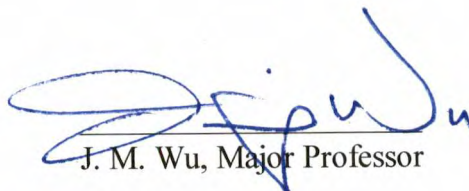


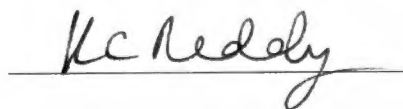
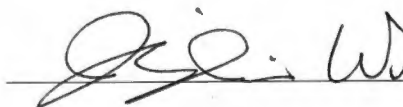
To the Graduate Council:

I am submitting herewith a dissertation written by Andrew Gregory Denny entitled "Numerical Study of Lift Augmentation in Massively Separated Turbulent Flows with Forcing." I have examined the final copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Aerospace Engineering.



J. M. Wu, Major Professor

We have read this dissertation
and recommend its acceptance:



Accepted for the Council:



Vice Provost
And Dean of The Graduate School

NUMERICAL STUDY OF LIFT AUGMENTATION IN MASSIVELY
SEPARATED TURBULENT FLOWS WITH FORCING

A Dissertation

Presented for the

Doctor of Philosophy

Degree

The University of Tennessee, Knoxville

Andrew Gregory Denny

December 1997

ACKNOWLEDGMENTS

Over the years that I have worked on this effort, many people have provided support, encouragement, and consolation along the way. I am very grateful for the patience and support of Professors J. M. Wu and J. Z. Wu; and I am indebted to Professor R. L. Roach for the original code which was modified for this work and for the countless hours spent in discussions of numerical modeling. Dr. R. H. Nichols was also a source of much practical information about computational fluid dynamics.

I also want to express my thanks to my parents, Andrew and Irene Denny, and to my parents-in-law, Max and Virginia Conner. All of the leaf-raking, yard-mowing, plumbing, child-care, and general support made focusing on this task much easier than it otherwise would have been.

I would be grievously remiss if I did not express my deep gratitude and love to my wife Janice. On the day that I write these words we are celebrating our twelfth wedding anniversary, and I am painfully aware that most of those years have been occupied with work on this degree. Without her unconditional love and support this effort would have foundered long ago. During the course of this effort, we have added four wonderful children (Joshua, Allison, Andrew, and Kristin) to our family and dealt with all the various trials of daily life. Through all of it, Janice never once complained about the time I spent on this endeavor. She has always been encouraging (even after being up all night with sick children), and I will spend the rest of my days making up this lost time to her and my children.

Finally, I must acknowledge my deep debt to the Lord and Savior, Jesus Christ.

As James wrote many years ago, "Every good and perfect gift is from above, coming down from the Father of the heavenly lights, Who does not change like shifting shadows." (James 1:17, NIV). In my life I have received many "good and perfect gifts" including the opportunity to pursue this degree, and it is only proper to express gratitude at the constant, unchanging love and unfailing support of the Lord.

ABSTRACT

At high angles of attack the flow over any lifting body experiences massive separation. The resulting shear layers naturally tend to roll up into large vortical structures. The presence and motion of these structures has a significant impact on the continuing flowfield development. By applying an artificial forcing to the shedding shear layer the rollup can be intensified or reduced, and the entire vorticity field may be altered. Control of these vortical structures by forcing to enhance lift has been the goal of numerous investigations, and some progress has been made in understanding how an unsteady forcing can be used to advantage.

This study describes work done in determining the effectiveness of using computational simulations to model massively separated, turbulent, unsteady flowfields subject to small-amplitude mechanical forcing. Algorithm development and validation highlight certain concerns that arise in the modeling of such unsteady flowfields using the Reynolds-Averaged Navier-Stokes equations, and questions of grid density, numerical diffusion, timestep size, and turbulence modeling are addressed with a suite of sample calculations.

Simulation of flowfields undergoing mechanical forcing by flap is accomplished using an overset grid methodology with the flow solver. Excellent agreement is demonstrated for flow over a NACA 63₃-018 airfoil with a small flap placed near the leading edge separation point. Computed values of the unforced lift and drag coefficients agree with given experimental values to less than 2%. Forcing frequencies for the

experimental configuration were in the range of global shedding frequency, and the maximum variation of the aerodynamic coefficients was seen when the forcing frequency was equal to the shedding frequency. Excellent agreement with experiment is also found for the cases involving forcing, and the data is used to explain the effect of forcing in terms of a global receptivity mechanism.

TABLE OF CONTENTS

CHAPTER	PAGE
I. INTRODUCTION	1
II. DEVELOPMENT OF NUMERICAL MODEL	7
Governing Equations.....	7
Turbulence Modeling	11
Modifications for Overset Grids	16
III. VALIDATION	22
Flow Over a Circular Cylinder.....	22
Flow Over a NACA 0012 Airfoil.....	32
IV. RESULTS	66
Flow Over a Backward-facing NACA 0025 Airfoil.....	66
Flow Over a NACA 63 ₃ -018 Airfoil	92
V. CONCLUSIONS AND RECOMMENDATIONS.....	167
BIBLIOGRAPHY	170
APPENDIX.....	175
VITA.....	193

LIST OF TABLES

TABLE	PAGE
1. Drag and Strouhal Data for Cylinder Calculations.....	25
2. Effect of Timestep Variation for Reynolds Number = 6×10^6	49
3. Effect of Spatial Resolution for Reynolds Number = 2×10^6 , $\alpha = 45^\circ$	51
4. Effect of Spatial Resolution for Reynolds Number = 5×10^5 , $\alpha = 18^\circ$	59
5. Effect of Timestep Variation for Reynolds Number = 5×10^5 , $\alpha = 18^\circ$	62

LIST OF FIGURES

FIGURE	PAGE
1. An Airfoil at High Angle of Attack with a Leading Edge Flap.....	5
2. Two-Grid Configuration for a Leading Edge Flap/Airfoil Combination.....	17
3. Two-Grid Configuration with Hole Points Removed	18
4. 357 x 129 Grid Used for Circular Cylinder Calculations.....	24
5. Variation of Drag Coefficient with Reynolds Number For Circular Cylinder Flow	26
6. Force Histories for Cylinder Flow.....	27
7. Variation of Strouhal Number with Reynolds Number for Circular Cylinder Flow	31
8. Surface Pressure Coefficient for Circular Cylinder Flow	33
9. 257 x 129 NACA 0012 Computational Grid - Near Field.....	38
10. Comparison of Lift Coefficient for the NACA 0012 Airfoil, $Re = 6 \times 10^6$	39
11. Comparison of Drag Coefficient for NACA 0012 Airfoil, $Re = 6 \times 10^6$	40
12. Variation of Force Coefficients Due to Turbulence Tripping.....	41
13. Variation of Force Coefficients with Grid Density	43
14. Force History for NACA 0012 at 18° , 129 x 65 Grid, $Re = 6 \times 10^6$	45
15. Surface Vorticity Comparison for NACA 0012 at 18° , $Re = 6 \times 10^6$	46
16. Force History for NACA 0012 at 18° , 129 x 65 Grid, Small Artificial Viscosity (AV).....	48
17. Variation of Force Coefficients with Grid Density for NACA 0012 at 45° , $Re = 2 \times 10^6$	52
18. Lift Coefficient Comparison for NACA 0012 at 45° , $Re = 2 \times 10^6$	54

FIGURE	PAGE
19. Surface Vorticity Comparison for NACA 0012 at 45° , $Re = 2 \times 10^6$	55
20. Effect of Reducing Artificial Viscosity on Lift Coefficient for NACA 0012 at 45°	56
21. Comparison of Lift Coefficient Due to Timestep Variation, $\alpha = 45^\circ$, $Re = 2 \times 10^6$	58
22. Lift Coefficient Comparison for NACA 0012 at 18° , $Re = 5 \times 10^5$	60
23. Variation of Mean Force Coefficients with Grid Density for NACA 0012 at 18° , $Re = 5 \times 10^5$	61
24. Force History for NACA 0012 at 18° , $Re = 5 \times 10^5$, 129×65 Grid - Small AV	63
25. Tentative Low-Speed Flow Map for NACA 0012 Airfoil	64
26. 513×257 , 201×169 Grid Set Used in Computing Flow in Zhou Case 1	68
27. Force Coefficient History for Zhou Case 1, $Re = 6.6 \times 10^5$	69
28. Mean Surface Pressure Coefficient Comparison, Zhou Case 1, No Forcing.....	70
29. Force Coefficient Comparison Between Zhou Case1 and Case 2, No Forcing.....	72
30. Mean Surface Pressure Coefficient Comparison Between Zhou Case 1 and Case 2, No Forcing.....	73
31. Tunnel Grid Set Used in Computing Wall Interference for Zhou Cases	74
32. Comparison of Pressure Coefficients Between Zhou Case 1 and Tunnel Calculation.....	76
33. Instantaneous Vorticity Contours for Zhou Tunnel Case.....	77
34. Mean Surface Pressure Coefficient for Case 2, With and Without Forcing	79

FIGURE

PAGE

35. Experimental Mean Surface Pressure Coefficient for NACA 0025/Flap Combination.....	80
36. Computed Lift Coefficient Behavior for Zhou Case 2 With Forcing	81
37. Mean Surface Pressure Comparison Between Zhou Case 1 and Case 2, With Forcing	82
38. Eddy Viscosity Contours for Zhou Case 2	84
39. Comparison of Stream Traces for Laminar and Turbulent Calculations	87
40. Mean Surface Pressure Coefficient for Zhou Laminar Case 3, No Forcing	88
41. Lift Coefficient History for Zhou Laminar Case 3, No Forcing	89
42. Variation of Mean Surface Pressure Coefficient Due to Forcing for Zhou Laminar Case 3	91
43. NACA 63 ₃ -018 Airfoil/Leading Edge Flap Grids Used for Hsiao Computations.....	94
44. Force Coefficients for Hsiao Baseline Calculation, $Re = 1.9 \times 10^5$	95
45. Mean Surface Pressure Coefficient for Hsiao Baseline	97
46. Mean Surface Pressure Coefficient Comparison With Forcing	98
47. Variation of Lift Coefficient with Forcing Frequency	99
48. Variation of Drag Coefficient with Forcing Frequency	100
49. Dependence of Lift Increment on Flap Amplitude and Initial Position	101
50. Dependence of Drag Increment on Flap Amplitude and Initial Position	102
51. Lift Coefficient Histories	104
52. Drag Coefficient Histories	107
53. Stream Traces for Baseline Flowfield	111

FIGURE	PAGE
54. Vorticity Contours for Baseline Flowfield.....	120
55. Vorticity Contours for Case 1, $f^* = 1$	128
56. Vorticity Contours for Case 2, $f^* = 2$	136
57. Vorticity in Leading Edge Region for Baseline	144
58. Vorticity in Trailing Edge Region for Baseline.....	145
59. Vorticity in Leading Edge Region for Case 1, $f^* = 1$	147
60. Vorticity in Trailing Edge Region for Case 1, $f^* = 1$	148
61. Vorticity in Leading Edge Region for Case 2, $f^* = 2$	149
62. Vorticity in Trailing Edge Region for Case 2, $f^* = 2$	150
63. Velocity Phase Plots for Baseline Case	152
64. Velocity Phase Plots for Case 1, $f^* = 1$	153
65. Velocity Phase Plots for Case 2, $f^* = 2$	154
66. Leading Edge Shedding Behavior.....	156
67. Vorticity Behavior Due to Flap Motion	161

LIST OF SYMBOLS AND ABBREVIATIONS

English Symbols

a_{ij}	entries in the matrix $\mathbf{X}^{-1}$, which is used in the development of a non-reflective boundary condition
a_n	normal component of the surface acceleration
a_0	freestream stagnation sound speed
$\mathbf{A}$	inviscid flux Jacobian
Amp	flap amplitude
ΔV	general representation of artificial viscosity
$\mathbf{\Delta V}$	vector of artificial viscosity components
$\mathbf{b}$	velocity of the solid surface
$\mathbf{B}$	inviscid flux Jacobian
c	chord length of airfoil
c_{b1}	= 0.1355, constant in the Spalart-Allmaras turbulence model
c_{b2}	= 0.622, constant in the Spalart-Allmaras turbulence model
c_{t1}	= 1, constant in the Spalart-Allmaras turbulence model
c_{t2}	= 2, constant in the Spalart-Allmaras turbulence model
c_{t3}	= 1.1, constant in the Spalart-Allmaras turbulence model
c_{t4}	= 2, constant in the Spalart-Allmaras turbulence model
c_{v1}	= 7.1, constant in the Spalart-Allmaras turbulence model
c_{w2}	= 0.3, constant in the Spalart-Allmaras turbulence model

C_{w3}	= 2, constant in the Spalart-Allmaras turbulence model
C_D	drag coefficient
$C_{D\text{amplitude}}$	amplitude of periodically varying drag coefficient
$C_{D\text{exp}}$	experimental value of drag coefficient
$C_{D\text{mean}}$	time-average, or mean, value of periodically varying drag coefficient
C_{D0}	drag coefficient for unforced flow
C_L	lift coefficient
$C_{L\text{exp}}$	experimental value of lift coefficient
$C_{L\text{mean}}$	time-average, or mean, value of periodically varying lift coefficient
C_{L0}	lift coefficient for unforced flow
C_p	pressure coefficient, also given as CP in certain figures
CFL	Courant-Friedrichs-Lewy number
d	distance from field point to the nearest solid wall, used in turbulence model
d_t	distance from field point to the point of transition, used in turbulence model
D	cylinder diameter
e_t	total energy per unit volume
f	forcing frequency, Hz
f^*	non-dimensional frequency, with non-dimensionalization factor given as used in text
f_{t1}	tripping term in the Spalart-Allmaras turbulence model
f_{t2}	tripping term in the Spalart-Allmaras turbulence model
f_{v1}	term in the Spalart-Allmaras turbulence model

f_{v2}	term in the Spalart-Allmaras turbulence model
f_w	term in the Spalart-Allmaras turbulence model
F	inviscid flux vector
g	term in the Spalart-Allmaras turbulence model
g_t	term in the Spalart-Allmaras turbulence model
G	inviscid flux vector
H_i	normal wave propagation function for non-reflective boundary condition
i,j	dummy indices
I	identity matrix
$I_{\max}$	maximum number of grid points in circumferential direction, also given as II
J	Jacobian of coordinate transformation
$J_{\max}$	maximum number of grid points in body-outward direction, also given as JJ
L	general length scale used in non-dimensionalization, typically taken as airfoil chord or cylinder diameter
LE	leading edge
$\mathbf{n}$	surface normal vector
p	pressure
Φ	angular measure around cylinder, measured from leading stagnation point
Pr	Prandtl number
Pr_t	turbulent Prandtl number
Q	vector of conservative variables
ΔQ	incremental, or “delta” change in Q

r	term in the Spalart-Allmaras turbulence model
$\mathbf{r}$	vector distance from the center of rotation to a point on the body surface
R	gas constant
$\mathbf{R}$	viscous flux vector
Re	Reynolds number
Re_D	Reynolds number calculated with the diameter, D
RHS	right-hand side of numerical algorithm
$\mathbf{S}$	viscous flux vector
St	Strouhal number
St_{exp}	experimental value of Strouhal number
t	time
T	non-dimensional time; also temperature
T_{cycle}	time in periodic cycle
T_i	time marking start of flap motion
TE	trailing edge
u	Cartesian component of velocity in the x-direction
$\mathbf{u}$	velocity vector
U	contravariant component of velocity
U_∞	magnitude of freestream velocity
v	Cartesian component of velocity in the y-direction
$\mathbf{v}_{cr}$	translational velocity of the instantaneous center of rotation
V	contravariant component of velocity

V	viscous flux Jacobian
VOR	used for vorticity label in contour plots
W	characteristic variable
W	viscous flux Jacobian
x	Cartesian spatial coordinate
X	matrix of the eigenvectors of $\tilde{B}$
X^{-1}	inverse of matrix of eigenvectors of $\tilde{B}$
x_s	x-coordinate of flap center of rotation
y	Cartesian spatial coordinate
y^+	boundary layer coordinate normal to solid surface
$y^+_{\max}$	maximum value of y^+ defined by grid spacing adjacent to solid boundary
y_s	y-coordinate of flap center of rotation

Greek Symbols

α	angle of attack
γ	ratio of specific heats, taken as 1.4 for air
$\delta_{\xi,\eta}$	central difference operator
$\delta_{x,y}$	viscous flux terms in energy equation
ε	internal energy per unit mass
ε_2	second-order artificial diffusion coefficient
ε_4	fourth-order artificial diffusion coefficient

η	curvilinear spatial coordinate normal to body surface
$\eta_{\max}$	maximum value of η , location of farfield boundary
θ	initial flap position relative to freestream direction
$\Delta\theta$	flap rotational displacement
Θ_A	user-defined maximum amplitude of flap motion
κ	= 0.41, constant in the Spalart-Allmaras turbulence model
λ	second coefficient of viscosity
$\lambda^{(i)}$	i^{th} entry on the diagonal of Λ
$\lambda_1, \lambda_2, \lambda_3, \lambda_4$	eigenvalues of $\tilde{\mathbf{B}}$
Λ	diagonal matrix containing the eigenvalues of $\tilde{\mathbf{B}}$
μ	molecular viscosity
μ_t	eddy viscosity
ν	kinematic molecular viscosity
ν_t	eddy viscosity
$\hat{\nu}$	modified eddy viscosity in Spalart-Allmaras turbulence model
ξ	curvilinear spatial coordinate in circumferential direction
$\xi_{\min}$	minimum value of ξ
$\xi_{\max}$	maximum value of ξ
ρ	fluid density
ρ_0	freestream stagnation density
σ	= 2/3, constant in the Spalart-Allmaras turbulence model

$\sigma_{\xi,\eta}$	scaled spectral radii of inviscid flux Jacobians
τ	time
$\Delta\tau$	time step
τ_w	shear stress on solid surface
τ_{xx}	normal stress component in x-direction
τ_{xy}, τ_{yx}	shear stress components
τ_{yy}	normal stress component in y-direction
υ	specific volume
ϕ	flag to designate whether a point on a grid is a field point or a hole point
χ	function of eddy viscosity used in Spalart-Allmaras turbulence model
ω	magnitude of vorticity; also magnitude of angular velocity
$\dot{\omega}$	angular acceleration
$\vec{\omega}$	vorticity vector; also angular velocity vector
ω_s	shear layer instability frequency, rad/s
ω_t	magnitude of the vorticity at the transition point
Ω	Fourier transform of vorticity

Mathematical Symbols

∇	gradient symbol; also backward difference operator
Δ	forward difference
$\sim$	designates quantities transformed from Cartesian to curvilinear coordinates

Subscripts

Note: The use of subscripts denoting differentiation should be clear from the context used.

o	stagnation conditions
t	partial derivative with respect to time
x	partial derivative with respect to x
y	partial derivative with respect to y
η	partial derivative with respect to η
ξ	partial derivative with respect to ξ
∞	freestream conditions

Superscripts

n	present time step
$n+1$	next time step
$n-1$	last time step

CHAPTER I

INTRODUCTION

At high angles of attack the flow over stream-lined bodies can experience massive separation. The resulting shear layers naturally tend to roll up into large vortical structures. The presence and motion of these structures has a significant impact on the continuing flowfield development. By applying an artificial forcing to the shedding shear layer the rollup can be intensified or reduced, and the vorticity field may be altered. This results in a (hopefully useful) modification of the instantaneous and time-averaged loads on the body. Control of these vortical structures to enhance the lift and lift-to-drag ratio has been the goal of numerous investigations.

The ability to effect large changes in separated flowfields through small disturbances has long been known. Roshko [1] noted that the presence of an interference element in the near wake behind a circular cylinder could dramatically change the wake structure and the forces on the cylinder. Beginning in the 1980's, J. M. Wu and A. D. Vakili studied the problem of controlling separated flow. Wu and Wu [2] present a good discussion of the theory of lift enhancement at high angles of attack where the flow is massively separated and unstable. Vakili [3] also provides a good review of relevant physical theory, with an emphasis on the manipulation of vortex flows using various forms of passive and active excitation. The underlying theme of these works is the use of unsteady control mechanisms in concert with the natural unsteadiness in the flow to create a desired effect.

Numerous researchers have focused on the use of acoustic excitation to enhance lift. The results of Zaman, Bar-Sever, and Mangalam [4] are typical. In their study, acoustic forcing from an external microphone was applied to the flow over an airfoil for a range of angles of attack in a low Reynolds number regime. The results indicated that for angles of attack up to stall, low amplitude acoustic forcing was effective in removing laminar separation by causing the flow to transition to turbulence. At higher angles of attack, forcing at higher frequencies and larger amplitudes was required to cause any significant increase in lift. In fact, tunnel resonance was found to play a key role in creating large amplitude transverse velocities from the low-amplitude acoustic forcing in the region of the airfoil leading edge. Another example is the work Chang, et. al. [5], in which internal acoustic excitation channeled through a small hole in the leading edge of a NACA 63₃-018 airfoil was used to improve its performance in the region of static stall.

Mechanical oscillations have also been used to modify the structure of the unsteady flowfield and alter the loads on a body immersed in the flow. Karim and Acharya [6] investigated the possibility of controlling the dynamic stall vortex over a pitching airfoil using leading edge suction. Parameter ranges were identified which could yield partial or complete suppression of the stall vortex by energizing the oncoming shear layer. Gopalkrishnan, et. al. [7] considered flow over a transversely oscillating D-cylinder and found that the presence of a pitching airfoil located in the wake of the cylinder could substantially alter the vortex structure of the wake. Hsiao, Wang, and Zohar [8] demonstrated active control of the separated flow over an airfoil using an oscillating flap located near the point of separation near the leading edge. With the appropriate

combination of flap geometric and excitation parameters, the time-averaged lift coefficient was increased by up to approximately 50%. Zhou, et. al. [9] found large increases in lift for the flow over a NACA 0025 airfoil positioned backwards in the flow, at an angle of attack of 27° . The unsteady excitation was provided by a small flap positioned at the effective leading edge of the airfoil. Perhaps the most unique demonstration of unsteady forcing used to advantage can be found in the work of Tryantafallou, et. al. [10].

Studying the swimming behavior of fish and using a robotic tuna, their work demonstrated how the vortex shedding sequence generated by a flapping tail motion could be used to generate a strong thrust force. In this manner, fish can swim at higher speeds than would otherwise be possible. The body motion of the fish also enabled it to extract energy from the wakes of obstacles upstream of the fish by creating counter-rotating vortices to weaken those in the oncoming wake flow. It should be noted that in these studies, the focus was not on suppressing separation, but on utilizing the energy present in the shedding process.

To better understand the flow physics involved in controlling a separated flowfield, several studies have been performed which utilized computational methods. A numerical analysis of buffet alleviation using a porous airfoil was performed by Gillan and Cooper [11]. This Navier-Stokes analysis clearly showed the effectiveness of passive control in reducing periodic leading edge shedding in a low-speed flow, leading to alleviation of buffeting. Laurie and Farokhi [12] utilized a discrete vortex method to simulate the flow over a NACA 0012 airfoil equipped with a leading edge flap. In their study, the flap had a length equal to approximately 20% chord, and was attached in such a way that the sharp

leading edge of the flap served as the leading edge of the configuration. Results indicated that enhanced lift could be obtained by tailoring the oscillatory behavior of the flap. Kim and Rho [13] performed a Navier-Stokes simulation of the flowfield around an airfoil with a stationary and oscillating spoiler. Their results indicated that resonance developed when the oscillation frequency of the spoiler equaled the vortex shedding frequency. Numerous studies, such as those by Visbal and Shang [14], Ko and McCroskey [15] and others have demonstrated the feasibility of using Navier-Stokes simulations to model pitching airfoils, dynamic stall, vortex shedding, and other unsteady fluid dynamic behavior.

The primary objective of this work is a computational investigation into the active control of the separated flowfield over an airfoil at high angles of attack. The focus is on an analysis of the physical mechanisms involved in vortex control using an oscillating flap to perturb the basic flow. The sketch in Figure 1 is illustrative of such an arrangement. Flow striking the high-angle-of-attack airfoil separates at the leading and trailing edges. A small flap located near the leading edge is in intimate contact with the shear layer forming there. In this figure the shear layers are represented by constant vorticity surfaces taken from a numerical solution. By oscillating the flap about an axis parallel the leading edge, the shear layer can be perturbed. The resulting development of the shear layer and the overall flowfield is the concern of this study.

To perform the analysis, the two-dimensional, compressible Navier-Stokes equations were solved in a time-accurate manner using an approximate-factorization, finite-difference algorithm. A domain decomposition approach utilizing overlapped grids allowed gridding of the complex geometries. For turbulent flow calculations, the

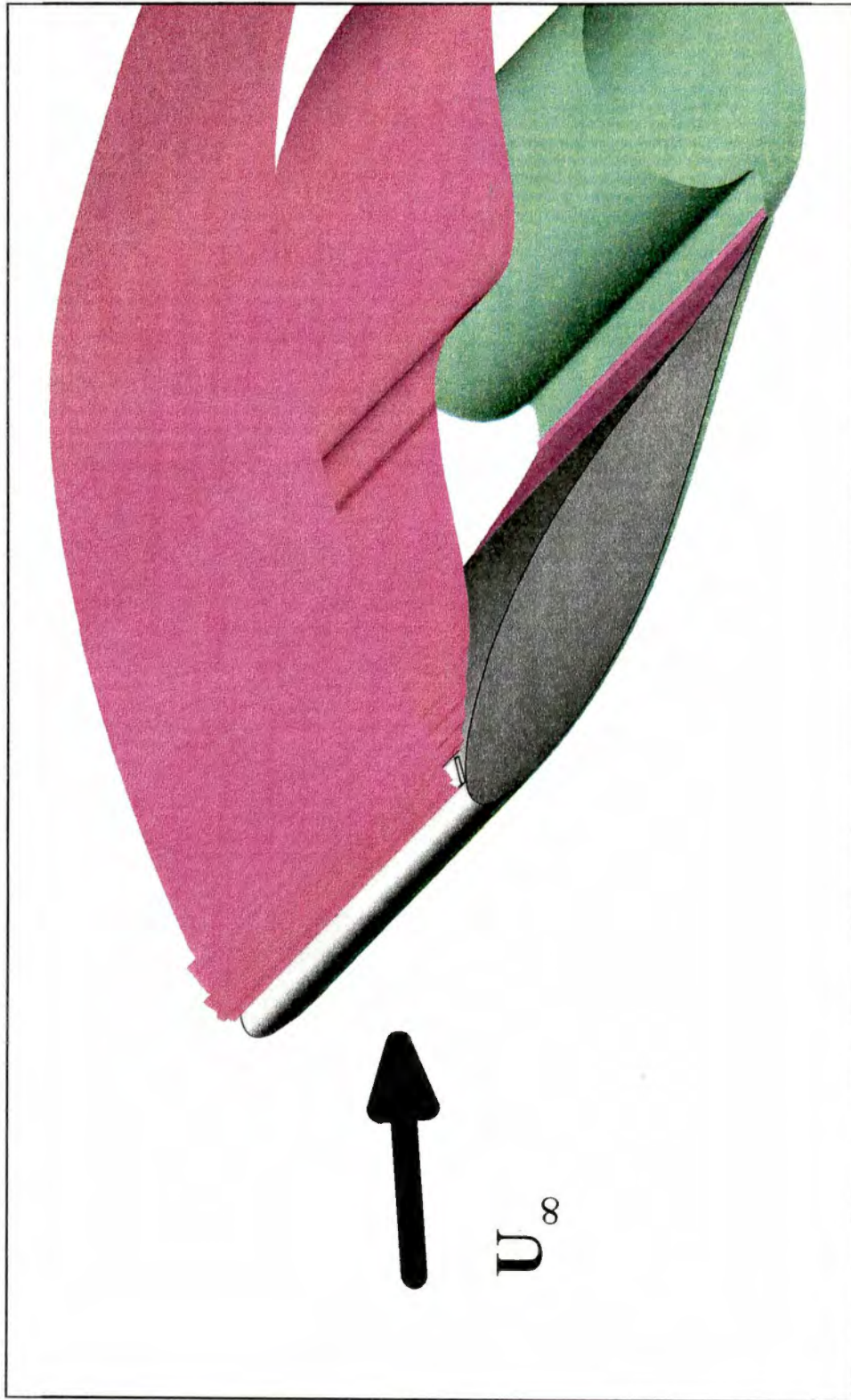


Figure 1: An Airfoil at High Angle of Attack with a Leading Edge Flap

Reynolds-averaged form of the equations was used with a one-equation turbulence model for closure. Validation of the algorithm was performed using a range of test cases. These cases were selected to examine the accuracy of the code, and in particular the ability to simulate turbulent, massively separated, unsteady flowfields. The need for this is not trivial, since there is not a large body of experimental data relating to the focus of this study, and the computational results that have been published can be confusing. Barton and Pulliam [16] showed that non-physical oscillatory solutions could be generated using an Euler solver. A more careful treatment of the solid surface boundary condition eliminated some of this behavior. The inclusion of the viscous terms in the equations of motion also served to remove non-physical behavior. The choice of a turbulence model can also have a pronounced effect on the nature of a computer-generated solution. The behavior of the flow solver when modeling turbulent, separated, unsteady flows needs to be quantified before any computational results can be trusted.

CHAPTER II

DEVELOPMENT OF THE NUMERICAL MODEL

Governing Equations

It is assumed that the two-dimensional flow of a compressible, viscous fluid over a body is described by the Reynolds-averaged Navier-Stokes equations, which can be written in conservation form as

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial (\mathbf{F} - \mathbf{R})}{\partial x} + \frac{\partial (\mathbf{G} - \mathbf{S})}{\partial y} = \mathbf{0}. \quad (1)$$

The vector $\mathbf{Q}$ of conservative variables is

$$\mathbf{Q} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ e_t \end{bmatrix} \quad (2)$$

where ρ is the fluid density, and (u, v) are the Cartesian components of the velocity. The total energy per unit volume e_t is the sum of the internal energy per unit volume ρe and the kinetic energy per unit volume of the fluid,

$$e_t = \rho e + \frac{1}{2} \rho (u^2 + v^2). \quad (3)$$

The inviscid flux vectors $\mathbf{F}$ and $\mathbf{G}$ are given by

$$\mathbf{F} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ (e_t + p)u \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ (e_t + p)v \end{bmatrix}$$
(4)

where the pressure p is related to the conservative variables as

$$p = (\gamma - 1) \left[e_t - \frac{1}{2} \rho (u^2 + v^2) \right].$$
(5)

Here it has been assumed that the fluid is an ideal gas satisfying the equation of state $p = \rho RT$, where T is the temperature and R is the gas constant. The ratio of specific heats is γ , and is treated as a constant in this study. The corresponding viscous flux vectors $\mathbf{R}$ and $\mathbf{S}$ are

$$\mathbf{R} = \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \delta_x \end{bmatrix}, \quad \mathbf{S} = \begin{bmatrix} 0 \\ \tau_{yx} \\ \tau_{yy} \\ \delta_y \end{bmatrix},$$
(6)

where

$$\begin{aligned} \tau_{xx} &= 2\mu \frac{\partial u}{\partial x} + \lambda \nabla \cdot \mathbf{u} \\ \tau_{yy} &= 2\mu \frac{\partial v}{\partial y} + \lambda \nabla \cdot \mathbf{u} \\ \tau_{xy} &= \tau_{yx} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \\ \delta_x &= \frac{\gamma\mu}{Pr} \frac{\partial \epsilon}{\partial x} + u \tau_{xx} + v \tau_{xy} \end{aligned}$$
(7)

$$\delta_y = \frac{\gamma\mu}{Pr} \frac{\partial \varepsilon}{\partial y} + u \tau_{xy} + v \tau_{yy}$$

The Stokes hypothesis can be invoked to give $\lambda = -2\mu/3$. In the above equations the velocity components are non-dimensionalized by freestream stagnation sound speed a_0 , density by freestream stagnation density ρ_0 , and the pressure and total energy by $\rho_0 a_0^2$. For the remainder of this report, all variables should be considered non-dimensional unless otherwise explicitly stated. The Boussinesq assumption has been used to express the Reynolds stresses in terms of an eddy viscosity μ_t multiplying the velocity gradient. The viscous terms are modified with the substitutions

$$\begin{aligned} \mu &\rightarrow \mu + \mu_t \\ \frac{\mu}{Pr} &\rightarrow \frac{\mu}{Pr} + \frac{\mu_t}{Pr_t} \end{aligned} \quad (8)$$

where Pr_t is the turbulent Prandtl number.

A detailed derivation of the algorithm is given in the Appendix. Using second-order accurate central differences for the spatial derivatives and a second-order accurate backward difference for the time derivative, the conservation equations can be discretized in an implicit, approximate factorization fashion to give

$$\begin{aligned} \left[\frac{\mathbf{I}}{\Delta\tau} + \frac{2}{3} \delta_\xi \left(\left(\tilde{\mathbf{A}} - \tilde{\mathbf{V}} + \frac{\partial \tilde{\mathbf{V}}_\xi}{\partial \xi} \right) [-] - \delta_\xi (\tilde{\mathbf{V}}_\xi [-]) - \varepsilon_2 \nabla_\xi \left(\frac{\sigma_\xi}{J} \Delta_\xi \right) [-] \right) \right] \Delta \tilde{\mathbf{Q}}^* &= \frac{2}{3} \frac{\mathbf{RHS}'^n}{\Delta\tau} \\ \left[\frac{\mathbf{I}}{\Delta\tau} + \frac{2}{3} \delta_\eta \left(\left(\tilde{\mathbf{B}} - \tilde{\mathbf{W}} + \frac{\partial \tilde{\mathbf{W}}_\eta}{\partial \eta} \right) [-] - \delta_\eta (\tilde{\mathbf{W}}_\eta [-]) - \varepsilon_2 \nabla_\eta \left(\frac{\sigma_\eta}{J} \Delta_\eta \right) [-] \right) \right] \Delta \tilde{\mathbf{Q}}^* &= \frac{\Delta \tilde{\mathbf{Q}}^*}{\Delta\tau} \end{aligned} \quad (9)$$

with the right-hand side given by

$$\begin{aligned} \mathbf{RHS}'^n = \mathbf{RHS}^n + \frac{\varepsilon_2}{J} \left[\nabla_\xi \left(\sigma_\xi \Delta_\xi J \tilde{\mathbf{Q}}^n \right) + \nabla_\eta \left(\sigma_\eta \Delta_\eta J \tilde{\mathbf{Q}}^n \right) \right] - \\ \frac{\varepsilon_4}{J} \left[\nabla_\xi \left(\sigma_\xi \Delta_\xi \nabla_\xi \Delta_\xi J \tilde{\mathbf{Q}}^n \right) + \nabla_\eta \left(\sigma_\eta \Delta_\eta \nabla_\eta \Delta_\eta J \tilde{\mathbf{Q}}^n \right) \right]. \end{aligned} \quad (10)$$

The operators ∇ and Δ signify the appropriate backward and forward differences, respectively. It should be understood that the system of equations is now expressed in body-fitted curvilinear coordinates (ξ, η) . The blended second-fourth order dissipation of Pulliam [17] is added to the system to ensure stability and avoid odd-even decoupling. The quantities in brackets are taken as operators acting on the variables which are multiplied by them. Definitions of the quantities σ_ξ , σ_η , ε_2 , and ε_4 are given in the Appendix. In particular the form of the unmodified **RHS** can also be found there.

Situations involving body motion can be treated assuming the grid moves as a part of the solid body. As Thomas and Lombard [18] showed, grid motion can cause additional conservation errors. Applying eq. (9) to a uniform stream gives

$$\frac{3\tilde{\mathbf{Q}}_\infty^{n+1} - 4\tilde{\mathbf{Q}}_\infty^n + \tilde{\mathbf{Q}}_\infty^{n-1}}{2\Delta\tau} + \left(\tilde{\mathbf{F}}_\infty^{n+1} \right)_\xi + \left(\tilde{\mathbf{G}}_\infty^{n+1} \right)_\eta = 0 \quad (11)$$

This can be simplified to yield

$$\frac{3\left(\frac{1}{J^{n+1}}\right) - 4\left(\frac{1}{J^n}\right) + \left(\frac{1}{J^{n-1}}\right)}{2\Delta\tau} + \left(\left(\frac{\xi_t}{J} \right)^{n+1} \right)_\xi + \left(\left(\frac{\eta_t}{J} \right)^{n+1} \right)_\eta = 0 \quad (12)$$

which indicates that an error can arise unless the transformation Jacobian is updated using the same time integration scheme used for the conservation variables.

For the O-type grids used in this study, three specific boundary conditions are needed. On solid surfaces, the no-slip condition is imposed, along with the condition of

adiabaticity. At farfield boundaries a non-reflective condition based on the work of Thomson [19] is used, which automatically applies upwinding as needed based on the state of the flow. Along grid cut lines, where the grid re-enters itself, averaging of the flow variables is used. A complete discussion of the boundary condition development is given in the Appendix.

Turbulence Modeling

In solving the Reynolds-averaged equations, the “Reynolds stress” term must be evaluated in order to close the equation set. A host of candidate models exists, but the applicability of turbulence models to massively separated flows is a subject of current research. As observed by Huang and Lin [20], the behavior of a separated shear layer depends strongly on the nature of the boundary layer at separation. The location of transition, the position of the separation point, and the strength of the turbulence at separation all affect the development of the shed shear layer. The ability of existing turbulence models to represent these effects adequately is still an open question. In fact, experimental evidence suggests that the Boussinesq assumption may not even be valid for flows in which large streamline curvature is present (Warsi, [21]). The Boussinesq assumption implies that the principal axes of the Reynolds stress tensor are parallel to the principal axes of the mean strain rate tensor. But experimental data shows that while the axes of the mean strain rate tensor can be changed instantly by the application of some force, a non-zero relaxation time is required for the axes of the Reynolds stress tensor to

follow. Because of these concerns, a secondary focus of this study is to gain some insight on the applicability of turbulence modeling for massively separated, unsteady flows.

Some work has been done in critically evaluating various turbulence models for separated flows. Srinivasan, et. al. [22] applied a number of typical models to the computation of dynamic stall. In this study, the following models were considered: a Baldwin-Lomax (BL) algebraic model, a Re-Normalization Group (RNG) algebraic model, a Johnson-King (JK) one-half equation model, and the Spalart-Allmaras (SA) and Baldwin-Barth (BB) one-equation models. The results indicated that none of the models provided satisfactory results over the entire range of flows considered. Perhaps not surprisingly, the Baldwin-Lomax algorithm performed poorly for this class of separated flows. The RNG algebraic model provided a significantly better simulation at a comparable computation expense. However, for deep stall the Baldwin-Barth and Spalart-Allmaras one-equations yielded better behavior over the entire pitching cycle. The Spalart-Allmaras model did exhibit a tendency to predict early separation.

Rumsey and Vatsa [23] investigated four different turbulence models for two- and three-dimensional transonic flows. The Baldwin-Barth and Spalart-Allmaras models performed well, with the Spalart-Allmaras model providing slightly better results. The Johnson-King model used in the study also gave good results. The worst predictions came from a Baldwin-Lomax model.

Rogers, et. al. [24] examined the performance of the Spalart-Allmaras and Baldwin-Barth one-equation models, and Menter's Shear Stress Transport (SST) and Durbin-Mansour two-equation models in computing a three-element airfoil flow. All but

the Durbin-Mansour model performed similarly well, with the Spalart-Allmaras and SST models giving better comparison to experiment than the Baldwin-Barth.

In a later study, Kern [25] applied the Spalart-Allmaras, Baldwin-Barth, and SST models to a multi-element high lift airfoil computation. His findings were inconclusive. Each of the models did a good job of predicting the lift coefficient up to the stall angle of attack. In the post-stall regime the Baldwin-Barth and SST models compared more favorably to the experimental lift trend. But the Spalart-Allmaras and SST models provided a better correlation of stagnation pressure loss in the outer boundary layer.

For this study, the Spalart-Allmaras model was selected to model the turbulence of the flow field. The available data indicates that it should perform comparably to a two-equation model, but with fewer programming difficulties. The Spalart-Allmaras model also appears to handle adverse pressure gradients better than the Baldwin-Barth model.

The Spalart-Allmaras turbulence model computes a modified eddy viscosity as the solution of the transport equation [26]

$$\begin{aligned} \frac{D\hat{\nu}}{Dt} = & c_{b1}[1 - f_{t2}]\hat{S}\hat{\nu} + \frac{1}{\sigma}\left[\nabla \cdot ((\nu + \hat{\nu})\nabla\hat{\nu}) + c_{b2}(\nabla\hat{\nu})^2\right] \\ & - \left[c_{w1}f_w - \frac{c_{b1}}{\kappa^2}f_{t2}\right]\left[\frac{\hat{\nu}}{d}\right]^2 + f_{t1}\Delta U^2 \end{aligned} \quad (13)$$

where the tripping terms are given by

$$\begin{aligned} f_{t1} = & c_{t1}g_t \exp\left(-c_{t2}\frac{\omega_t^2}{\Delta U^2}\left[d^2 + g_t^2 d_t^2\right]\right) \\ f_{t2} = & c_{t3} \exp(-c_{t4}\chi^2) \end{aligned} \quad (14)$$

with

$$g_t \equiv \min \left(0.1, \frac{\Delta U^2}{\omega_t \Delta x} \right). \quad (15)$$

The quantity d is the distance from a field point to the nearest solid wall, d_t is the distance from a field point to the point of transition, and ω_t is the magnitude of the vorticity at the transition point. The constants are given in the original paper as $c_{b1} = 0.1355$, $\sigma = 2/3$, $c_{b2} = 0.622$, $\kappa = 0.41$, $c_{w2} = 0.3$, $c_{w3} = 2$, $c_{v1} = 7.1$, $c_{t1} = 1$, $c_{t2} = 2$, $c_{t3} = 1.1$, $c_{t4} = 2$, and $c_{w1} = c_{b1}/\kappa^2 + (1 + c_{b2})/\sigma$. The following definitions apply:

$$\begin{aligned} \hat{S} &\equiv |\omega| + \frac{\hat{v}}{\kappa^2 d^2} f_{v2} \\ f_{v2} &= 1 - \frac{\chi}{1 + \chi f_{v1}} \\ f_{v1} &= \frac{\chi^3}{\chi^3 + c_{v1}^3} \\ \chi &= \frac{\hat{v}}{v} \\ f_w &= g \left[\frac{1 + c_{w3}^6}{g^6 + c_{w3}^6} \right] \\ g &= r + c_{w2} (r^6 - r) \\ r &= \frac{\hat{v}}{\hat{S} \kappa^2 d^2} \end{aligned} \quad (16)$$

The eddy viscosity is then obtained from

$$\nu_t = \hat{v} f_{v1} \quad (17)$$

and from this the kinematic eddy viscosity can be computed for use in eq. (8).

The tripping terms can be set to zero for flows where the transition locations are not known a priori. Although the turbulence model is not designed to predict the transition location, this procedure provides a reasonable approach for flows where typical transition models are not applicable, i.e. massively separated flows. The validity of this approximation is discussed in a later section.

The transport equation (13) is solved numerically using an approximate factorization scheme similar to that used for the conservation equations. However, the convective terms are treated using first-order upwind differences. This provides sufficient diffusion for stability. Central differences with added diffusion could be used, but the criterion used in computing the diffusion coefficients for the conservation equations does not work well for turbulence algorithms. The non-linear production and destruction operators can be handled explicitly, or in an implicit fashion. Preliminary results showed that although the implicit approach made the model somewhat more robust, the overall behavior was similar for either treatment. The model is implemented in an uncoupled fashion, with the computation of the turbulence field occurring after the update of the flow field.

Along solid surfaces the eddy viscosity is identically zero. From eq. (17) this corresponds to $\hat{v} = 0$. Along far field boundaries, a Neumann condition is used for the eddy viscosity:

$$\frac{\partial \hat{v}}{\partial n} = 0. \quad (18)$$

Grid cuts are handled by averaging. The turbulence field is initialized to some fraction of the kinematic viscosity, typically one-tenth.

Modifications for Overset Grids

When the body geometry is sufficiently complex, the development of a suitable body-fitted coordinate system can be difficult. To alleviate this problem an overset grid methodology may be used. In brief, smaller grids are constructed around pieces of the geometry of interest. These small grids collectively cover the entire domain and overlap one another, much the way swatches of cloth are pieced together to form a quilt. The solution may then be obtained on each of these smaller domains and the complete solution is reconstructed accordingly. It is important to recognize that these grid patches can be entire grids, or grid fragments in which some sections have been removed. By judicious grid generation and the removal of grid sections fairly complicated geometries may be discretized using simple grid topologies.

Figure 2 illustrates how an airfoil with a small leading edge flap could be represented. Grids are generated for both bodies, as shown. Some grid points fall within the solid boundaries of the bodies, and are removed to produce holes in the grids as seen in Figure 3. The points that make up the holes are called hole points. In this way the problem of grid generation is simplified, but the question of grid-to-grid communication must be addressed. Information from one grid must be passed to another across inter-grid boundaries. On each of these boundaries the solution must be updated much in the manner of a physical grid boundary. A straightforward method is to interpolate the

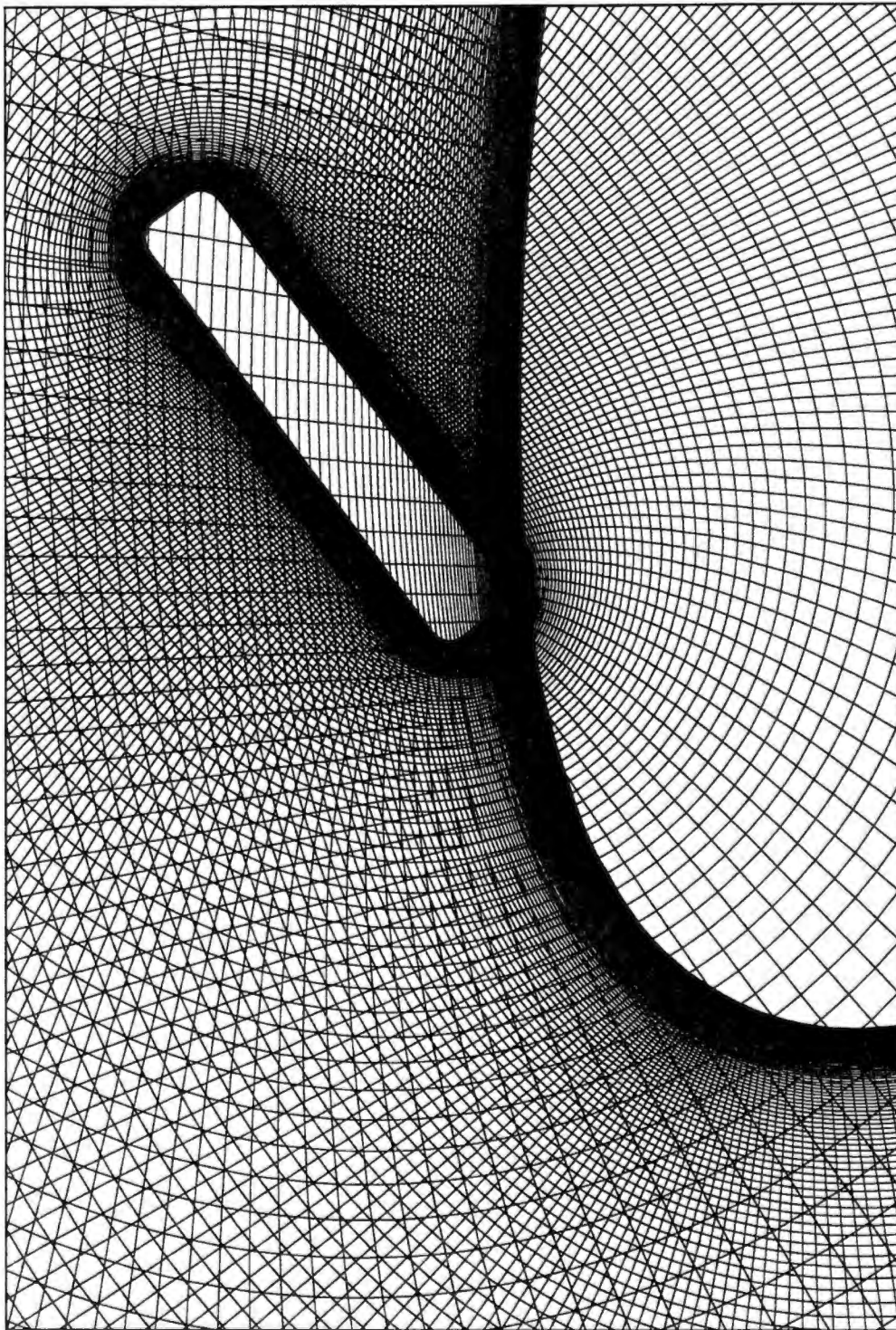


Figure 2: Two-Grid Configuration for a Leading Edge Flap/Airfoil Combination

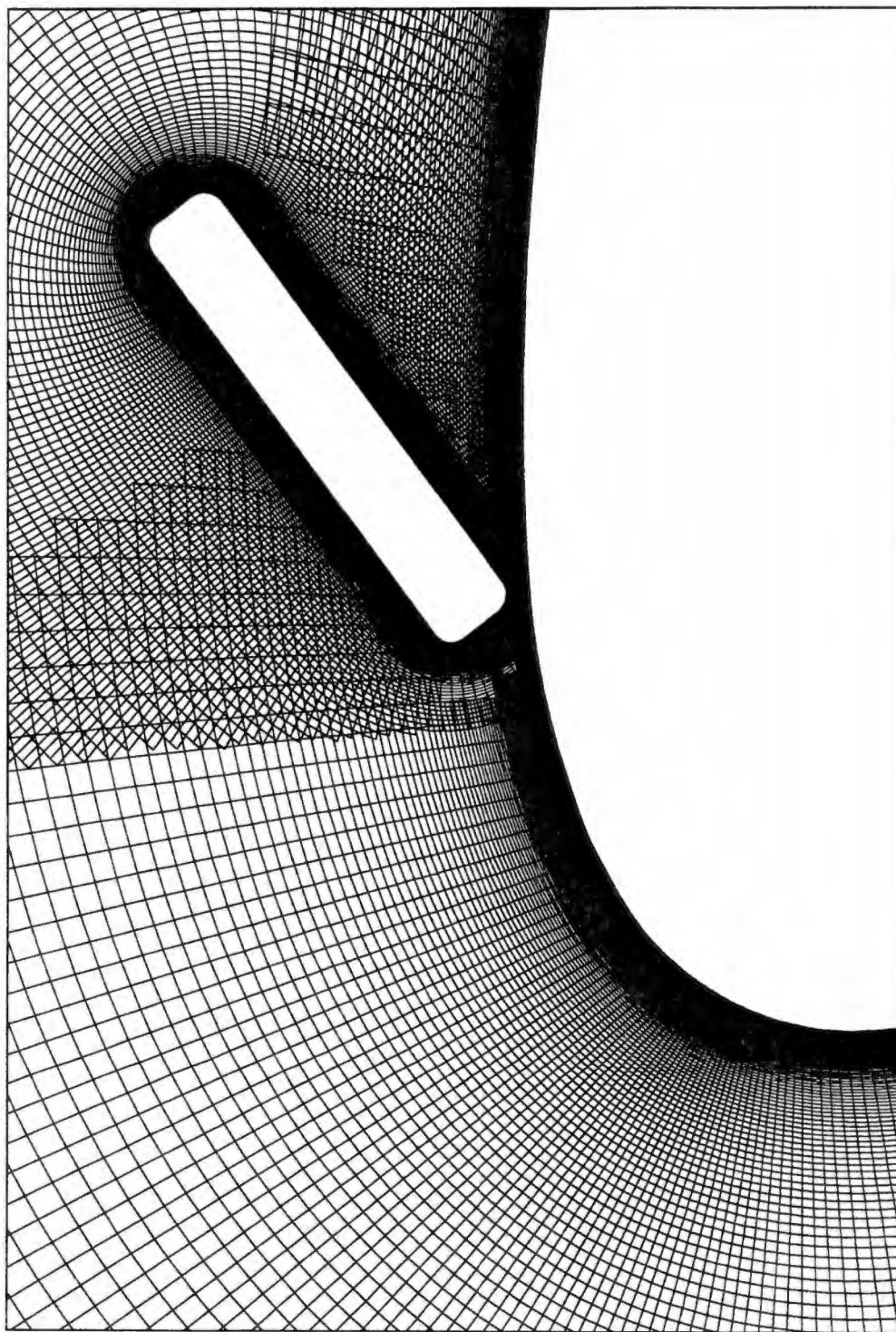


Figure 3: Two-Grid Configuration with Hole Points Removed

solution to a hole boundary point in one grid from valid field points in a “donor” grid. It is recognized that this procedure does raise questions of conservation and mismatch in signal propagation speeds, but it has been found to work well in practice. A bilinear interpolation algorithm is used to perform the update. More details on the overset grid method can be found in Steger and Benek [27].

The method of locating holes and hole boundaries is straightforward. Consider two grids, the cutting grid and the hole grid. A set of level surfaces, given by $\xi = \text{constant}$ or $\eta = \text{constant}$, within the cutting grid are selected. These surfaces define the basic cutting surface. A search algorithm finds all points in the hole grid that fall within the cutting surface. These points are defined as holes by setting a flag ϕ to zero. For a given point ϕ is defined as

$$\phi = \begin{cases} 1 & \text{for a field point} \\ 0 & \text{for a hole point} \end{cases} .$$

Once the hole points are defined a sweep is made across the hole grid, and points adjacent to the hole are defined as hole boundary points. In order to provide a full five-point difference stencil for the numerical diffusion, two layers of hole boundary points are generated. The hole boundary points and the points within holes do not need to be updated by the flow solver. The approximate factorization algorithm, given by eq. (9), can be modified as

$$\begin{aligned}
& \left[\frac{\mathbf{I}}{\Delta\tau} + \frac{2\phi}{3} \delta_{\xi} \left(\left(\tilde{\mathbf{A}} - \tilde{\mathbf{V}} + \frac{\partial \tilde{\mathbf{V}}_{\xi}}{\partial \xi} \right) [-] - \phi \delta_{\xi} (\tilde{\mathbf{V}}_{\xi} [-]) - \phi \varepsilon_2 \nabla_{\xi} \left(\frac{\sigma_{\xi}}{J} \Delta_{\xi} \right) [-] \right) \right] \Delta \tilde{\mathbf{Q}}^* = \\
& \quad \frac{2\phi \mathbf{RHS}'^n}{3 \Delta\tau} \\
& \left[\frac{\mathbf{I}}{\Delta\tau} + \frac{2\phi}{3} \delta_{\eta} \left(\left(\tilde{\mathbf{B}} - \tilde{\mathbf{W}} + \frac{\partial \tilde{\mathbf{W}}_{\eta}}{\partial \eta} \right) [-] - \phi \delta_{\eta} (\tilde{\mathbf{W}}_{\eta} [-]) - \phi \varepsilon_2 \nabla_{\eta} \left(\frac{\sigma_{\eta}}{J} \Delta_{\eta} \right) [-] \right) \right] \Delta \tilde{\mathbf{Q}}^n = \\
& \quad \frac{\phi \Delta \tilde{\mathbf{Q}}^*}{\Delta\tau}
\end{aligned} \tag{19}$$

to prevent the inadvertent update of a hole or hole boundary point.

After the hole boundary points have been obtained, the donor grid (which is usually the same as the cutting grid) is searched for suitable interpolation stencils. Hole boundary points for which no suitable stencil may be found are deemed orphan points, and are updated via a weighted averaging of the surrounding field points. Where a valid stencil does exist, a bilinear interpolation is used.

The use of bilinear interpolation introduces an error into the solution scheme that is generally referred to in the literature as a non-conservation problem, which is a convenient term that conveys little information. The problem is connected to the fact that the linear approximations used for the flow variables cannot represent the correct functional forms for all of variables and still satisfy the conservation equations. This difficulty can be lessened significantly by having overlap regions made up of similarly-resolved grids. In this case the functional reconstruction used in the bilinear interpolation is not too different from the functional representation introduced by Taylor series approximation on an individual grid.

A second concern that arises in the use of overset grids is sometimes lumped into the non-conservation category. In regions of overlap the signal transfer process typically

involves the passage of information from grid A to grid B, travel over a region common to both grids, and then passage from grid B to grid A. If the grid characteristics are substantially different in the common region, phase differences in the propagation of information will develop. The value of the signal reaching a point in grid B due to the flow solver will not agree with the value interpolated from grid A to the same point in B. This error arises regardless of how the information is passed between grids, and thus is not strictly speaking a conservation error. It can be minimized by reducing the common region of signal propagation between grids to a minimum, and by placing regions of overlap in areas of similar grid resolution. These are not hard and fast rules, but rather the outgrowth of experience.

For problems that do not involve relative motion between grids, these hole boundary points and interpolation stencils may be computed once at the beginning of a run and stored for future use. However, when relative motion occurs, the hole and stencil definition procedure must be performed at every timestep. Hole boundary points are recomputed as part of the overall boundary condition procedure, for both the flow solver and the turbulence model.

The development of an algorithm to provide numerical solutions the Navier-Stokes equations is not an overly difficult task. However, obtaining an algorithm that can provide accurate and reliable results can take more effort. The next section focuses on the validation of the algorithm described above.

CHAPTER III

VALIDATION

Flow Over a Circular Cylinder

Before the numerical model can be applied to a study of massively separated, turbulent flows, it must be validated. The purpose of this validation effort is multifold. At a bare minimum, the code should faithfully reproduce well-established experimental data for cases where such results exist. This is particularly important in situations where simulations are to be made and no such data exist. In these cases the code is being used as a tool basically to extrapolate from an established baseline of known solutions to the problem of interest. The veracity of the results can be determined only indirectly. The ability to reproduce a solid baseline serves to build confidence in the computational results.

The algorithm's behavior should also be examined as the timestep and grid resolution are varied. The solution for a calculation should be independent of timestep and grid spacing if the results are to be physically meaningful. The effect of changes in other input parameters, such as artificial viscosity and turbulence transition location, should also be considered.

All of the validation calculations were performed on single grids. That is, no overset grids were used in the validation. However, Meakin [28] found that the error behavior of computations on overset grids followed that of single grid calculations if the flowfield is sufficiently resolved on the individual component grids.

For this study a set of flow simulations was selected which should be representative of unsteady, massively separated, turbulent flows. The first case involves flow over a circular cylinder at various Reynolds numbers, Re_D , based on cylinder diameter. The main quantities of interest for comparison are the drag coefficient C_D , the surface pressure coefficient C_p , and the Strouhal number (St) based on vortex shedding frequency. The shedding frequency was obtained from spectral analyses of the drag histories. Four runs, at Reynolds numbers of 5×10^5 , 7.5×10^5 , 1×10^6 , and 8.4×10^6 , respectively, were made. The range of Reynolds numbers was selected to focus on the transitional range, where the drag coefficient and Strouhal number exhibit significant variations. The same O-grid, composed of 357 points in the circumferential direction and 129 points in the radial direction, was used for these four runs. The initial spacing in the radial direction was such that $y^+ = \frac{y\sqrt{\rho\tau_w}}{\mu} < 5$ for the highest Re_D run. Figure 4 shows this grid. The grid outer boundary is roughly 15 diameters from the cylinder. A Mach number of 0.2 was used to represent (nearly) incompressible flow, and the artificial viscosity coefficients were set at $(\epsilon_2, \epsilon_4) = (0.2, 0.04)$. For these simulations the data of Jones, et. al. [29] was used to obtain the transition information needed as input for the turbulence model. Transition points were selected using plots of experimental pressure coefficient, at locations where the C_p curve indicated the formation of a separation bubble. At the highest Reynolds number of 8.4×10^6 , transition was assumed to occur at a point midway between the forward stagnation point and the point of boundary layer separation.

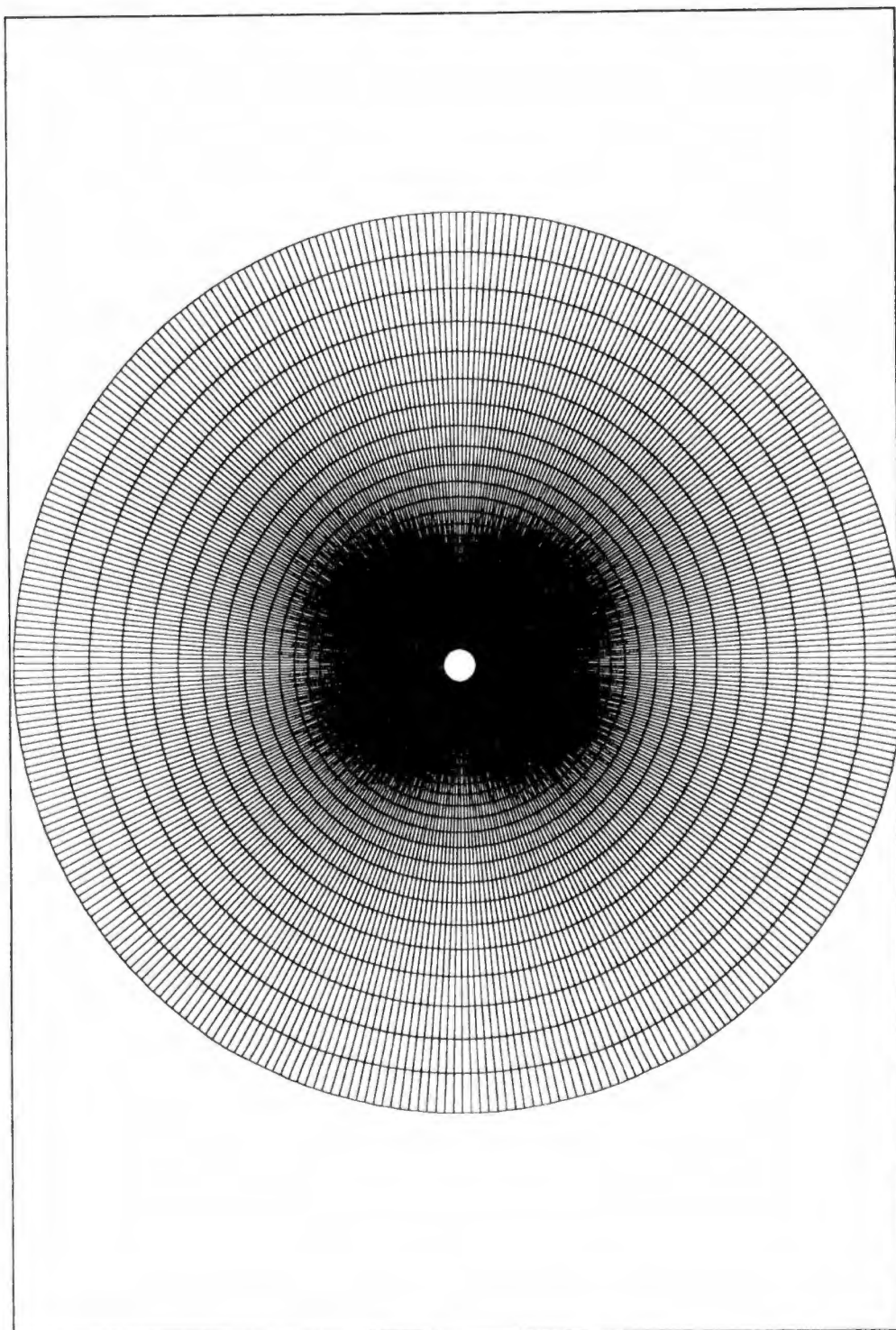


Figure 4: 357 x 129 Grid Used for Circular Cylinder Calculations

Figure 5 shows the computed mean drag coefficients plotted along with experimental data from White [30]. The curve representing the empirical data is his fairing of data taken from Roshko [1], Jones, et. al. [29], and other sources. The computed mean C_D does show a drop between Reynolds numbers of 10^5 and 10^6 , and increases for Re_D beyond 1×10^6 . However, the agreement with experiment is only qualitatively good. Time histories of the lift and drag coefficients are given in Figure 6. The amplitudes of C_L and C_D drop as the Reynolds number moves through transition from the sub-critical to super-critical regime, and then increase again as Re_D increases past transition. Experimental evidence indicates that in this transitional regime periodic behavior may disappear completely, although the data are not conclusive. The computed results at least qualitatively represent this variation in the force amplitudes.

The variation in Strouhal number with Reynolds number is shown in Figure 7. The faired curve is taken from White [30]. Again the qualitative behavior is in accord with the experimental data, and the computed values fall within the variation of the experimental value. The Strouhal number reaches a maximum as the transitional region is broached, and then falls with increasing Re_D to a uniform value of approximately 0.26-0.27. The drag and Strouhal data is also summarized in Table 1.

