

August 6, 1963

To the Graduate Council:

I am submitting herewith a thesis written by Herman Leslie LaNieve III entitled "Entrance Effects in Non-Newtonian Pipe Flow." I recommend that it be accepted for nine quarter hours of credit in partial fulfillment of the requirements for the degree of Master of Science, with a major in Chemical Engineering.

DeBryne
Major Professor

We have read this thesis and
recommend its acceptance:

M. P. Pables

John W. Prados

G. B. Albert

Accepted for the Council:

Hilton A. Smith
Dean of the Graduate School

ENTRANCE EFFECTS IN NON-NEWTONIAN PIPE FLOW

**A Thesis
Presented to
the Graduate Council of
The University of Tennessee**

**In Partial Fulfillment
of the Requirements for the Degree
Master of Science**

**by
Herman Leslie LaNieve III
August 1963**

ACKNOWLEDGEMENT

The author wishes to acknowledge the efforts of Dr. D. C. Bogue, his major professor, whose advice and guidance throughout this investigation have been invaluable.

The author is grateful to Dr. H. F. Johnson, Head of the Department of Chemical and Metallurgical Engineering, for providing financial aid in the form of graduate and teaching assistantships to enable the author to pursue his graduate studies.

The experimental equipment was constructed by Mr. O. V. Prater and modified by Mr. Prater and Mr. Howard Freeman of the Chemical and Metallurgical Engineering Laboratories, under the direction of Mr. E. H. Honeycutt. Their work is gratefully acknowledged.

Sincere appreciation is extended to Mrs. W. L. Norton for her diligence and speed in typing this manuscript.

The author is indeed grateful to his wife, whose inspiration and help throughout this period of graduate work are greatly appreciated.

ABSTRACT

The prediction of the excess pressure loss accompanying the flow of a fluid through a sharp-edged contraction (from a large reservoir into a small tube) has been the subject of many theoretical analyses. Recent analyses have been concerned with a general "power law" non-Newtonian fluid. However, few experimental measurements, either of pressure losses or of velocity profiles, have been made, especially for non-Newtonian fluids.

In this investigation the entrance region pressure profiles were measured for an ethylene glycol-water solution, which was Newtonian, and for Carbopol-water gels of several concentrations, which were non-Newtonian. The flow was in the laminar regime with Reynolds numbers ranging from 100 to 1500.

The experimental results were reported in the form of an entrance correction to the Poiseuille equation. An attempt was made to separate the upstream correction from the downstream correction, since the theoretical developments have ignored the upstream pressure effects.

The following values were found for the total entrance pressure correction:

Power Law Exponent (n)	Total Pressure Correction (velocity heads)
1	2.53
.62	2.30

(n)	(velocity heads)
.47	2.27
.42	2.20

The maximum experimental error was estimated to be 3 per cent. The entrance correction was found to decrease with decreasing values of n in qualitative but not quantitative agreement with theory. The quantitative discrepancy probably was caused by an increased upstream pressure loss with decreasing n . A vena contracta was found to exist at the entrance for Reynolds numbers as low as 400, in disagreement with the plug flow assumption.

From this experimental investigation it was concluded that the pressure effects upstream from and immediately at the entrance, which have been neglected in the theory, are in fact significant, but that the downstream pressure effects are in reasonable agreement with the theoretical analyses.

TABLE OF CONTENTS

CHAPTER	PAGE
I. INTRODUCTION	1
II. BACKGROUND AND THEORY	3
Developed Flow	3
Flow in the Entrance Region	6
The Boundary Layer Approach	11
III. DESCRIPTION OF THE INVESTIGATION	14
Experimental Equipment	14
Test Materials	20
Experimental Procedure	20
IV. PRESENTATION AND DISCUSSION OF RESULTS	24
V. CONCLUSIONS AND RECOMMENDATIONS	41
LIST OF REFERENCES	43
APPENDICES	46
A. SAMPLE CALCULATIONS	47
B. ANALYSIS OF SOURCES OF EXPERIMENTAL ERROR	52
C. DESCRIPTION OF VISCOMETRY	55
D. TABULATION OF DATA	60
LIST OF SYMBOLS	77
VITA	81

LIST OF TABLES

TABLE	PAGE
I. Location of Pressure Taps	17
II. The Entrance Pressure Correction	27
III. Newtonian Data Processing Program	48
IV. Non-Newtonian Data Processing Program	49
V. Error Analysis of a Typical Newtonian Data Point	53
VI. Reduced Data for Newtonian Run A	61
VII. Reduced Data for Newtonian Run B	62
VIII. Reduced Data for Newtonian Run C	63
IX. Reduced Data for Newtonian Run D	64
X. Reduced Data for Newtonian Run E	65
XI. Reduced Data for non-Newtonian Run F	66
XII. Reduced Data for non-Newtonian Run G	67
XIII. Reduced Data for non-Newtonian Run H	68
XIV. Reduced Data for non-Newtonian Run I	69
XV. Reduced Data for non-Newtonian Run J	70
XVI. Reduced Data for non-Newtonian Run K	71
XVII. Reduced Data for non-Newtonian Run L	72
XVIII. Reduced Data for non-Newtonian Run M	73
XIX. Reduced Data for non-Newtonian Run N	74
XX. Reduced Data for non-Newtonian Run O	75
XXI. Reduced Data for non-Newtonian Run P	76

LIST OF FIGURES

FIGURE	PAGE
1. Schematic Diagram of Test Section	15
2. Diagram of Flow System	19
3. Evaluation of Entrance Pressure Coefficient from Typical Data	25
4. Comparison of Entrance Pressure Corrections . . .	28
5. Entrance Region Pressure Profile $n=1$	30
6. Entrance Region Pressure Profile $n=.62$	31
7. Entrance Region Pressure Profile $n=.47$	32
8. Entrance Region Pressure Profile $n=.42$	33
9. Expanded Entrance Profile $n=1$	35
10. Expanded Entrance Profile $n=.62$	36
11. Expanded Entrance Profile $n=.47$	37
12. Expanded Entrance Profile $n=.42$	38
13. Determination of Power Law Constants	58

CHAPTER I

INTRODUCTION

As a fluid passes from a large container into a small pipe an excessive pressure drop occurs near the contraction. This entrance pressure effect must be considered in capillary viscometry or in flow through a short pipe where the entrance effect is not negligible. Several mathematical analyses have predicted the amount of this pressure drop for Newtonian fluids (3, 4, 7, 10, 12) and for non-Newtonian fluids (2, 5, 15, 16), but there have been few experimental measurements. The several Newtonian measurements (9, 12, 17) were not in close agreement, whereas the only precise power law non-Newtonian measurements to date (6) were made entirely downstream from the entrance without reference to the pressure above the entrance and, therefore, did not measure the complete entrance effect.

The present investigation was undertaken to obtain much needed experimental data concerning the pressure drop in the entrance region for Newtonian and for several power law non-Newtonian fluids. The specific objectives of the investigation were:

1. To construct a flow system including a test contraction with appropriate pressure measuring apparatus.

2. To measure the downstream entrance region pressure profiles for a Newtonian and several non-Newtonian fluids.

3. To measure the overall entrance pressure drop for these fluids.

CHAPTER II

BACKGROUND AND THEORY

I. DEVELOPED FLOW

The pressure drop accompanying streamline flow through a smooth pipe has been described for Newtonian fluids by the Poiseuille equation (1):

$$\frac{\Delta P}{(\int v^2/2g_c)} = 32 (L/R)/N_{Re} . \quad (1)$$

Although the Newtonian shear stress-shear rate relationship,

$$\tau_{rx} = - \mu \frac{\partial v_x}{\partial r} , \quad (2)$$

was inherent in the derivation of the Poiseuille equation, the Poiseuille equation can be modified to describe pressure effects for many non-Newtonian fluids.

Metzner and Reed (11) presented a method for correlating non-Newtonian pipe flow data. Their development began with a consideration of the perfectly general Mooney-Rabinowitsch equation (18) for the rate of shear of a fluid at a wall,

$$- \left(\frac{dv_x}{dr} \right)_w = \frac{3}{4} \left(\frac{8V}{D} \right) + \frac{1}{4} \left(\frac{8V}{D} \right) \frac{d \ln \frac{8V}{D}}{d \ln \frac{D \Delta P}{4L}} , \quad (3)$$

which is entirely independent of fluid properties (excluding time dependent fluids). The term $d \ln (8V/D) / d \ln (D\Delta P/4L)$ was designated as $1/n'$ and was found to be constant over a large range of shear stresses for many fluids for which data could be found. For fluids with constant n' , Equation 3 could be arranged to

$$-\left(\frac{d v_x}{dr}\right)_w = \frac{3n'+1}{4n'} \frac{8V}{D}, \quad (4)$$

and since n' is constant,

$$\frac{D \Delta P}{4L} = K' \left(\frac{8V}{D}\right)^{n'} \quad (5)$$

where K' is a constant. Since $D \Delta P/4L$ is the definition of the wall shear stress (1), τ_w , substituting Equation 4 into 5 yields

$$\tau_w = K' \left(\frac{4n'}{3n'+1}\right)^{n'} \left(-\frac{d v_x}{dr}\right)_w^{n'}, \quad (6)$$

which is simply a power law relation at the wall.

The usual definition of the Fanning friction factor is
(1)

$$f = \frac{(D \Delta P/4L)}{(\rho V^2/2g_c)} \quad (7)$$

Substitution of $D \Delta P/4L$ from Equation 5 into Equation 7 gives

$$f = \frac{16}{\frac{D^{n'} v^{2-n'} \rho}{g_c K' 8^{n'-1}}} \quad (8)$$

By letting $f=16/N_{Re}^n$ as for Newtonian fluids in laminar flow, one defines a generalized Reynolds number:

$$N_{Re}^n = \frac{D^{n'} v^{2-n'} \rho}{g_c K' 8^{n'-1}} \quad (9)$$

A general form of the Poiseuille equation in terms of the general Reynolds number, N_{Re}^n , can be obtained by equating Equations 7 and 8 and rearranging.

Wilkinson (18) showed that the power law fluid model,

$$\tau_{rx} = K \left(-\frac{\partial v_x}{\partial r} \right)^n, \quad (10)$$

was described by Metzner and Reed's (11) general equations. By comparing Equations 6 and 10 the following equalities of constants were observed:

$$K = K' \left(\frac{4n'}{3n'+1} \right)^{n'} \quad (11)$$

and

$$n = n' \quad (12)$$

A power law Reynolds number, N_{Re}^n , was obtained by

substituting the power law constants n and K into the equation for the general Reynolds number, N_{Re}'' (Equation 9), as follows:

$$N_{Re}' = \frac{D^n v^{2-n} f}{g_c K \left(\frac{3n+1}{4n}\right)^n 8^{n-1}} \quad (13)$$

The power law Poiseuille equation was then found by setting $f=16/N_{Re}'$ and rearranging Equation 7:

$$\frac{\Delta P}{f v^2 / 2g_c} = \frac{L/R}{N_{Re}'} \quad (14)$$

II. FLOW IN THE ENTRANCE REGION

As a fluid enters a pipe from a large container an excessive entrance pressure drop occurs. This excessive entrance pressure drop has been measured (12, 17) and reported as a pressure correction, C_{tot} in units of dimensionless velocity heads, to the Poiseuille equation (Equation 1),

$$\frac{P_0 - P}{f v^2 / 2g_c} = 32 \frac{L/R}{N_{Re}'} + C_{tot} \quad (15)$$

where $(P_0 - P)$ is the pressure drop from above the entrance of the pipe to a point within the developed flow region (where the effects of the entrance are no longer felt). In the case of power law fluids the pressure loss can be

expressed as a correction to the power law Poiseuille equation (Equation 14) as follows:

$$\frac{P_o - P}{f V^2 / 2g_c} = 32 \frac{X/R}{N_{Re}} + C_{tot} . \quad (16)$$

The excess pressure drop in the entrance region is caused by several effects. The most obvious effect can be derived from the following mechanical energy balance for an incompressible fluid traversing a change in flow area (1):

$$\frac{P_1 - P_2}{f} = \frac{V_2^2 - V_1^2}{2g_c} . \quad (17)$$

The subscripts 1 and 2 indicate positions upstream and downstream respectively from the change in area. In Equation 17 the positions denoted by 1 and 2 are assumed close enough together to have negligible friction loss between them. From the continuity relationship if the upstream flow area is much larger than the downstream area, the upstream velocity, V_1 , will be negligible with respect to the downstream velocity, V_2 . Equation 17 can then be rearranged to give

$$\frac{P_1 - P_2}{f V_2^2 / 2g_c} = 1 . \quad (18)$$

From Equation 18 it can be seen that the change in the flow area at the entrance for a large contraction ratio contributes a pressure correction of 1 velocity head to C_{tot} .

In theoretical analyses of the entrance region (2, 3, 5, 7, 10, 12, 15, 16) the fluid entering the tube has been viewed as a plug retarded by the walls to produce the developed velocity profile downstream. The excess pressure loss in the entrance region has been attributed to the increase of kinetic energy of the fluid as it changes its velocity profile and to an increased viscous friction. This portion of the entrance pressure correction caused by the development of the velocity profile will be called C_{down} .

Ignored in past theoretical analyses of the entrance region is the possible pressure loss upstream from and immediately at the entrance. Such a pressure loss could be caused by deviation of the velocity profile at the entrance from the assumed plug shape. This additional upstream pressure drop will be denoted by C_{up} .

Finally then, C_{tot} represents the following combination of entrance pressure corrections for an infinitely large upstream vessel:

$$C_{\text{tot}} = C_{\text{down}} + 1 + C_{\text{up}} \quad . \quad (19)$$

Several mathematical analyses and experimental investigations have predicted the amount of excess pressure loss in the entrance region. The most important of these will be briefly discussed, and their results will be presented in the form of the predicted pressure corrections. Because of the

assumption of plug flow at the entrance the mathematical analyses will give values only for C_{down} or $C_{\text{down}+1}$, whereas the experimental investigations yield values for C_{tot} .

In 1891 Boussinesq (3) presented a mathematical analysis for a Newtonian fluid predicting the downstream entrance pressure effect. He linearized the governing differential equation for the entrance flow by neglecting several small terms. Then assuming the velocity profile to be describable as a series function of radius and distance from the entrance, he was able to solve the equations for the velocity function. The pressure loss in the entrance was calculated directly from the velocity profile, resulting in the prediction of a value of downstream correction, $C_{\text{down}+1}$, of 2.24.

Later Schiller (12) utilized the Prandtl boundary layer concept (13) to simplify the Navier-Stokes equations governing entrance flow. An integral method (13) was then used to solve the differential equations assuming a parabolic equation for the boundary layer velocity profile. This development yielded a pressure correction, $C_{\text{down}+1}$, of 2.16. Experimentally, measuring from above the contraction (in the large vessel) to beyond the entrance length Schiller obtained values of the overall correction, C_{tot} , of 2.35, 2.36, 2.115, and 2.45.

Langhaar (10) by means of a linearizing approximation solved the governing equations obtaining a value of 2.28 for the correction, $C_{\text{down}+1}$.

