dot{m}_0+\dot{m}_f\right)\tau_\lambda}{\dot{m}_0}$$

$$f\frac{\eta_b h_{PR}}{c_{pc}T_0}+\tau_r\tau_c=\left(1+f\right)\tau_\lambda$$

$$f\left(\frac{\eta_b h_{PR}}{c_{pc}T_0}-\tau_\lambda\right)=\tau_\lambda-\tau_r\tau_c$$

Equation (3.19) is obtained by isolating for the fuel-air ratio:

$$f = \frac{\tau_\lambda - \tau_r \tau_c}{\left(\frac{\eta_b h_{PR}}{c_{pc} T_0} - \tau_\lambda \right)} \tag{3.19}$$

this juncture, the only unknown value in the specific thrust equation is the ratio of exit to entrance temperature. This value can be calculated by applying the following transformation, using the temperature ratio definitions and the isentropic relations:

$$\frac{T_9}{T_0} = \frac{T_9}{T_{t9}} \frac{T_{t9}}{T_0} = \frac{1}{\frac{T_{t9}}{T_9}} \frac{T_{t5}}{T_0} = \frac{\frac{T_{t5}}{T_{t4}} \frac{T_{t4}}{T_0}}{\left(\frac{P_{t9}}{P_9} \right)^{\frac{\gamma_9 - 1}{\gamma_9}}} = \frac{\tau_t \tau_\lambda \frac{c_{pc}}{c_{pt}}}{\left(\frac{P_{t9}}{P_9} \right)^{\frac{\gamma_9 - 1}{\gamma_9}}}$$

3.4.3. Thrust-Specific Fuel Consumption and the Fuel Flow Rate

The thrust-specific fuel consumption is a useful indicator that displays the fuel economy of the turbojet engine. Equation (3.20) shows the relationship between the thrust-specific fuel consumption, the fuel-air ratio, the thrust, and the fuel flow rate:

$$S = \frac{f}{F_T / \dot{m}_0} = \frac{\dot{m}_f}{F_T} \tag{3.20}$$

3.5. Off-Design Engine Performance

When a turbojet engine operates outside the prescribed on-design atmospheric and flight conditions, it is considered to be off-design. Since the geometry of the turbojet engine is determined and fixed during the on-design analysis, the performance of the turbojet will change accordingly during off-design analysis. The following assumptions are applied to off-design engine performance analysis as defined by Mattingly[1].

- Ideal and perfect gas
- Flow chocked at turbine entrance and the nozzle throat
- Burner and nozzle pressure ratios are constant
- Component efficiencies are constant
- Power produced from the turbine is transferred completely to the compressor
- Fuel-air ration is constant

Off-design calculations are similar to on-design calculations, with the exceptions of the mass flow and the compressor pressure and temperature ratios. These values are obtained using the referencing technique described below.

3.5.1. Mass Flow Parameter

To begin the off-design analysis, the concept of the mass flow parameter is introduced. The purpose of the mass flow parameter is to help simplify and aid the development of the equations used to analyze off-design performance.

To start, the basic mass flow is represented in terms of density, area, and velocity:

$$\dot{m} = \rho A u$$

Using a combination of the equation (2.10) and (2.25) the following algebra transformation is applied:

$$\frac{\dot{m}}{A} = \rho u = \frac{Pu}{RT} = \frac{Pu}{RT}\frac{\sqrt{\gamma RT}}{a} = MP\frac{\sqrt{\gamma}}{\sqrt{RT}} = MP\frac{\sqrt{\gamma}}{\sqrt{RT}}\frac{P_t}{P_t}\sqrt{\frac{T_t}{T_t}} = M\sqrt{\frac{\gamma}{R}}\frac{P}{P_t}\sqrt{\frac{T_t}{T}}\frac{P_t}{\sqrt{T_t}}$$

From the above form, like terms are collected and equation (2.26) and (2.27) are applied. This results in the definition of the mass flow parameter, and it is a function of the Mach number, specific heat ratio, and the gas constant:

$$\frac{\dot{m}}{A} = M\sqrt{\frac{\gamma}{R}}\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{-\gamma}{\gamma-1}}\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{2}}\frac{P_t}{\sqrt{T_t}}$$

$$MFP = \frac{\dot{m}\sqrt{T_t}}{AP_t} = M\frac{\sqrt{\gamma}}{\sqrt{R}}\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1-\gamma}{2(\gamma-1)}} \tag{3.21}$$

3.5.2. Off-Design Mass Flow

To determine the off-design mass flow, the total mass flow rate is redefined in terms of the mass flow parameter:

$$\dot{m}_4 = \dot{m}_0 + \dot{m}_f = \dot{m}_0\left(1+f\right) = \frac{A_4 P_{t4}}{\sqrt{T_t}} MFP\left(M_4\right)$$

Isolating $\dot{m}_0$, the expression can now be expressed in terms of the pressure ratio definitions from Table 1:

$$\dot{m}_0 = \frac{A_4 P_{t4}}{\sqrt{T_t}\left(1+f\right)} MFP\left(M_4\right) = \frac{A_4 P_0 \pi_r \pi_d \pi_c \pi_b}{\sqrt{T_t}\left(1+f\right)} MFP\left(M_4\right)$$

In the above expression, certain variables can be grouped and considered to be constant:

$$\frac{A_4 \pi_b}{\left(1+f\right)} MFP\left(M_4\right) = \text{constant}$$

Consequently, by isolating the above constant, a different grouping of parameter variables is formed. This new group of variables in their ratio form can also be considered to be a constant. The constant is referred to as the reference value, calculated from on-design parameters.

$$\frac{\dot{m}_0\sqrt{T_t}}{P_0\pi_r\pi_d\pi_c} = \text{constant} = \left(\frac{\dot{m}_0\sqrt{T_t}}{P_0\pi_r\pi_d\pi_c}\right)_{\text{reference}}$$

Isolating $\dot{m}_0$, a relationship is formed between on-design and off-design mass flows. Equation (3.22) shows the referencing technique for calculating off-design mass flow:

$$\dot{m}_0 = \frac{P_0\pi_r\pi_d\pi_c}{\sqrt{T_t}}\left(\frac{\dot{m}_0\sqrt{T_t}}{P_0\pi_r\pi_d\pi_c}\right)_{\text{reference}} \tag{3.22}$$

3.5.3. Off-Design Compressor Temperature Ratio

Using the same turbine and compressor power balance as before, the following expression is obtained:

$$\eta_m\left(f+1\right)\frac{c_{pt}}{c_{pc}}\frac{T_{t4}}{T_{t2}}\left(1-\tau_t\right) = \tau_c - 1$$

By collecting the user-defined design values, a constant is formed. As previously, the remaining parameter variables are isolated to form the reference term.

$$\eta_m\left(f+1\right)\frac{c_{pt}}{c_{pc}}\left(1-\tau_t\right) = \text{constant} = \frac{T_{t2}}{T_{t4}}\left(\tau_c - 1\right) = \left(\frac{T_{t2}}{T_{t4}}\left(\tau_c - 1\right)\right)_{\text{reference}}$$

Equation (3.23) relates the on- and off-design conditions to calculate the compressor temperature ratio:

$$\frac{T_{t2}}{T_{t4}}\left(\tau_c - 1\right) = \left(\frac{T_{t2}}{T_{t4}}\left(\tau_c - 1\right)\right)_{reference}$$

$$\left(\tau_c - 1\right) = \frac{T_{t4}}{T_{t2}}\left(\frac{T_{t2}}{T_{t4}}\left(\tau_c - 1\right)\right)_{reference}$$

$$\tau_c = 1 + \frac{T_{t4}/T_{t2}}{\left(T_{t4}/T_{t2}\right)_{reference}}\left(\tau_c - 1\right)_{reference} \tag{3.23}$$

3.5.4. Off-Design Compressor Pressure Ratio

Equation (3.24) is derived from the definition of the compressor isentropic efficiency:

$$\eta_c = \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\tau_c - 1}$$

$$\eta_c\left(\tau_c - 1\right) = \pi_c^{\frac{\gamma-1}{\gamma}} - 1$$

$$\left(1 + \eta_c\left(\tau_c - 1\right)\right)^{\frac{\gamma}{\gamma-1}} = \pi_c \tag{3.24}$$

3.6. Component Maps

It is important to note that equations used by GasTurb 11 and GSP 11 will take slightly different forms from the equations presented thus far. The difference is due to the inclusion of component map data. As it has already been stated that GSP 11 and GasTurb 11 model real gas via specific heats that are constant with pressure but vary with temperature. These programs also utilize component maps to make off-design calculations, unlike in the previous section where the performances of the turbojet engine components at off-design conditions were calculated by directly referencing the on-design condition.

The component maps are typically created from experimental data, and they specifically show the component behavior at different operating conditions. Every component map is different; therefore, GasTurb 11 and GSP 11 use a scaling method by matching the user-design point parameters with the design point of the component map. To properly compare the turbojet simulations, both GasTurb 11 and GSP 11 used with the same following compressor and turbine maps for off-design analysis.

This illustrates an important difference between PERF and that of GasTurb 11 and GSP 11. With PERF, the off-design assumptions allow constant component efficiencies for off-design flight conditions. With GasTurb 11 and GSP 11, however, the inclusion of component maps allows the variations of component efficiencies, with respect to changing flight conditions. As a result, the varying component efficiencies coupled with varying specific heats will have an impact on turbojet performance.

3.6.1. Compressor Map

The compressor map describes the behavior of the compressor for different operating conditions. Figure 4 shows the particular compressor map used by both GasTurb 11 and GSP 11 for off-design analysis.[4, 22] The red and black contour lines are compressor efficiency and spool speed, respectively. The horizontal axis shows the corrected mass flow as defined by equation (3.25), where $T_{ref} = 288.2[K]$, $P_{ref} = 101.3[\text{kPa}]$. The vertical axis shows the compressor pressure ratio.

$$\dot{m}_c \equiv \frac{\dot{m}\sqrt{T_t/T_{ref}}}{P_t/P_{ref}} \tag{3.25}$$

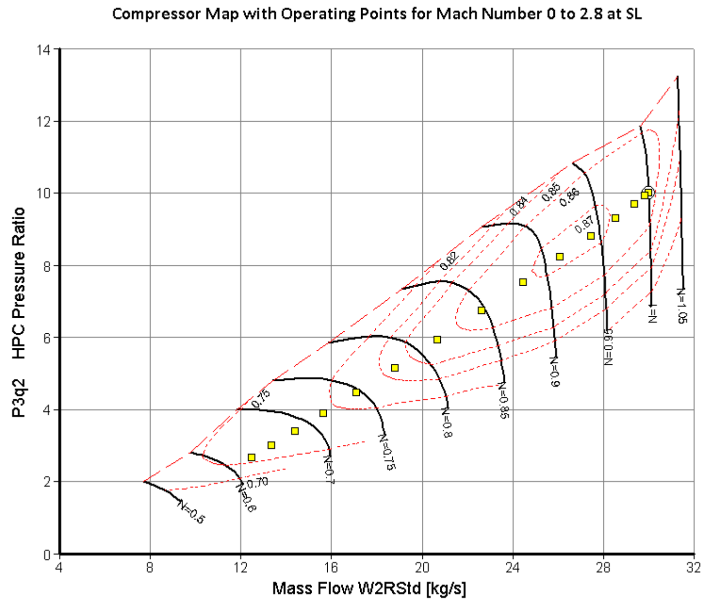

Figure 4. *NASA TM 101433 Example Compressor Map* Taken from GasTurb 11.

The yellow points in Figure 4 represent the different operating points ranging from flight Mach number of 0 to 2.8 at sea level static. Together they form the operating line. These particular operating points are used to clearly demonstrate the changing nature of the efficiency in response to changing flight conditions.

3.6.2. Turbine Map

The turbine map describes the behavior of the turbine for various operating conditions. Figure 5 shows the particular turbine map used by both GasTurb 11 and GSP 11 to perform the off-design analysis[4, 22]. The red and black contour lines represent the efficiency and the percentage of the on-design corrected spool speed, respectively. The corrected spool speed is defined by equation (3.26). The horizontal axis represents the percentage of the on-design corrected spool speed while the vertical axis represents the turbine pressure ratio.

$$N_c \equiv \frac{N}{\sqrt{T_t/T_{ref}}} \tag{3.26}$$

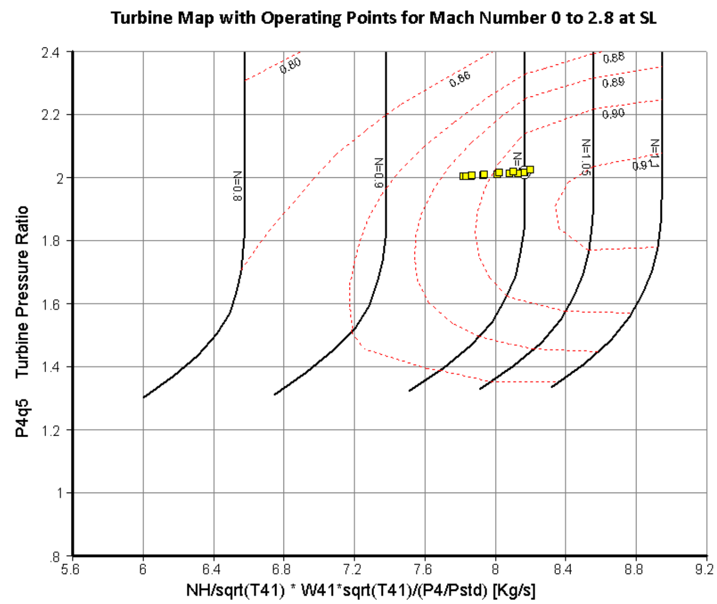

Figure 5. *High Work Low Aspect Ratio Turbine NASA TM83655* Taken from GasTurb 11.

