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I am submitting herewith a thesis written by Ali R. Hajialigol entitled "Non-Parametric Spectral Analysis for Tube Leak Detection." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

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Non-Parametric Spectral Analysis for Tube Leak Detection

**A Thesis Presented
for the Master of Science Degree
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ABSTRACT

Because forced outages of utilities in the United States are primarily the result of boiler tube failures in fossil-fueled power plant--failures that have placed tremendous financial burdens on the industry--it is no surprise that the interest in spectral analysis methods for tube leak detection has reached an epic high.

Therefore, in this thesis, non-parametric spectral estimation methods were developed to investigate their effectiveness for tube leak detection. Such methods included the following: (1) the minimum variance method and (2) eigenanalysis methods, utilizing both the EV and MUSIC techniques. Findings suggest that although all of the PSD estimation methods performed well enough to detect leaks, eigenanalysis methods proved to be far superior over both classical and parametric methods, and the EV method showed the most promise regarding early leak detection.

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CHAPTER I

INTRODUCTION

1.1. Problem Statement

Boiler tube failures are the primary cause of utility forced outages in the United States. These outages cause two main problems: (1) the obvious cessation of power generation for the units and their dependent communities; and (2) the cost. Because the lost power must be replaced, that is, purchased from other utilities, not to mention the additional cost of repairing the leaks, the financial burden that is placed on the outage utility is tremendous--so much so that the cost per year actually exceeds \$5 billion. But despite expansive engineering commitments to preventing these failures, approximately 41,000 incidents still occur every year [1].

It would be naive to expect that boiler tube leaks simply will not occur. The fact is that over time, the heat, pressure, and wear on the boiler and its apparatus give way to leakage. Consequently, early detection, as well as location and severity of the tube leakage, is most desirable in order to reduce the number of forced outages of power generating electric utilities.

Therefore, an automated boiler tube leak detection system which provides the above features, indicating both reliably and timely the onset of even a slight leakage, is not just desirable but necessary to the successful operation of an electric utility.

1.2. Overview

In order to create an effective tube leak detection system, it is beneficial to understand the nature of a leak. Stulen's work on the characteristics of leak detection [2] helps to understand this phenomenon. When a leak occurs in a boiler tube, pressurized water or steam flows into the opening and is discharged as a jet of steam. The jet's turbulent composition generates a continuous broad-band random vibration which adds to the boiler's ambient noise. It is this vibration which needs to be detected.

Stulen also offers three significant characteristics of jet-generated noise which must be considered in any leak detection system. They are the following: (1) acoustic power increases with the mass flow rate and the velocity of the jet; (2) the noise generated is random and has a power density spectrum with a peak amplitude at a given frequency; and (3) the radiation of noise from the jet is not uniform but is directional.

The leak's acoustic power is created by a conversion of some of the mechanical energy of the jet. Thus, the composition of the acoustic power level can be expressed in decibels (dB) as the following:

$$L_w = 10 \log_{10} \left(\frac{1}{2} \dot{m} V^2 \xi 10^{12} \right), \quad (1.1)$$

where $\dot{m}$ is the mass flow rate, V is the exit velocity of the jet, ξ is the efficiency constant, and 10^{12} refers to the logarithmic sound

scale. Thus, as the equation above shows, the acoustic power is proportional to the mechanical power, shown as $1/2\dot{m}V^2$ in watts.

Also, it should be noted that the acoustic sound generated varies according to the size of the hole. The expression, which is as follows,

$$f_p = \frac{S_t V}{D}, \quad (1.2)$$

illustrates, where S_t is the Strouhal number, i.e., the ratio of inertial and vibrational forces, that f_p , peak frequency (Hz) is determined by the jet's diameter, D , when discharged [2].

The acoustic power of the leak is broad-band and continuous, but it is not concentrated at the frequency given in the preceding equation. Instead, the acoustic power falls off on either side of the peak frequency [2]. Furthermore, it is not uniformly radiated but, in fact, has direction. And, according to data given by Stulen [2], most of the sound pressure occurs between 30° to 45° from the jet's axis.

Now that the nature of leaks has been discussed, the question is, "How can practical applications of this knowledge in terms of designing a tube-leak detection system be made?" One commonly used detection system incorporates an acoustic method. Obviously, in order to make use of an acoustic leak detection system, pressure-sensitive sensors, which recognize airborne soundwaves, must be placed at critical sites throughout the boiler unit, usually behind a metal waveguide which is normally attached to the boiler wall. Each

sensor's signal passes through a high-pass and low-pass filter and is used to compute a mean-square value for the sensor signal. The high-pass filter passes signal frequencies above 1,000 Hertz (Hz) because combustion roar dominates the lower frequencies. The low-pass filter passes signal frequencies below 10,000 Hz to eliminate any high-frequency transients. Thus, the mean-square value calculated to determine leak presence considers signal content only between 1,000 and 10,000 Hz. Each sensor has threshold values set to activate an alarm if the mean-square value of a sensor's signal exceeds this threshold for a specified period of time [3].

Normally, the signal obtained from a sensor is dominated by background noise whose spectrum has peaks at low frequencies and is flat at high frequencies. Background noise is mainly generated by the combustion process. If there is a leak, the leak noise is added to the background noise.

In early 1991, the Tennessee Valley Authority (TVA) started a joint effort with the Communications, Information, and Signal Processing (CISP) Group of the University of Tennessee, Knoxville, to develop spectral analysis techniques for tube leak detection. It was believed that a more reliable and less expensive leak detection system could be developed by investigating spectral analysis techniques.

Both classical and parametric Power Spectral Density (PSD) estimation techniques were employed by the venture's members [4,5] to develop the detection algorithm in the LabVIEW software environment from National Instruments. Above works show that leaks have been detected by both classical and parametric methods

in which parametric methods gave earlier detection than classical methods.

The main purpose of this project is to develop non-parametric spectral analysis methods--specifically, minimum variance and eigenanalysis--for tube leak detection. In order to compare the results of this work with the previous ones, the data in Wan's report [5] is used.

All programming is done in the LabVIEW software environment. LabVIEW is a visual programming language that provides a graphical programming environment with a library of data acquisition and digital signal processing analysis functions for use in application development [6]. An application program in LabVIEW is called a Virtual Instrument (VI) and consists of a front panel, a block diagram, and an icon/connector. The front panel is the interface between an operator and the VI. The diagram, which is created by the graphical programming language, is the VI's source code. A subVI is an embedded VI within another VI. When the VI is used as a subVI in another VI's block diagram, it is represented by its icon. Icon acts as a port through which data can be transferred between the VI and the subVI.