Atkinson and Goldstein (7) extended the range of Schiller's mathematical work to larger values of dimensionless length from the entrance and arrived at a value of 2.41 for the pressure correction, $C_{\text{down}+1}$.

More recently Bogue (2) applied Schiller's method to a non-Newtonian fluid describable by the power law model. The method was modified by the assumption of a cubic equation for the velocity profile in the boundary layer. Bogue's results were identical to Schiller's for the Newtonian case and predicted that in the power law case the pressure correction would be a function of the power law exponent.

Tomita (16) obtained results similar to those of Bogue using almost the same method except for the assumption of a different velocity function. He later used a variational method to arrive at very similar results (15).

Dodge (6) obtained experimental data for Carbopol-water gels of several concentrations which agreed very well with Bogue's prediction for the pressure profile below the entrance, although Dodge's data give no information about the overall entrance loss since he made no pressure measurements above or at the entrance.

Weltmann and Keller (17) reported pressure drop

measurements for non-Newtonian fluids in the entrance region which do not differ appreciably from those for Newtonian fluids. They reported a value for C_{tot} of 2.2 for both cases.

In the most recent theoretical treatment of a power law fluid in the entrance region, Collins (5) presented a digital computer solution of the boundary layer equations of motion predicting a Newtonian correction, C_{down}^{+1} , of 2.34 and non-Newtonian corrections correspondingly greater than those of Bogue. This work represents the most exact solution of the differential equations to date. Collins' method will be described more fully in the following section.

III. THE BOUNDARY LAYER APPROACH

The generally accepted picture of flow through a contraction lends itself to the methods of boundary layer theory (13) where the flow is thought of in two parts:

1. A laminar boundary layer building up from zero thickness at the entrance until the entire flow is represented by the laminar velocity profile at the end of the entrance region.

2. The core of the fluid which begins at the entrance as a flat profile and remains flat until consumed by the boundary layer.

Actually a rounding off of the core profile might be

expected near the end of the entrance region, but in that region the flow approaches laminar flow, and little error should be introduced.

Assuming the boundary layer flow model, the equation of motion describing flow in the entrance region of a tube may be simplified using the well known boundary layer order of magnitude analysis (13) to

$$v_r \frac{\partial v_x}{\partial r} + v_x \frac{\partial v_x}{\partial x} = - \frac{g_c}{\gamma} \frac{\partial p}{\partial x} - \frac{g_c}{\gamma} \frac{\partial (r \tau_{rx})}{\partial r} \quad (20)$$

and the continuity equation to

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_x}{\partial x} = 0 \quad . \quad (21)$$

The entrance region boundary layer equations have been solved for the Newtonian case (12) and more recently for the general power law case (2, 5, 15, 16) where τ_{rx} was defined by Equation 10.

Collins (5) used finite difference methods to solve Equations 20 and 21. Near the entrance he assumed the velocity in the boundary layer, $v_x(r,x)$, to be a series function of r and x . The boundary conditions for this portion of the problem were plug flow at the entrance, zero velocity at the wall, and core flow outside the boundary layer. To account for a slight rounding of the core near the end of the entrance region the velocity profile in this region was

approximated by describing it in terms of a small perturbation to the fully developed profile. The two solutions were then joined together in the section where both were applicable. The pressure profile was calculated from the calculated velocity profile, $v_x(r,x)$. Collins' results will be presented for comparison with the present experimental results in Chapter IV.

CHAPTER III

DESCRIPTION OF THE INVESTIGATION

I. EXPERIMENTAL EQUIPMENT

The Entrance Region

The test section consisted of a sharp-edged (zero length) contraction from a 6-in. inside diameter tube to a tube of 0.547-in. inside diameter. The large contraction ratio, 12:1, was chosen so that the upstream velocity would be negligible with respect to the downstream velocity and that the results of the investigation could be compared with those for $\infty:1$ contractions.

Figure 1 shows a detailed cross section of the test section. Upstream was a 6-ft length of straight 6-in. diameter thin walled brass tubing, the first 1 foot of which contained 3/4-in. diameter thin walled brass tubes as straightening vanes. The downstream section consisted of a 1/2-in. inside diameter (.547 in. finished) brass tube 5 ft in length. The contraction was formed by silversoldering the 1/2-in. tube into a hole drilled through the center of a flat brass plate perpendicular to the surface. The entrance was polished smooth so that a constant diameter of .547 in. was maintained inside the tube and so that the entrance was sharp-edged. The brass plate was then connected by flange with

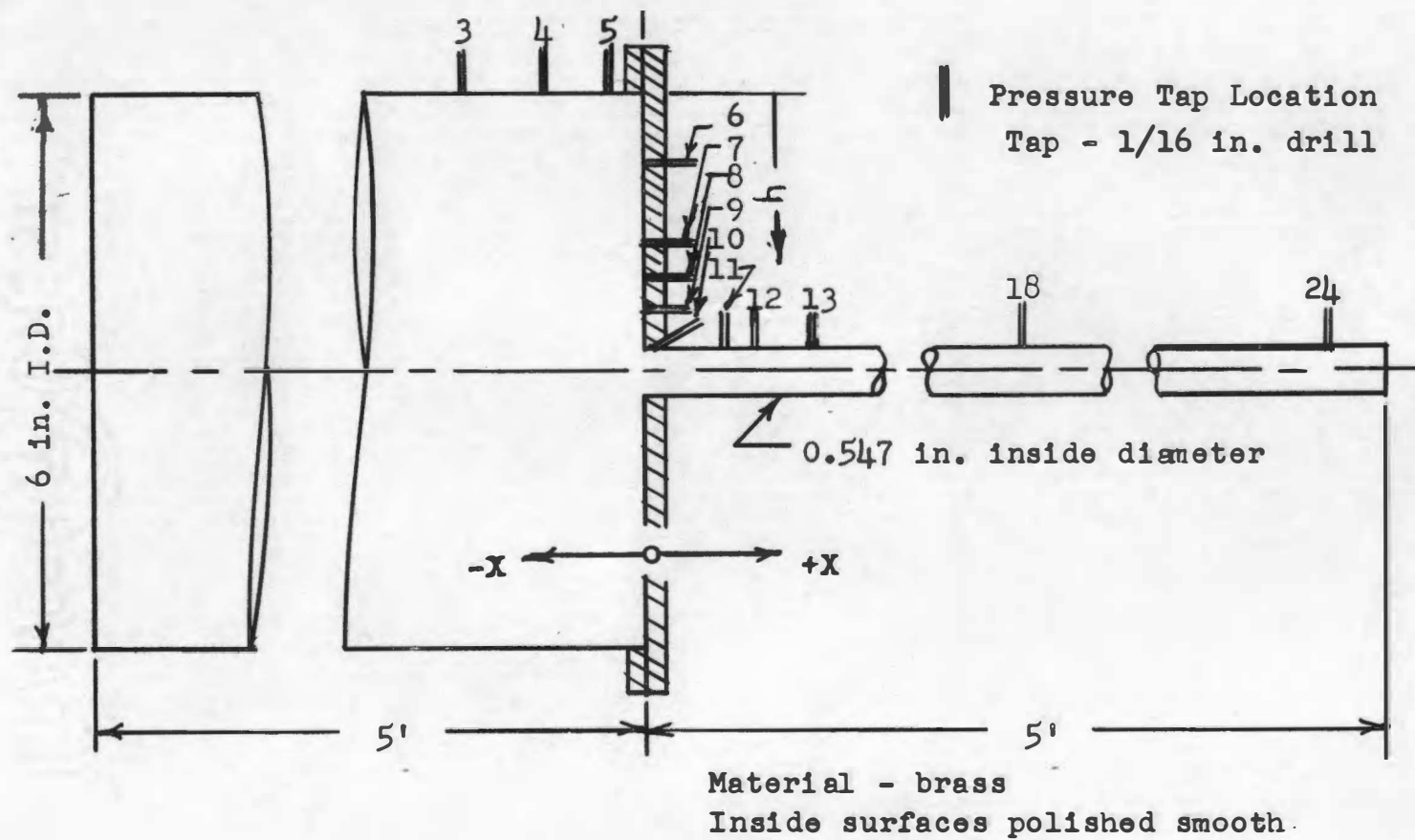


Figure 1. Schematic Diagram of Test Section.

O-ring seal to the 6-in. diameter tube.

Wall pressure tap holes of 1/16-in. diameter were drilled on the upper side of the test section, spaced as indicated in Table I. The horizontal position (X , in in.) was measured from the inside surface of the plate forming the contraction, with the positive direction downstream from the contraction. Taps in the 1/2-in. tube were more closely spaced at the upstream end in order that the pressure effect during the development of the velocity profile might be measured. The taps were spaced farther apart downstream as the profile developed. The downstream taps were so located that for every run at least two taps were located in the developed flow region. Taps were provided in the vertical plate (zero distance) to measure any radial effects at the entrance. The positions of these taps were measured vertically from the centerline of the tube, but were reported in Table I as vertical distance from Tap 1 in the 6-in. tube (h , in in.) since calculations were based on this distance. Several taps were also provided in the 6-in. diameter tube as a reference for the downstream measurements and for measuring any upstream pressure effect.

The Pressure Measuring System

The pressure drop was measured with a 120-cm U-tube manometer using fluids of 1.75 and 1.2 specific gravity

TABLE I

LOCATION OF PRESSURE TAPS

Tap	X (in.)	h (in.)
1	-24.057	0
2	- 5.057	0
3	- 2.057	0
4	- 1.057	0
5	- 0.611	0
6	0	1.21
7	0	1.92
8	0	2.27
9	0	2.63
10	0.2	2.714
11	0.5	2.714
12	1.0	2.714
13	1.5	2.714
14	2.5	2.714
15	4.5	2.714
16	6.5	2.714
17	8.5	2.714
18	10.5	2.714
19	14.5	2.714
20	18.5	2.714
21	26.5	2.714
22	36.5	2.714
23	46.5	2.714
24	56.5	2.714

depending on the magnitude of the pressure drop. This manometer in most cases could be read to the nearest millimeter. Since the fluids investigated were very viscous, they were unsuitable for use in the manometer leads; therefore it was necessary to purge the leads with water.

Brass 1/8-in. fittings were soldered above the tap holes so that a valve could be placed above each tap. Each valve was fitted with a brass nozzle to which the Tygon tubing manometer leads could be attached.

The Flow System

The entire system was constructed of brass and stainless steel since these metals were fairly unreactive with the test fluids. The flow was regulated by a 1-1/2-in. diaphragm valve and measured with a 10 gal/min-capacity rotameter. To avoid parallax in reading the rotameter, a sight wire was installed for each rotameter setting used. A Yeomans Brothers centrifugal pump rated at 15 gal/min at 30-ft dynamic head was used in the system. A schematic diagram of the flow system is shown in Figure 2.

A bypass was provided for rotameter calibration leading to a 55-gal stainless steel drum situated on a balance. The flow during calibration was controlled by a 1-1/2-in. globe valve.

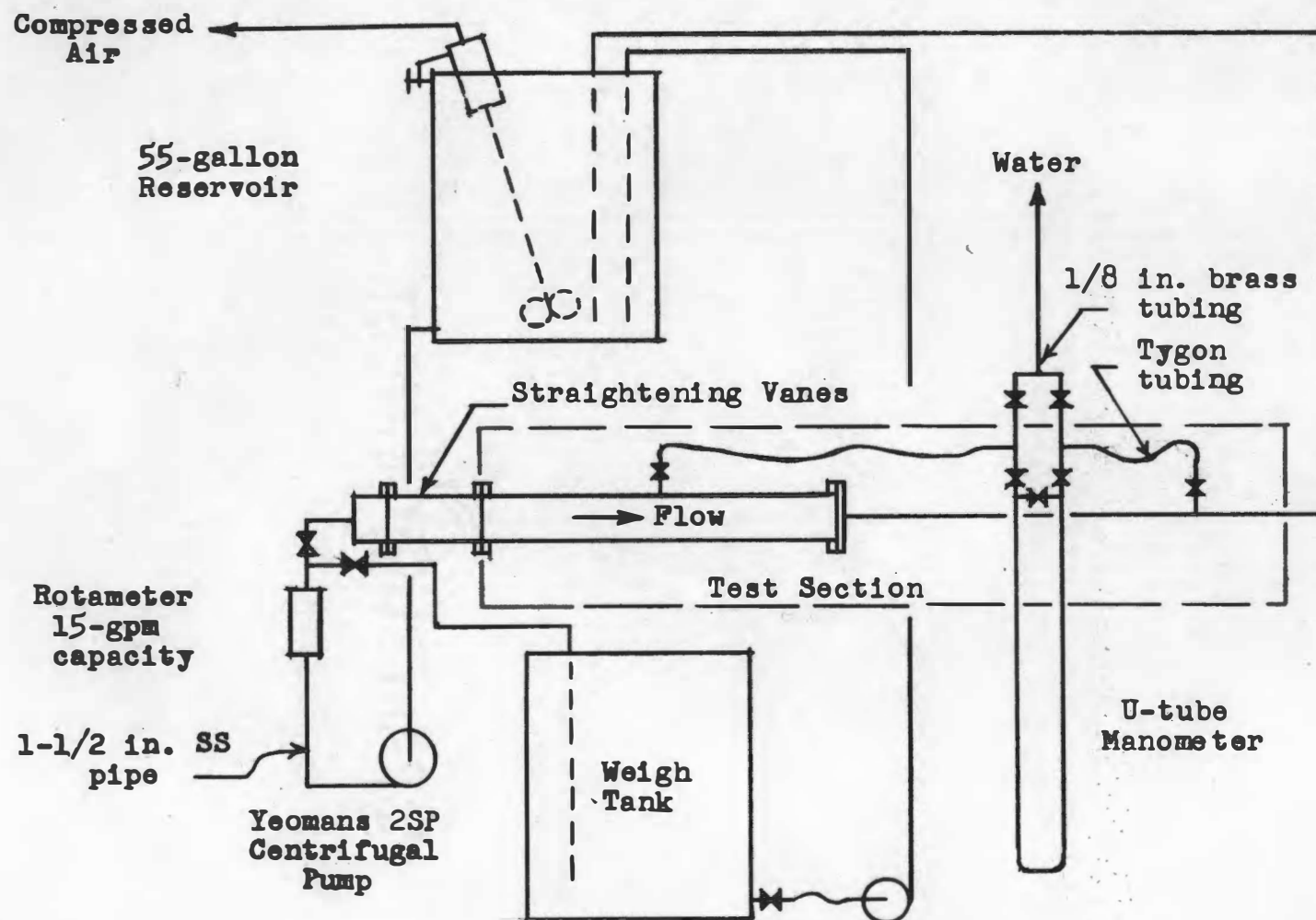


Figure 2. Diagram of Flow System.

II. TEST MATERIALS

International Harvester antifreeze which is 94.9 per cent ethylene glycol was used for the Newtonian fluid. It was chosen because its viscosity was high enough for use in the flow system and because it was relatively inexpensive. Its Newtonian behavior was verified with a rotational viscometer.

Through purging the manometer leads with water during the initial runs the ethylene glycol concentration was decreased to about 85 per cent when the actual test runs were made, reducing the viscosity at room temperature to about 8 centipoise.