Once again, the yellow points in Figure 5represent the operating points from Mach number 0 to 2.8 at sea level static. The operating points in Figure 5, in fact, correspond to the operating points in Figure 5. This illustrates the changing nature of the turbine efficiency in response to the change in flight condition.

Chapter 4 Results and Discussion

With the turbojet performance equations defined, it is now appropriate to compare the effects of ideal-perfect gas assumptions to real gas calculations coupled with the use of component maps and varying efficiencies as performed by GasTurb 11 and GSP 11. The raw data containing the results of the calculations are included in the appendix.

4.1. On-Design Simulation Comparison

In preparation for the simulations, a custom engine is created with its parameters listed in Table 4. The custom engine is used by all three programs. The specific heat and ratio values listed in the table can be considered ‘textbook standards’. They are necessary for the calculations listed in Section 3.4; therefore, they must be properly defined. GasTurb 11 and GSP 11, however, do not ask for user-defined specific heats and ratios. The on-design simulations are calculated at sea level static, meaning both altitude and Mach number are zero.

In order to ensure that GasTurb 11 and GSP 11 are indeed comparing simulations of the same turbojet, the inlets are specified to Military Specification 5008B standard.[1] The geometry of the nozzle is specified to achieve optimal on-design expansion. Throughout the simulations, the turbojet will fly at full throttle, meaning a constant combustor exit temperature.

The turbojet performance predicted by using the equations derived in Section 3.4 will be used as a base of reference. The computer program Engine Performance Analysis Program (PERF) uses the equations from section 3.4 and it is an accompaniment of Mattingly’s textbook[1], courtesy of the AIAA. Therefore, PERF will run alongside GasTurb 11 and GSP 11. PERF calculates turbojet performance using ideal and perfect gas assumptions, with constant user-defined specific heats and ratios. GasTurb 11 and GSP 11 use a pseudo-perfect gas model where the specific heats and ratios are not constants and are functions of temperature and gas composition but not pressure. This is how GasTurb 11 and GSP 11 approximate real gas effects.

Table 4. Custom Engine Component Parameters.

Parameter	Units	Value	Parameter	Units	Value
c_{pc}	[kJ/kg*K]	1.004	γ_c	[-]	1.4
c_{pt}	[kJ/kg*K]	1.235	γ_t	[-]	1.3
N	[rpm]	15000	T_{t4}	[K]	1800
$\dot{m}_0$	[kg/s]	30	π_b	[-]	0.95
π_d	[-]	1	η_b	[-]	0.99
e_c	[-]	0.9	e_t	[-]	0.9
π_c	[-]	10	π_n	[-]	1
h_{PR}	[MJ/kg]	42.1	P_0/P_9	[-]	1
η_m	[-]	0.99			

The results of the on-design simulation are summarized in Table 5. The results between GasTurb 11 and GSP 11 are remarkably close, demonstrating the two programs to be in agreement with their approximations of real gas effects on turbojet performance. Compared to GasTurb 11 and GSP 11, the results from PERF show a slightly lower thrust and slightly higher fuel flow rate. As a result PERF has a higher thrust-specific fuel consumption rate. Since component maps are not used for on-design analysis, the differences are, therefore, caused by the variations in specific heat.

Table 5. On-Design Performance Comparison of Custom Engine

Program	Thrust [kN]	Thrust % Difference From PERF	$\dot{m}_f$ [kg/s]	$\dot{m}_f$ % Difference from PERF	TSFC [g/kN*s]	TSFC % Difference from PERF
GasTurb 11	33.71	0.78	1.13093	8.32	33.5461	9.04
GSP 11	33.761	0.94	1.133	8.15	33.5611	9.00
PERF	33.447	0	1.2336	0	36.8797	0

With the initial on-design performance of the custom turbojet established, it is prudent to compare other on-design performances using different parameters. Table 6 lists several different component parameters with varying levels of technology, summarized from Mattingly's textbook. The levels of technology represent the progress of technology over increments of 20 years starting from Level 1 in 1945 through Level 4 in 2005.

Table 6. Technology Level Component Parameters.

Parameter	Level 1	Level 2	Level 3	Level 4
e_c	0.8	0.84	0.88	0.9
T_{t4}	1110	1390	1780	2000
π_b	0.9	0.92	0.94	0.95
η_b	0.88	0.94	0.99	0.999
e_t	0.8	0.85	0.89	0.9
η_m	0.95	0.97	0.99	0.995

Figure 6 reveals that at lower technology levels, PERF calculates a higher thrust than GasTurb 11 and GSP 11 by 3.75% and 2.78% respectively for Level 1. But at higher technology levels, PERF calculates a lower thrust by 0.99% and 1.18% respectively for Level 4. This shows that the efficiency levels have a direct impact on the comparison of ideal-perfect to real gas calculations.

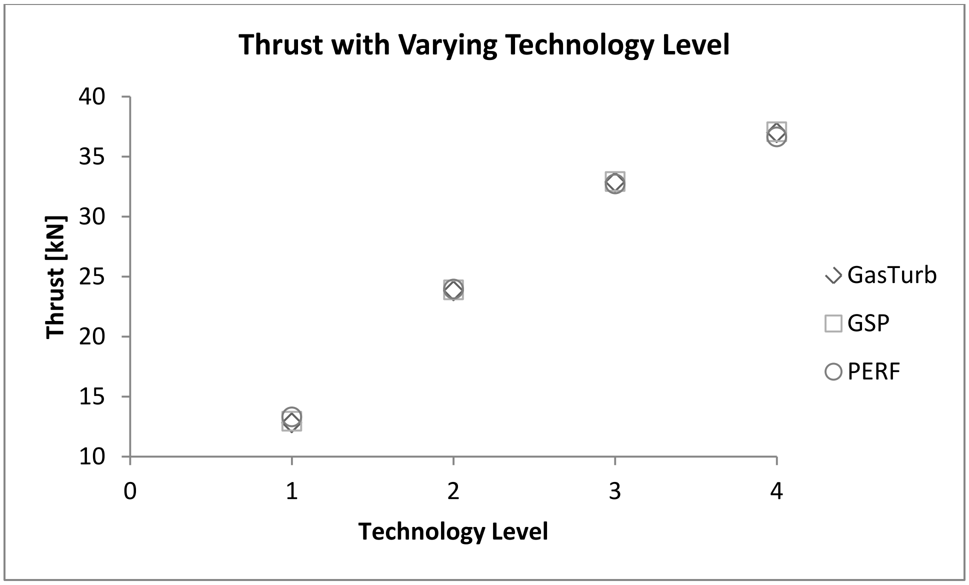

Figure 6. On-Design Thrust Comparison.

In Figure 7, GasTurb 11 and GSP 11 both produced near identical results for the fuel flow rate. But unlike the on-design thrust comparisons, there is a much more distinct difference between PERF and that of GasTurb 11 and GSP 11. At Level 1, the difference is 27.14% and 26.62% respectively. The difference at Level 4 is 5.04% and 4.95% respectively.

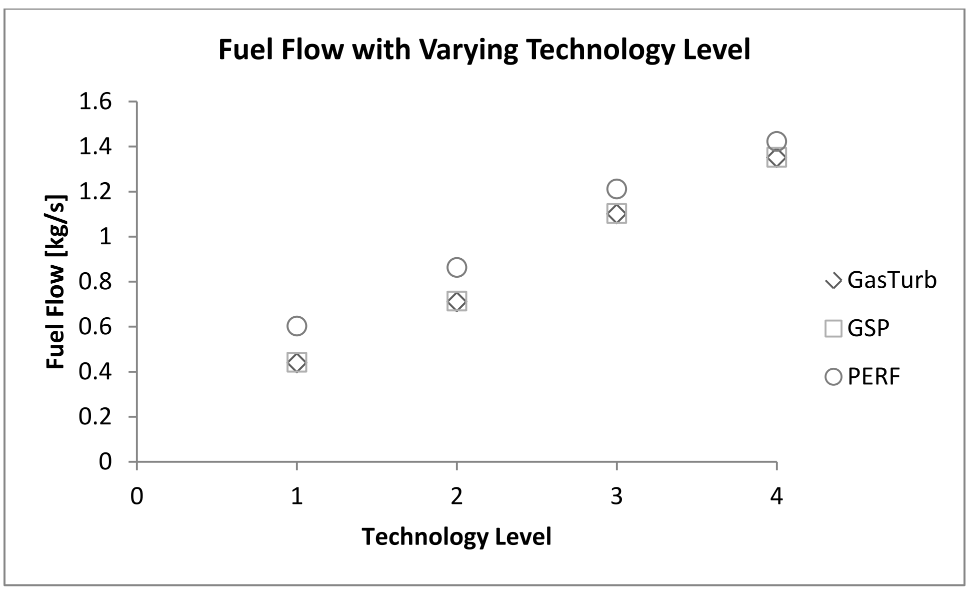

Figure 7. On-Design Fuel Flow Comparison.

The thrust-specific fuel consumption is a ratio between the fuel flow rate and the thrust, and it is an indication of the fuel efficiency of the turbojet engine. Therefore, due to higher fuel flow rates, the thrust-specific fuel consumption calculated by PERF is higher than the values calculated by GasTurb 11 and GSP 11 for all technology levels. Figure 8 shows that the lower the technology level, the greater the discrepancy. At Level 1, GasTurb 11 and GSP 11 have differences of 24.29% and 24.53% respectively from PERF. At Level 4, GasTurb 11 and GSP 11 have differences of 5.96% and 6.06% respectively from PERF.

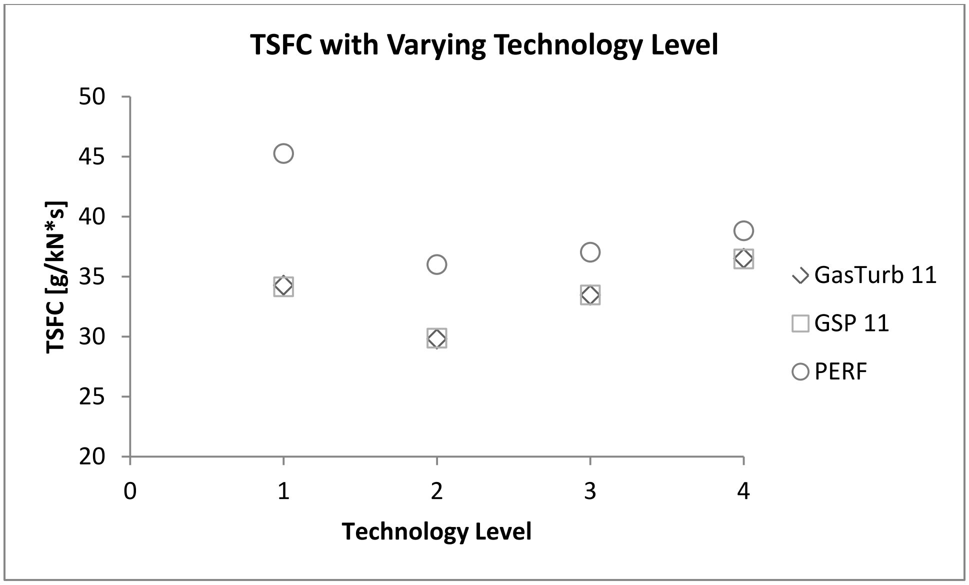

Figure 8. On-Design TSFC Comparison.

Based on the on-design simulations, it is evident that all three programs follow the same trend. However, the effects of the different gas assumptions on turbojet performance are greatly magnified by the change in efficiencies. It would appear that the lower the efficiency, the greater this difference.

4.2. Off-Design Simulation Comparison

The off-design simulation of the custom turbojet is conducted in two parts. The first part is to simulate flight conditions at Mach 0.5 for varying altitudes. The second part is to simulate flight conditions at a constant 8000 meters, a typical cruising altitude of commercial aircraft, while varying the flight Mach number. In addition to the different gas assumptions, the off-design simulations will also serve to illustrate the effects of the use of component maps, meaning varying component efficiencies, as compared to the referencing technique.

4.2.1. Varying Altitudes at a Mach Number of 0.5

Figure 9 shows that thrust decreases with increasing altitude at a fixed Mach number. The thrust calculated by PERF, however, decreases more dramatically with increasing altitude. It is apparent that the further the operating point is away from the design point, the greater the difference between PERF and GasTurb 11 and GSP 11. At 0[m], GasTurb 11 and GSP 11 have differences of 1.32% and 0.69% respectively from PERF. At 8000[m], GasTurb 11 and GSP 11 have differences of 21.63% and 21.34% respectively from PERF.