Table 1: Drag and Strouhal Data for Cylinder Calculations

Reynolds Number	C_{Dmean}	$C_{Damplitude}$	St
5.0×10^5	0.497	0.071	0.270
7.5×10^5	0.430	0.043	0.276
1.0×10^6	0.389	0.039	0.286
8.4×10^6	0.504	0.060	0.265

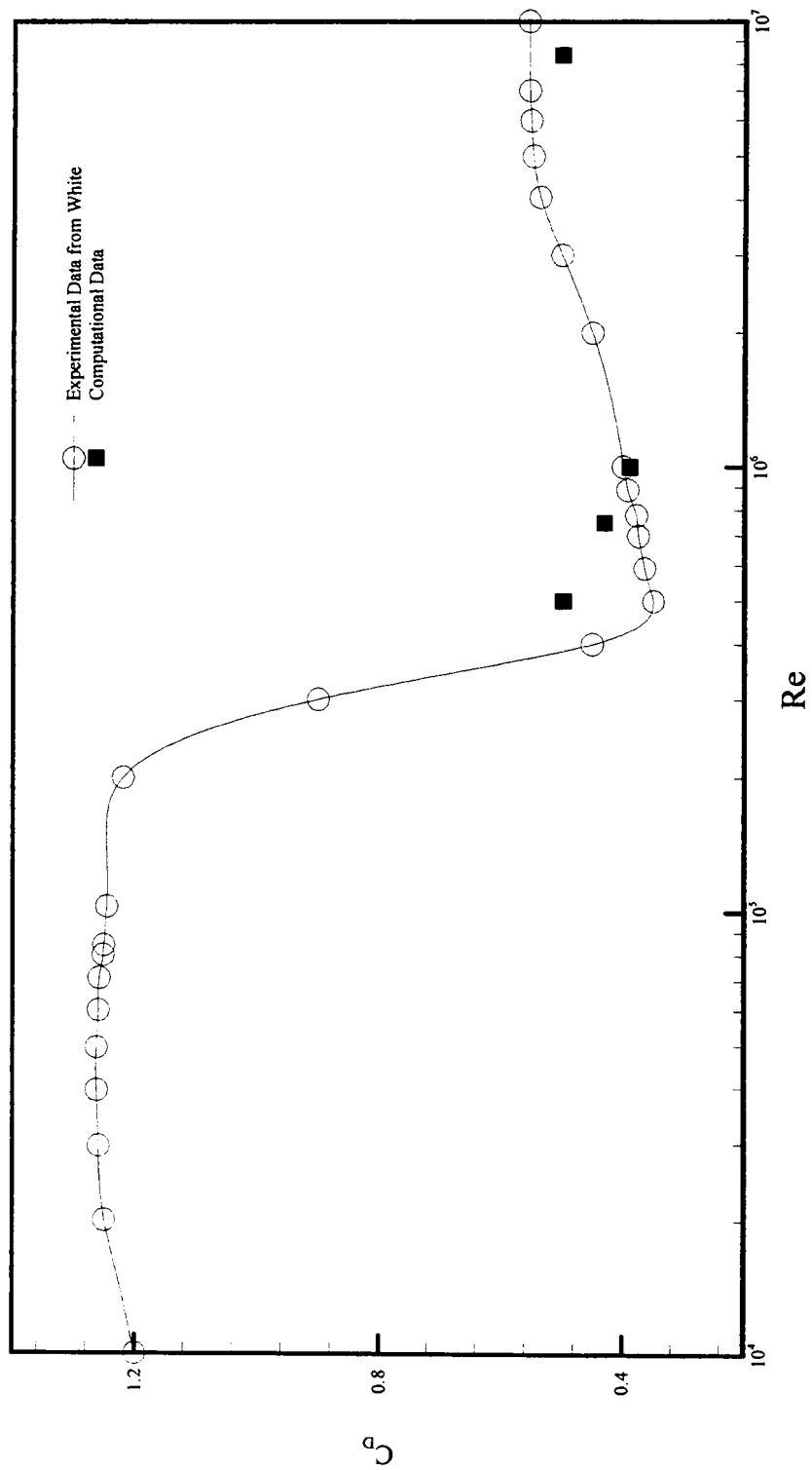


Figure 5: Variation of Drag Coefficient with Reynolds Number For Circular Cylinder Flow

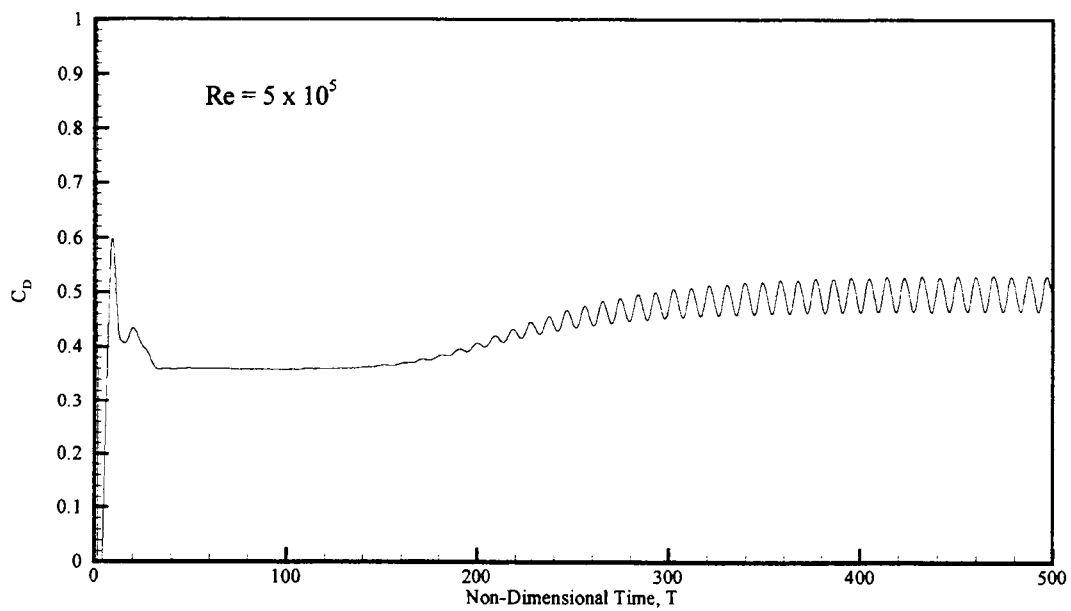
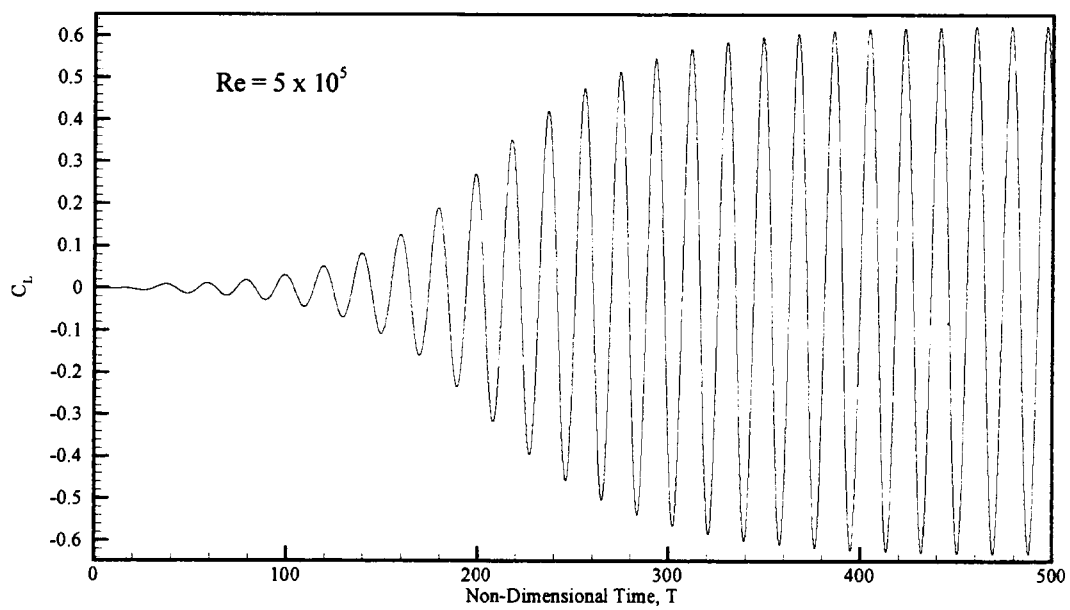


Figure 6: Force Histories for Cylinder Flow

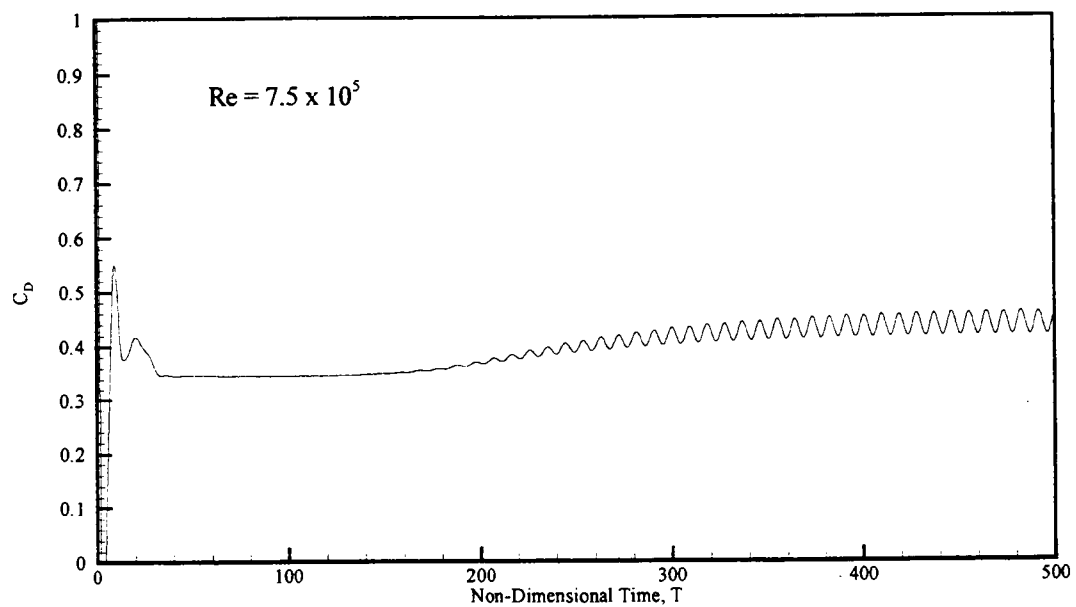
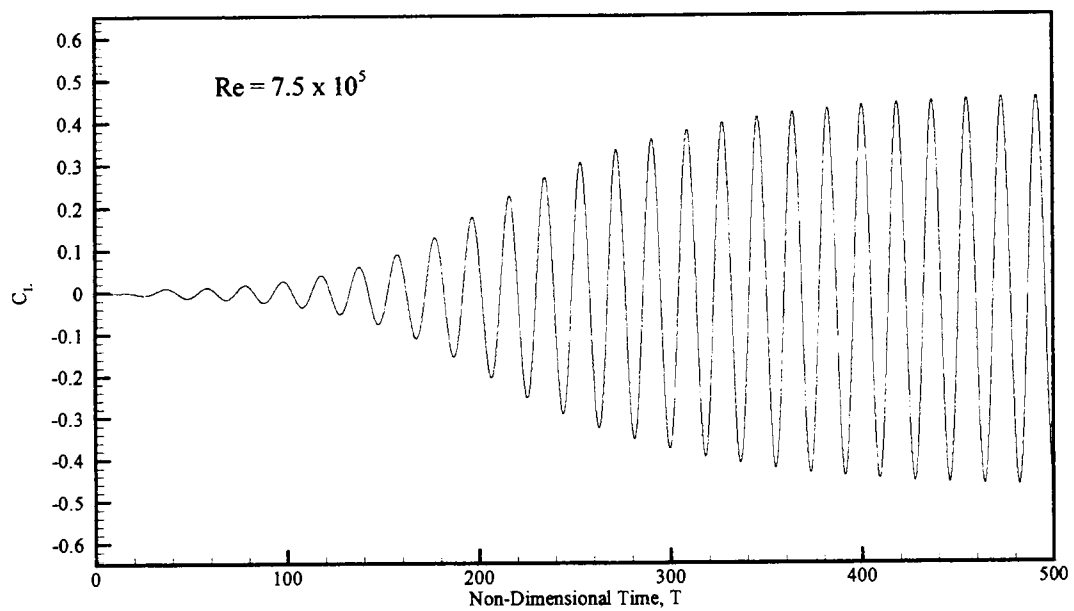


Figure 6: Force Histories for Cylinder Flow (Continued)

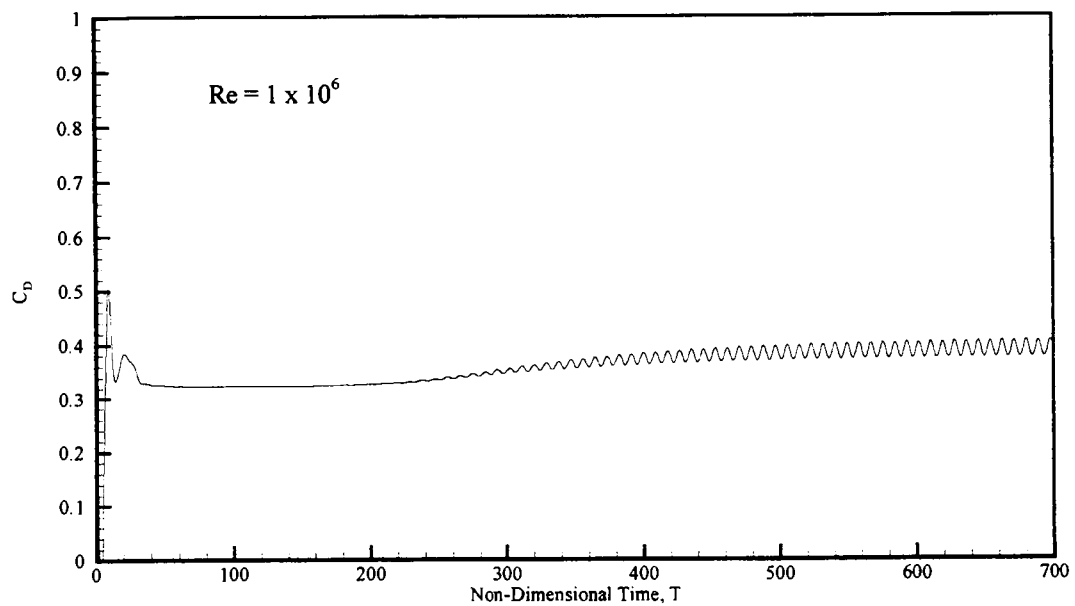
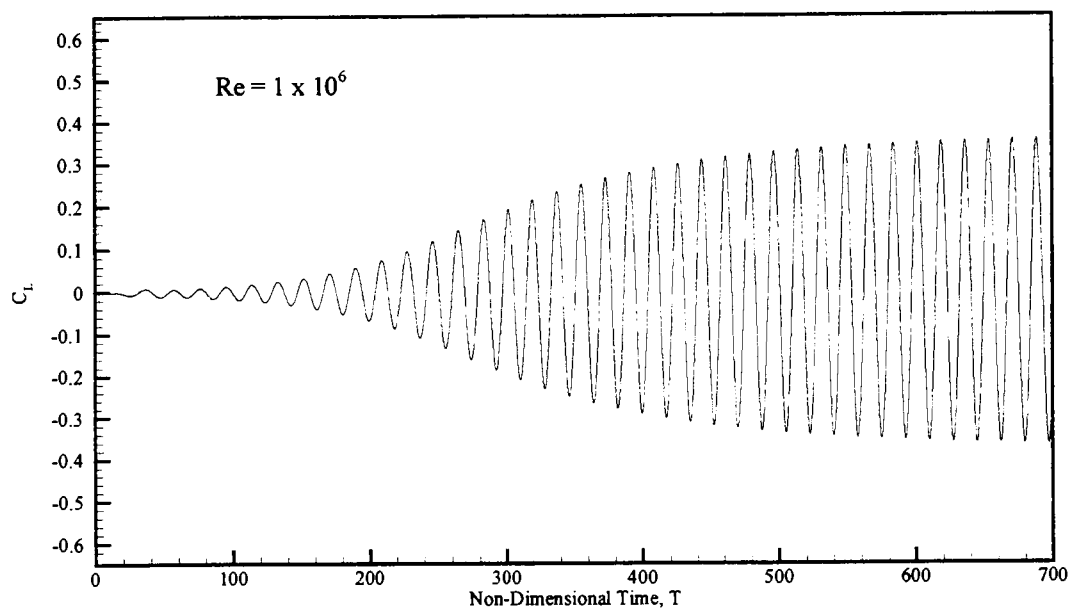


Figure 6: Force Histories for Cylinder Flow (Continued)

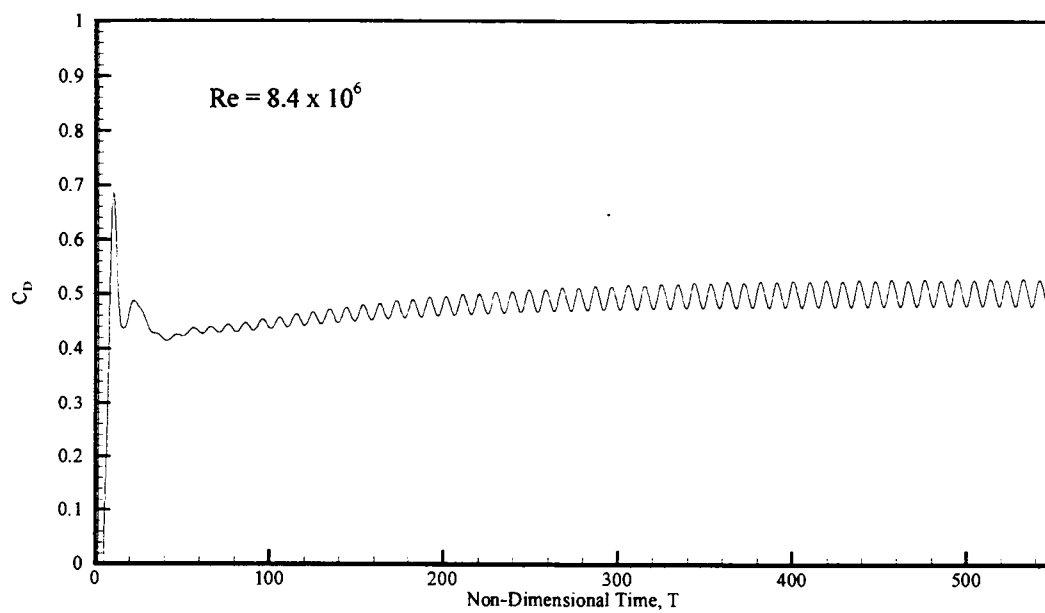
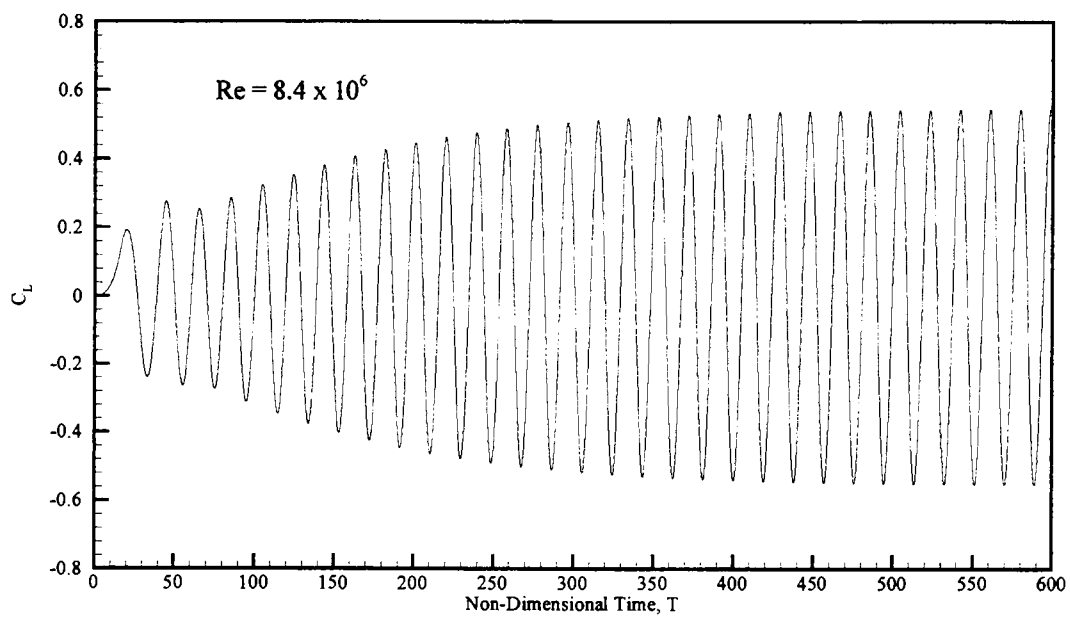


Figure 6: Force Histories for Cylinder Flow (Concluded)

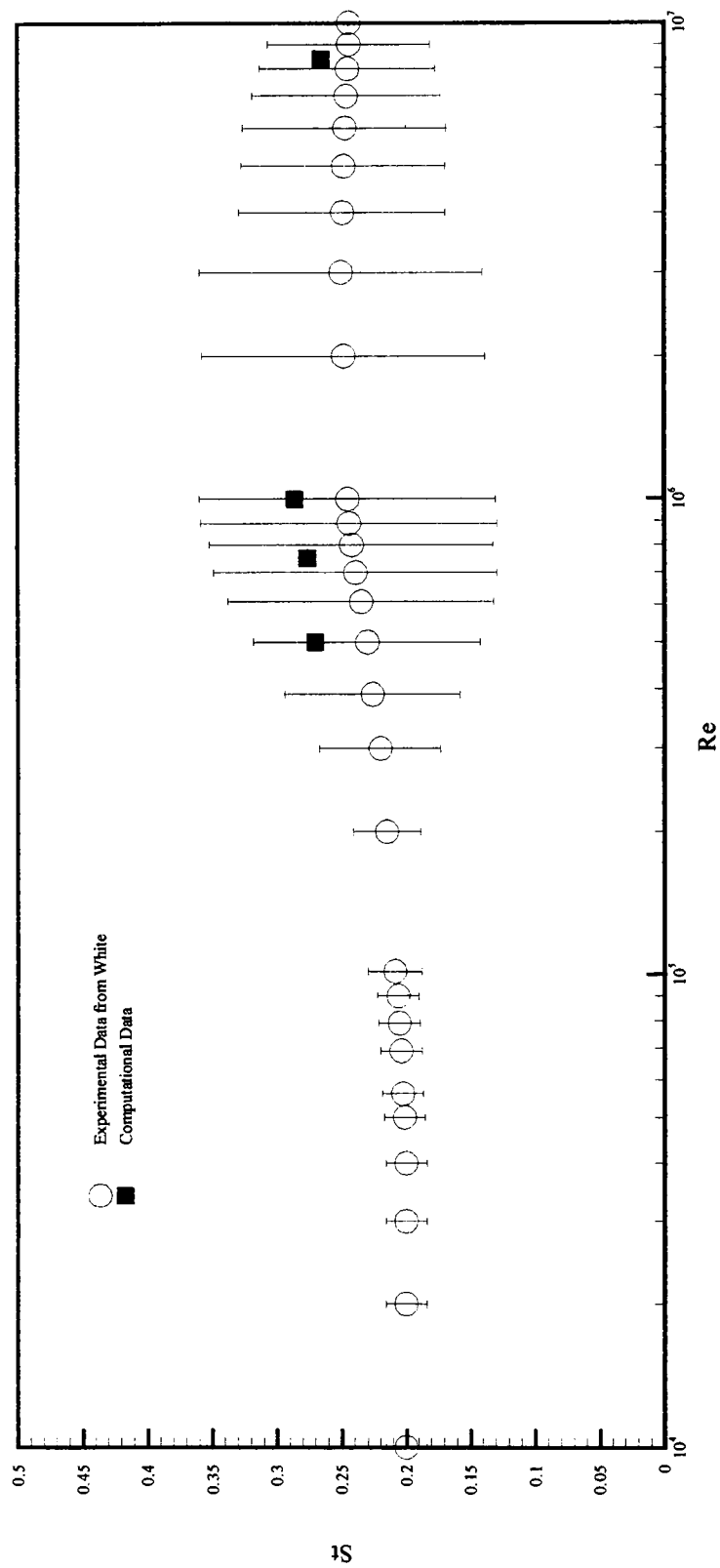


Figure 7: Variation of Strouhal Number with Reynolds Number for Circular Cylinder Flow

Figure 8 shows the surface pressure coefficients for each of the runs. The experimental data is taken from Jones, et. al. [29]. The flow solver tends to under-predict the suction on the leeward side of the cylinder for $Re_D < 1 \times 10^6$. This could be related to the observed tendency of the Spalart-Allmaras turbulence model to predict early separation. Early separation also produces a thicker wake, resulting in a larger drag. Conversely, at the highest Reynolds number the flow solver over-predicts the suction peak, and appears to retard separation slightly. This would result in a thinner wake and consequent lower drag coefficient. However, the overall qualitative agreement is good, and the computed base pressure coefficients match the experimental data well. No grid effects were considered in this phase of the validation. These cylinder calculations were performed as a first check on the performance of the flow solver.

Flow Over a NACA 0012 Airfoil

Flow over a circular cylinder is a good basic test of a solver's ability to handle unsteady separated flows. In order to quantify further the behavior of the flow solver, flow over a NACA 0012 airfoil at various Reynolds numbers and grid densities was considered. To begin, flow at a Reynolds number of 6×10^6 was computed for angles of attack from 0° to 20° . The objective of this set of computations was to reproduce the lift and drag curves for the NACA 0012. The Mach number for these calculations was again set to 0.2, and an O-grid composed of 257×129 points was used for each angle of attack. The initial grid point spacing normal to the airfoil provided a y^+ of 2 or less over most of

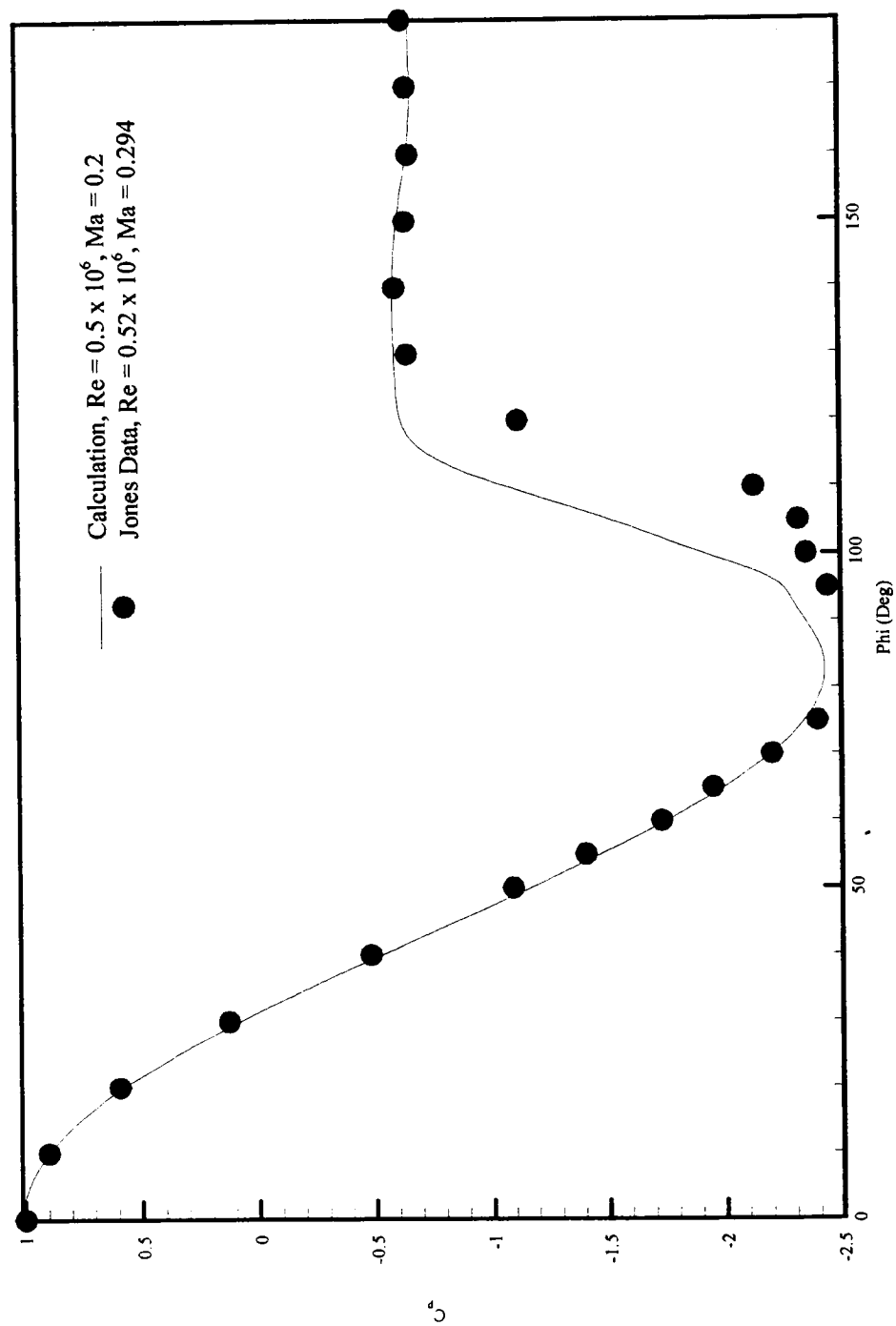


Figure 8: Surface Pressure Coefficient for Circular Cylinder Flow

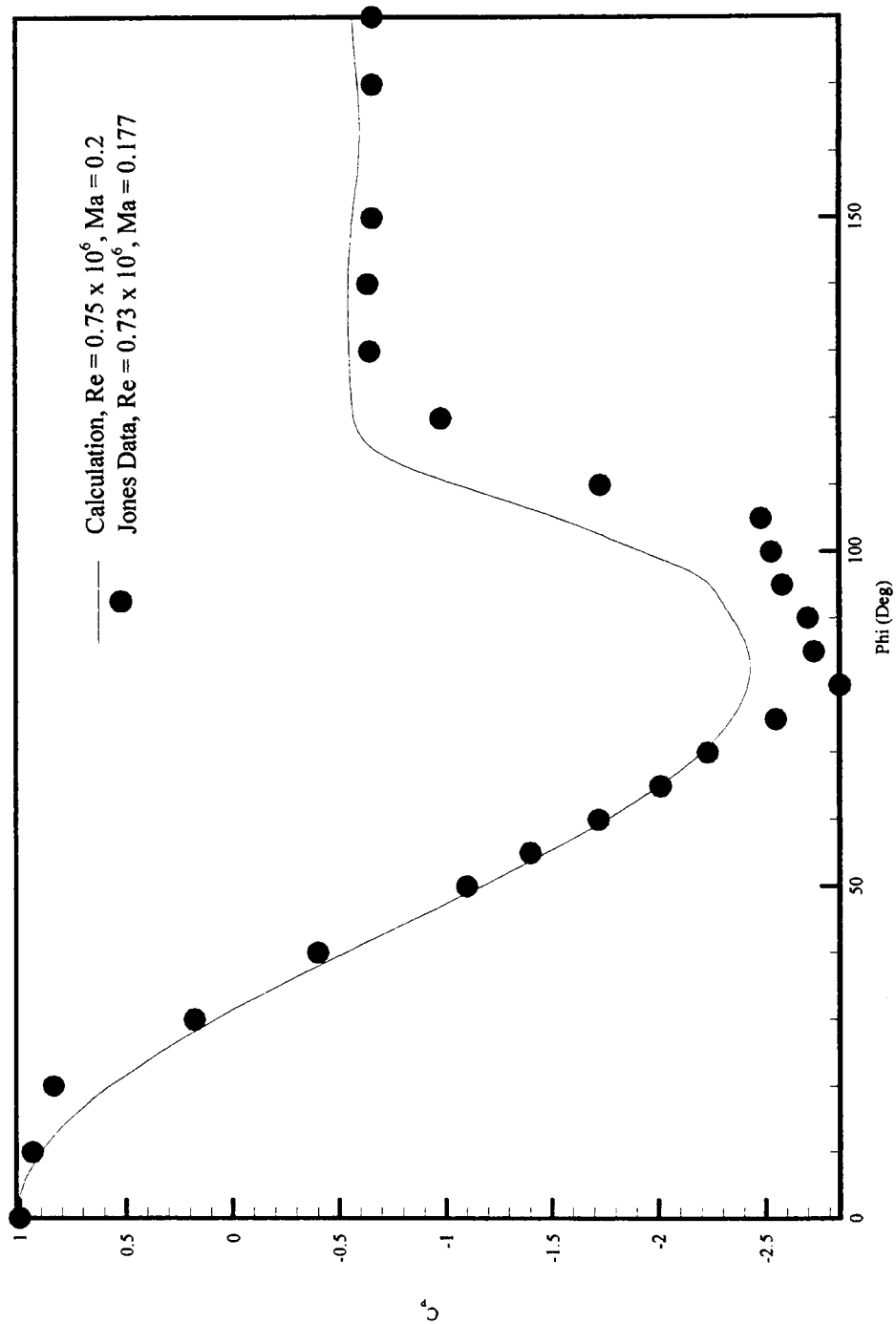


Figure 8: Surface Pressure Coefficient for Circular Cylinder Flow (Continued)

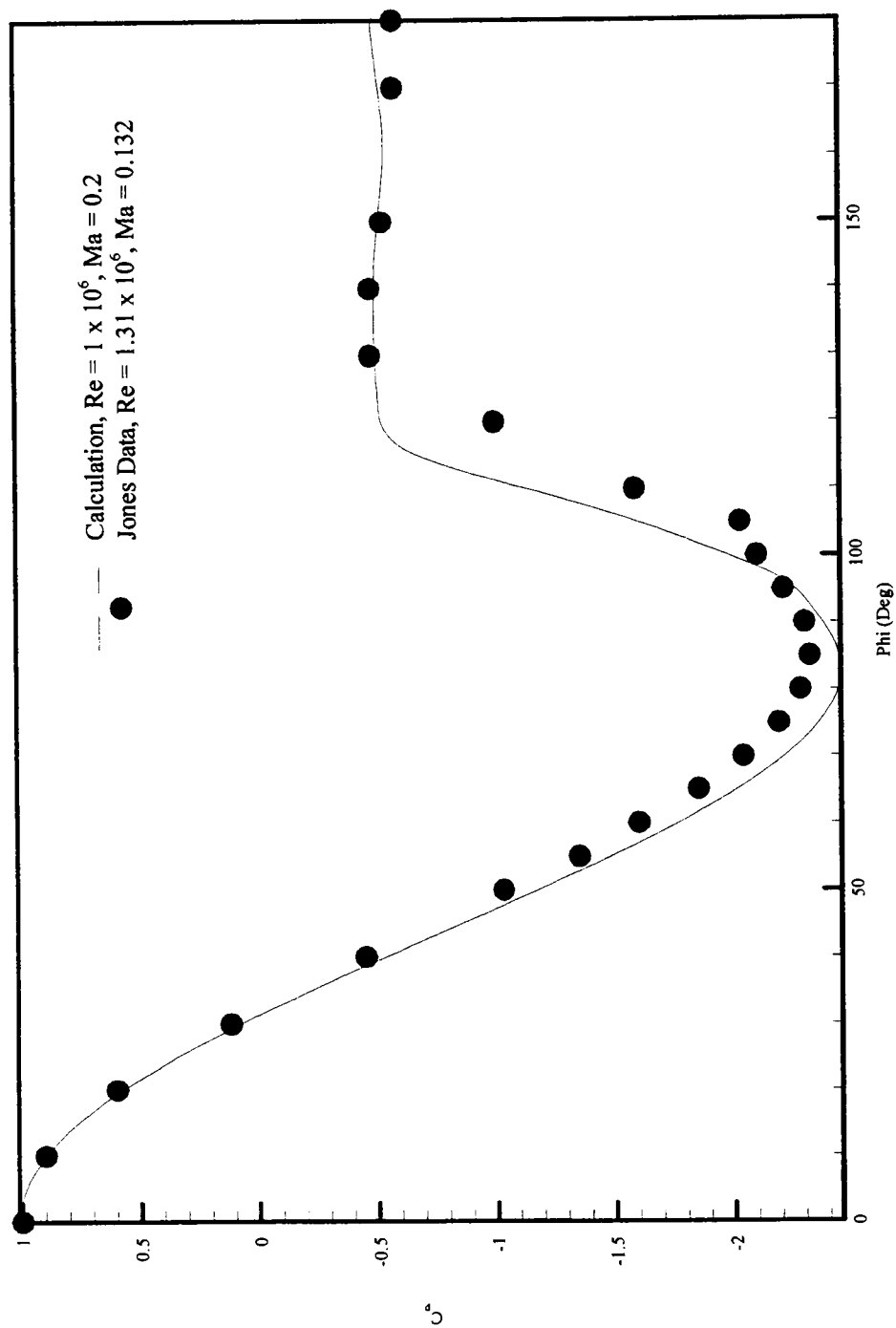


Figure 8: Surface Pressure Coefficient for Circular Cylinder Flow (Continued)

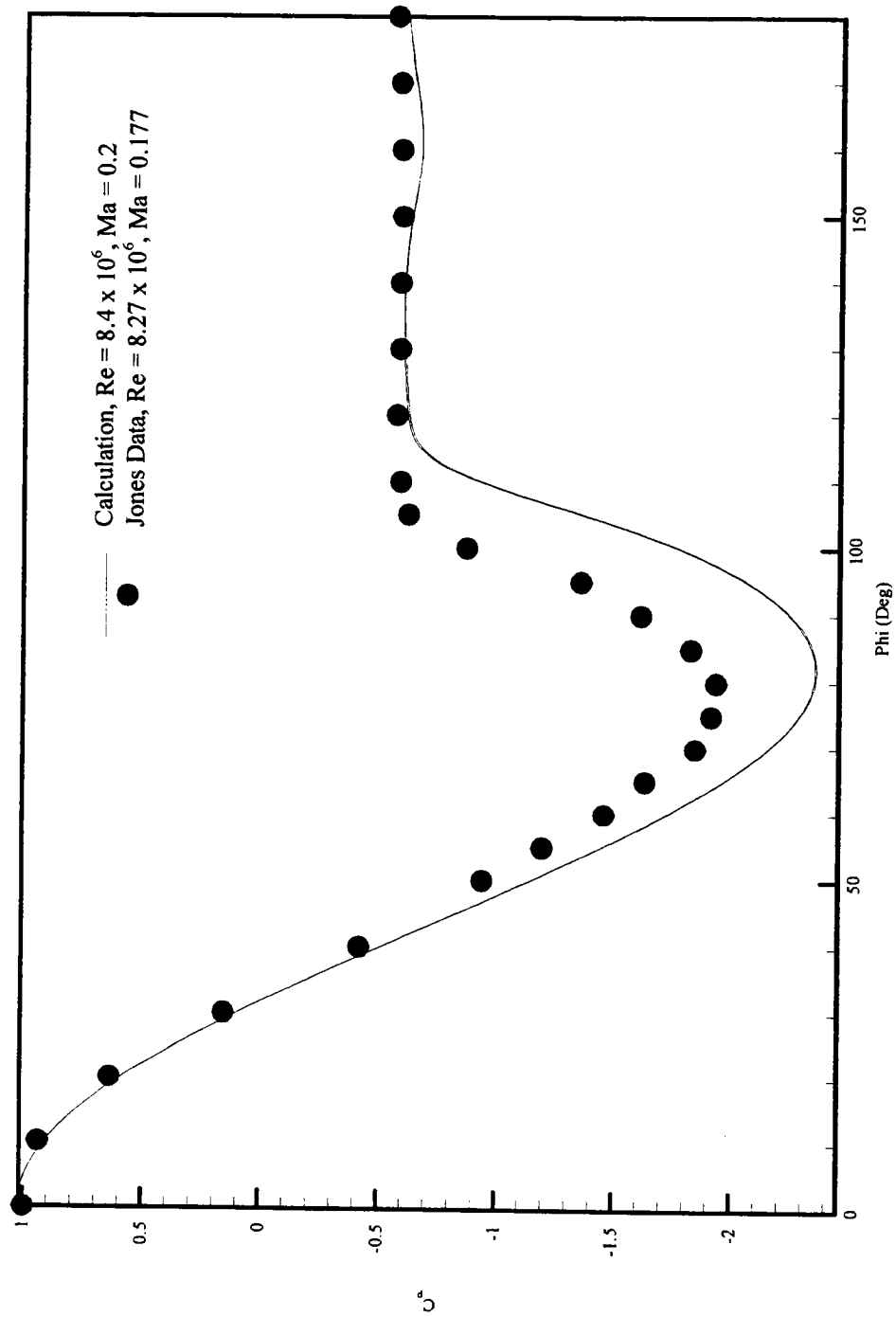


Figure 8: Surface Pressure Coefficient for Circular Cylinder Flow (Concluded)

the airfoil surface, with a maximum y^+ of 5 near the leading edge stagnation point. The outer boundary was located approximately 15 chords away from the airfoil surface. Figure 9 shows the near-field region of the grid. The artificial viscosity coefficients were set at $(\varepsilon_2, \varepsilon_4) = (0.2, 0.04)$. The location of transition was set at 1% of the chord for the 0° angle of attack case. At other angles of attack the transition points were shifted along the airfoil surface to keep the distance between the leading edge stagnation and transition points roughly constant. This procedure ensured that along the upper surface the transition point was located downstream of the suction peak, as it should be (see Schlichting [31], for example).

The computed lift curve is shown in Figure 10, along with experimental data from Abbott and von Doenhof [32] and Jacobs and Sherman [33]. The drag coefficient is presented in Figure 11. The agreement in C_L is excellent, and the computed drag also matches the experimental data well. For these runs, the variations in lift and drag were less than 10^{-5} at convergence.

For situations where the transition points are not known a priori, the Spalart-Allmaras turbulence model can be modified by setting the tripping terms to zero. The assumption then is that the turbulence model will simulate transition with some degree of fidelity. In order to evaluate this assumption runs at 6° , 10° , 16° , and 18° angles of attack were repeated, using no explicitly set trips. Figure 12 shows the original tripped data plotted along with the untripped results. Although the results show some variation, it appears that allowing the model to transition naturally is a satisfactory approximation for a typical airfoil shape. In fact, the largest variation at 18° moves the computed lift

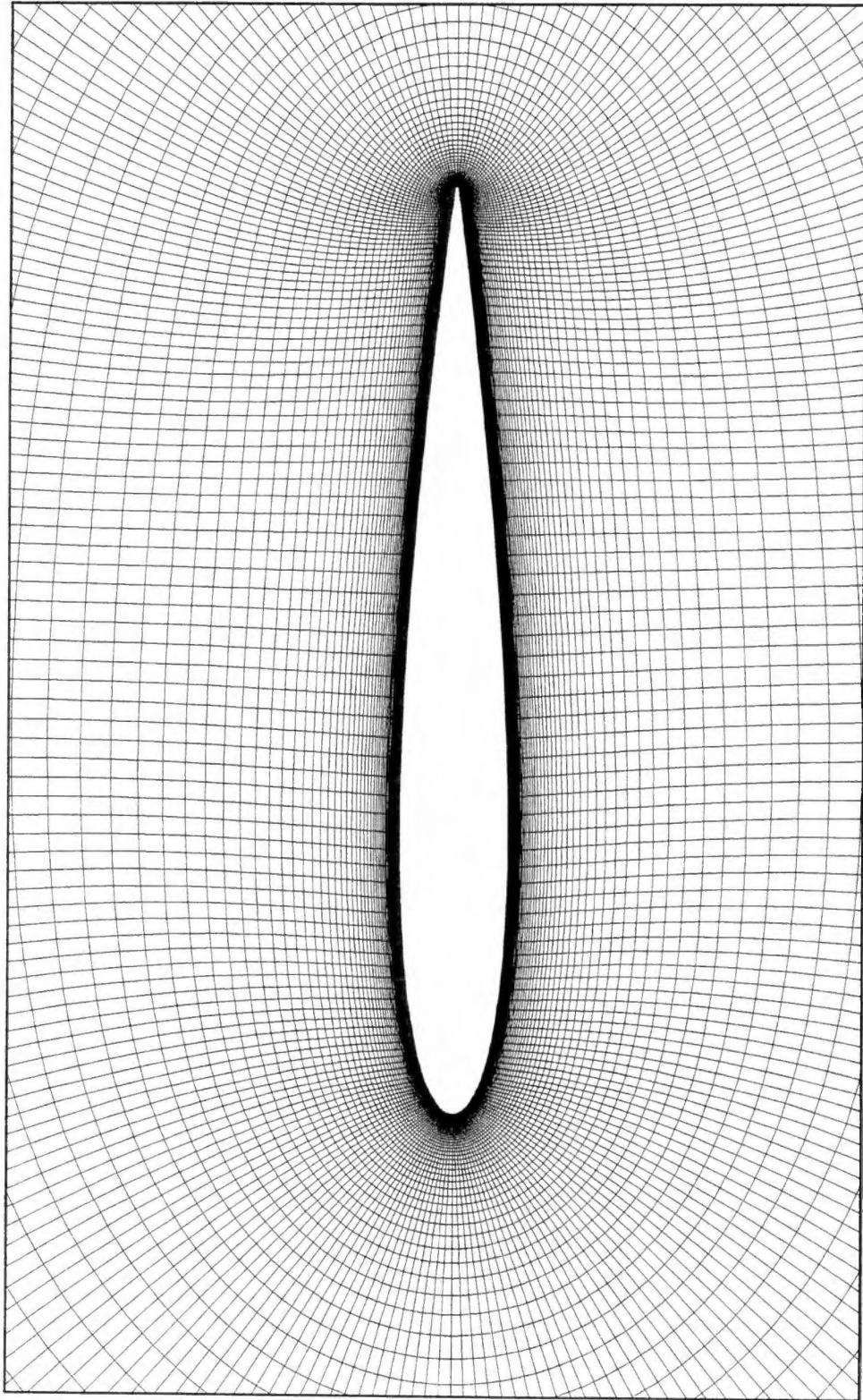


Figure 9: 257 x 129 NACA 0012 Computational Grid - Near Field

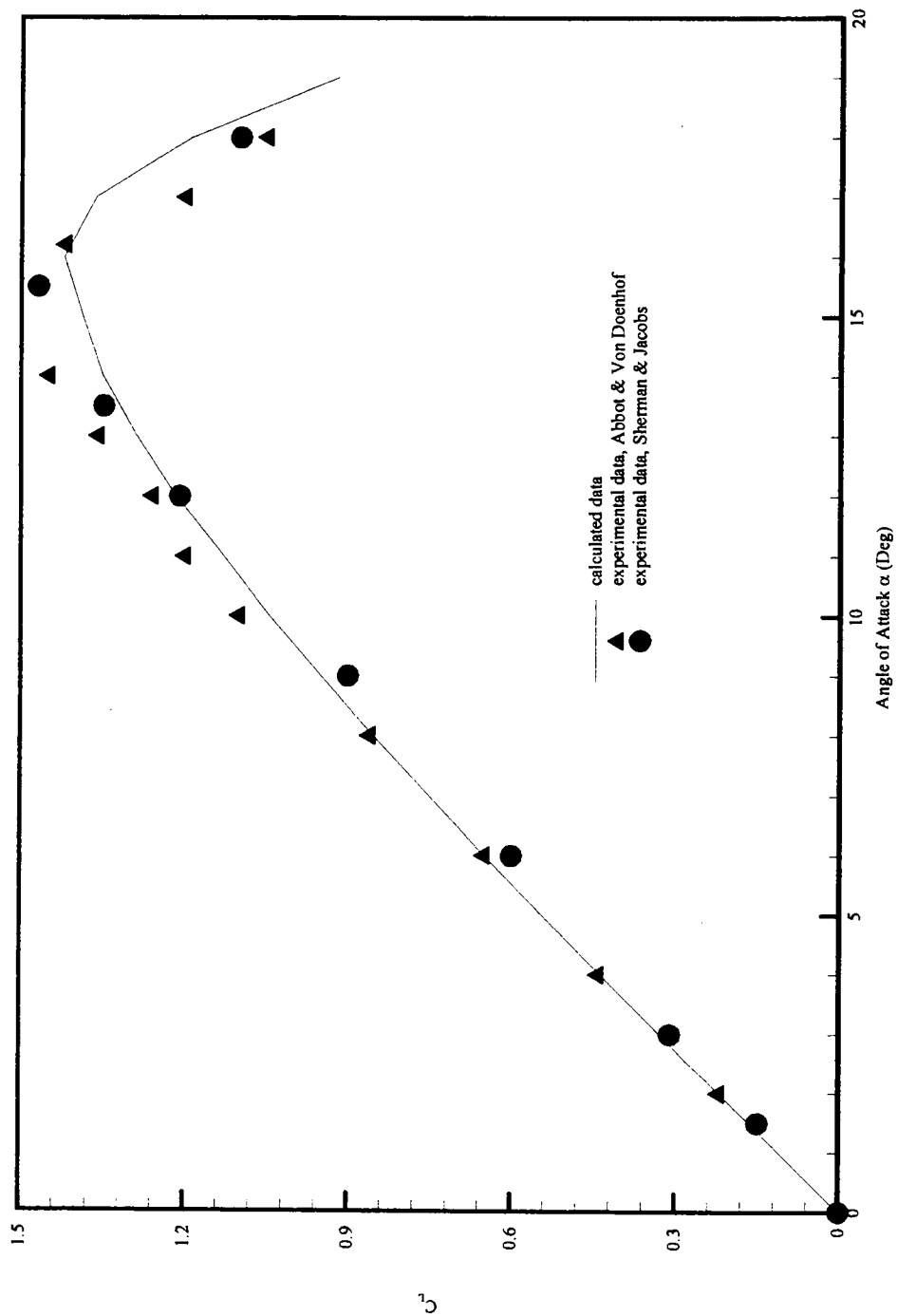


Figure 10: Comparison of Lift Coefficient for the NACA 0012 Airfoil, $Re = 6 \times 10^6$

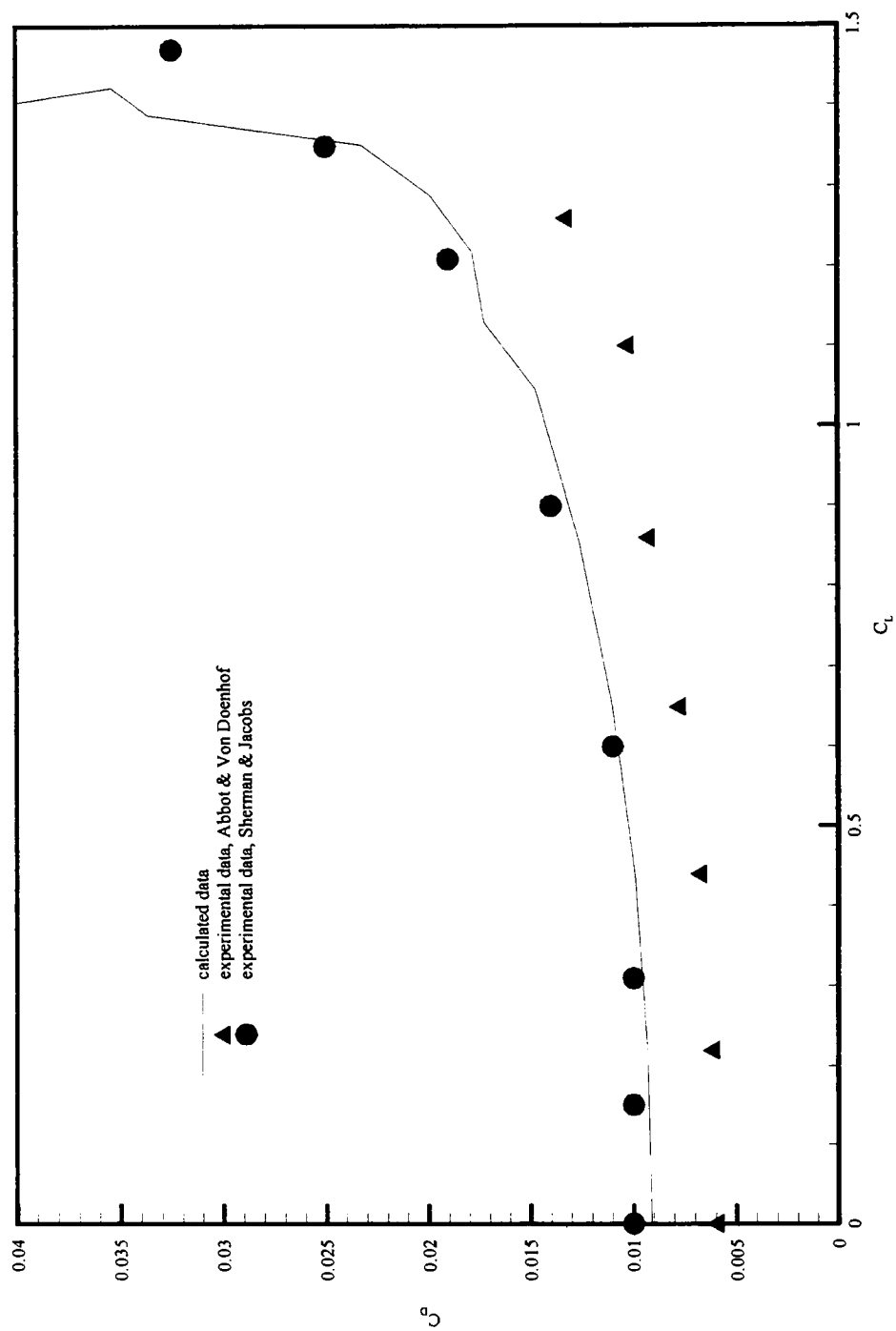


Figure 11: Comparison of Drag Coefficient for NACA 0012 Airfoil, $Re = 6 \times 10^6$

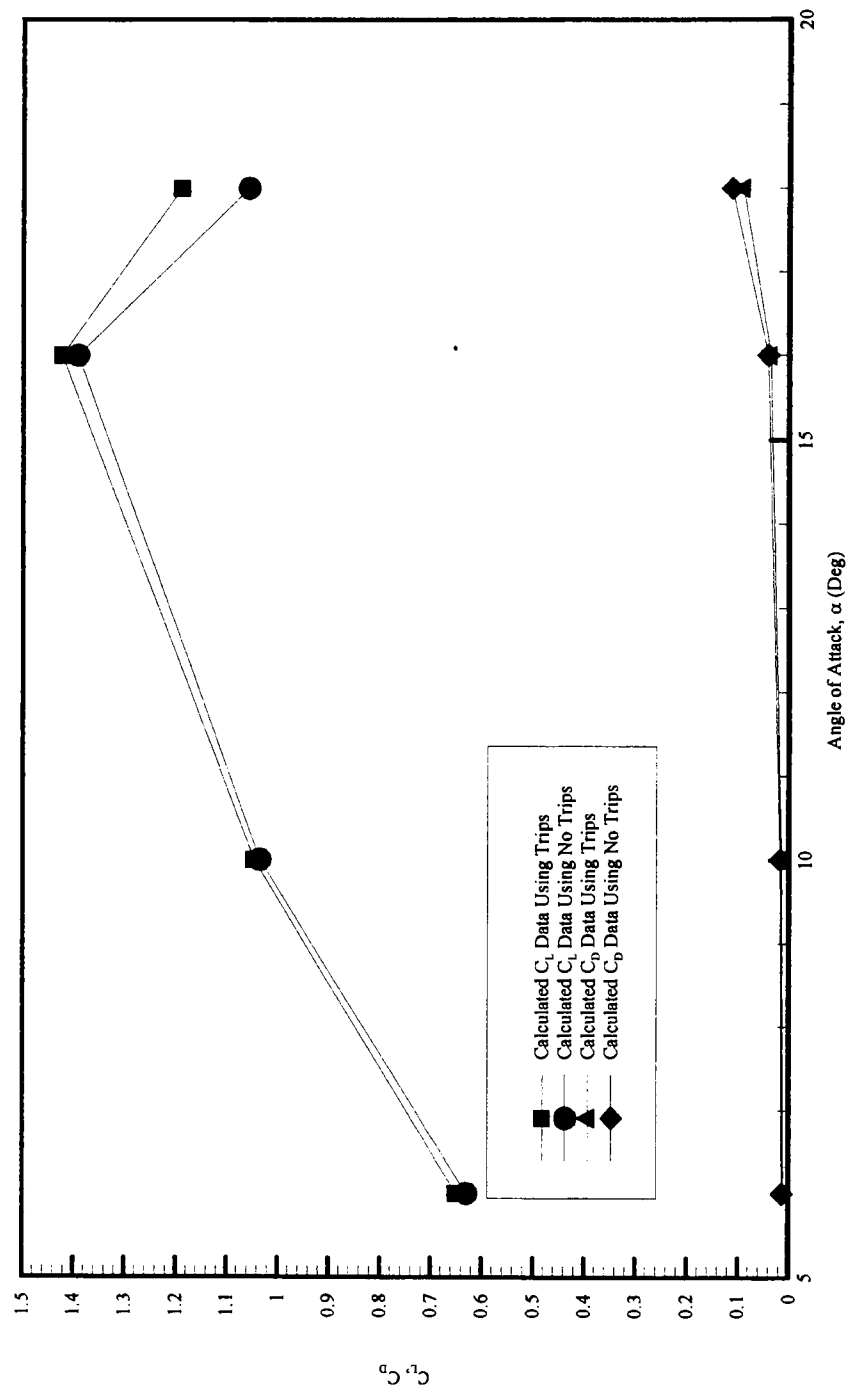


Figure 12: Variation of Force Coefficients Due to Turbulence Tripping

coefficient closer to the experimental data. However, this should not be misconstrued as evidence that the turbulence model can accurately predict transition.

An examination of grid density effects was also made using the NACA 0012 airfoil. A set of runs was made using the original 257×129 grid along with a fine grid of 513×257 points and a coarse grid of 129×65 points. At a Reynolds number of 6×10^6 the corresponding values of $y_{\max}^+$ are 3.5 and 8 for the fine and coarse grids, respectively. Two angles of attack (6° and 16°) were chosen for the calculations, and the timestep was held constant at $\Delta\tau = 0.004$ (non-dimensional) as the grid density was varied. The CFL number ranged from roughly 250 to 1000 as a result. The resulting lift and drag coefficients are plotted in Figure 13, where the ordinate is the number of grid points in a particular grid normalized by the number of points in the finest grid.

For the attached flows at 6° , the results are essentially grid-independent. The variations in force coefficients could be attributed to the error in the numerical integration used to compute the aerodynamic forces. For the 16° case, the results are clearly not grid-independent as the grid density falls below that of the medium grid. A further calculation at 18° using the coarse grid was made at this point, as a matter of interest. In this case not only were the force coefficients in error, but the flowfield exhibited massive unsteadiness, as shown in the coefficient histories in Figure 14.

A comparison of the surface vorticity for the 18° coarse and medium grid cases is shown in Figure 15. These results indicate that the flow separates much too early in the coarse grid case. Whereas the flow separates at approximately the airfoil quarter-chord in the medium grid case, the coarse grid calculation produces separation between 5% and

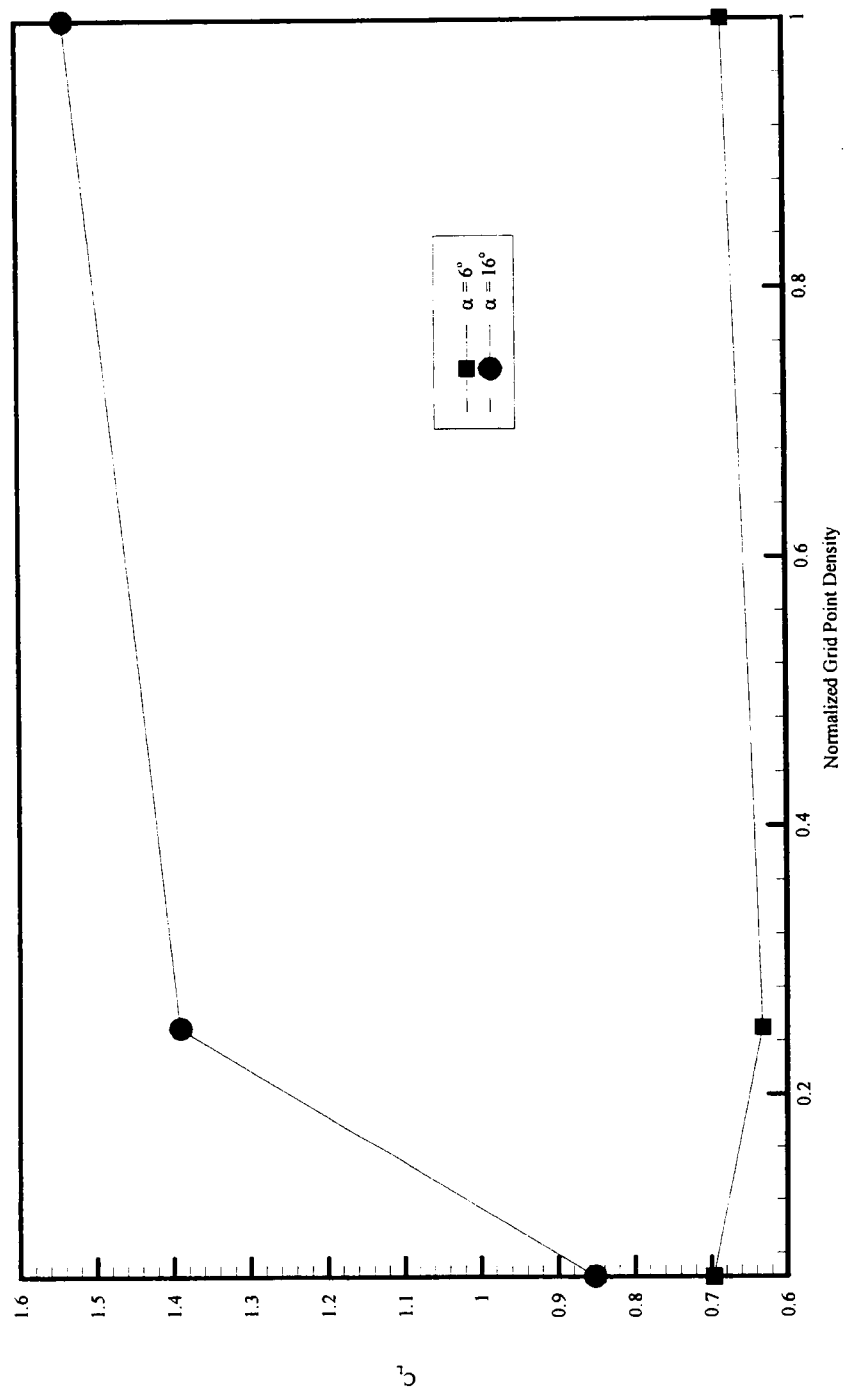


Figure 13: Variation of Force Coefficients with Grid Density

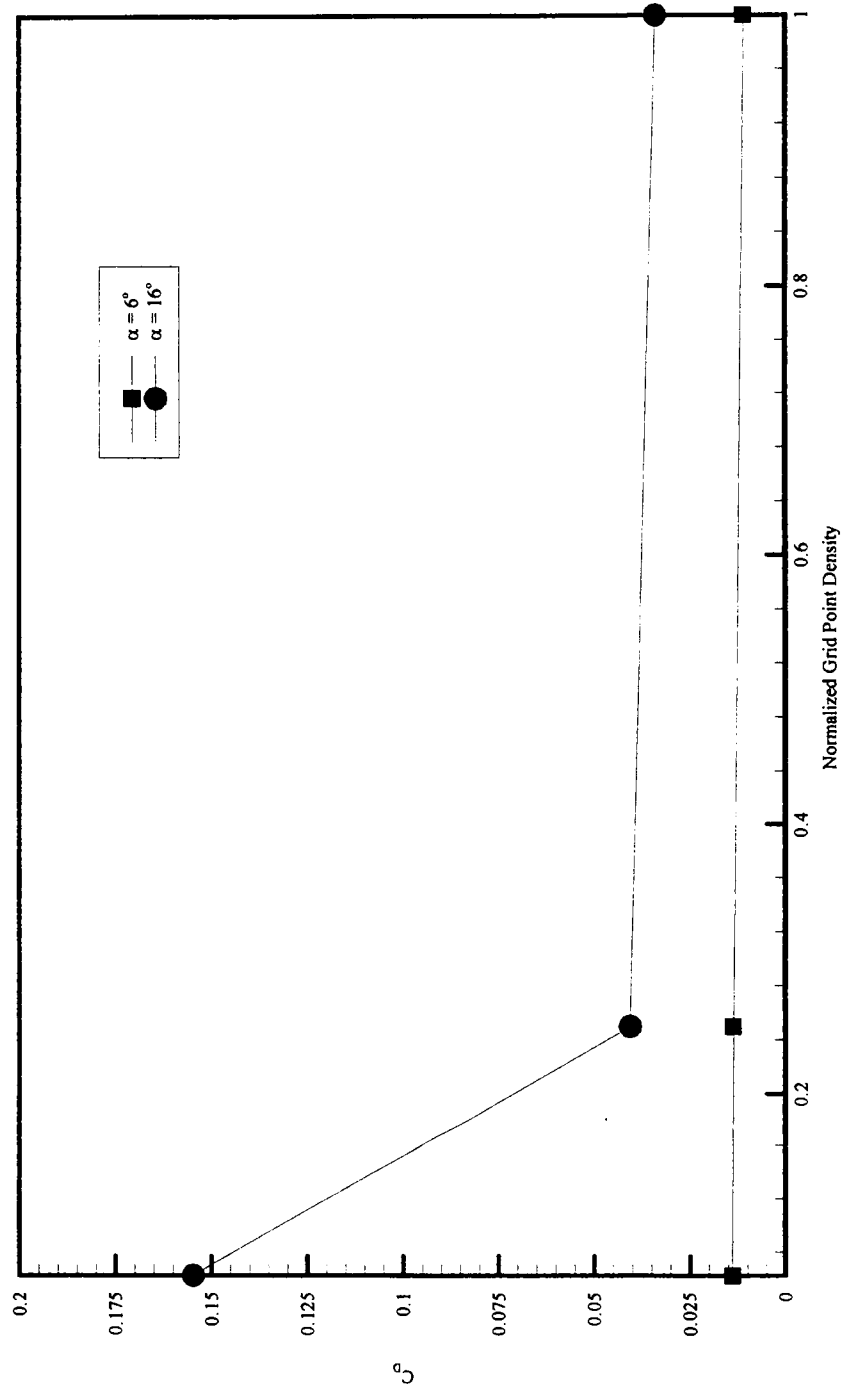


Figure 13: Variation of Force Coefficients with Grid Density (Concluded)

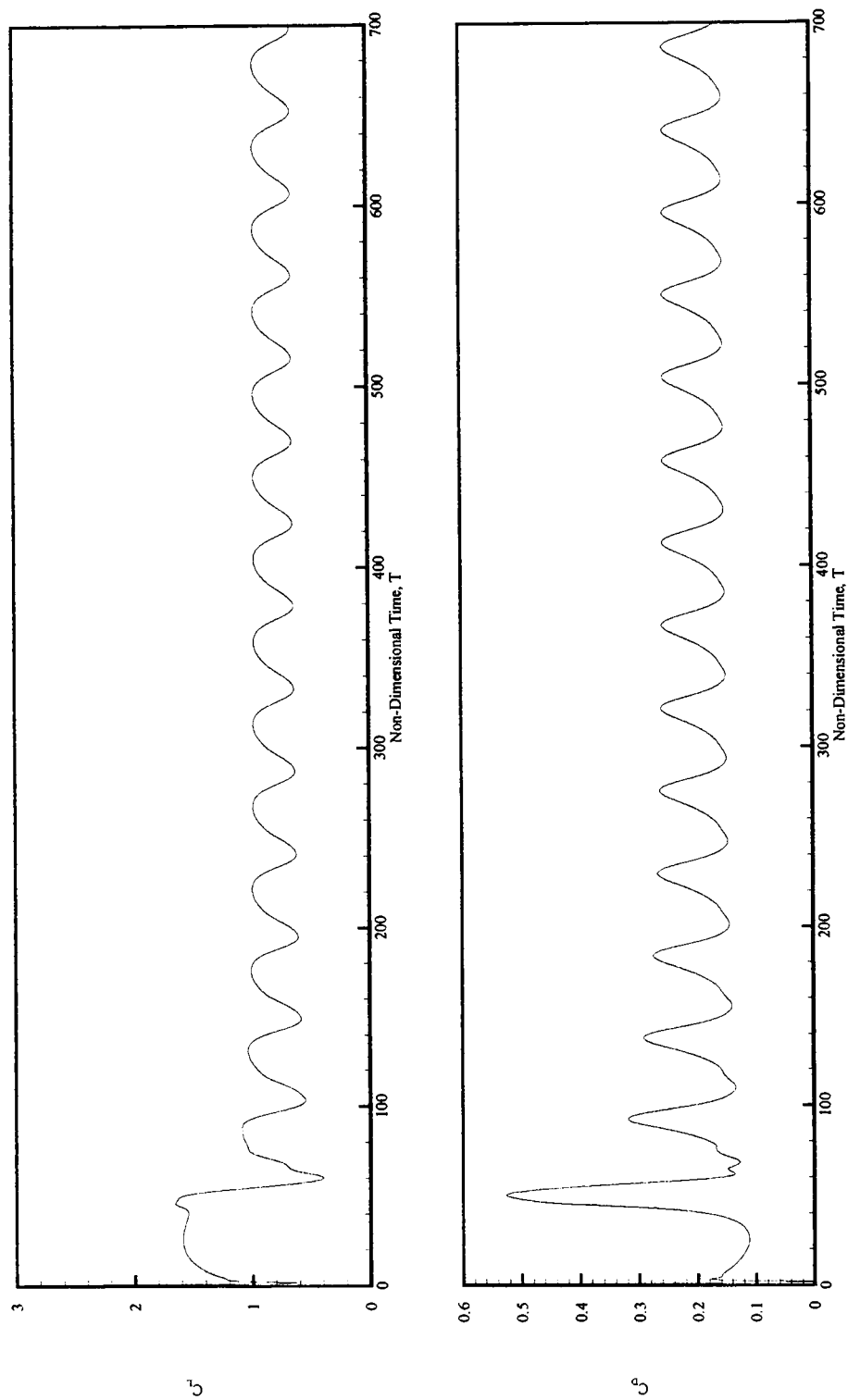


Figure 14: Force History for NACA 0012 at 18° , 129×65 Grid, $Re = 6 \times 10^6$

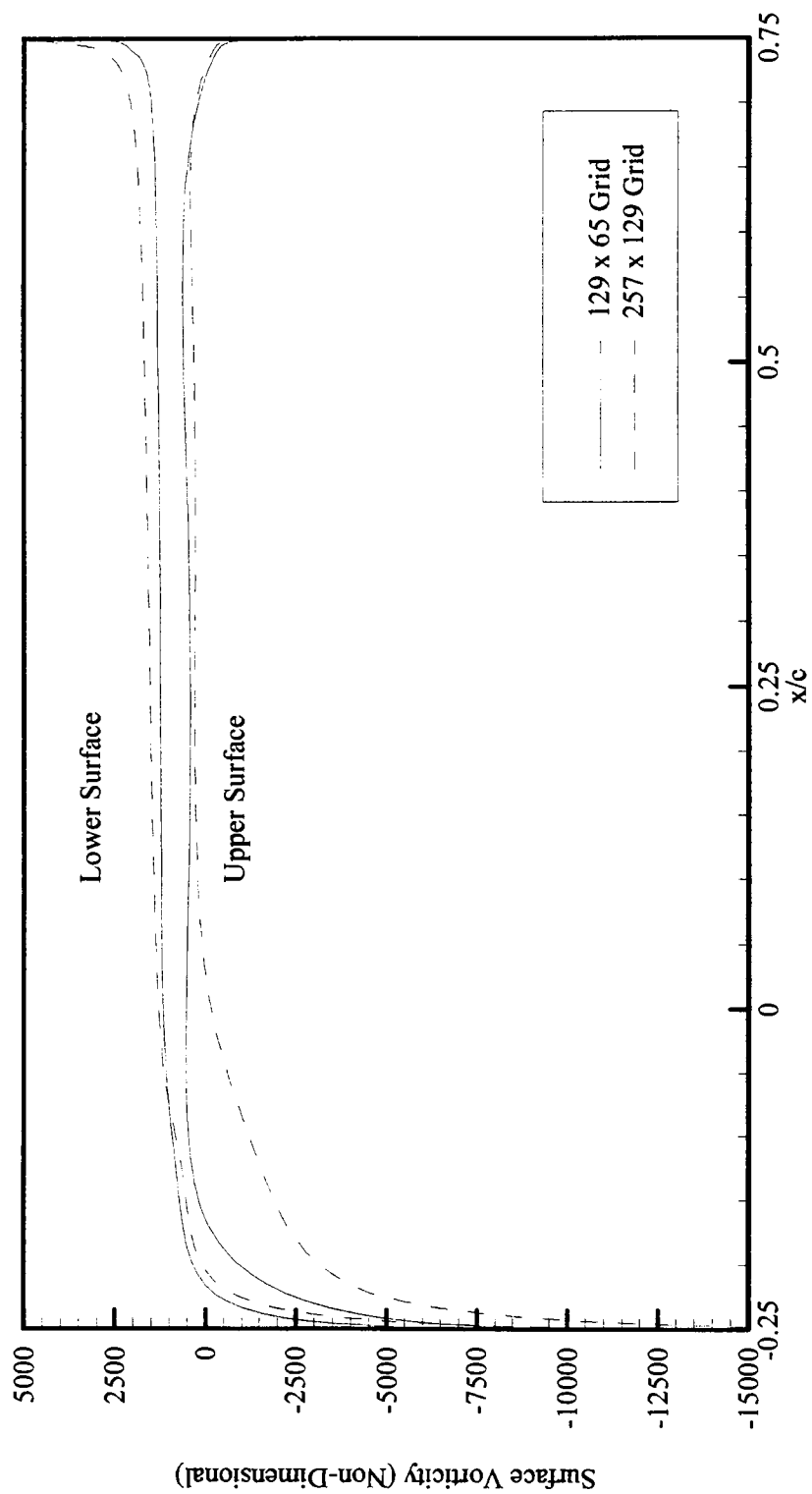


Figure 15: Surface Vorticity Comparison for NACA 0012 at 18° , $Re = 6 \times 10^6$

10% chord. The results are similar to what might be expected at a much lower (laminar) Reynolds number. This prompted another calculation using the coarse grid, but the artificial viscosity coefficients were reduced by a factor of four from the previous calculation. This change resulted in dramatically improved behavior in the force coefficients, as shown in Figure 16. The 16° coarse grid case was also run using the reduced values of artificial viscosity. The resulting lift and drag coefficients of 1.297 and 0.047 agree much better with the medium and fine grid results. However, the pressure contours show oscillations at the solid surface, indicating that the solution is not without some problems.