1.3 Review of Power Spectrum Estimation Techniques

Power spectrum estimation has progressed through several stages since the turn of the century. Perhaps the first estimator to be used extensively was the periodogram which was based on the direct approach via an FFT operation on the data. Although the periodogram method is known to be a poor estimator because the

standard deviation of the estimator exceeds its mean value, it is still used today. In 1958, Blackman and Tukey [7] published their autocorrelative method for power spectrum estimation, the steps of which include estimating the autocorrelation function from the observed data, windowing the autocorrelation estimate in an appropriate manner, and then Fourier transforming the windowed autocorrelation function to finally obtain the estimated power spectrum.

Many of the problems of the above classical PSD estimation techniques can be traced to the assumptions made about the data outside the measurement interval. The assumptions of data being zero outside the window decreases the resolution of the PSD estimate. An additional disadvantage of windowing is that unless one performs good window carpentry, excessive side lobes (leakage) may be introduced in the PSD estimate. Leakage may be reduced by employing well-designed windows, but again at the expense of spectral resolution, particularly when the data record is short.

In order to overcome the problems in classical PSD estimation, parametric PSD estimation was developed. This class of PSD estimation includes autoregressive (AR), moving average (MA), and autoregressive moving average (ARMA) methods. With these methods, one is able to make a more reasonable assumption than to assume the data are zero outside the window. Use of *a priori* information (assumptions) may permit selection of an exact model for the process that generated the data samples, or at least a model that is a good approximation to the actual underlying process. It is

then usually possible to obtain a better spectral estimate based on the model by determining the parameters of the model from the observations. Thus spectrum analysis, in the context of modeling, becomes a three-step procedure. The first step is to select a time series model (AR, MA, or ARMA). The second step is to estimate the parameters of the assumed model using either the available data samples or autocorrelation lags (either known or estimated from the data). The third step is to obtain the spectral estimate by substituting the estimated model parameters into the theoretical PSD implied by the model. One major motivation for the current interest in the modeling approach to spectral estimation is the higher frequency resolution achievable with these modern techniques over that of the classical techniques previously discussed.

A detailed comparison of classical and parametric PSD estimation has been given in [5]. From this work, one can easily see that the ARMA PSD estimation method is the best among all techniques.

1.4 Thesis Outline

This thesis contains five sections. Chapter II of this thesis gives background and discussion of the minimum variance PSD estimation method. Chapter III presents background and discussion of the eigenanalysis PSD estimation method. Chapter IV describes the experimental results and a comparison of what each method's relative performance might be as a part of the leak detection algorithm. It also compares the result of the non-parametric

methods to the result of both classical and parametric methods. Finally, Chapter V presents conclusions and recommendations for future study.

CHAPTER II

MINIMUM VARIANCE SPECTRAL ESTIMATION

2.1 Introduction

The minimum variance (MV) spectral estimator was originally developed by Capon [8] for seismic array frequency-wavenumber analysis. Capon promoted the method as a high-resolution spectral analysis technique, although it exhibits more resolution than Fourier estimators but less than that of an AR spectral estimator. The inverse transform of the MV estimator does not match the autocorrelation sequence that is used to create the MV estimate, unlike AR methods that do match. Thus, MV will be a spectral estimate that describes relative component strengths over frequency, and it does have the virtue of yielding peak heights that are linearly proportional to the power of sinusoids present in the process.

The MV spectral estimator is given by

$$P_{MV}(f) = \frac{T}{\mathbf{e}^H(f) \mathbf{R}_p^{-1} \mathbf{e}(f)}, \quad (2.1)$$

where $\mathbf{R}_p^{-1}$ is the inverse of the estimated autocorrelation matrix of dimension $(p+1) \times (p+1)$, $\mathbf{e}(p)$ is the complex sinusoid vector

$$\mathbf{e}(f) = \begin{bmatrix} 1 \\ \exp(j2\pi fT) \\ \cdot \\ \cdot \\ \cdot \\ \exp(j2\pi fpT) \end{bmatrix}, \quad (2.2)$$

and T is the sample interval.

Both Yule-Walker and Burg algorithms will be used to compute the autocorrelation parameters.

2.2 Derivation of the Minimum Variance Spectral Estimator

Suppose that a discrete time series is to be filtered by a realizable linear convolutional filter of length $p+1$. The sampled output is obtained as

$$y[n] = \sum_{k=0}^p a[k]x[n-k], \quad (2.3)$$

where x is the input and $a[k]$ are the filter coefficients. Assume that both the input to the filter and the filter weights can be complex and that

$$x[n] = A \exp[j2\pi f_0 nT] + z[n], \quad (2.4)$$

where A is a real constant and $z[n]$ is complex Gaussian noise with zero mean.

The filter weights $a[k]$ can be chosen to pass $A \exp[j2\pi f_0 nT]$ and to reject $z[n]$. If $y[n]$ is to be an unbiased estimate of $A \exp[j2\pi f_0 nT]$,

$$A \exp[j2\pi f_0 nT] = \sum_{k=0}^p a[k] A \exp[j2\pi f_0 T(n-k)] \quad (2.5)$$

or

$$1 = \sum_{k=0}^p a[k] A \exp[-j2\pi f_0 pT]. \quad (2.6)$$

Equations (2.3) and (2.6) can be written in matrix form

$$y[n] = \mathbf{x}^T[n] \mathbf{a} \quad (2.7)$$

and

$$1 = \mathbf{e}^H(f_0) \mathbf{a}, \quad (2.8)$$

in which the vectors $\mathbf{x}[n]$ and $\mathbf{a}$ of dimension $p+1$ are defined as

$$\begin{aligned} \mathbf{x}[n] &= [x[n], x[n-1], \dots, x[n-p]]^T \\ \mathbf{a} &= [a[0], a[1], \dots, a[p]]^T \end{aligned} \quad (2.9)$$

and the vector $\mathbf{e}(f_0)$ was defined by Eq. (2.2). The variance of $y[n]$ is simply

$$\begin{aligned} \rho &= E\{|y[n]|^2\} \\ &= E\{\mathbf{a}^H \mathbf{x}^*[n] \mathbf{x}^T[n] \mathbf{a}\} \\ &= \mathbf{a}^H E\{\mathbf{x}^*[n] \mathbf{x}^T[n]\} \mathbf{a} \\ &= \mathbf{a}^H \mathbf{R}_p \mathbf{a} \end{aligned} \quad (2.10)$$

in which the E is the expectation operator and the $(p+1) \times (p+1)$ Toeplitz autocorrelation matrix is

$$\mathbf{R}_p = \begin{bmatrix} r_{xx}[0] & \dots & r_{xx}^*[p] \\ \vdots & \ddots & \vdots \\ r_{xx}[p] & \dots & r_{xx}[0] \end{bmatrix} . \quad (2.11)$$