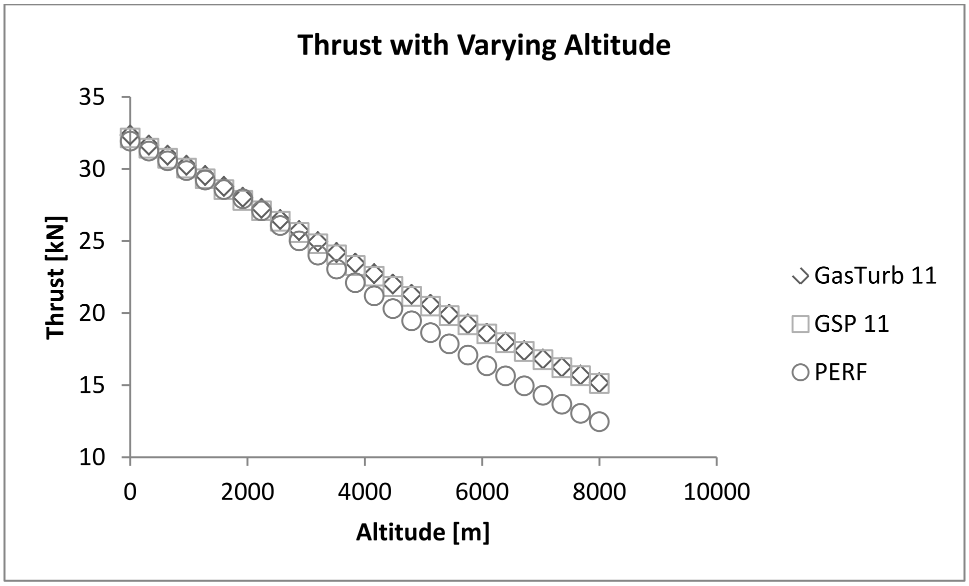

Figure 9. Off-Design Thrust with Varying Altitude at M=0.5.

Since the thrust decreases with increasing altitude, the same trend is expected of the fuel flow. Figure 10 shows that at a fixed flight Mach number, the fuel flow rate calculated by PERF, using ideal and perfect gas assumptions, results in a steeper drop with increasing altitude. Using the components maps, on the other hand, results in a more gradual decrease. At 0[m], GasTurb 11 and GSP 11 have differences of 7.80% and 8.10% respectively from PERF. At 8000[m], GasTurb 11 and GSP 11 have differences of 17.43% and 17.66% respectively from PERF.

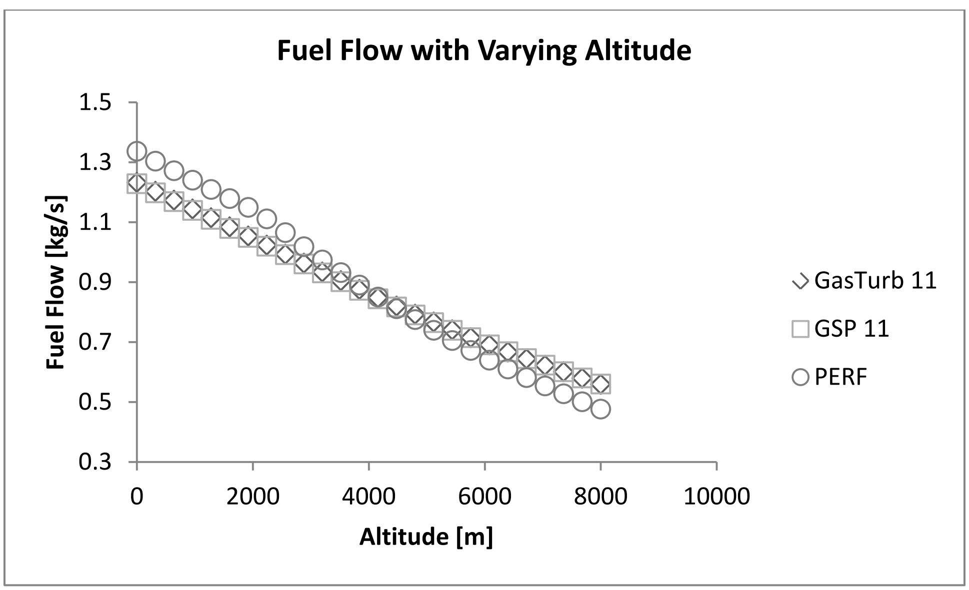

Figure 10. Off-Design Fuel Flow with Varying Altitude at M=0.5.

Once again, PERF calculations result in higher thrust-specific fuel consumption. Figure 11 shows that the effect of constant specific heats and ratios produces a much more dramatic decrease in the thrust specific fuel consumption. The combination of component maps and real gas effects, on the other hand, yields a gentler and more gradual change with increasing altitude. At 0[m], GasTurb 11 and GSP 11 have differences of 9.00% and 8.73% respectively from PERF. At 8000[m], GasTurb 11 and GSP 11 have differences of 3.45% and 3.03% respectively from PERF.

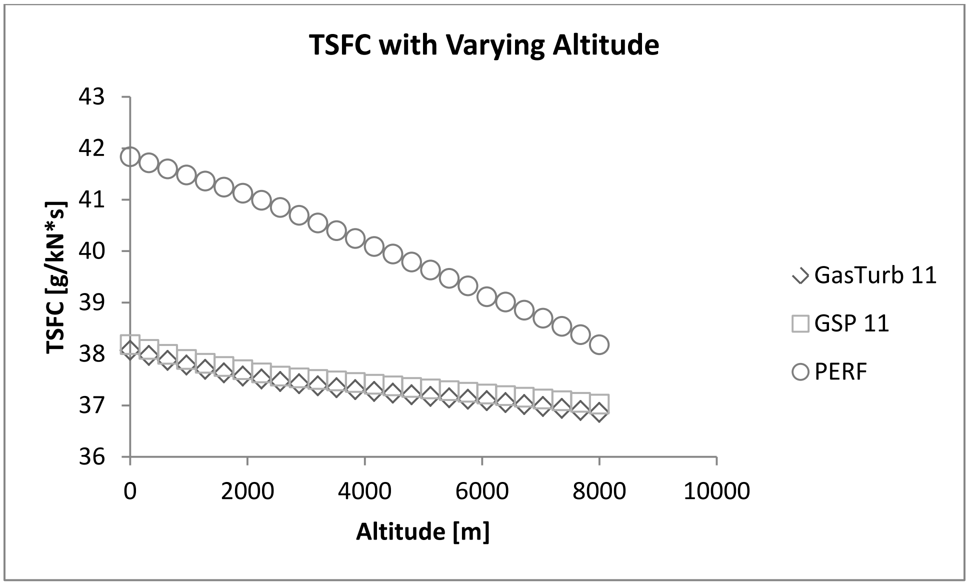

Figure 11. Off-Design TSFC with Varying Altitude at M=0.5.

Figure 9, Figure 10, and Figure 11 visually illustrates the differences between the ideal-perfect gas assumption with constant component efficiencies as calculated by PERF and the real gas assumptions coupled with the use of component maps with varying component efficiencies as calculated by GasTurb 11 and GSP 11.

4.2.2. Varying Mach Numbers at an Altitude of 8000[m]

Figure 12 shows that at constant altitude, the thrust increases with Mach number. Similar to the pervious analysis, PERF's calculations exhibit a more dramatic change with the change in Mach number. Compared the GasTurb 11 and GSP 11, the increase in thrust is more substantial. There is a slight discontinuity at Mach 1 with PERF's calculation. This is most likely due to the effect of change in the inlet pressure ratio specified by the Military Specification 5008B standard. However, the combination of component maps and real gas approximations eliminates the discontinuity. Both GasTurb 11 and GSP 11 once again show a smooth and gradual change in thrust with increasing Mach number. At Mach 0, GasTurb 11 and GSP 11 have differences of 29.24% and 29.40% respectively from PERF. At Mach 2, GasTurb 11 and GSP 11 have differences of 14.52% and 15.68% respectively from PERF.

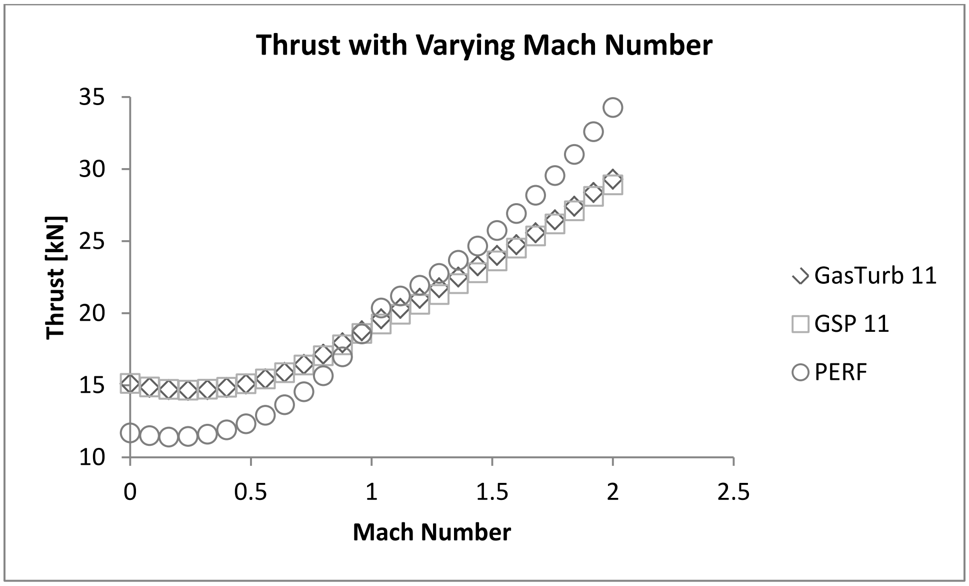

Figure 12. Off-Design Thrust with Varying Mach Numbers at h=8000 [m].

The fuel flow rate follows a similar trend compared to thrust. In Figure 13, PERF's calculations once again produce a more dramatic increase. In the same fashion as thrust, there appears to be a slight discontinuity at Mach 1 due to the change in inlet pressure ratio. However, this discontinuity is again eliminated in GasTurb 11 and GSP 11's calculations. At Mach 0, GasTurb 11 and GSP 11 have differences of 27.62% and 28.15% respectively from PERF. At Mach 2, GasTurb 11 and GSP 11 have differences of 13.34% and 14.35% respectively from PERF.

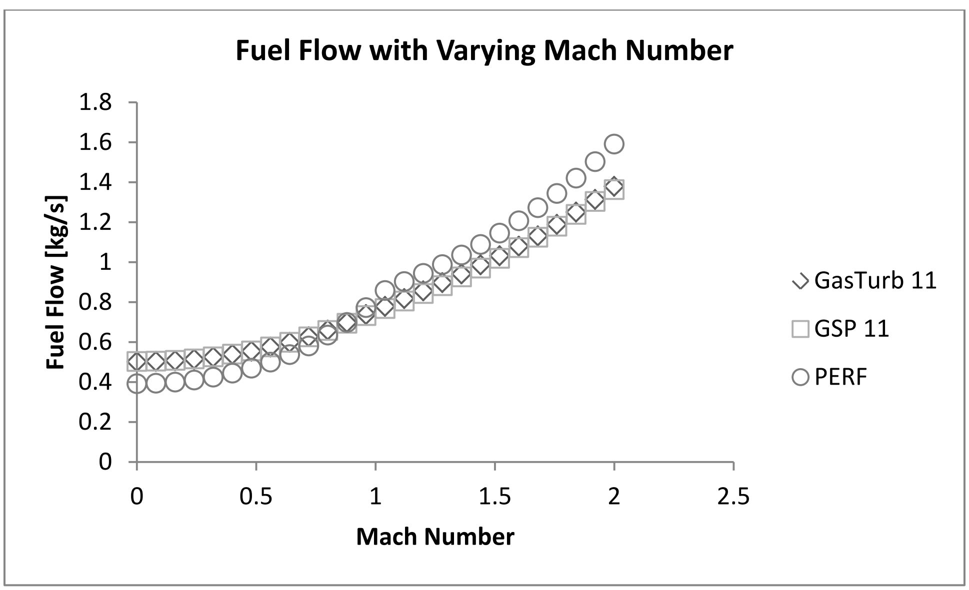

Figure 13. Off-Design Fuel Flow with Varying Mach Numbers at h=8000[m].

Based on the previous simulations, one would expect that PERF would once again have higher thrust-specific fuel consumption as compared to GasTurb 11 and GSP 11. However this is only true at lower Mach numbers. Due to the substantial increase in thrust as calculated by PERF, the resulting thrust-specific fuel consumption is actually better than with GasTurb 11 and GSP 11. The biggest difference between PERF and that of GasTurb 11 and GSP 11 occurs at Mach 1, by 5.85% and 5.41% respectively, where the discontinuity happens. Figure 14 once again shows that the combination of component maps and real gas effects produces a more gradual and gentler change. At Mach 0, GasTurb 11 and GSP 11 have differences of 1.05% and 0.78% respectively from PERF. At Mach 2, GasTurb 11 and GSP 11 have differences of 1.37% and 1.57% respectively from PERF.