Thus, the coarse grid results can be brought in line with the results for the finer grids by adjusting the artificial viscosity coefficients. In fact, some grid-dependent function for the artificial viscosity could probably be found which would eliminate the variation in force coefficients across the grid family. However, this is not really an desirable practice. In fact, the sensitivity of the solution to these changes in the artificial viscosity coefficients for a given grid renders the entire solution questionable, if an arbitrary change has the ability to effect dramatic changes on the nature of a solution. This is especially unsettling since the artificial viscosity coefficients then become tuning constants. The solution procedure for an arbitrary case would have to include a detailed study of parametric changes in these constants. It is better to set these parameters to some reasonable values and recognize the limitation in grid density to ensure grid-independency. Of course, these values should be as low as possible without producing spurious oscillations in the surface pressure behavior.

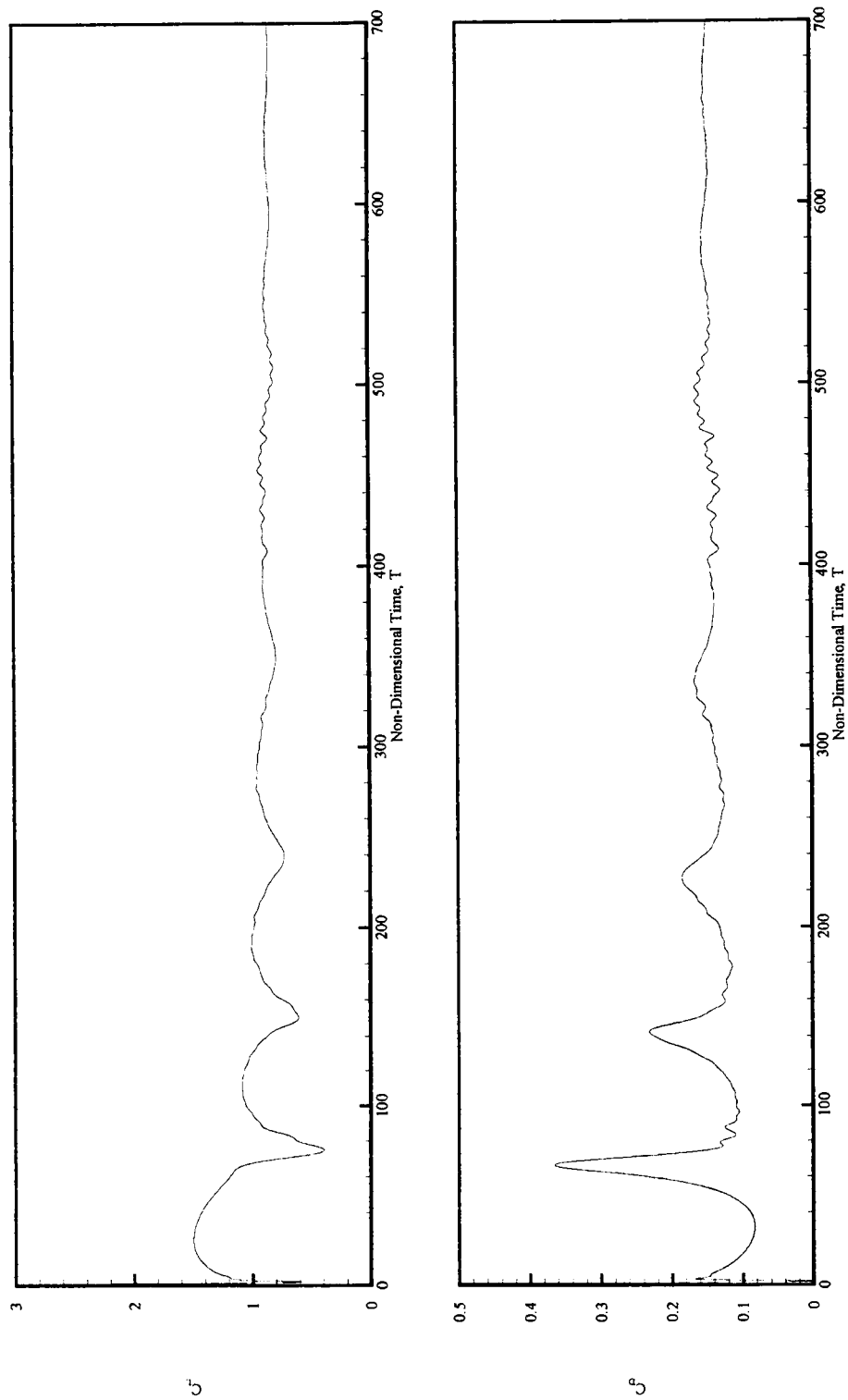


Figure 16: Force History for NACA 0012 at 18°, 129 x 65 Grid, Small Artificial Viscosity (AV)

The effect of varying the timestep size was also examined for the 6° and 16° cases described above, although the procedure was less formal in that a variety of timestep sizes were used. Table 2 contains the results for the 6° calculation. Results for the 16° case are also summarized in Table 2. The data do indicate some variation as $\Delta\tau$ is changed, but for a given grid density the results are reasonably consistent. In fact, changing the timestep does little to improve the 16° results, implying that the artificial viscosity and grid resolution seem to be the dominant factors affecting the quality of the results.

Table 2: Effect of Timestep Variation for Reynolds Number = 6×10^6

Angle of Attack	Grid Size	$\Delta\tau$	C_L	C_D
6°	129 x 65	0.0040	0.696	0.014
	129 x 65	0.0080	0.696	0.014
	257 x 129	0.0010	0.650	0.011
	257 x 129	0.0040	0.632	0.013
	257 x 129	0.0060	0.659	0.011
16°			$C_{Lexp} = 0.65$	$C_{Dexp} = 0.008$
	129 x 65	0.0040	0.852	0.155
	129 x 65	0.0080	1.000	0.113
	257 x 129	0.0008	1.423	0.035
	257 x 129	0.0040	1.392	0.041
	257 x 129	0.0080	1.360	0.040
			$C_{Lexp} = 1.41$	$C_{Dexp} = 0.028$

It is clear that for flows that involve little or no separation, the results are reasonably insensitive to variations in the grid density. However, in regions where flow separation is a dominant feature of the flow, more care must be taken to ensure that grid

resolution does not adversely affect solution quality. In other words, massively separated flows require finer grids than do flows with little or no separation.

This observation concerning grid resolution effects on separated flows is not altogether surprising, since a number of critical details in a separated flow can be lost or corrupted by a low resolution grid. One such quantity is the tangential pressure gradient on the surface. It controls the location of separation along a smooth surface, drives the creation of vorticity at a solid surface, and can have a strong impact on the development of turbulence. These factors in turn directly affect the structure and behavior of the wake, and the wake behavior can have a global effect on the flowfield. At high angles of attack, the pressure gradient after the suction peak can be quite strong, and can oscillate due to separation bubbles. Insufficient grid resolution will smear this gradient. This smearing can result in poor predictive ability by the flow solver.

A second, more subtle concern stems from the fact that large-scale separated flowfields can be quite unstable to small perturbations. In flow over a cylinder, Tryantafallou, et. al. [34] showed that the instability of the mean separated velocity profile is critical to the development of a vortex street and the frequency selection of the shedding. Oertel [35] performed a similar analysis for the flow separating from a finite-thickness flat plate. In particular, the presence of a small region of absolute instability is necessary in order for a large-scale instability to form. In order to obtain the correct unsteady behavior in a simulation, the algorithm needs to capture the velocity profile accurately, and not introduce spurious perturbations which would trigger a non-physical wake instability. Conversely, numerical damping of physical modes that should exist can

make a steady flow out of what should be an unsteady flow. A failure in either aspect, or both, will lead to non-physical results.

To complete the validation two unsteady cases were also considered. At very high angles of attack it is known that vortex shedding will take place from a NACA 0012 airfoil. To begin this part of the validation, flow over the NACA 0012 airfoil at a 45° angle of attack was computed. The flow parameters were selected to correspond to the experimental data of Critzos, et. al. [36]. Due to damping of the force balance, Critzos' report shows effectively time-averaged behavior, but includes lift and drag coefficients for a NACA 0012 airfoil in a $0^\circ - 360^\circ$ angle of attack range. The Reynolds number for the computation was 2×10^6 , and the same coarse-medium-fine grid set from the previous calculations was used, with the timestep fixed at 0.002. The Mach number was set to 0.2, and the artificial viscosity coefficients were held fixed at $(\epsilon_2, \epsilon_4) = (0.2, 0.04)$. At this Reynolds number the maximum values of y^+ for the grid set were 3.0, 0.8, and 0.5, respectively. A comparison of the mean lift and drag coefficients is given in Figure 17, and the data is summarized in Table 3.

Table 3: Effect of Spatial Resolution for Reynolds Number = 2×10^6 , $\alpha = 45^\circ$

Grid Size	C_{Lmean}	C_{Dmean}	St
513 x 257	1.487	1.508	0.148
257 x 129	1.327	1.363	0.148
129 x 65	0.879	0.891	na
	$C_{Lexp} = 1.20$	$C_{Dexp} = 1.24$	not available

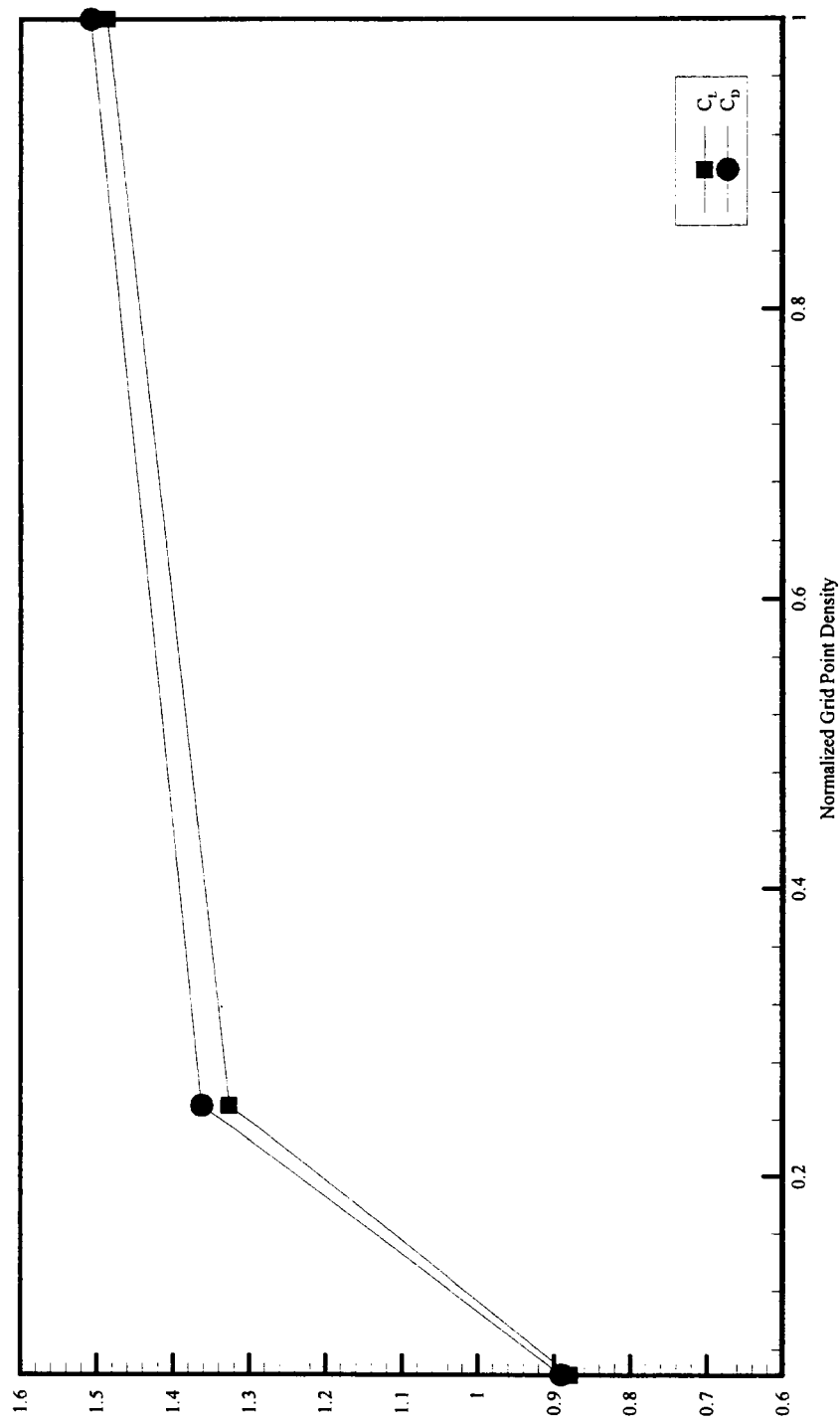


Figure 17: Variation of Force Coefficients with Grid Density for NACA 0012 at 45° , $Re = 2 \times 10^6$

Critzos, et. al. reported a value of approximately 1.2 for the lift coefficient, while the drag coefficient was given as roughly 1.24. Agreement with experiment is reasonable, except for the coarse grid calculation. The over-prediction of lift and drag at this angle of attack is probably related to the same effect that caused the over-prediction of suction pressure in the cylinder calculations discussed previously. More interesting results are presented in Figure 18, which shows the lift coefficient behavior for the three cases. The most striking feature is the diminishing of oscillatory behavior in the coarse grid computation. In the previous calculations coarsening of the grid induced oscillatory behavior through a (presumed) pseudo-laminar shift in the separation point. However, in the present case all oscillatory behavior has been suppressed due to coarsening of the grid with the artificial viscosity held fixed. A comparison of the surface vorticity in the leading edge region is shown in Figure 19. It indicates that the change in location of the separation point is much smaller than in the lower angle of attack calculations. This is not surprising, since the separation point will be very close to the leading edge at a high angle of attack, and there is correspondingly less room for the separation point to travel. The implication is that the near-field, mean velocity profiles should be similar for the varying grid densities. Regarding the oscillation suppression, the supposition is that the high frequency, low level perturbations required to initiate the wake instability are being damped out. This assumption is supported by the results shown in Figure 20. Here, the coarse grid case has been recomputed with artificial viscosity coefficients reduced by a factor of four. The lift coefficient now shows a nicely oscillatory behavior in agreement

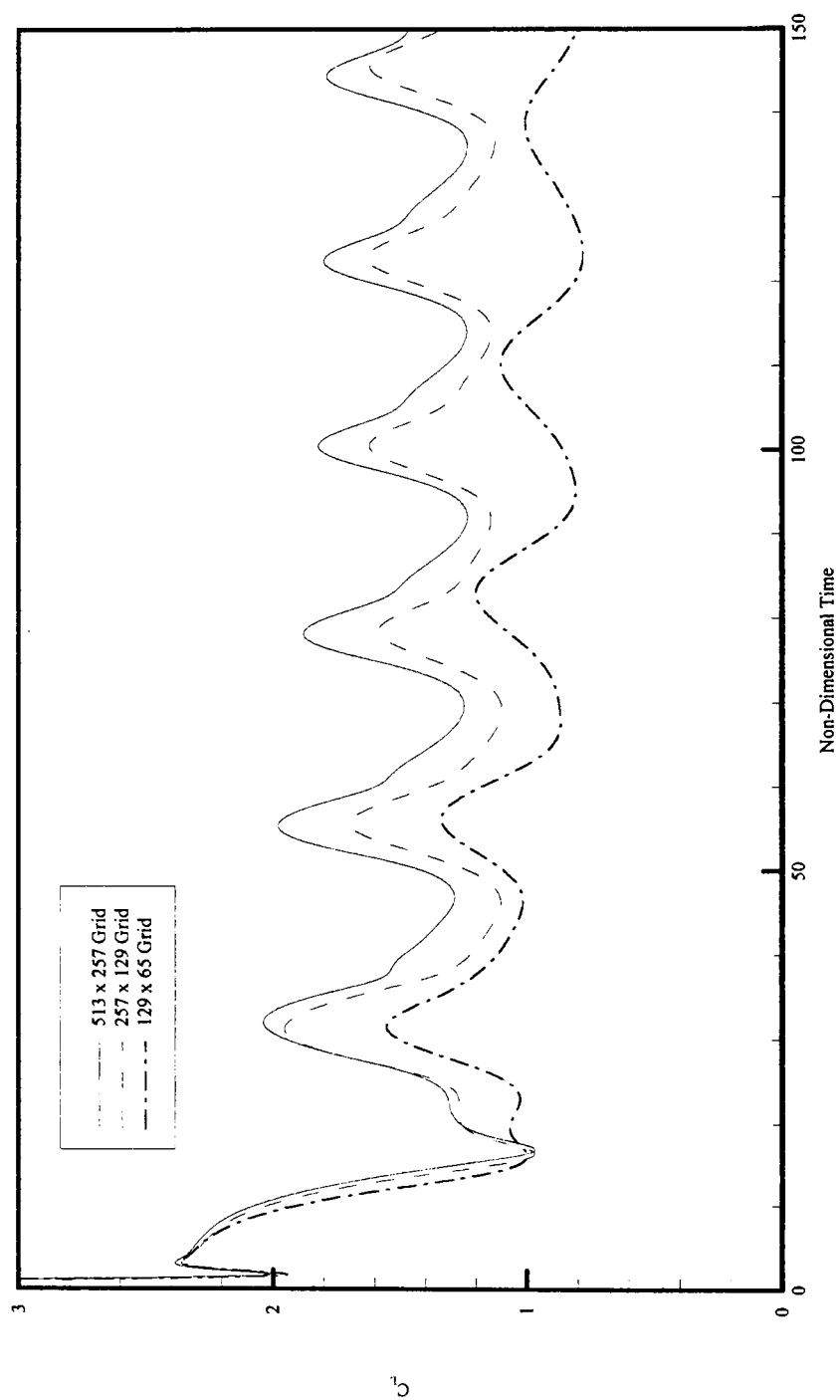


Figure 18: Lift Coefficient Comparison for NACA 0012 at 45° , $Re = 2 \times 10^6$

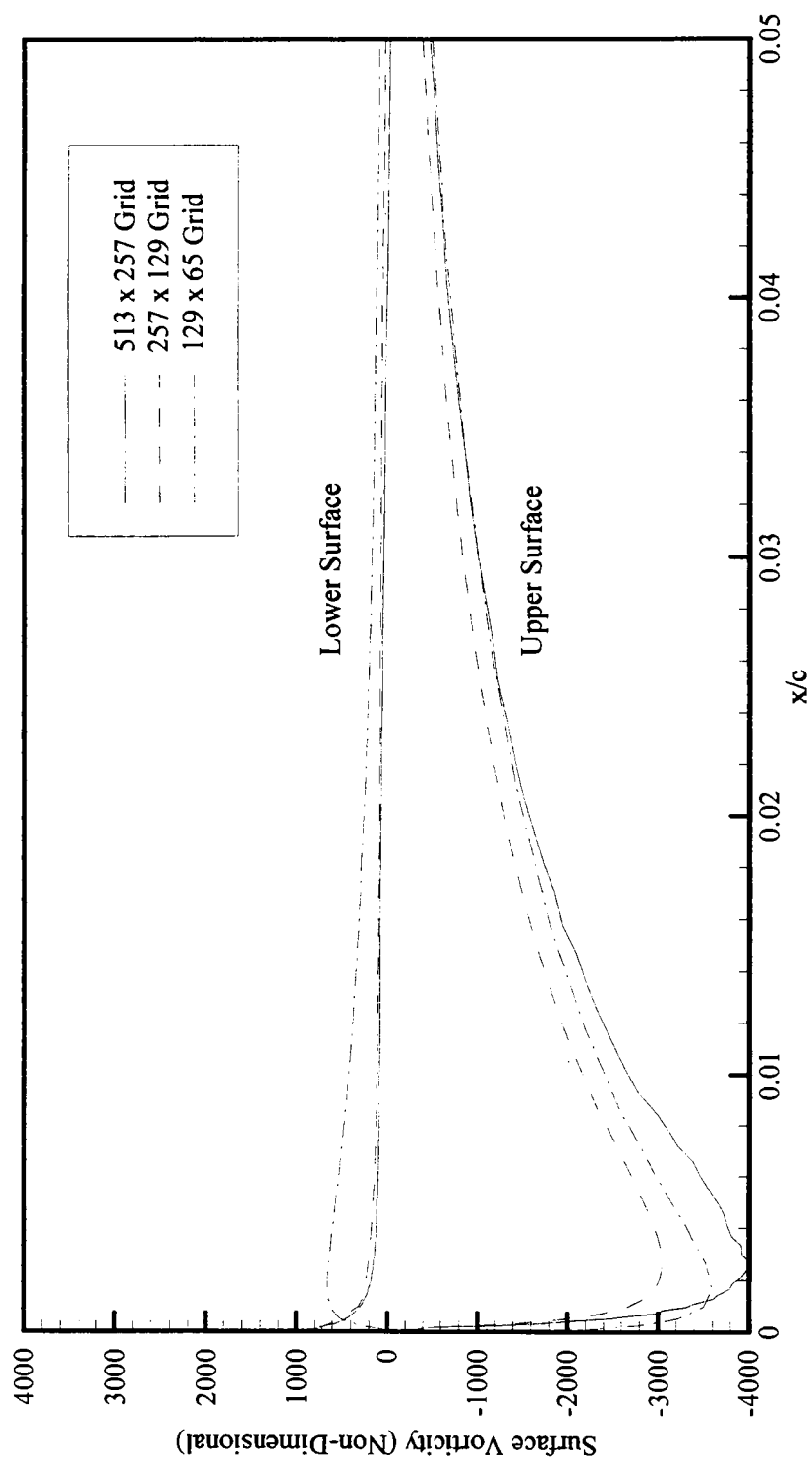


Figure 19: Surface Vorticity Comparison for NACA 0012 at 45° , $Re = 2 \times 10^6$

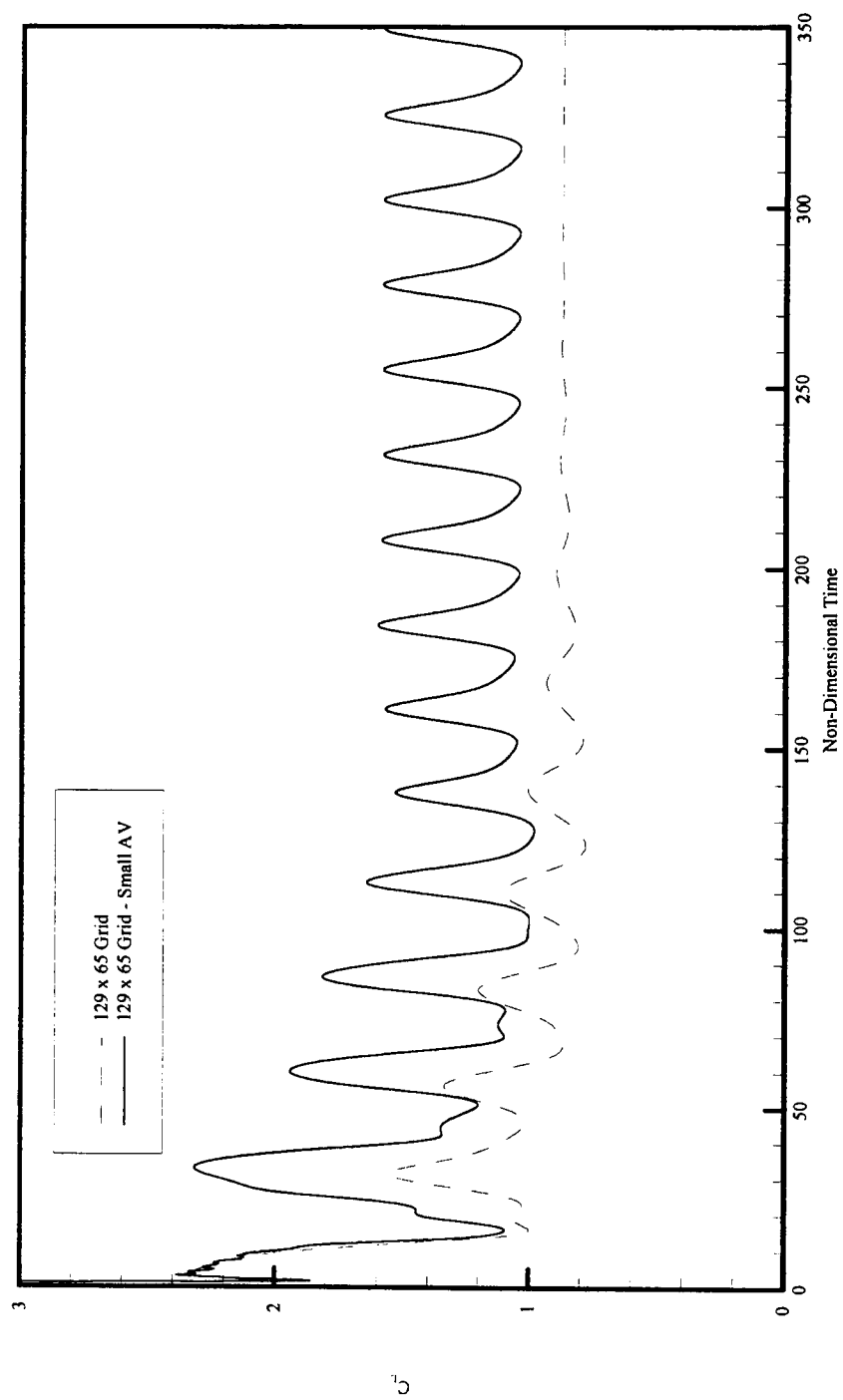


Figure 20: Effect of Reducing Artificial Viscosity on Lift Coefficient for NACA 0012 at 45°

with the results from the other grids, although once again pressure oscillations were found at the solid surface.

The medium grid case was also run at a timestep $\Delta\tau = 0.0008$. A comparison of the lift histories for the two medium grid cases is given in Figure 21. Some small variation is present, but the results are essentially independent of the timestep size.

Based on earlier works, a set of calculations using a NACA 0012 at an 18° angle of attack was also performed. A Reynolds number of 5×10^5 was selected, and the Mach number was set to 0.3. Again the fine-medium-coarse set of grids was used, and the artificial viscosity coefficients were held constant at $(\epsilon_2, \epsilon_4) = (0.2, 0.04)$. A timestep of 0.002 was used for this set of computations. The choice of this Reynolds number and angle of attack is significant, because a small body of experimental data exists which indicates that the developed wake instability occurs at a much lower frequency than the Strouhal frequency in a neighborhood slightly past the onset of stall. Zaman, et. al. [5] performed a thorough study of low Reynolds number flow over airfoil shapes in which anomalously low shedding frequencies were found. In a second study (Zaman, McKinzie, and Rumsey [37]) the effects of various external sources of forcing were examined, and it was determined that the low frequency was caused by a self-induced flow instability. This instability was the result of the transitional state of the boundary layer at separation. Rumsey [38] performed calculations of the flow over a NACA 0012 at a Reynolds number of 1×10^6 and an 18° angle of attack and also detected low frequency oscillations. As far back as the 1930's, Jones [39] was describing his observations of similar low frequency

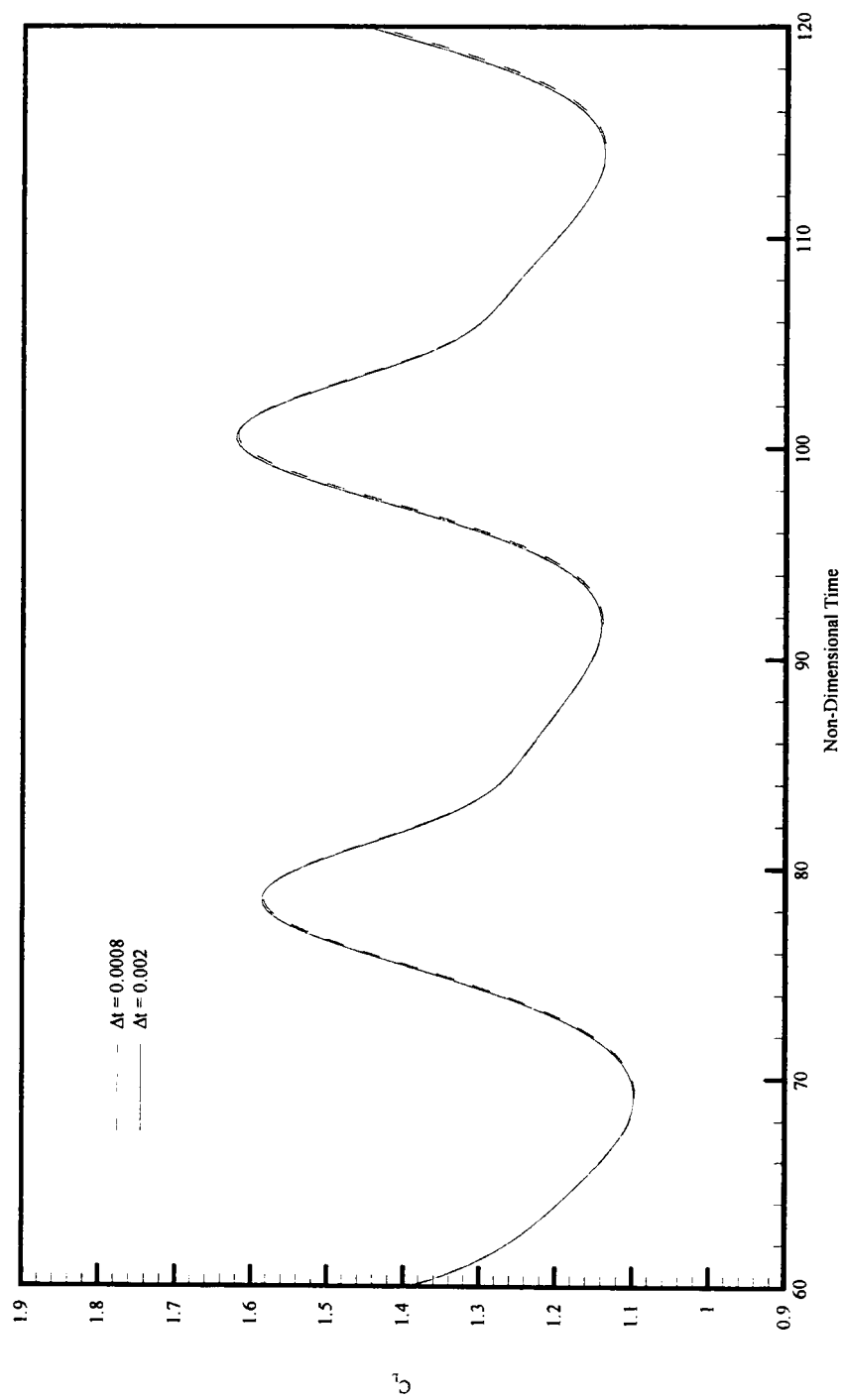


Figure 21: Comparison of Lift Coefficient Due to Timestep Variation, $\alpha = 45^\circ$, $Re = 2 \times 10^6$

shedding behavior. The experimental data of Critzos, et. al. also contain results at a Reynolds number of 5×10^5 for comparison.

The lift coefficient histories are shown in Figure 22. Figure 23 presents a comparison of the mean lift and drag coefficients. The data in Figure 23 is summarized in Table 4. The coarse grid solution is oscillatory, with mean force coefficients that are not unreasonable. However, the Strouhal number is larger than the value of 0.022 suggested by the data of Zaman, et. al. [37]. The medium and fine grid solutions yield Strouhal numbers in good agreement with the findings of Zaman, et. al., and also with computations of a similar nature performed by Rumsey as a part of the same study. The mean force coefficients also agree nominally with the data of Critzos, who reported values of approximately 0.67 and 0.24 for the lift and drag, respectively.

Table 4: Effect of Spatial Resolution for Reynolds Number = 5×10^5 , $\alpha = 18^\circ$

Grid Size	C_{Lmean}	C_{Dmean}	St
513 x 257	0.711	0.235	0.022
257 x 129	0.695	0.210	0.023
129 x 65	0.700	0.225	0.034
	$C_{Lexp} = 0.67$	$C_{Dexp} = 0.24$	$St_{exp} \sim 0.022$

The upward frequency shift in the coarse grid calculations is similar to the behavior seen in the higher Reynolds number solutions at this angle of attack. This implies that the base velocity profile is incorrect for the coarse grid solution, leading to unphysical stability characteristics in the flowfield. The effect of changing the artificial viscosity coefficients

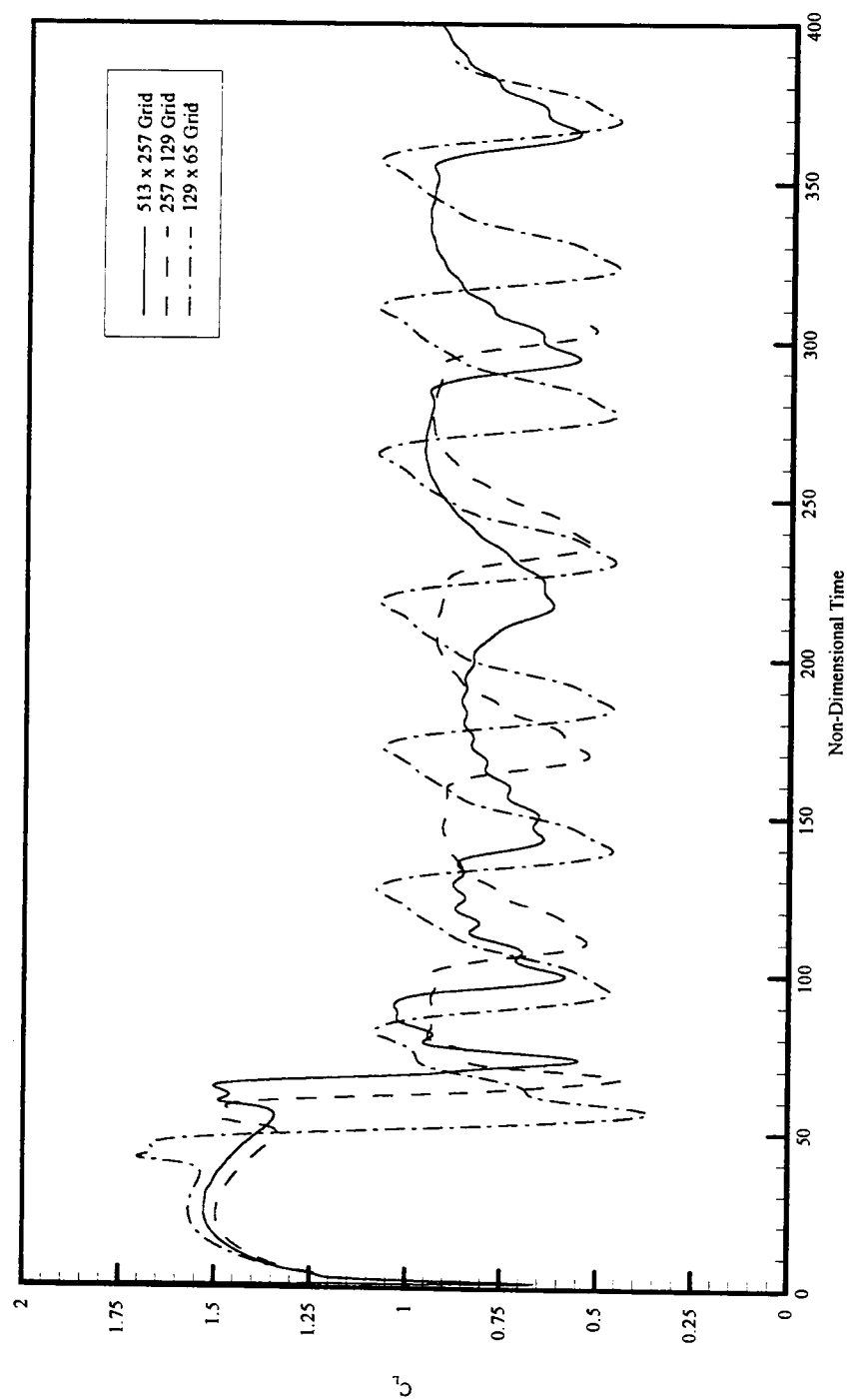


Figure 22: Lift Coefficient Comparison for NACA 0012 at 18° , $Re = 5 \times 10^5$

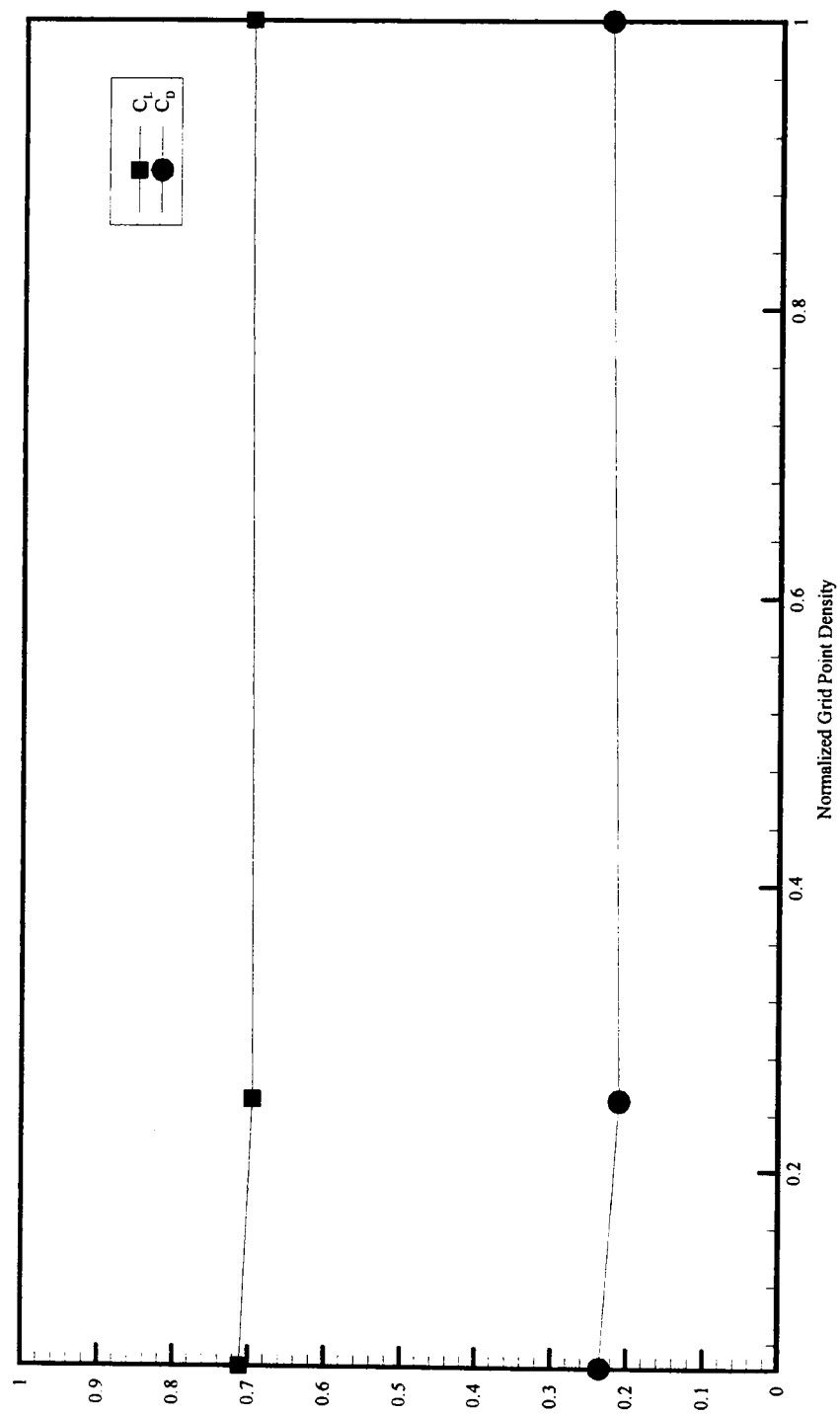


Figure 23: Variation of Mean Force Coefficients with Grid Density for NACA 0012 at 18° , $Re = 5 \times 10^5$

to (0.05,0.01) is shown in Figure 24. The Strouhal number for this run is 0.025, which is much closer to the (assumed) correct values.

Variations in the timestep size were also considered for this configuration. The results are collected in Table 5, and show slight variations in forces and frequency for a fixed grid density, even for the largest $\Delta\tau$ considered. This indicates that the physics of the flow appear to be well-simulated if the grid resolution is sufficient.

Table 5: Effect of Timestep Variation for Reynolds Number = 5×10^5 , $\alpha = 18^\circ$

Grid Size	$\Delta\tau$	C_{Lmean}	C_{Dmean}	St
129 x 65	0.002	0.799	0.224	0.034
129 x 65	0.006	0.801	0.220	0.037
129 x 65	0.016	0.746	0.225	0.032
257 x 129	0.002	0.792	0.191	0.023
257 x 129	0.008	0.777	0.200	0.020
		$C_{Lexp} = 0.67$	$C_{Dexp} = 0.24$	$St_{exp} \sim 0.022$

All of the results obtained above, along with data from other sources, can be represented in a flow map for the NACA 0012. Such a map is shown in Figure 25. The region descriptions and boundaries are tentative. However, this figure serves to illustrate how complex the flow (and its calculation) over even a simple geometry can be.

An additional caveat should be noted here. All these calculations are two-dimensional. As is well-known, turbulence can introduce significant three-dimensionality into a flow. Thus, a map such as that in Figure 25 must be considered in light of such a

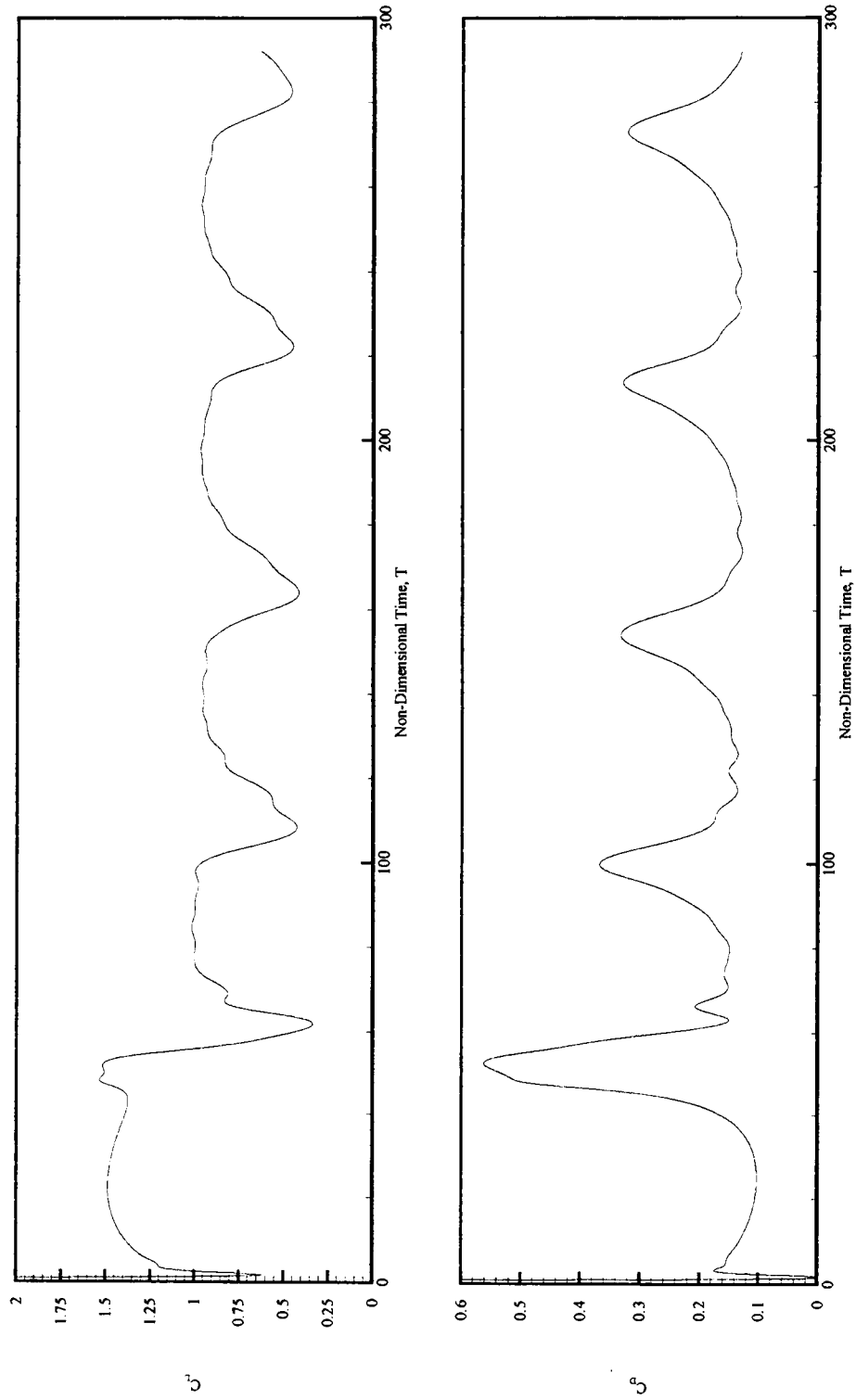


Figure 24: Force History for NACA 0012 at 18° , $Re = 5 \times 10^5$, 129×65 Grid - Small AV

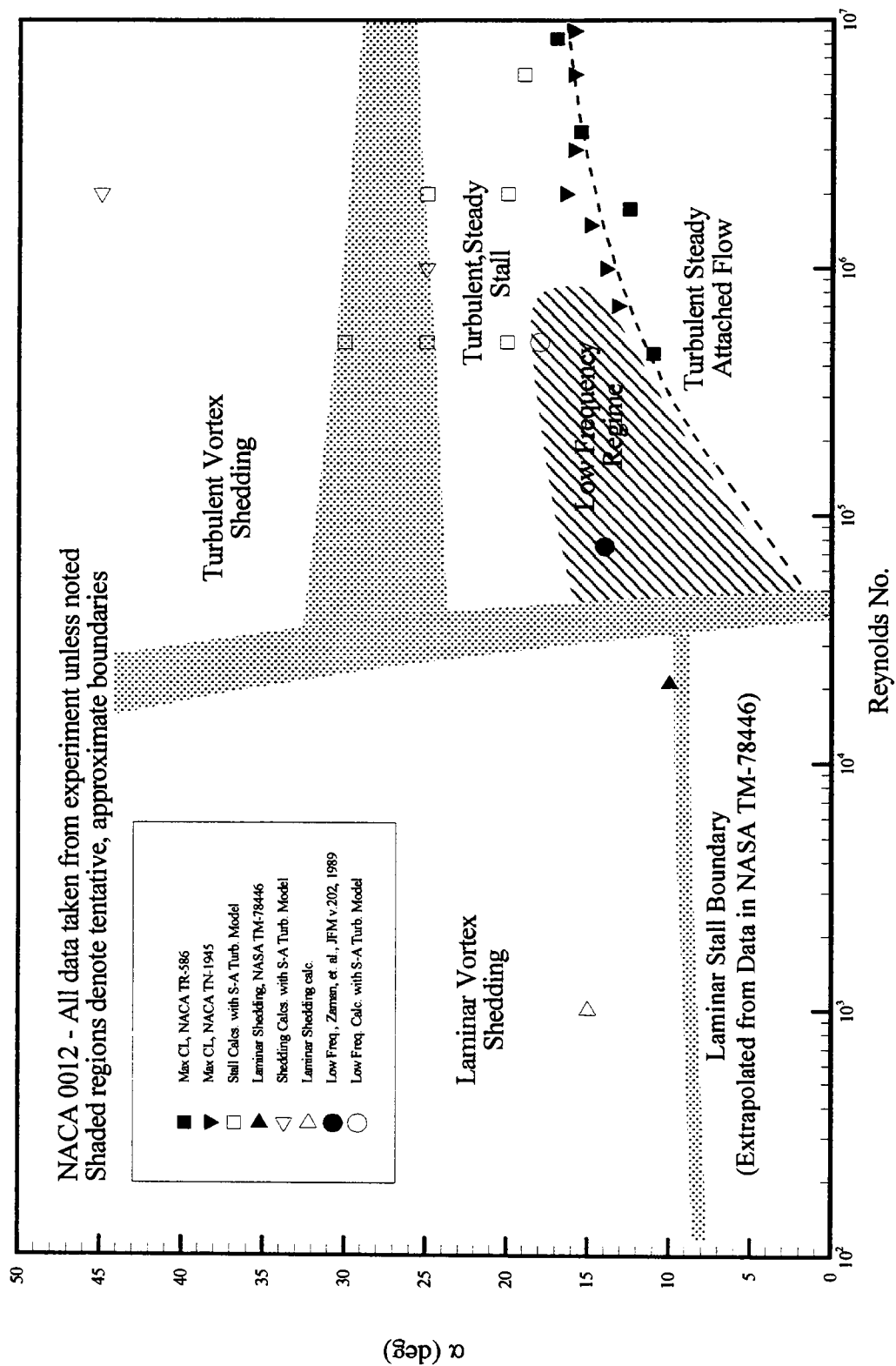


Figure 25: Tentative Low-Speed Flow Map for NACA 0012 Airfoil

limitation. Although grid refinement and timestep tests can establish the numerical validity of a solution, physical validity can only be founded in a solid connection to empirical data.

This rather lengthy validation effort has shown that the algorithm works well in simulating steady, attached and separated flows, and massively separated, unsteady flows. With suitable grid fineness, the unsteady behavior predicted by the algorithm seems to be a fair approximation of the flow physics represented by this level of modeling, and is independent of the various numerical parameters. Although this effort has not been exhaustive, the findings can serve as a guideline in determining grids and bounds on the artificial viscosity for the calculations described in the remainder of this work.

CHAPTER IV

RESULTS

Flow Over a Backward-Facing NACA 0025 Airfoil

The focus of this study is the analysis of mechanically-excited lift augmentation. To this end two cases involving such forcing were examined. The first case was taken from the experimental work of M. D. Zhou and H. H. Fernholz, as cited in [9]. Zhou, et. al. considered the flow over a NACA 0025 airfoil that was rotated such that the sharp edge became the effective leading edge, with an angle of attack of 27° . In this way, the separation line was well-defined. A small flap placed near the effective leading edge was oscillated at various frequencies. With forcing amplitudes as low as 1.3° at a test Reynolds number of 6.6×10^5 , an increase in the lift of as much as 60-70% was reported. These results showed that in a time-averaged sense a vortex was “captured” over the airfoil when the forcing frequency was of the same order of magnitude as the convective frequency scale, given by U_∞/L .

The airfoil section in the empirical study had an aspect ratio of 1, and was centrally mounted in the wind tunnel. The data was obtained in the 2m x 1.4m (W x H) wind tunnel at the Hermann-Foettinger Institute, Technical University Berlin. This is a low-speed wind tunnel with a solid-wall test section, according to a personal communication with Prof. Zhou. Nominally the tunnel blockage was 8.9% for zero angle of attack. However, at the 27° airfoil angle of attack used, the effective blockage was probably closer to 16%. End-plates were used to suppress the development of three-dimensional flow behavior.

In the hope of gaining insight into the physical mechanisms at work in this experiment, a set of computations following the experimental configuration was performed. A baseline flow was computed. Once the startup transients disappeared and oscillatory behavior had been established, the flap motion was initiated according to a prescribed behavior given by

$$\Delta\theta = \Theta_A \sin 2\pi f(t - T_i) \quad (20)$$

Here Θ_A is a user-defined amplitude, f is the forcing frequency, and T_i is the time that the flap motion begins.

A two-dimensional calculation was used in this study, since the end-plates should have suppressed the development of three-dimensional flow. It is recognized that three-dimensionality of the flow could have been strong. The initial calculation was performed on a two-grid set where the O-grid wrapped around the airfoil possessed 513 points in the circumferential direction and 257 points in the transverse direction. The flap grid was composed of 201 x 169 points. Figure 26 shows the grids with the holes created by the overset grid algorithm. As can be seen, the flap is quite small, having a length equal to 2.2% of the airfoil chord. The maximum value of y^+ was 0.7 for both the airfoil and flap. A non-dimensional timestep of 0.005 ($CFL = 500$) was used for the computation. This calculation will be referred to as Case 1.

Baseline results, i.e. with no forcing, for Case 1 are shown in Figures 27 and 28. Figure 27 gives the lift and drag histories. The flow is clearly unsteady, with a Strouhal number of approximately 0.1. The mean lift coefficient is 0.6. Zhou, et. al. reported a mean lift coefficient of roughly 0.7, and the experimental Strouhal number was reported as

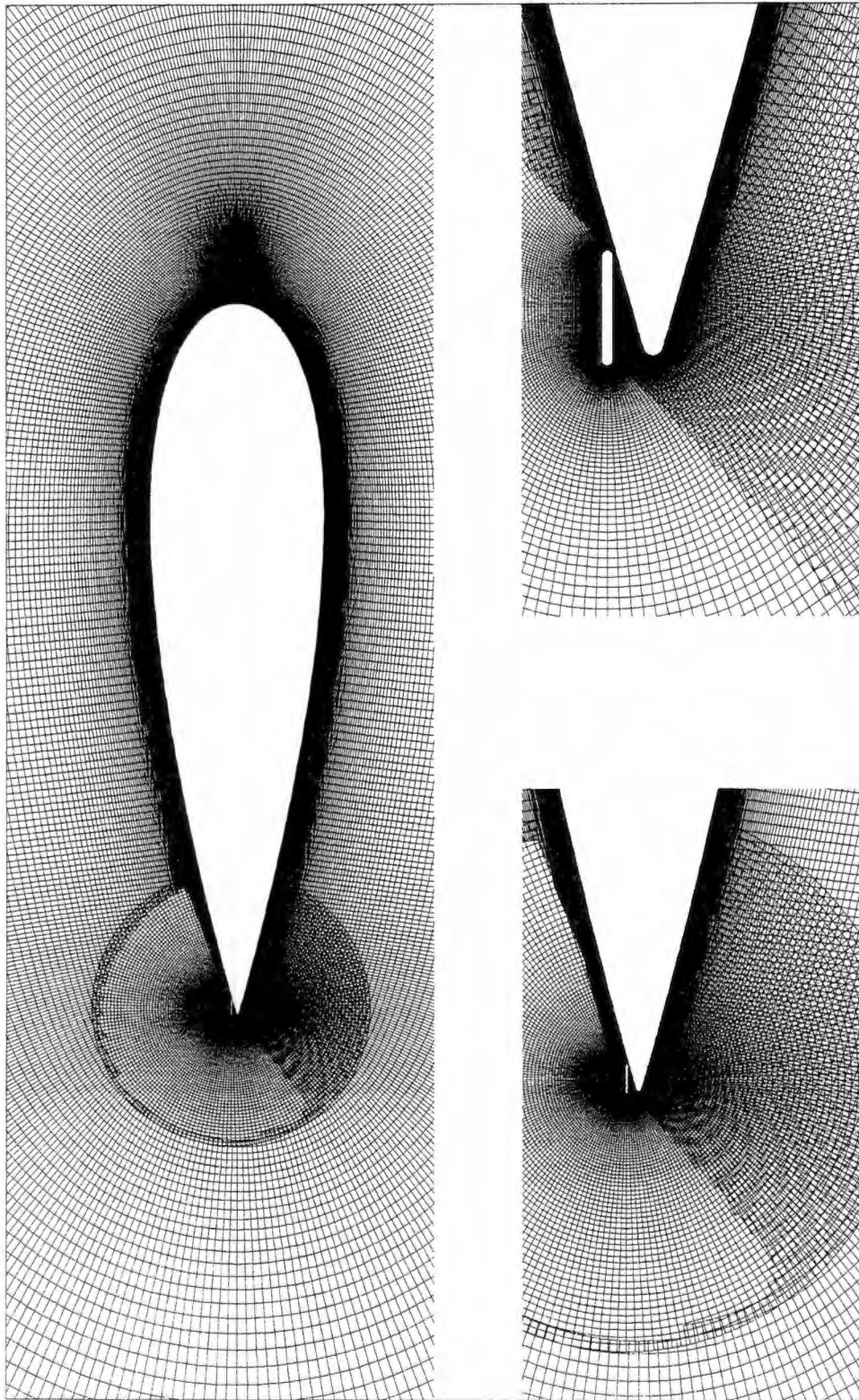


Figure 26: 513 x 257, 201 x 169 Grid Set Used in Computing Flow in Zhou Case 1

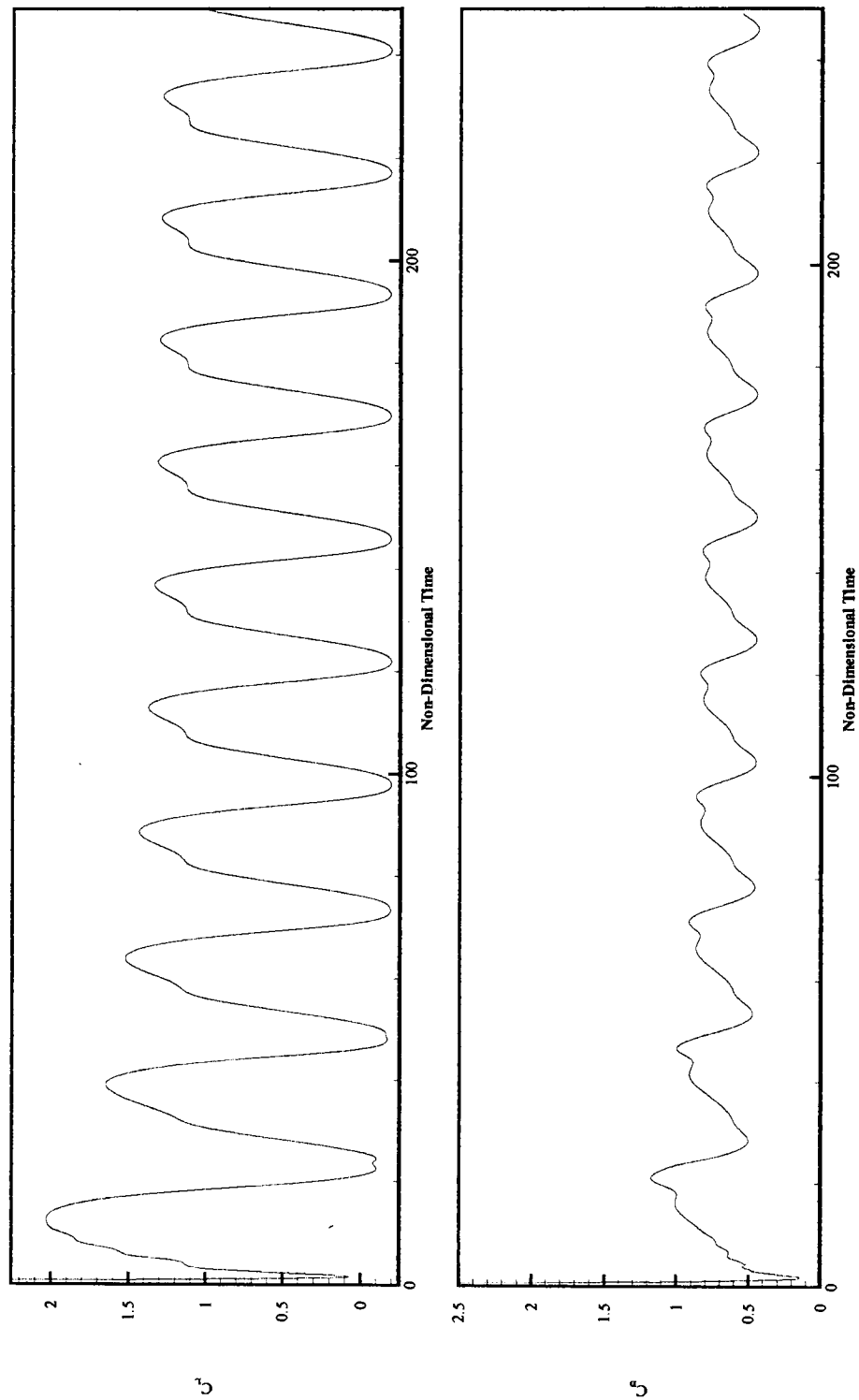


Figure 27: Force Coefficient History for Zhou Case 1, $Re = 6.6 \times 10^5$

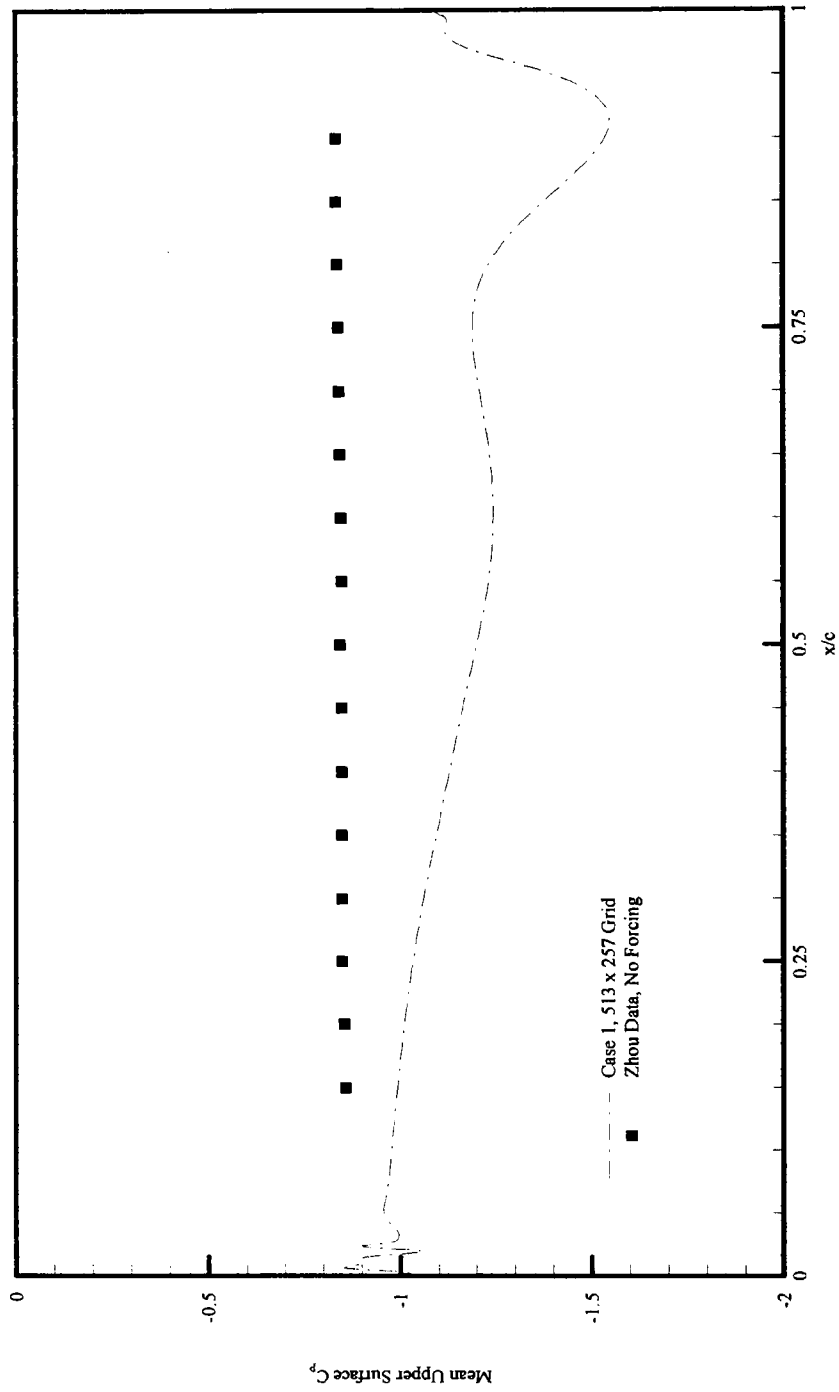


Figure 28: Mean Surface Pressure Coefficient Comparison, Zhou Case 1, No Forcing

0.23. For further comparison, the mean pressure coefficient on the upper surface of the airfoil is shown in Figure 28.

In order to minimize the amount of computer processor time required to calculate the solution in the presence of a moving flap, a second case, hereafter referred to as Case 2, was developed in which the airfoil grid density was reduced to 301×201 points. For this grid the spacing off the body was set to the same value used in the finer grid, so the maximum y^+ value was essentially unchanged. The maximum CFL number was held fixed at approximately 500, so the effective timestep was increased. All other parameters were held fixed. The results for this run are given in Figures 29 and 30. The comparison of force coefficient histories in Figure 29 indicates that the agreement between solutions of Cases 1 and 2 is quantitatively excellent. The computed mean pressure coefficients are in good agreement also, as seen in Figure 30.