The output variance of Equation (2.10) is similar to the linear prediction error filter variance [9] except that $a[0]$ is arbitrary here rather than constrained to be unity. The coefficients are to be selected so that, at a frequency f_0 under consideration, the frequency response of the filter has unit gain. This constraint can be represented by Equation (2.6) or (2.8). A sinusoid of frequency f_0 , that is, input to the filter, would therefore be undistorted at the filter output. In order to reject components of the spectrum not near f_0 , one simply should minimize the output variance, Equation (2.10). The constrained minimum variance solution for the filter coefficients can be shown [10] to satisfy

$$\mathbf{a}_{MV} = \frac{\mathbf{R}_p^{-1} \mathbf{e}(f_0)}{\mathbf{e}^H(f_0) \mathbf{R}_p^{-1} \mathbf{e}(f_0)} . \quad (2.12)$$

Eq. (2.12) can be substituted into Eq. (2.10) to obtain the minimum variance

$$\rho_{MV} = \frac{1}{\mathbf{e}^H(f_0)\mathbf{R}_p^{-1}\mathbf{e}(f_0)} \quad (2.13)$$

The output variance serves as a good indicator of power in the input spectrum in the vicinity of f_0 . A different optimum variance is obtained for each f_0 selected. The filter adapts its shape across the frequency range to the process autocorrelation function.

An appropriate *minimum variance (MV) spectral estimator* that may be deduced from Equation (2.13) is

$$\begin{aligned} P_{MV} &= T\rho_{MV} \\ &= \frac{T}{\mathbf{e}^H(f)\mathbf{R}_p^{-1}\mathbf{e}(f)} \end{aligned} \quad (2.14)$$

defined for $-1/2T \leq f \leq 1/2T$. The sample interval T gives Equation (2.14) units of power per Hz, appropriated for a power spectral density.

It can be shown [11] that a complex sinusoid in white noise with autocorrelation

$$r_{xx}[k] = P \exp(j2\pi f_0 kT) + \rho_\omega \delta[k] \quad (2.15)$$

has a MV spectral estimator value at $f = f_0$ of

$$P_{MV} = \frac{T\rho_\omega}{P} \left(1 + pP/\rho_\omega\right) \quad (2.16)$$

If the signal-to-noise ratio P/ρ_ω is very high, then

$$P_{MV}(f_0) \approx PT, \quad (2.17)$$

which is the power of the sinusoid scaled by T . Peaks in the MV spectral estimator will be, therefore, approximately linearly proportional to power when the signal-to-noise ratio is high and the spectral lines are sufficiently separated, unlike the AR under the PSD estimator that was proportional to the square of the power. However, the area under the MV PSD curve is proportional to the square root of the sinusoid power, an indicator that the MV PSD is not a true PSD. Capon and Goodman have shown that the MV PSD has a mean and a variance that behaves much like the averaged periodogram (i.e., the variance is reduced linearly as the number of data segments averaged increases).

2.3 Comparison of the MV and AR Spectral Estimator

An explicit relationship exists between the MV and AR spectral estimators when the autocorrelation sequence is known. The inverse of the Toeplitz autocorrelation matrix can be expressible [9] as

$$\mathbf{R}_p^{-1} = \mathbf{A}_p \mathbf{P}_p^{-1} \mathbf{A}_p^H, \quad (2.18)$$

where $\mathbf{A}_p$ is a $(p+1) \times (p+1)$ matrix of the AR parameters from order 0 to order p

$$\mathbf{A}_p = \begin{bmatrix} 1 & 0 & \dots & 0 \\ a_p[1] & 1 & \dots & 0 \\ a_p[2] & a_{p-1}[1] & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ a_p[p] & a_{p-1}[p-1] & \dots & 0 \end{bmatrix}, \quad (2.19)$$

and $\mathbf{P}_p^{-1}$ is a $(p+1) \times (p+1)$ diagonal matrix of inverted white noise variance for the corresponding orders

$$\mathbf{P}_p^{-1} = \begin{bmatrix} 1/\rho_p & 0 & \dots & 0 \\ 0 & 1/\rho_{p-1} & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \dots & 1/\rho_0 \end{bmatrix}. \quad (2.20)$$

Substitution of Eq. (2.18) into the reciprocal MV spectral function (2.1) yields

$$\begin{aligned} \frac{1}{P_{MV}(p, f)} &= \frac{1}{T} \mathbf{e}^H(f) \mathbf{R}_{xx}^{-1} \mathbf{e}(f) \\ &= \frac{1}{T} \mathbf{e}^H(f) \mathbf{A}_p \mathbf{P}_p^{-1} \mathbf{A}_p^H \mathbf{e}(f) \\ &= \frac{1}{T} \sum_{k=0}^p \frac{1}{\rho_k} \left| \sum_{m=0}^k a_k[m] \exp(-j2\pi f m T) \right|^2 \\ &= \sum_{k=0}^p \frac{1}{P_{AR}(k, f)} \end{aligned} \quad (2.21)$$

where notation $P(k,f)$ is used to indicate a PSD of order k with frequency f [12]. The assumption $a_k[0]=1$ is made for all orders $0 \leq k \leq p$. Thus, the reciprocal of the minimum variance spectral estimator is equal to the average of the reciprocals of the autoregressive spectral estimators. The lower resolution of the MV spectral estimate relative to the AR estimate is due to the averaging effect of combining the low order AR spectra of least resolution with the high order AR spectra of the highest resolution. The MV method, though, still has better resolution than classical PSD estimators in higher signal-to-noise-level conditions. The variance of the MV spectral estimator has been observed for large data records to be less than AR PSD estimator of identical order p [11].

Equation (2.21) represents a *frequency-domain* relationship between the MV and AR spectral estimators. A *time-domain* relationship can also be developed [13]. The p th order AR PSD estimator may alternatively be expressed as

$$\begin{aligned}
 P_{AR}(f) &= \frac{T\rho_p}{\left|1 + \sum_{k=0}^p a_p[k] \exp(-j2\pi f k T)\right|^2} \\
 &= \frac{T}{\sum_{k=-p}^p \psi_{AR}[k] \exp(-j2\pi f k T)}
 \end{aligned} \tag{2.22}$$

by expanding its denominator. The $\psi_{AR}[k]$ coefficients are calculated from the correlation of the AR parameters

$$\Psi_{AR}[k] = \begin{cases} \frac{1}{\rho_p} \sum_{i=0}^p a_p[k+i] a_p^*[i] & \text{for } 0 \leq k \leq p \\ \Psi_{AR}^*[-k] & \text{for } -1 \geq k \geq -p \end{cases}, \quad (2.23)$$

in which $a_p[0]=1$ is assumed, by definition.