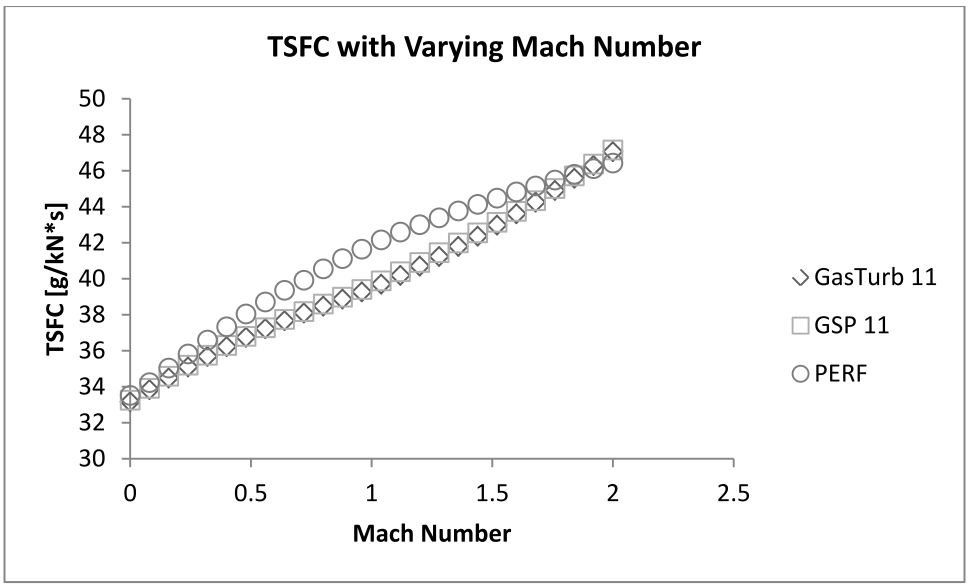

Figure 14. Off-Design TSFC with Varying Mach Numbers at h=8000[m].

Chapter 5 User Experience

The results calculated by both GasTurb 11 and GSP 11 are very similar and at times nearly identical. The experiences of using the two programs, however, are vastly different.

5.1. Using GasTurb 11

GasTurb 11 is very user-friendly, and it comes with an extensive manual [4] and numerous tutorials.[23-32] The user starts by selecting the type of engine (turbojet, turbofan, etc.) and the type of calculation (on-design or off-design). This is illustrated by Figure 15.

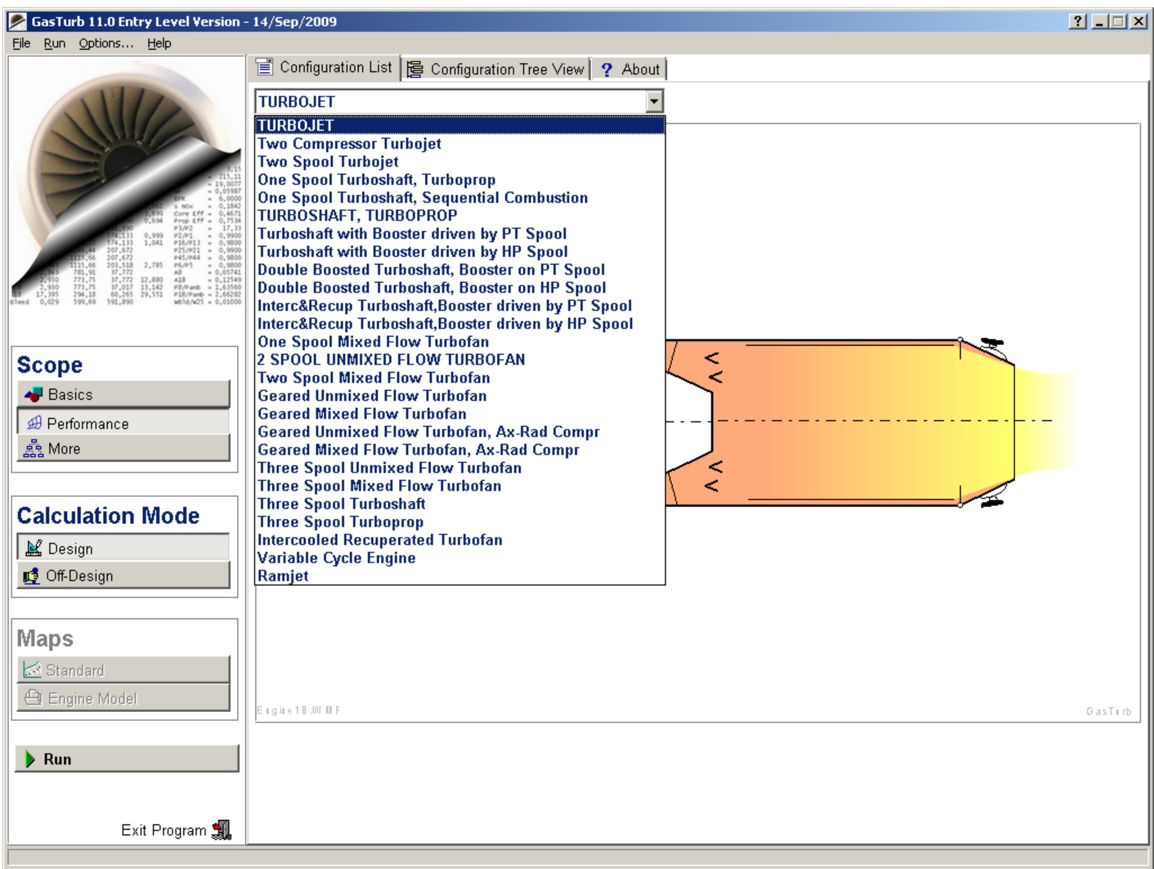

Figure 15 GasTurb 11 Gas Turbine Engine Selection Taken from GasTurb 11

The user is then presented with the opportunity to input the engine parameters. The parameters are organized into tabs where they can be easily found. The number of input parameters is noticeably fewer than those of GSP 11. Despite the lower number of input parameters, however, the available input parameters in GasTurb 11 are the ones that the user will typically want to change and use most often. As a result, GasTurb 11 is easier to navigate and appears to be more intuitive. This is illustrated by Figure 16.

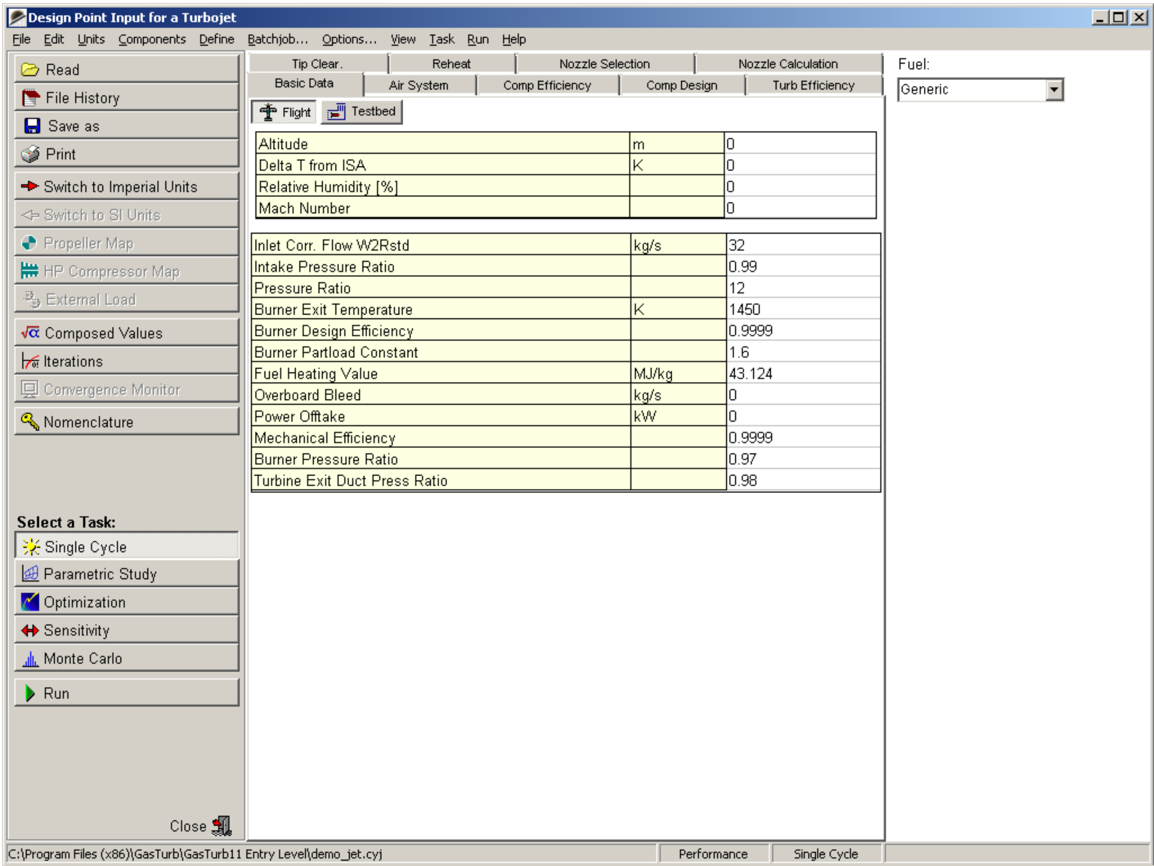

Figure 16 GasTurb 11 Parameter Inputs taken from GasTurb 11

5.2. Using GSP 11

GSP 11 is very customizable. This is the biggest difference between GasTurb 11 and GSP 11. Instead of simply selecting the type of gas turbine engine, the user has the option to build the desired gas turbine engine from a list of components. This is illustrated by Figure 17.

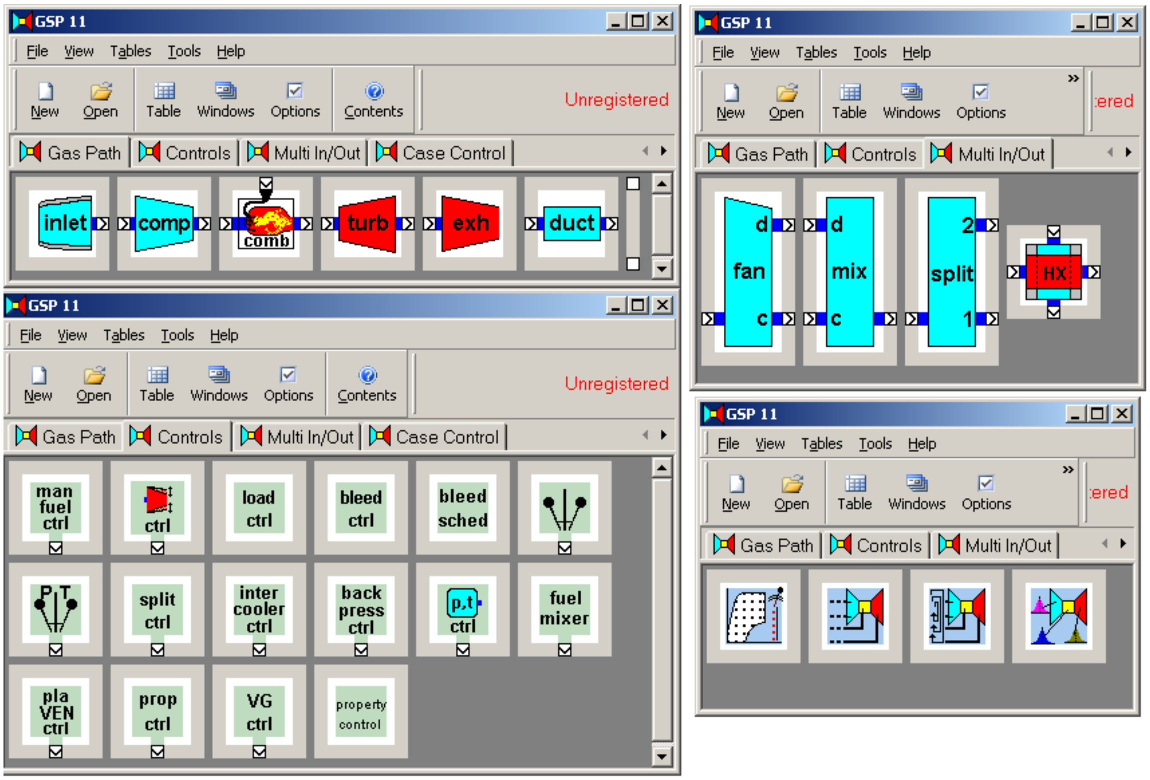

Figure 17 GSP 11 Component Library Windows taken from GSP 11

Each component, in turn, has a list of adjustable parameters, also organized into tabs. This feature allows GSP 11 to offer a wider range of parameter adjustments than GasTurb 11. However, the user is then required to be extra vigilant with GSP 11, as the customizability feature forces more user involvement. This is illustrated by Figure 18.