Carbopol 934, a product of the B. F. Goodrich Chemical Company, dissolved in water and neutralized with sodium hydroxide, was used as the non-Newtonian test fluid. This solution was used because it has been found to follow the power law relation over a large range of shear stress (6). Since the power law constants vary with Carbopol concentration, solutions of several concentrations were used.

III. EXPERIMENTAL PROCEDURE

Pressure Measurement

In beginning a test run the fluid was circulated through the system and the flow rate adjusted to a previously

chosen rotameter reading. Periodic adjustment of the flow control valve was necessary to maintain the desired rotameter reading. The temperature of the test fluid was held constant near room temperature by flowing cold water over the pump housing. The manometer legs were connected by Tygon tubing to the taps between which the pressure drop was to be measured. Where possible the pressure drop was measured with respect to Tap 1 located 24 in. upstream from the contraction. During a run the pressure at every tap was measured and at most taps was remeasured. If the pressure reading for a tap could not be reproduced during the run or if the temperature varied more than $1/2^{\circ}\text{C}$, the data were discarded, and a new run was begun.

The manometer leads were purged with water before each pressure reading. Purging was accomplished by allowing the manometer fluid to reach its approximate level with some test fluid in the leads, closing the valves between the manometer and the water inlet, opening the water valve purging the test fluid back into the test section, closing the water valve and reopening the manometer to the flow system. This procedure was repeated for each tap until no change in manometer reading was noticed upon purging.

Flow Rate Determination

Since the pressure drop was so sensitive to fluid

velocity and since the rotameter calibration was found to vary with composition of Carbopol, it was necessary to calibrate the rotameter after each run. The rotameter was calibrated by flowing the test fluid through the test section bypass into a tared barrel for a measured length of time. To increase the accuracy of calibration it was necessary to pour the fluid into a second barrel while the control valve was adjusted to maintain the desired rotameter reading and to begin timing when the flow was diverted into the tared barrel. After the measured time flow was returned to the second barrel until the control valve could be closed, the tared barrel was weighed and the fluid was returned to the flow system reservoir by another pump. Several calibration runs were made, and the best ones were averaged. The error in calibration was estimated to be less than 1 per cent.

Sample Analysis

Samples of the test fluid were removed from the system at a downstream tap immediately before and after a run. Since the non-Newtonian fluid tended to deteriorate with time, the samples were examined soon after they were taken. Sample examination consisted of measuring the density with a Westphal balance and measuring the rheological properties with a rotational viscometer. If the properties of the samples taken before and after the run varied, the run was discarded.

Viscometry

All viscometric measurements were made with a Haake Rotovisco manufactured by Gebruder Haake K. G. Berlin-Steglitz, Germany, using concentric cylinder attachments SVI and NV. Operating details are in Appendix C.

Preparation of the Carbopol Gel

Carbopol is delivered as an acid powder and must be dissolved in water and neutralized to form the characteristic gel. Solutions of less than 1 per cent by weight Carbopol were investigated. In order to introduce the Carbopol into the system as a homogeneous bubble-free gel it was necessary to use a step-wise procedure as follows:

1. The flow system was filled with water and all air was expelled.
2. Carbopol powder was poured onto the surface of the water in the reservoir.
3. The solution was agitated in the reservoir by an air powered stirrer and was also pumped through the system until the Carbopol had completely dissolved.
4. The solution was neutralized with 10 per cent by weight sodium hydroxide to a pH of about 7 as measured by pHDrion paper.

When the solution concentration was to be changed water was added, and the solution was stirred overnight or until homogeneous.

CHAPTER IV

PRESENTATION AND DISCUSSION OF RESULTS

Equations 15 and 16 show that pressure drop data for the entrance region when plotted in terms of pressure difference in velocity heads, $(P_0 - P)/(\rho V^2/2g_c)$, versus reduced distance from the entrance, $(X/R)/N_{Re}^1$, are independent of the equipment size (so long as the contraction ratio is the same) and fluid velocity.

Figure 3 presents typical pressure drop data and demonstrates the method used to find the entrance pressure correction, $C_{down}+1$. The reference pressure tap for this plot was located on the vertical plate forming the contraction (zero distance from the entrance) at a radial distance of 1/2 in. from the centerline. Since this pressure tap was located on the upstream side of the entrance, the downstream pressure readings contained an additional 1 velocity head caused by the change in cross section. The equipment was designed so that a tap located 0.2 in. downstream from the entrance might be used for the reference tap to separate the downstream pressure drop from the upstream drop, but for most runs a vena contracta effect rendered the use of this tap not meaningful.

Equations 15 and 16 predict that the data plotted in terms of the co-ordinates of Figure 3 will approach a constant

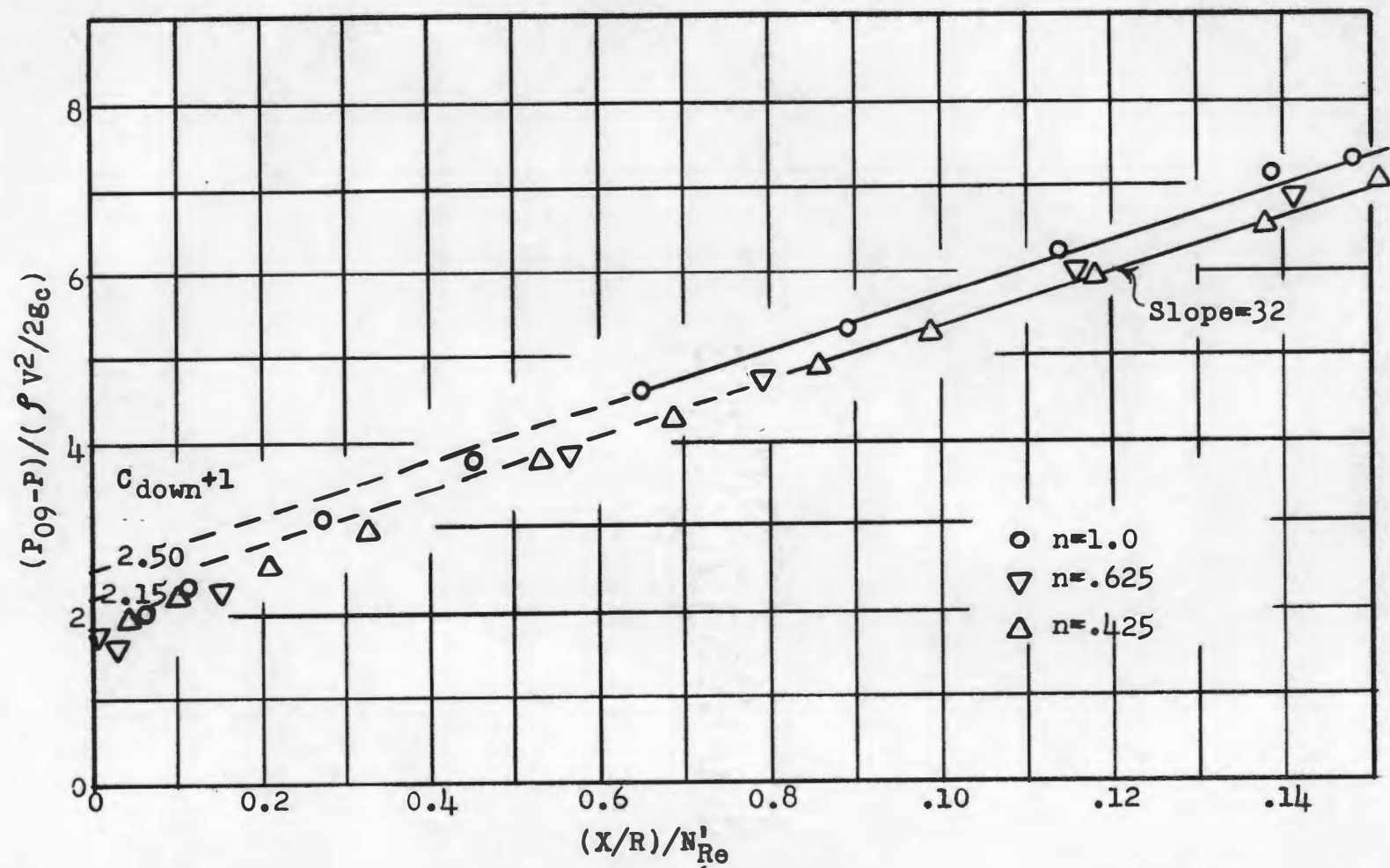


Figure 3. Evaluation of Entrance Coefficient from Typical Data.

slope of 32 as the fluid obtains its laminar profile, therefore the downstream entrance correction was found by extrapolating to zero distance a line of slope 32 through the far downstream data points. Much of the experimental error came from the difficulty of locating this line since any slight error in slope of the data would have a large lever arm effect at the far downstream points; therefore, any run for which the slope of the data far downstream differed much from 32 had to be discarded.

The remaining upstream pressure drop (above the previously mentioned reference tap) was measured with respect to a tap located in the 6-in. diameter pipe 24 in. upstream from the entrance.

The downstream including the entrance and the total pressure drop corrections, $C_{\text{down}+1}$ and C_{tot} , respectively, for each run, as calculated from the experimental pressure profiles, are presented in Table II along with the rheological data for each run and the literature values for the corrections. The experimental values of $C_{\text{down}+1}$ are higher by about 15 per cent than those predicted by Collins' downstream development. The comparison of the experimental values of $C_{\text{down}+1}$ with Collins' and Bogue's values are shown more clearly in Figure 4.

For Newtonian fluids ($n=1$) the experimental value of C_{tot} of 2.53 agrees within experimental accuracy with the

TABLE II
THE ENTRANCE PRESSURE CORRECTION

Run	N'_{Re}	n	K $(\frac{lb_f sec^n}{ft^2})$	Exp. $C_{down}+1$	Exp. C_{tot}	Bogue(2) $C_{down}+1$	Collins(5) $C_{down}+1$	Schiller(12) C_{tot} Exp.	Weltmann(17) C_{tot} Exp.
A	580	1.0	8.3 cp	2.55		2.16	2.34	{ 2.35 2.36 2.45 2.115	2.2
B	580	1.0	8.3 cp	2.55	2.525				
C	1395	1.0	8.48 cp	2.50	2.53				
D	1492	1.0	7.96 cp	2.50	2.56				
E	1495	1.0	8.46 cp	2.50	2.53				
F	1469	.62	.00334	2.26	2.302	1.86	2.0		2.2
G	1220	.62	.00334	2.26	2.306				
H	4710	.60	.00366						
I	2616	.60	.00366						
J	1647	.475	.0259	2.23	2.274	1.70	1.81		2.2
K	1056	.475	.0259	2.23	2.274				
L	930	.470	.0284	2.23	2.28				
M	343	.470	.0284	2.23	2.26				
N	173	.425	.0715	2.15	2.24	1.60	1.74		2.2
O	162	.425	.0715	2.15	2.37				
P	449	.41	.0595	2.15	2.19				

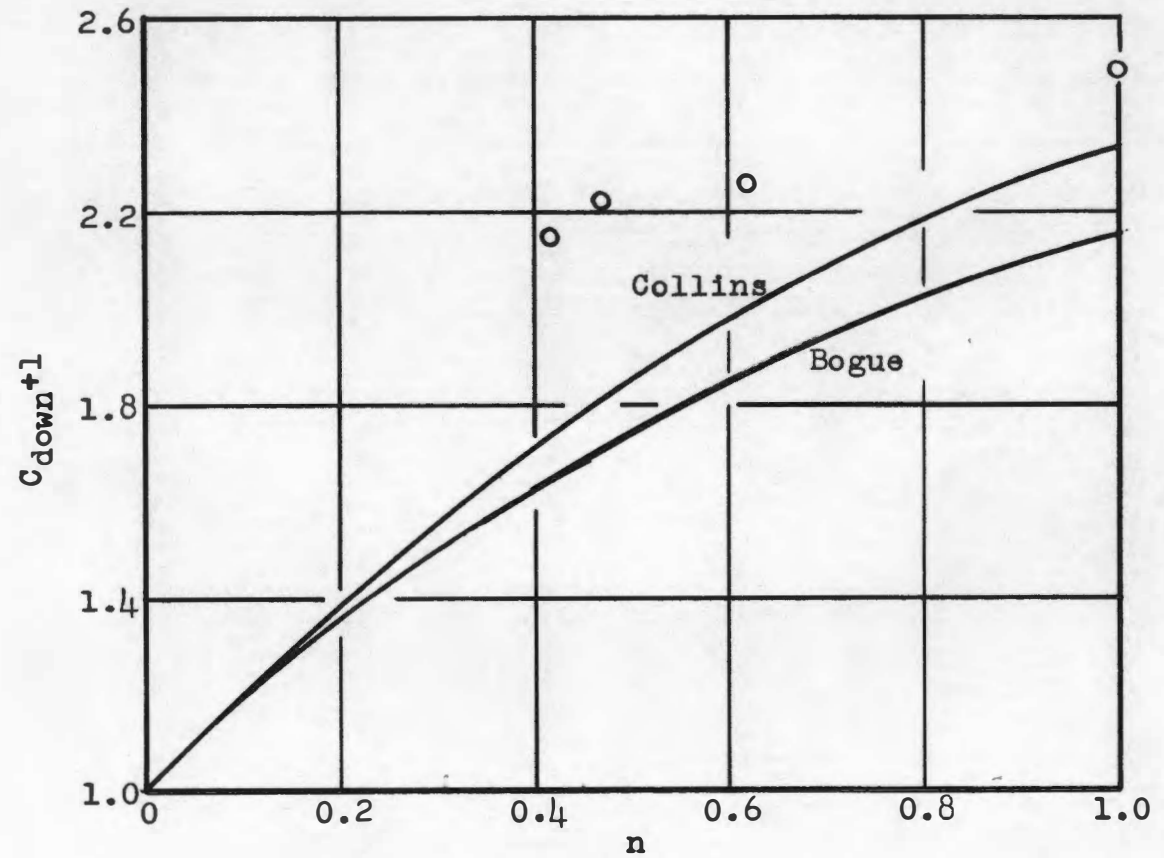


Figure 4. Comparison of Entrance Pressure Correction.

highest of Schiller's four experimental values of 2.45, 2.36, 2.35, and 2.115 (12).

The experimental data for each fluid were compared with Collins' (5) theoretical downstream results in Figures 5, 6, 7, and 8. The experimental pressure drop data were calculated from the reference tap on the vertical plate forming the contraction and therefore include the 1 velocity head caused by the change in cross section. The scale for the theoretical results (right hand ordinate) was displaced until the theoretical curve best fit the data points in the developed profile region. The amount of displacement indicated the difference between the experimental and theoretical (Collins') values of $C_{down}+1$. From these plots it can be seen that the theoretical curves fit the experimental points fairly well downstream from the entrance.