The MV spectral estimator can be evaluated in a similar way,

$$\mathbf{R}_p^{-1} = \frac{1}{\rho_p} \mathbf{T}_p \mathbf{T}_p^H - \frac{1}{\rho_p} \mathbf{S}_p \mathbf{S}_p^H, \quad (2.24)$$

where the triangular Toeplitz matrix $\mathbf{T}_p$ is

$$\mathbf{T}_p = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 \\ a_p[1] & 1 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_p[p-1] & a_p[p-2] & \dots & 1 & 0 \\ a_p[p] & a_p[p-1] & \dots & a_p[1] & 1 \end{bmatrix}, \quad (2.25)$$

and the triangular Toeplitz matrix $\mathbf{S}_p$ is

$$\mathbf{S}_p = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ a_p^*[p] & 0 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_p^*[2] & a_p^*[3] & \dots & 0 & 0 \\ a_p^*[1] & a_p^*[2] & \dots & a_p^*[p] & 0 \end{bmatrix}. \quad (2.26)$$

Substituting Eq. (2.24) into Eq. (2.14) for the inverse R_p^{-1} will yield, after extensive algebra,

$$P_{MV} = \frac{T}{\sum_{k=-p}^p \psi_{MV}[k] \exp(-j2\pi f k T)} . \quad (2.27)$$

The $\psi_{MV}[k]$ coefficients are calculated as a linearly weighted (windowed) correlation of AR parameters

$$\psi_{MV}[k] = \begin{cases} \frac{1}{\rho_p} \sum_{i=0}^{p-k} (p+1-k-2i) a_p[k+i] a_p^*[i] & \text{for } 0 \leq k \leq p \\ \psi_{MV}^*[-k] & \text{for } -1 \geq k \geq -p . \end{cases} \quad (2.28)$$

The FFT can be used to evaluate the alternative representation (2.28) of the MV spectral estimator over a range of frequencies. This is more computationally efficient than direct evaluation of the original MV spectral estimator function (2.14).

2.4 Estimating the MV Coefficients

In order to compute the MV power spectrum using Eq. (2.27) and (2.28), one must determine first of all the length of the required prediction filter p (or equivalently the order of the MV process) and, second, the coefficients themselves. Since the method of determining p assumes a knowledge of the coefficients, the estimation of these parameters will be discussed first. After finding

the MV's coefficients, then an approach will be presented to find the best model order.

There are several methods which can be used to obtain the coefficients. Two commonly used methods will be discussed here.

2.4.1 Yule-Walker Method

The most obvious approach to the AR PSD estimation from data samples is to use the autocorrelation function of the Yule-Walker (Y.W.) algorithm. If the unbiased autocorrelation estimate is selected, it may result in a nonpositive-definite autocorrelation matrix, so that a stable AR filter will not be assured. The biased correlation estimates will always yield positive-semidefinite autocorrelation matrices, so that a stable filter will be assured.

A model can be represented by its linear difference equation as

$$x[n] = -\sum_{k=1}^p a[k]x[n-k] + u[n] , \quad (2.29)$$

where $x[n]$ is an AR process of order p . The process is termed an autoregression in that the sequence $x[n]$ is a linear regression on itself with $u[n]$ representing the error. If Eq. (2.29) is multiplied by $x^*[n-m]$ and then the expectation is taken, then the result is

$$E\{x[n]x^*[n-m]\} = -\sum_{k=1}^p a[k]E\{x[n-k]x^*[n-m]\} \quad (2.30)$$

or

$$r_{xx}[m] = -\sum_{k=1}^p a[k]r_{xx}[m-k] . \quad (2.31)$$

The relationship between the autocorrelation sequence and a pure autoregressive model may be found as

$$r_{xx}[m] = \begin{cases} -\sum_{k=1}^p a[k]r_{xx}[m-k] & \text{for } m > 0 \\ -\sum_{k=1}^p a[k]r_{xx}[-k] + \rho_{\omega} & \text{for } m = 0 \\ r_{xx}^*[-m] & \text{for } m < 0 \end{cases} \quad (2.32)$$

This relationship may be evaluated for the $p + 1$ lag indices $0 \leq m \leq p$ and formed into the matrix expression

$$\begin{bmatrix} r_{xx}[0] & r_{xx}[-1] & \dots & r_{xx}[-p] \\ r_{xx}[1] & r_{xx}[0] & \dots & r_{xx}[-p+1] \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ r_{xx}[p] & r_{xx}[p-1] & \dots & r_{xx}[0] \end{bmatrix} \begin{bmatrix} 1 \\ a[1] \\ \cdot \\ \cdot \\ \cdot \\ a[p] \end{bmatrix} = \begin{bmatrix} \rho_{\omega} \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}, \quad (2.33)$$

which is recognized as the biased autocorrelation function (ACF) estimator. The matrix in (2.31) is Hermitian ($r_{xx}[-k] = r_{xx}^*[k]$) and Toeplitz. The Levinson algorithm [9] then can be used to obtain the solution for ρ_{ω} , $a[1]$, ..., $a[p]$.

The Yule-Walker algorithm represents the most direct approach for finding the MV model parameters and white noise variance ρ_{ω} of Eq. (2.32). Biased autocorrelation estimates $\hat{R}_{xx}[m]$ are substituted for the unknown autocorrelation function $R_{xx}[m]$. $\hat{R}_{xx}[m]$ ensures a stable MV filter, since $\hat{R}_{xx}[0] \geq R_{xx}[|m|]$. The

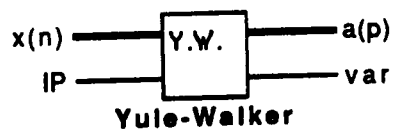
Levinson algorithm recursively solves matrix Eq. (2.33) for $a[1], \dots, a[p]$ and ρ_ω given $\hat{R}_x[m]$ for $0 \leq m \leq p$. For a long data record, the Y.W. PSD estimate yields reasonable resolution; however, poor resolution PSD estimates will result for short data records.

Figure 2.1 shows the connector pane, front panel, and block diagram of the Y.W. VI created in LabVIEW to estimate MV model parameters for acoustic data. Reading from left to right on the block diagram, a double-extended-precision floating-point array of data is passed from the connector pane (or front panel) into the node labeled " $x[n]$." The desired model order is passed in as a 32-bit integer through the node labeled " IP ." An autocorrelation is performed on the input $x[n]$ to yield $R_{xx}[m]$, which is biased by dividing by N (the size of $x[n]$). This is done in a VI called "auto-correl." The model order IP and the $IP+1$ lags of the biased $R_{xx}[m]$ are then passed to the icon labeled "Lvsn." This VI solves the matrix equation (2.33) by using the Levinson recursive algorithm to estimate the model parameters $a[k]$ where $0 \leq k \leq p$ and the noise variance ρ_ω .