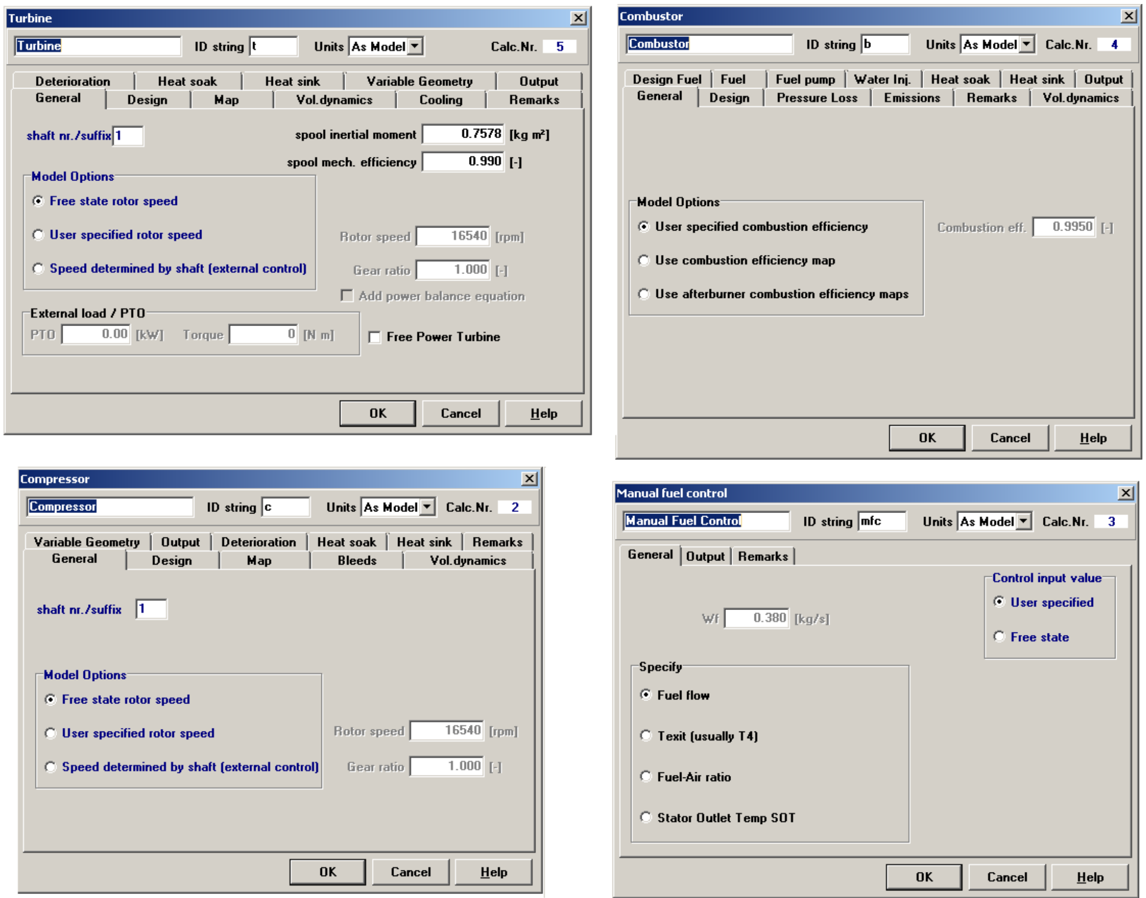

Figure 18 GSP 11 Component Data Windows taken from GSP 11

Once the user has constructed the desired gas turbine engine, he or she has the option of saving the engine as the reference model. GSP 11 is organized in a project tree format, where the reference model is the parent capable of spawning child configurations. Making changes to the child configuration will not affect the parent configuration, but changes made to the parent configuration will propagate down to the children. This allows the user to organize and run all desired simulations in one window. This is illustrated by Figure 19.

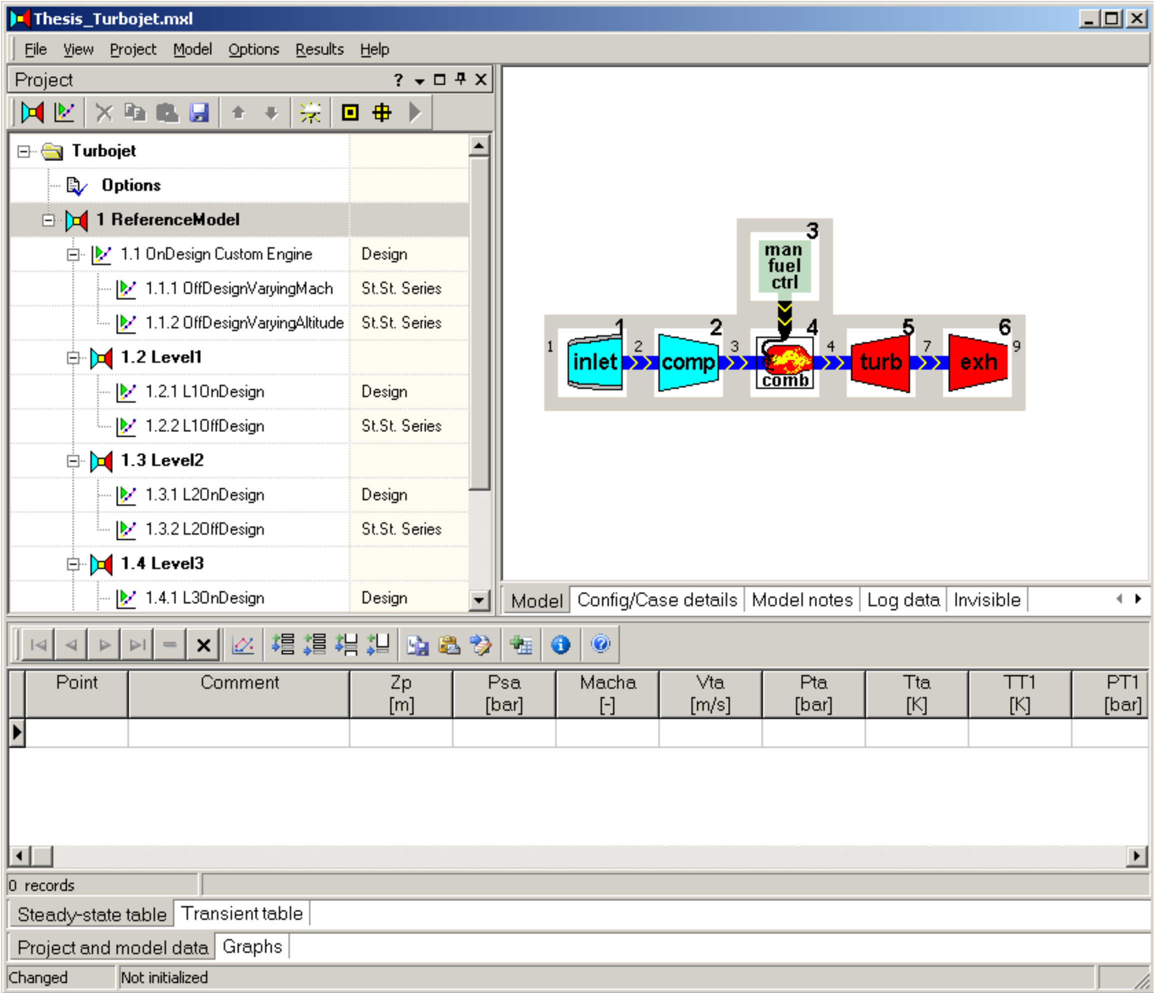

Figure 19 GSP 11 Project Window taken from GSP 11

Chapter 6 Conclusion

For on-design calculations, the differences between GasTurb 11, GSP 11, and PERF are not particularly large and they all follow the same trend. However, the differences are noticeably interesting. Low component efficiencies effect PERF's calculations by creating a larger discrepancy with the other two programs. This means that ideal-perfect gas calculations are more susceptible to changes in component efficiency.

For off-design calculations with a fixed Mach number and varying altitudes, the thrust calculated by PERF experiences a noticeably more dramatic decrease with altitude. The same is true with the fuel flow rate. For off-design calculations with a fixed altitude and varying Mach numbers, on the other hand, the thrust and the fuel flow rate calculated by PERF experience a much more dramatic increase.

In all cases, the ideal and perfect gas calculations have the tendency to exhibit more dramatic changes with varying parameters of user-defined efficiencies, altitude, and Mach number. The combination of component maps and real gas effects, however, seem to produce a more gradual change. It can also be concluded that PERF, using ideal-perfect assumptions, tends to produce higher fuel flow rates than GasTurb 11 and GSP 11 at low altitudes, and lower fuel flow rates at high altitudes. While the thrust calculated by PERF is always lower than GasTurb 11 and GSP 11, The discrepancy increases with increasing altitude. The exception is with high Mach numbers. At high Mach numbers, the ideal-perfect gas calculations will quickly overtake real gas calculations toyield a higher thrust and a higher fuel flow rate.

6.1. Recommendation for Future Work

In addition to turbojet engines, GasTurb 11 and GSP 11 are both capable of running simulations for several different configurations of turbojet and turbofan engines with or without afterburners. It would be interesting to compare these simulations from GasTurb 11 and GSP 11 to that of ideal-perfect gas calculations.

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Appendix

10.1. On-Design Custom Engine GasTurb 11 Data

```
Turbojet SL static, ISA


          W         T         P        WRstd
Station  kg/s       K        kPa       kg/s       FN       =     33.71 kN
 amb              288.15   101.325                TSFC     =   33.5461 g/(kN*s)
  1     30.000    288.15   101.325                FN/W2    =   1123.75 m/s
  2     30.000    288.15   101.325    30.000
  3     30.000    592.35  1013.250     4.301      Prop Eff =    0.0000
 31     30.000    592.35  1013.250                eta core =    0.3834
  4     31.131   1800.00   962.588     8.168
 41     31.131   1800.00   962.588     8.168      WF       =   1.13093 kg/s
 49     31.131   1568.18   477.831                s NOx    =   0.21992
  5     31.131   1568.18   477.831    15.359      XM8      =    1.0000
  6     31.131   1568.18   477.831                A8       =    0.0656 m²
  8     31.131   1568.18   477.831    15.359      P8/Pamb  =    4.7158
Bleed    0.000    288.15   101.325                WBld/W2  =   0.00000
--------------------------------------------
P2/P1 = 1.0000  P4/P3 = 0.9500  P6/P5 1.0000      CD8      =    1.0000
Efficiencies:   isentr  polytr    RNI    P/P      W_NGV/W2 =   0.00000
 Compressor     0.8648  0.9000  1.000 10.000      WCL/W2   =   0.00000
 Burner         0.9900                 0.950      Loading  =    100.00 %
 Turbine        0.9068  0.9000  1.114  2.014      e45 th   =   0.90676
--------------------------------------------      far7     =   0.03770
Spool mech Eff  0.9900  Nom Spd    15000 rpm      PWX      =      0.00 kW
--------------------------------------------
Con-Di Nozzle:                                    A9/A8    =   1.36379
 A9*(Ps9-Pamb)       -0.008                       CFGid    =   1.00000
--------------------------------------------


hum [%]     war0       FHV     Fuel
   0.0   0.00000    42.100    JP-10

 Composed Values:
 1:  V9                                                      = 1083.18
 2:  (((V9^2)*W8/2)-((V0^2)*ZW2Rstd/2))/(WF*42100000)        = 0.383572
```

Input Data File:
C:\Users\Rayne\Desktop\Turbojet Thesis\Turbojet Thesis Data\GasTurb Parameters\OnDesign Custom Engine.CYJ

10.1.1. On-Design Level 1 Technology Engine GasTurb 11 Data

```
  Turbojet SL static, ISA


         W          T          P         WRstd
Station  kg/s        K         kPa        kg/s        FN        =      12.82 kN
 amb               288.15    101.325                  TSFC      =    34.2684 g/(kN*s)
  1      30.000    288.15    101.325                  FN/W2     =     427.18 m/s
  2      30.000    288.15    101.325     30.000
  3      30.000    645.98   1013.250      4.492       Prop Eff  =     0.0000
 31      30.000    645.98   1013.250                  eta core  =     0.1459
  4      30.439   1110.00    911.925      6.631
 41      30.439   1110.00    911.925      6.631       WF        =    0.43916 kg/s
 49      30.439    778.62    154.096                  s NOx     =    0.28979
  5      30.439    778.62    154.096     32.867       XM8       =     0.9672
  6      30.439    778.62    154.096                  A8        =     0.1428 m²
  8      30.439    778.62    154.096     32.867       P8/Pamb   =     1.5208
Bleed     0.000    288.15    101.325                  WBld/W2   =    0.00000
---------------------------------------------
P2/P1 = 1.0000  P4/P3 = 0.9000  P6/P5 1.0000          CD8       =     1.0000
Efficiencies:    isentr  polytr    RNI     P/P        W_NGV/W2  =    0.00000
 Compressor      0.7312  0.8000  1.000 10.000         WCL/W2    =    0.00000
 Burner          0.8800                 0.900         Loading   =     100.00 %
 Turbine         0.8343  0.8000  1.848  5.918         e45 th    =    0.83426
---------------------------------------------         far7      =    0.01464
Spool mech Eff  0.9500  Nom Spd    15000 rpm          PWX       =       0.00 kW
---------------------------------------------
Con-Di Nozzle:                                        A9/A8     =    1.03497
 A9*(Ps9-Pamb)     -3.19E-7                           CFGid     =    1.00000
---------------------------------------------

hum [%]      war0       FHV     Fuel
   0.0    0.00000    42.100    JP-10

 Composed Values:
 1:  V9                                                            = 421.016
 2:  (((V9^2)*W8/2)-((V0^2)*ZW2Rstd/2))/(WF*42100000)              = 0.145913
```

Input Data File:
C:\Users\Rayne\Desktop\Thesis Datav1\OnDesignLevel 1 Parameters.CYJ