Because of the insensitivity of pressure measurements no definite values for the distance required for the complete velocity profile development can be obtained from this investigation. Values of entrance length predicted by Bogue (2) and Collins (5) are as follows:

n	$(X_d/R)/(N_{Re}')$ Collins	$(X_d/R)/(N_{Re}')$ Bogue
1	0.1	0.0575
0.6	0.096	0.040
0.47	0.088	0.032
0.41	0.083	0.027

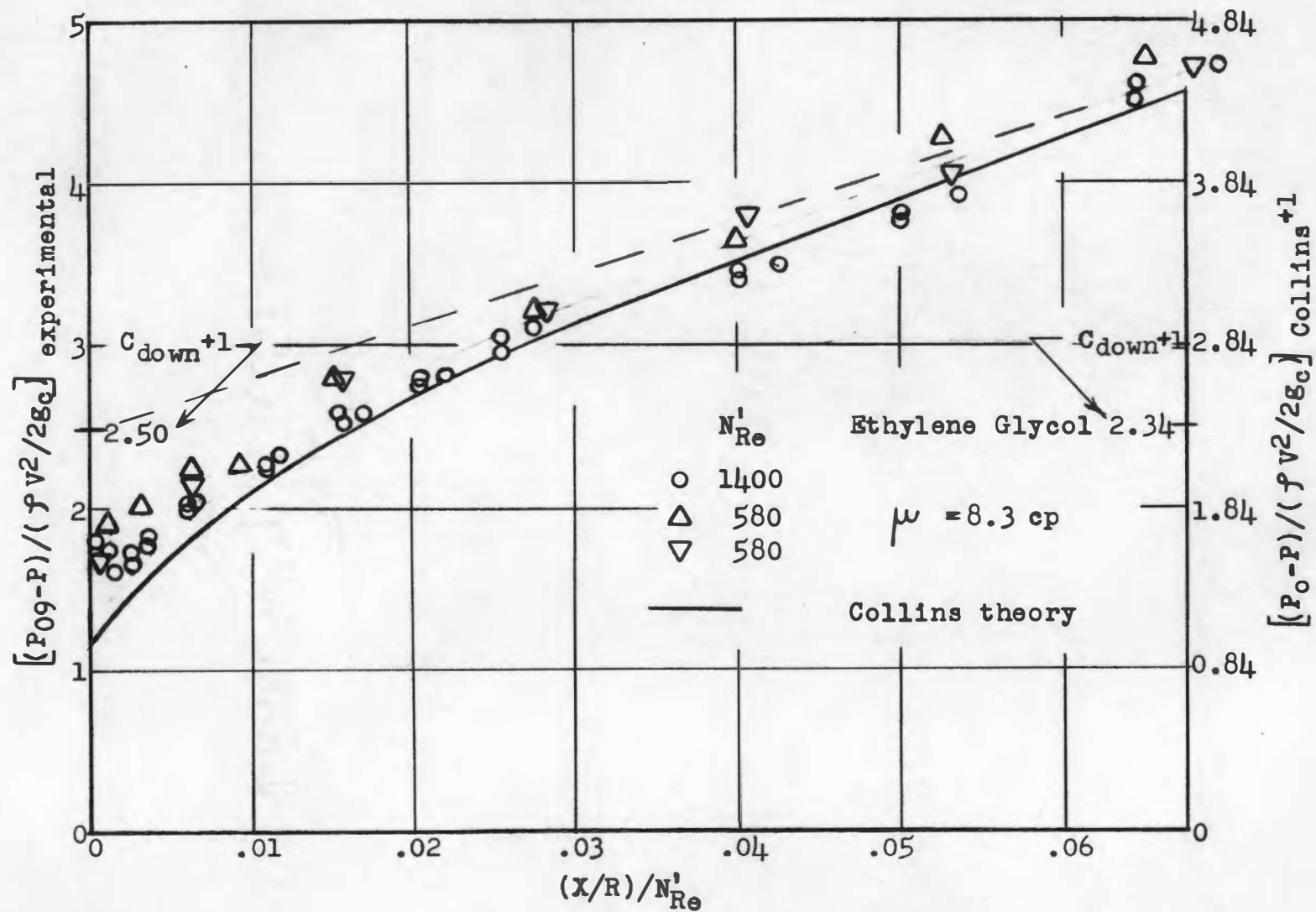


Figure 5. Entrance Region Pressure Profile $n=1$.

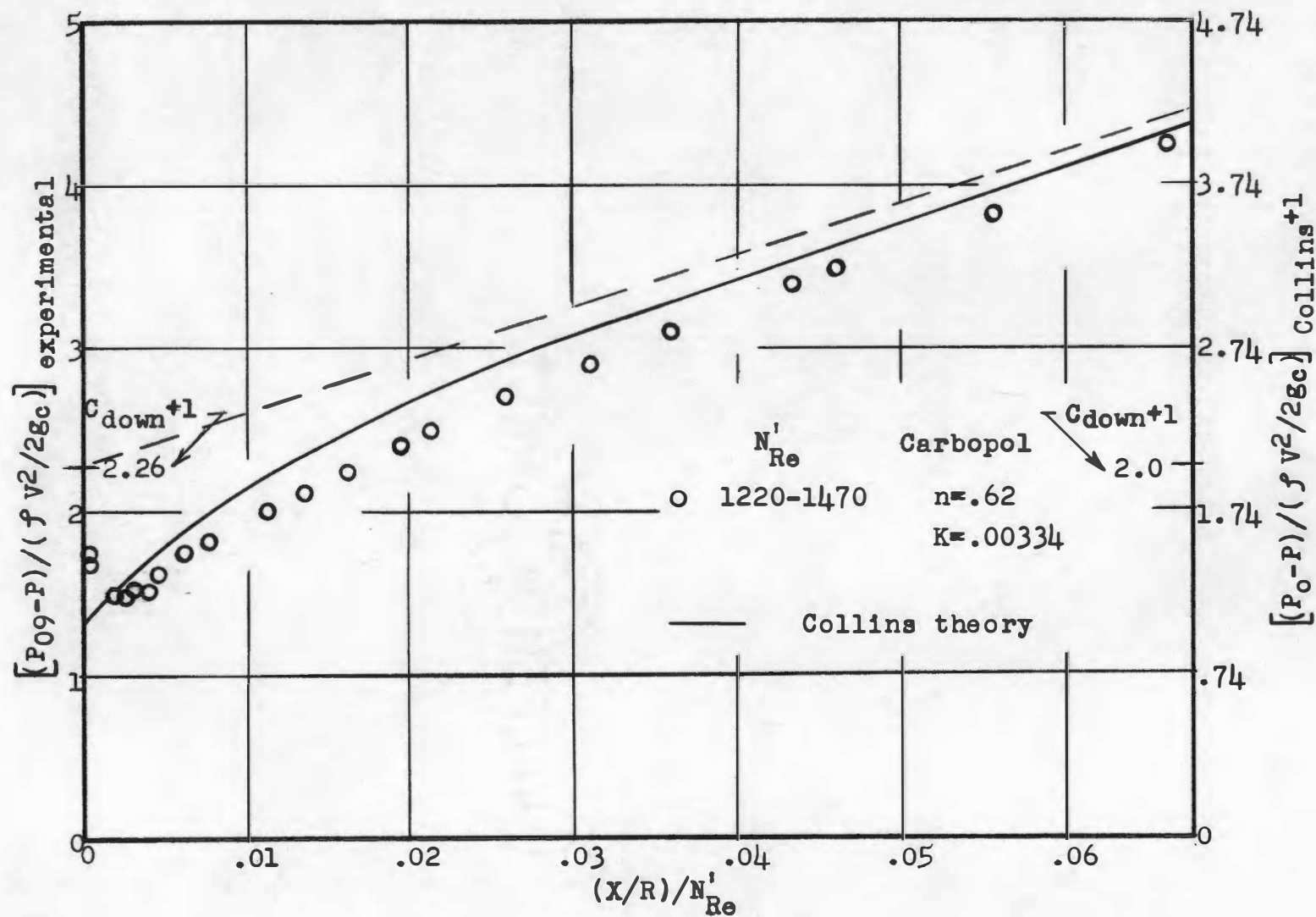


Figure 6. Entrance Region Pressure Profile $n=.62$.

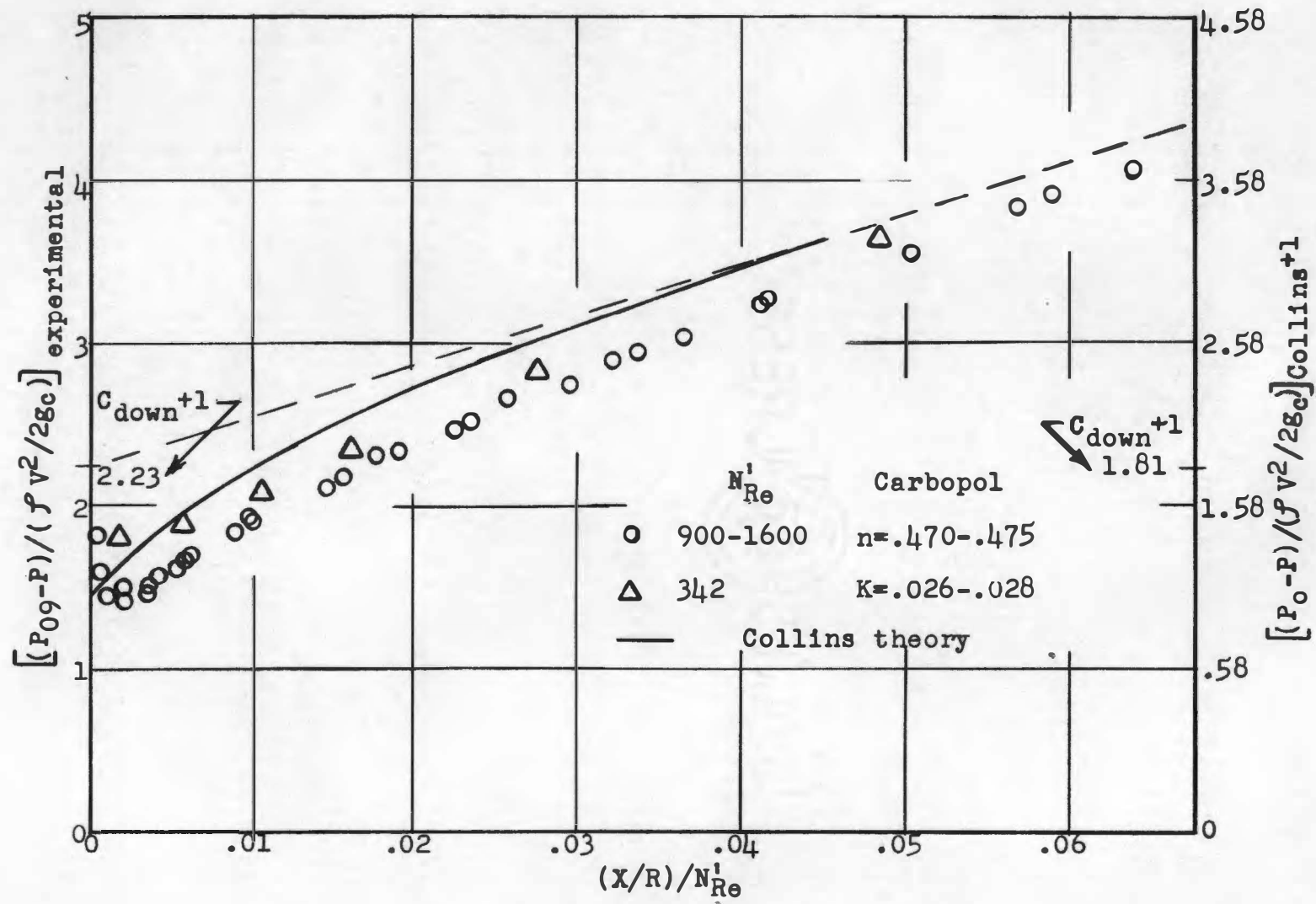


Figure 7. Entrance Region Pressure Profile $n=.47$.

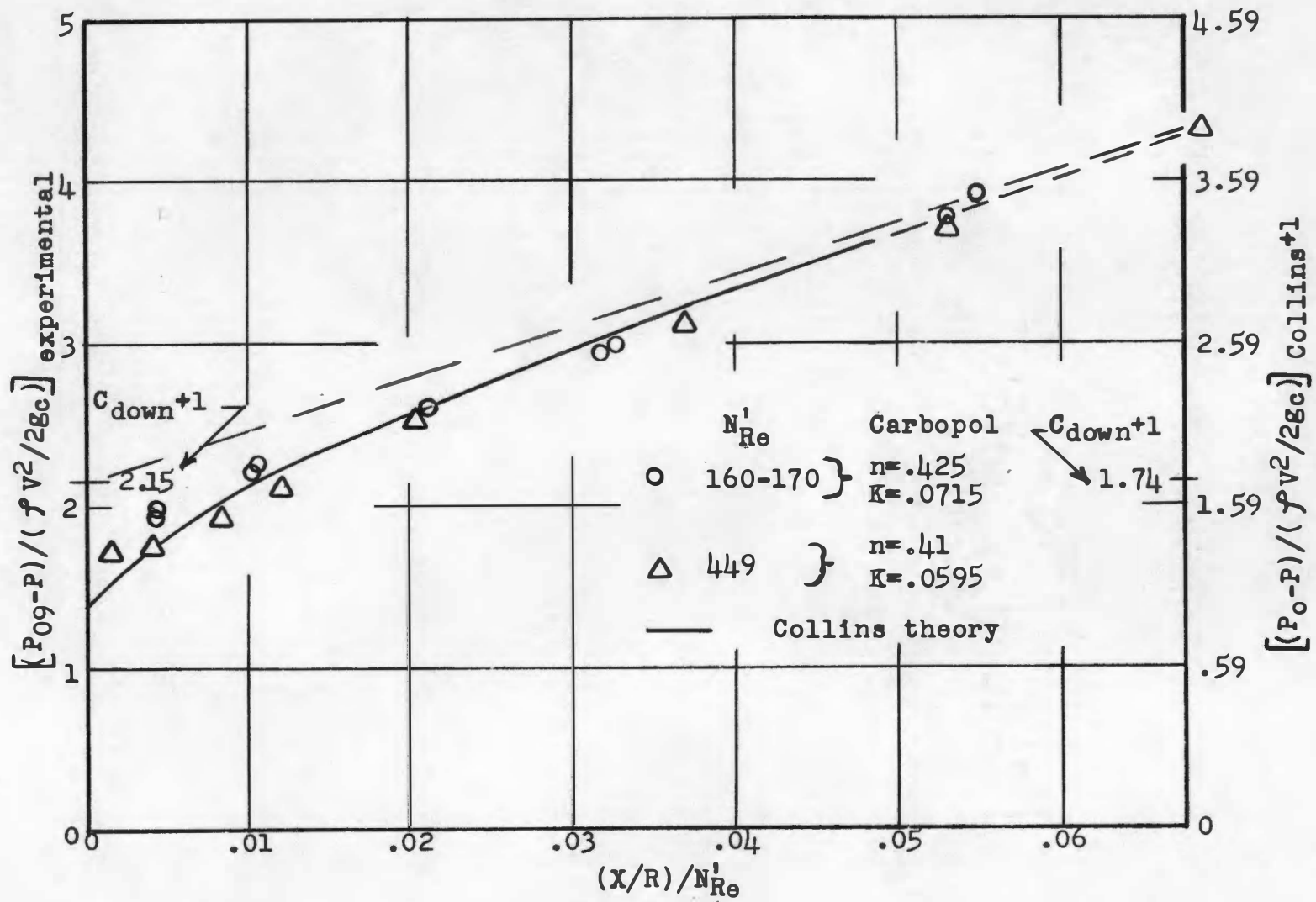


Figure 8. Entrance Region Pressure Profile $n=.42$.