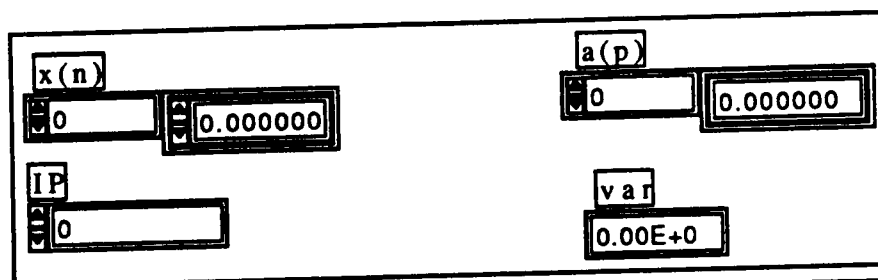
2.4.2 Burg Method

In contrast to the Yule-Walker method which estimates the AR parameters directly, the Burg method estimates the reflection coefficients and then uses the Levinson recursion to obtain the AR parameter estimates. The reflection coefficient estimates are obtained by minimizing estimates of the prediction error power for different order predictors in a recursive manner. Specifically, based

Connector Pane



Front Panel



Block Diagram

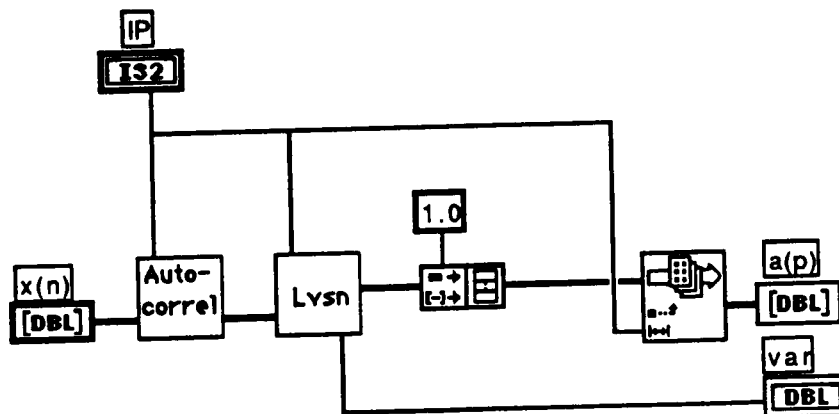


Figure 2.1 Connector Pane, Front Panel, and Block Diagram of Yule-Walker VI

on the Levinson algorithm [9], if estimates of the reflection coefficients $\{k_1, k_2, \dots, k_p\}$ are available, the AR parameters may be estimated as follows:

$$\begin{aligned} r_{xx}[0] &= \frac{1}{N} \sum_{n=1}^N |x[n]|^2 \\ a[1] &= \hat{k}_1 \\ \rho_1 &= (1 - |a_1[1]|^2) r_{xx}[0] . \end{aligned}$$

For $n=2, 3, \dots, p$,

$$a_p[n] = \begin{cases} a_{p-1}[n] + k_p a_{p-1}^*[p-n] & \text{for } n = 0, 1, \dots, p-1 \\ k_p & \text{for } n = p \end{cases} \quad (2.34)$$

and

$$\rho_p = (1 - |a_p[p]|^2) \rho_{p-1} . \quad (2.35)$$

The estimates of the AR filter parameters are $\{a_p[1], a_p[2], \dots, a_p[p]\}$ and the white noise variance estimates is ρ_p . It remains only to obtain estimates of the reflection coefficients. In deriving the p th reflection coefficient estimates, Burg assumed that the $(p-1)$ st order prediction error filter coefficients had already been estimated as $\{a_{p-1}[1], a_{p-1}[2], \dots, a_{p-1}[p-1]\}$, having been obtained by minimizing the $(p-1)$ st order prediction error power. Using the Levinson recursion, the coefficients of the k th order prediction error filter depend only on k_p according to (2.34), and hence, the k th order prediction error power estimate also depends only on k_p . Burg proposed to estimate k_p by minimizing the *average* of the estimates of the forward and backward prediction error powers. This

approach is a *constrained minimization* of ρ . The constrained or recursive minimization will not always produce a global minimum. Hence, to obtain the estimate of k_p , one must minimize

$$\rho_p = \frac{1}{2}(\rho_p^f + \rho_p^b) \quad (2.36)$$

where

$$\rho_p^f = \frac{1}{N} \sum_{n=p+1}^N \left| x[n] + \sum_{m=1}^p a_p[m] x[n-m] \right|^2 \quad (2.37)$$

$$\rho_p^b = \frac{1}{N} \sum_{n=p+1}^N \left| x[n-p] + \sum_{m=1}^p a_p^*[m] x[n+m-p] \right|^2 \quad (2.38)$$

ρ_p^f and ρ_p^b are functions only of k_p since the $(p-1)$ st order prediction coefficients are assumed to have already been estimated by minimizing ρ_{p-1} . Defining the forward and backward prediction error [9] as

$$e_p^f[n] = x[n] + \sum_{m=1}^p a_p[m] x[n-m] \quad (2.39)$$

$$e_p^b[n] = x[n-p] + \sum_{m=1}^p a_p^*[m] x[n+m-p], \quad (2.40)$$

substitution of Eq. (2.34) into these two definitions yields the recursive relationships

$$\begin{aligned}
e_p^f[n] &= e_{p-1}^f[n] + k_p e_{p-1}^b[n-1] \\
e_p^b[n] &= e_{p-1}^b[n-1] + k_p^* e_p^f[n]
\end{aligned} \tag{2.41}$$

where

$$e_0^f[n] = e_0^b[n] = x[n] \quad .$$

When these relations are substituted into (2.37) and (2.38) and then into (2.36), the average estimated prediction error power becomes

$$\rho_p^b = \frac{1}{2} \left[\frac{1}{N} \sum_{n=p+1}^N |e_p^f[n]|^2 + \frac{1}{N} \sum_{n=p+1}^N |e_p^b[n]|^2 \right] . \tag{2.42}$$

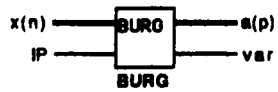
Differentiating ρ_p^b with respect to the real and imaginary parts of k_p by using the complex gradient and setting the result equal to zero and solving for k_p , yields

$$\hat{k}_p = \frac{-2 \sum_{n=p+1}^N e_{p-1}^f[n] e_{p-1}^{b*}[n-1]}{\sum_{n=p+1}^N |e_{p-1}^f[n]|^2 + \sum_{n=p+1}^N |e_{p-1}^b[n-1]|^2} \tag{2.43}$$

which is the Burg method for estimating of the p th coefficient. The estimates are given as $\{a_p[1], a_p[2], \dots, a_p[p], \rho_p\}$.