The variation between calculation and experiment is not insignificant. One possible source of the discrepancies is the interference of the tunnel walls. Zhou did not provide any information on correction factors or the level of tunnel interference, but an estimate may be determined by recomputing the flow over the airfoil/flap combination in the presence of solid walls. This was done using the grid set from Case 2 and a tunnel grid, shown in Figure 31. The tunnel grid was extended for five chords upstream and downstream of the airfoil. It can be seen that the airfoil represents a significant obstacle to the tunnel flow. For this calculation the flow solver was slightly modified to include appropriate boundary conditions at the tunnel inlet and exit. At the inlet, the momentum and temperature were held fixed and the pressure was extrapolated from the interior using

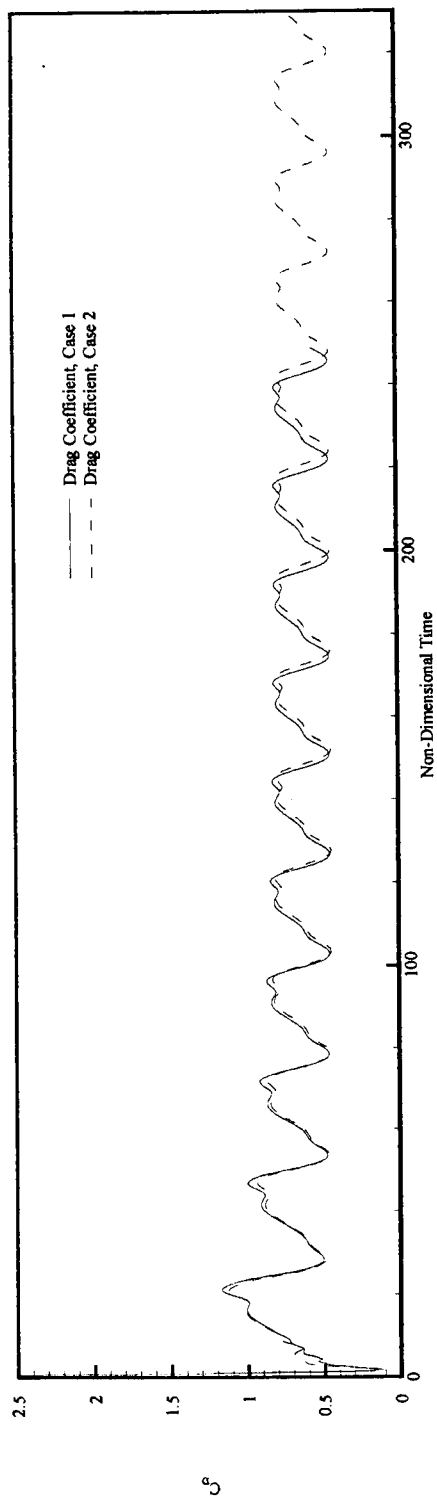
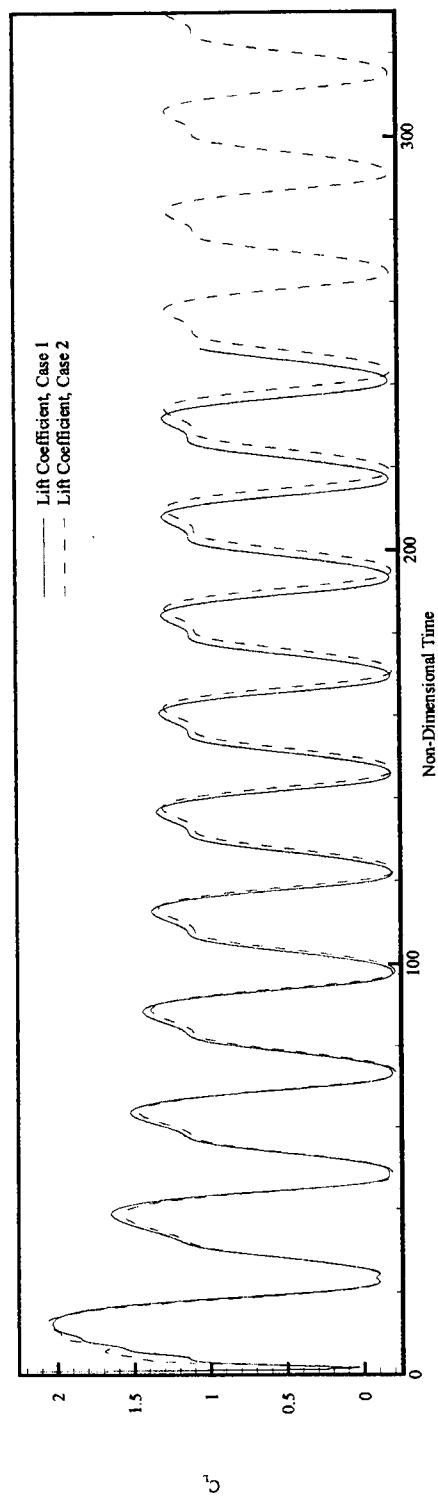


Figure 29: Force Coefficient Comparison Between Zhou Case1 and Case 2, No Forcing

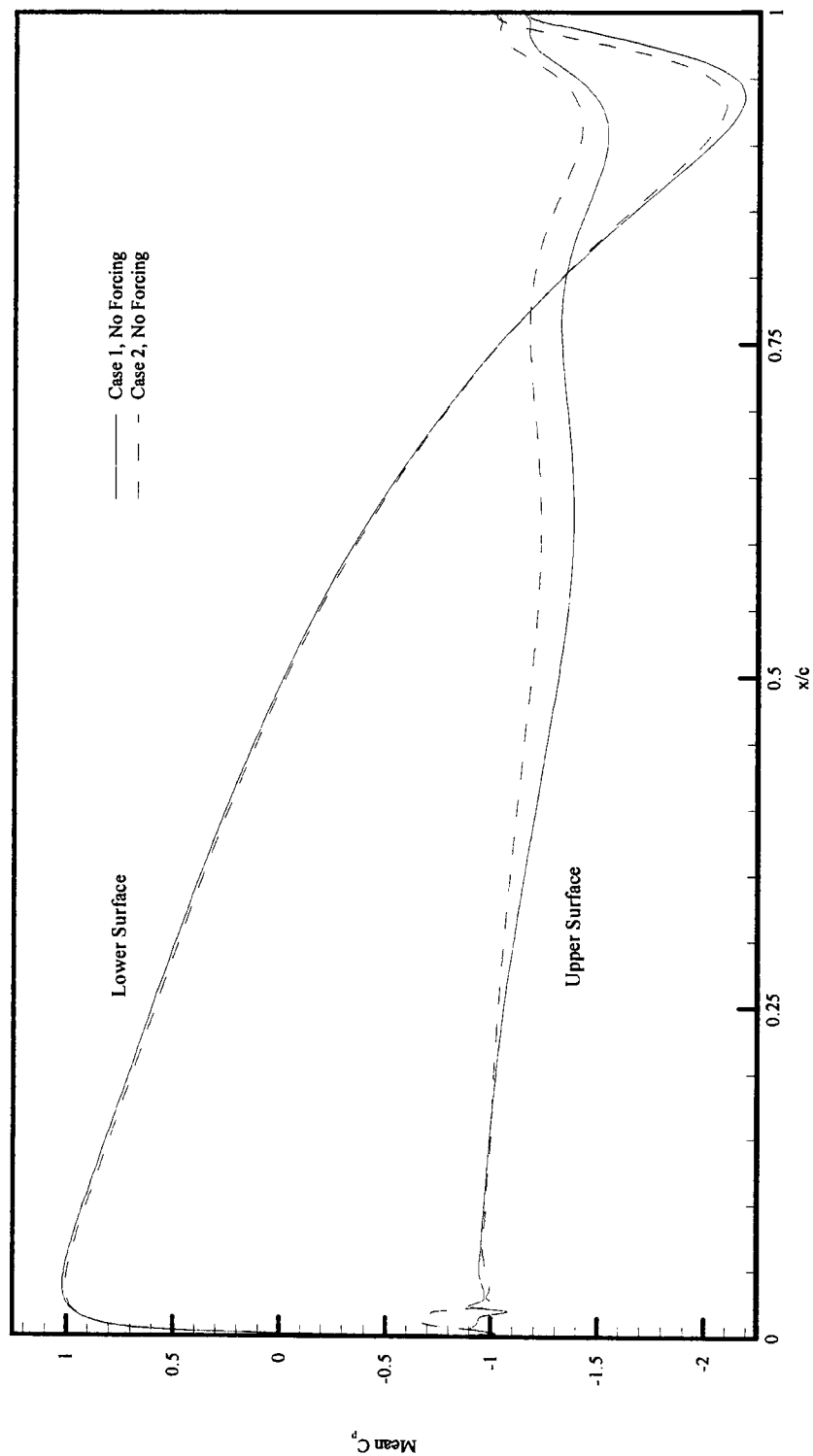


Figure 30: Mean Surface Pressure Coefficient Comparison Between Zhou Case 1 and Case 2, No Forcing

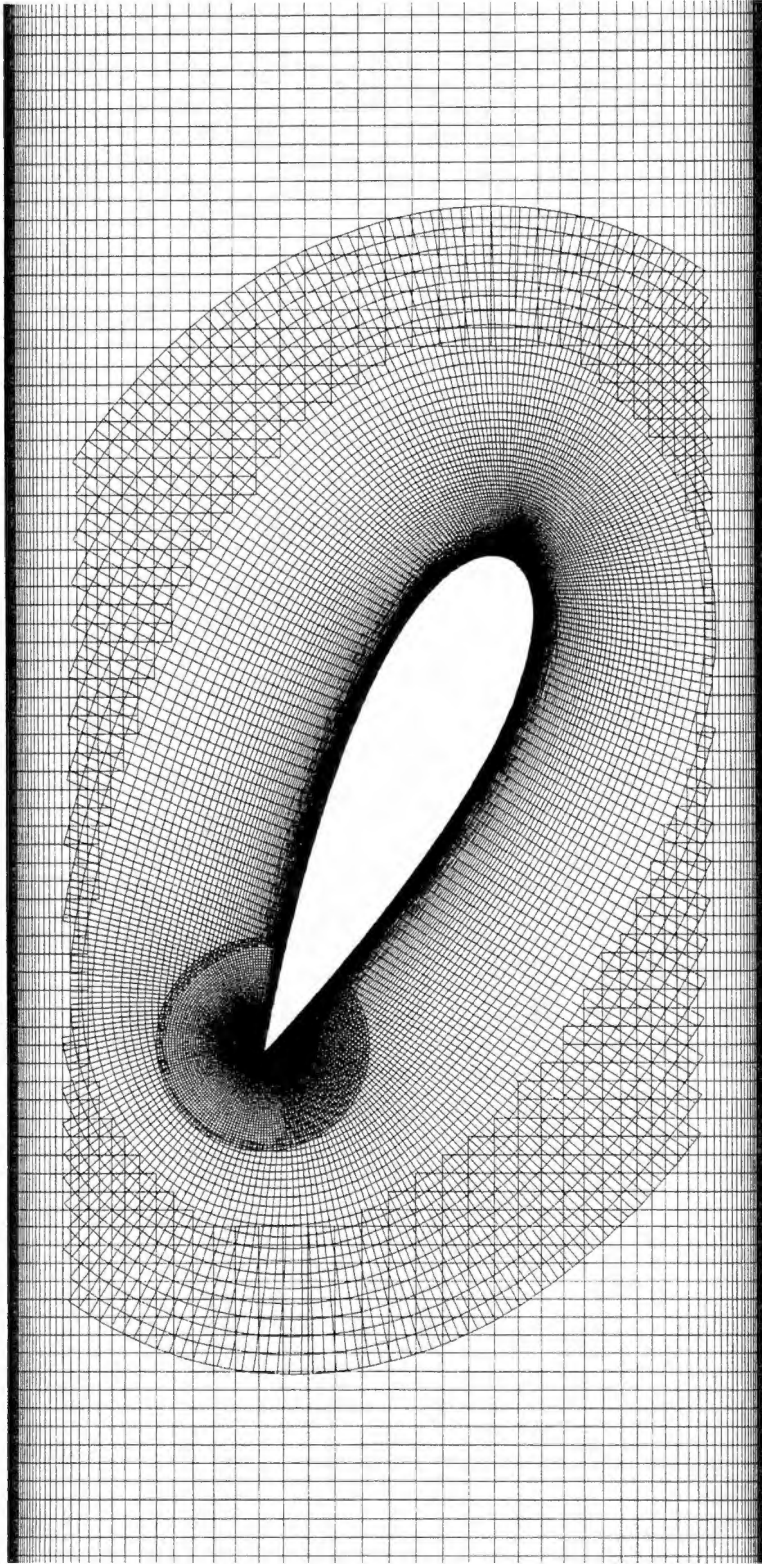


Figure 31: Tunnel Grid Set Used in Computing Wall Interference for Zhou Cases

a first-order scheme. Density was then computed from the equation of state. At the exit, Neumann conditions were applied to the temperature and velocity, and the pressure was extrapolated from the interior. The density was updated via the equation of state.

Mean surface pressure coefficients are shown in Figure 32. The effect of the wall presence is obvious. The lower surface and tunnel wall produce a significant acceleration in the trailing edge flow, resulting in lower pressure and altered separation characteristics in the trailing edge region. For comparison the experimental upper surface C_p is included in the plot. In the leading edge region, the separated shear layer is stronger, due to the upper wall. The separation vortex trajectory also changes, and the vortex travels closer to the upper airfoil surface. The instantaneous vorticity contours in Figure 33 suggest that separation occurs along the tunnel walls. It should be pointed out that the blockage effects computed in this way are approximate, since the actual tunnel flow was probably three-dimensional. However, the results provide some insight, and the effects described above are at least qualitatively correct. Given the level of tunnel interference that was probably present in the experiments, the discrepancies between the free-air calculations and experiment were deemed to be less important than first thought.

With a baseline flowfield established, the flap motion was initiated. In the experiment, a flap oscillation with a linear amplitude of as little as 0.05% of the flap chord was used. This translates to an angular amplitude of approximately 1.3° . A range of frequencies was investigated in the experimental study. The maximum effect occurred for a non-dimensional frequency $f^* = fL/U_\infty$ between 1 and 2. For the computations

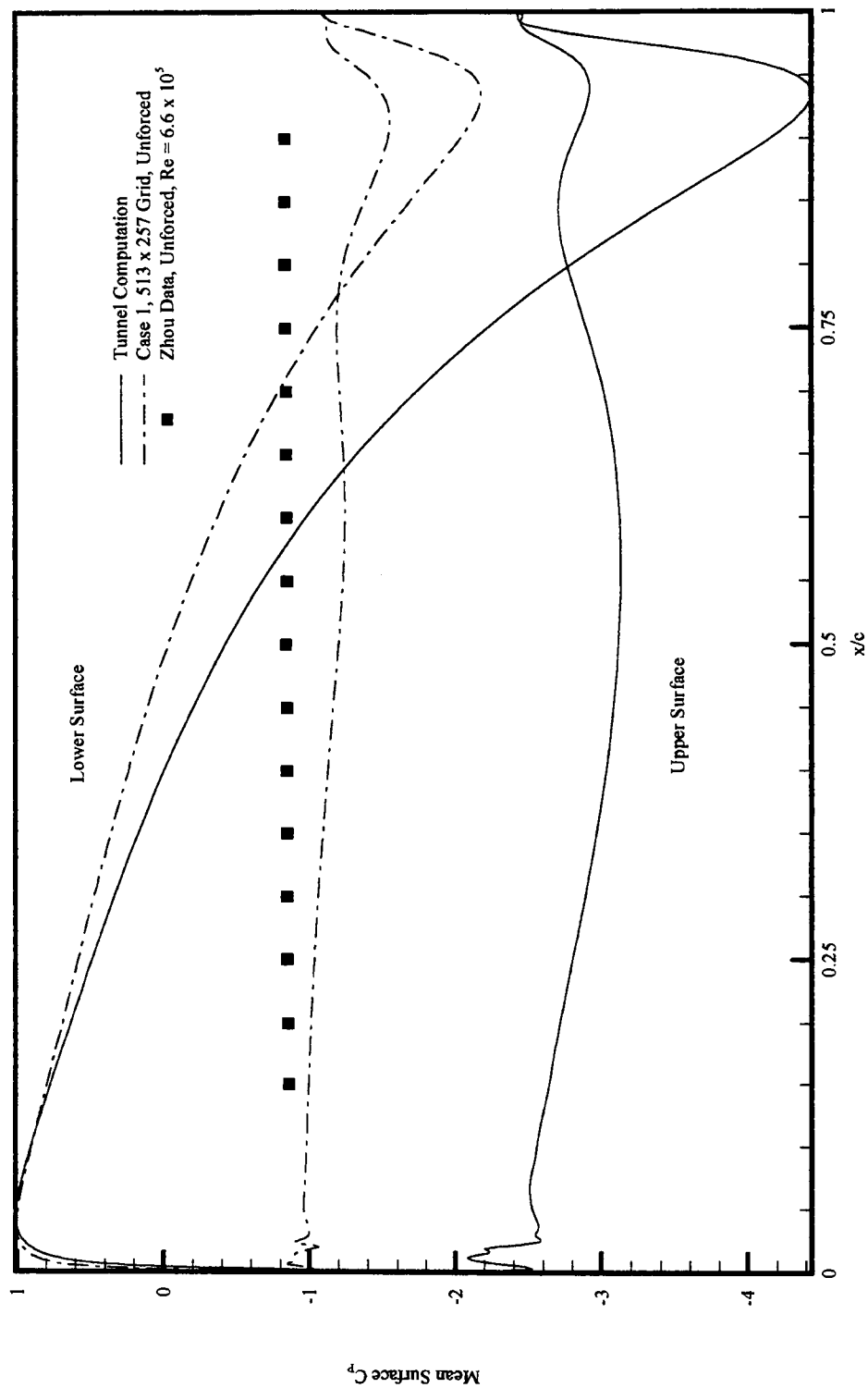


Figure 32: Comparison of Pressure Coefficients Between Zhou Case 1 and Tunnel Calculation

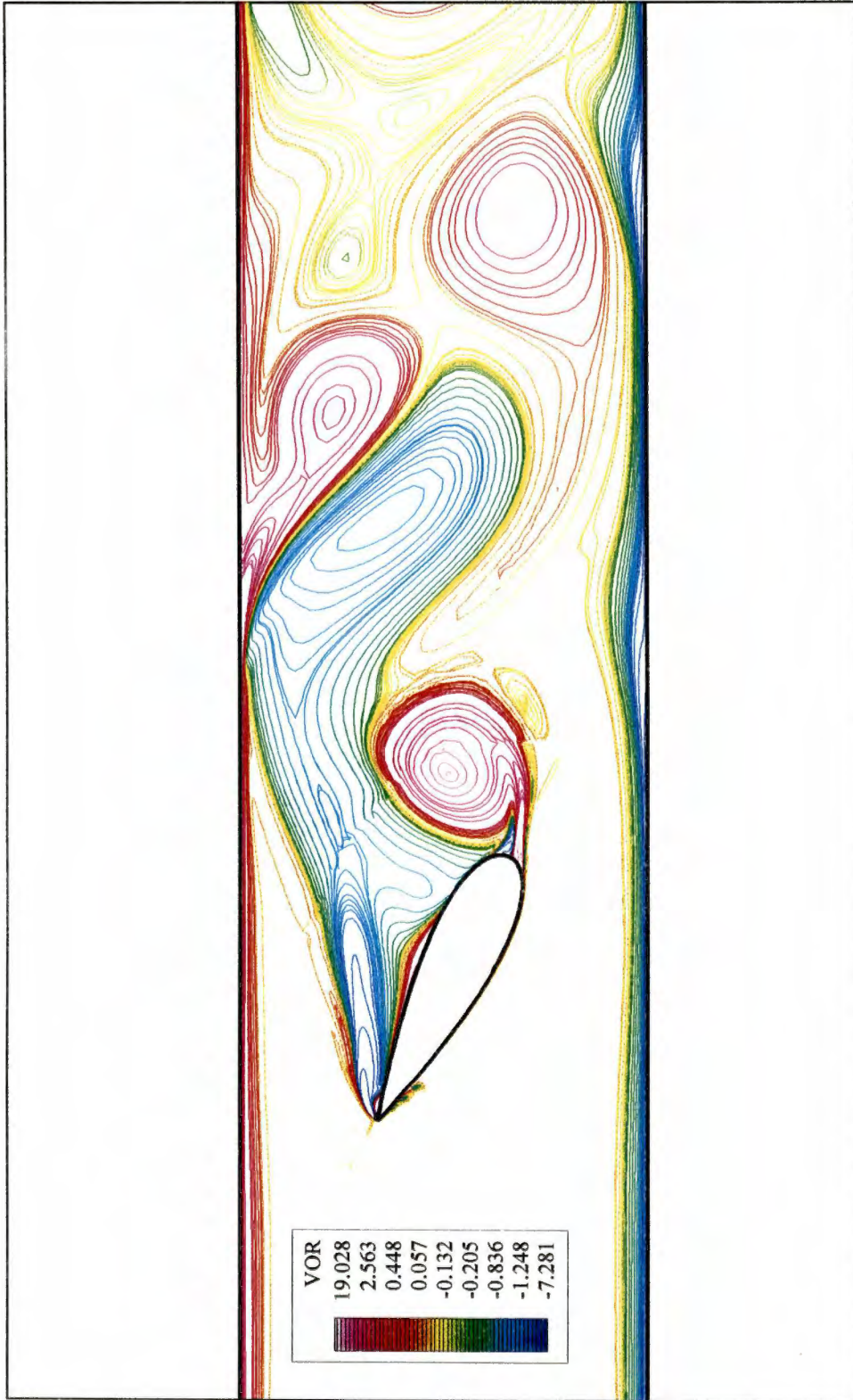


Figure 33: Instantaneous Vorticity Contours for Zhou Tunnel Case

discussed below, a non-dimensional frequency of 2 was used, with an angular amplitude of 1.3° .

Figure 34 shows the computed mean pressure coefficient on the airfoil upper surface with and without forcing for Case 2. Little change is seen between the calculations. This is in sharp contrast to the published data, shown in Figure 35. The experimental data seems to indicate that the flap oscillation has resulted in a time-averaged vortex positioned approximately over the airfoil quarter-chord. This effect is not reproduced by the computation. The computed effect on the lift is also negligible, as seen in Figure 36. This is in strong disagreement with the empirical result that the forcing produced a 70% increase in the lift coefficient.

The grids used in Case 2 were developed to reduce the time required for a computation involving a moving flap. Although the baseline results were good, it is possible that the grid density was insufficient for capturing the flap effect. Thus, a short moving flap calculation was performed using the grid set for Case 1, until it was obvious that no significant differences developed between the cases. The mean pressure coefficient is given in Figure 37. Again the results are disappointing. The higher grid density had no real effect on the solution. The C_p curve is still relatively flat and shows no change from the baseline due to forcing.

The failure of the computation to agree with the experimental data could result for a variety of reasons. Certainly, questions about the experimental procedure can be raised, given the cursory results on wall interference discussed above. However, this merely avoids the issue, and the author feels that changes of the magnitude described by Zhou and

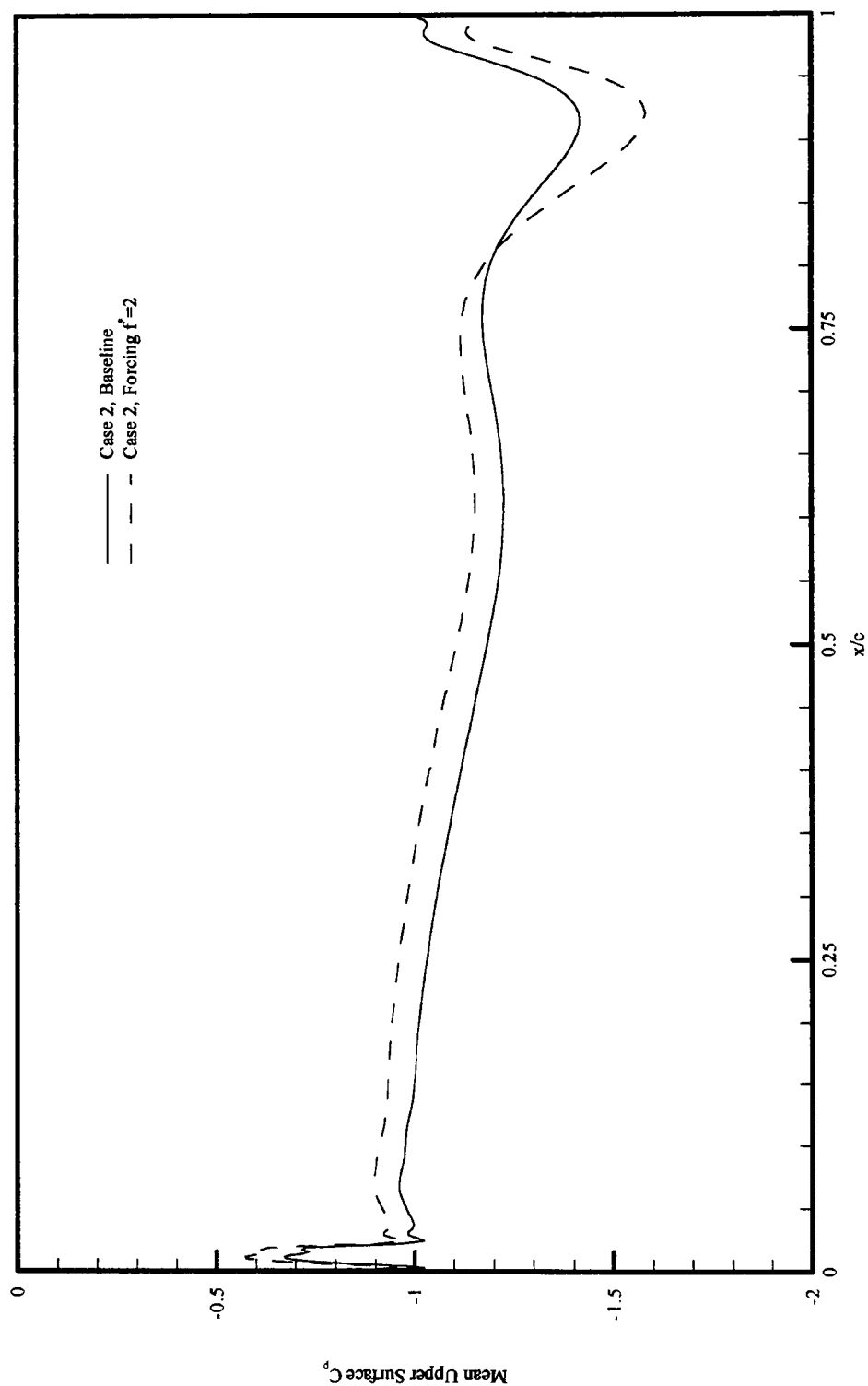


Figure 34: Mean Surface Pressure Coefficient for Case 2, With and Without Forcing

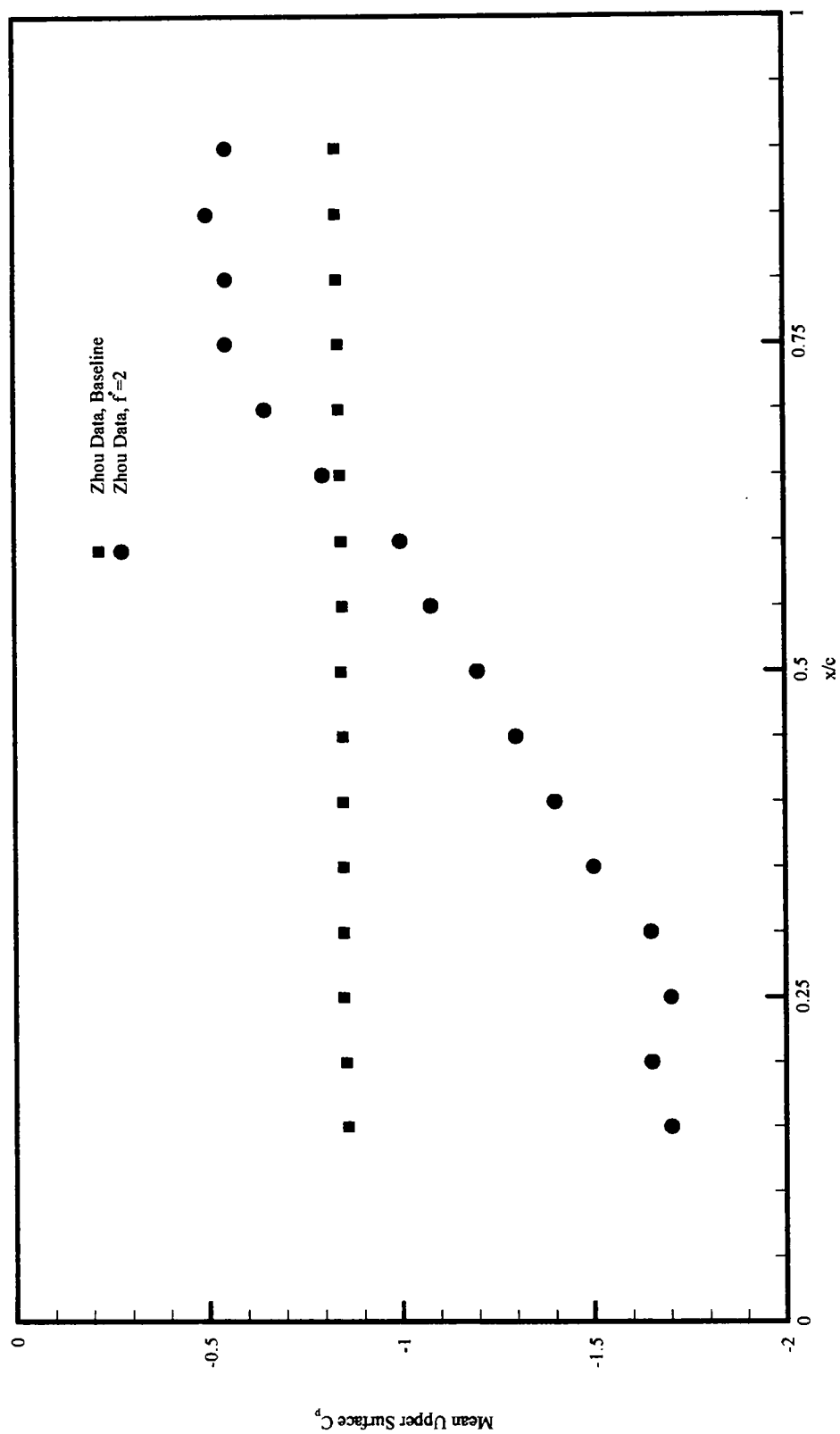


Figure 35: Experimental Mean Surface Pressure Coefficient for NACA 0025/Flap Combination

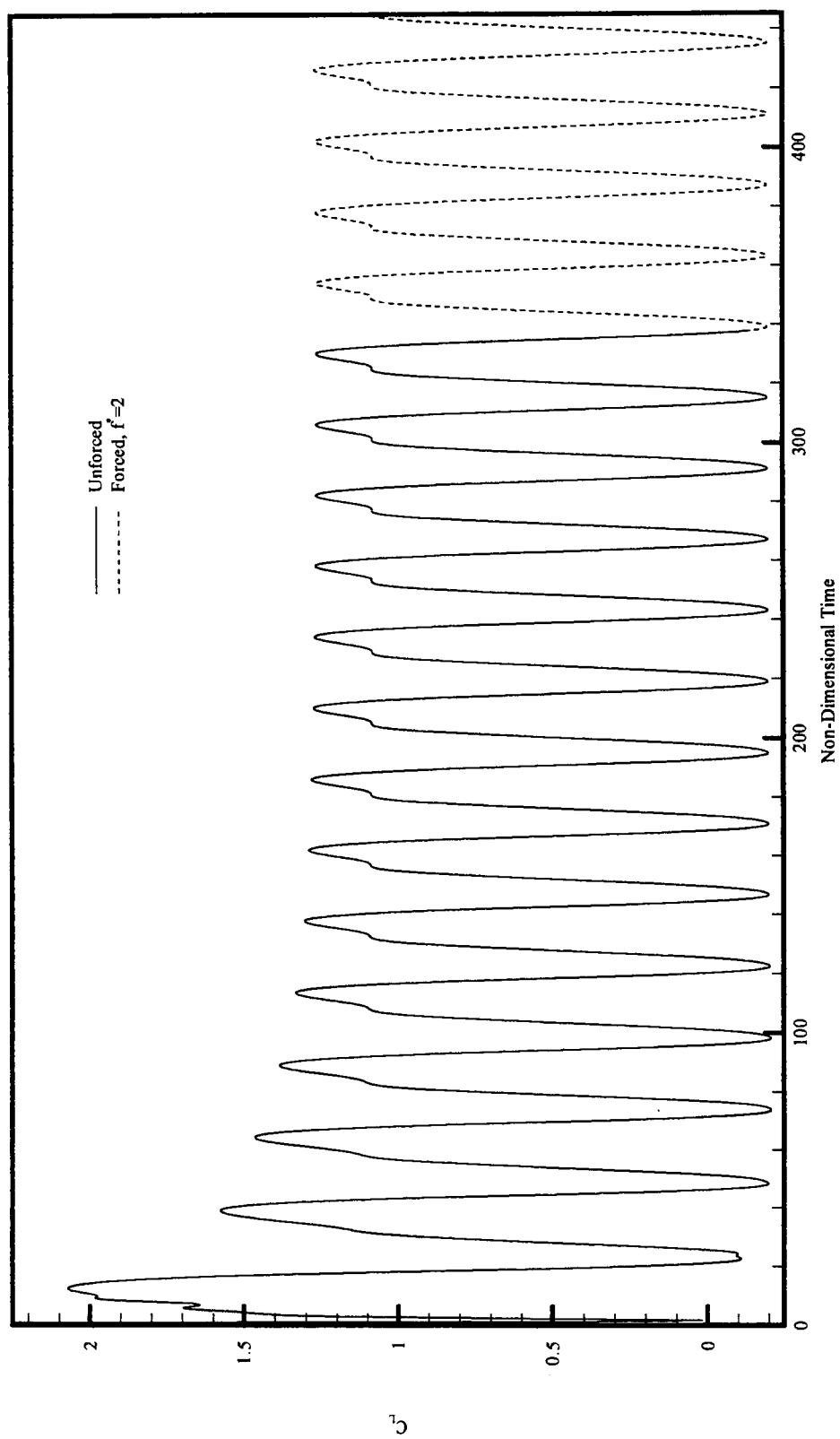


Figure 36: Computed Lift Coefficient Behavior for Zhou Case 2 With Forcing

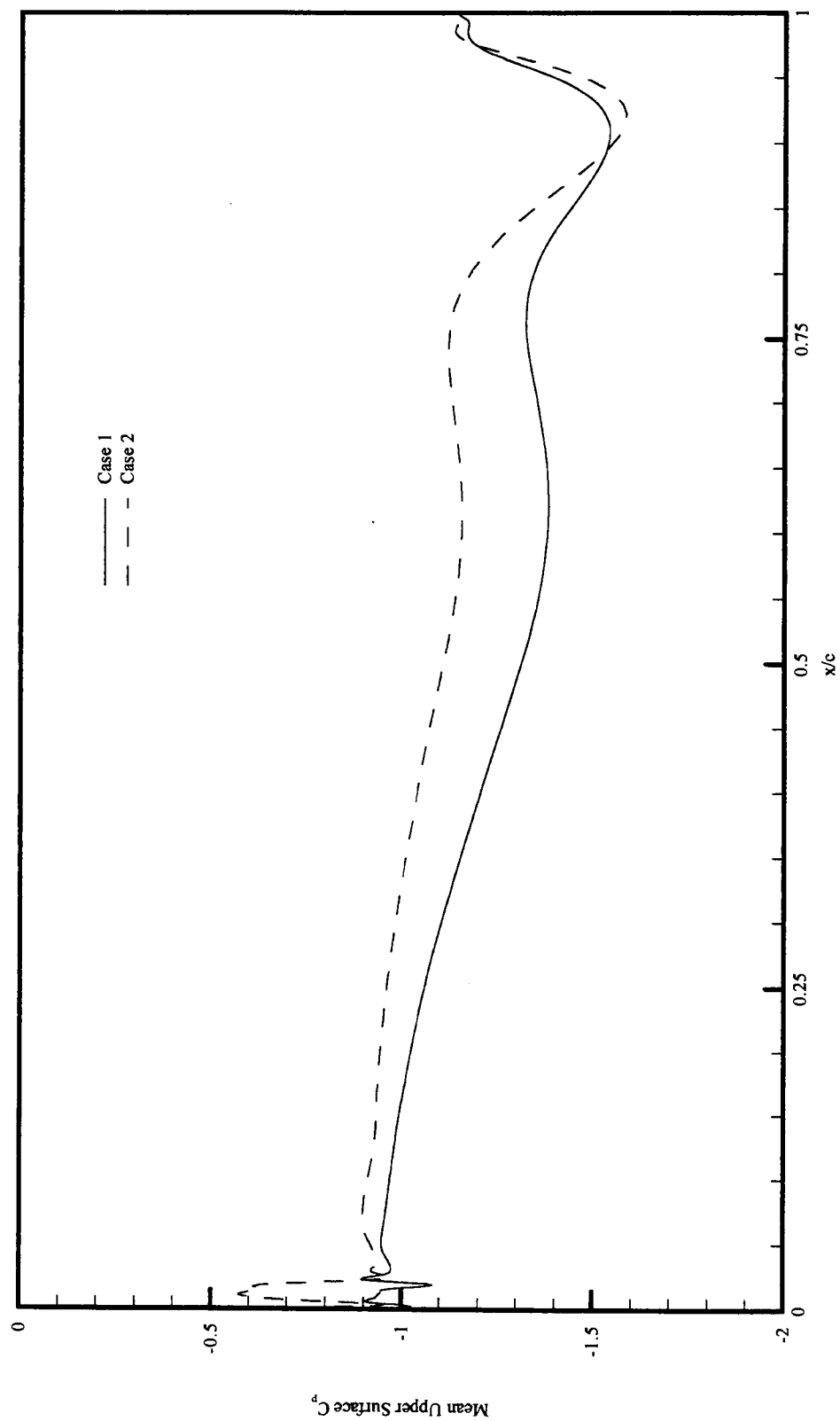


Figure 37: Mean Surface Pressure Comparison Between Zhou Case 1 and Case 2, With Forcing

Fernholz are not simply the result of the experimental technique. A second possibility is that the presence of three-dimensional flow is a prerequisite for the flap effect to be manifested. Although the wind tunnel flow probably was three-dimensional over a large region, the main physical effect due to the flap should have been concentrated along the leading edge region in a fairly two-dimensional manner. This assumes that the flap oscillation served primarily to promote the excitation of the shear layer, initiate an earlier roll-up, and create a more ordered sequence of vortices in the developing layer. This assumption is consistent with the size of the input amplitude used by Zhou and Fernholz, which seems to be too small to act on any scale larger than the shear layer. Thus a two-dimensional computation should be sufficient.

A larger concern can be associated with the turbulence modeling. A turbulence model is used to simulate the smallest length and time scales associated with turbulence, without resorting to extremely fine grids. These scales are essentially removed from the problem, and replaced with an "average" behavior, represented as an eddy viscosity. Most turbulence models in use at present use some function of the vorticity as a production source term. This generally should not cause any difficulties in computational simulations. However, for separation at a sharp leading edge, the vorticity becomes quite large. This can lead to an over-production of eddy viscosity in the region of the flap, damping the very effect which is sought in the computation. In Figure 38, the eddy viscosity contours give an indication of the level of eddy viscosity in the leading edge region. Values of eddy viscosity in the region of the flap tip and immediately downstream range from 1-100 times

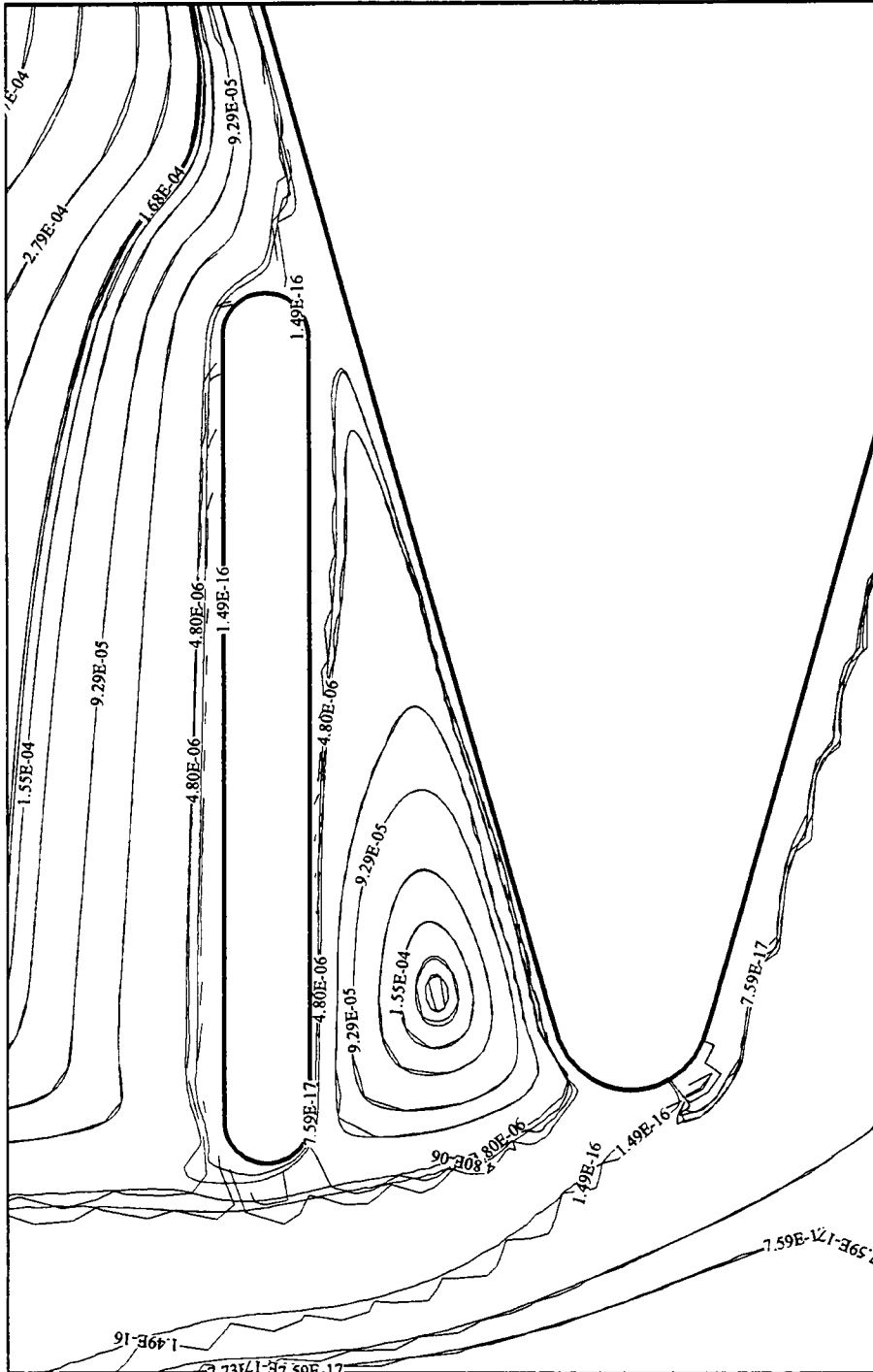


Figure 38: Eddy Viscosity Contours for Zhou Case 2

the value of the kinematic viscosity, creating very large gradients in the diffusive coefficients.

In many computations no problem arises due to over-production of eddy viscosity in a limited region. However, in this case, the limited region could be critical to the flowfield behavior under the effect of forcing. In their experiment, Zhou and Fernholz used excitation frequencies in a range around the shedding frequency. The presence of multiple frequencies in a flow system can give rise to sideband frequencies, due to the convective nonlinearity. For example, if an initial flow state can be represented as a sum of two harmonic sinusoidal terms, the convective terms in the governing equations will give birth to components given by products of the sinusoids, or equivalently, a sum and difference of the two distinct frequencies. These sums and differences constitute the sideband modes. If the amplitudes of these sideband modes grow sufficiently, further nonlinear interaction will produce additional side frequencies.

In these experiments, the relative closeness of the forcing frequency and shedding frequency would generate sideband frequencies that span a frequency range from nearly zero to roughly twice the shedding frequency (for the frequencies where the largest lift increment was found). Any unstable modes (i.e. the shear layer instability or a cavity resonance caused by the flap/TE cutout) in this range could potentially be excited, and create a significant variation in the evolution of the shear layer downstream of the flap. It is possible that some of this frequency range was over-damped by the introduction of the turbulence model, killing off the growth of the essential modes.

As stated earlier, it is also possible that the Boussinesq assumption is not valid in flow regions possessing large streamline curvature. If this assumption is invalid in the region of the sharp leading edge, then no eddy viscosity model will provide adequate modeling of the turbulence.

To partially address these concerns, a third calculation was performed using the grids of Case 2 and assuming laminar flow. Although the flow was most likely turbulent over much of the airfoil, an argument can be made that at the sharp leading edge the flow could very well have been laminar. According to Schlichting [31], the location of initial flow instability generally lies between a minimum pressure point and the separation point. Given the test geometry, it is reasonable to assume that between the forward stagnation point and the fixed separation point, the pressure gradient along the surface must have been favorable, with the pressure steadily decreasing as the leading edge was approached. This favorable gradient would serve to suppress transition, leading to a laminar separation. Therefore, a laminar calculation, denoted as Case 3, should provide some illumination.

Results of the baseline laminar calculation are given in Figures 39 through 41. The instantaneous stream traces in Figure 39 clearly indicate a series of small, strong vortices forming just downstream of the separation point. This behavior is noticeably absent in the results for Cases 1 and 2. For comparison, Figure 39 also shows the stream traces for Case 2. The mean surface pressure coefficient, plotted in Figure 40, reflects this with a pronounced suction peak over the forward part of the airfoil. The lift coefficient, shown in Figure 41, demonstrates a degree of apparent randomness, but a spectral analysis

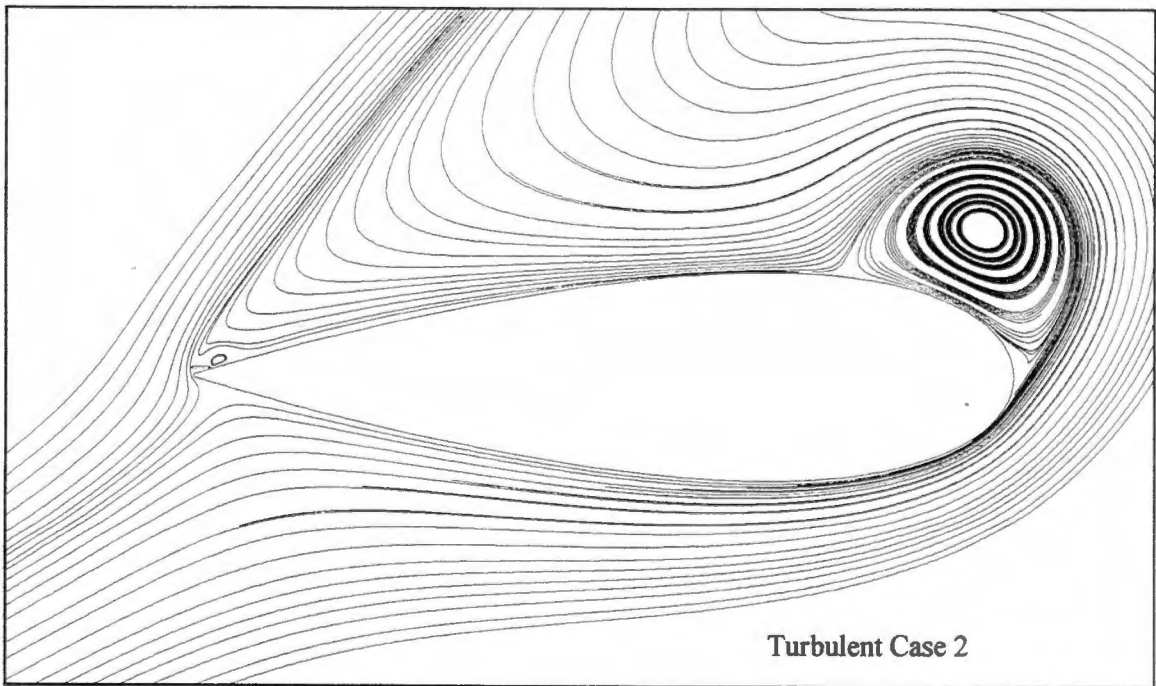
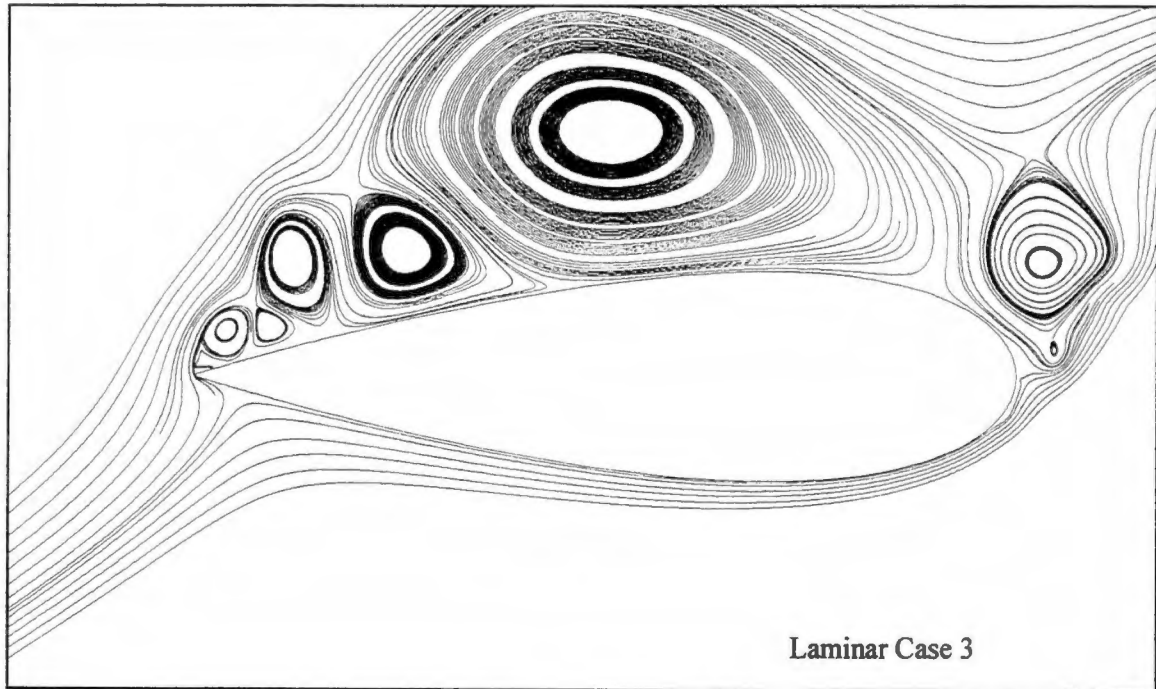


Figure 39: Comparison of Stream Traces for Laminar and Turbulent Calculations

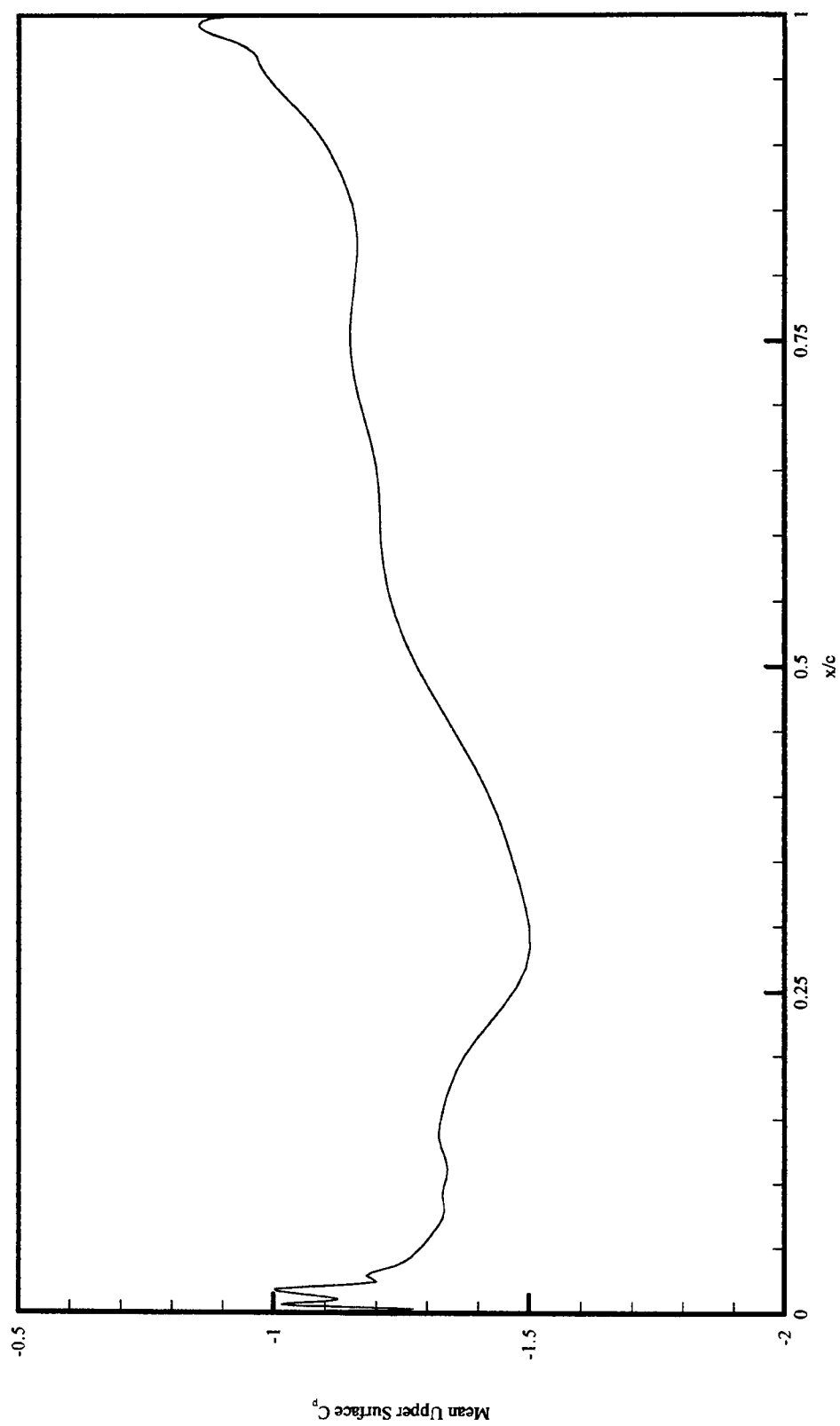


Figure 40: Mean Surface Pressure Coefficient for Zhou Laminar Case 3, No Forcing

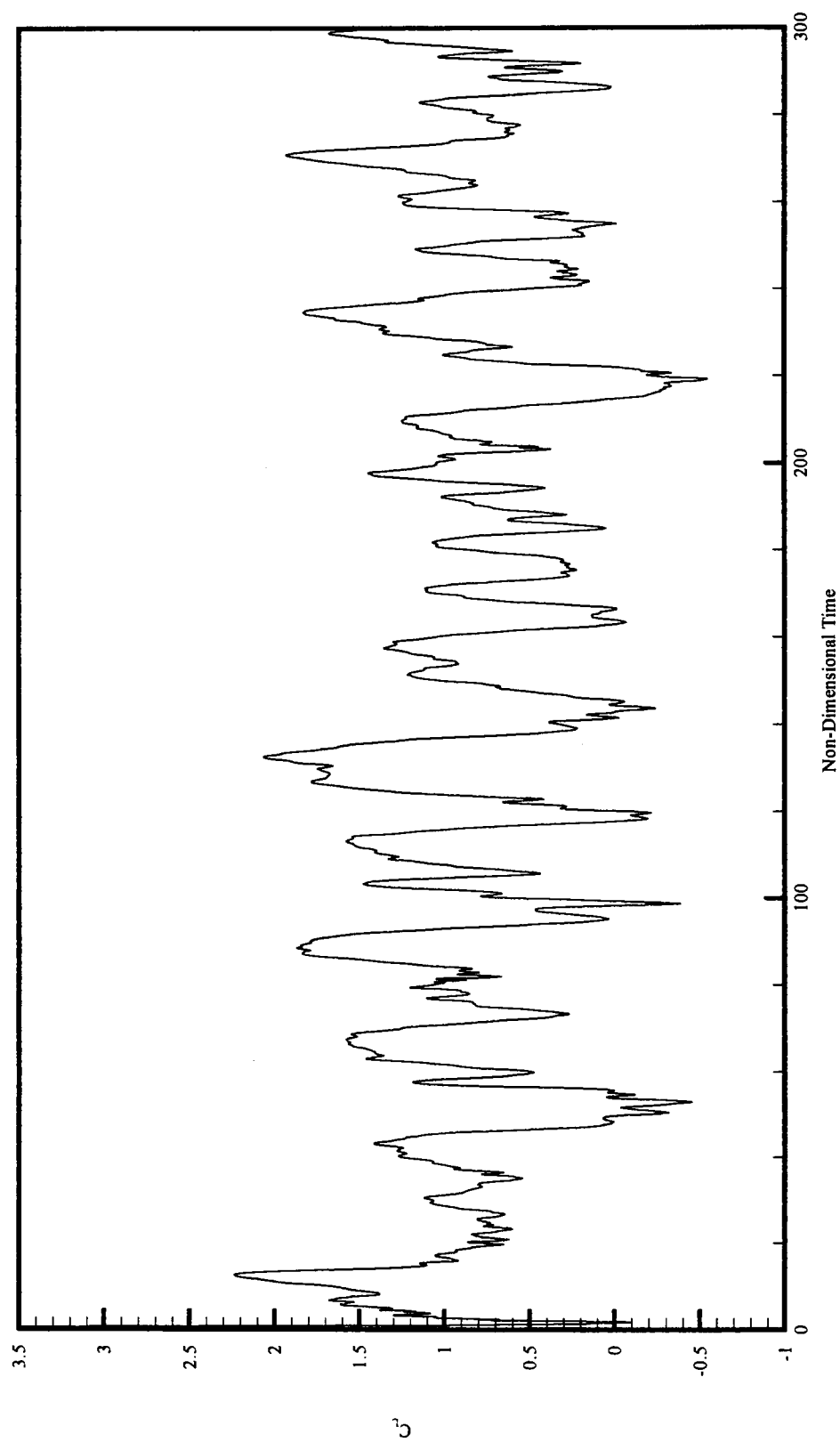


Figure 41: Lift Coefficient History for Zhou Laminar Case 3, No Forcing

revealed a first harmonic at a Strouhal number of approximately 0.1. The mean lift coefficient is 0.86, showing some agreement with the experimental baseline.

The addition of flap motion does have some effect on the flow in this calculation, albeit a small one. The mean pressure coefficient shows an increase in the suction peak, as seen in Figure 42. However, the pressure increases more rapidly as the trailing edge is approached. This does indicate some modulation in the vortex formation. Variations in the force coefficients are essentially lost in the noise. But the results do tend to support the argument that the forcing effect is overwhelmed by the eddy viscosity in the turbulent calculations.

The actual effect would seem to lie somewhere between the cases considered here. The unforced baseline tends to be reflected more accurately by the turbulent calculations. However, physical arguments support the notion of laminar separation, probably followed by a transition to turbulence. The laminar calculation certainly indicates that a small flap oscillation can have some effect on the flow behavior. This would make some sense, because the shear layer instability frequency range for laminar flows is quite different from the turbulent range. Perhaps the laminar calculation was capable of capturing some part of the instability growth. As an aside, in a personal communication with Prof. Zhou he echoed his belief that the experimentally observed flowfields did fall somewhere between those plotted in Figure 39.

It is possible that the real flow was turbulent from the separation point onward, and the numerical scheme does not adequately capture the instability of the mean flowfield owing to excessive dissipation. Then the flap motion could not trigger the critical

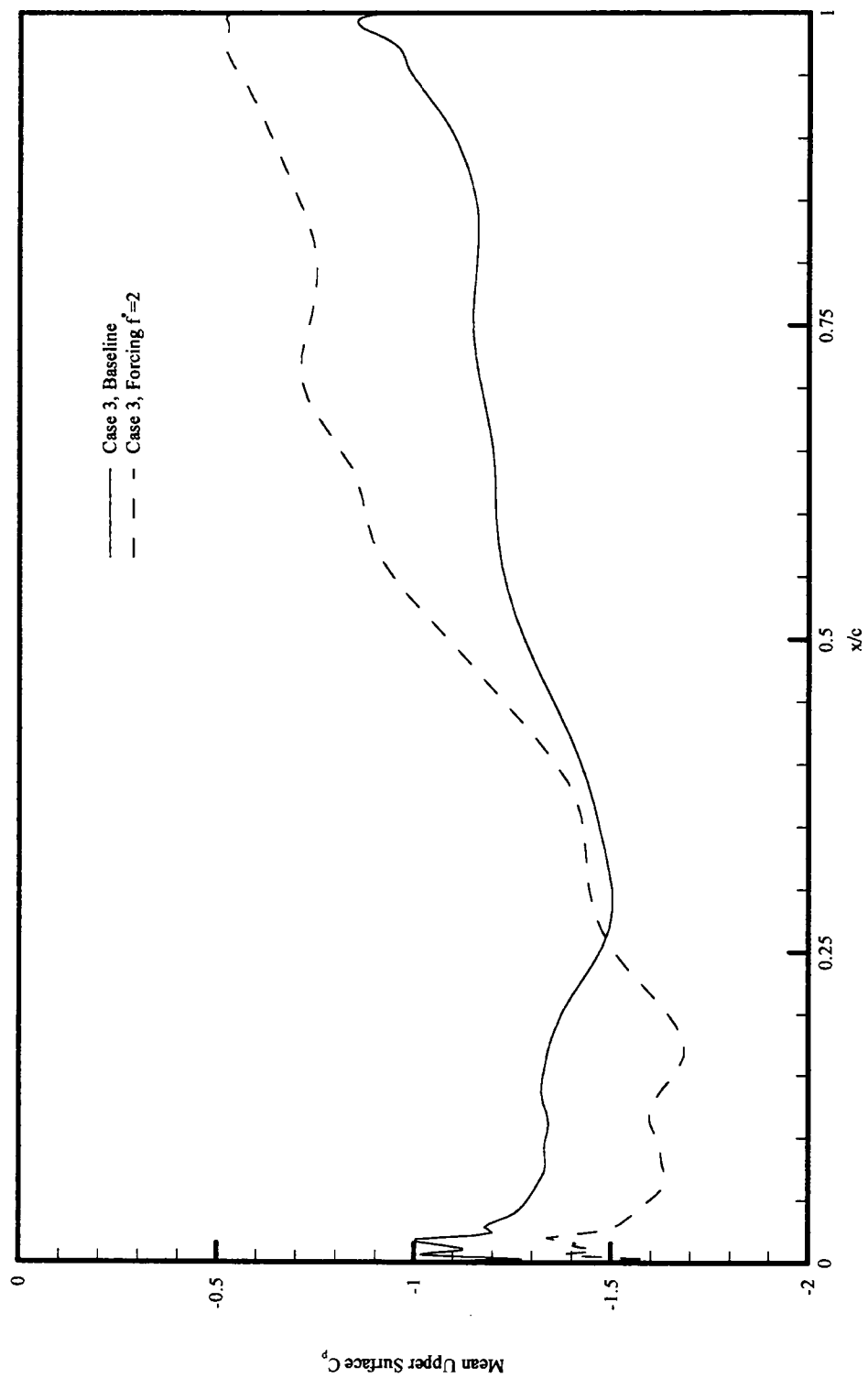


Figure 42: Variation of Mean Surface Pressure Coefficient Due to Forcing for Zhou Laminar Case 3

instability which gives rise to the vortex enhancement. Additional calculations using a flap amplitude four times larger than the experimental value proved fruitless, indicating that the flap amplitude was not a critical parameter in this failure.