Bogue's values were dictated by the boundary layer analysis while Collins' values were based on 98 per cent development of the center line velocity. As can be seen from Figures 5, 6, 7, and 8 the data approach a constant slope of 32 (the criterion for developed flow) very near the values of entrance length predicted by Bogue.

Vena contracta and other effects immediately at the entrance are shown more clearly in expanded pressure profiles at the entrance (Figures 9, 10, 11, and 12). The reference pressure tap for these figures was located in the large pipe 24 in. upstream from the entrance. Several runs for each fluid were presented in each figure so that the figures might demonstrate the effect of Reynolds number on the entrance pressure drop. For the same fluid at lower Reynolds numbers the net pressure drop at the entrance was greater. Although the vena contracta peaks for the runs at higher Reynolds numbers were great very near the entrance, the fluid regained much of its lost energy resulting in a smaller net loss than was experienced in the lower Reynolds number runs. The vena contracta was not noticed in runs with Reynolds numbers below 400, although at low Reynolds numbers the vena contracta would be expected to move closer to the entrance so that the nearest tap (0.2 in. downstream) might not detect it.

The vena contracta, recognized by the large pressure drop at the entrance, has not been accounted for in the

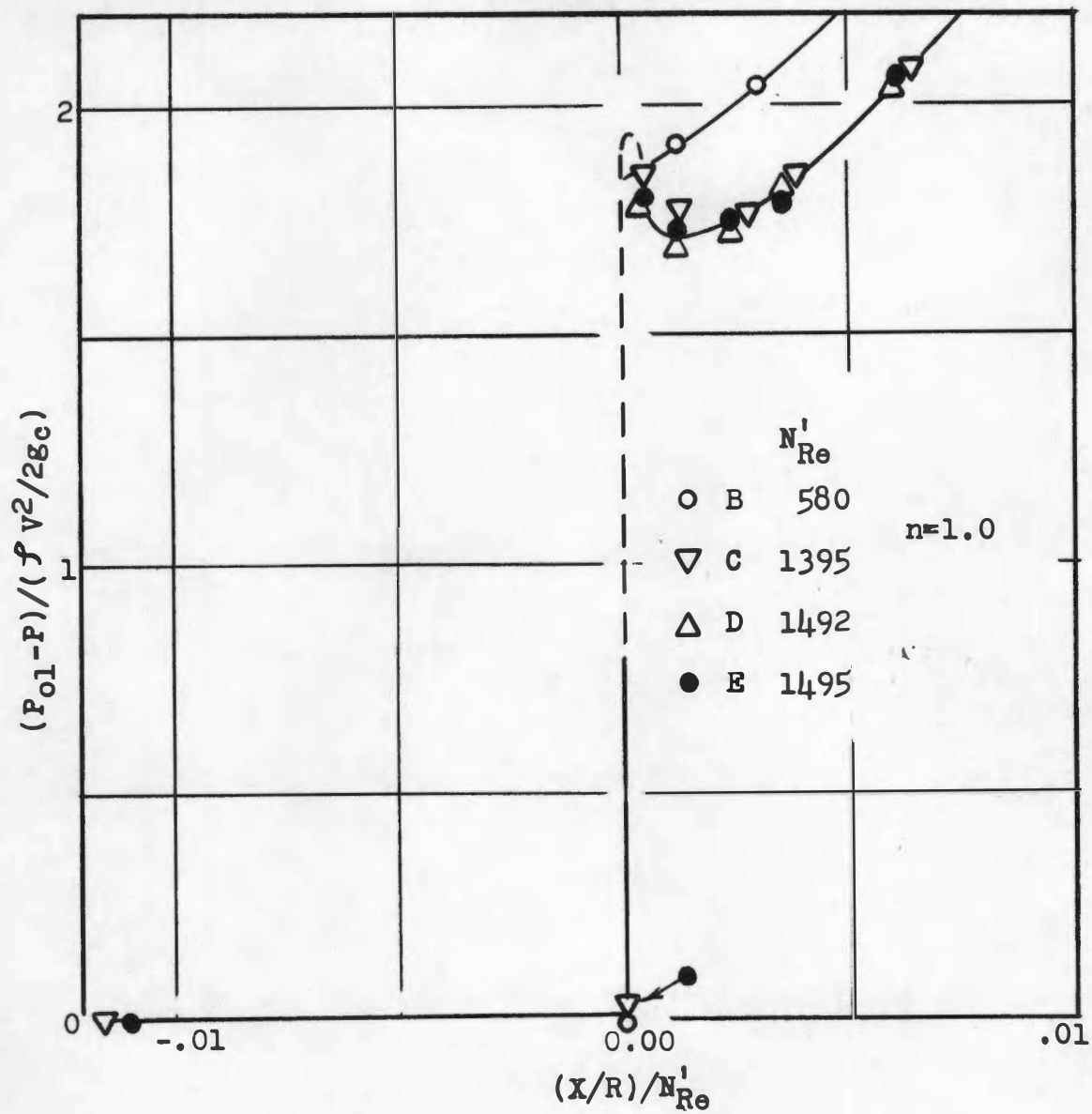


Figure 9. Expanded Entrance Profile $n=1.0$.

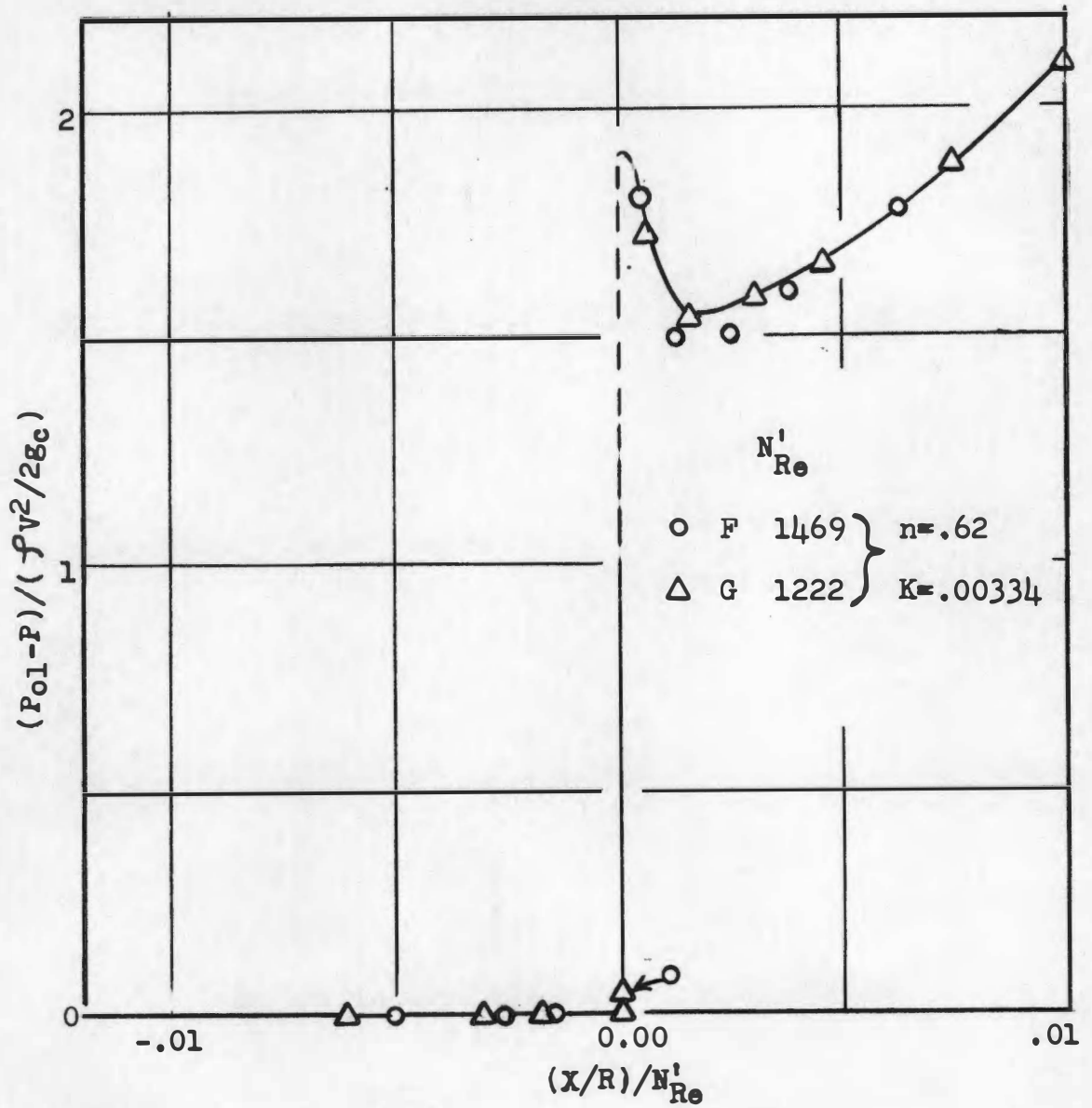


Figure 10. Expanded Entrance Profile $n=.62$.

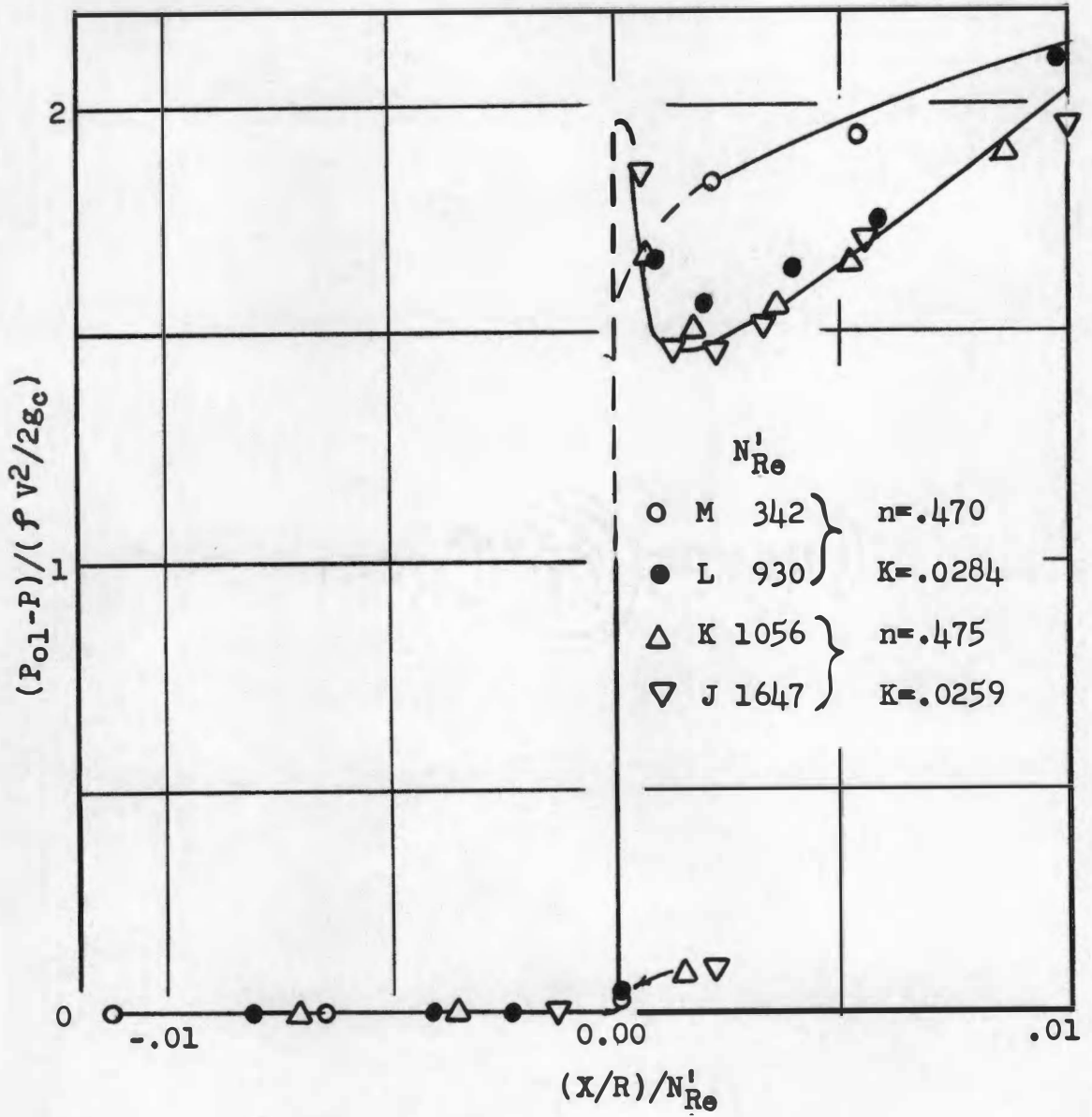


Figure 11. Expanded Entrance Profile $n=.47$.

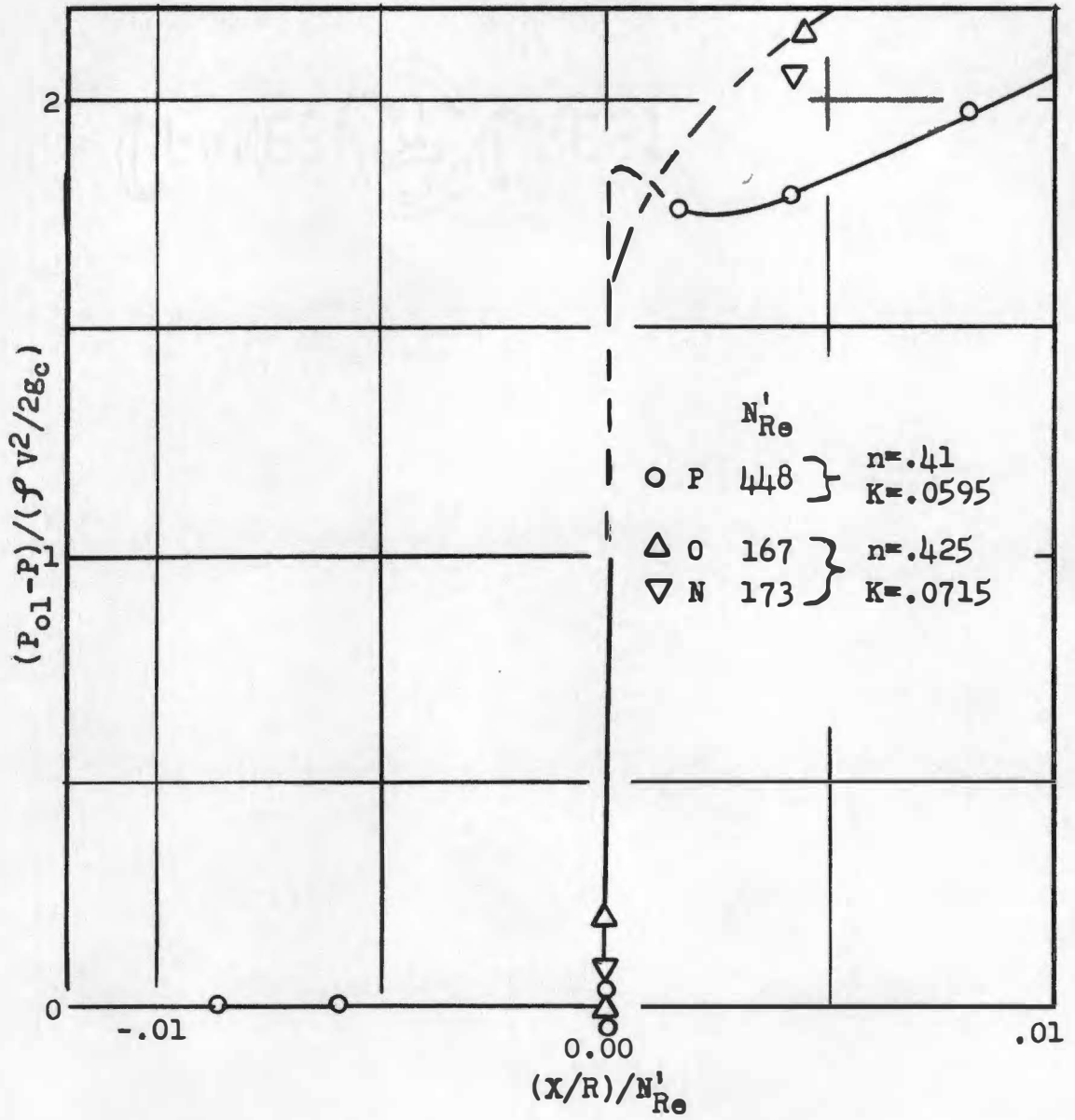


Figure 12. Expanded Entrance Profile $n=.42$.

previous theoretical works. Although much of its resulting pressure loss is recovered, this effect seems worthy of consideration in treating the overall contraction problem in that its formation indicates radial flow slightly above the entrance and shows that the entering fluid is not in plug form.