Figure 2.2 shows the connector pane, front panel, and block diagram of the VI, referred to as Burg, which was created to use the

Connector Pane



Front Panel



Block Diagram

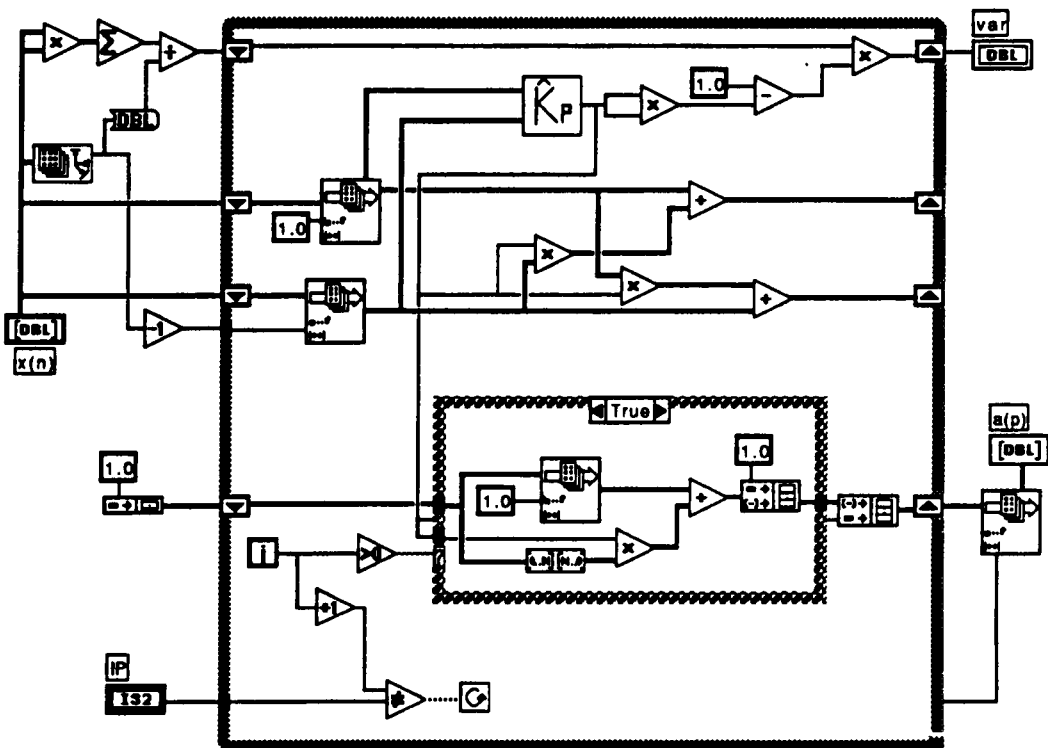


Figure 2.2 Connector Pane, Front Panel, and Block Diagram of Burg VI

Burg algorithm in order to find the MV parameters. Comparing these two algorithms, it is clear that the noise variance, ρ_ω , is much smaller in the Burg algorithm than the Yule-Walker algorithm, which explains why a better PSD resolution is apparent when using the Burg algorithm.

2.5 Model Order Selection

The selection of the model order in AR or MV spectral estimation is a critical one. Many techniques have been proposed to estimate model order. Nearly all model order estimators are based on the estimated prediction error power. The estimated prediction error power is guaranteed to decrease or stay the same as the model order increases for all the AR parameter estimation methods. One cannot simply monitor the decrease in power as a means of determining model order but must also account for the increase in variance of a spectral estimate based on an increasing number of parameters. Two methods proposed by Akaike [14] adhere to this philosophy. The first one, termed the *final prediction error* (FPE), estimates the model order as the value that minimized [15]

$$\text{FPE}(p) = \frac{N+p}{N-p} \rho_p, \quad (2.44)$$

where ρ_p , as described in the previous section, is the estimate of white noise variance (prediction error power) for the p th order AR model. It is seen that whereas ρ_p decreases with p , the term $(N+p)/(N-p)$ increases with p . The FEP is an estimate of the

prediction error power when the prediction coefficients must be estimated from data. The term $(N+p)/(N-p)$ accounts for the increase in the variance of the prediction error power estimator due to the inaccuracies in the prediction coefficient estimates. A second criterion, which appears to be in more general usage, is the *Akaike information criterion* (AIC). It is defined as [14]

$$\text{AIC}(p) = N \ln \rho_p + 2p . \quad (2.45)$$

As before, the order selected is the one that minimizes the AIC. This method is not limited to AR model order determination but may be used more generally for choosing a model order among computing models. Kashyab [16] has pointed out that the AIC is *not* a consistent estimator. This tendency to overestimate the true model order has allowed researchers to propose methods that replace the $2p$ factor in (2.45) by one that increases faster with N , typically $p \ln N$ [17].

The performance of the AIC and FEP is similar. For larger data records ($N \rightarrow \infty$), the two estimators will yield identical model order estimates [18].

2.6 Implementation of the MV Spectral Estimator

The most efficient approach to implement the MV spectral estimator uses the following sequence of computations:

- Compute the autocorrelation estimates from the ac data.

- Solve the Yule-Walker equations using the Levinson algorithm.
- Correlate the parameters to yield the $\Psi_{MV}[k]$ coefficients; make use of Eq. (2.28).
- Solve Eq. (2.27) to find the MV spectral estimator.

However, better results can be obtained by replacing the first two steps above with the following:

- Estimate the coefficients with the Burg algorithm.

The flowchart of the MV spectral estimator is shown in Figure 2.3.