A set of computations using the Baldwin-Lomax turbulence model with the Degani-Schiff modification was also made. This turbulence model is an algebraic relation that is not generally recognized as being useful for massively separated flows, but it is simple to program. The results were likewise disappointing, showing no ability to capture the experimental behavior. Thus, the detailed results are not presented.

Flow Over a NACA 63₃-018 Airfoil

The results using the NACA 0025 configuration were disappointing. However, it was felt that the underlying problems in the solution were not the result of an error in the code, but rather a limitation on the level of modeling detail and possibly problems in the experimental data. A second case was selected based on the work of Hsiao, et. al. [8]. Hsiao and his colleagues considered the flow over a NACA 63₃-018 airfoil with a small flap positioned near the leading edge separation point. The flap length was equal to 4% of the airfoil chord. Angles of attack were between 28° and 35°, so the flow was massively separated and unsteady, with periodic vortex shedding. Reynolds numbers of 1.9×10^5 and 3.1×10^5 were used. By oscillating the flap at a frequency equal to the shedding frequency, an increase in the lift and drag of as much as 50% was obtained. The effect was strongly frequency-dependent, with the increase narrowly centered around the shedding frequency. At the highest angle of attack, much smaller increases also appeared

at the first harmonic above and below the shedding frequency. Amplitude effects were also examined in the experimental work, where the flap amplitude was varied from 6.7° to 21.2° . The key finding was that the flap needed to be in physical contact with the separating shear layer in order for the force increase to occur.

Hsiao, et. al. performed the work in a wind tunnel with a 90 x 120 cm test section. The airfoil model had a chord of 30 cm and a span of 60 cm. The resulting blockage was approximately 2% at an airfoil angle of attack of zero, which was significantly smaller than the blockage in the experiments of Zhou and Fernholz. Additionally, the flap was located along a smooth separation edge, suggesting that adverse effects from the turbulence model would be much smaller. Because of these differences it was felt that a numerical simulation would have a good chance of success.

For the simulation, an O-grid with 413 x 257 points was used for the airfoil. The flap was modeled with an O-grid having 201 x 151 points. For both grids, the initial y^+ spacing was less than 0.5 over all solid surfaces at a Reynolds number of 1.9×10^5 . Figure 43 shows these grids. The Mach number was set to 0.2, as in other computations. A single angle of attack of 28° was selected from those used by Hsiao and his co-workers. The initial flap position was chosen such that the flap chord made an angle of 30° with the freestream direction.

A baseline flowfield, i.e. with no forcing, was computed first. A time history of the force coefficients is given in Figure 44. These coefficients clearly show the unsteady nature of the flow. The Strouhal number, computed from the lift coefficient history, is 0.153. The mean lift coefficient is 0.84, and the time-averaged drag coefficient is 0.49.

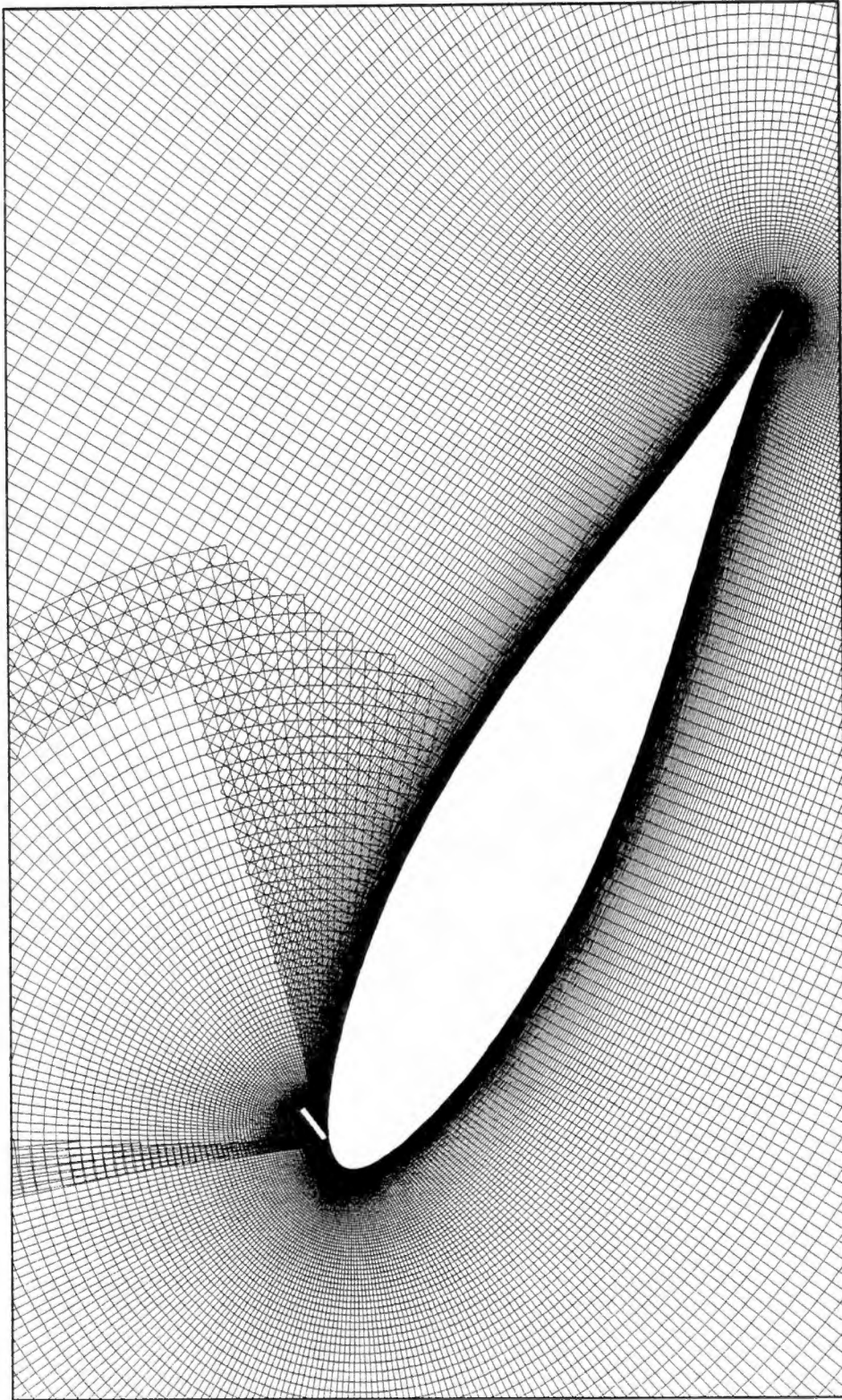


Figure 43: NACA 63₃-018 Airfoil/Leading Edge Flap Grids Used for Hsiao Computations

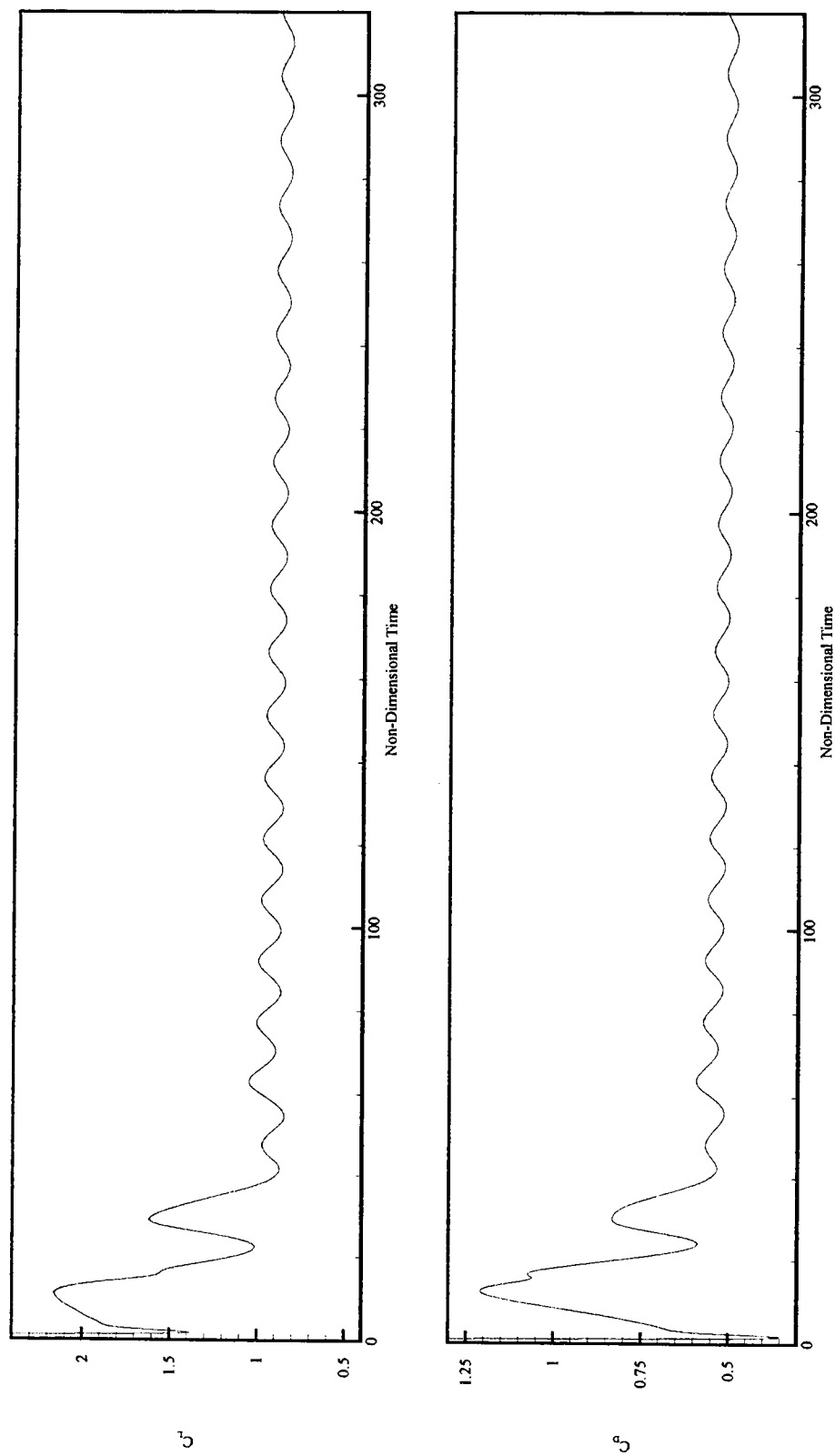


Figure 44: Force Coefficients for Hsiao Baseline Calculation, $Re = 1.9 \times 10^5$

The experimental value of Strouhal number was approximately 0.16 at this Reynolds number, and the mean lift and drag coefficients were 0.83 and 0.48, respectively. The time-averaged mean surface pressure coefficient is given in Figure 45, along with experimental data. The agreement between computation and experiment is good. The small peaks in the computed pressure distribution near the leading edge are caused by the flap presence.

Once a good baseline flowfield was established, the flap motion was initiated. For this study, three dynamic cases were selected. These were chosen to examine variations in the forcing frequency and flap amplitude. The first case, Case 1, consisted of a flap amplitude $\Delta\theta = 12.5^\circ$ and a non-dimensional forcing frequency $f^* = 1$. Here f^* is defined as the ratio of the forcing frequency to the vortex shedding frequency. Case 2 corresponded to a flap amplitude of 12.5° and a forcing frequency of 2. The last case, referred to as Case 3, used a forcing frequency of 1, but the flap amplitude $\Delta\theta$ was 6.7° . These amplitudes were taken from the experiments, as were the non-dimensional forcing frequencies.

The results can be summarized in Figures 46 to 50. The variation in the time-averaged surface pressure coefficient is presented in Figure 46. For the experimental results a flap amplitude of 12.5° was used. The mean force coefficients are shown in Figures 47 and 48, plotted with the test data for comparison. The variation with respect to forcing frequency is well-represented by the numerical simulation. The effect of varying the flap amplitude, presented in Figures 49 and 50, is also captured by the computation.

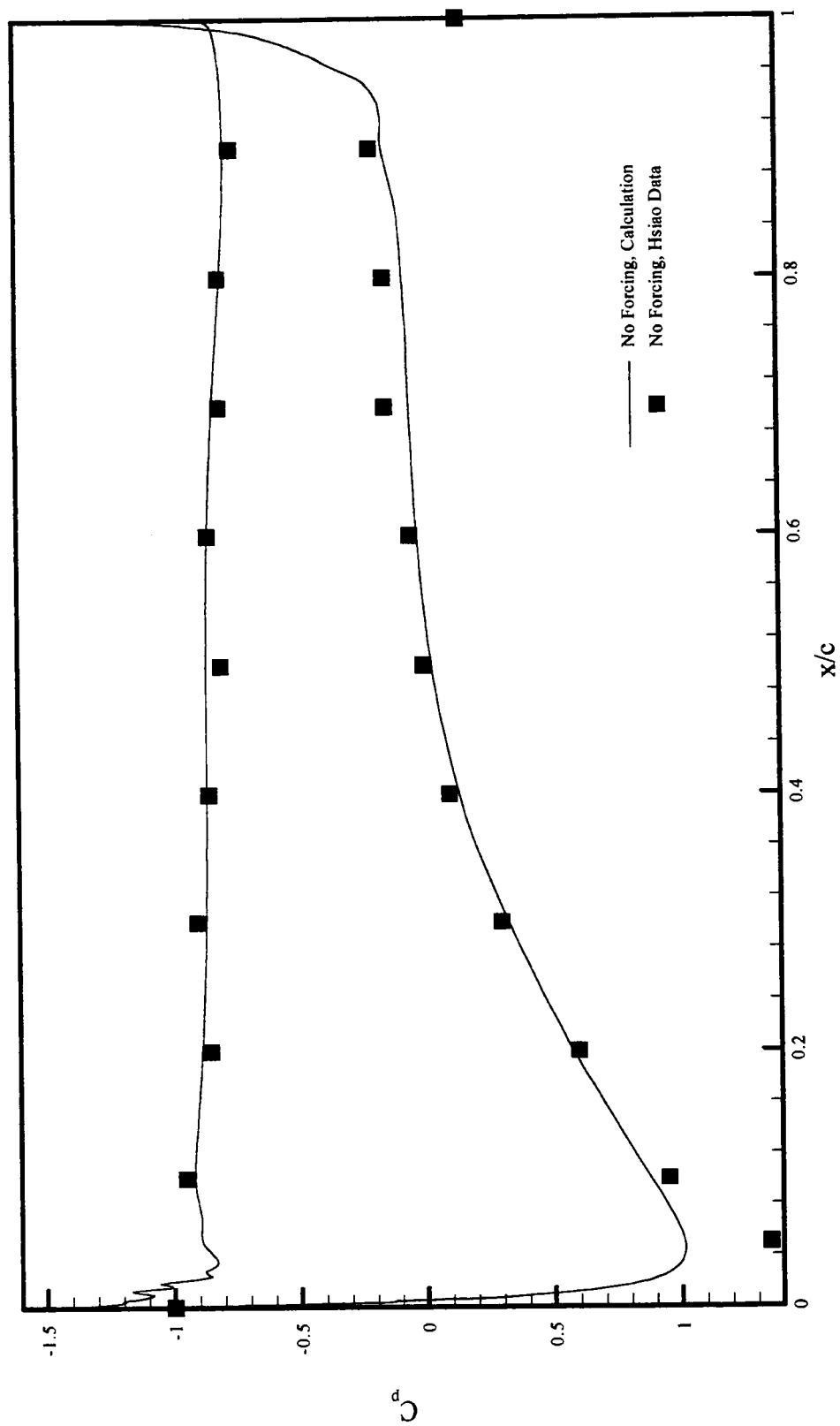


Figure 45: Mean Surface Pressure Coefficient for Hsiao Baseline

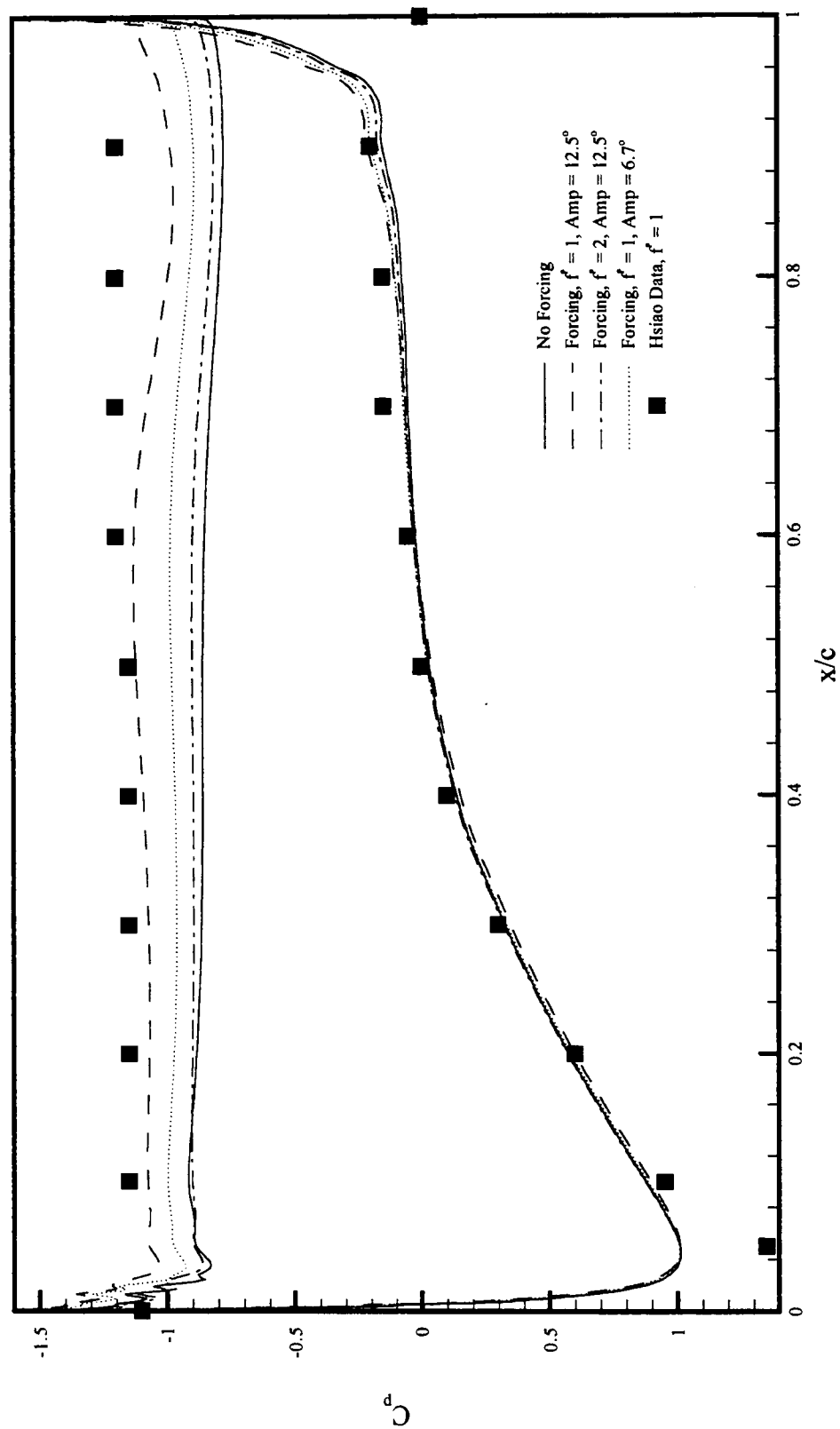


Figure 46: Mean Surface Pressure Coefficient Comparison With Forcing

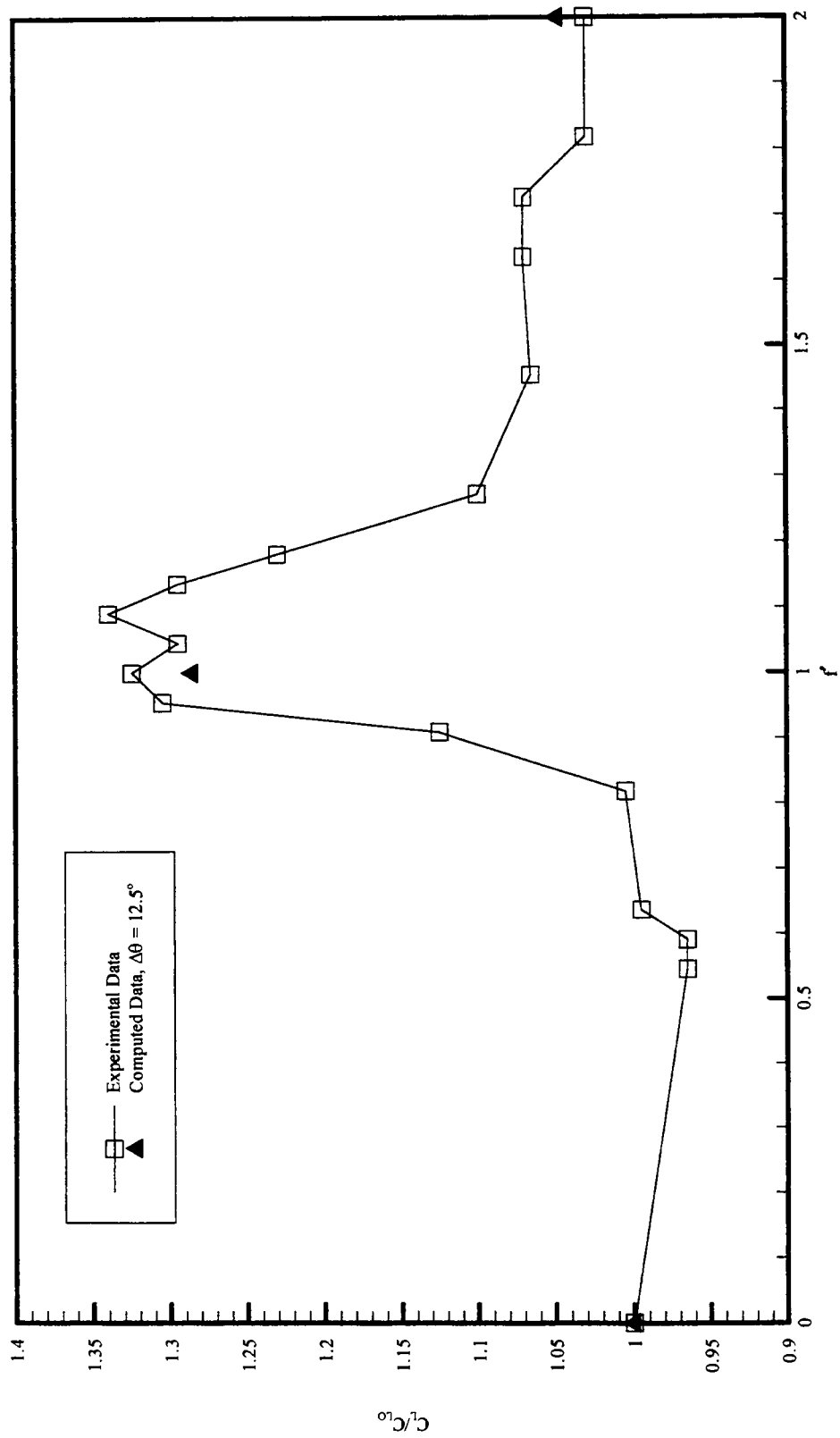


Figure 47: Variation of Lift Coefficient with Forcing Frequency

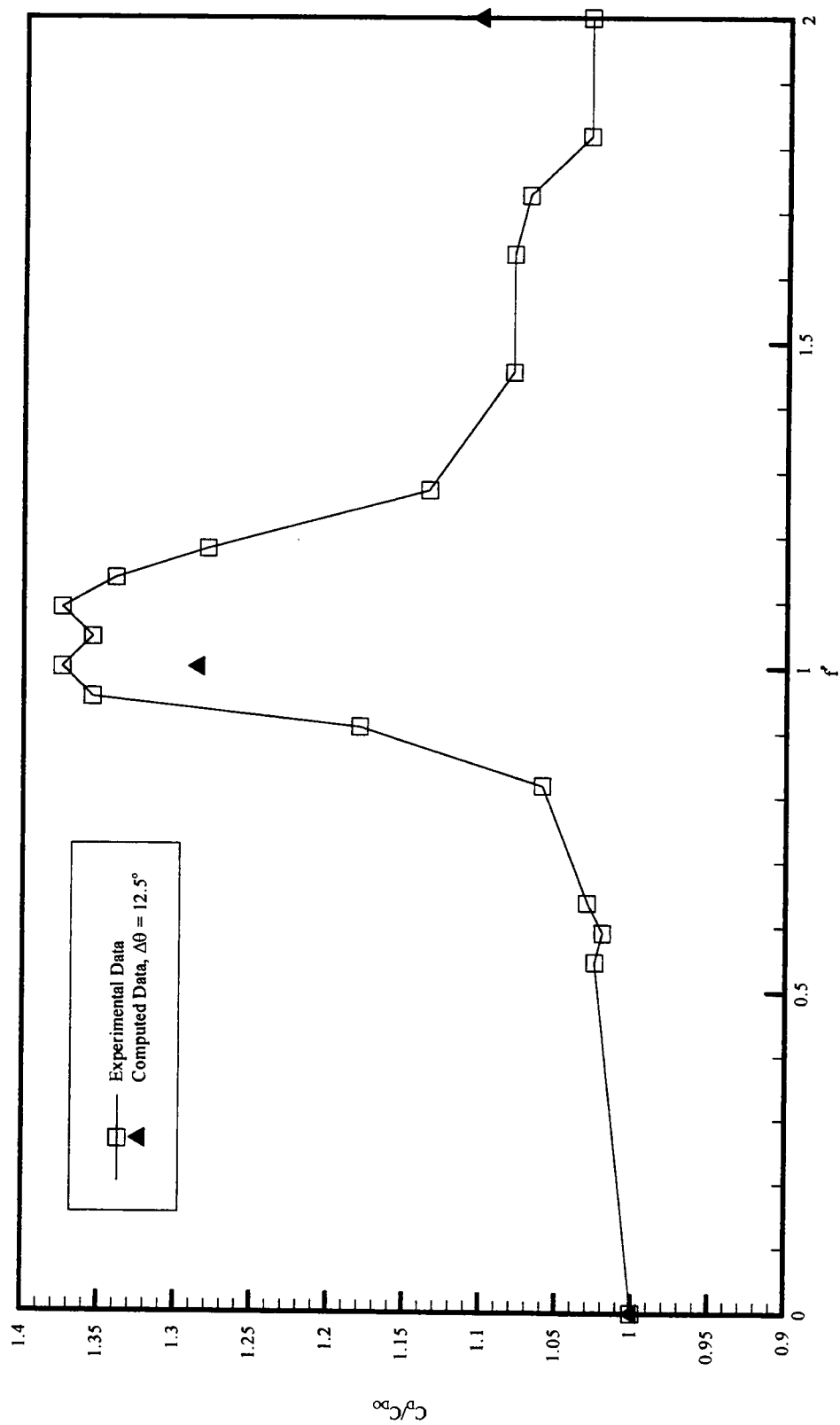


Figure 48: Variation of Drag Coefficient with Forcing Frequency

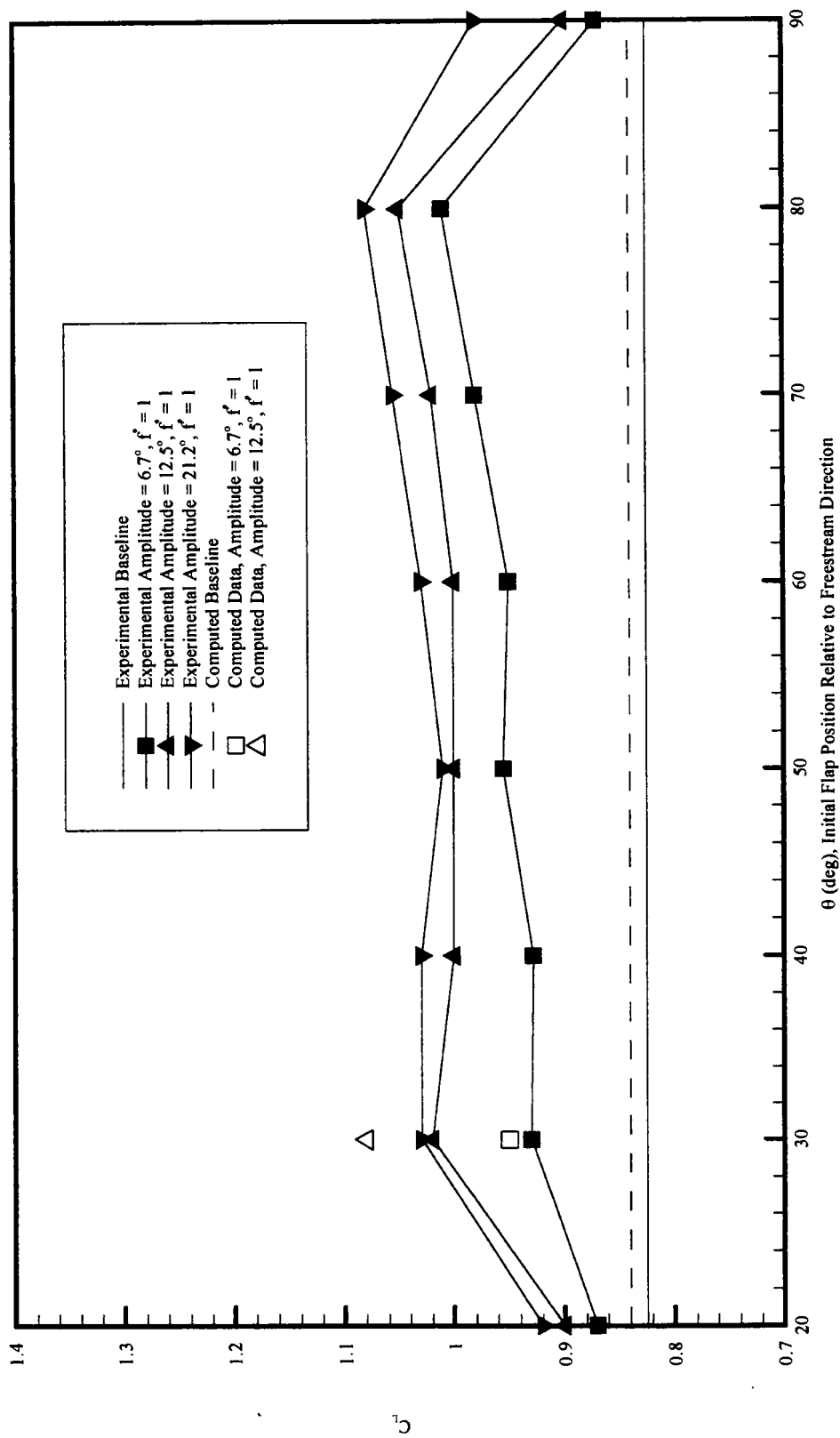


Figure 49: Dependence of Lift Increment on Flap Amplitude and Initial Position

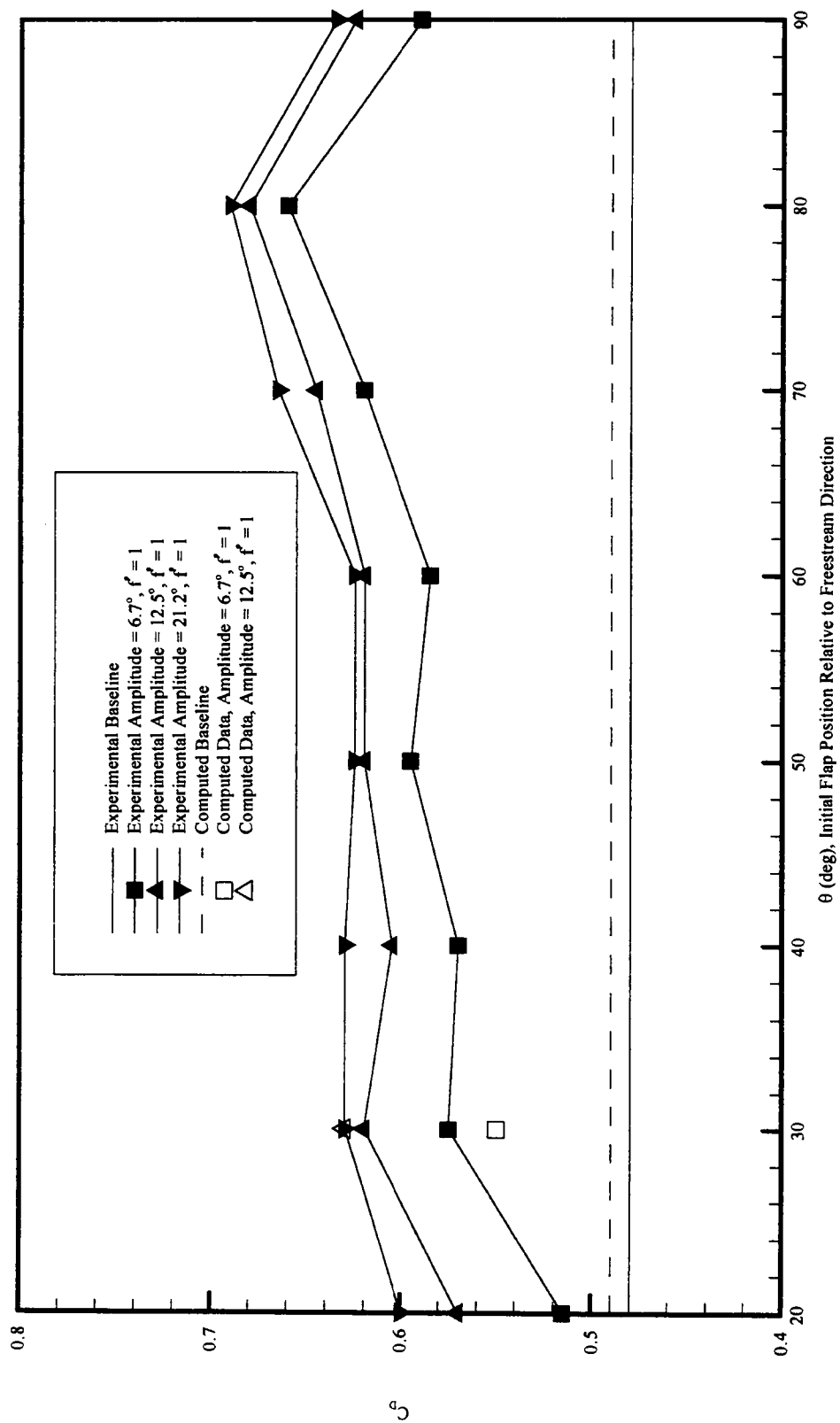


Figure 50: Dependence of Drag Increment on Flap Amplitude and Initial Position

The lift coefficient histories are shown in Figure 51. The forcing by the flap significantly alters the instantaneous behavior of the lift as well as the time average. In Case 1, the flap forcing creates a higher average value of C_L , and the amplitude of the lift oscillations also increases by a sizable amount. For Case 2, the higher frequency flapping appears to have very little effect on the time average. The pattern of the lift coefficient suggests the introduction of higher harmonics into the flowfield, along with a weakening of the fundamental periodic behavior present in the unforced flow. In Case 3, the forcing does result in a slight increase in the lift coefficient. The oscillations are also well-ordered, as in Case 1. The drag coefficient histories are shown in Figure 52. The behavior is similar to that of the lift coefficient. Forcing at the shedding frequency increases both the mean drag value and the amplitude of the instantaneous drag. The periodicity of the drag also appears to be reinforced. Forcing at twice the shedding frequency does not provide any real increase in the time-averaged drag, and introduces a degree of apparent randomness in the instantaneous drag data.

The behavior of the aerodynamic loads tends to support the conclusion of Hsiao, that the principal effect of the flapping is to organize and energize the large-scale vortical structures passing over the upper surface of the airfoil. The time-averaged surface pressure distribution shows no trace of a vortex "capture" phenomenon, which would be marked by a characteristic suction peak on the airfoil. However, it is not obvious how a small perturbation at a specific frequency can trigger such a large effect. For a forcing frequency $f^* = 1$, the ratio of maximum flap-tip velocity to freestream velocity is approximately 1.5% when the flap amplitude is 12.5° . The level of forcing rises to 3% in

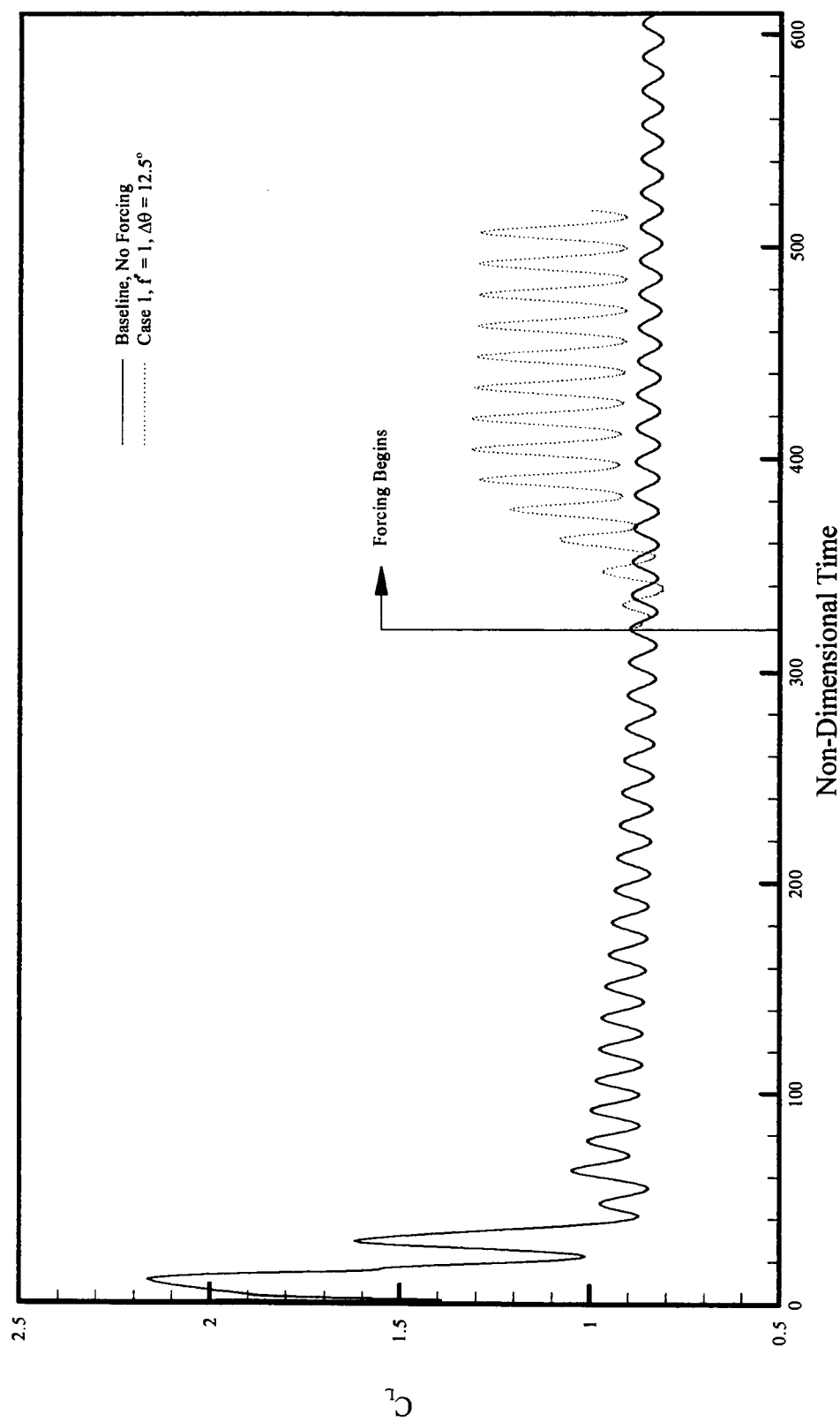


Figure 51: Lift Coefficient Histories

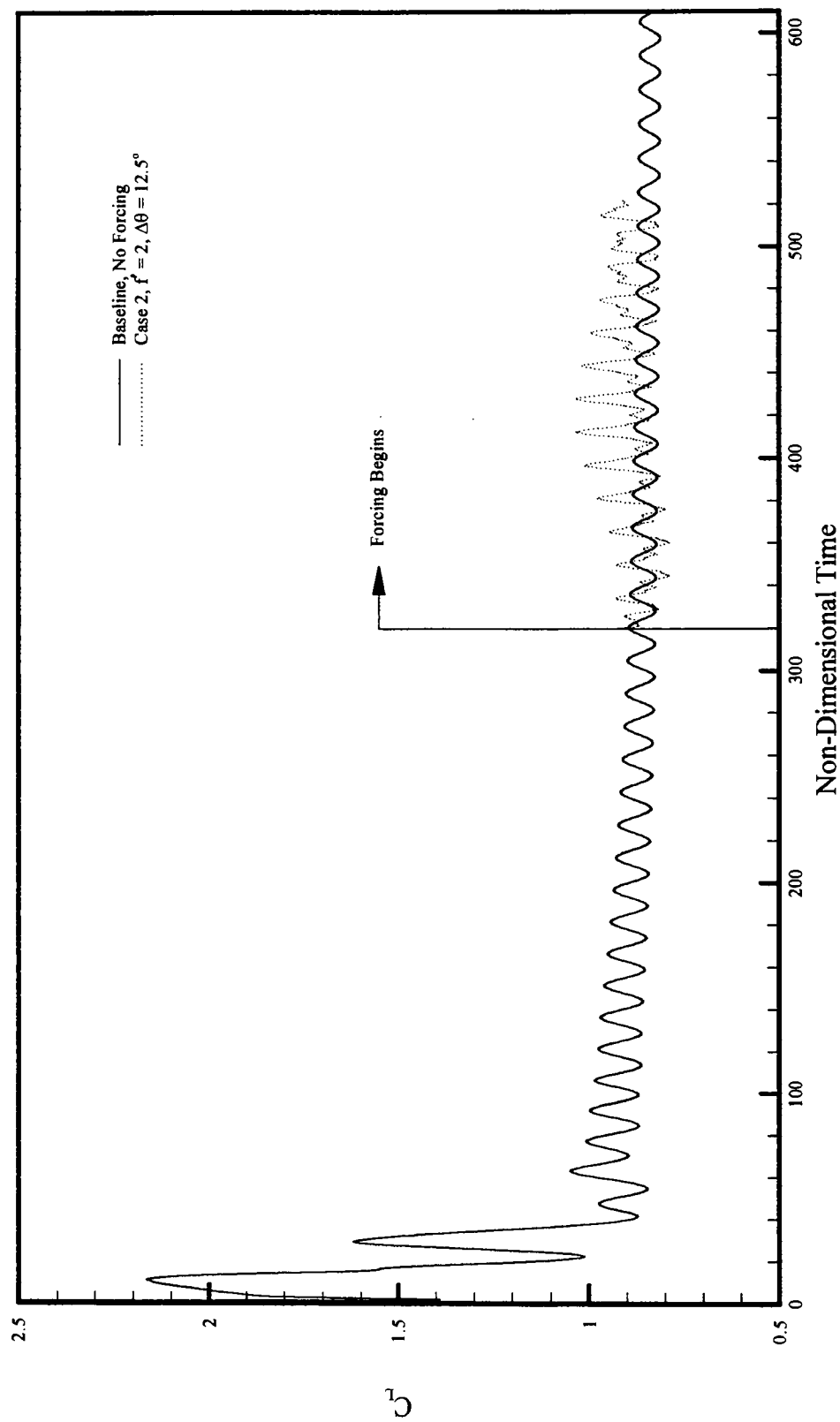


Figure 51: Lift Coefficient Histories (Continued)

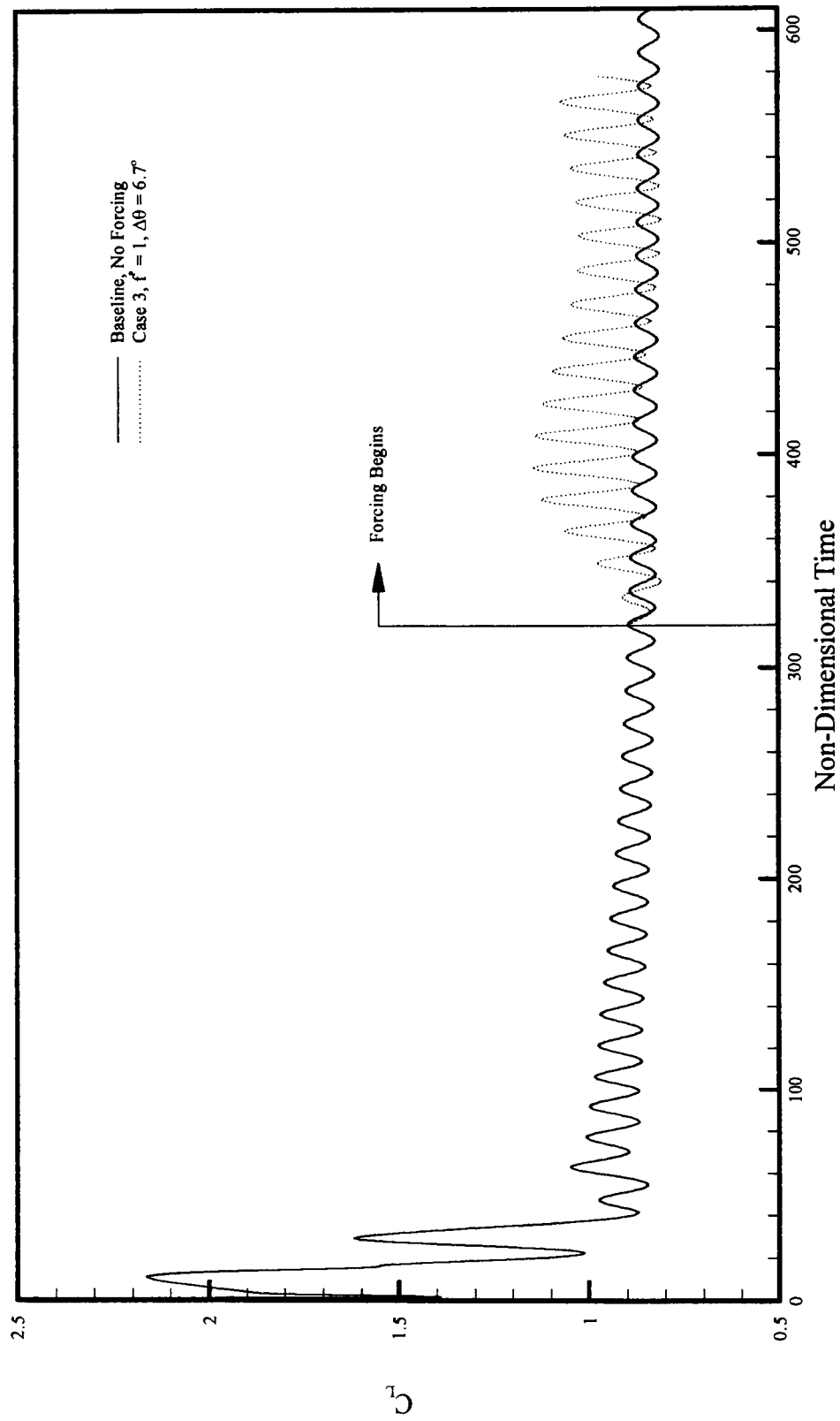


Figure 51: Lift Coefficient Histories (Concluded)

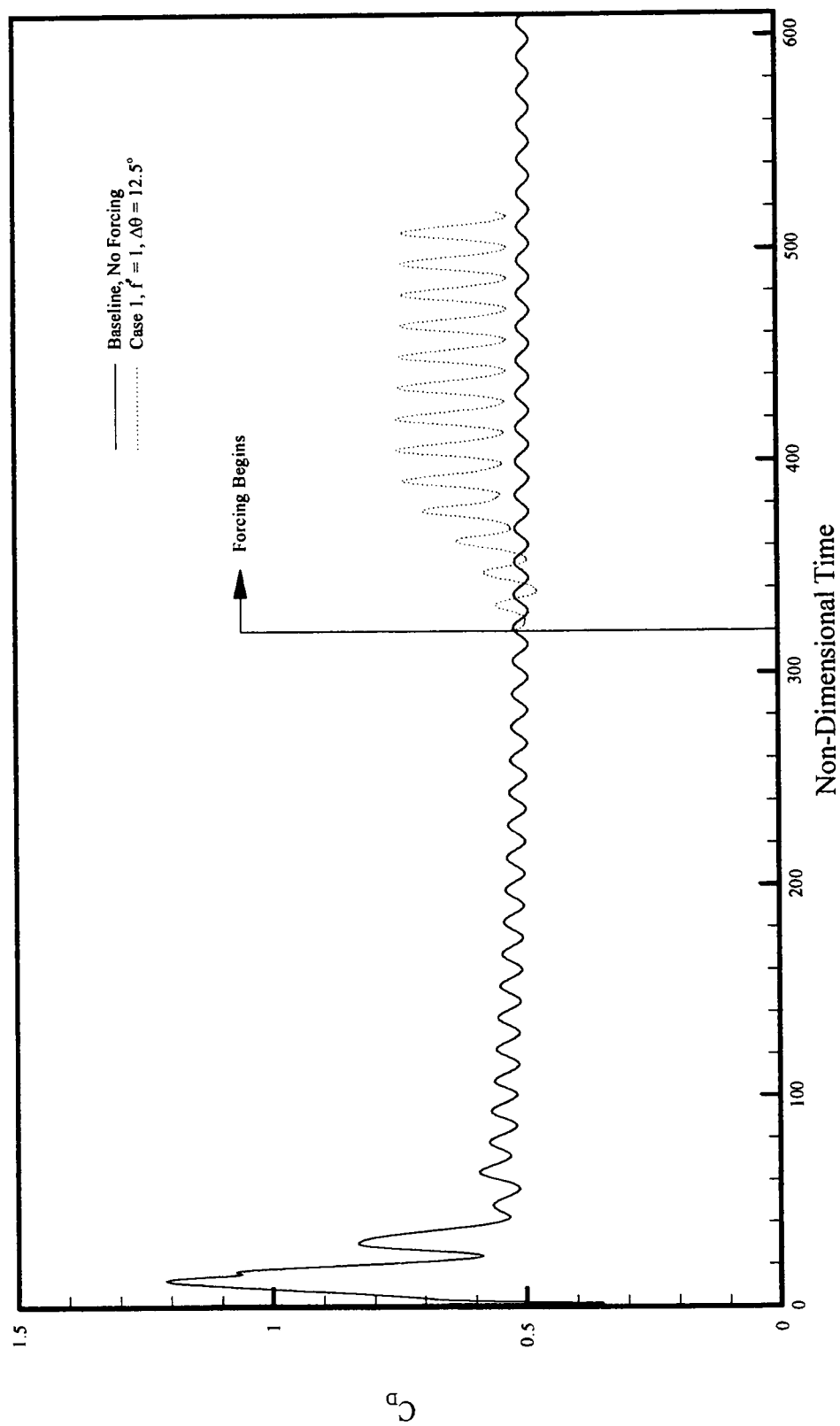


Figure 52: Drag Coefficient Histories

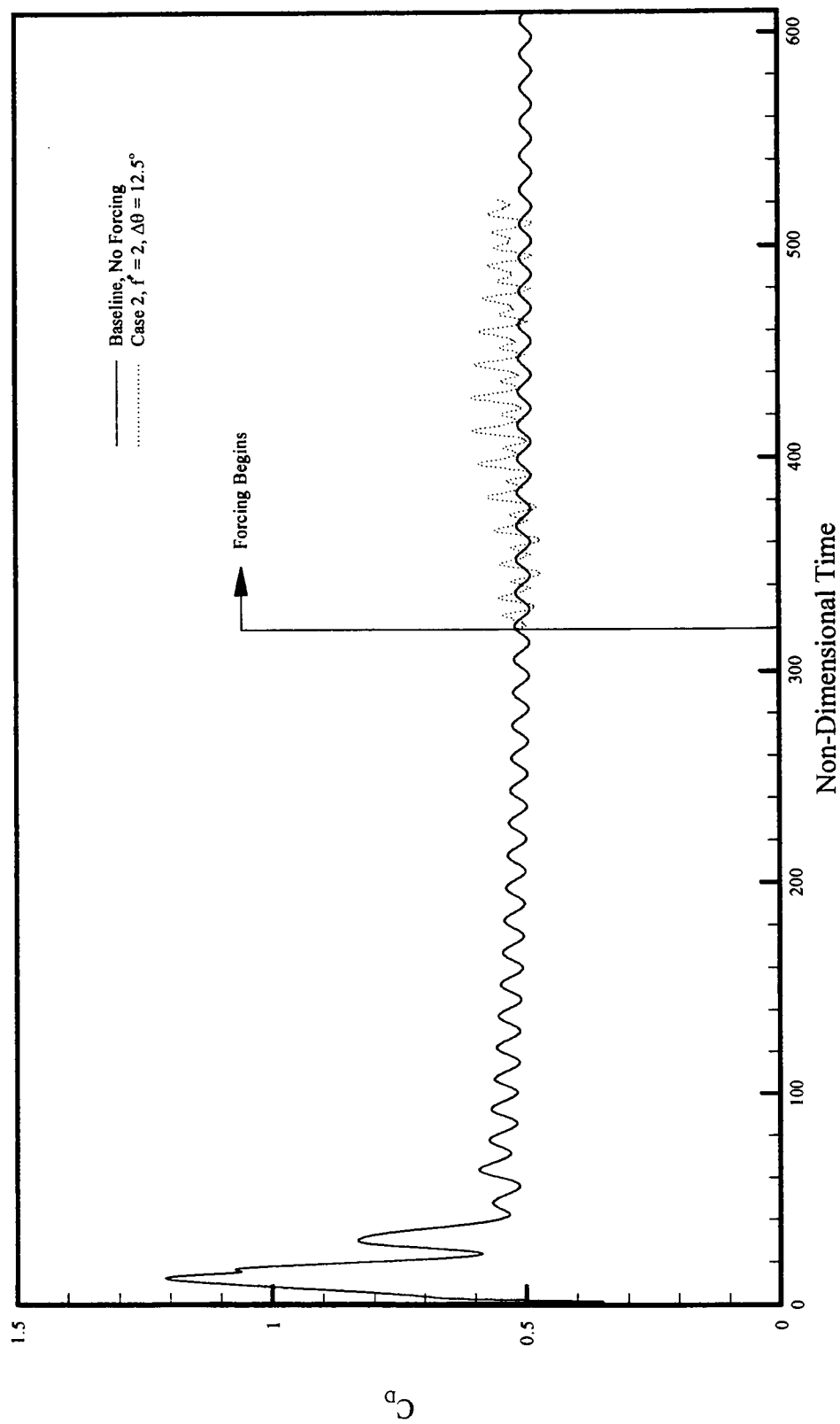


Figure 52: Drag Coefficient Histories (Continued)

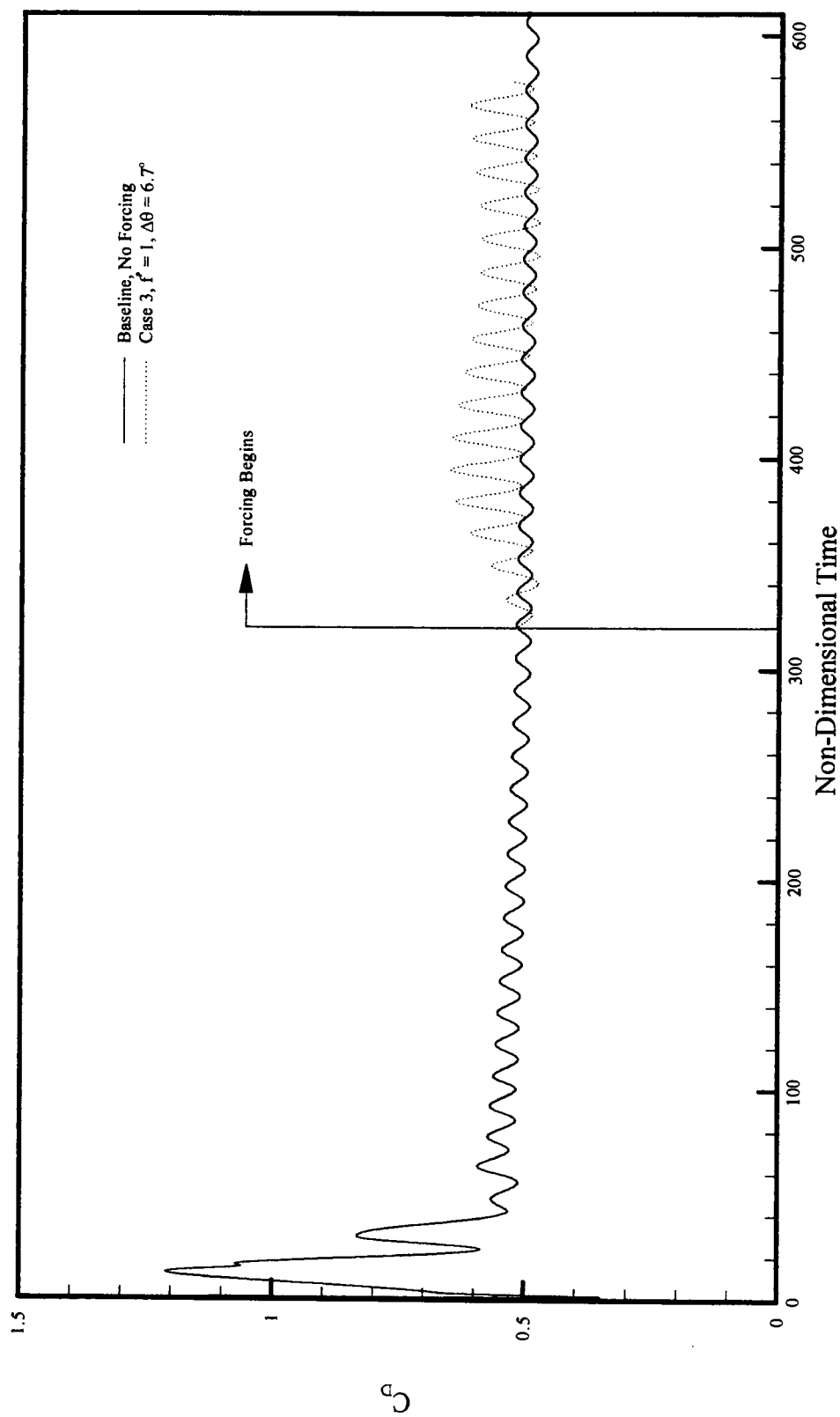


Figure 52: Drag Coefficient Histories (Concluded)

Case 2, and drops to roughly 0.8% for Case 3. These forcing levels are quite small compared to the magnitude of the flowfield changes due to the forcing. A wealth of data can be extracted from the numerically simulated flowfield. This data is next used in conjunction with the experimental findings to provide a better understanding of the physics involved in this vortex control.