10.1.2. On-Design Level 2 Technology Engine GasTurb 11 Data

```
  Turbojet SL static, ISA


          W          T          P         WRstd
Station  kg/s        K         kPa        kg/s         FN        =     23.82 kN
 amb               288.15    101.325                   TSFC      =   29.8143 g/(kN*s)
  1     30.000     288.15    101.325                   FN/W2     =    794.05 m/s
  2     30.000     288.15    101.325     30.000
  3     30.000     622.50   1013.250      4.409        Prop Eff  =    0.0000
 31     30.000     622.50   1013.250                   eta core  =    0.3090
  4     30.710    1390.00    932.190      7.319
 41     30.710    1390.00    932.190      7.319        WF        =   0.71022 kg/s
 49     30.710    1107.91    297.929                   s NOx     =   0.25682
  5     30.710    1107.91    297.929     20.446        XM8       =    1.0000
  6     30.710    1107.91    297.929                   A8        =    0.0865 m²
  8     30.710    1107.91    297.929     20.446        P8/Pamb   =    2.9403
Bleed    0.000     288.15    101.325                   WBld/W2   =   0.00000
--------------------------------------------
P2/P1 = 1.0000  P4/P3 = 0.9200  P6/P5 1.0000           CD8       =    1.0000
Efficiencies:   isentr  polytr    RNI     P/P          W_NGV/W2  =   0.00000
 Compressor     0.7844  0.8400  1.000  10.000          WCL/W2    =   0.00000
 Burner         0.9400                  0.920          Loading   =    100.00 %
 Turbine        0.8666  0.8500  1.460   3.129          e45 th    =   0.86657
--------------------------------------------           far7      =   0.02367
Spool mech Eff  0.9700  Nom Spd    15000 rpm           PWX       =      0.00 kW
--------------------------------------------
Con-Di Nozzle:                                         A9/A8     =   1.10043
 A9*(Ps9-Pamb)       -0.009                            CFGid     =   1.00000
--------------------------------------------

hum [%]     war0       FHV     Fuel
   0.0   0.00000    42.100    JP-10

 Composed Values:
 1:  V9                                                            = 775.968
 2:  (((V9^2)*W8/2)-((V0^2)*ZW2Rstd/2))/(WF*42100000)              = 0.309218
```

Input Data File:
C:\Users\Rayne\Desktop\Thesis Datav1\OnDesign Data\OnDesign Level 2 Parameters.CYJ

10.1.3. On-Design Level 3 Technology Engine GasTurb 11 Data

```
Turbojet SL static, ISA


           W          T          P         WRstd
Station  kg/s         K         kPa        kg/s       FN        =     32.89 kN
 amb                288.15    101.325                 TSFC      =   33.4655 g/(kN*s)
  1      30.000     288.15    101.325                 FN/W2     =   1096.22 m/s
  2      30.000     288.15    101.325     30.000
  3      30.000     601.90   1013.250      4.336      Prop Eff  =    0.0000
 31      30.000     601.90   1013.250                 eta core  =    0.3753
  4      31.101    1780.00    952.455      8.202
 41      31.101    1780.00    952.455      8.202      WF        =   1.10056 kg/s
 49      31.101    1539.72    453.399                 s NOx     =   0.23099
  5      31.101    1539.71    453.399     16.025      XM8       =    1.0000
  6      31.101    1539.71    453.399                 A8        =    0.0684 m²
  8      31.101    1539.71    453.399     16.025      P8/Pamb   =    4.4747
Bleed     0.000     288.15    101.325                 WBld/W2   =   0.00000
--------------------------------------------
P2/P1 = 1.0000  P4/P3 = 0.9400  P6/P5 1.0000          CD8       =    1.0000
Efficiencies:   isentr  polytr    RNI     P/P         W_NGV/W2  =   0.00000
 Compressor     0.8379  0.8800  1.000 10.000          WCL/W2    =   0.00000
 Burner         0.9900                 0.940          Loading   =    100.00 %
 Turbine        0.8978  0.8900  1.117  2.101          e45 th    =   0.89782
--------------------------------------------          far7      =   0.03669
Spool mech Eff  0.9900  Nom Spd    15000 rpm          PWX       =      0.00 kW
--------------------------------------------
Con-Di Nozzle:                                        A9/A8     =   1.32788
 A9*(Ps9-Pamb)        -0.007                          CFGid     =   1.00000
--------------------------------------------

hum [%]     war0       FHV     Fuel
   0.0   0.00000    42.100    JP-10

 Composed Values:
 1:  V9                                                           = 1057.64
 2:  (((V9^2)*W8/2)-((V0^2)*ZW2Rstd/2))/(WF*42100000)             = 0.375418
```

Input Data File:
C:\Users\Rayne\Desktop\Thesis Datav1\ OnDesign Level 3 Parameters.CYJ

10.1.4. On-Design Level 4 Technology Engine GasTurb 11 Data

```
 Turbojet SL static, ISA


           W         T         P        WRstd
Station  kg/s        K        kPa       kg/s         FN       =     37.01 kN
 amb                288.15   101.325                 TSFC     =   36.5087 g/(kN*s)
  1      30.000     288.15   101.325                 FN/W2    =   1233.58 m/s
  2      30.000     288.15   101.325    30.000
  3      30.000     592.35  1013.250     4.301       Prop Eff =    0.0000
 31      30.000     592.35  1013.250                 eta core =    0.3840
  4      31.351    2000.00   962.588     8.667
 41      31.351    2000.00   962.588     8.667       WF       =   1.35109 kg/s
 49      31.351    1776.90   519.188                 s NOx    =   0.21992
  5      31.351    1776.90   519.188    15.145       XM8      =    1.0000
  6      31.351    1776.90   519.188                 A8       =    0.0649 m²
  8      31.351    1776.90   519.188    15.145       P8/Pamb  =    5.1240
Bleed     0.000     288.15   101.325                 WBld/W2  =   0.00000
--------------------------------------------
P2/P1 = 1.0000  P4/P3 = 0.9500  P6/P5 1.0000         CD8      =    1.0000
Efficiencies:   isentr  polytr    RNI     P/P        W_NGV/W2 =   0.00000
 Compressor     0.8648  0.9000  1.000 10.000         WCL/W2   =   0.00000
 Burner         0.9990                 0.950         Loading  =    100.00 %
 Turbine        0.9059  0.9000  0.981  1.854         e45 th   =   0.90592
--------------------------------------------         far7     =   0.04504
Spool mech Eff  0.9950  Nom Spd    15000 rpm         PWX      =      0.00 kW
--------------------------------------------
Con-Di Nozzle:                                       A9/A8    =   1.43073
 A9*(Ps9-Pamb)       -0.005                          CFGid    =   1.00000
--------------------------------------------


hum [%]     war0       FHV     Fuel
   0.0   0.00000    42.100    JP-10

 Composed Values:
 1:  V9                                                           = 1180.59
 2:  (((V9^2)*W8/2)-((V0^2)*ZW2Rstd/2))/(WF*42100000)             = 0.384109
```

Input Data File:
C:\Users\Rayne\Desktop\Thesis Datav1\ OnDesign Level 4 Parameters.CYJ

10.2. On-Design Custom Engine GSP 11 Data

```
GSP 11     Thesis_Turbojet.mxl                                12:31  June 22, 2013
OPERATING POINT OUTPUT
================================================================================

Ambient/Flight conditions:
================================================================================
Zp            =              0 [m]      Psa           =        1.01325 [bar]
Macha         =          0.000 [-]      Vta           =            0.0 [m/s]
Pta           =        1.01325 [bar]    Tta           =         288.15 [K]
================================================================================

Global system performance data:                                      DesignPoint
================================================================================
FN            =         33.761 [kN]     WF            =          1.133 [kg/s]
W             =         30.000 [kg/s]   TSFC          =        0.12082 [kg/N h]
Rotor speeds:
N1            =          15000 [rpm]                  =         100.00 [%]
================================================================================

Engine station data:                                                 DesignPoint
================================================================================

Station    W[kg/s]      TT[K]      TS[K]   PT[bar]   PS[bar]  WC[kg/s]
------- ---------- ---------- ---------- --------- --------- ---------
 a       *********     288.15  *********  1.013250  1.013250 *********
 1        30.00000     288.15  *********  1.013250 ********* *********
 2        30.00000     288.15  *********  1.013250 *********  30.00000
 3        30.00000     592.37  ********* 10.132500 ********* *********
 4        31.13304    1800.00  *********  9.625875 ********* *********
 41      *********  *********  ********* ********* ********* *********
 7        31.13304    1571.65  *********  4.780155 ********* *********
 8       *********  *********  ********* ********* ********* *********
 9        31.13304    1571.65  *********  4.780155  1.013250 *********
------- ---------- ---------- ---------- --------- --------- ---------

Station   H[kJ/kg] Cp[J/kg K]  S[J/kg K]    FAR[-]   Mach[-]    V[m/s]     A[mｲ]
------- ---------- ---------- ---------- --------- --------- --------- ---------
 a       *********  *********  ********* ********* 0.0000000 ********* *********
 1       *********  *********  ********* ********* ********* ********* *********
 2       *********  *********  ********* ********* ********* ********* *********
 3       *********  *********  ********* ********* ********* ********* *********
 4       *********  *********  ********* ********* ********* ********* *********
 41      *********  *********  ********* ********* ********* ********* *********
 7       *********  *********  ********* ********* ********* ********* *********
 8       *********  *********  ********* ********* ********* ********* *********
 9       *********  *********  ********* ********* *********    1084.4 *********
------- ---------- ---------- ---------- --------- --------- --------- ---------
"*********" = parameter not added to data output set!
================================================================================

Calculated expressions
================================================================================
PropulsivePower=     0.0000E+00
FlowPower     =    1.8305E+07
PropulsiveEfficiency=     0.0000E+00
PowerFuel     =    4.7701E+07          TSFCv2        =     3.3561E+01
ThermalEfficiency=     3.8375E-01
================================================================================
```

10.2.1. On-Design Level 1 Technology Engine GSP 11 Data

```
GSP 11     Thesis_Turbojet.mxl                              13:24  June 22, 2013
OPERATING POINT OUTPUT
================================================================================

Ambient/Flight conditions:
================================================================================
Zp           =              0 [m]      Psa          =       1.01325 [bar]
Macha        =          0.000 [-]      Vta          =           0.0 [m/s]
Pta          =        1.01325 [bar]    Tta          =        288.15 [K]
================================================================================

Global system performance data:                                     DesignPoint
================================================================================
FN           =         12.950 [kN]     WF           =        0.4423 [kg/s]
W            =         30.000 [kg/s]   TSFC         =       0.12297 [kg/N h]
Rotor speeds:
N1           =          15000 [rpm]                 =        100.00 [%]
================================================================================

Engine station data:                                                DesignPoint
================================================================================

Station    W[kg/s]      TT[K]      TS[K]   PT[bar]   PS[bar]  WC[kg/s]
------- ---------- ---------- ---------- --------- --------- ---------
 a       *********     288.15  *********  1.013250  1.013250 *********
 1        30.00000     288.15  *********  1.013250 ********* *********
 2        30.00000     288.15  *********  1.013250 *********  30.00000
 3        30.00000     646.20  ********* 10.132500 ********* *********
 4        30.44233    1110.00  *********  9.119250 *********   6.83481
 41      *********  *********  ********* ********* ********* *********
 7        30.44233     793.20  *********  1.542404 *********  33.82772
 8       *********  *********  ********* ********* ********* *********
 9        30.44233     793.20     711.56  1.542404  1.013250 *********
------- ---------- ---------- ---------- --------- --------- ---------

Station   H[kJ/kg] Cp[J/kg K]  S[J/kg K]    FAR[-]   Mach[-]    V[m/s]    A[m2]
------- ---------- ---------- ---------- --------- --------- --------- ---------
 a       *********  *********  ********* ********* 0.0000000 ********* *********
 1       *********  *********  ********* ********* ********* ********* *********
 2       *********  *********  ********* ********* ********* ********* *********
 3       *********  *********  ********* ********* ********* ********* *********
 4       *********  *********  ********* ********* ********* ********* *********
 41      *********  *********  ********* ********* ********* ********* *********
 7       *********  *********  ********* ********* ********* ********* *********
 8       *********  *********  ********* ********* ********* ********* *********
 9       *********  *********  ********* ********* *********     425.4 *********
------- ---------- ---------- ---------- --------- --------- --------- ---------
"*********" = parameter not added to data output set!
================================================================================

Calculated expressions
================================================================================
PropulsivePower=     0.0000E+00
FlowPower     =    2.7543E+06
PropulsiveEfficiency=     0.0000E+00
PowerFuel     =    1.8622E+07          TSFCv2       =     3.4158E+01
ThermalEfficiency=     1.4790E-01
================================================================================
```