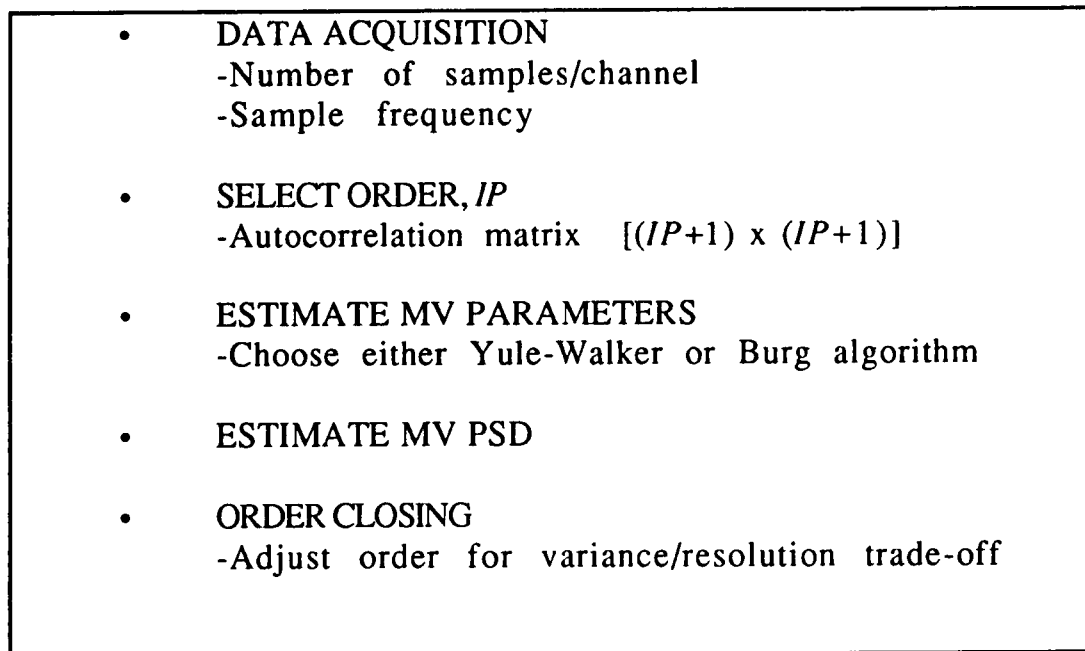


Figure 2.3 Flowchart of the Minimum Variance Estimator

Improved spectral resolution can be obtained by using eigenanalysis methods which will be discussed in the following chapter.

CHAPTER III

EIGENANALYSIS SPECTRAL ESTIMATION

3.1 Introduction

A class of spectral estimation procedures based on an eigenanalysis of the spatial and autocorrelation matrix has been developed in the past decade. The single most important property of this class is that it produces unbiased frequency estimates with infinite resolution, regardless of the signal-to-noise ratio. This property is not shared by the older methods. For example, the resolution of the classical and parametric methods degenerates with decreasing SNR. Some support for this claim has been demonstrated in [9]. Singular value decomposition (SVD) will be used here to provide more accurate estimates of the frequency for a sinusoid process in white noise. The basis for the improved performance of the eigenanalysis techniques is presented in this chapter. Key to the performance is the division of the information in the autocorrelation matrix or the data matrix into two vector subspaces; one a *signal* subspace and the other a *noise* subspace.

Functions of the vectors in either the signal or noise subspaces can be used to create frequency estimators that, when plotted, show sharp peaks at the frequency locations of sinusoids or other narrowband spectral components. Included in this class of eigenanalysis spectral estimators are the multiple signal classification (MUSIC) and the eigenvector (EV) algorithms.

3.2 Properties of the Autocorrelation Matrix for Sinusoids

Consider the wide-sense stationary process consisting of m complex sinusoids in additive white Gaussian noise,

$$x[n] = \sum_{i=1}^m A_i \exp(j2\pi f_i n + \phi_i) + z[n], \quad (3.1)$$

for $n = 0, 1, \dots, N-1$. The variable $z[n]$ is zero mean white Gaussian noise of variance ρ_ω . Assuming the phase to be an independent random variable uniformly distributed on $[0, 2\pi]$, then the ACS for the process given by (3.1) is easily shown to be [9]

$$r_{xx}[k] = \sum_{i=1}^m A_i^2 \exp(j2\pi f_i k) + \rho_\omega \delta[k]. \quad (3.2)$$

Let $P_i = A_i^2$ denote the power of the i th sinusoid so that the $(p+1) \times (p+1)$ autocorrelation matrix for $p > m$ is

$$\begin{aligned} \mathbf{R}_p &= \sum_{i=1}^m P_i \mathbf{s}_i \mathbf{s}_i^H + \rho_\omega \mathbf{I} \\ &= \mathbf{S}_p + \mathbf{W}_p \end{aligned} \quad (3.3)$$

where $\mathbf{I}$ is a $(p+1) \times (p+1)$ identity matrix and $\mathbf{s}_i$ is a signal vector of dimension $p+1$ carrying the frequency information of the i th sinusoid

$$\mathbf{s}_i = \begin{bmatrix} 1 \\ \exp(j2\pi f_i T) \\ \cdot \\ \cdot \\ \cdot \\ \exp(j2\pi f_i pT) \end{bmatrix}. \quad (3.4)$$

$\mathbf{R}_p$ is seen to be the sum of a signal autocorrelation matrix $\mathbf{S}_p$ and a noise autocorrelation matrix $\mathbf{W}_p$. $\mathbf{S}_p$ and $\mathbf{W}_p$ are $(p+1) \times (p+1)$ matrices which can be written as

$$\mathbf{S}_p = \sum_{i=1}^m P_i \mathbf{s}_i \mathbf{s}_i^H \quad (3.5)$$

$$\mathbf{W}_p = \rho_\omega \mathbf{I}. \quad (3.6)$$

$\mathbf{S}_p$ is not full rank since it is of dimension $(p+1) \times (p+1)$, and the rank is only m . The total autocorrelation matrix $\mathbf{R}_p$ is, however, full rank due to the inclusion of the $\rho_\omega \mathbf{I}$ term which has full rank $p+1$. The frequency information is contained in the signal vectors, the $\mathbf{s}_i$'s. If one is given $\mathbf{R}_p$ or can estimate it, then it would be desirable to be able to effect the decomposition of Eq. (3.3). The frequency estimates could then be obtained from the signal vectors. This does not appear to be possible in practice, although a related decomposition based on the eigenvectors $\mathbf{V}_i$ and eigenvalues λ_i of $\mathbf{S}_p$ is feasible.