A small radial pressure effect was noticed along the vertical plate forming the contraction which was greater for the more non-Newtonian fluids and negligible for the Newtonian. This radial effect did not extend beyond 1 in. from the centerline. Since the tap nearest the entrance on the vertical plate was $1/2$ in. from the centerline (approximately $1/4$ in. from the tube opening), some of the pressure drop measured between this and the first downstream tap (0.2 in. from the entrance) may well have been due to upstream radial effects especially in the case of the more non-Newtonian fluids. This upstream effect could be the reason that the more non-Newtonian fluids yielded pressure corrections disproportionately greater than the theoretical downstream corrections.

Although the Reynolds number was noticed to influence the pressure profile at the entrance no significant variation of C_{tot} with Reynolds number was found.

The overall experimental error was estimated to be about 3 per cent resulting in a value of C_{tot} for the

Newtonian fluid of $2.53 \pm .08$. The error analysis is presented in Appendix B.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The following conclusions were drawn from this investigation:

1. For the Newtonian and non-Newtonian fluids investigated, the theoretical analyses describe accurately the shape of the pressure profile from slightly below the entrance throughout the remainder of the downstream entrance region.
2. The experimental data were in reasonable agreement with the theoretically predicted length required for the development of the velocity profile.
3. The presence of a vena contracta at the entrance for Reynolds numbers above 400 indicated radial flow immediately at the entrance in disagreement with the entrance plug flow assumption.
4. Significant pressure losses above and immediately at the entrance were observed, the losses being greater for the more non-Newtonian fluids.
5. The measured values for the downstream entrance correction, $C_{down}+1$, were about 15 per cent higher than Collins' theoretical values. This

discrepancy is believed to be due to the above mentioned pressure effects immediately at the entrance.

6. The overall pressure correction, C_{tot} , was found to decrease with power law exponent, n , in qualitative but not quantitative agreement with theory.

7. The overall correction did not vary appreciably with Reynolds number.

Recommendations

It is recommended that future experimental investigations extend this work to include other non-Newtonian fluids, particularly slurries. Measurements for thicker fluids at higher flow rates would be useful also.

Future theoretical non-Newtonian analyses should include the upstream and entrance effects perhaps by carrying out a complete numerical solution of the equations of motion.

LIST OF REFERENCES

LIST OF REFERENCES

1. Bird, R. B., Stewart, W. E. and Lightfoot, E. N. "Transport Phenomena" pp. 45, 187 New York, John Wiley and Sons, Inc. (1960)
2. Bogue, D. C. "Entrance Effects and Prediction of Turbulence in Non-Newtonian Flow" Ind. Eng. Chem. 51 874-8 (1959)
3. Boussinesq, J. V. "Sur la manière dont les vitesses, dans un tube cylindrique de section circulaire, évasé à son entrée, se distribuent depuis cette entrée jusqu'aux endroits où se trouve établi un régime uniforme" Compt. Rend. 113 9-15 (1891)
4. Campbell, W. D. and Slattery, J. C. "Flow in the Entrance of a Tube" Trans. A.S.M.E. Journal of Basic Engr. 85 Series D 41-46 (1963)
5. Collins, M. "Application of Boundary Layer Theory to Laminar Flow of Pseudoplastic Fluids" Ph.D. thesis, Princeton University, Princeton, New Jersey (1962)
6. Dodge, D. W. "Turbulent Flow of Non-Newtonian Fluids in Smooth Round Tubes" Ph.D. thesis, University of Delaware, Newark, Delaware (1957)
7. Goldstein, S. "Modern Developments in Fluid Dynamics" Vol. 1 pp. 299-308 London, Oxford Press (1938)
8. Keen, J. D. "Computer Studies of Inelastic Non-Newtonian Flow in Channels of Varying Cross-section" M.S. thesis, University of Tennessee, Knoxville, Tennessee (1962)
9. Kreith, F. and Eisenstadt, R. "Pressure Drop and Flow Characteristics of Short Capillary Tubes at Low Reynolds Numbers" Trans. A.S.M.E. 79 1070-8 (1957)
10. Langhaar, H. L. "Steady Flow in the Transition Length of a Straight Tube" J. Applied Mechanics 9 No. 2 A55-9 (1942)
11. Metzner, A. B. and Reed, J. C. "Flow of Non-Newtonian Fluids - Correlation for the Laminar, Transition, and Turbulent Flow Regions" A.I.Ch.E. Journal 1 434 (1955)

12. Schiller, L. "Die Entwicklung der laminarin Geschwindigkeitsverteilung und ihre Bedeutung für Zähigkeitsmessungen." Z. angew. Mathematik und Mechanik 2 94-106 (1922)
13. Schlichting, H. "Boundary Layer Theory" pp. 94-7, 123-4, 206 New York, McGraw-Hill Book Co., Inc. (1955)
14. Shapiro, A. H. and Smith, R. "Friction Coefficients in the Inlet Length of Smooth Round Tubes" Natl. Advisory Comm. Aeronaut. TN 1785 (1948)
15. Tomita, Y. "Analytical Treatments of Non-Newtonian Fluid Flow by Introducing the Conception of Boundary Layer" J.S.M.E. Vol. 4, No. 13 (Feb. 1961)
16. _____. "On the Non-Newtonian Laminar Flow in the Region near the Entrance of a Circular Pipe" Chem. Engineering Japan 23 525-9 (1959)
17. Weltmann, R. and Keller, T. "Pressure Losses of Titania and Magnesium Slurries in Pipes and Pipeline Transitions" Natl. Advisory Comm. Aeronaut. TN 3889 (1957)
18. Wilkinson, W. L. "Non-Newtonian Fluids" pp. 130-134, 30 New York, Pergamon Press (1960)

APPENDICES

APPENDIX A

SAMPLE CALCULATIONS

The calculations for this investigation consisted mainly of processing the experimental data into the form of the dimensionless parameters $(P_{01}-P)/(\rho V_2/2g_c)$ and $(X/R)/N'_{Re}$. The data processing was programmed in Forgo for the IBM 1620 digital computer. Programs for the Newtonian and non-Newtonian runs are presented in Tables III and IV, respectively.

A sample calculation for one pressure tap reading for a non-Newtonian run is shown below:

Sample Run F, Sample Tap 19

Reference Tap 1

1. Velocity in small tube:

$$V = W / \rho_c A$$

ρ_c = density of Carbopol-water solution

$$V = .59 / (0.9965)(62.43)(.0016312)$$

$$V = 5.82 \text{ ft/sec}$$

2. Power law Reynolds number:

$$N'_{Re} = D^n v^{2-n} \rho_c / K g_c \left(\frac{3n+1}{4n} \right)^n 8^{n-1} \quad (12)$$

The calculation of n and K is described in Appendix C.

TABLE III

NEWTONIAN DATA PROCESSING PROGRAM

```

C  C  H L LANIEVE  24588
C      MS THESIS
      DIMENSION X(24),          T(24),DELH(24),DELCH(24),B(24)
      DIMENSION C(24),VISC(24),VEL(24),REN(24),DIST(24),PRES(24)
      READ, (X(I),DELCH(I),I=1,24)
      READ, (T(I),I=1,24)
      READ, (DELH(I),I=1,24)
      READ,VISC1,T1,DENW,DENEG,DENO,DENWD,PPS
      R=0.02279
      A=0.0016312
      GC=32.174
      DO 30 I=1,24
      D=6.72E-4*VISC1
      VISC(I)=D*EXP(3.05E3*((1./(T(I)+273.))-(1./(T1+273.))))
      REN(I)=(PPS/(DENEG*A))*2.*R*DENEG/VISC(I)
      DIST(I)=X(I)/(R*REN(I)*12.)
      B(I)=(DENO-DENWD)*DELH(I)*62.43/(12.*2.54)
      C(I)=(DENEG-DENW)*DELCH(I)*62.43/12.
      VELC=PPS/(DENEG*62.43*A)
      PRES(I)=(B(I)+C(I))*2.*GC/(DENEG*62.43*(VELC**(2.)))
30 PUNCH,REN(I),DIST(I),PRES(I)
      STOP
      END
56.5 2.71 46.5 2.71 36.5 2.71 26.5 2.71 18.5 2.71
14.5 2.71 10.5 2.71 8.5 2.71 6.5 2.71 4.5 2.71
2.5 2.71 1.5 2.71 1. 2.71 .5 2.71 .2 2.71
0. 2.63 56.5 2.71 46.5 2.71 36.5 2.71 26.5 2.71
0. 2.27 0. 1.92 0. 1.21 -.611 0.
24.8 24.8 24.9 24.9 25.0 25.0 25.1 25.2 25.2 25.3
25.4 25.4 25.4 25.5 25.5 25.6 25.6 24.8 24.8 24.9
25.6 25.5 25.4 25.3
114.3 99.9 85.7 72.5 59.6 52.7 46.5 42.2 38.2 34.1
29.8 26.0 24.8 24.8 25.6 -2.8 114.3 99.9 85.7 72.5
-2.8 -2.8 -2.8 -2.8
8.48 25.0 .9988 1.1034 1.2124 .9988 .2865

```

TABLE IV

NON-NEWTONIAN DATA PROCESSING PROGRAM

```

C  C  H. L. LA NIEVE  NN DATA MAY 15,63 (1)
C      MS THESIS NN DATA
      DIMENSION X(24),DELH(24),DELCH(24)
      DIMENSION DIST(24),PRES(24),B(24),C(24)
      DIMENSION TW(15)
      READ,(X(I),DELCH(I),I=1,24)
      READ,(DELH(I),I=1,24)
      READ,DENW,DENO,DENC,CK,CN,PPS
      R=0.02279
      A=0.0016312
      GC=32.174
      VEL=PPS/(DENC*62.43*A)
      Z=((R*2.0)**CN)*(VEL**(2.0-CN))
      Y=((3.0*CN)+1.0)/(4.0*CN)**CN
      RENP=(Z*DENC*62.43)/((GC*CK*Y)*(8.0**(CN-1.0)))
      PUNCH ,RENP,CN,CK
      DO 30 I=1,24
      DIST(I)=X(I)/(R*RENP*12.0)
      B(I)=(DENO-DENW)*DELH(I)*62.43/(12.0*2.54)
      C(I)=(DENC-DENW)*DELCH(I)*62.43/12.0
      PRES(I)=(B(I)+C(I))*2.0*GC/(DENC*62.43*(VEL**(2.0)))
30    PUNCH ,DIST(I),PRES(I)
      DO 20 I=1,15
      TW(I)=(B(I)+C(I)-(B(I+1)+C(I+1)))*6.0*R/((X(I)-X(I+1))*2.09E-3)
20    PUNCH, TW(I)
      STOP
      END
      56.5,2.71,46.5,2.71,36.5,2.71,26.5,2.71,18.5,2.71
      14.5,2.71,10.5,2.71,8.5,2.71,6.5,2.71,4.5,2.71
      2.5,2.71,1.5,2.71,1.0,2.71,0.5,2.71,0.2,2.71
      0.,2.63,0.,2.27,0.,1.92,0.,1.21,-.611,0.
      -1.057,0.,-2.057,0.,-5.057,0.,14.5,2.71
      0. 315.9 257.7 199.2 152.6 129.1 105.5 94.1 81.4 79.0
      55.5 46.9 43.4 39.2 38.7 1.7 1.7 1.7 1.5 1.1
      .9 .7 .9 129.8
.9965 1.733 .9965 .0595 .41 .59

```

$$N'_{Re} = \frac{[(2)(.02279)]^{.41} (5.82)^{2-.41} (.9965)(62.43)}{(.0595)(32.174) \left[\frac{(3)(.41)+1}{(4)(.41)} \right]^{.41} 8^{(.41-1)}}$$

$$N'_{Re} = 448.8$$

3. Dimensionless distance from entrance:

$$(X/R)/N'_{Re} = (14.5/12)/(.02279)(448.8)$$

$$(X/R)/N'_{Re} = 0.11814$$

4. Manometer differential in lb_f/ft^2 :

$$\Delta P = (\rho_{oil} - \rho_{water}) H \rho_{water} (g/g_c)$$

$$\Delta P = (1.733 - .9965)(129.1)(62.43) / (2.54)(12)$$

$$\Delta P = 195 \text{ lb}_f/\text{ft}^2$$

5. Manometer correction:

Because the height of some taps differed from that of the reference tap and because water was used in the manometer leads, a correction for the density difference of the water and test fluid was necessary. For the Carbopol-water test fluid the density difference was negligible.

$$\Delta P_{\text{correction}} = (\rho_{\text{Carbopol}} - \rho_{\text{water}}) h \rho_{\text{water}} (g/g_c) / 12$$

$$\Delta P_{\text{correction}} = (0)(2.714)(62.43) / 12$$

$$\Delta P_{\text{correction}} = 0$$

6. Dimensionless pressure difference:

$$(P_{01} - P) / (\rho V^2 / 2g_c) = (\Delta P + \Delta P_{\text{correction}}) 2g_c / (\rho_c)(V^2)$$

$$(P_{01}-P)(\int V^2/2g_c) = (195)(2)(32.174) / (.9965)(62.43)(5.82)^2$$

$$(P_{01}-P)(\int V^2/2g_c) = 6.0203$$

7. Entrance pressure correction:

The pressure correction was calculated graphically from the plot of dimensionless pressure versus dimensionless length. Its value is the intercept at zero length of a line of slope 32 drawn through the far downstream experimental data points.