The baseline flowfield is examined first. Figure 53 is a sequence of instantaneous streamline traces spanning approximately one shedding cycle. For unsteady flows the instantaneous streamlines do not convey as much information as particle traces or vorticity contours, but they can be useful in visualizing the flowfield. The streamlines are colored by pressure coefficient as an aid in visualizing the low-pressure vortex cores. Time during the shedding period is given by the cycle time, denoted by T_{cycle} in the figures. For a Strouhal number of 0.16, the non-dimensional shedding period is approximately 15-16. The periodic behavior of the flowfield can be broken down into several distinct phases. In the first illustration, a large leading edge (LE) eddy can be seen over the airfoil surface. The leading edge eddy expands and begins to convect away from the surface in the next two frames. During this time, a trailing edge (TE) vortex is formed as the flow at the trailing edge separates and is entrained by the LE eddy. As the trailing edge vortex grows, the LE vortex is caught between the strengthening TE vortex and the outer flow, and is stretched and flattened by the opposing streams. Meanwhile, the trailing vortex moves upstream under the action of the remnants of the LE vortex. When the TE vortex reaches a certain size, the shear layer and the remnants of the LE eddy are entrained down toward the airfoil surface under the induction of the TE vortex. This occurs at approximately

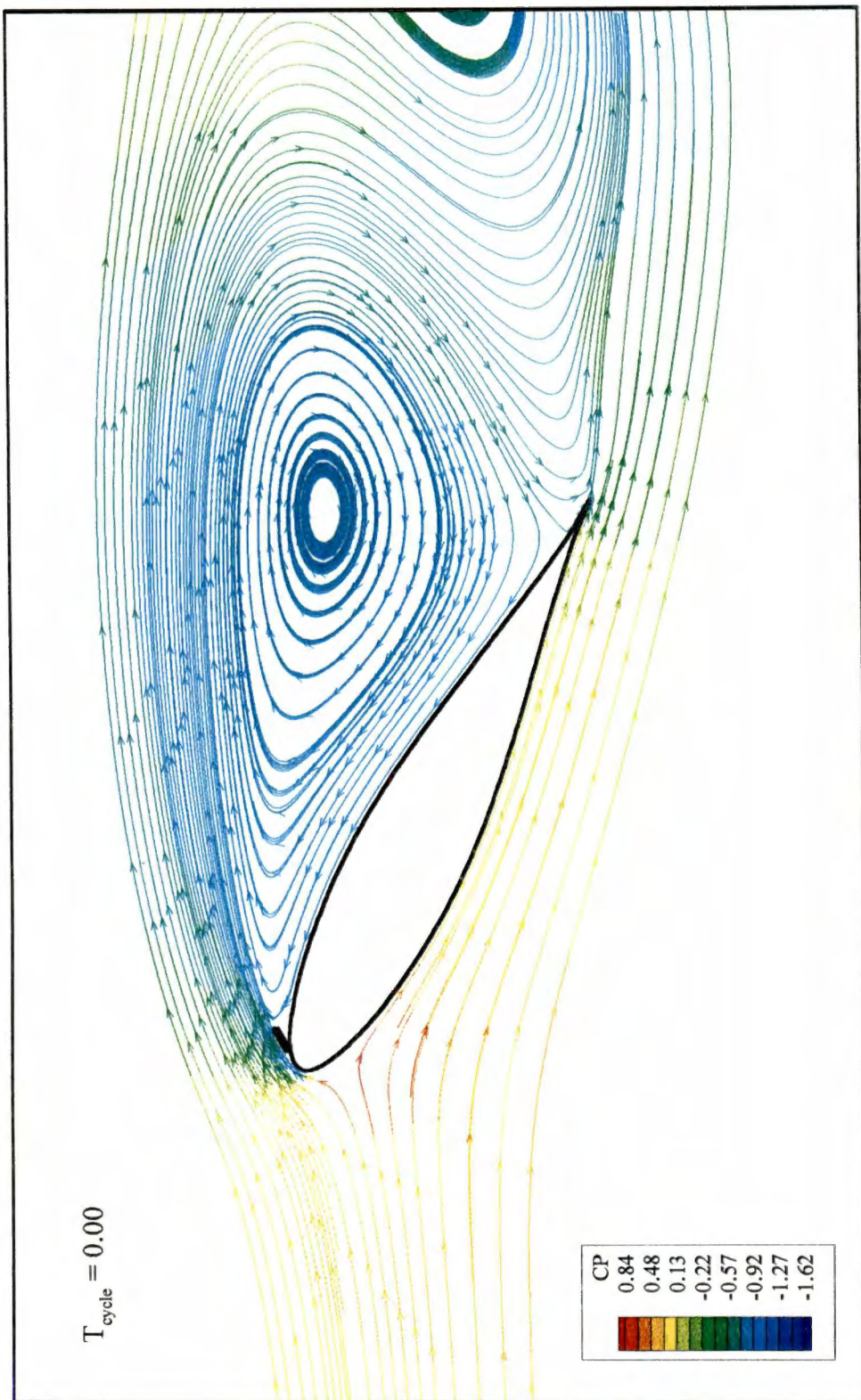


Figure 53: Stream Traces for Baseline Flowfield

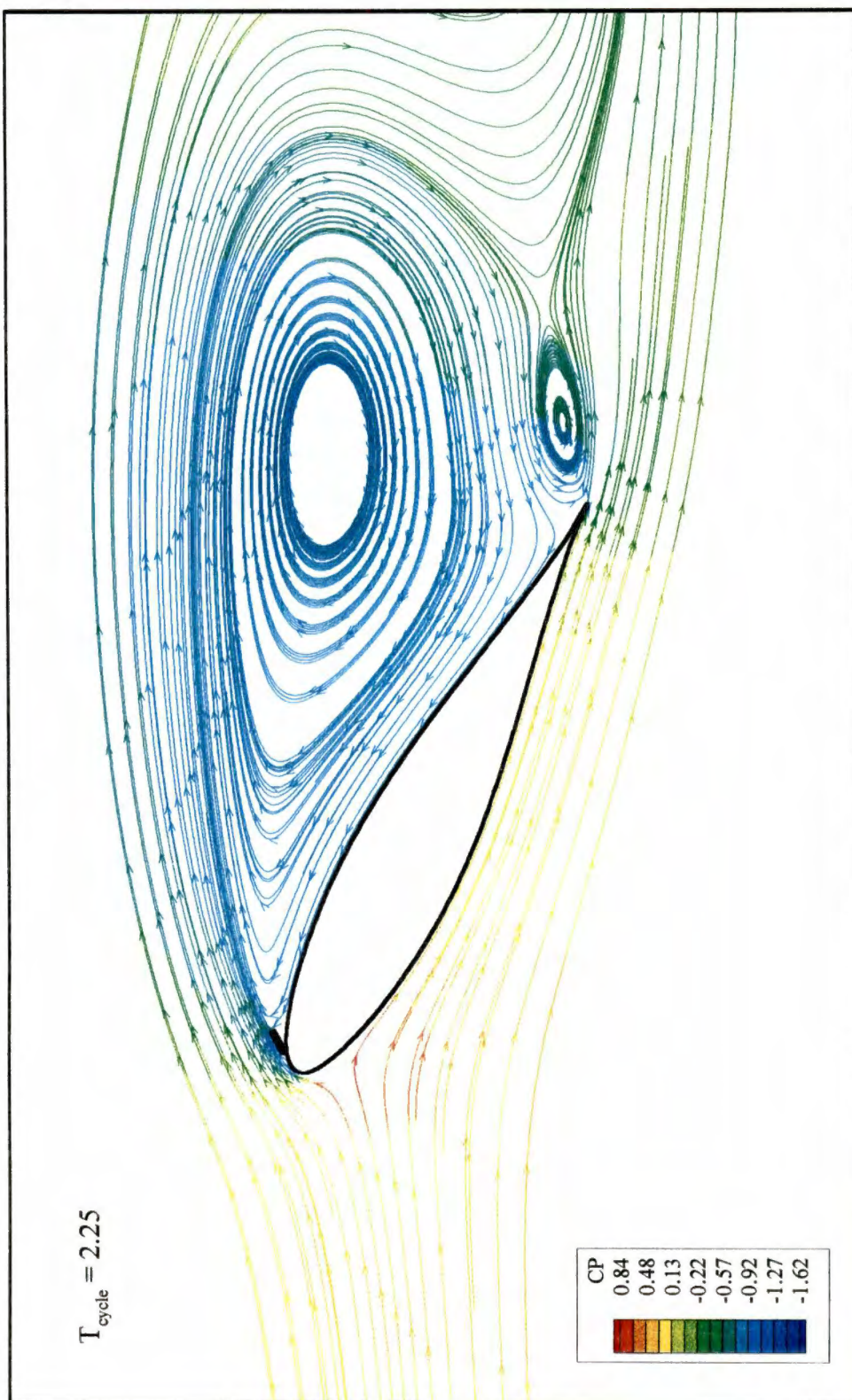


Figure 53: Stream Traces for Baseline Flowfield (Continued)

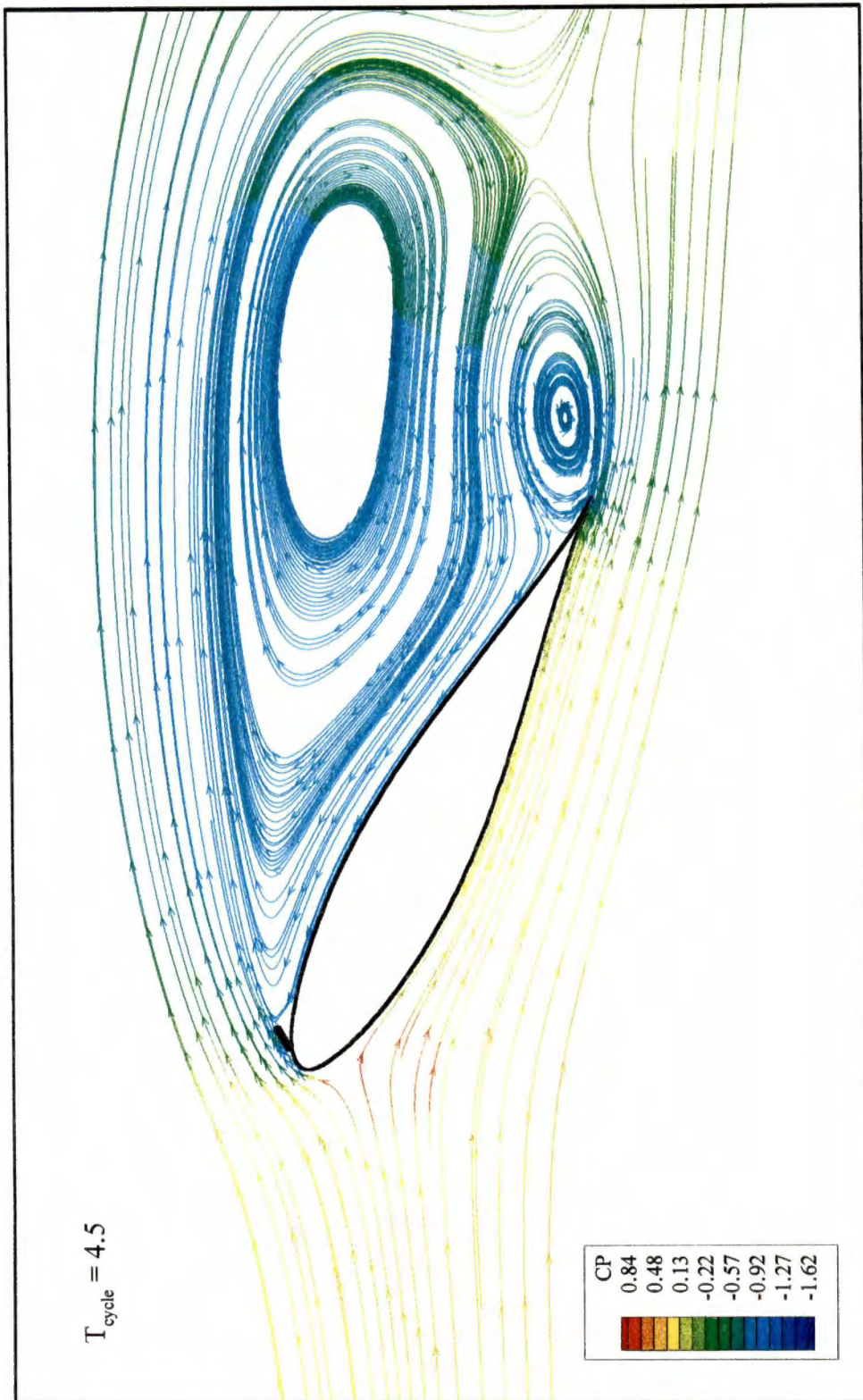


Figure 53: Stream Traces for Baseline Flowfield (Continued)

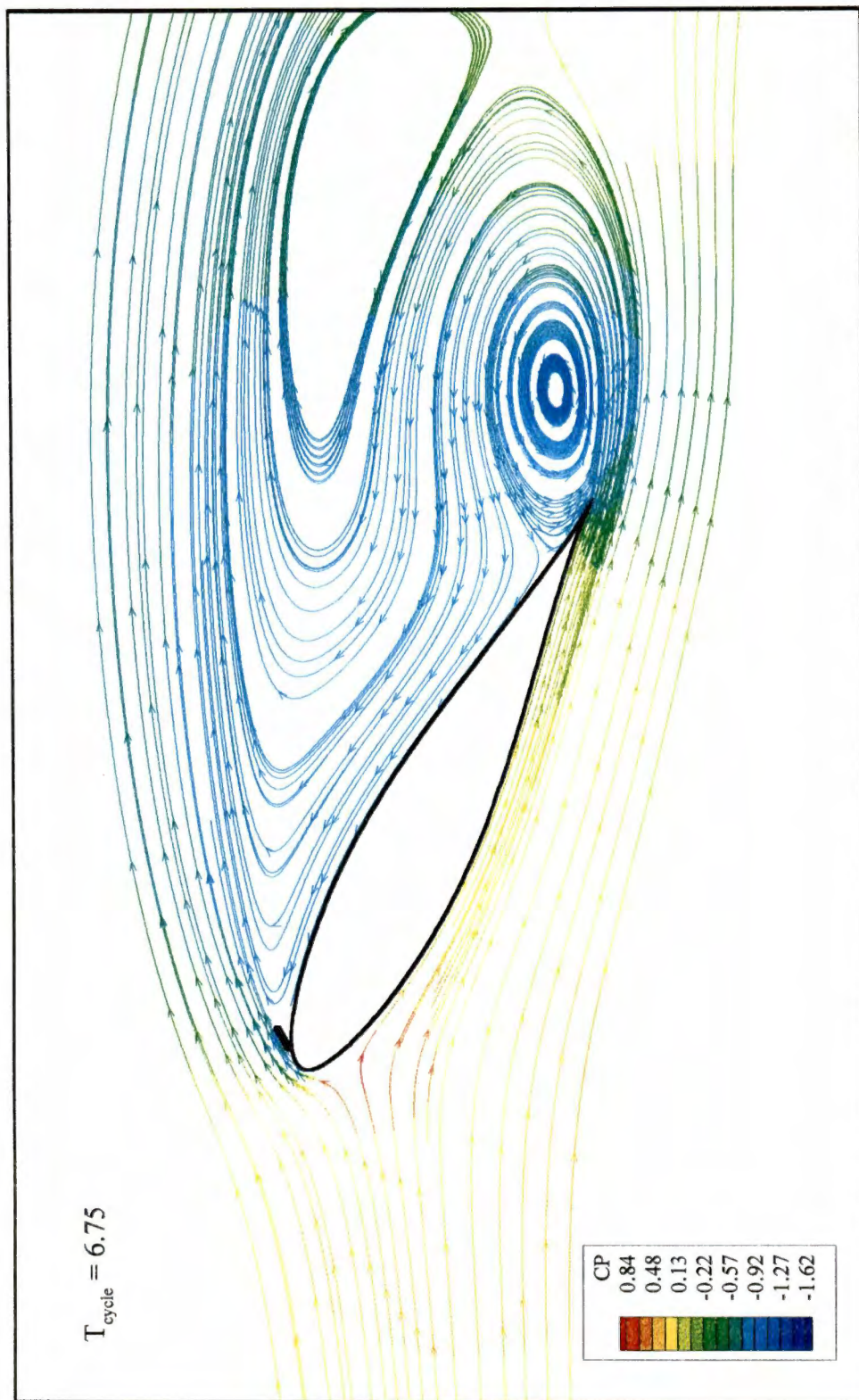


Figure 53: Stream Traces for Baseline Flowfield (Continued)

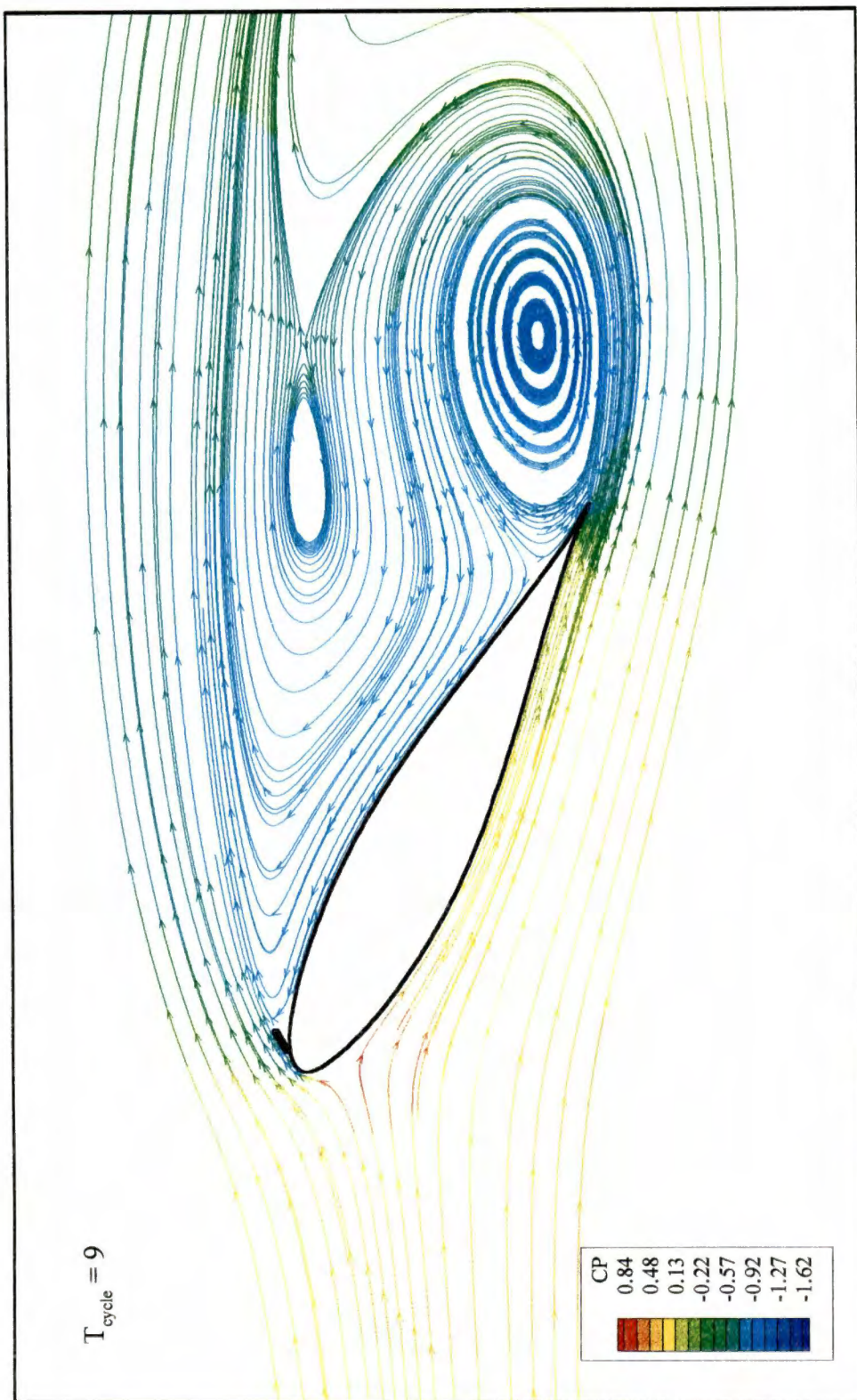


Figure 53: Stream Traces for Baseline Flowfield (Continued)

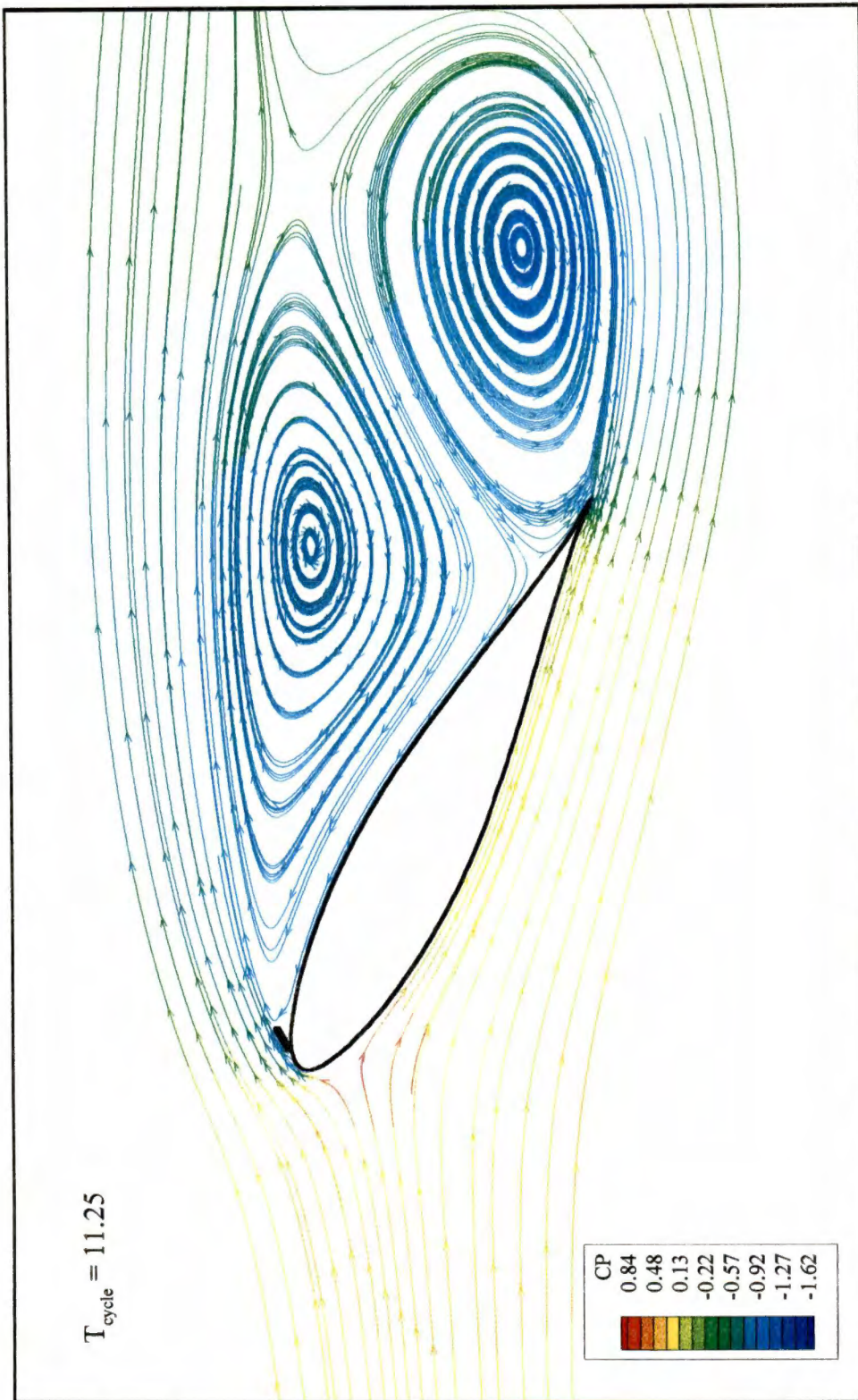


Figure 53: Stream Traces for Baseline Flowfield (Continued)

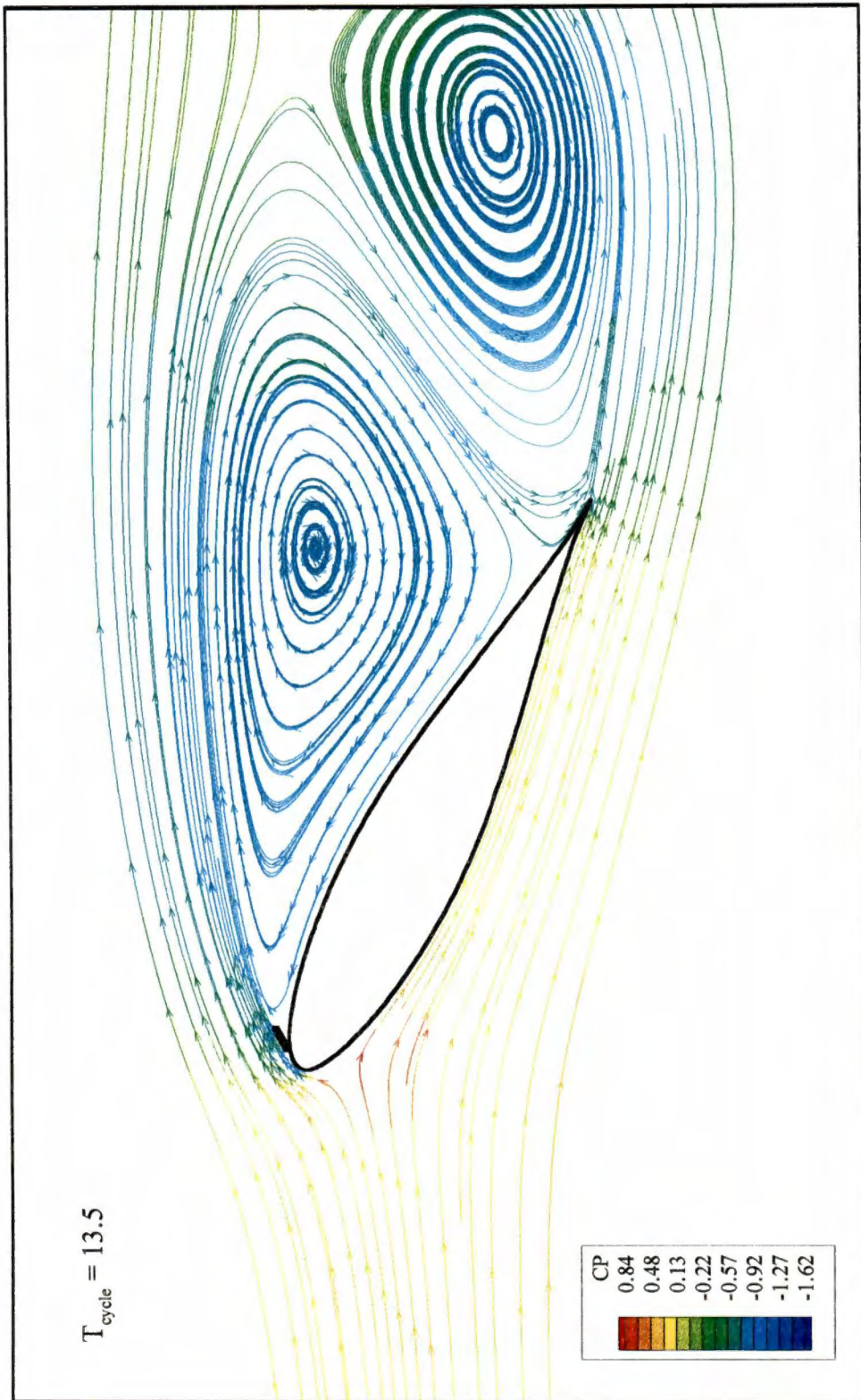


Figure 53: Stream Traces for Baseline Flowfield (Continued)

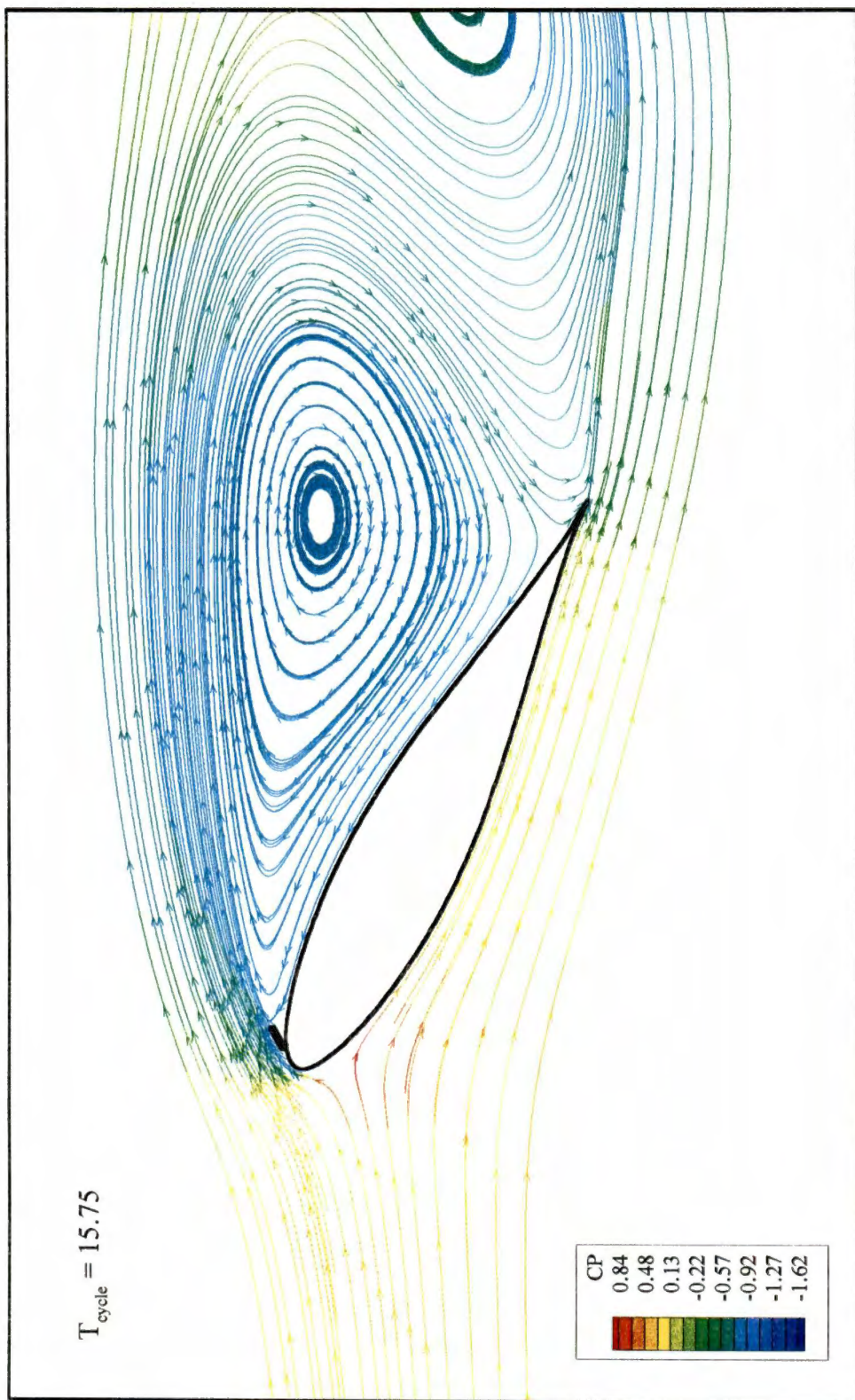


Figure 53: Stream Traces for Baseline Flowfield (Concluded)

$T_{\text{cycle}} = 9$ in the sequence. This induced turning marks the beginning of the formation of a new leading edge vortex. It also marks the end of the upstream motion of the trailing edge vortex. The trailing edge eddy begins to move downstream under the action of its image vortex located within the airfoil surface. As the TE vortex moves away, it also helps to accelerate the flow in the leading edge vortex core. The leading edge vortex grows as it entrains more of the shear layer leaving the leading edge. This vortex grows along the surface, bringing the cycle back to the starting point, at roughly $T_{\text{cycle}} = 15.75$.

Figure 54 shows vorticity contours for the baseline flowfield during the same sequence presented above. The contours show the periodic growth of vortex eddies at the leading and trailing edges. However, the structure of the eddies is not strongly defined. In fact, the leading edge eddy shows very little variation during the cycle. The leading edge shear layer is slightly displaced upward into the oncoming flow as the trailing edge vortex grows, but this motion is nearly imperceptible.

The addition of flap forcing is revealed in the next two sets of vorticity contours. Figure 55 shows the flowfield behavior for Case 1, with $f^* = 1$. The flowfield is marked by well-defined vortical eddies that develop and convect away from the airfoil. Compared to the baseline contours, the leading and trailing edge vortices can be easily observed. The illustrations begin as the trailing edge vortex is starting to convect away from the airfoil. The leading edge shear layer shows evidence of coalescence into strong, individual eddies. At $T_{\text{cycle}} = 5.4$, the TE vortex is moving out of the picture, and the leading edge shear layer shows a well-defined eddy developing. By $T_{\text{cycle}} = 8.1$, a strong leading edge vortex has formed and is beginning to convect away. The trailing edge vortex has moved out of



Figure 54: Vorticity Contours for Baseline Flowfield

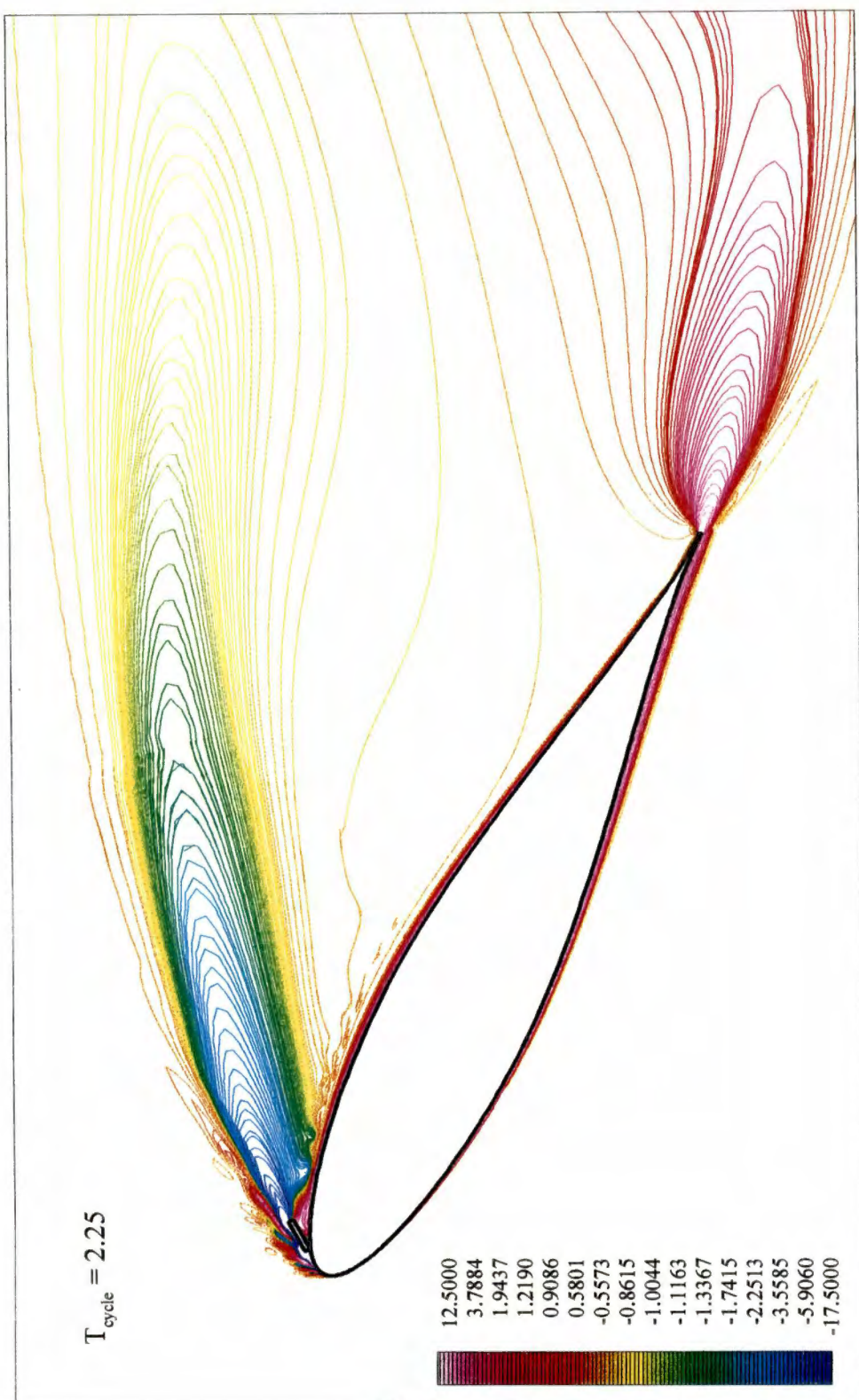


Figure 54: Vorticity Contours for Baseline Flowfield (Continued)

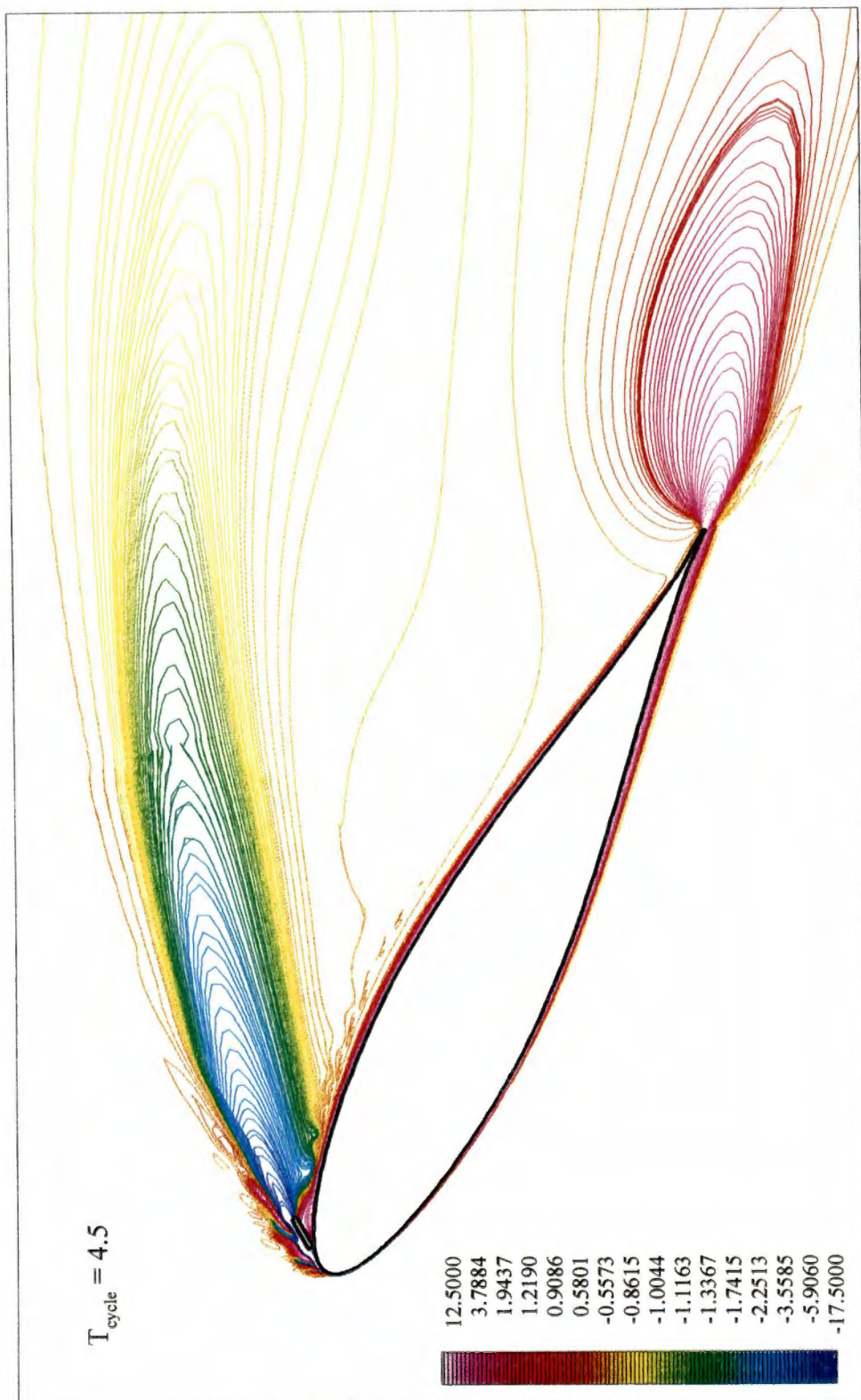


Figure 54: Vorticity Contours for Baseline Flowfield (Continued)



Figure 54: Vorticity Contours for Baseline Flowfield (Continued)

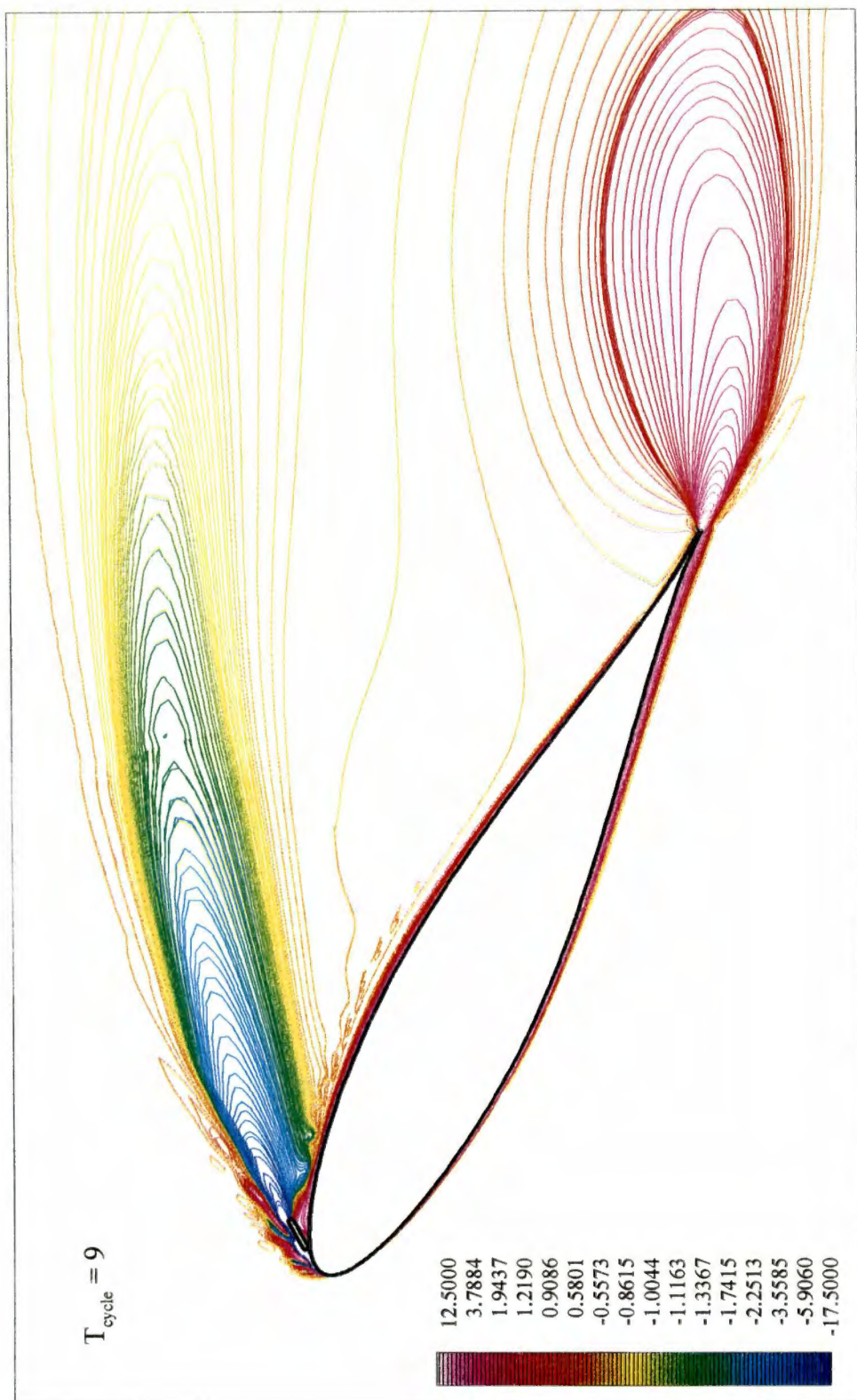


Figure 54: Vorticity Contours for Baseline Flowfield (Continued)

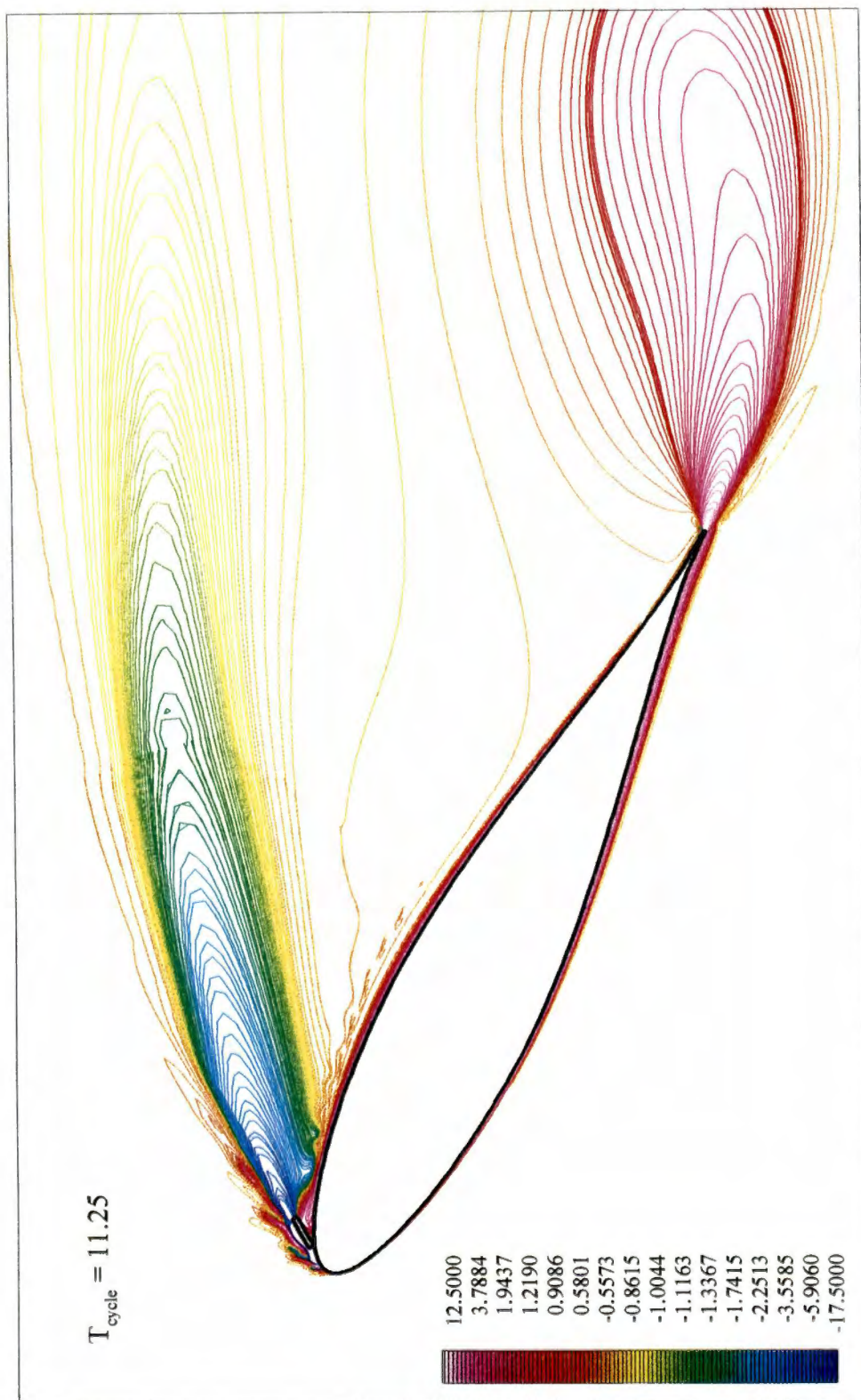


Figure 54: Vorticity Contours for Baseline Flowfield (Continued)



Figure 54: Vorticity Contours for Baseline Flowfield (Continued)

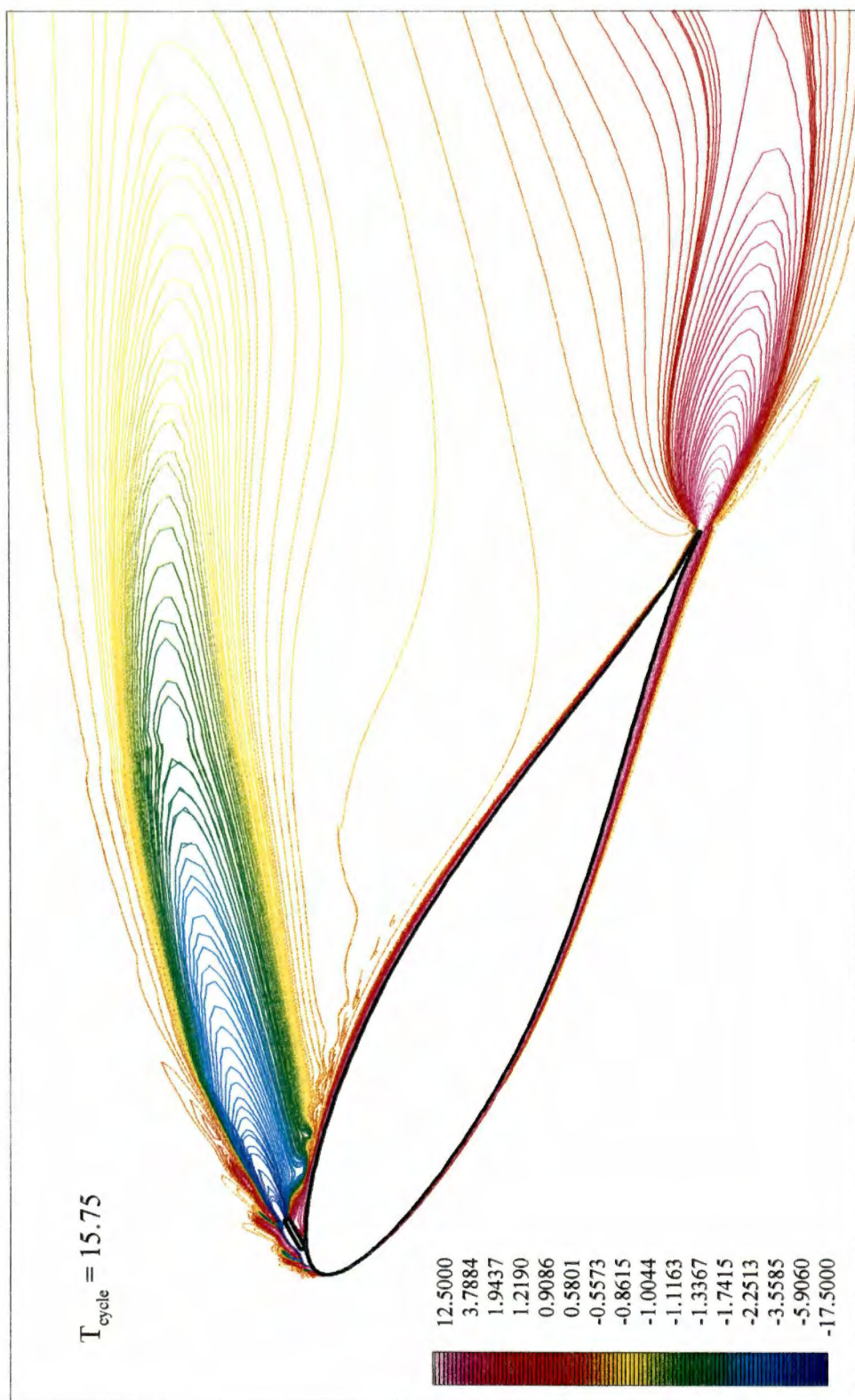


Figure 54: Vorticity Contours for Baseline Flowfield (Concluded)

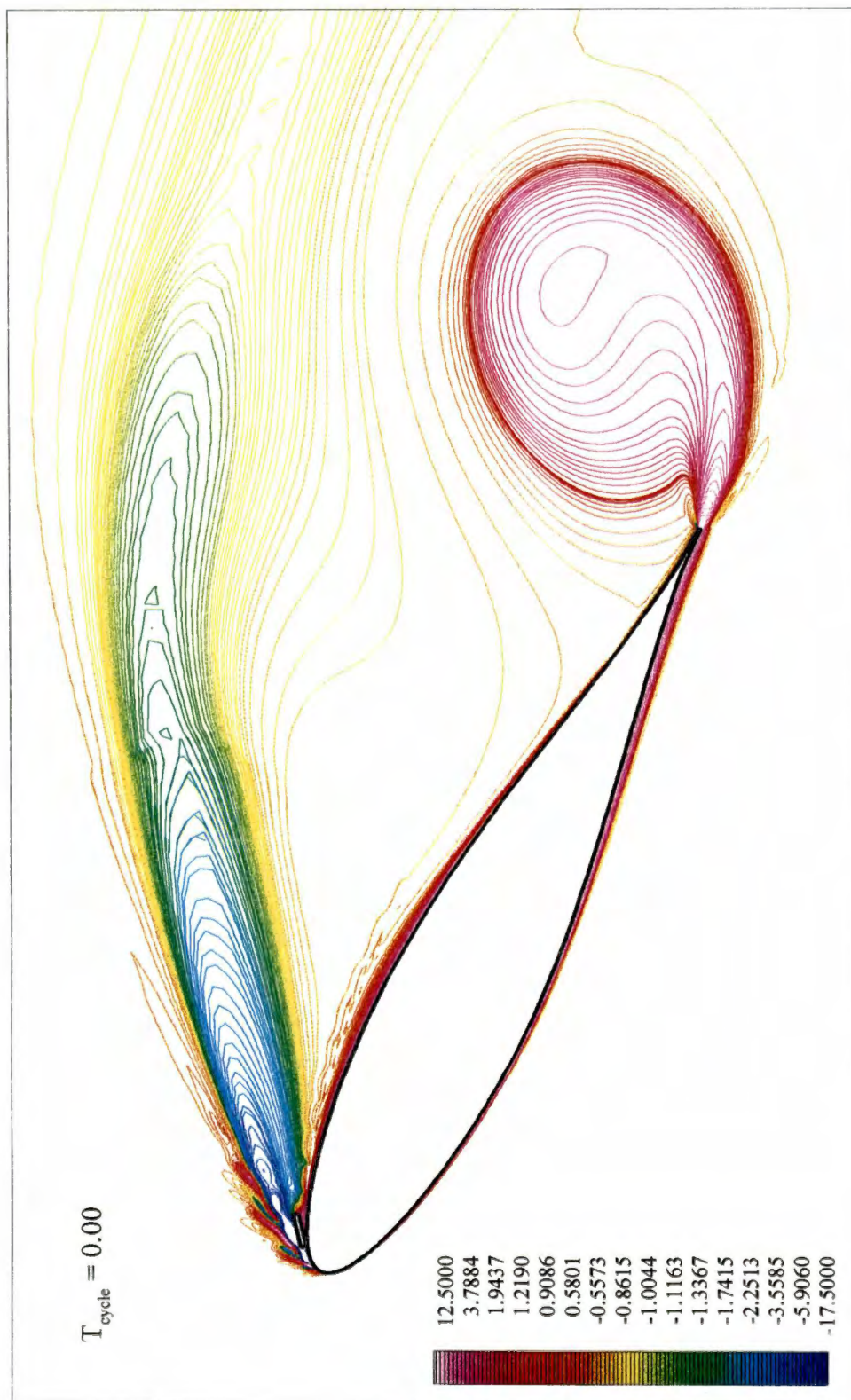


Figure 55: Vorticity Contours for Case 1, $f^* = 1$

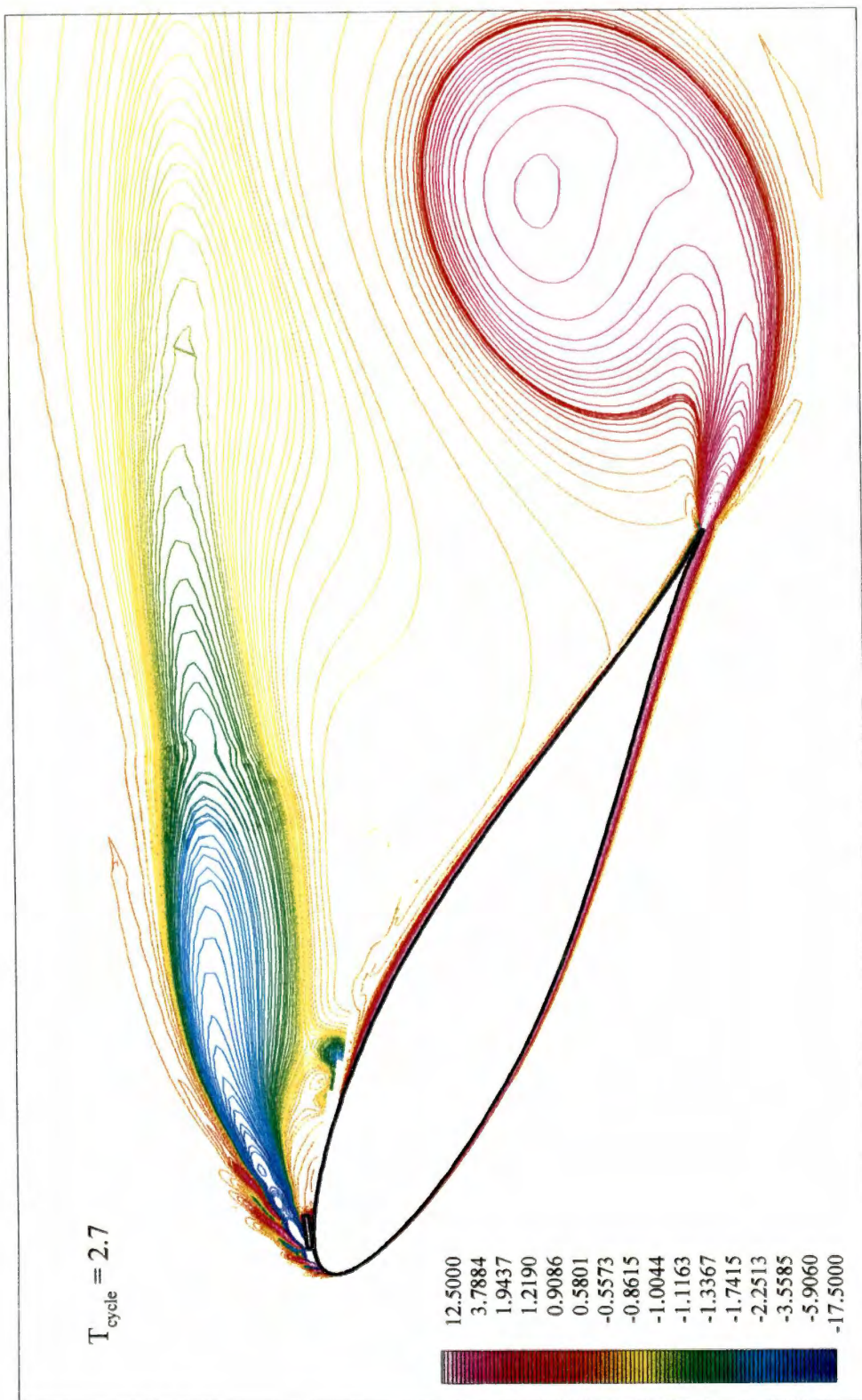


Figure 55: Vorticity Contours for Case 1, $f' = 1$ (Continued)



Figure 55: Vorticity Contours for Case 1, $f^* = 1$ (Continued)



Figure 55: Vorticity Contours for Case 1, $f' = 1$ (Continued)



Figure 55: Vorticity Contours for Case 1, $f' = 1$ (Continued)

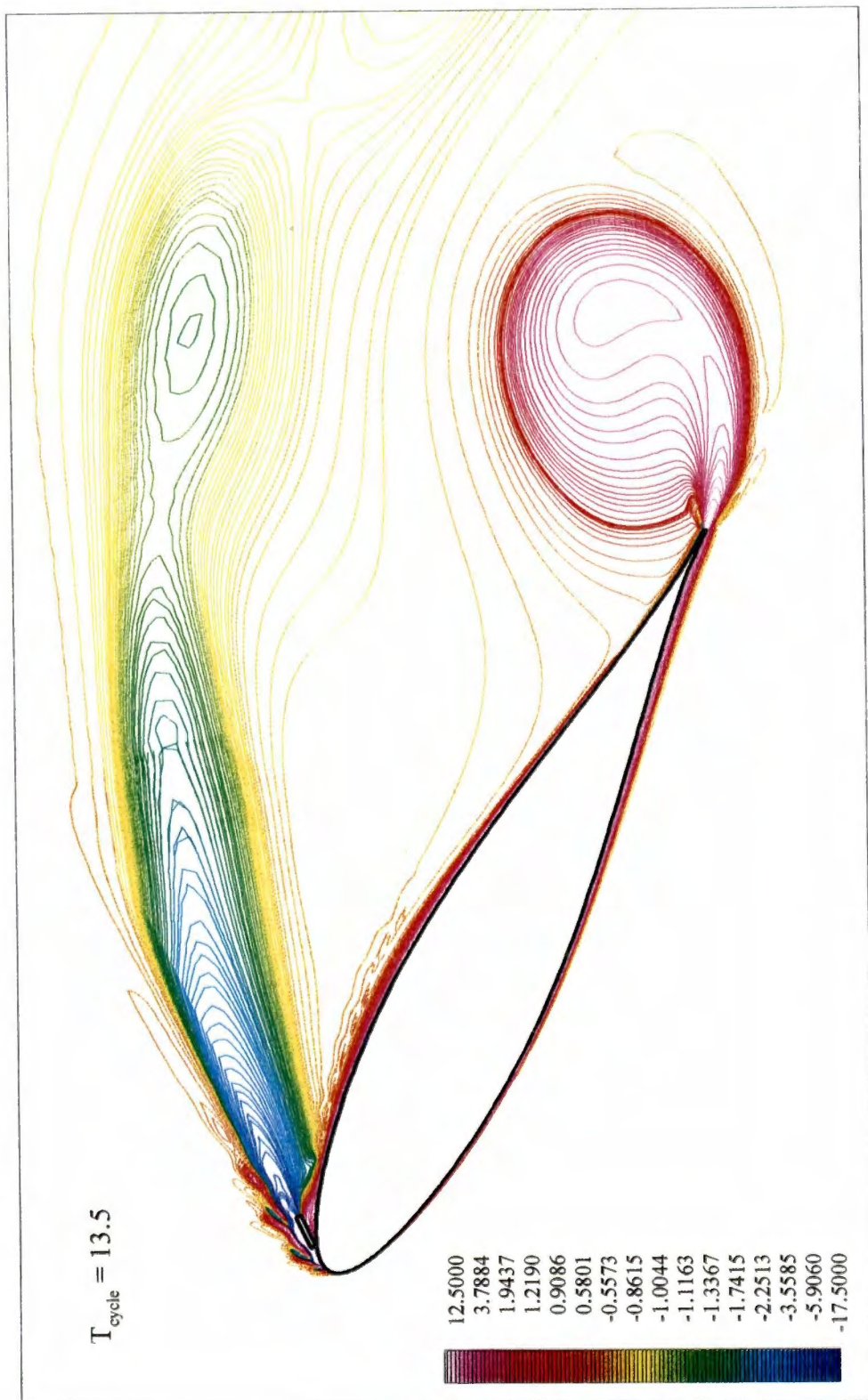


Figure 55: Vorticity Contours for Case 1, $f' = 1$ (Continued)

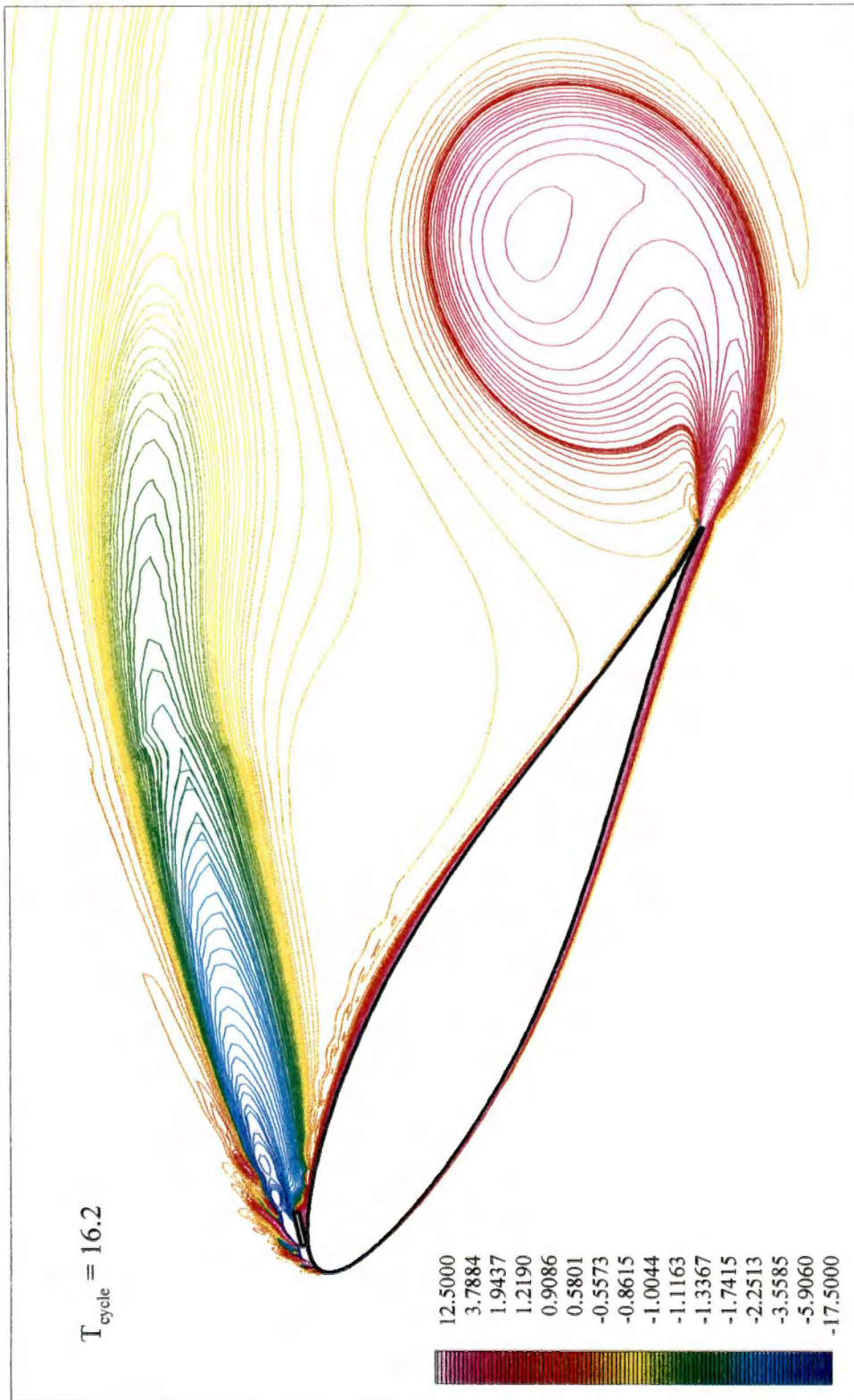


Figure 55: Vorticity Contours for Case 1, $f' = 1$ (Concluded)

the picture entirely, and a new TE eddy is beginning to form. As the trailing edge vortex increases in strength, the fluid in the LE shear layer is stretched out between the TE vortex and the oncoming flow. Distinct eddies can be seen forming just downstream of the flap. By the time $T_{\text{cycle}} = 13.5$ the trailing edge vortex has reached maximum growth and begins to convect away, and in the final frame the flowfield has essentially returned to the starting point in the cycle.

In Figure 56, vorticity contours for Case 2, $f^* = 2$, are shown. The cycle is entered at a point when a strong leading edge vortex is moving downstream, and a trailing edge vortex is beginning to form. Unlike Case 1, a second strong vortex is generated at the leading edge during the development of the trailing edge vortex, at $T_{\text{cycle}} = 5.4$. This effectively negates the impact of the trailing edge vortex on the aerodynamic coefficients. The growth of the trailing edge vortex is also limited from that seen in Case 1. As the cycle progresses, the trailing edge vortex grows and moves away from the airfoil, and another leading edge vortex develops, as seen at $T_{\text{cycle}} = 13.5$. Thus, two strong leading edge vortices are formed during the shedding cycle for the case where the flap excitation is at twice the shedding frequency.

Based on the qualitative behavior of the flowfield seen in the vorticity contour plots, the most significant effect of the flap motion seems to be a modulation of the vorticity feeding from the leading edge into the LE vortex. If the vorticity is introduced into the flowfield at the right instant in the global cycle, the leading edge vortex is strengthened, and the trailing edge vortex is not destructively affected. This "global receptivity" appears to be the key mechanism controlling the flowfield behavior.