10.2.2. On-Design Level 2 Technology Engine GSP 11 Data

```
GSP 11     Thesis_Turbojet.mxl                            13:21  June 22, 2013
OPERATING POINT OUTPUT
==============================================================================

Ambient/Flight conditions:
==============================================================================
Zp           =              0 [m]      Psa          =        1.01325 [bar]
Macha        =          0.000 [-]      Vta          =            0.0 [m/s]
Pta          =        1.01325 [bar]    Tta          =         288.15 [K]
==============================================================================

Global system performance data:                                    DesignPoint
==============================================================================
FN           =         23.896 [kN]     WF           =          0.714 [kg/s]
W            =         30.000 [kg/s]   TSFC         =        0.10757 [kg/N h]
Rotor speeds:
N1           =          15000 [rpm]                 =         100.00 [%]
==============================================================================

Engine station data:                                               DesignPoint
==============================================================================

Station    W[kg/s]      TT[K]      TS[K]   PT[bar]   PS[bar]  WC[kg/s]
------- ---------- ---------- ---------- --------- --------- ---------
 a       *********     288.15  ********* 1.013250  1.013250 *********
 1        30.00000     288.15  ********* 1.013250 ********* *********
 2        30.00000     288.15  ********* 1.013250 *********  30.00000
 3        30.00000     622.63  ********* 10.132500 ********* *********
 4        30.71400    1390.00  ********* 9.321900 *********   7.60259
 41      *********  *********  ********* ********* ********* *********
 7        30.71400    1113.88  ********* 2.980396 *********  21.17221
 8       *********  *********  ********* ********* ********* *********
 9        30.71400    1113.88     856.62 2.980397  1.013250 *********
------- ---------- ---------- ---------- --------- --------- ---------

Station   H[kJ/kg] Cp[J/kg K]  S[J/kg K]    FAR[-]   Mach[-]    V[m/s]     A[m2]
------- ---------- ---------- ---------- --------- --------- --------- ---------
 a       *********  *********  ********* ********* 0.0000000 ********* *********
 1       *********  *********  ********* ********* ********* ********* *********
 2       *********  *********  ********* ********* ********* ********* *********
 3       *********  *********  ********* ********* ********* ********* *********
 4       *********  *********  ********* ********* ********* ********* *********
 41      *********  *********  ********* ********* ********* ********* *********
 7       *********  *********  ********* ********* ********* ********* *********
 8       *********  *********  ********* ********* ********* ********* *********
 9       *********  *********  ********* ********* *********     778.0 *********
------- ---------- ---------- ---------- --------- --------- --------- ---------
"*********" = parameter not added to data output set!
==============================================================================

Calculated expressions
==============================================================================
PropulsivePower=     0.0000E+00
FlowPower     =    9.2954E+06
PropulsiveEfficiency=     0.0000E+00
PowerFuel     =    3.0059E+07          TSFCv2       =     2.9880E+01
ThermalEfficiency=     3.0924E-01
==============================================================================
```

10.2.3. On-Design Level 3 Technology Engine GSP 11 Data

```
GSP 11     Thesis_Turbojet.mxl                                13:22  June 22, 2013
OPERATING POINT OUTPUT
================================================================================

Ambient/Flight conditions:
================================================================================
Zp            =               0 [m]      Psa           =        1.01325 [bar]
Macha         =           0.000 [-]      Vta           =            0.0 [m/s]
Pta           =         1.01325 [bar]    Tta           =         288.15 [K]
================================================================================

Global system performance data:                                     DesignPoint
================================================================================
FN            =          32.935 [kN]     WF            =         1.1027 [kg/s]
W             =          30.000 [kg/s]   TSFC          =        0.12053 [kg/N h]
Rotor speeds:
N1            =           15000 [rpm]                  =         100.00 [%]
================================================================================

Engine station data:                                                DesignPoint
================================================================================

Station    W[kg/s]      TT[K]      TS[K]   PT[bar]   PS[bar]  WC[kg/s]
------- ---------- ---------- ---------- --------- --------- ---------
 a       *********     288.15  ********* 1.013250  1.013250 *********
 1        30.00000     288.15  ********* 1.013250 ********* *********
 2        30.00000     288.15  ********* 1.013250 *********  30.00000
 3        30.00000     601.86  ********* 10.132500 ********* *********
 4        31.10266    1780.00  ********* 9.524550 *********   8.58759
 41      *********  *********  ********* ********* ********* *********
 7        31.10266    1543.37  ********* 4.535458 *********  16.74798
 8       *********  *********  ********* ********* ********* *********
 9        31.10266    1543.37    1098.10 4.535458  1.013250 *********
------- ---------- ---------- ---------- --------- --------- ---------

Station   H[kJ/kg] Cp[J/kg K]  S[J/kg K]    FAR[-]   Mach[-]    V[m/s]    A[m²]
------- ---------- ---------- ---------- --------- --------- --------- ---------
 a       *********  *********  ********* ********* 0.0000000 ********* *********
 1       *********  *********  ********* ********* ********* ********* *********
 2       *********  *********  ********* ********* ********* ********* *********
 3       *********  *********  ********* ********* ********* ********* *********
 4       *********  *********  ********* ********* ********* ********* *********
 41      *********  *********  ********* ********* ********* ********* *********
 7       *********  *********  ********* ********* ********* ********* *********
 8       *********  *********  ********* ********* ********* ********* *********
 9       *********  *********  ********* ********* *********    1058.9 *********
------- ---------- ---------- ---------- --------- --------- --------- ---------
"*********" = parameter not added to data output set!
================================================================================

Calculated expressions
================================================================================
PropulsivePower=     0.0000E+00
FlowPower     =     1.7438E+07
PropulsiveEfficiency=     0.0000E+00
PowerFuel     =     4.6422E+07         TSFCv2        =     3.3480E+01
ThermalEfficiency=     3.7564E-01
================================================================================
```

10.2.4. On-Design Level 4 Technology Engine GSP 11 Data

GSP 11 Thesis_Turbojet.mxl 13:23 June 22, 2013

OPERATING POINT OUTPUT

Ambient/Flight conditions:

Zp	=	0 [m]	Psa	=	1.01325 [bar]
Macha	=	0.000 [-]	Vta	=	0.0 [m/s]
Pta	=	1.01325 [bar]	Tta	=	288.15 [K]

Global system performance data: DesignPoint

FN	=	37.079 [kN]	WF	=	1.3523 [kg/s]
W	=	30.000 [kg/s]	TSFC	=	0.13129 [kg/N h]
Rotor speeds:					
N1	=	15000 [rpm]		=	100.00 [%]

Engine station data: DesignPoint

Station	W[kg/s]	TT[K]	TS[K]	PT[bar]	PS[bar]	WC[kg/s]
a	*********	288.15	*********	1.013250	1.013250	*********
1	30.00000	288.15	*********	1.013250	*********	*********
2	30.00000	288.15	*********	1.013250	*********	30.00000
3	30.00000	592.37	*********	10.132500	*********	*********
4	31.35228	2000.00	*********	9.625875	*********	9.10962
41	*********	*********	*********	*********	*********	*********
7	31.35228	1783.05	*********	5.192324	*********	15.91472
8	*********	*********	*********	*********	*********	*********
9	31.35228	1783.05	1245.68	5.192324	1.013250	*********

Station	H[kJ/kg]	Cp[J/kg K]	S[J/kg K]	FAR[-]	Mach[-]	V[m/s]	A[m^2]
a	*********	*********	*********	*********	0.0000000	*********	*********
1	*********	*********	*********	*********	*********	*********	*********
2	*********	*********	*********	*********	*********	*********	*********
3	*********	*********	*********	*********	*********	*********	*********
4	*********	*********	*********	*********	*********	*********	*********
41	*********	*********	*********	*********	*********	*********	*********
7	*********	*********	*********	*********	*********	*********	*********
8	*********	*********	*********	*********	*********	*********	*********
9	*********	*********	*********	*********	*********	1182.7	*********

"*********" = parameter not added to data output set!

Calculated expressions

PropulsivePower= 0.0000E+00

FlowPower = 2.1926E+07

PropulsiveEfficiency= 0.0000E+00

PowerFuel = 5.6931E+07 TSFCv2 = 3.6470E+01

ThermalEfficiency= 3.8514E-01

10.3. On-Design Custom Engine PERF Data

```
 PERF (Ver. 4.2)         Turbojet - Single Spool - CSH
Date:6/22/2013 12:47:08 PM
 Engine File: C:\Program Files (x86)\EOP\Custom Engine.EPA

   Input Constants
   Pidmax= 1.0000   Pi b  = 0.9500   Eta b = 0.9900   Pi n  = 1.0000
   cp c  = 1.0049   cp t  = 1.2351   Gam c = 1.4000   Gam t = 1.3000
   Eta c = 0.8641   Eta t = 0.9072   Eta m = 0.9900   Eta P = 1.0000
   hPR   =  42100

   Control Limits:  Tt4   = 1800.0   Pi c  =  10.00

   Parameter                       Reference**            Test**
   Mach Number @ 0                    0.0100            0.0100
   Temperature @ 0                    288.15            288.15 K
   Pressure    @ 0                  101.3250            101.3250 kPa
   Altitude    @ 0                         0                 0 m
   Total Temp  @ 4                   1800.00           1800.00 K
   Pi r  / Tau r            1.0001/ 1.0000     1.0001/ 1.0000
   Pi d                               1.0000            1.0000
   Pi c / Tau c            10.0000/ 2.0771    10.0000/ 2.0771
   Pi t / Tau t             0.4944/ 0.8639     0.4944/ 0.8639
   Control Limit                                          Tt4max
   Spool RPM (% of Reference Point)  100.00            100.00
   Pt9/P9                             4.6970            4.6970
   P0/P9                              1.0000            1.0000
   Mach Number @ 9                    1.6912            1.6912
   Mass Flow Rate @ 0                  30.00             30.00
   Corr Mass Flow @ 0                  30.00             30.00 kg/sec
   Flow Area      @ 0                  7.197             7.197 m^2
   Flow Area*     @ 0                  0.124             0.124 m^2
   Flow Area      @ 9                  0.089             0.089 m^2
   MB - Fuel/Air Ratio (f)           0.04110           0.04110
   Overall Fuel/Air Ratio (fo)       0.04110           0.04110
   Specific Thrust  (F/m0)           1113.86           1114.62 m/sec
   Thrust Spec Fuel Consumption (S) 36.9027           36.8776 mg/(N-sec)
   Thrust (F)                          33416             33439 N
   Fuel Flow Rate                       4439              4439 kg/hr
   Propulsive Efficiency (%)            0.63              0.63
   Thermal Efficiency    (%)           44.52             44.58
   Overall Efficiency    (%)            0.28              0.28
```

10.3.1. On-Design Level 1 Technology Engine PERF Data

```
 PERF (Ver. 4.2)         Turbojet - Single Spool - CSH
Date:6/22/2013 2:18:50 PM
 Engine File: C:\Program Files (x86)\EOP\Level 1 Engine.EPA

   Input Constants
   Pidmax= 1.0000   Pi b  = 0.9000   Eta b = 0.8800   Pi n  = 1.0000
   cp c  = 1.0040   cp t  = 1.2390   Gam c = 1.4000   Gam t = 1.3000
   Eta c = 0.7295   Eta t = 0.8310   Eta m = 0.9500   Eta P = 1.0000
   hPR   =  42100

   Control Limits:  Tt4   = 1110.0   Pi c  =  10.00

   Parameter                        Reference**            Test**
   Mach Number @ 0                     0.0100            0.0100
   Temperature @ 0                     288.15            288.15 K
   Pressure    @ 0                   101.3250            101.3250 kPa
   Altitude    @ 0                          0                 0 m
   Total Temp  @ 4                    1110.00           1109.99 K
   Pi r  / Tau r             1.0001/ 1.0000     1.0001/ 1.0000
   Pi d                                1.0000            1.0000
   Pi c / Tau c             10.0000/ 2.2758     9.9999/ 2.2758
   Pi t / Tau t              0.1726/ 0.7231     0.1726/ 0.7231
   Control Limit                                        PIC Max
   Spool RPM (% of Reference Point)  100.00            100.00
   Pt9/P9                              1.5539            1.5539
   P0/P9                               1.0000            1.0000
   Mach Number @ 9                     0.8449            0.8449
   Mass Flow Rate @ 0                   30.00             30.00
   Corr Mass Flow @ 0                   30.00             30.00 kg/sec
   Flow Area      @ 0                   7.194             7.194 m^2
   Flow Area*     @ 0                   0.124             0.124 m^2
   Flow Area      @ 9                   0.143             0.143 m^2
   MB - Fuel/Air Ratio (f)            0.02010           0.02010
   Overall Fuel/Air Ratio (fo)        0.02010           0.02010
   Specific Thrust  (F/m0)             443.69            443.99 m/sec
   Thrust Spec Fuel Consumption (S) 45.2918           45.2612 mg/(N-sec)
   Thrust (F)                           13311             13320 N
   Fuel Flow Rate                        2170              2170 kg/hr
   Propulsive Efficiency (%)             1.54              1.54
   Thermal Efficiency    (%)            14.88             14.90
   Overall Efficiency    (%)             0.23              0.23
```

10.3.2. On-Design Level 2 Technology Engine PERF Data

```
 PERF (Ver. 4.2)         Turbojet - Single Spool - CSH
Date:6/22/2013 2:20:14 PM
 Engine File: C:\Program Files (x86)\EOP\Level 2 Engine.EPA

   Input Constants
   Pidmax= 1.0000   Pi b  = 0.9200   Eta b = 0.9400   Pi n  = 1.0000
   cp c  = 1.0040   cp t  = 1.2390   Gam c = 1.4000   Gam t = 1.3000
   Eta c = 0.7831   Eta t = 0.8662   Eta m = 0.9700   Eta P = 1.0000
   hPR   =  42100

   Control Limits:  Tt4   = 1390.0   Pi c  =  10.00

   Parameter                      Reference**            Test**
   Mach Number @ 0                   0.0100           0.0100
   Temperature @ 0                   288.15           288.15 K
   Pressure    @ 0                 101.3250           101.3250 kPa
   Altitude    @ 0                        0                0 m
   Total Temp  @ 4                  1390.00          1390.00 K
   Pi r  / Tau r            1.0001/ 1.0000     1.0001/ 1.0000
   Pi d                              1.0000           1.0000
   Pi c / Tau c            10.0000/ 2.1884    10.0000/ 2.1884
   Pi t / Tau t             0.3205/ 0.7999     0.3205/ 0.7999
   Control Limit                                       Tt4max
   Spool RPM (% of Reference Point)  100.00           100.00
   Pt9/P9                            2.9485           2.9485
   P0/P9                             1.0000           1.0000
   Mach Number @ 9                   1.3746           1.3746
   Mass Flow Rate @ 0                 30.00            30.00
   Corr Mass Flow @ 0                 30.00            30.00 kg/sec
   Flow Area      @ 0                 7.194            7.194 m^2
   Flow Area*     @ 0                 0.124            0.124 m^2
   Flow Area      @ 9                 0.097            0.097 m^2
   MB - Fuel/Air Ratio (f)          0.02877          0.02877
   Overall Fuel/Air Ratio (fo)      0.02877          0.02877
   Specific Thrust  (F/m0)           798.54           799.08 m/sec
   Thrust Spec Fuel Consumption (S) 36.0310          36.0064 mg/(N-sec)
   Thrust (F)                         23956            23972 N
   Fuel Flow Rate                      3107             3107 kg/hr
   Propulsive Efficiency (%)           0.87             0.87
   Thermal Efficiency    (%)          33.16            33.20
   Overall Efficiency    (%)           0.29             0.29
```