APPENDIX B

ANALYSIS OF SOURCES OF EXPERIMENTAL ERROR

Since the most significant results of this experiment were expressed in terms of the pressure correction found by extrapolating a line of slope 32 through the downstream Poiseuille flow data points back to the entrance, the experimental accuracy may be determined by investigating the error in the determination of the values of the measured pressure loss, $(P_{01}-P)/(\mathcal{F} V^2/2g_c)$ in reduced velocity head units, and the slope of the data plot,

$$\frac{P_{01}-P}{\mathcal{F} V^2/2g_c} \cdot \frac{R N'_{Re}}{X} = 32 . \quad (B-1)$$

The degree of accuracy of measurement and the relative error of each measured variable are presented in Table V along with the resulting relative error in $(P_{01}-P)/(\mathcal{F} V^2/2g_c)$ and slope.

For the Newtonian case for typical measurement errors the slope was found to be 32 ± 1.4 . The relative error in the pressure loss $(P_{01}-P)/(\mathcal{F} V^2/2g_c)$ for each measurement was 2.6 per cent. Since actually the slope was held to closer tolerance than the above value, the entrance pressure correction, C_{tot} , should be $2.53 \pm .08$ or should have a maximum relative error of 3 per cent.

TABLE V

ERROR ANALYSIS OF A TYPICAL NEWTONIAN DATA POINT

Measured Variable	Accuracy	Typical Relative Error (%)	Slope (%)	Relative Error in $(P_{09}-P)/(\rho v^2/2g_c)$ (%)
D	.001 in.	.18	.73	.73
H	.1 cm	.2	.2	.2
X	.01 in.	.1	.1	-
W	.005 lb _m /sec	.5	.5	1
f	.001	.1	.1	.1
* $f_{oil} - f_{water}$	.001	.13	.13	.13
** $f_{oil} - f_{water}$	.001	.5	.5	.5
μ	perhaps conservative	2	2	-
**Total relative error			4.2	2.6

*Specific gravity of oil = 1.75.

**Specific gravity of oil = 1.2.

The non-Newtonian error should not be much greater than the Newtonian error particularly for values of n approaching 1. Any difference in error would be caused by the viscometry, and it is believed that, except in the case of $n=0.4$ where the fluid rheology did not exactly follow the power law model, the accuracy was similar for both cases.

APPENDIX C

DESCRIPTION OF VISCOMETRY

A Haake Rotovisco viscometer built by Gebruder Haake K. G., Berlin-Steglitz, Germany, was used for all rheological measurements. The operating instructions were provided with the instrument. Two different procedures were used for the treatment of the Newtonian and non-Newtonian data.

Newtonian Measurements

For Newtonian fluids bob and cup attachment NV was used. From measurements of speed of rotation, the force required (scale reading) to maintain the speed and the dimensions of the bob and cup, the viscosity could be determined (18). National Bureau of Standards oils H, I, and J of the same viscosity range as the samples were used to check the calibration of the viscometer. The viscosity of the experimental fluid, ethylene glycol and water, was found to be the same at several shear rates, verifying that the fluid was Newtonian. Since the pressure measurement runs were made at various temperatures, viscosity-temperature data were measured for the ethylene glycol-water solution and recorded as log viscosity versus the reciprocal of the absolute temperature. The viscosity measurements were then made at 25°C and the data converted to the experimental temperatures. The

scale reading of the viscometer limited the accuracy of the viscosity determination to a relative error of about 2 per cent.

Non-Newtonian Measurements

Since the non-Newtonian fluids were more viscous, their viscosity measurements were made with cylindrical bob and cup attachment SVI. The shear stress, τ , and shear rate, $\dot{\gamma}$, were required for the non-Newtonian fluid. After relating the gear setting to revolutions per minute the shear rate was calculated from the relationship (18)

$$\dot{\gamma} = \frac{\omega R_{avg}}{\Delta R}, \quad (C-1)$$

where R_{avg} is the average of the inner and outer radii and ΔR is the gap thickness.

Because end effects complicated the direct calculation of the shear stress, τ was evaluated indirectly through calibration with National Bureau of Standards Oils. The standard oils gave a reliable value of $\tau / \dot{\gamma}$ (viscosity) and the value of $\dot{\gamma}$ calculated from Equation C-1 was assumed to be reliable; therefore, τ was easily evaluated. A summary of pertinent data follows:

Cup and Bob SVI:

Cup diameter: 23.09 mm

Bob diameter: 20.19 mm

$$\begin{array}{lll} \mathcal{T} \text{ (dynes/cm}^2\text{)} & 11.0S & 50 \text{ Head} \\ \mathcal{D} \text{ (sec}^{-1}\text{)} & 456/U & \end{array}$$

where

S = scale reading, 0-100

U = gear setting, 1, 2, 3 . . . 162

Since the test gels had a tendency to change their rheological properties with time, the viscometry measurements were made immediately after the run and at the same temperature as the run. The shear stress was measured at every possible shear rate for the fluid (10 speeds), and the data were plotted as log shear stress versus log shear rate. The slope was the desired power law exponent, n , while extrapolation to zero log shear rate gave the power law K . For all fluids tested this plot was a nearly straight line except at very low shear rates where the shear stress became higher than predicted by the power law. The viscometry plots for some of the Carbopol-water gels are shown in Figure 13.

The wall shear stress was determined in the developed flow section of the test section by the equation (1)

$$\mathcal{T}_w = \frac{D}{4L} \Delta P \quad (C-2)$$

and was found to be of the same order of magnitude as the highest shear stress measured with the viscometer, assuring

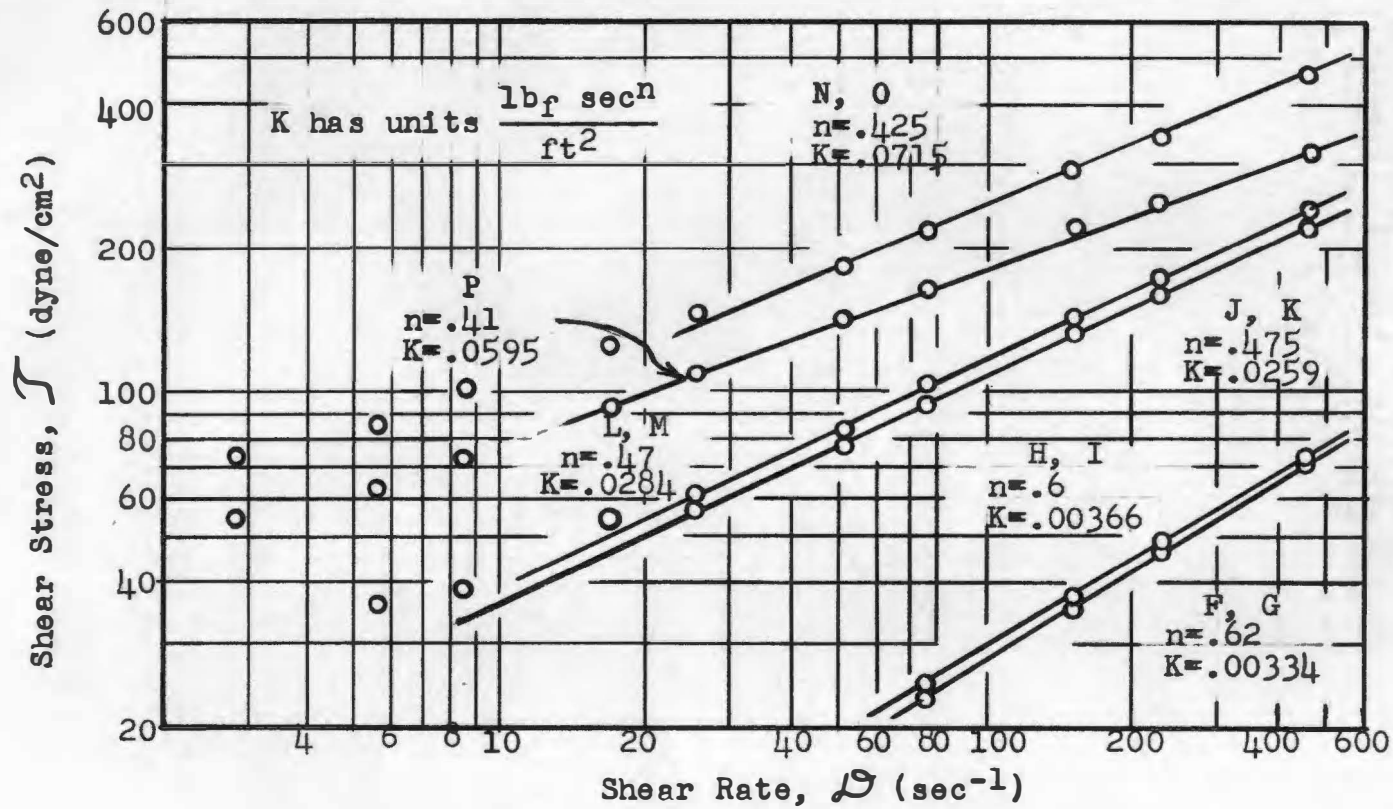


Figure 13. Determination of Power Law Constants.

that the viscometry data were applicable to the actual flow situation.

APPENDIX D

TABULATION OF DATA

The experimental data are presented in the form of the reduced coordinates of distance from the entrance, $(X/R)/N_{Re}^*$, and pressure difference, $(P_{01}-P)/(\rho V^2/2g_c)$. For the Newtonian runs the tap numbers along with the Reynolds number for each tap and the fluid specific gravity (SG), viscosity and mass flow rate for each run are given. For the non-Newtonian runs Reynolds number, mass flow rate, and a value for each of the power law constants n and K are given for each run.

TABLE VI
REDUCED DATA FOR NEWTONIAN RUN A

Tap	N_{Re}	$(X/R)/N_{Re}$	$(P_{01}-P)/(\rho v^2/2g_c)$
24	578.0	.3574	15.232
23	578.0	.2942	12.823
22	580.0	.2301	10.497
21	582.0	.1665	8.299
20	582.0	.1162	6.862
19	582.0	.0911	6.016
18	583.9	.0658	5.171
17	583.9	.0532	4.494
16	583.9	.0407	4.241
15	585.9	.0281	3.649
14	585.9	.0156	3.226
13	585.9	.0094	2.677
12	585.9	.0062	2.592
11	585.9	.0031	2.423
10	587.9	.0012	2.169
9	587.9	.0000	0.436
24	589.9	.3502	15.274
23	589.9	.2882	12.315
22	589.9	.2263	10.624

$W = 0.1085 \text{ lb}_m/\text{sec}$
 $\mu = 8.3 \text{ cp}$
 $SG_{EG} = 1.1013$

TABLE VII
REDUCED DATA FOR NEWTONIAN RUN B

Tap	N_{Re}	$(X/R)/N_{Re}$	$(P_{01}-P)/(\rho v^2/2g_c)$
24	580.0	.3562	14.055
23	582.0	.2922	12.223
22	586.0	.2277	10.221
21	586.0	.1654	8.186
20	588.1	.1150	6.489
19	590.1	.0898	5.641
18	590.1	.0561	4.759
17	592.2	.0525	4.284
16	592.2	.0401	3.673
15	594.2	.0277	3.232
14	596.2	.0153	2.825
13	598.3	.0092	2.248
12	598.3	.0061	2.248
11	598.3	.0031	2.044
10	600.4	.0012	1.909
9	602.4	.0000	-0.026
24	621.3	.3325	13.337
23	621.3	.2736	11.952
22	619.2	.2155	9.950
21	617.1	.1570	7.982
20	615.0	.1100	6.489
19	612.9	.0865	5.539
18	610.8	.0629	4.725
17	610.8	.0509	4.199

$$\begin{aligned}
 W &= .1215 \text{ lb}_m/\text{sec} \\
 \mu &= 8.3 \text{ cp} \\
 SG_{EG} &= 1.1020
 \end{aligned}$$

TABLE VIII
REDUCED DATA FOR NEWTONIAN RUN C

Tap	N_{Re}	$(X/R)/N_{Re}$	$(P_{01}-P)/(\rho v^2/2g_c)$
24	1395	.1481	7.397
23	1395	.1219	6.492
22	1400	.0953	5.599
21	1400	.0692	4.770
20	1405	.0482	3.959
19	1405	.0377	3.525
18	1410	.0272	3.135
17	1414	.0220	2.865
16	1414	.0168	2.613
15	1419	.0116	2.356
14	1424	.0064	2.085
13	1424	.0038	1.846
12	1424	.0026	1.771
11	1429	.0013	1.771
10	1429	.0005	1.821
9	1434	.0000	0.030
24	1434	.1441	7.397
23	1395	.1219	6.492
22	1395	.0957	5.599
21	1400	.0692	4.770
8	1434	.0000	0.001
7	1429	.0000	-0.026
6	1424	.0000	-0.081

$$\begin{aligned}
 W &= 0.2865 \text{ lb}_m/\text{sec} \\
 \mu &= 8.48 \text{ cp} \\
 SG_{EG} &= 1.1034
 \end{aligned}$$

TABLE IX
REDUCED DATA FOR NEWTONIAN RUN D

Tap	N_{Re}	$(X/R)/N_{Re}$	$(P_{01}-P)/(f v^2/2g_c)$
24	1492	.1384	7.179
23	1498	.1135	6.300
22	1503	.0888	5.427
21	1503	.0645	4.554
20	1508	.0449	3.830
19	1508	.0352	3.453
18	1508	.0255	3.024
17	1513	.0205	2.809
16	1513	.0157	2.594
15	1513	.0109	2.278
14	1513	.0060	2.037
13	1518	.0036	1.815
12	1523	.0024	1.721
11	1523	.0012	1.688
10	1523	.0005	1.775
9	1528	.0000	0.063
24	1586	.1303	6.810
23	1591	.1068	6.038
22	1597	.0836	5.272
21	1597	.0607	4.373
20	1597	.0424	3.688
19	1597	.0332	3.339
18	1597	.0240	2.936
9	1528	.0000	0.059

$$\begin{aligned}
 W &= 0.272 \text{ lb}_m/\text{sec} \\
 \mu &= 7.96 \text{ cp} \\
 SG_{EG} &= 1.1012
 \end{aligned}$$

TABLE X
REDUCED DATA FOR NEWTONIAN RUN E

Tap	N_{Re}	$(X/R)/N_{Re}$	$(P_{01}-P)/(\rho V^2/2g_c)$
24	1495	.1382	6.990
23	1495	.1138	6.283
22	1495	.0893	5.484
21	1500	.0646	4.631
20	1510	.0448	3.854
19	1510	.0351	3.486
18	1510	.0254	3.108
17	1521	.0204	2.871
16	1521	.0156	2.644
15	1521	.0108	2.288
14	1521	.0060	2.050
13	1526	.0036	1.791
12	1526	.0024	1.748
11	1526	.0012	1.726
10	1531	.0005	1.796
9	1536	.0000	0.031
24	1547	.1335	6.947
23	1547	.1099	6.159
22	1547	.0863	5.365
21	1536	.0626	4.550
8	1536	.0000	0.006
7	1536	.0000	-0.007
6	1536	.0000	0.003
5	1536	-.0014	-0.011

$$\begin{aligned}
 W &= 0.311 \text{ lb}_m/\text{sec} \\
 \mu &= 8.46 \text{ cp} \\
 SG_{EG} &= 1.1058
 \end{aligned}$$