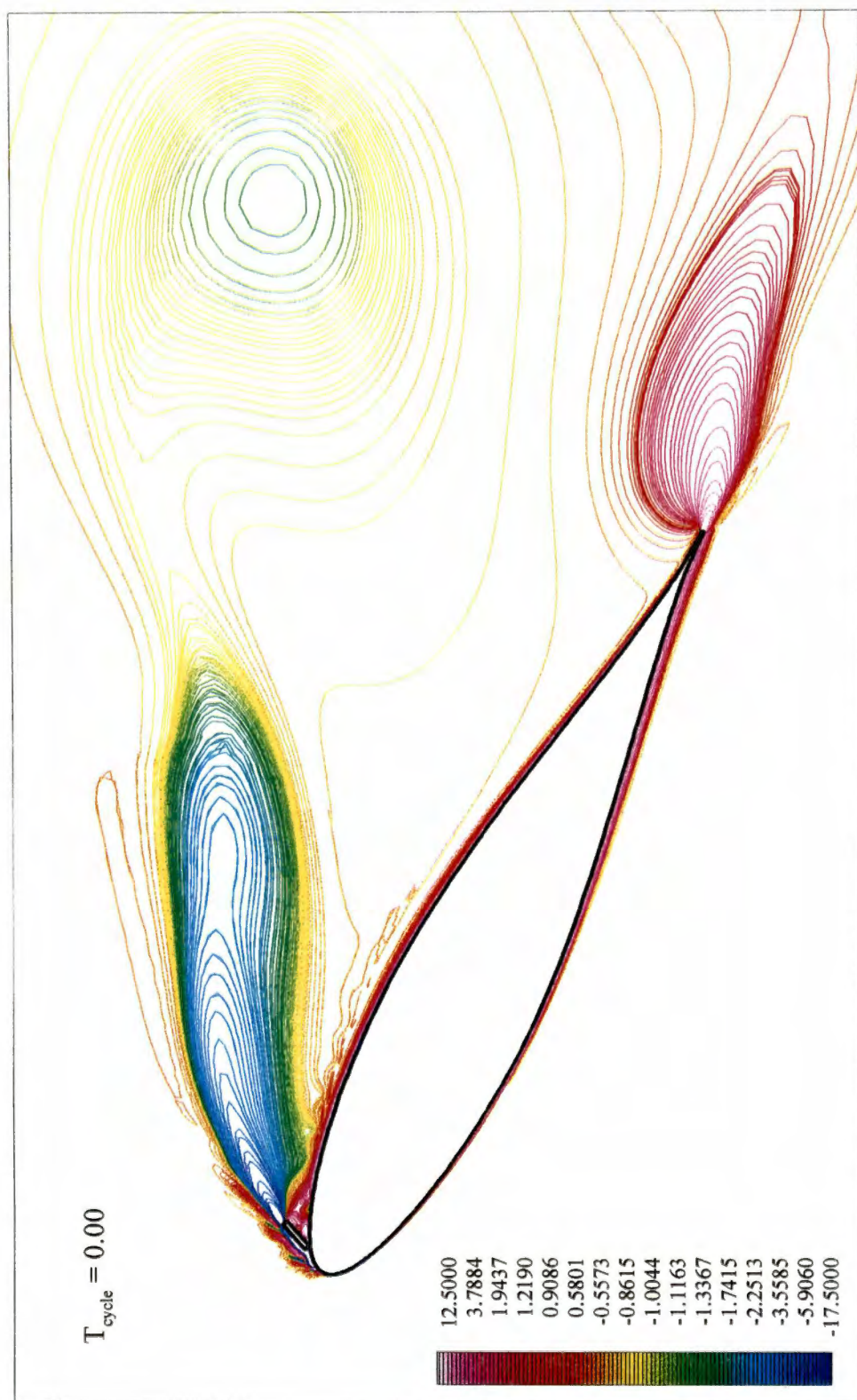


Figure 56: Vorticity Contours for Case 2, $f^* = 2$

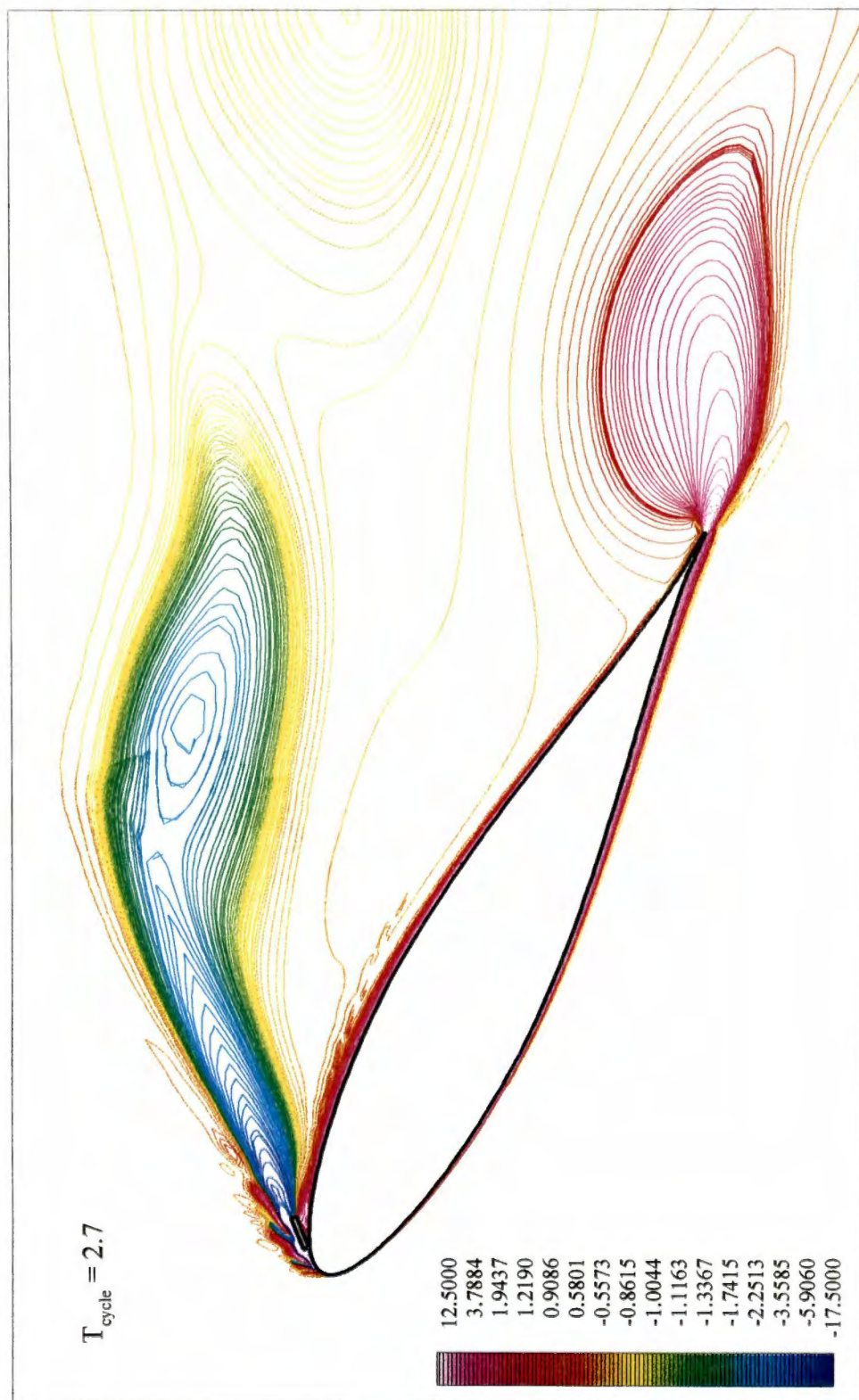


Figure 56: Vorticity Contours for Case 2, $f' = 2$ (Continued)

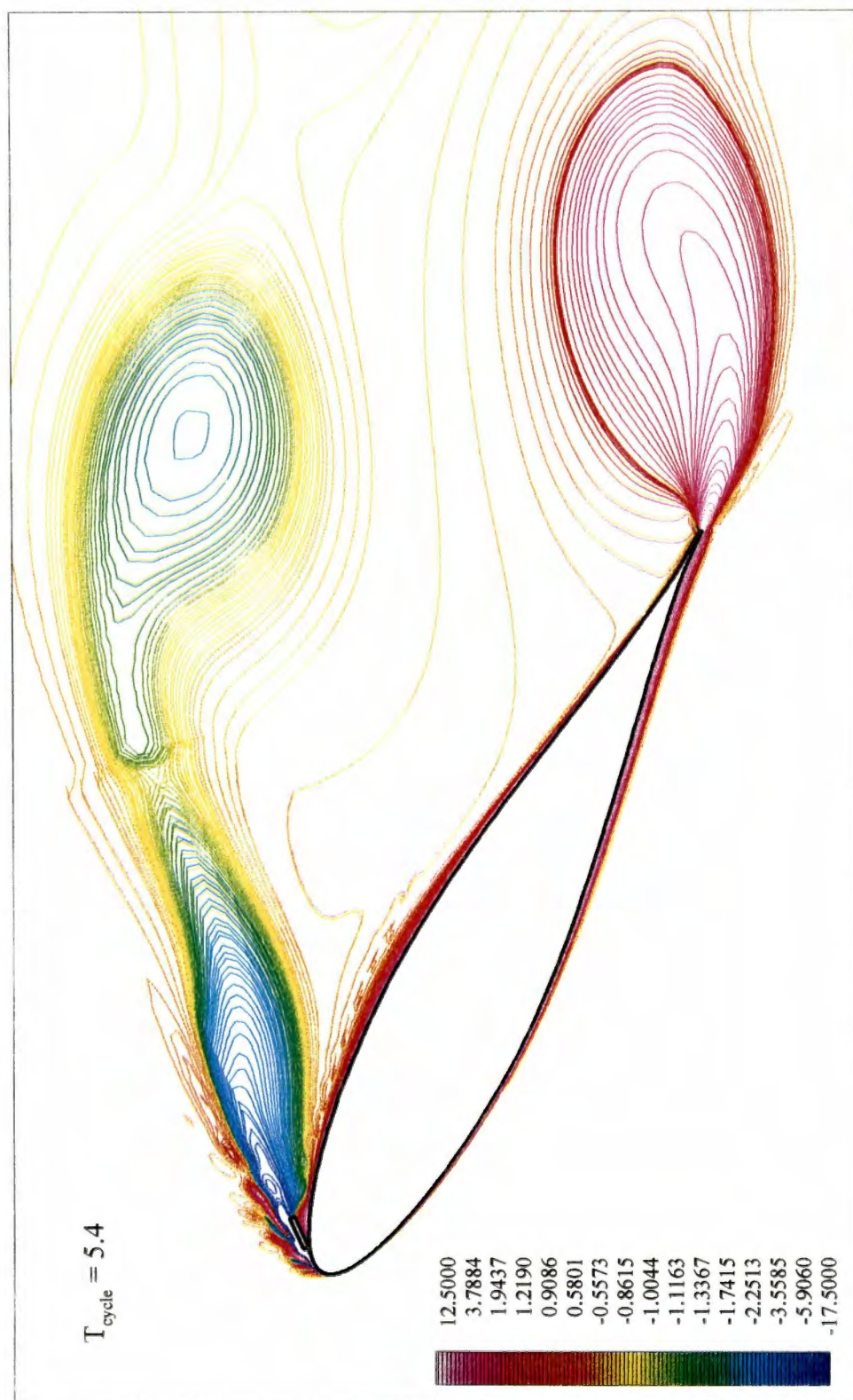


Figure 56: Vorticity Contours for Case 2, $f^* = 2$ (Continued)

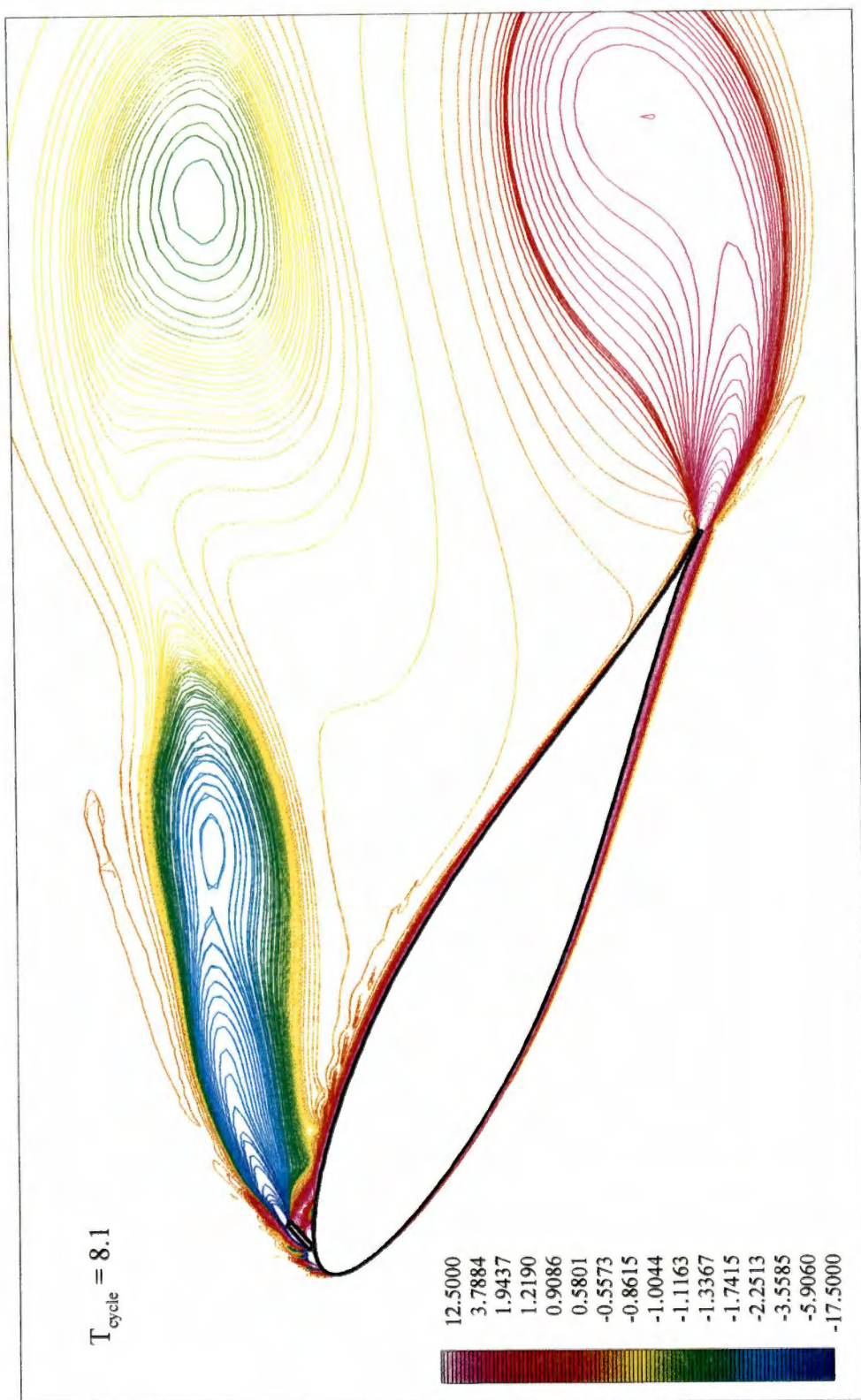


Figure 56: Vorticity Contours for Case 2, $f' = 2$ (Continued)

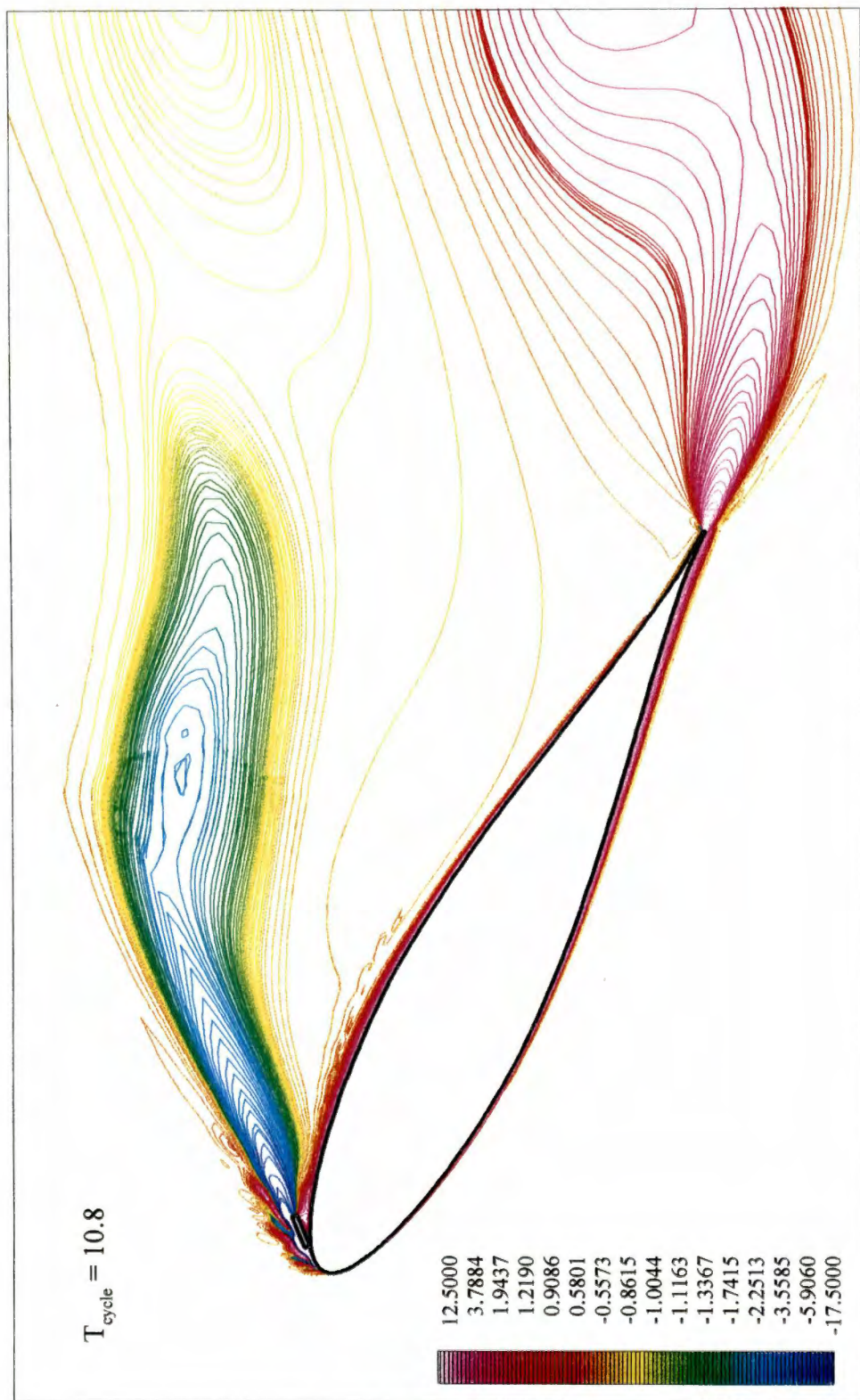


Figure 56: Vorticity Contours for Case 2, $f^* = 2$ (Continued)



Figure 56: Vorticity Contours for Case 2, $f^* = 2$ (Continued)



Figure 56: Vorticity Contours for Case 2, $f' = 2$ (Concluded)

Receptivity describes the ability of a particular system to respond to an input. In order for a forcing function to drive a system response, the forcing signal must occur in a way that the system can "see" it. In the study of vibrations, the concept of receptivity is similar to the transmissibility of a signal, which indicates how much of an input signal can be transmitted to a vibrational system. As a simple example, consider a spring-mass-damper system. Such a system possesses a natural frequency. Forcing at this frequency leads to a condition of resonance, even if the forcing amplitude is quite small. Forcing at frequencies far above the natural frequency have little or no effect on the system. In fact, the amplitude response of such a system asymptotically approaches zero as the input frequency goes to infinity.

An inspection of vorticity and velocity data in the leading edge and trailing edge regions can serve to quantify some of these observations. Figure 57 shows the vorticity in the proximity of the leading edge separation point for the baseline flowfield. This data is presented as a time history and in a Fourier decomposition, where Ω is the transformed vorticity. As expected, the spectral analysis reveals a harmonic component at the shedding Strouhal number of approximately 0.16. A second harmonic component exists at a Strouhal number of 7.2. This high frequency corresponds to a periodic behavior seen in the shear layer at the separation point, and is in fact the shear layer instability frequency, as discussed below. The existence of the sidebands surrounding the high frequency peak appears to be related to the non-linear interaction between the shedding frequency and the shear layer instability frequency. The trailing edge vorticity content is shown in Figure 58.

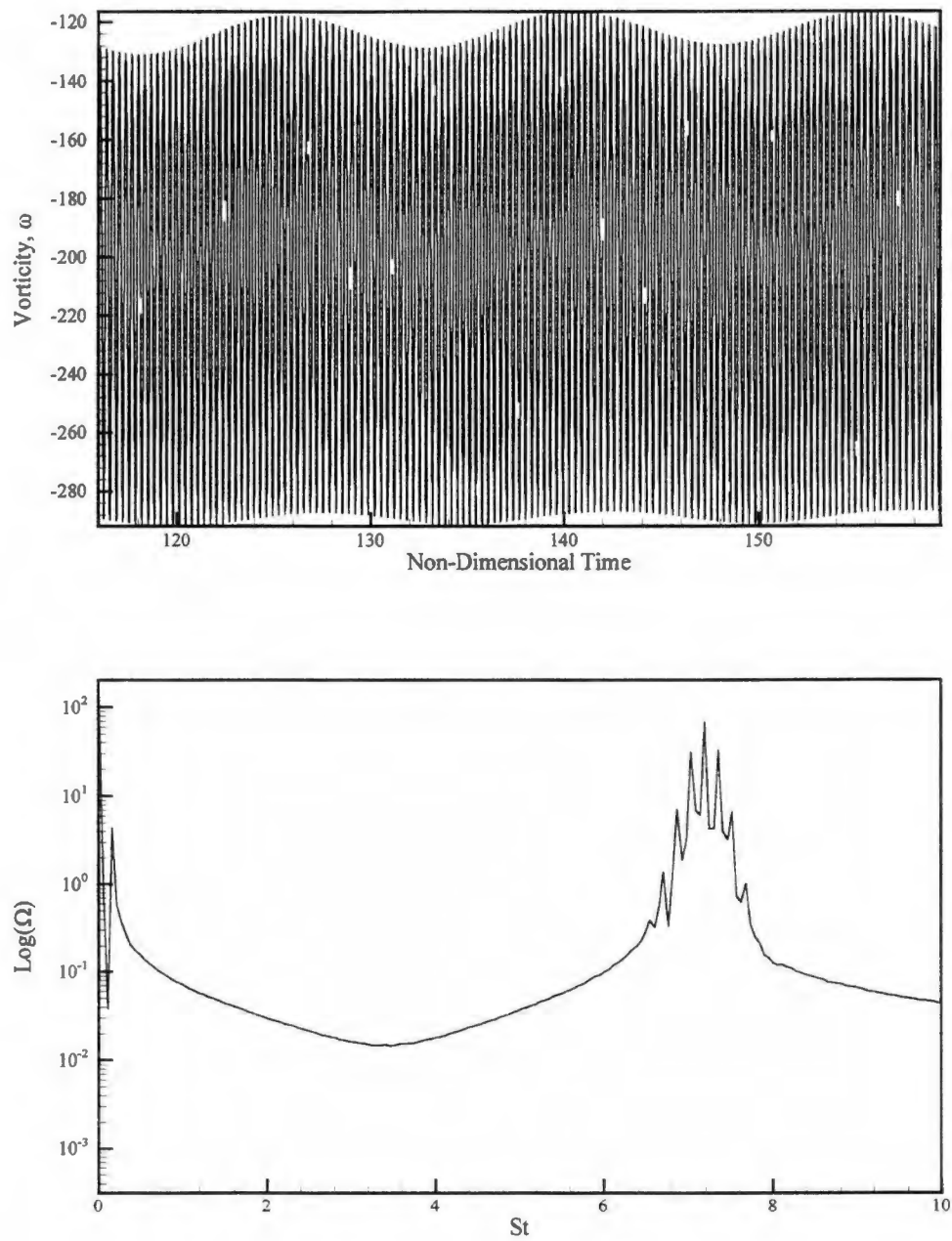


Figure 57: Vorticity in Leading Edge Region for Baseline

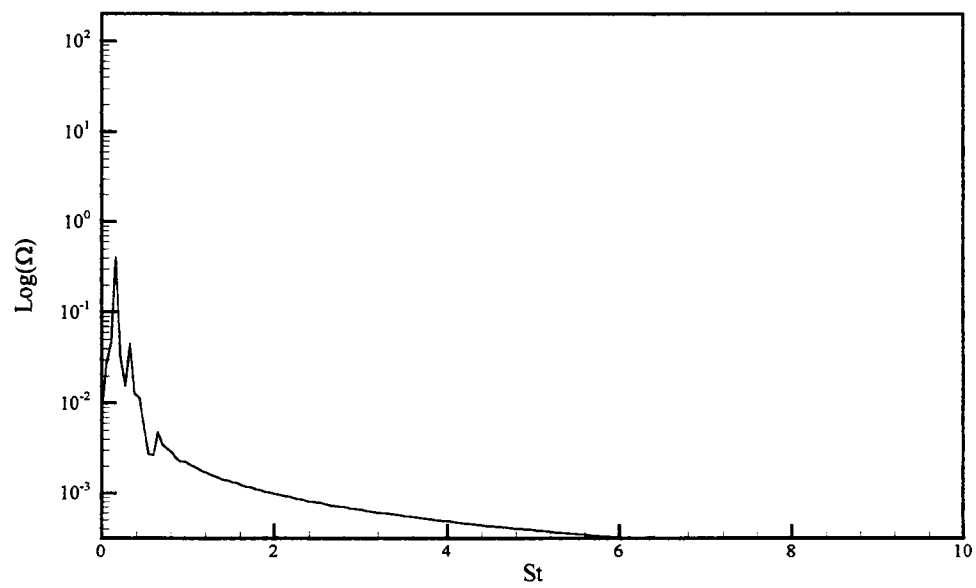
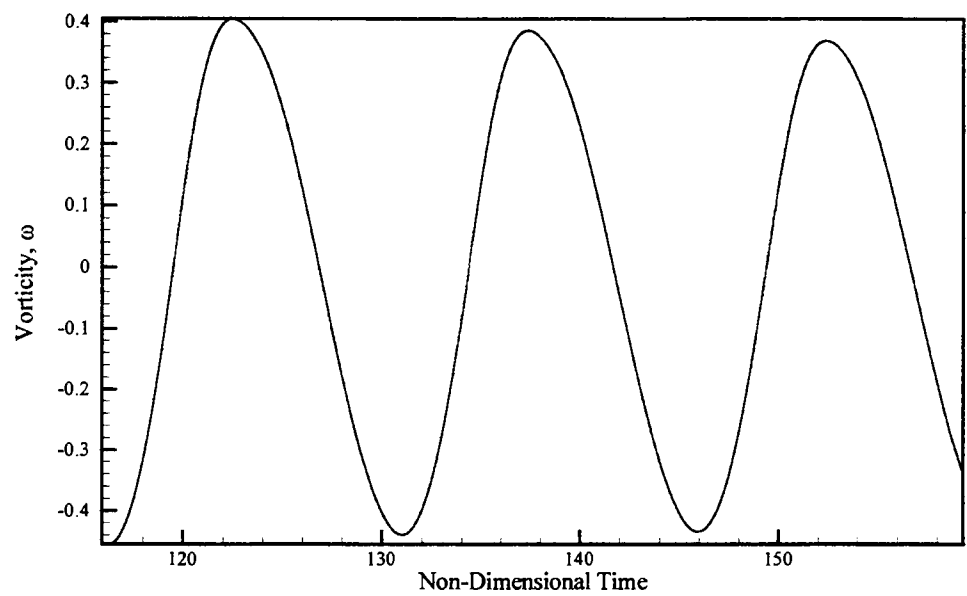


Figure 58: Vorticity in Trailing Edge Region for Baseline

At the trailing edge, it can be seen that the vorticity is very nearly unimodal, at a Strouhal number of 0.16. A super-harmonic at a Strouhal number of 0.32 is also present.

The introduction of flap motion results in the leading and trailing edge vorticity behavior shown in Figures 59 to 62. For $f^* = 1$, the harmonic component at the shedding frequency is significantly increased at the leading and trailing edges. The super-harmonics of the shedding frequency are also larger for $f^* = 1$. For $f^* = 2$, the leading edge vorticity shows some increase at the fundamental harmonic, although this increase is not as large as that for $f^* = 1$. In addition, the bandwidth spread at the shear layer instability frequency is somewhat larger than that for $f^* = 1$, due to the injection of a third separate frequency into the system. However, the trailing edge vorticity shows a sizable decrease in the harmonic component when the flap forcing frequency is 2. The trailing edge vortex experiences significant interference due to the presence of the co-developing leading edge vortex at this forcing frequency. However, in all cases the fundamental harmonic frequency does not change, i.e. the shedding frequency is basically unaltered.

It should also be noted that the shedding and shear layer instability modes remain clearly distinct, even with forcing. Some small variation of the harmonic content at the shear layer instability frequency is seen. However, nothing is present to indicate a significant change in the shear layer instability growth beyond that seen in the baseline case. For example, the spectral peak at $St \sim 7.2$ remains nearly constant for all cases, and there is no indication of any pairing and merging of vortices created by the shear layer rollup. Such behavior would be revealed by a spectral peak around $St \sim 3.6$, but no such peak is observed. The main observable change is the increase of energy in the shedding

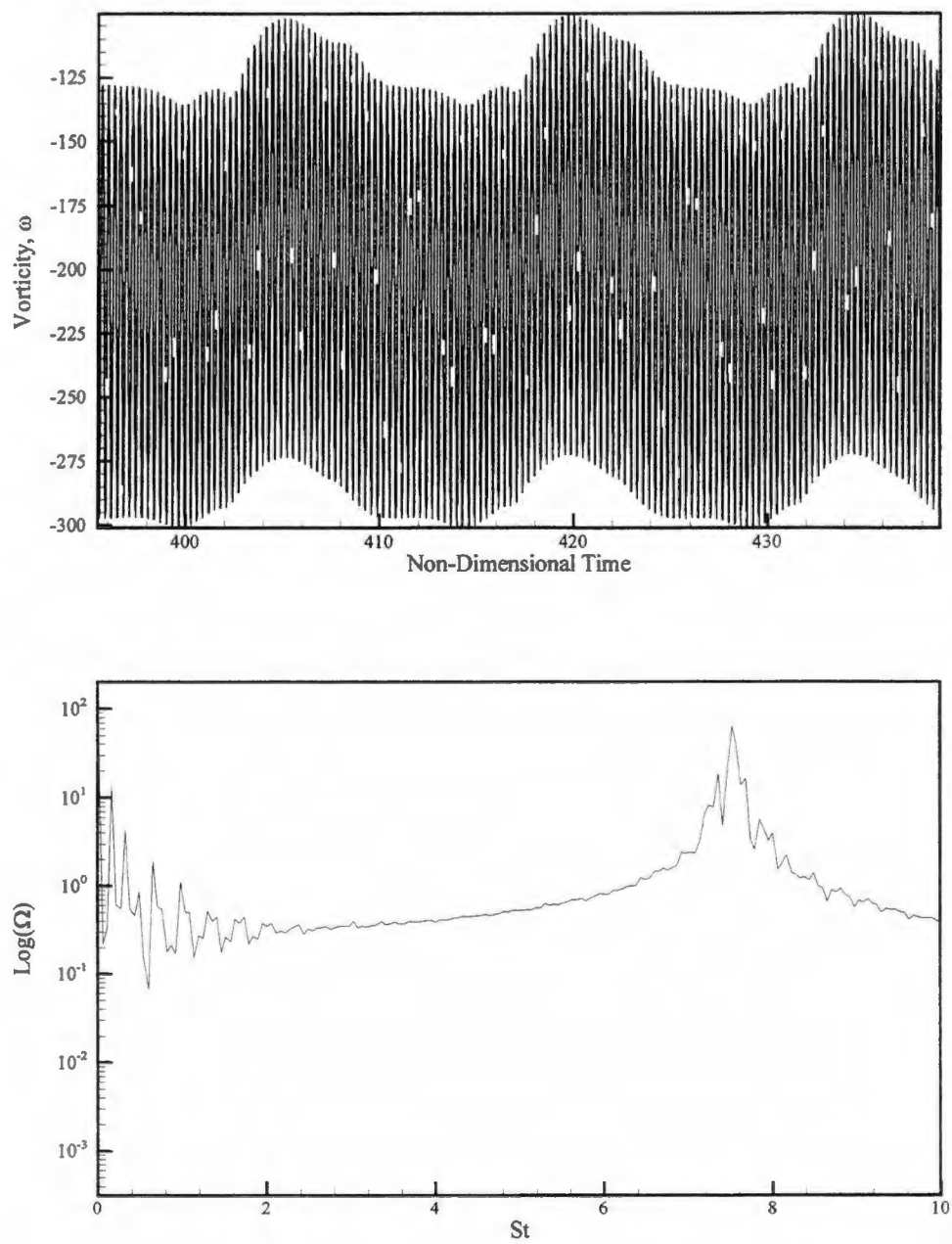


Figure 59: Vorticity in Leading Edge Region for Case 1, $f^* = 1$

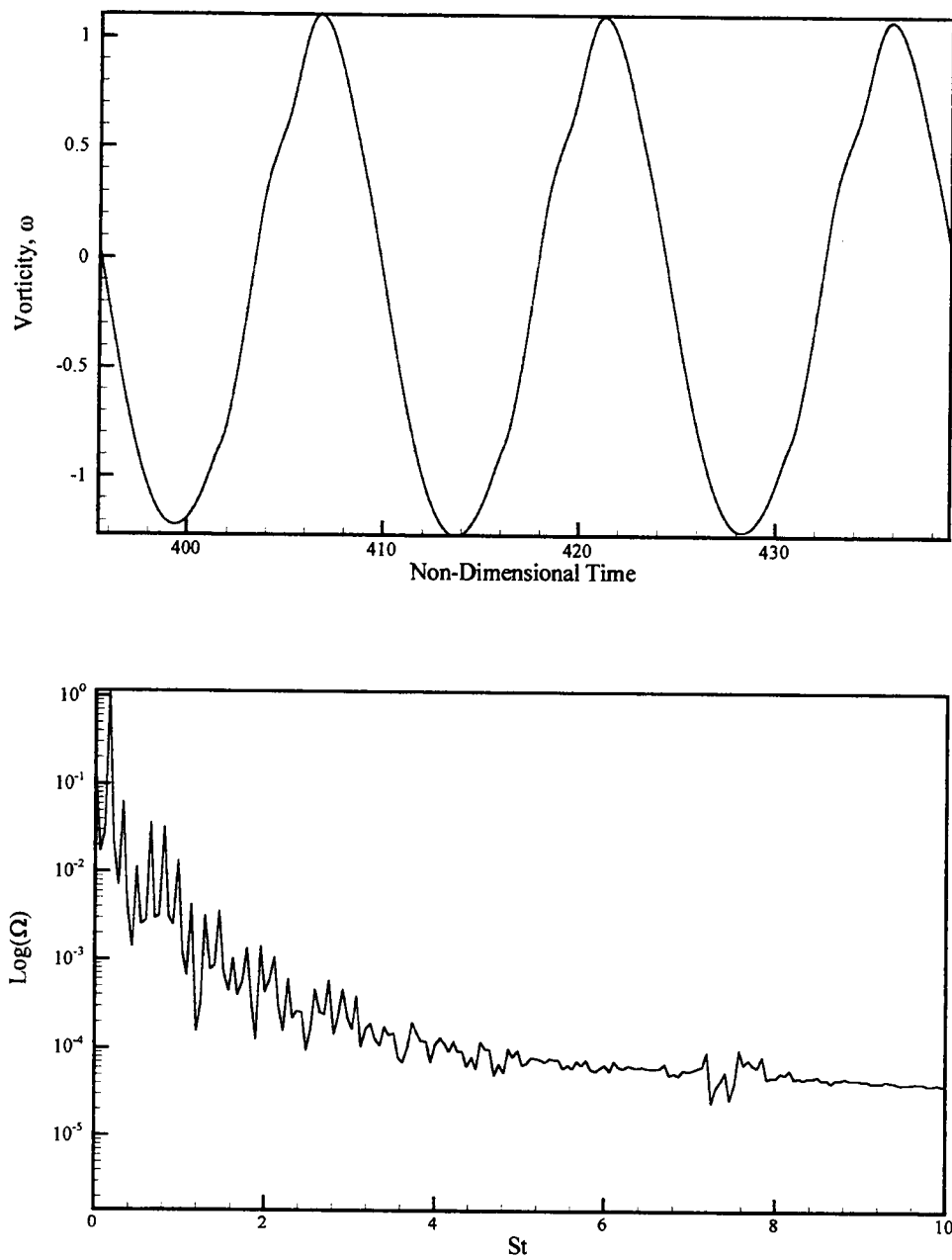


Figure 60: Vorticity in Trailing Edge Region for Case 1, $f^* = 1$

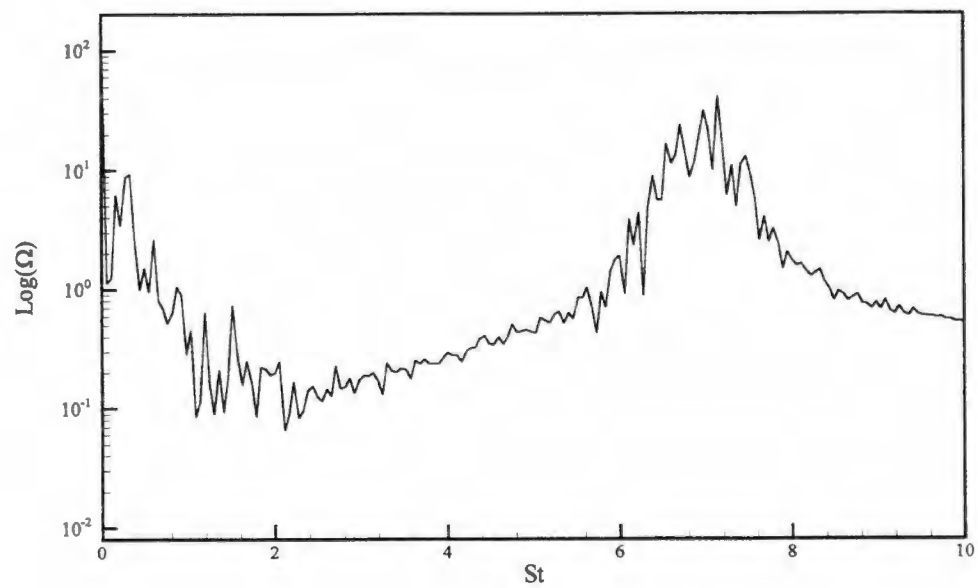
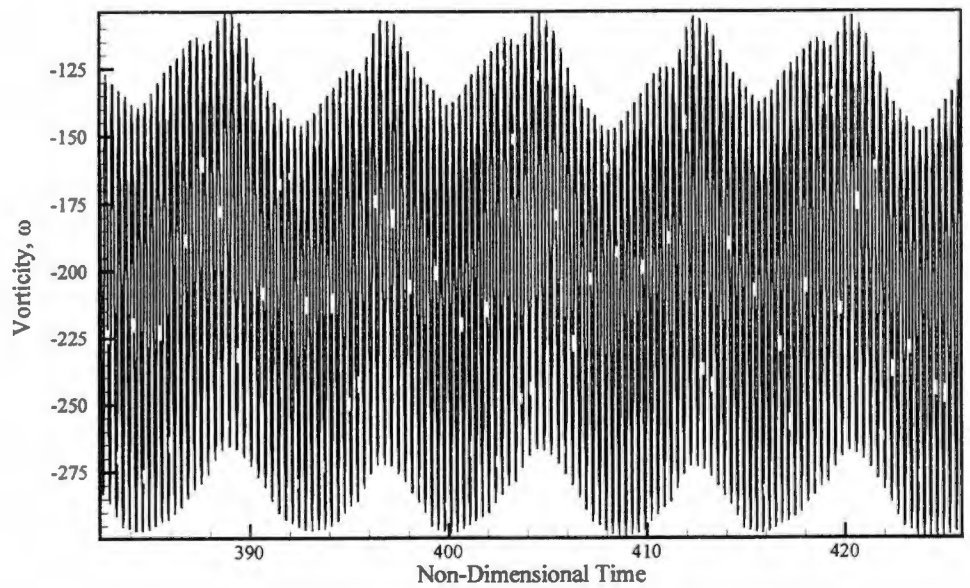


Figure 61: Vorticity in Leading Edge Region for Case 2, $f^* = 2$

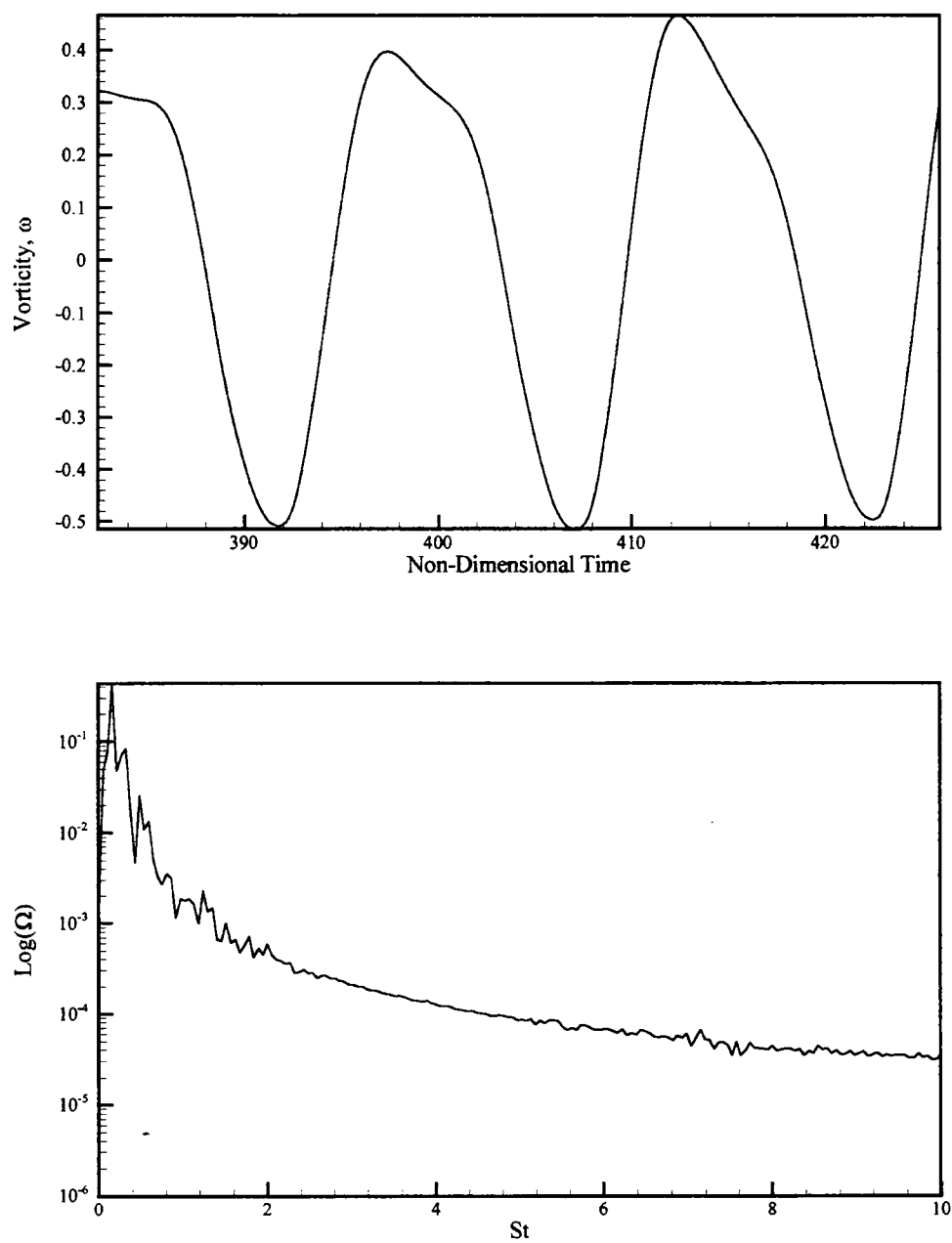


Figure 62: Vorticity in Trailing Edge Region for Case 2, $f^* = 2$

frequency mode and several of its superharmonics. If anything, the forcing at $f^* = 1$ appears to reduce some of the energy around the shear layer instability frequency. The small amplitude forcing at $f^* = 2$ produces a low-level growth of a wide spread of frequencies, which is commensurate with the injection of a third distinct frequency into the flowfield, but there is no substantial change in the overall mode shapes.

The velocity phase plots shown in Figures 63 through 65 reinforce the previous discussion. The baseline flowfield shows a fairly well-ordered behavior at the shedding edges. Forcing at a frequency of unity yields a sizable increase in the velocity amplitudes. Forcing at $f^* = 2$ results in only a slight change in the velocity component amplitudes compared to the $f^* = 1$ case. However, in both dynamic cases the forcing introduces additional frequencies at multiples of the forcing frequency into the flowfield. This accounts for the increased spread seen in the leading edge phase plots.

The source of the high frequency signal seen in the figures above is found in the behavior of the leading edge shear layer at the separation point. The leading edge flow separation possesses its own cyclical behavior, due to the shear layer instability at separation. A small, unstable leading edge bubble forms a distance of approximately one to two wavelengths from the separation point, where the wavelength is that of the most unstable shear layer perturbation. This wavelength is computed from $\lambda_s \sim U/2\omega_s$, where U is the freestream velocity and ω_s is the shear layer instability frequency in rad/s. It is assumed that the velocity in the shear layer varies from zero to freestream. Using the high frequency corresponding to a Strouhal number of 7.2, this wavelength is about 0.5% of the airfoil chord. The bubble grows to form a small, strong eddy. This eddy leaves the

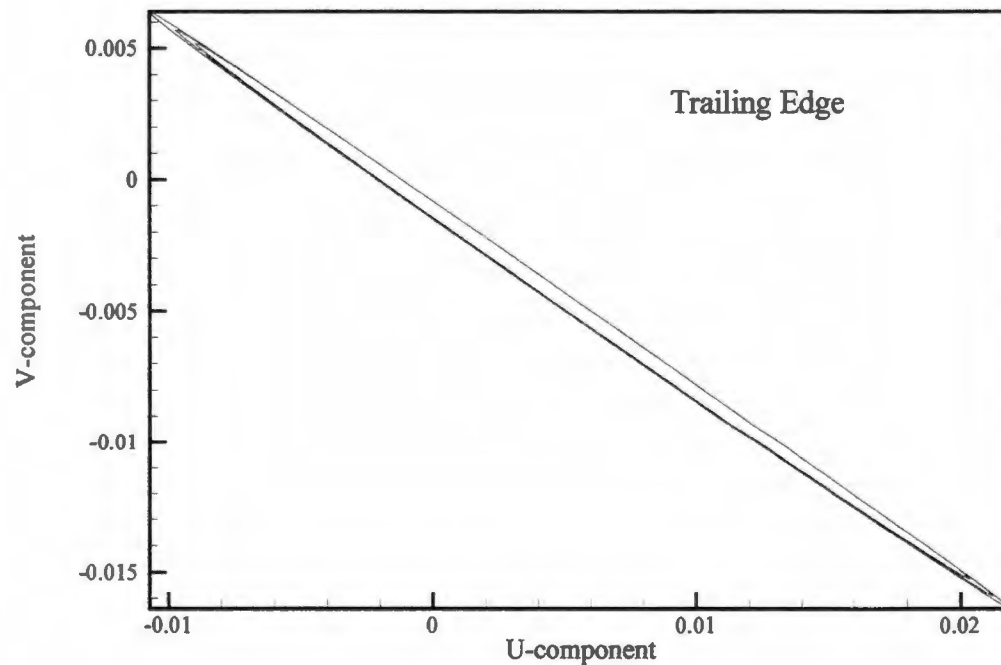
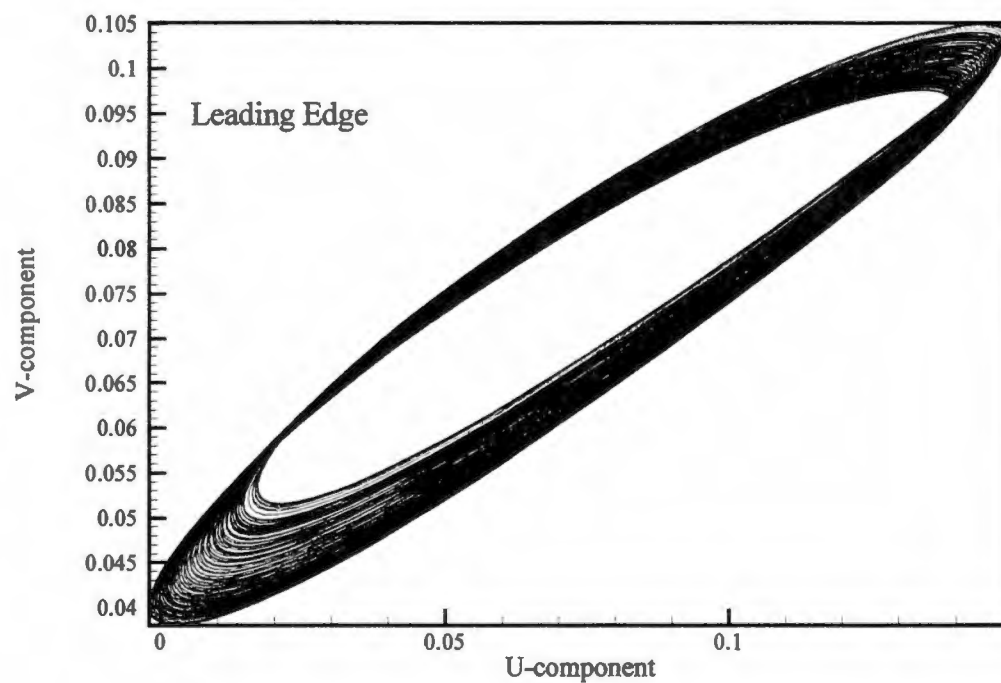


Figure 63: Velocity Phase Plots for Baseline Case

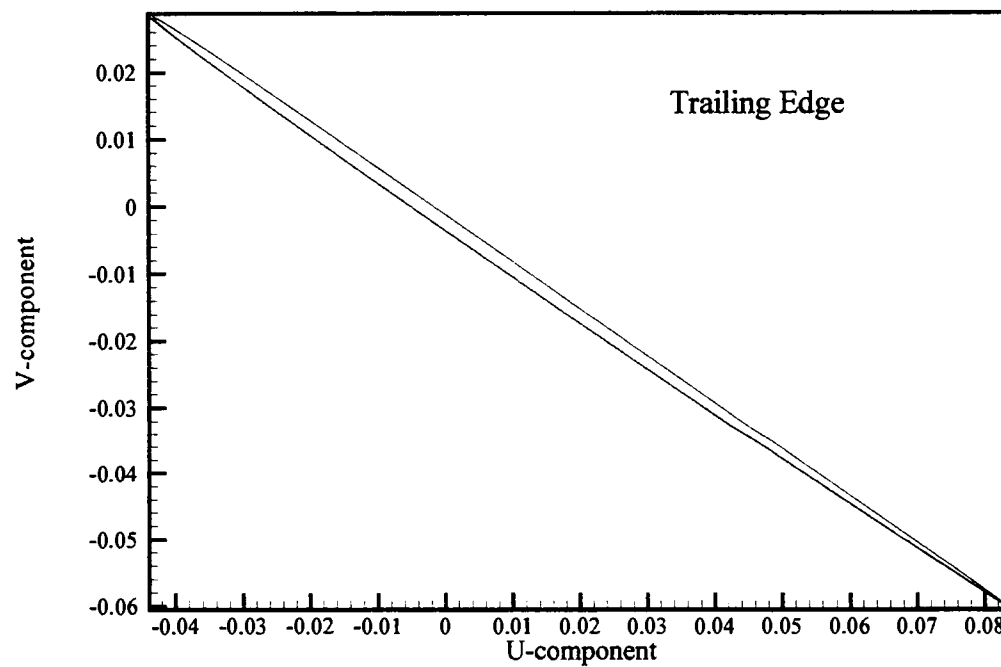
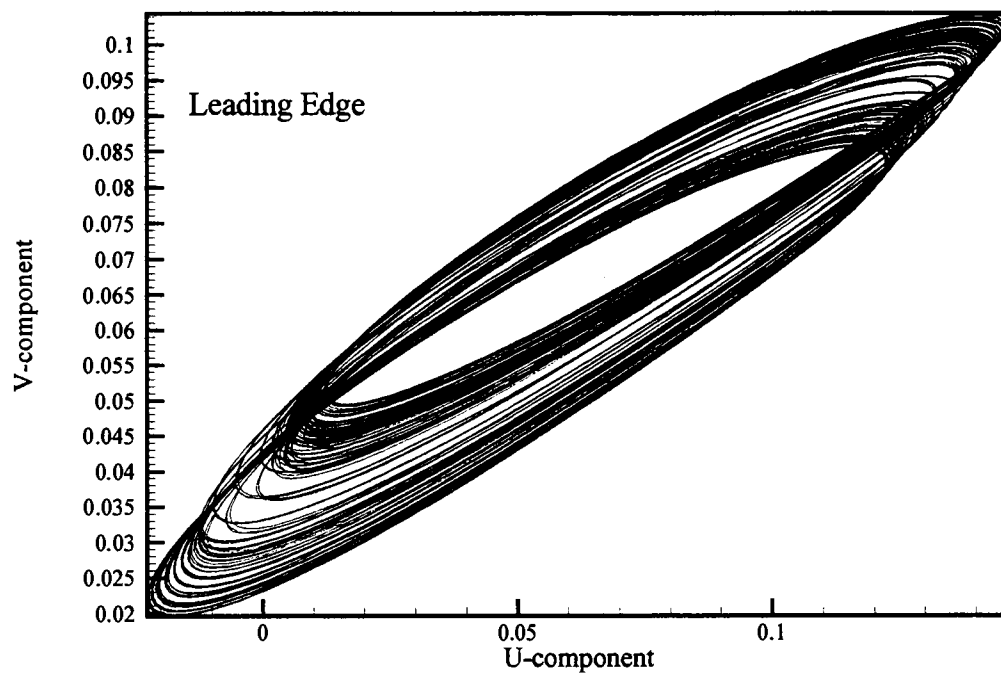


Figure 64: Velocity Phase Plots for Case 1, $f^* = 1$

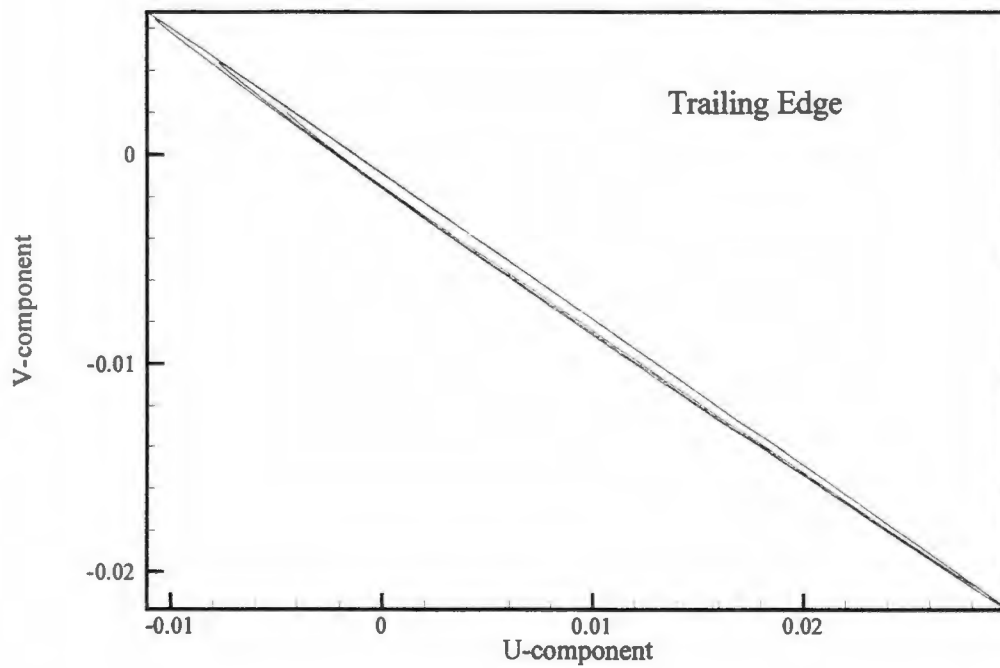
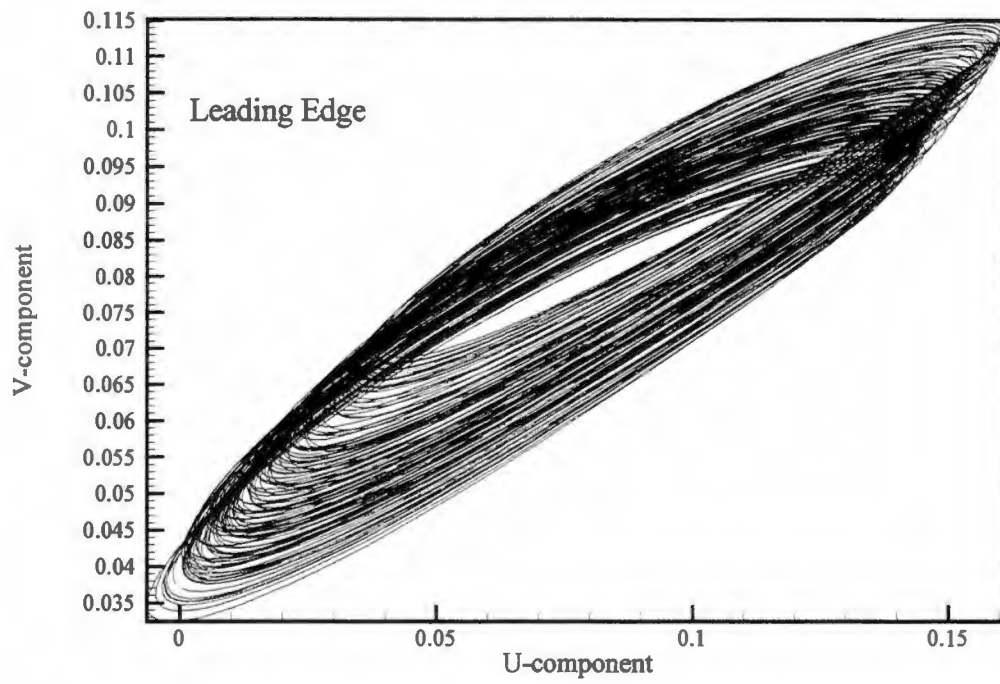


Figure 65: Velocity Phase Plots for Case 2, $f^* = 2$

surface and travels downstream, where it merges with an eddy trapped by the juncture of the leading edge flap and airfoil surface. As this merged eddy grows, part of it spills over into the shear layer moving downstream. The formation of the leading edge separation eddies is what causes the high frequency pressure and vorticity fluctuations. The stream traces in Figure 66 show the typical leading edge flow structure at several instants in the high frequency cycle. It should be noted that the eddy viscosity in the region of the flap was generally an order of magnitude smaller than the kinematic viscosity for these calculations, unlike the computations for the backward NACA 0025 airfoil with sharp leading edge separation.

It is interesting that the baseline solution does seem to capture the higher frequency behavior of the shear layer, unlike the solutions using Zhou's configuration. The separation behavior occurs in a region of nearly laminar flow, where the eddy viscosity computed by the turbulence model is quite small. The flow remains nearly laminar over the flap length, and the momentum thickness does not vary greatly from the separation point to the end of the flap. A rough calculation of the momentum thickness using the velocity distribution just downstream of the flap was performed. With this value of momentum thickness, an estimate of the most unstable shear layer mode was calculated. The computation yielded a Strouhal number for the most unstable mode between approximately 6.8 and 8.5, depending on whether the formula for laminar or turbulent flow was used. The value from the numerical simulation was ~ 7.2 .

To further explain the effect of the flap motion, consider the data presented in Figure 67. The first curve shows the vorticity in the flow at a point slightly downstream

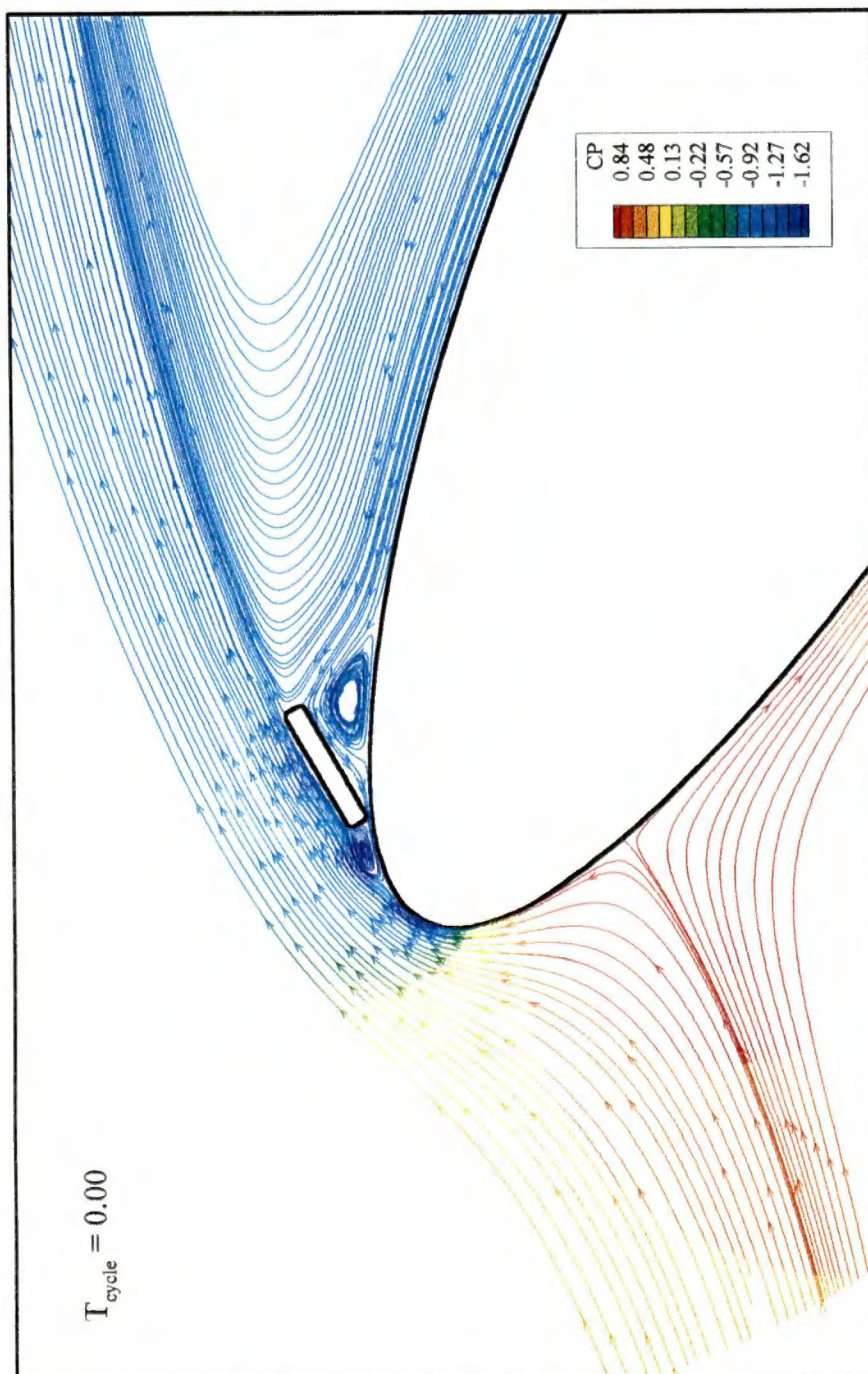


Figure 66: Leading Edge Shedding Behavior

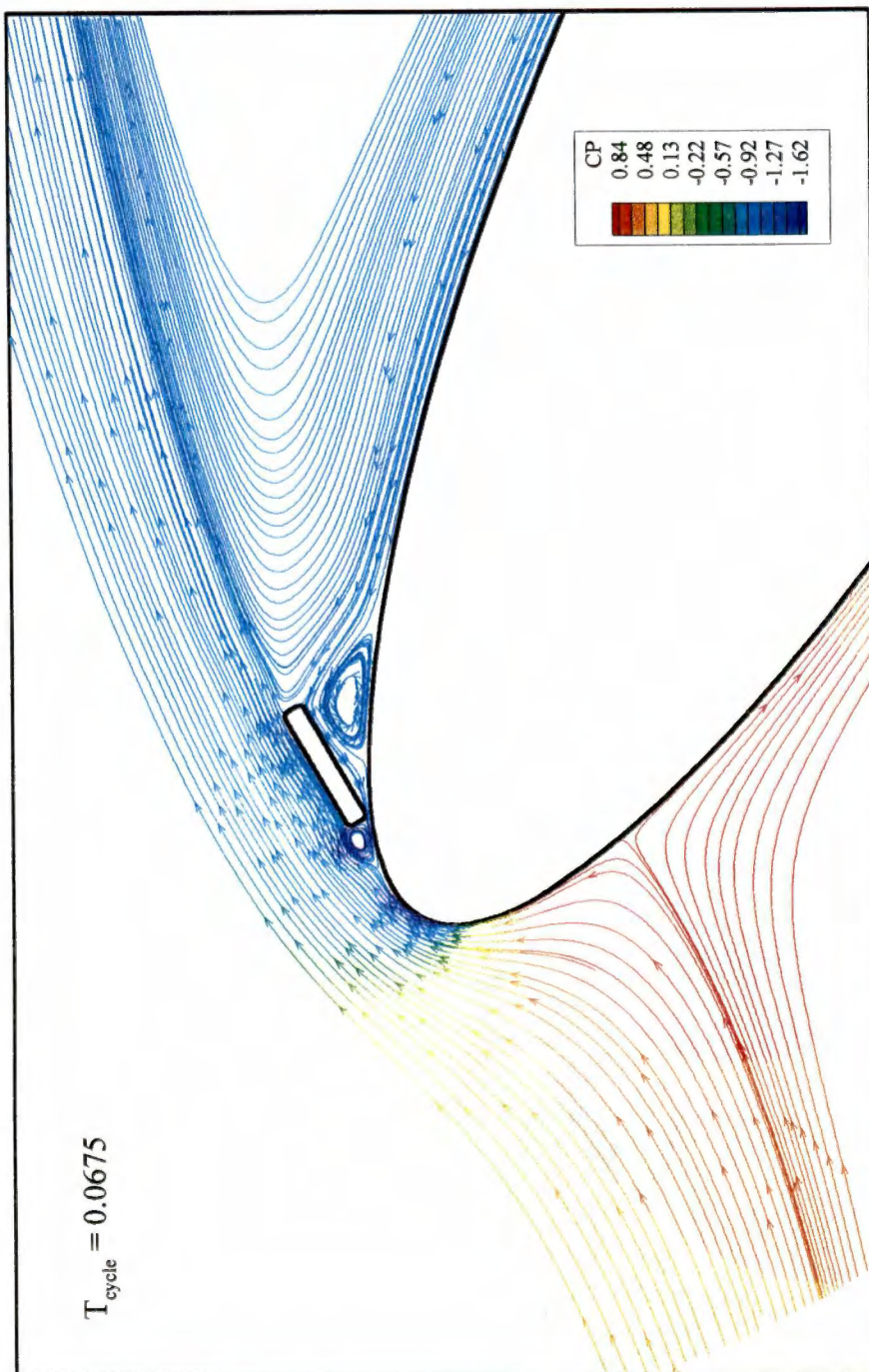


Figure 66: Leading Edge Shedding Behavior (Continued)

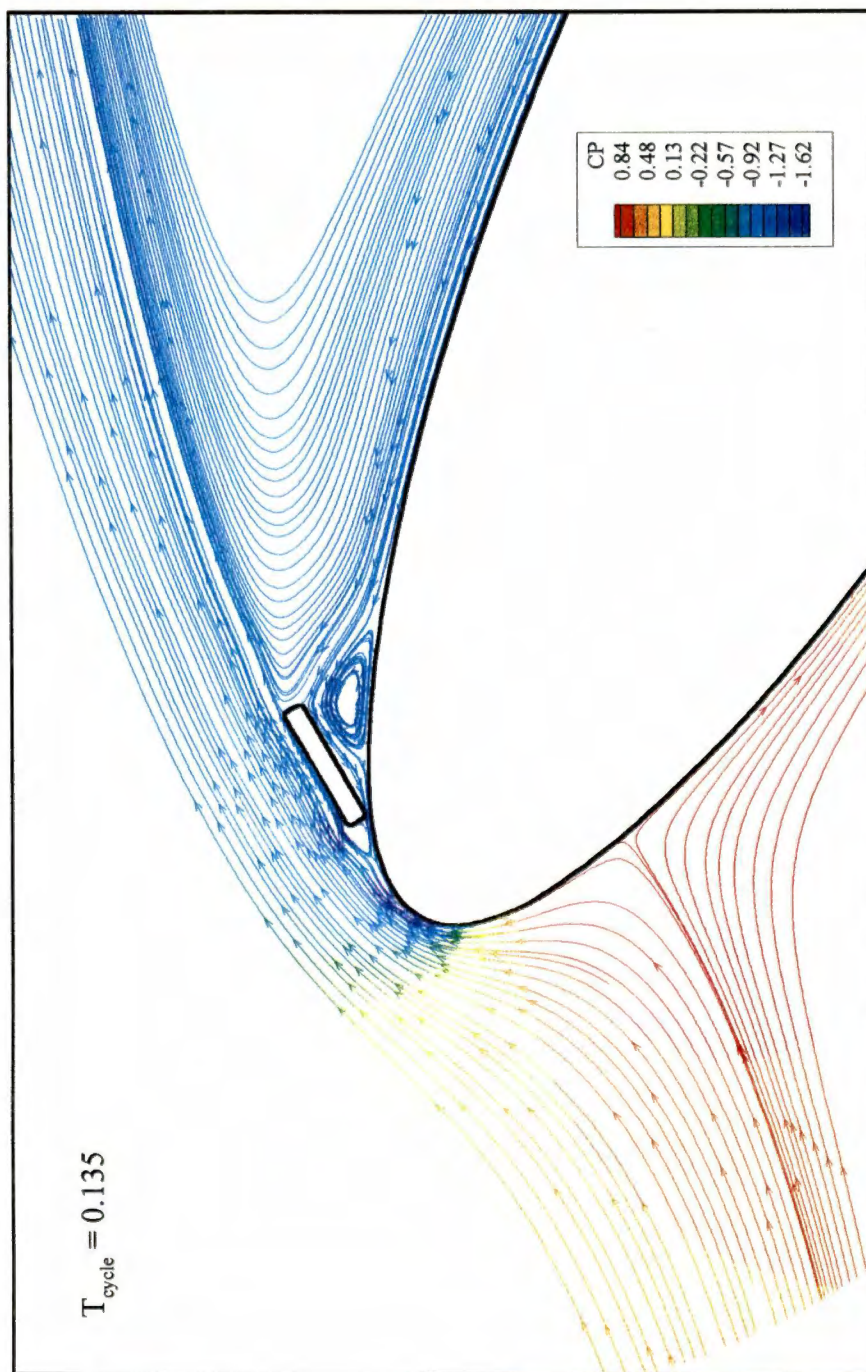


Figure 66: Leading Edge Shedding Behavior (Continued)