10.3.3. On-Design Level 3 Technology Engine PERF Data

```
 PERF (Ver. 4.2)         Turbojet - Single Spool - CSH
Date:6/22/2013 2:21:29 PM
 Engine File: C:\Program Files (x86)\EOP\Level 3 Engine.EPA

   Input Constants
   Pidmax= 1.0000   Pi b  = 0.9400   Eta b = 0.9900   Pi n  = 1.0000
   cp c  = 1.0040   cp t  = 1.2390   Gam c = 1.4000   Gam t = 1.3000
   Eta c = 0.8370   Eta t = 0.8982   Eta m = 0.9900   Eta P = 1.0000
   hPR   =  42100

   Control Limits:  Tt4   = 1780.0   Pi c  =  10.00

   Parameter                       Reference**            Test**
   Mach Number @ 0                    0.0100            0.0100
   Temperature @ 0                    288.15            288.15 K
   Pressure    @ 0                  101.3250            101.3250 kPa
   Altitude    @ 0                         0                 0 m
   Total Temp  @ 4                   1780.00           1780.00 K
   Pi r  / Tau r            1.0001/ 1.0000     1.0001/ 1.0000
   Pi d                               1.0000            1.0000
   Pi c / Tau c            10.0000/ 2.1119    10.0000/ 2.1119
   Pi t / Tau t             0.4755/ 0.8584     0.4755/ 0.8584
   Control Limit                                          Tt4max
   Spool RPM (% of Reference Point)  100.00            100.00
   Pt9/P9                            4.4696            4.4696
   P0/P9                             1.0000            1.0000
   Mach Number @ 9                   1.6588            1.6588
   Mass Flow Rate @ 0                 30.00             30.00
   Corr Mass Flow @ 0                 30.00             30.00 kg/sec
   Flow Area      @ 0                 7.194             7.194 m^2
   Flow Area*     @ 0                 0.124             0.124 m^2
   Flow Area      @ 9                 0.091             0.091 m^2
   MB - Fuel/Air Ratio (f)          0.04039           0.04039
   Overall Fuel/Air Ratio (fo)      0.04039           0.04039
   Specific Thrust  (F/m0)          1090.08           1090.82 m/sec
   Thrust Spec Fuel Consumption (S) 37.0545           37.0293 mg/(N-sec)
   Thrust (F)                         32702             32725 N
   Fuel Flow Rate                      4362              4362 kg/hr
   Propulsive Efficiency (%)           0.65              0.64
   Thermal Efficiency    (%)          43.42             43.48
   Overall Efficiency    (%)           0.28              0.28
```

10.3.4. On-Design Level 4 Technology Engine PERF Data

```
 PERF (Ver. 4.2)         Turbojet - Single Spool - CSH
Date:6/22/2013 2:22:27 PM
 Engine File: C:\Program Files (x86)\EOP\Level 4 Engine.EPA

   Input Constants
   Pidmax= 1.0000   Pi b  = 0.9500   Eta b = 0.9990   Pi n  = 1.0000
   cp c  = 1.0040   cp t  = 1.2390   Gam c = 1.4000   Gam t = 1.3000
   Eta c = 0.8641   Eta t = 0.9063   Eta m = 0.9950   Eta P = 1.0000
   hPR   =  42100

   Control Limits:  Tt4   = 2000.0   Pi c  =  10.00

   Parameter                       Reference**            Test**
   Mach Number @ 0                    0.0100            0.0100
   Temperature @ 0                    288.15            288.15 K
   Pressure    @ 0                  101.3250            101.3250 kPa
   Altitude    @ 0                         0                 0 m
   Total Temp  @ 4                   2000.00           2000.00 K
   Pi r  / Tau r              1.0001/ 1.0000     1.0001/ 1.0000
   Pi d                               1.0000            1.0000
   Pi c / Tau c              10.0000/ 2.0771    10.0000/ 2.0771
   Pi t / Tau t               0.5384/ 0.8793     0.5384/ 0.8793
   Control Limit                                         Tt4max
   Spool RPM (% of Reference Point)  100.00            100.00
   Pt9/P9                             5.1153            5.1153
   P0/P9                              1.0000            1.0000
   Mach Number @ 9                    1.7463            1.7463
   Mass Flow Rate @ 0                  30.00             30.00
   Corr Mass Flow @ 0                  30.00             30.00 kg/sec
   Flow Area      @ 0                  7.194             7.194 m^2
   Flow Area*     @ 0                  0.124             0.124 m^2
   Flow Area      @ 9                  0.092             0.091 m^2
   MB - Fuel/Air Ratio (f)           0.04742           0.04742
   Overall Fuel/Air Ratio (fo)       0.04742           0.04742
   Specific Thrust  (F/m0)           1220.76           1221.59 m/sec
   Thrust Spec Fuel Consumption (S) 38.8488           38.8224 mg/(N-sec)
   Thrust (F)                          36623             36648 N
   Fuel Flow Rate                       5122              5122 kg/hr
   Propulsive Efficiency (%)            0.58              0.58
   Thermal Efficiency    (%)           46.04             46.11
   Overall Efficiency    (%)            0.27              0.27
```

10.4. Off-Design Custom Engine Data with Varying Altitude at M=0.5

	GasTurb			GSP			PERF		
Altitude [m]	Thrust [kN]	$\dot{m}_f$ [kg/s]	TSFC [g/kN*s]	Thrust [kN]	$\dot{m}_f$ [kg/s]	TSFC [g/kN*s]	Thrust [kN]	$\dot{m}_f$ [kg/s]	TSFC [g/kN*s]
0	32.36621	1.232177	38.06984	32.16393	1.22814	38.18333	31.944	1.336409	41.836
320	31.66557	1.202359	37.97056	31.45242	1.19796	38.08889	31.251	1.303698	41.717
640	30.96696	1.172871	37.87492	30.75588	1.16845	37.99167	30.569	1.27164	41.599
960	30.26603	1.143542	37.78303	30.07013	1.13951	37.89444	29.897	1.240128	41.48
1280	29.54919	1.114014	37.70033	29.33473	1.10942	37.81944	29.236	1.209259	41.362
1600	28.79436	1.083604	37.6325	28.57741	1.07897	37.75556	28.586	1.179001	41.244
1920	28.03556	1.053295	37.56996	27.83425	1.04917	37.69444	27.945	1.149266	41.126
2240	27.27328	1.023096	37.51274	27.10519	1.01999	37.63056	27.111	1.11128	40.99
2560	26.49817	0.992757	37.46511	26.39006	0.99143	37.56944	26.08	1.065368	40.85
2880	25.72695	0.962743	37.42156	25.59657	0.96079	37.53611	25.023	1.018361	40.697
3200	24.96573	0.933236	37.38069	24.81352	0.93064	37.50556	24.024	0.974101	40.547
3520	24.21217	0.904113	37.34127	24.04908	0.90123	37.475	23.061	0.931595	40.397
3840	23.4697	0.875521	37.30431	23.30297	0.87254	37.44444	22.117	0.890099	40.245
4160	22.73852	0.84746	37.26978	22.5749	0.84458	37.41111	21.206	0.85017	40.091
4480	22.01812	0.819866	37.23598	21.86362	0.81728	37.38056	20.316	0.811482	39.943
4800	21.30962	0.792814	37.20449	21.16931	0.79066	37.35	19.479	0.77505	39.789
5120	20.61275	0.766275	37.17478	20.4923	0.76473	37.31944	18.65	0.739137	39.632
5440	19.92822	0.740279	37.14729	19.83231	0.73948	37.28611	17.871	0.705368	39.47
5760	19.25647	0.714801	37.12002	19.18904	0.71488	37.25556	17.092	0.67216	39.326
6080	18.60418	0.690064	37.09189	18.56219	0.69093	37.22222	16.346	0.639357	39.114
6400	17.99338	0.666742	37.05487	17.95147	0.66761	37.18889	15.648	0.610444	39.011
6720	17.39745	0.644008	37.01737	17.3566	0.64492	37.15833	14.965	0.581435	38.853
7040	16.81624	0.621867	36.98012	16.77727	0.62284	37.125	14.307	0.553609	38.695
7360	16.25205	0.600369	36.94111	16.21321	0.60137	37.09167	13.682	0.527263	38.537
7680	15.70194	0.579439	36.90239	15.66413	0.58048	37.05833	13.05	0.50082	38.377
8000	15.16566	0.559067	36.86399	15.12974	0.56017	37.025	12.469	0.476079	38.181

10.5. Off-Design Custom Engine Data with Varying Mach Numbers at h=8000[m]

	GasTurb			GSP			PERF		
Mach	Thrust [kN]	$\dot{m}_f$ [kg/s]	TSFC [g/kN*s]	Thrust [kN]	$\dot{m}_f$ [kg/s]	TSFC [g/kN*s]	Thrust [kN]	$\dot{m}_f$ [kg/s]	TSFC [g/kN*s]
0	15.11201	0.501124	33.16068	15.13155	0.50313	33.25	11.693	0.391868	33.513
0.08	14.85089	0.502547	33.83954	14.88178	0.5046	33.90833	11.505	0.39377	34.226
0.16	14.69688	0.506863	34.48781	14.71437	0.50886	34.58333	11.411	0.39975	35.032
0.24	14.64588	0.514112	35.10286	14.66143	0.51607	35.2	11.446	0.41003	35.823
0.32	14.69532	0.524375	35.68314	14.72438	0.5263	35.74444	11.609	0.424785	36.591
0.4	14.84312	0.537748	36.22875	14.86757	0.53947	36.28611	11.902	0.444325	37.332
0.48	15.08951	0.554395	36.74039	15.10675	0.55588	36.79722	12.332	0.469122	38.041
0.56	15.43513	0.574499	37.22022	15.44177	0.57568	37.28056	12.915	0.499965	38.712
0.64	15.88148	0.598279	37.67147	15.87478	0.59908	37.73889	13.649	0.537238	39.361
0.72	16.4362	0.626155	38.09612	16.40886	0.62637	38.17222	14.55	0.580821	39.919
0.8	17.15228	0.660033	38.48078	17.0475	0.65783	38.58889	15.665	0.635216	40.55
0.88	17.9388	0.69723	38.86713	17.81099	0.69412	38.97222	16.976	0.698036	41.119
0.96	18.77714	0.737283	39.26494	18.59349	0.73258	39.4	18.561	0.77288	41.64
1.04	19.61	0.7784	39.693	19.25575	0.76789	39.87778	20.362	0.858442	42.159
1.12	20.33	0.81695	40.1797	19.93309	0.80507	40.38889	21.191	0.902588	42.593
1.2	21.03	0.85602	40.6978	20.61546	0.84323	40.90278	21.942	0.943462	42.998
1.28	21.76	0.89738	41.2348	21.2929	0.88243	41.44167	22.768	0.987858	43.388
1.36	22.51	0.94069	41.7911	22.05403	0.92603	41.98889	23.674	1.036093	43.765
1.44	23.29	0.98651	42.3635	22.80942	0.9704	42.54444	24.666	1.088486	44.129
1.52	24.01	1.03201	42.9741	23.62696	1.01927	43.13889	25.746	1.145208	44.481
1.6	24.75	1.07948	43.6066	24.52066	1.07244	43.73611	26.918	1.206546	44.823
1.68	25.57	1.1313	44.2505	25.36636	1.12461	44.33333	28.186	1.272739	45.155
1.76	26.47	1.18853	44.9036	26.20122	1.17938	45.01111	29.554	1.344116	45.48
1.84	27.42	1.2499	45.5758	27.10498	1.23901	45.71111	31.023	1.420791	45.798
1.92	28.37	1.31348	46.2928	28.10782	1.30337	46.36944	32.597	1.503113	46.112
2	29.3	1.37899	47.0567	28.90265	1.36285	47.15278	34.278	1.591288	46.423

Vita

Rayne Sung was born on February 25th, 1987, the only son of Jefferson and Cecilia Sung. He graduated from Earl Haig Secondary School and went on to obtain a B.S.E. degree in Aerospace Engineering from the University of Michigan, Ann Arbor. Rayne attended the University of Tennessee Space Institute pursuing a degree in Aerospace Engineering. He was awarded the M.S. Degree in Aerospace Engineering from the University of Tennessee, Knoxville in August 2013.