TABLE XI
REDUCED DATA FOR NON-NEWTONIAN RUN F

Tap	$(X/R)/N'_{Re}$	$(P_{01}-P)/(\rho V^2/2g_c)$
24	.1406	6.950
23	.1157	6.043
22	.0908	5.164
21	.0660	4.285
20	.0460	3.545
19	.0361	3.147
18	.0261	2.756
17	.0212	2.547
16	.0162	2.310
15	.0112	2.073
14	.0062	1.787
13	.0037	1.605
12	.0025	1.514
11	.0012	1.501
10	.0005	1.808
9	.0000	.04916
8	.0000	.007251
7	.0000	.007209
6	.0000	.007123
5	-.0015	.006978
4	-.0026	.006978
3	-.0051	.006978
2	-.0126	.006978

$$\begin{aligned}
 N'_{Re} &= 1469.2 \\
 W &= 0.480 \text{ lb}_m/\text{sec} \\
 n &= 0.62 \\
 K &= 0.00334 \text{ lb}_f \text{ sec}^n/\text{ft}^2
 \end{aligned}$$

TABLE XII
REDUCED DATA FOR NON-NEWTONIAN RUN G

Tap	$(X/R)/N'_{Re}$	$(P_{01}-P)/(\rho V^2/2g_c)$
24	.1691	7.893
23	.1392	6.854
22	.1092	5.778
21	.0793	4.739
20	.0554	3.883
19	.0434	3.436
18	.0314	2.962
17	.0254	2.725
16	.0194	2.452
15	.0135	2.188
14	.0075	1.878
13	.0045	1.659
12	.0030	1.586
11	.0015	1.540
10	.0006	1.732
9	.0000	.05510
8	.0000	.01858
7	.0000	.009415
6	.0000	.009304
5	-.0018	.009114
4	-.0032	.009114
3	-.0062	.009114
2	-.0151	.009114

$$N'_{Re} = 1221.9$$

$$W = 0.42 \text{ lb}_m/\text{sec}$$

$$n = 0.62$$

$$K = 0.00334 \text{ lb}_f \text{ sec}^n/\text{ft}^2$$

TABLE XIII
REDUCED DATA FOR NON-NEWTONIAN RUN H

Tap	$(X/R)/N_{Re}^{'}$	$(P_{01}-P)/(\rho V^2/2g_c)$
24	.0439	4.426
23	.0361	3.866
22	.0283	3.318
21	.0206	2.818
20	.0144	2.474
19	.0113	2.292
18	.0082	2.089
17	.0066	1.980
16	.0050	1.942
15	.0035	1.720
14	.0019	1.561
13	.0012	1.427
12	.0007	1.378
11	.0004	1.325
10	.0002	1.955
9	.0000	.02799
8	.0000	.00564
7	.0000	.004235
6	.0000	.004218
5	-.0005	.004189
4	-.0008	.004189
3	-.0016	.002793
2	-.0039	.002793

$$N_{Re}^{'} = 4710$$

$$W = 1.073 \text{ lb}_m/\text{sec}$$

$$n = 0.60$$

$$K = 0.00366 \text{ lb}_f \text{sec}^n/\text{ft}^2$$

TABLE XIV
REDUCED DATA FOR NON-NEWTONIAN RUN I

Tap	$(X/R)/N_{Re}^1$	$(P_{01}-P)/(\rho V^2/2g_c)$
24	.0790	5.166
23	.0650	4.584
22	.0510	4.004
21	.0370	3.416
20	.0259	2.931
19	.0203	2.672
18	.0147	2.378
17	.0119	2.219
16	.0091	2.067
15	.0063	1.876
14	.0035	1.653
13	.0021	1.475
12	.0014	1.420
11	.0007	1.588
10	.0003	2.035
9	.0000	.03249
8	.0000	.009830
7	.0000	.009811
6	.0000	.009771
5	-.0008	.009704
4	-.0015	.009704
3	-.0029	.006469
2	-.0071	.006469

$$N_{Re}^1 = 2616.2$$

$$W = 0.705 \text{ lb}_m/\text{sec}$$

$$n = 0.60$$

$$K = 0.00366 \text{ lb}_f\text{sec}^n/\text{ft}^2$$

TABLE XV
REDUCED DATA FOR NON-NEWTONIAN RUN J

Tap	$(X/R)/N_{Re}^1$	$(P_{01}-P)/(\rho V^2/2g_c)$
24	.1254	6.394
23	.1032	5.581
22	.0810	4.776
21	.0588	3.959
20	.0411	3.292
19	.0322	2.943
18	.0233	2.572
17	.0189	2.382
16	.0144	2.178
15	.0100	1.956
14	.0056	1.710
13	.0033	1.530
12	.0022	1.468
11	.0011	1.480
10	.0004	1.856
9	.0000	.04147
8	.0000	.01430
7	.0000	.01430
6	.0000	.01430
5	-.0014	.01430
4	-.0023	.01430
3	-.0046	.01430
2	-.0112	.00850

$$N_{Re}^1 = 1646.8$$

$$W = 1.065 \text{ lb}_m/\text{sec}$$

$$n = 0.475$$

$$K = 0.0259 \text{ lb}_f \text{sec}^n/\text{ft}^2$$

TABLE XVI
REDUCED DATA FOR NON-NEWTONIAN RUN K

Tap	$(X/R)/N'_{Re}$	$(P_{01}-P)/(\rho V^2/2g_c)$
22	.1263	6.340
21	.0917	5.127
20	.0640	4.124
19	.0502	3.619
18	.0363	3.092
17	.0294	2.808
16	.0225	2.529
15	.0156	2.229
14	.0086	1.902
13	.0052	1.661
12	.0035	1.574
11	.0017	1.515
10	.0007	1.679
9	.0000	.05375
8	.0000	.03072
7	.0000	.03072
6	.0000	.03072
5	-.0021	.03072
4	-.0037	.03072
3	-.0071	.01792
2	-.0175	.01024

$$N'_{Re} = 1056.4$$

$$W = 0.796 \text{ lb}_m/\text{sec}$$

$$n = 0.475$$

$$K = 0.0259 \text{ lb}_f \text{ sec}^n/\text{ft}^2$$

TABLE XVII
REDUCED DATA FOR NON-NEWTONIAN RUN L

Tap	$(X/R)/N'_{Re}$	$(P_{01}-P)/(\int v^2/2g_c)$
23	.1829	8.412
22	.1435	7.000
21	.1042	5.627
20	.0728	4.500
19	.0570	3.926
18	.0413	3.328
17	.0334	3.028
16	.0256	2.717
15	.0177	2.370
14	.0098	2.006
13	.0059	1.751
12	.0039	1.650
11	.0020	1.565
10	.0008	1.669
9	.0000	.05355
8	.0000	.01966
7	.0000	.01968
6	.0000	.01407
5	-.0024	.01412
4	-.0042	.01412
3	-.0081	.01412
2	-.0199	.008475

$$N'_{Re} = 929.8$$

$$W = 0.76 \text{ lb}_m/\text{sec}$$

$$n = 0.47$$

$$K = 0.0284 \text{ lb}_f \text{sec}^n/\text{ft}^2$$

TABLE XVIII
REDUCED DATA FOR NON-NEWTONIAN RUN M

Tap	$(X/R)/N'_{Re}$	$(P_{01}-P)/(\rho V^2/2g_c)$
23	.4977	19.221
22	.3907	15.456
21	.2836	11.796
20	.1980	8.857
19	.1552	7.414
18	.1124	5.971
17	.0910	5.249
16	.0696	4.496
15	.0482	3.722
14	.0268	2.959
13	.0161	2.394
12	.0107	2.185
11	.0054	1.976
10	.0021	1.871
9	.0000	.07274
8	.0000	.05188
7	.0000	.05195
6	.0000	.05207
5	-.0065	.05229
4	-.0113	.05229
3	-.0220	.05229
2	-.0541	.04183

$$N'_{Re} = 341.6$$

$$W = 0.395 \text{ lb}_m/\text{sec}$$

$$n = 0.47$$

$$K = 0.0284 \text{ lb}_f \text{sec}^n/\text{ft}^2$$

TABLE XIX
REDUCED DATA FOR NON-NEWTONIAN RUN N

Tap	$(X/R)/N'_{Re}$	$(P_{01}-P)/(\rho V^2/2g_c)$
21	.5607	20.303
20	.3914	14.878
19	.3068	12.171
18	.2222	9.486
17	.1799	8.133
16	.1375	6.768
15	.0952	5.371
14	.0529	4.006
13	.0317	3.181
12	.0212	2.829
11	.0106	2.476
10	.0042	2.190
9	.0000	.2206
8	.0000	.1325
7	.0000	.1324
6	.0000	.1323
5	-.0129	.1320
4	-.0224	.1320
3	-.0435	.1320
2	-.1070	.1320

$$N'_{Re} = 172.8$$

$$W = 0.384 \text{ lb}_m/\text{sec}$$

$$n = .425$$

$$K = 0.0715 \text{ lb}_f \text{sec}^n/\text{ft}^2$$

TABLE XX
REDUCED DATA FOR NON-NEWTONIAN RUN 0

Tap	$(X/R)/N'_{Re}$	$(P_{01}-P)/(\int v^2/2g_c)$
23	1.0168	35.535
22	.7981	28.414
21	.5795	21.281
20	.4045	15.626
19	.3171	12.764
18	.2296	9.960
17	.1859	8.540
16	.1421	7.132
15	.0984	5.655
14	.0547	4.258
13	.0328	3.346
12	.0219	1.846
11	.0109	2.585
10	.0044	2.285
9	.0000	.3457
8	.0000	.1842
7	.0000	.1843
6	.0000	.1847
5	-.0134	.1847
4	-.0231	.1847
3	-.0450	.1847
2	-.1106	.1270

$$N'_{Re} = 167.2$$

$$W = 0.376 \text{ lb}_m/\text{sec}$$

$$n = .425$$

$$K = 0.0715 \text{ lb}_f \text{sec}^n/\text{ft}^2$$

TABLE XXI
REDUCED DATA FOR NON-NEWTONIAN RUN P

Tap	$(X/R)/N'_{Re}$	$(P_{01}-P)/(\rho V^2/2g_c)$
23	.3789	14.731
22	.2974	12.017
21	.2159	9.289
20	.1507	7.116
19	.1181	6.020
18	.0856	4.920
17	.0693	4.388
16	.0530	3.796
15	.0367	3.684
14	.0204	2.588
13	.0122	2.187
12	.0081	2.024
11	.0041	1.828
10	.0002	1.805
9	.0000	.07928
8	.0000	.07928
7	.0000	.07928
6	.0000	.06995
5	-.0050	.05130
4	-.0086	.04197
3	-.0168	.03264
2	-.0412	.04197

$$N'_{Re} = 448.8$$

$$W = 0.59 \text{ lb}_m/\text{sec}$$

$$n = 0.41$$

$$K = 0.0595 \text{ lb}_f \text{sec}^n/\text{ft}^2$$

LIST OF SYMBOLS

LIST OF SYMBOLS

- A - cross section area, ft^2 or in^2
- C_{down} - entrance pressure correction downstream from the entrance (experimentally measured from Tap 9),
dimensionless
- C_{tot} - total entrance pressure correction (experimentally measured from Tap 1), dimensionless
- C_{up} - pressure correction upstream from and immediately at the entrance, dimensionless
- D - diameter, in., ft, or cm
- $\dot{D}$ - shear rate, sec^{-1}
- f - Fanning friction factor (see Equation 7), dimensionless
- g_c - force-mass conversion factor, $\text{lb}_m \text{ ft} / \text{lb}_f \text{ sec}^2$
- H - manometer differential, cm
- h - vertical distance measured from Tap 1 located on top of the 6-in. tube, in.
- K - power law coefficient, $\text{lb}_f \text{ sec}^n / \text{ft}^2$
- K' - proportionality constant (see Equation 5), $\text{lb}_f \text{ sec}^n / \text{ft}^2$
- L - horizontal distance between two pressure taps or a characteristic length, ft or in.
- N_{Re} - Reynolds number, dimensionless
- N'_{Re} - power law Reynolds number (see Equation 13),
dimensionless
- N_{Re}^n - a general non-Newtonian Reynolds number (see Equation 9), dimensionless

- n - power law exponent, dimensionless
- n' - fluid constant (see Equation 5), dimensionless
- P - static pressure at a given position along a pipe,
 lb_f/ft^2
- P₀ - static pressure at a reference tap upstream from a
 contraction, lb_f/ft^2
- P₀₁ - static pressure at reference Tap 1 located 24 in.
 upstream from the contraction, lb_f/ft^2
- P₀₉ - static pressure at reference Tap 9 located on the
 vertical plate forming the contraction, lb_f/ft^2
- ΔP - static pressure difference between two points
 separated by a distance L, lb_f/ft^2
- p - pressure, lb_f/ft^2
- R - radius, ft or in.
- r - radial distance, ft
- V - average velocity, ft/sec
- v_r - radial point velocity, ft/sec
- v_x - longitudinal point velocity, ft/sec
- X - distance from entrance, ft or in.
- X_d - distance from entrance required for velocity profile
 development, ft
- x - longitudinal distance, ft
- W - mass flow rate, lb_m/sec

Greek Symbols

ω - angular velocity, sec^{-1}

μ - viscosity, $\text{lb}_f \text{ sec/ft}^2$

ρ - density, lb_m/ft^3

τ - shear stress, dyne/cm^2

τ_{rx} - shear stress in the x direction acting on the plane
perpendicular to the r direction, lb_f/ft^2

τ_w - shear stress at the wall, lb_f/ft^2

VITA

The author was born in Knoxville, Tennessee, on December 13, 1938. His parents are Mr. and Mrs. H. L. LaNieve, Jr. He has one younger sister, Frances.

The author attended Knoxville West High School, graduating in June, 1956. He entered The University of Tennessee in September, 1956, and received a B.S. in Chemical Engineering in June, 1960. While at The University of Tennessee he was a member of Alpha Chi Sigma chemistry fraternity and Tau Beta Pi national honorary fraternity. He also became an associate member of Sigma Xi national research society.

The author worked during the summers of 1959 and 1960 for the Isotopes Division of Oak Ridge National Laboratories in Oak Ridge, Tennessee.

In the fall of 1960 he entered the graduate school of The University of Tennessee as a graduate assistant in the Department of Chemical and Metallurgical Engineering. In September, 1961, he accepted a teaching assistantship in the same department, which he holds at the present time. He plans to work toward a Ph. D. degree in Chemical Engineering.

The author was married on June 9, 1959, to Alice Fielden of Knoxville. His wife received a B. S. in Education from The University of Tennessee in June of 1960 and since

that time has been art teacher at Holston High School in Knoxville. The author's hobbies are fishing, hunting, and other outdoor sports.