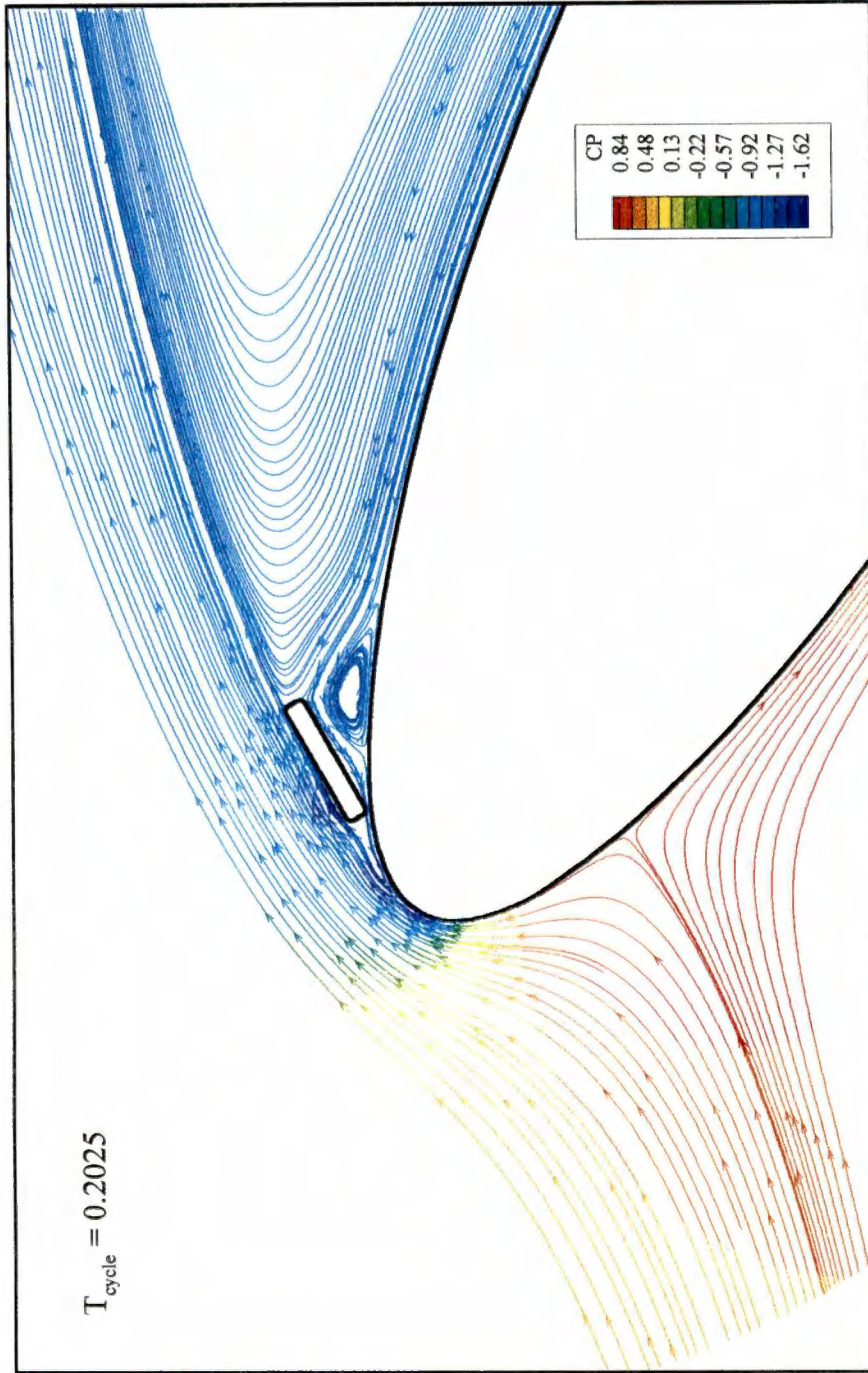


Figure 66: Leading Edge Shedding Behavior (Continued)

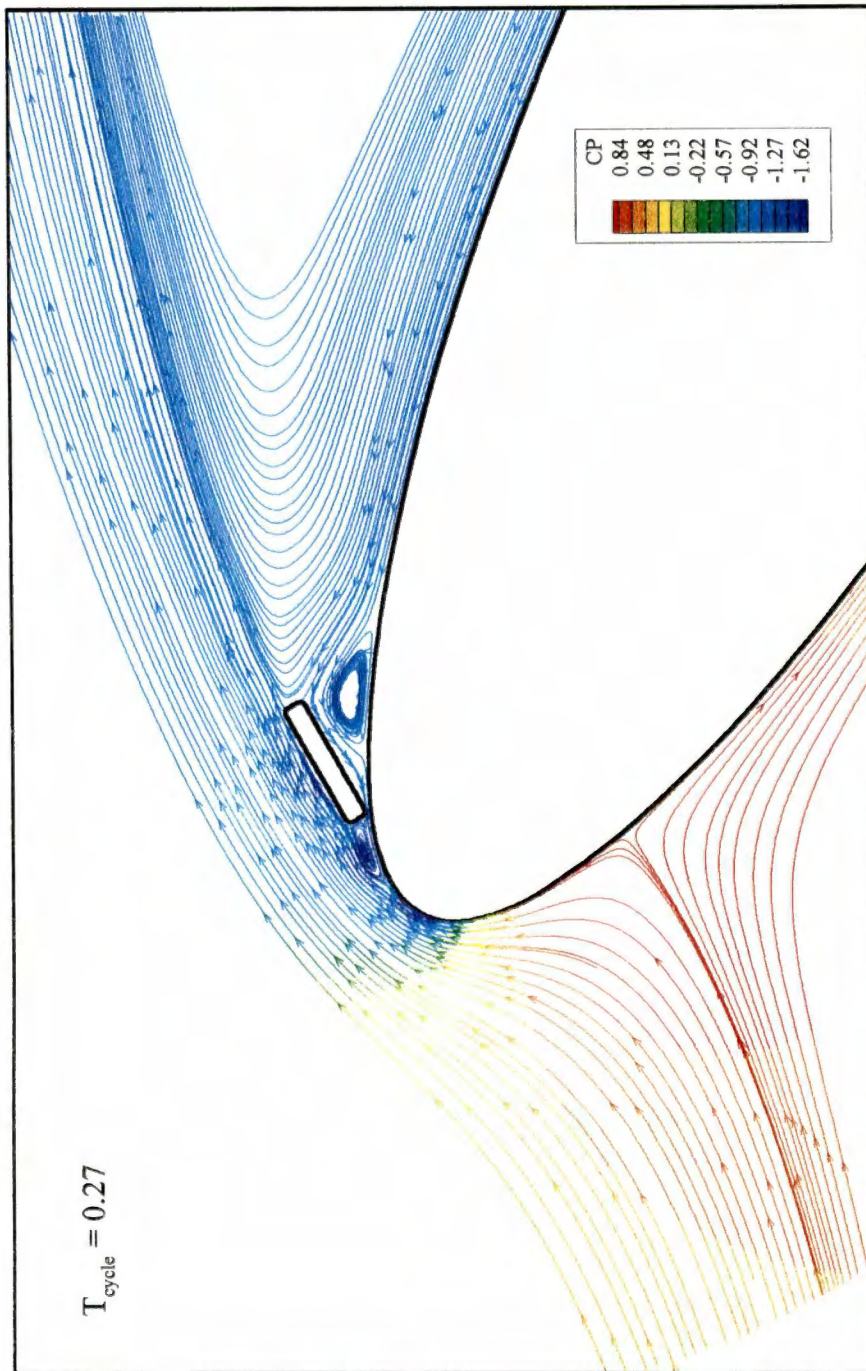


Figure 66: Leading Edge Shedding Behavior (Concluded)

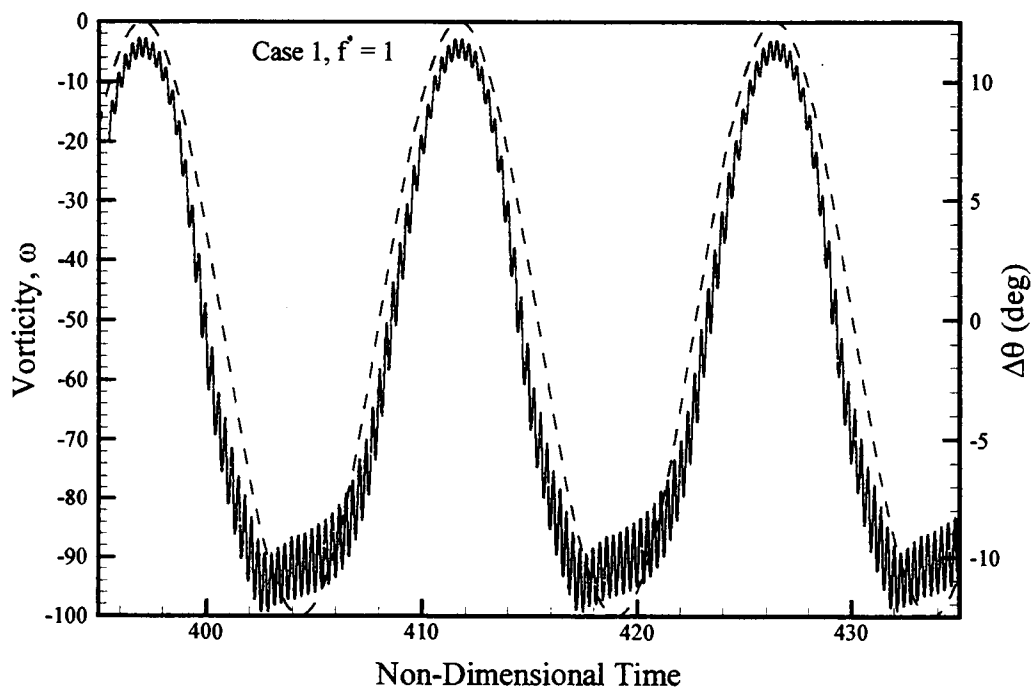
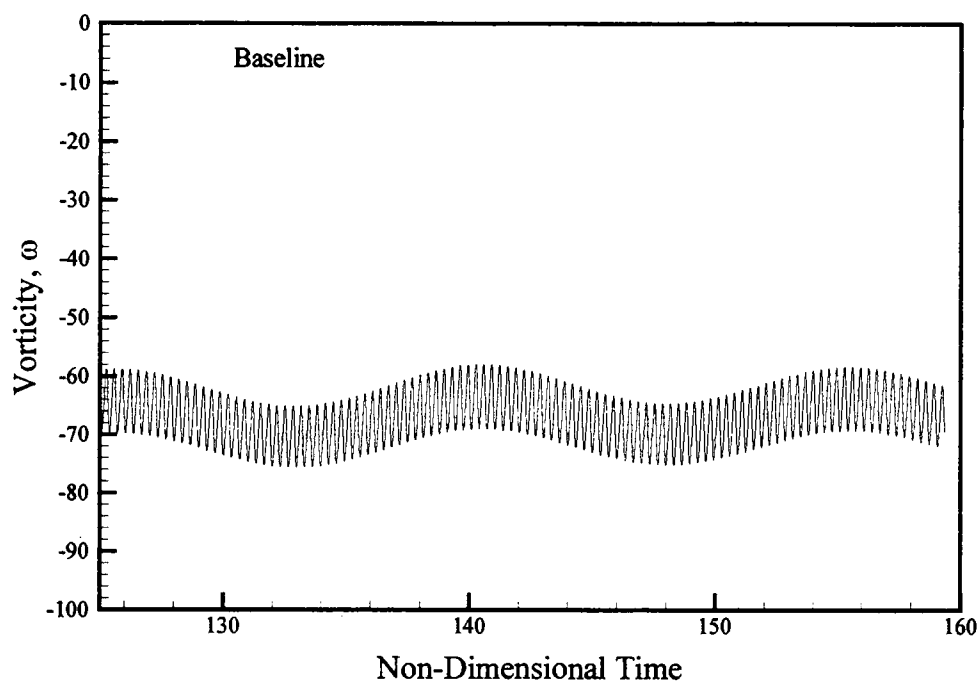


Figure 67: Vorticity Behavior Due to Flap Motion

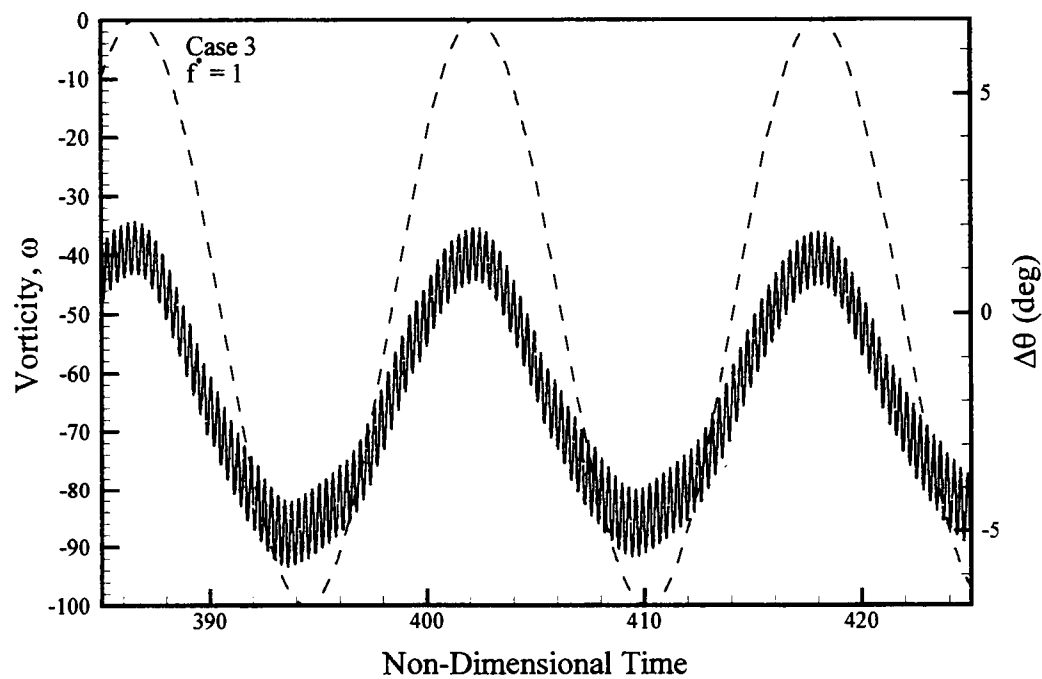
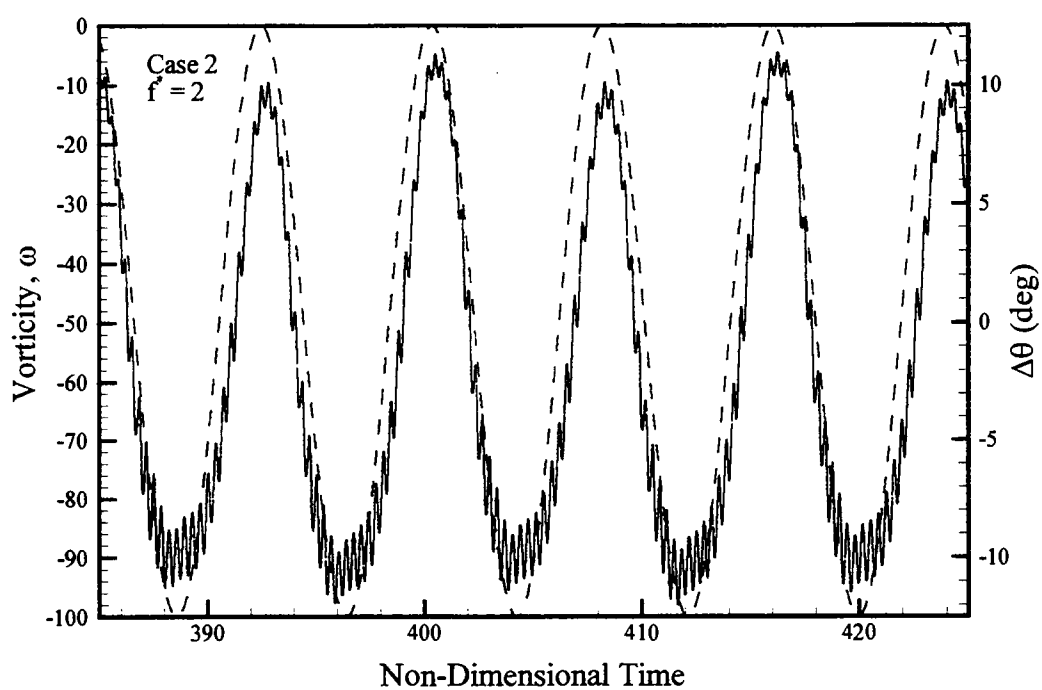


Figure 67: Vorticity Behavior Due to Flap Motion (Concluded)

of the flap trailing edge, and parallel to the flap upper surface, for the baseline. The remaining curves show the vorticity for the three forcing cases. It is this vorticity that is convected downstream into the main body of the flowfield over the airfoil surface and feeds into the LE vortex. The flap position $\Delta\theta$ is also given as a dashed line for the three dynamic cases. It should be noted that a positive flap displacement corresponds to a downward flap stroke, due to the definition of positive $\Delta\theta$ used in this study.

For the baseline, the vorticity possesses harmonic components at the shedding frequency and the shear layer instability frequency. The component of vorticity at the shedding frequency varies between -59 and -75. The addition of forcing has a dramatic effect on the vorticity downstream of the flap. In all the dynamic cases the flap motion synchronizes the vorticity to the flap forcing frequency. The vorticity amplitudes are very similar in the two cases having an amplitude of 12.5° , with the vorticity varying between approximately -5 and -99. As the flap moves up ($\Delta\theta$ becomes negative) the vorticity becomes strongly negative. This makes sense, as the flap rotation increases the relative velocity between the flow and the flap solid surface. On its upstroke, the flap pushes into the shear layer and into the freestream. This forces the flow to separate from the flap edge, in a manner similar to the flow around a spoon which is suddenly stirred in a cup of cocoa. The separated flow rolls up quickly to form the small eddies observed in the vorticity contours above. These small eddies possess significant vorticity, which is shown in the figures. The downstroke of the flap causes the separation to vanish, with the result that the vorticity (contained in the eddies) reduces nearly to zero. The results for Case 3, given by the last curve, are qualitatively similar to the results of Case 1. However, it can

be seen that the smaller flap amplitude has less of an effect on the vorticity amplitude. For the small-amplitude excitation the vorticity varies between -40 and -95. For all cases, the result is the creation of a series of well-defined eddies with enhanced strength. This stream of eddies constitutes a periodic vortical signal, and these eddies, with their enhanced vorticity, are carried downstream and into the leading edge vortex. It should be pointed out that the vorticity levels shown in the figures are approximate, since the trajectory of the eddies will vary from case to case and the eddies may not pass through the measurement point. Thus, the similarity in vorticity levels for Case 1 and Case 2 may be nothing more than a coincidence.

It is the receptivity of the fluid/solid system that allows a vortical signal of a given frequency to trigger growth in the aerodynamic coefficients, and essentially lead to resonance in the system. The fluid/solid system possesses a natural frequency given by the shedding frequency. The system is receptive to appropriate disturbances at this frequency. A vortical signal at the shedding frequency supplies vortices at the right instant to strengthen a developing leading edge vortex, and diminishes the leading edge vorticity during trailing edge vortex development. A signal that is not synchronized to the natural frequency of the system cannot reinforce the natural harmonic behavior of the system. It is important to note that this is a global receptivity mechanism. It is not based on the receptivity of the shear layer, as Hsiao and his colleagues contended. The shear layer possesses a receptivity behavior of its own, but the typical response spectrum of the shear layer is in the frequency range of $St \sim 7.2$. The forcing has little interaction with the high frequency harmonic at $St \sim 7.2$, as discussed above. However, shear layer instability does

play a part, since the rolling-up of the separated flow created by the moving flap is similar to the shear layer instability. But in this case the flap motion does not act directly on the shear layer instability. In fact, any other mechanism which could introduce vortical eddies into the flow at the proper phase could cause the same global flowfield modulation, without resorting to any shear layer instability.

The existence of this global receptivity explains other features of the experimental data. The frequency spread over which a significant increase in the aerodynamic coefficients occurs because the developing leading edge vortex can still entrain and utilize a significant portion of the vortical signal, for flap forcing at frequencies slightly below and above the shedding frequency. In other words, the global system can receive and utilize a significant portion of the energy over a small range of frequencies, with the maximum amount occurring at the shedding frequency.

In summary, the flap motion serves to synchronize and strengthen the vortices when the motion occurs at the shedding frequency. As the flap moves up, it creates a significant increase in the development of negative vorticity in the leading edge region due to flow separation from the flap edge. This separated flow rolls up into small eddies that convect downstream into the LE vortex region. As the flap moves down, the strength of this vorticity drops to almost zero, providing a zone of demarcation between the strong vortical eddies created by the upward flap motion. At a forcing frequency of 1, the eddies thus formed merge with the developing LE vortex to strengthen and synchronize it, because of the receptivity of the fluid/solid system. The stronger LE vortex in turn generates a stronger TE vortex. When the flap motion occurs at twice the shedding

frequency, the energy input via the vorticity signal cannot be received by the system, i.e. the leading edge eddies do not merge effectively with the developing LE vortex. In addition, the trailing edge vortex growth is diminished through destructive interference from the leading edge vortices. Although shear layer instability leads to the creation of the eddies by the flap, the main element in the flowfield modulation is the global receptivity.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Within the restrictions of the physical assumptions contained in the numerical modeling scheme, this study has demonstrated that computational fluid dynamics provides a valuable tool for examining massively separated, turbulent flows. By incorporating an overset grid methodology, problems involving relative body motion can be analyzed. In this study two cases involving vortex control by mechanical oscillation were considered.

Excellent results were obtained for the case involving vortex control by mechanical excitation taken from work done by Hsiao, et. al. [8]. In this case the fundamental physics involve the perturbation of a shear layer which can be realistically modeled under the assumptions involved in the Reynolds-averaged Navier Stokes equations. The interaction of the shear layer frequency and the shedding/forcing frequencies is quite limited, owing to the large separation in these frequencies. Computational results demonstrated good agreement with the experimental data compiled by Hsiao and his coworkers. The numerical results were used to explain the effect of the flap motion on the global flowfield behavior in terms of a global receptivity mechanism. The separated flow from the leading edge provides a source of energy to the flowfield. By periodically perturbing the shear layer at the shedding frequency, the flap motion produces a vortical signal that the global flowfield can easily receive. In this way, the flap oscillation synchronizes and therefore strengthens the vortices formed at the leading and trailing edges.

The successful simulation of the experiments of Hsiao, et. al. using overset grids would seem to vindicate the domain decomposition methodology. Ironically, the simulation of the results of Hsiao and his colleagues may lend additional support to the presumed cause of failure in the computation of the backward-facing airfoil. In the case of the NACA 63₃-018 airfoil, a high frequency shear layer instability frequency was computed. This flow involved separation from a smooth surface, and the eddy viscosities computed by the turbulence model were not as large as those from the sharp leading edge separation of Zhou, et. al.

Future work could follow one of several paths. In order to investigate the physics involved in backward-facing NACA 0025 configuration, a Large Eddy Simulation (LES) or Direct Numerical Simulation (DNS) technique should be considered, since current turbulence models are incapable of adequately rendering the small-scale motions that appear to play a role. These methods are time-consuming, but given the success of the simpler computational model in the case of the NACA 63₃-018, it is felt that numerical modeling can play a significant role in analyzing more complex flowfields if the proper physics are included. In conjunction with a more detailed numerical study, an effort to establish a more rigorous set of experimental data should also be considered.

For practical applications DNS or LES computations are still too expensive. Thus, the question of applicability of existing turbulence models to massively separated, unsteady flows should continue to be pursued. Such computations are becoming more commonplace, as the perception of the power of computational fluid dynamics grows.

However, the results obtained in this study certainly raise suspicions about the ability of turbulence modeling to capture the flow physics accurately, in some cases.

Given the way the flap works to control the flow in the NACA 63₃-018 configuration, a logical extension of the work presented here would be to investigate other methods of excitation. The appropriate use of a periodically pulsing jet could provide vorticity perturbations similar to those produced by the flap. A recent numerical study by Wu, et. al. [40] provides some interesting results for such forcing applied to flow over a NACA 0012. With a validated numerical model (compared to experimental data for carefully controlled experiments), such designs could be explored in a cost-effective manner in the computer. In addition, examinations of other airfoil geometries should be considered. A different airfoil geometry might possess secondary and tertiary separations, which would alter the global receptivity and could significantly affect the flowfield behavior under forcing. Also, forcing at a frequency compatible with the high frequency shear layer instability should be explored.

This current work underscores the fact that the term "vorticity control" is a broadly defined concept that still needs refinement. The cases considered in this effort included one case that clearly depended on a global receptivity mechanism, and another case that appeared to be a manifestation of a localized shear layer receptivity. In addition, the dependence of such control on geometry is still poorly defined in many cases. A validated numerical simulation provides data which could be obtained experimentally only with great difficulty, if at all. An analysis of this data can yield a much deeper understanding of the different aspects of vorticity control.

BIBLIOGRAPHY

BIBLIOGRAPHY

1. Roshko, A., "Experiments on the Flow Past a Circular Cylinder at Very High Reynolds Number," *Journal of Fluid Mechanics* **10**, 345-356.
2. Kawamura, R., and Aihara, Y. (eds.), *Fluid Dynamics of High Angle of Attack Flow*, Springer-Verlag, 1993, 35-63.
3. Vakili, A. D., "Review of Vortical Flow Utilization," *AIAA-90-1429*.
4. Zaman, K. B. M. Q., Bar-Sever, A., and Mangalam, S. M., "Effect of Acoustic Excitation on the Flow Over a Low-Re Airfoil," *Journal of Fluid Mechanics* **182**, 127-148.
5. Chang, R. C., Hsiao, F.-B., and Shyu, R.-N., "Forcing Level Effects of Internal Acoustic Excitation on the Improvement of Airfoil Performance," *Journal of Aircraft* **29**, No. 5, 823-829.
6. Karim, M. A., and Acharya, M., "Control of the Dynamic-Stall Vortex Over a Pitching Airfoil by Leading-Edge Suction," *AIAA-93-3267*.
7. Gopalkrishnan, R., Triantafyllou, M. S., Triantafyllou, G. S., and Barrett, D., "Active Vorticity Control in a Shear Flow Using a Flapping Foil," *Journal of Fluid Mechanics* **274**, 1-21.
8. Hsiao, F.-B., Wang, T.-Z., and Zohar, Y., "Flow Separation Control of a 2-D Airfoil by a Leading-Edged Oscillating Flap," Pacific International Conference on Aerospace Science and Technology, Taiwan, Dec. 6-9, 1993.
9. Zhou, M. D., Fernholz, H. H., Ma, H. Y., Wu, J. Z., and Wu, J. M., "Vortex Capture by a Two-Dimensional Airfoil with a Small Oscillating Leading-Edge Flap," *AIAA-93-3266*.
10. Tryantafallou, M. S., and Tryantafallou, G. S., "An Efficient Swimming Machine," *Scientific American*, 64-70, March 1995.
11. Gillan, M. A., and Cooper, R. K., "Computational Analysis of Buffet Alleviation Over a Porous Airfoil at High Angle-of-Attack," *AIAA-94-1818-CP*.
12. Laurie, K. B., and Farokhi, S., "Separated Flowfield and Lift on an Airfoil with an Oscillating Leading-Edge Flap," *AIAA-93-3422-CP*.

13. Kim, J.-H., and Rho, O. H., "Numerical Simulation of the Flowfield Around an Airfoil with a Stationary and Oscillating Spoiler," *AIAA-96-2509*.
14. Visbal, M. R., and Shang, J. S., "Numerical Investigation of the Flow Structure Around a Rapidly Pitching Airfoil," *AIAA-87-1424*.
15. Ko, S. and McCroskey, W. J., "Computations of Unsteady Separating Flows Over an Oscillating Airfoil," *AIAA Journal* **35**, No. 7, 1235-1238.
16. Barton, J. T., and Pulliam, T. H., "Airfoil Computation at High Angles of Attack, Inviscid and Viscous Phenomena," *AIAA-84-0524*.
17. Pulliam, T. H., "Artificial Dissipation Models for the Euler Equations," *AIAA Journal* **24**, No. 12, 1931-1940.
18. Thomas, P. D., and Lombard, C. K., "Geometric Conservation Law and Its Application to Flow Computations on Moving Grids," *AIAA Journal* **17**, No. 10, 1030-1037.
19. Thomson, K. W., "Time Dependent Boundary Conditions for Hyperbolic Systems," *Journal of Computational Physics* **68**, 1-24.
20. Huang, R. F., and Lin, C. L., "Vortex Shedding and Shear-Layer Instability of a Cantilever Wing at Low Reynolds Number," *AIAA-95-0589*.
21. Warsi, Z. U. A., Fluid Dynamics: Theoretical and Computational Approaches, CRC Press, Boca Raton, FL, 1993.
22. Srinivasan, G. R., Ekaterinaris, J. A., and McCroskey, W. J., "Dynamic Stall of an Oscillating Wing Part 1: Evaluation of Turbulence Models," *AIAA-93-3403*.
23. Rumsey, C. L., and Vatsa, V. N., "A Comparison of the Predictive Capabilities of Several Turbulence Models Using Upwind and Central Difference Computer Codes," *AIAA-93-0192*.
24. Rogers, S. E., Menter, F. R., Durbin, P. A., and Mansour, N. N., "A Comparison of Turbulence Models in Computing Multi-Element Airfoil Flows," *AIAA-94-0291*.
25. Kern, S. B., "Evaluation of Turbulence Models for High-Lift Military Airfoil Flowfields," *AIAA-96-0057*.
26. Spalart, P. R., and Allmaras, S. R., "A One-Equation Turbulence Model for Aerodynamic Flows," *AIAA-92-0439*.

27. Steger, J. L., and Benek, J. A., "On the Use of Composite Grid Schemes in Computational Aerodynamics," *Computer Methods in Applied Mechanics and Engineering* **64**, Nos. 1-3, 301-320, 1987.
28. Meakin, R., "On the Spatial and Temporal Accuracy of Overset Grid Methods for Moving Body Problems," *AIAA-94-1925*.
29. Jones, G. W., Cincotta, J. J., and Walker, R. W., "Aerodynamic Forces on a Stationary and Oscillating Circular Cylinder at High Reynolds Numbers," *NASA Technical Report R-300*.
30. White, F. M., *Fluid Mechanics*, McGraw-Hill Book Company, New York, NY, 1979.
31. Schlichting, H., *Boundary Layer Theory*, McGraw-Hill Book Company, New York, NY, 1987.
32. Abbott, I. H., and von Doenhoff, A. E., *Theory of Wing Sections*, Dover Publications, Inc., New York, NY, 1959.
33. Jacobs, E. N., and Sherman, A., "Airfoil Section Characteristics as Affected by Variations of the Reynolds Number," *NACA Technical Report 586*.
34. Tryantafallou, G. S., Tryantafallou, M. S., and Chrysostomidis, C., "On the Formation of Vortex Streets Behind Stationary Cylinders," *Journal of Fluid Mechanics* **170**, 461-477.
35. Oertel, H., "Wakes Behind Blunt Bodies," *Annual Review of Fluid Mechanics* **1990**, **22**, 539-564.
36. Critzos, C. C., Heyson, H. H., and Boswinkle, R. W., "Aerodynamic Characteristics of NACA 0012 Airfoil Section at Angles of Attack from 0° to 180° ," *NACA Technical Note 3361*.
37. Zaman, K. B. M. Q., McKinzie, D. J., and Rumsey, C. L., "A Natural Low-Frequency Oscillation of the Flow Over an Airfoil Near Stalling Conditions," *Journal of Fluid Mechanics* **202**, 403-442.
38. Rumsey, C. L., "Time-Dependent Navier-Stokes Computations of Separated Flows Over Airfoils," *AIAA-85-1684*.
39. Jones, B. M., "Stalling," *Journal of the Royal Aeronautic Society* **285**, 753-769.

40. Wu, J.-M., Lu, X.-Y., Denny, A. G., Fan, Meng, and Wu, J.-Z., "Post-Stall Flow Control on an Airfoil by Local Unsteady Forcing I - Lift, Drag, and Pressure Characteristics," *AIAA-97-2063*.
41. Hoffman, K. A., and Chang, S. T., Computational Fluid Dynamics for Engineers - Vol. II, Engineering Education System, Wichita, KS, 1993.
42. Beam, R. M., and Warming, R. F., "An Implicit Finite-Difference Algorithm for Hyperbolic Systems in Conservation Law Form," *Journal of Computational Physics* **22**, 87-110.
43. Visbal, M. R., "Evaluation of an Implicit Navier-Stokes Solver for Some Unsteady Separated Flows," *AIAA-86-1053*.
44. Roach, R. L., and Jenkins, J. E., "Improvements to the Numerical Simulation of Pitching Delta Wings," *AIAA-95-0761*.
45. Chakravarthy, S. R., "Euler Equations - Implicit Schemes and Boundary Conditions," *AIAA Journal* **21**, No. 5, 699-706.
46. Poinso, T. J., and Lele, S. K., "Boundary Conditions for Direct Simulations of Compressible Viscous Flows," *Journal of Computational Physics* **101**, 104-129.

APPENDIX

APPENDIX

ALGORITHM DEVELOPMENT

The equations of motion are written as

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial(\mathbf{F} - \mathbf{R})}{\partial x} + \frac{\partial(\mathbf{G} - \mathbf{S})}{\partial y} = \mathbf{0}. \quad (\text{A1})$$

where the vectors $\mathbf{Q}$, $\mathbf{F}$, $\mathbf{G}$, $\mathbf{R}$, and $\mathbf{S}$ are as defined in Chapter II. In this form, the equations are expressed in terms of a rectangular Cartesian coordinate system (x,y) in physical space. In the interest of computational efficiency and accuracy, and to simplify boundary condition development, the equations are transformed into a computational domain (ξ,η) . This is also referred to as a body-fitted coordinate system, since one or more of the coordinate planes $\xi = \text{constant}$ or $\eta = \text{constant}$ are coincident with the surfaces of the body. The details of the coordinate transformation can be found in a number of references, such as Hoffman and Chang [41]. The results are given here without derivation for the convenience of the reader.

A general mapping between the (x,y,t) system and the (ξ,η,τ) system can be written as

$$\begin{aligned} x &= x(\xi, \eta, \tau) \\ y &= y(\xi, \eta, \tau) \\ t &= t(\xi, \eta, \tau). \end{aligned} \quad (\text{A2})$$

The Jacobian of the transformation is

$$J = \begin{vmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \tau} \\ \frac{\partial y}{\partial \xi} & \frac{\partial y}{\partial \eta} & \frac{\partial y}{\partial \tau} \\ \frac{\partial t}{\partial \xi} & \frac{\partial t}{\partial \eta} & \frac{\partial t}{\partial \tau} \end{vmatrix}. \quad (\text{A3})$$

Assuming that $t = \tau$, this reduces to

$$J = \frac{\partial x}{\partial \xi} \frac{\partial y}{\partial \eta} - \frac{\partial y}{\partial \xi} \frac{\partial x}{\partial \eta}. \quad (\text{A4})$$

The condition $J > 0$ is sufficient for the mapping to be well-defined over the domain. The partial derivatives in the Jacobian are the metrics of the transformation. Given the metrics of the transformation from one system to the other, say (x,y) to (ξ,η) , the reciprocal set can be computed as

$$\xi_x = \frac{y_\eta}{J}, \xi_y = \frac{-x_\eta}{J}, \xi_t = \frac{1}{J} (x_\eta y_\tau - y_\eta x_\tau) \quad (\text{A5})$$

$$\eta_x = \frac{-y_\xi}{J}, \eta_y = \frac{x_\xi}{J}, \eta_t = \frac{1}{J} (x_\tau y_\xi - y_\tau x_\xi) \quad (\text{A6})$$

and the subscript notation refers to the partial derivative. Using the definitions in equations (A5) and (A6) and the chain rule of calculus the conservation equations can be written as

$$\frac{\partial \tilde{\mathbf{Q}}}{\partial \tau} + \frac{\partial (\tilde{\mathbf{F}} - \tilde{\mathbf{R}})}{\partial \xi} + \frac{\partial (\tilde{\mathbf{G}} - \tilde{\mathbf{S}})}{\partial \eta} = \mathbf{0} \quad (\text{A7})$$

after some algebraic manipulation. The transformed vectors $\tilde{\mathbf{Q}}$, $\tilde{\mathbf{F}}$, $\tilde{\mathbf{G}}$, $\tilde{\mathbf{R}}$, and $\tilde{\mathbf{S}}$ are given by

$$\tilde{\mathbf{Q}} = \frac{1}{J} \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ e_t \end{bmatrix} \quad (\text{A8})$$

$$\tilde{\mathbf{F}} = \frac{1}{J} \begin{bmatrix} \rho U \\ \rho u U + \xi_x p \\ \rho v U + \xi_y p \\ (e_t + p)U - \xi_t p \end{bmatrix} \quad (\text{A9})$$

$$\tilde{\mathbf{G}} = \frac{1}{J} \begin{bmatrix} \rho V \\ \rho u V + \eta_x p \\ \rho v V + \eta_y p \\ (e_t + p)V - \eta_t p \end{bmatrix} \quad (\text{A10})$$

$$\tilde{\mathbf{R}} = \frac{1}{J} \begin{bmatrix} 0 \\ \xi_x \tau_{xx} + \xi_y \tau_{xy} \\ \xi_x \tau_{xy} + \xi_y \tau_{yy} \\ \xi_x \delta_x + \xi_y \delta_y \end{bmatrix} \quad (\text{A11})$$

$$\tilde{\mathbf{S}} = \frac{1}{J} \begin{bmatrix} 0 \\ \eta_x \tau_{xx} + \eta_y \tau_{xy} \\ \eta_x \tau_{xy} + \eta_y \tau_{yy} \\ \eta_x \delta_x + \eta_y \delta_y \end{bmatrix} \quad (\text{A12})$$

and the quantities U and V are the contravariant velocity components,

$$U = \xi_t + \xi_x u + \xi_y v \quad (\text{A13})$$

$$V = \eta_t + \eta_x u + \eta_y v. \quad (\text{A14})$$

The conservation equations are solved using an implicit Beam-Warming [42] type approximate factorization scheme. To begin the development, the time derivative is approximated using a second-order backward difference, and the convective flux vectors

are expressed at the (n+1) time level. Additionally, the viscous flux vectors are split such that the parallel terms (denoted below as $\tilde{\mathbf{R}}_1, \tilde{\mathbf{S}}_1$) are also evaluated at the (n+1) time level. The cross terms in the viscous fluxes ($\tilde{\mathbf{R}}_2, \tilde{\mathbf{S}}_2$) are lagged at the n^{th} time level. This results in

$$\frac{3\tilde{\mathbf{Q}}^{n+1} - 4\tilde{\mathbf{Q}}^n + \tilde{\mathbf{Q}}^{n-1}}{2\Delta\tau} + \left(\tilde{\mathbf{F}}^{n+1} - \tilde{\mathbf{R}}_1^{n+1}\right)_\xi + \left(\tilde{\mathbf{G}}^{n+1} - \tilde{\mathbf{S}}_1^{n+1}\right)_\eta = \left(\tilde{\mathbf{R}}_2^n\right)_\xi + \left(\tilde{\mathbf{S}}_2^n\right)_\eta \quad (\text{A15})$$

where the metric quantities are also evaluated at the (n+1) time level. The convective flux vectors can be written as functions of the conservative variables $\mathbf{Q}$. Thus the convective flux vectors at the (n+1) time level are linearized as

$$\begin{aligned} \tilde{\mathbf{F}}^{n+1} &\approx \tilde{\mathbf{F}}^n + \left. \frac{\partial \tilde{\mathbf{F}}}{\partial \tilde{\mathbf{Q}}} \right|_n \cdot (\tilde{\mathbf{Q}}^{n+1} - \tilde{\mathbf{Q}}^n) \equiv \tilde{\mathbf{F}}^n + \tilde{\mathbf{A}} \cdot \Delta \tilde{\mathbf{Q}}^n \\ \tilde{\mathbf{G}}^{n+1} &\approx \tilde{\mathbf{G}}^n + \left. \frac{\partial \tilde{\mathbf{G}}}{\partial \tilde{\mathbf{Q}}} \right|_n \cdot (\tilde{\mathbf{Q}}^{n+1} - \tilde{\mathbf{Q}}^n) \equiv \tilde{\mathbf{G}}^n + \tilde{\mathbf{B}} \cdot \Delta \tilde{\mathbf{Q}}^n \end{aligned} \quad (\text{A16})$$

The viscous fluxes are functions of $\mathbf{Q}$ and gradients of $\mathbf{Q}$. Consider $\mathbf{R}_1$ for example.

Formally

$$\tilde{\mathbf{R}}_1 = \tilde{\mathbf{R}}_1(\tilde{\mathbf{Q}}, \tilde{\mathbf{Q}}_\xi) \quad (\text{A17})$$

so that a time linearization may be expressed as

$$\tilde{\mathbf{R}}_1^{n+1} \approx \tilde{\mathbf{R}}_1^n + \left. \frac{\partial \tilde{\mathbf{R}}_1}{\partial \tilde{\mathbf{Q}}} \right|_n \cdot \Delta \tilde{\mathbf{Q}} + \left. \frac{\partial \tilde{\mathbf{R}}_1}{\partial \tilde{\mathbf{Q}}_\xi} \right|_n \cdot \Delta \tilde{\mathbf{Q}}_\xi. \quad (\text{A18})$$

The last term can be rewritten to give

$$\tilde{\mathbf{R}}_1^{n+1} \approx \tilde{\mathbf{R}}_1^n + \frac{\partial \tilde{\mathbf{R}}_1}{\partial \tilde{\mathbf{Q}}} \Big|_n \cdot \Delta \tilde{\mathbf{Q}} + \frac{\partial}{\partial \xi} \left(\frac{\partial \tilde{\mathbf{R}}_1}{\partial \tilde{\mathbf{Q}}_\xi} \Big|_n \cdot \Delta \tilde{\mathbf{Q}} \right) - \frac{\partial}{\partial \xi} \left(\frac{\partial \tilde{\mathbf{R}}_1}{\partial \tilde{\mathbf{Q}}_\xi} \Big|_n \right) \cdot \Delta \tilde{\mathbf{Q}}. \quad (\text{A19})$$

Similarly,

$$\tilde{\mathbf{S}}_1^{n+1} \approx \tilde{\mathbf{S}}_1^n + \frac{\partial \tilde{\mathbf{S}}_1}{\partial \tilde{\mathbf{Q}}} \Big|_n \cdot \Delta \tilde{\mathbf{Q}} + \frac{\partial}{\partial \eta} \left(\frac{\partial \tilde{\mathbf{S}}_1}{\partial \tilde{\mathbf{Q}}_\eta} \Big|_n \cdot \Delta \tilde{\mathbf{Q}} \right) - \frac{\partial}{\partial \eta} \left(\frac{\partial \tilde{\mathbf{S}}_1}{\partial \tilde{\mathbf{Q}}_\eta} \Big|_n \right) \cdot \Delta \tilde{\mathbf{Q}}. \quad (\text{A20})$$

With the following definitions

$$\tilde{\mathbf{V}} = \frac{\partial \tilde{\mathbf{R}}_1}{\partial \tilde{\mathbf{Q}}} \Big|_n, \tilde{\mathbf{W}} = \frac{\partial \tilde{\mathbf{S}}_1}{\partial \tilde{\mathbf{Q}}} \Big|_n, \tilde{\mathbf{V}}_\xi = \frac{\partial \tilde{\mathbf{R}}_1}{\partial \tilde{\mathbf{Q}}_\xi} \Big|_n, \tilde{\mathbf{W}}_\eta = \frac{\partial \tilde{\mathbf{S}}_1}{\partial \tilde{\mathbf{Q}}_\eta} \Big|_n \quad (\text{A21})$$

eq. (A15) can be written in “delta” formulation as

$$\begin{aligned} \frac{3}{2} \Delta \tilde{\mathbf{Q}}^n + \Delta \tau \left(\left(\tilde{\mathbf{A}} - \tilde{\mathbf{V}} + \frac{\partial \tilde{\mathbf{V}}_\xi}{\partial \xi} \right) \Delta \tilde{\mathbf{Q}}^n - \frac{\partial (\tilde{\mathbf{V}}_\xi \Delta \tilde{\mathbf{Q}}^n)}{\partial \xi} \right)_{\xi} + \\ \Delta \tau \left(\left(\tilde{\mathbf{B}} - \tilde{\mathbf{W}} + \frac{\partial \tilde{\mathbf{W}}_\eta}{\partial \eta} \right) \Delta \tilde{\mathbf{Q}}^n - \frac{\partial (\tilde{\mathbf{W}}_\eta \Delta \tilde{\mathbf{Q}}^n)}{\partial \eta} \right)_{\eta} = \mathbf{RHS}^n \Delta \tau \end{aligned} \quad (\text{A22})$$

with the residual $\mathbf{RHS}^n$ given by

$$\mathbf{RHS}^n = -(\tilde{\mathbf{F}}^n - \tilde{\mathbf{R}}^n)_{\xi} - (\tilde{\mathbf{G}}^n - \tilde{\mathbf{S}}^n)_{\eta} + \frac{\Delta \tilde{\mathbf{Q}}^{n-1}}{2\Delta \tau}. \quad (\text{A23})$$

Complete expressions for the flux Jacobians in eqs. (A16) and (A21) can be found in

numerous references, particularly Visbal [43] and Hoffman and Chang [41]. The

derivatives of $\tilde{\mathbf{V}}_\xi$ and $\tilde{\mathbf{W}}_\eta$ that appear in the first set of terms in each of the large brackets

combine with the viscous Jacobian terms to cancel exactly for rectilinear Cartesian

coordinate systems, but for curvilinear systems there is no cancellation. For curvilinear

systems the remainder is proportional to spatial gradients of the metric terms and viscosity coefficients. To the best of the author's knowledge these terms are typically left out of developments of implicit treatments of the viscous terms, even though they do not disappear for curvilinear coordinate systems. Several runs were made with and without these terms during the development of the algorithm, and the results did not indicate any conclusive difference either way. However, for the sake of completeness these terms were included in all the results reported in this work.

Equation (A22) is now approximately factored to reduce the system to a set of block tridiagonal equations. The result is

$$\begin{aligned} \left[\frac{\mathbf{I}}{\Delta\tau} + \frac{2}{3}\delta_\xi \left(\left(\tilde{\mathbf{A}} - \tilde{\mathbf{V}} + \frac{\partial \tilde{\mathbf{V}}_\xi}{\partial \xi} \right) [-] - \delta_\xi (\tilde{\mathbf{V}}_\xi [-]) \right) \right] \Delta \tilde{\mathbf{Q}}^* &= \frac{2}{3} \frac{\mathbf{RHS}^n}{\Delta\tau} \\ \left[\frac{\mathbf{I}}{\Delta\tau} + \frac{2}{3}\delta_\eta \left(\left(\tilde{\mathbf{B}} - \tilde{\mathbf{W}} + \frac{\partial \tilde{\mathbf{W}}_\eta}{\partial \eta} \right) [-] - \delta_\eta (\tilde{\mathbf{W}}_\eta [-]) \right) \right] \Delta \tilde{\mathbf{Q}}^n &= \frac{\Delta \tilde{\mathbf{Q}}^*}{\Delta\tau} \end{aligned} \quad (\text{A24})$$

where the spatial derivatives are approximated using second-order central differences, designated by the operators $\delta_{\epsilon,\eta}$. The notation used above is meant to indicate that the terms in square brackets form operators which act on the Δ -quantities. It is understood that the independent variables (ξ, η) are discretized as (ξ_i, η_j) , $i = 1, 2, 3, \dots, I_{\max}$, $j = 1, 2, 3, \dots, J_{\max}$.

To avoid the introduction of conservation errors, the metric quantities are computed using central differences also. The blended second-fourth order dissipation of Pulliam [17] is added to the system to ensure stability and avoid odd-even decoupling. The modified equation is

$$\begin{aligned}
& \left[\frac{\mathbf{I}}{\Delta\tau} + \frac{2}{3} \delta_\xi \left(\left(\tilde{\mathbf{A}} - \tilde{\mathbf{V}} + \frac{\partial \tilde{\mathbf{V}}_\xi}{\partial \xi} \right) [-] - \delta_\xi (\tilde{\mathbf{V}}_\xi [-]) - \varepsilon_2 \nabla_\xi \left(\frac{\sigma_\xi}{J} \Delta_\xi \right) [-] \right) \right] \Delta \tilde{\mathbf{Q}}^* = \frac{2}{3} \frac{\mathbf{RHS}'^n}{\Delta\tau} \\
& \left[\frac{\mathbf{I}}{\Delta\tau} + \frac{2}{3} \delta_\eta \left(\left(\tilde{\mathbf{B}} - \tilde{\mathbf{W}} + \frac{\partial \tilde{\mathbf{W}}_\eta}{\partial \eta} \right) [-] - \delta_\eta (\tilde{\mathbf{W}}_\eta [-]) - \varepsilon_2 \nabla_\eta \left(\frac{\sigma_\eta}{J} \Delta_\eta \right) [-] \right) \right] \Delta \tilde{\mathbf{Q}}^n = \frac{\Delta \tilde{\mathbf{Q}}^*}{\Delta\tau}
\end{aligned} \tag{A25}$$

with the right-hand side given by

$$\begin{aligned}
\mathbf{RHS}'^n = \mathbf{RHS}^n & + \frac{\varepsilon_2}{J} \left[\nabla_\xi (\sigma_\xi \Delta_\xi J \tilde{\mathbf{Q}}^n) + \nabla_\eta (\sigma_\eta \Delta_\eta J \tilde{\mathbf{Q}}^n) \right] - \\
& \frac{\varepsilon_4}{J} \left[\nabla_\xi (\sigma_\xi \Delta_\xi \nabla_\xi \Delta_\xi J \tilde{\mathbf{Q}}^n) + \nabla_\eta (\sigma_\eta \Delta_\eta \nabla_\eta \Delta_\eta J \tilde{\mathbf{Q}}^n) \right].
\end{aligned} \tag{A26}$$

The operators ∇ and Δ signify the appropriate backward and forward differences, respectively. The constants are given by

$$\begin{aligned}
\sigma_\xi &= \alpha_\xi \left[1 + \left(\frac{\alpha_\eta}{\alpha_\xi} \right)^\beta \right], \sigma_\eta = \alpha_\eta \left[1 + \left(\frac{\alpha_\xi}{\alpha_\eta} \right)^\beta \right] \\
\alpha_\xi &= |U| + a \sqrt{\xi_x^2 + \xi_y^2}, \alpha_\eta = |V| + a \sqrt{\eta_x^2 + \eta_y^2}.
\end{aligned} \tag{A27}$$

The parameters ε_2 , ε_4 , and β are user-selected inputs. For most of the calculations discussed in this study, β was set to 0.67, and $(\varepsilon_2, \varepsilon_4) = (0.2, 0.04)$. This is the form of the equations used in this study.

Since the implementation of artificial viscosity seems to be something of an art, the actual difference scheme used in this scheme is now presented. For simplicity only the term in the ξ -direction is considered, but the η -term is similar. In the ξ -direction, the dissipation is given by

$$\mathbf{AV}_{ij} = \frac{1}{2} \left[\frac{(\sigma_\xi)_{i+1,j}}{J_{i+1,j}} + \frac{(\sigma_\xi)_{i,j}}{J_{i,j}} \right] \begin{bmatrix} \kappa_2^+ (Q_{i+1,j} - Q_{i,j}) - \\ \kappa_4^+ (Q_{i+2,j} - 3Q_{i+1,j} + 3Q_{i,j} - Q_{i-1,j}) \end{bmatrix} \\ - \frac{1}{2} \left[\frac{(\sigma_\xi)_{i,j}}{J_{i,j}} + \frac{(\sigma_\xi)_{i-1,j}}{J_{i-1,j}} \right] \begin{bmatrix} \kappa_2^- (Q_{i,j} - Q_{i-1,j}) - \\ \kappa_4^- (Q_{i+1,j} - 3Q_{i,j} + 3Q_{i-1,j} - Q_{i-2,j}) \end{bmatrix} \quad (\text{A28})$$

where

$$\begin{aligned} \kappa_2^+ &= \varepsilon_2 \cdot \max(\bar{P}_{i+1,j}, \bar{P}_{i,j}) \\ \kappa_2^- &= \varepsilon_2 \cdot \max(\bar{P}_{i,j}, \bar{P}_{i-1,j}) \\ \kappa_4^{+,-} &= \max(0, \varepsilon_4 - \kappa_2^{+,-}) \\ \bar{P}_{i,j} &= \text{abs} \left(\frac{p_{i+1,j} - 2p_{i,j} + p_{i-1,j}}{p_{i+1,j} + 2p_{i,j} + p_{i-1,j}} \right) \end{aligned} \quad (\text{A29})$$

It should be noted that the Q 's are not divided by the transformation Jacobian. Along boundaries the modifications described by Pulliam [17] are used, which basically consist of using second-order differences in place of the third-order differences where the difference stencil does not support the higher-order differencing.

Solutions to the system of equations (A25) are obtained using a block tridiagonal solver. The original code was developed by Roach and Jenkins [44], but for this study substantial modifications were made to the artificial viscosity, boundary conditions, and metrics. The Spalart-Allmaras turbulence model was also added, along with the overset grid capability. An implicit treatment of the parallel viscous terms was also developed, and the time integration was changed to second order.

It is well-known that the solution of a system of partial differential equations is determined by the particular boundary conditions which are imposed. Thus, it is essential to use numerical boundary conditions which are good approximations to the physical

conditions that exist in a given problem. For the external flow problems discussed in this work, boundary conditions must be applied at solid surfaces, along the farfield boundary, and along grid cut lines.

Along solid walls, the no-slip condition requires the fluid velocity to be equal to the velocity of the solid surface. For a prescribed surface motion the velocity of the fluid at the surface is then given by

$$\mathbf{u} = \mathbf{b} \quad (\text{A30})$$

where $\mathbf{b}$ is the velocity of the solid surface. Ignoring the possibility of surface deformation, i.e. considering only rigid bodies, the surface velocity $\mathbf{b}$ can be decomposed into a translational and a rotational part,

$$\mathbf{b} = \mathbf{v}_{cr} + \boldsymbol{\omega} \times \mathbf{r}. \quad (\text{A31})$$

Here $\mathbf{v}_{cr}$ is the translational velocity of the instantaneous center of rotation, $\boldsymbol{\omega} = \omega \mathbf{k}$ is the angular velocity of the body, and $\mathbf{r} = x_s \mathbf{i} + y_s \mathbf{j}$ is the vector distance from the center of rotation to a point on the body surface.

An equation for the pressure can be developed using the momentum equation. Taking the normal component of the momentum equation and neglecting the viscous terms yields

$$-\rho \mathbf{n} \cdot \frac{D\mathbf{b}}{Dt} = \frac{\partial p}{\partial n} \approx \frac{\frac{\partial p}{\partial \eta} (\eta_x^2 + \eta_y^2) + \frac{\partial p}{\partial \xi} (\eta_x \xi_x + \eta_y \xi_y)}{\sqrt{\eta_x^2 + \eta_y^2}} \quad (\text{A32})$$

It is assumed that the solid surface represents a surface of constant η in the boundary-fitted coordinate system, such that $\mathbf{n} = \nabla\eta/|\nabla\eta|$. The normal component of the surface acceleration is then given by

$$a_n \equiv \frac{\nabla\eta}{|\nabla\eta|} \cdot \frac{D\mathbf{b}}{Dt} = \eta_x (-\dot{\omega} y_s - \omega^2 x_s) + \eta_y (\dot{\omega} x_s - \omega^2 y_s). \quad (\text{A33})$$

If the grid lines are orthogonal at the surface, as will be the case for all grids in this study, surfaces of constant η are perpendicular to surfaces of constant ξ . Then the term $\eta_x \xi_x + \eta_y \xi_y$ is identically zero. Approximating the normal derivative using a second-order forward finite difference gives the following result for the pressure at the surface:

$$-\rho_{i,1} a_n = \frac{1}{2} \sqrt{\eta_x^2 + \eta_y^2} [-3p_{i,1} + 4p_{i,2} - p_{i,3}] \quad (\text{A34})$$

or

$$p_{i,1} = \frac{2\sqrt{\eta_x^2 + \eta_y^2} \cdot \rho_{i,1} a_n + 4p_{i,2} - p_{i,3}}{3} \quad (\text{A35})$$

where the solid surface is described by $j=1, i=1,2,3,\dots,I_{\max}$ in computational space. In the pressure equation, the density is held at its n^{th} level value. After the pressure is updated, the density on the surface is computed assuming the solid wall is adiabatic. This condition is expressed for a ideal gas as

$$\mathbf{n} \cdot \nabla T = 0 = \mathbf{n} \cdot \nabla \left(\frac{p}{\rho R} \right). \quad (\text{A36})$$

For simplicity the inverse of density, the specific volume ν , is actually solved for in the algorithm. Combining equations (A32), (A33) and (A36) gives the desired result for the specific volume,

$$\frac{\partial \nu}{\partial n} = \frac{a_n}{p}. \quad (\text{A37})$$

This equation is differenced in a manner analogous to the pressure equation and solved after the pressure update for the specific volume. In practice, the equations for the density and pressure are updated in an iterative cycle for each time step in order to reduce the effect of lagging the density in the pressure equation.

Thus, the flow at a solid surface is updated by first determining the pressure and density on the surface. Velocity components are computed based on the prescribed motion of the surface. The conservative variables are then evaluated from the new values of pressure, density, and velocity.

Along the far field, a non-reflecting boundary condition based on Thomson's [19] approach is employed. The basic idea is to diagonalize the conservation equations in the direction normal to the boundary. The information in the eigenvalues is then used to determine the appropriate direction of one-sided derivatives along the boundary.

Formally, consider the conservation equations,

$$\frac{\partial \tilde{\mathbf{Q}}}{\partial \tau} + \frac{\partial \tilde{\mathbf{F}}}{\partial \xi} + \frac{\partial \tilde{\mathbf{G}}}{\partial \eta} = \mathbf{0} \quad (\text{A38})$$

where it is assumed that the viscous terms are negligible in the far field. Following previous developments, these equations can be recast using the flux Jacobians to give

$$\frac{\partial \tilde{\mathbf{Q}}}{\partial \tau} + \tilde{\mathbf{A}} \frac{\partial \tilde{\mathbf{Q}}}{\partial \xi} + \tilde{\mathbf{B}} \frac{\partial \tilde{\mathbf{Q}}}{\partial \eta} = \mathbf{0}. \quad (\text{A39})$$

In this discussion, the far field boundary is assumed to be a boundary of constant η , given by $\eta = \eta_{\max}$ (or $j = J_{\max}$). In applying a difference scheme using equation (A39), derivatives in the ξ -direction can be approximated using centered differences. However, in the η -direction no points exist for $\eta > \eta_{\max}$. Thus only one-sided differences can be written. But applying one-sided differences in an arbitrary fashion can lead to significant errors and even instabilities, since the direction of signal propagation is not handled properly. The essence of Thomson's approach is to utilize the eigenstructure of the flux Jacobians (in this case $\tilde{\mathbf{B}}$) to select the appropriate differencing.

The flux Jacobian $\tilde{\mathbf{B}}$ can be diagonalized by means of a similarity transformation as

$$\tilde{\mathbf{B}} = \mathbf{X} \Lambda \mathbf{X}^{-1}. \quad (\text{A40})$$

The matrix $\mathbf{X}$ is composed of the eigenvectors of $\tilde{\mathbf{B}}$, and Λ is a diagonal matrix containing the eigenvalues of $\tilde{\mathbf{B}}$,

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{bmatrix} \quad (\text{A41})$$

where

$$\begin{aligned}
\lambda_1 &= \eta_t + \eta_x u + \eta_y v \\
\lambda_2 &= \eta_t + \eta_x u + \eta_y v \\
\lambda_3 &= \eta_t + \eta_x u + \eta_y v + a\sqrt{\eta_x^2 + \eta_y^2} \\
\lambda_4 &= \eta_t + \eta_x u + \eta_y v - a\sqrt{\eta_x^2 + \eta_y^2}
\end{aligned} \tag{A42}$$

Substituting (A40) into equation (A39) gives

$$\frac{\partial \tilde{Q}}{\partial \tau} + \tilde{A} \frac{\partial \tilde{Q}}{\partial \xi} + \mathbf{X} \Lambda \mathbf{X}^{-1} \frac{\partial \tilde{Q}}{\partial \eta} = 0 \tag{A43}$$

or

$$\frac{\partial \tilde{Q}}{\partial \tau} = -\tilde{A} \frac{\partial \tilde{Q}}{\partial \xi} - \mathbf{X} \Lambda \mathbf{X}^{-1} \frac{\partial \tilde{Q}}{\partial \eta} \tag{A44}$$

The right-hand side can be interpreted as the combination of terms that make up the local time derivative of $\tilde{Q}$ at a boundary point. Call

$$-\mathbf{X} \Lambda \mathbf{X}^{-1} \frac{\partial \tilde{Q}}{\partial \eta} = \left. \frac{\partial \tilde{Q}}{\partial \tau} \right|_{\eta} \tag{A45}$$

where the subscript notation is a convenience to indicate that this is the η -part of the time rate of change. This term represents the contribution to $\partial \tilde{Q} / \partial \tau$ due to waves passing through the boundary in the η -direction. Equation (A45) can be shown to be approximately equivalent to a set of one-dimensional characteristic equations in η . Let a_{ij} be the entries in $\mathbf{X}^{-1}$, and let $\lambda^{(i)}$ be the i^{th} entry on the diagonal of Λ . Then the i^{th} equation in the set (A45) can be written as

$$a_{ij} \frac{\partial \tilde{Q}_j}{\partial \tau} \bigg|_{\eta} + \lambda^{(i)} a_{ij} \frac{\partial \tilde{Q}_j}{\partial \eta} = 0. \quad (\text{A46})$$

Here the indices i and j refer to the number of entries in the vector of conservative variables. Now assume that a new variable W can be defined as

$$dW_i = a_{ij} d\tilde{Q}_j. \quad (\text{A47})$$

which can be expanded to give

$$\frac{\partial W_i}{\partial \tau} d\tau + \frac{\partial W_i}{\partial \eta} d\eta = a_{ij} \left(\frac{\partial \tilde{Q}_j}{\partial \tau} d\tau + \frac{\partial \tilde{Q}_j}{\partial \eta} d\eta \right), \quad (\text{A48})$$

where dependence on ξ has been suppressed. By comparing the terms in eq. (A48) that multiply the independent variations $d\tau$ and $d\eta$, eq. (A46) can be recast as

$$\frac{\partial W_i}{\partial \tau} + \lambda^{(i)} \frac{\partial W_i}{\partial \eta} = 0. \quad (\text{A49})$$

Then along curves where $\lambda^{(i)} = d\eta/d\tau$, clearly W_i is constant, indicating that (under the approximations made in eqs. (A47) and (A48)) W is a set of characteristic variables.

A non-reflective condition is imposed by requiring that incoming waves contribute nothing to the time rate of change of $\tilde{Q}$ at the boundary. The mechanism to enforce this is developed by examining the signs of the eigenvalues $\lambda^{(i)}$. Let

$$H_i \equiv a_{ij} \frac{\partial \tilde{Q}_j}{\partial \eta}. \quad (\text{A50})$$

Along an $\eta_{\max}$ boundary, an inward propagating wave corresponds to $\lambda^{(i)} < 0$. If $\lambda^{(i)} < 0$, then the corresponding H_i is set to zero. Otherwise the H_i is computed from equation (A50), using backward differences formed from points within the domain for the needed derivatives.

The H_i are then used to compute the η -part of the time rate of change of $\tilde{Q}$ using eq. (A45), and this result is then substituted back into equation (A44). Using centered-difference approximations for the ξ -derivatives, an algorithm for updating $\tilde{Q}$ on the far field boundary is thus obtained. The time integration is performed using a four-stage Runge-Kutta scheme.

In practice some modifications to the above procedure are necessary. The equations are expressed in curvilinear coordinates, involving metric terms that arise through the coordinate transformation. As discussed previously, the metrics must be computed in a fashion compatible with the differencing used for the spatial derivatives of the inviscid flux terms to avoid conservation errors. However, the metrics calculated for the flow solver do not maintain a uniform free-stream conservation when used with the boundary algorithm. This is due to the mixture of differencing used, as well as the fact that certain terms are reduced to zero as needed to maintain non-reflectivity. The simplest solution to this problem is to subtract out a uniform free-stream from the appropriate quantities. Since a uniform flow automatically satisfies the conservation equations this is equivalent to subtracting out zero.

A second difficulty can arise from the explicit time integration used in the farfield update procedure. All explicit procedures are subject to a CFL timestep limitation. If the

local CFL number for a boundary point (determined by the flow solver) exceeds the maximum CFL number for the Runge-Kutta scheme, the Runge-Kutta algorithm will go unstable. For many grid topologies this is not a problem, since the timestep used by the flow solver is determined using the smallest cell in the grid. This cell is usually several orders of magnitude smaller than any of the far field boundary cells, thus ensuring that the local explicit CFL restriction is not violated in the boundary update. For situations where this is not the case, a simple and effective fix is to break the flow solver timestep up into substeps which individually do not violate the local CFL constraint for the farfield update using the Runge-Kutta scheme. The integration for a boundary point then proceeds over a number of these substeps until the overall update is complete. This procedure is admittedly approximate, but it seems to work well in the situations where it has been applied. A second, and more exact, alternative would be to incorporate the boundary condition in the implicit scheme used for the interior points. Chakravarthy [45] describes this approach, which basically involves using the characteristic form of the equations along the direction of interest (as developed above) to obtain the necessary entries in the appropriate sweep equations.

Although the development of this update procedure relies on the inviscid conservation equations, the viscous terms can be carried along in the analysis. A formal procedure was developed by Poinso and Lele [46], which embodies much of the preceding analysis of Thomson's approach. A simpler method, suitable for problems where the viscous influence at the boundary is very small, is to compute directly the

viscous terms tangent to the boundary, and use an extrapolation from the interior for the normal viscous terms. Such an approximation is used in this work.

Lastly, since centered differences are used to represent the tangential derivatives, explicit artificial viscosity is added to the numerical recipe to promote stability. This artificial viscosity is the second/fourth order dissipation described previously, but is computed only in the tangential direction.

The final boundary to consider is the grid cut line. A grid cut line, where a grid re-enters itself, is handled by simple averaging. Assuming that the cut line is formed by the surfaces $\xi_{\min} = 1$ and $\xi_{\max} = \Pi$, over all η , the dependent variables along the cut are updated as

$$Q_{1,j} = Q_{\Pi,j} = \frac{1}{2}(Q_{2,j} + Q_{\Pi-1,j}), \quad (\text{A51})$$

where the update is applied to the untransformed dependent variables, and the subscript indices again refer to the grid points in computational space.

VITA

Andrew Gregory Denny was born and raised in Knoxville, Tennessee. After attending both elementary school and high school in the Knoxville area, he began his college career at the University of Tennessee, Knoxville, in September, 1981. Greg (as he prefers to be called) graduated in June, 1985, with a Bachelor of Science degree in Mechanical Engineering after successfully completing the prescribed course of study. Commencement was promptly followed by graduate school and his marriage to Janice Fay Conner. He received his Master of Science degree in Mechanical Engineering in August, 1987. After graduation, he and Janice moved to Tullahoma, Tennessee, where he was employed as an engineer with Calspan Corporation at Arnold Engineering Research and Development Center (AEDC). At that time, he began part-time work on his Ph.D. in Aerospace Engineering at the University of Tennessee Space Institute. Through two contract changes at AEDC, he is now employed by Sverdrup Technology, Inc. During this time of working full-time and pursuing a degree part-time, Greg was blessed with the addition of four children, Joshua, Allison, Andrew, and Kristin. Greg received his Doctor of Philosophy degree in Aerospace Engineering in December 1997.