The signal matrix will have the eigendecomposition

$$S_p = \sum_{i=1}^{p+1} \lambda_i \mathbf{v}_i \mathbf{v}_i^H, \quad (3.7)$$

where the eigenvalues have been ordered in decreasing value $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_{p+1}$, and the eigenvectors are orthonormal $\mathbf{v}_i^H \mathbf{v}_j = \delta[i-j]$. Noble and Daniell [19] have shown that a matrix of dimension $p+1$ with rank $m < p+1$ will have $p-m+1$ zero eigenvalues. Thus, Eq. (3.7) can be written as

$$S_p = \sum_{i=1}^m \lambda_i \mathbf{v}_i \mathbf{v}_i^H. \quad (3.8)$$

The eigenvectors $\mathbf{v}_1, \dots, \mathbf{v}_m$, known as the principle eigenvectors, span the same signal subspace as the signal vectors $\mathbf{s}_1, \dots, \mathbf{s}_m$. This means that any principal eigenvector must be expressible as a linear combination of the signal vectors

$$\mathbf{v}_i = \sum_{k=1}^m \alpha_{ik} \mathbf{s}_k, \quad (3.9)$$

for $1 \leq i \leq m$. To be an eigenvector of S_p , then

$$S_p \mathbf{v}_i = \lambda_i \mathbf{v}_i. \quad (3.10)$$

Substitution of Eq. (3.5) into Eq. (3.10) yields

$$\sum_{k=1}^m P_k \mathbf{s}_k \mathbf{s}_k^H \mathbf{v}_i = \lambda_i \mathbf{v}_i \quad (3.11)$$

or

$$\mathbf{v}_i = \sum_{k=1}^m \left(\frac{P_k}{\lambda_i} \mathbf{s}_k^H \mathbf{v}_i \right) \mathbf{s}_k, \quad (3.12)$$

for $1 \leq i \leq m$, which means

$$\alpha_{ik} = \frac{P_k}{\lambda_i} \mathbf{s}_k^H \mathbf{v}_i. \quad (3.13)$$

The identity matrix can be decomposed as

$$\mathbf{I} = \sum_{i=1}^{p+1} \mathbf{v}_i \mathbf{v}_i^H. \quad (3.14)$$

Substituting Eqs. (3.8) and (3.14) into Eq. (3.3) yields the eigendecomposition of the autocorrelation matrix

$$\begin{aligned} \mathbf{R}_p &= \sum_{i=1}^m \lambda_i \mathbf{v}_i \mathbf{v}_i^H + \rho_\omega \sum_{i=1}^{p+1} \mathbf{v}_i \mathbf{v}_i^H \\ &= \sum_{i=1}^m (\lambda_i + \rho_\omega) \mathbf{v}_i \mathbf{v}_i^H + \sum_{i=m+1}^{p+1} \rho_\omega \mathbf{v}_i \mathbf{v}_i^H. \end{aligned} \quad (3.15)$$

Therefore, the principal eigenvectors $\mathbf{v}_1, \dots, \mathbf{v}_m$ span the signal subspace of both $\mathbf{R}_p$ and $\mathbf{S}_p$, with eigenvalues of $\lambda_1 + \rho_\omega, \dots, \lambda_m + \rho_\omega$, and the nonprincipal eigenvectors $\mathbf{v}_{m+1}, \dots, \mathbf{v}_{p+1}$ span the noise subspace of $\mathbf{R}_p$, all with the identical eigenvalue ρ_ω . The eigenvalues of the principle eigenvectors are composed of powers of both signal and noise, and as a result, white noise does contribute to

the eigenvalue weighting of the noise-free signal subspace eigenvectors.

The eigendecomposition (3.15) of the autocorrelation matrix can be exploited in a way to generate improved spectral estimators, or more correctly, improved frequency estimators. Retaining only the information in the signal subspace eigenvectors, that is, forming a lower-rank approximation to R_p , effectively enhances the SNR because of the omission of the contribution of power in the noise subspace components [9]. This is the basis of *signal subspace frequency estimators*, discussed in the next section. Noting that the eigenvectors are orthogonal and that the principal eigenvectors span the same subspace of the signal vectors, then the signal vectors are orthogonal to all the vectors in the noise subspace, including any linear combination

$$\mathbf{s}_i^H \left(\sum_{k=m+1}^{p+1} \beta_k \mathbf{v}_k \right) = 0, \quad (3.16)$$

for $1 \leq i \leq m$. This property forms the basis of the *noise subspace frequency estimators*, discussed in Section 3.4.

3.3 Signal Subspace Frequency Estimation

It has been shown that retention of the signal subspace, or principal eigenvectors, effectively improves the SNR for processes consisting of sinusoids in white noise by eliminating most of the noise contribution to an autocorrelation matrix. A class of signal subspace frequency estimators may be formed from any of the

parametric or non-parametric PSD estimators simply by replacing the autocorrelation matrix by its reduced rank approximation in terms of the principle eigenvectors.

Recalling from Chapter II, the MV method replies on the estimated autocorrelation matrix R_p for the definition of the spectral estimate

$$P_{MV} = \frac{T}{e^H(f) R_p^{-1} e(f)}, \quad (3.17)$$

where the vector $e(f)$ and matrix R_p have the same definition as Eqs. (2.2) and (2.11). If the orthonormal eigendecomposition of R_p is

$$R_p = \sum_{k=1}^p \lambda_k \mathbf{v}_k \mathbf{v}_k^H, \quad (3.18)$$

where the eigenvalues are ranked in decreasing magnitude $\lambda_1 \geq \lambda_2 \geq \dots, \geq \lambda_p$, and there are estimated to be m principal components ($m < p$), then the reduced rank principle eigenvector approximation to R_p is

$$\hat{R}_p = \sum_{k=1}^m \lambda_k \mathbf{v}_k \mathbf{v}_k^H, \quad (3.19)$$

and the inverse of $\hat{R}_p$ is

$$\hat{R}_p^{-1} = \sum_{k=1}^m \frac{1}{\lambda_k} \mathbf{v}_k \mathbf{v}_k^H. \quad (3.20)$$

The matrix R_p^{-1} can be substituted into Eq. (3.17) to create a spectral estimator with reduced noise contribution due to the omission of the noise subspace eigenvectors. Johnson [20] promoted the signal subspace improvements of the MV method.

3.4 Noise Subspace Frequency Estimation

The frequency estimation methods to be described in this section all rely on the property that the noise subspace eigenvectors of a Toeplitz autocorrelation matrix are orthogonal to the signal vectors. Frequency estimator functions can be devised, based on noise subspace principles, to yield spectrum-like plots with sharp peaks. The $p - m$ noise subspace eigenvectors $\mathbf{v}_{m+1}, \dots, \mathbf{v}_{p+1}$ of an autocorrelation matrix or modified covariance data matrix of p total eigenvectors and m principle eigenvectors will theoretically be orthogonal to the signal vectors, so that linear combinations with arbitrary weighting β_k such as

$$\sum_{k=m+1}^p \beta_k |\mathbf{e}^H(f) \mathbf{v}_k|^2 = \mathbf{e}^H(f) \left(\sum_{k=m+1}^p \beta_k \mathbf{v}_k \mathbf{v}_k^H \right) \mathbf{e}(f), \quad (3.21)$$

where $\mathbf{e}(f)$ will be zero whenever $\mathbf{e}(f_i) = \mathbf{s}_i$, one of the sinusoidal signal vectors. This means that the frequency estimator function

$$\frac{1}{\sum_{k=m+1}^p \beta_k |\mathbf{e}^H(f) \mathbf{v}_k|^2} \quad (3.22)$$

will theoretically have infinite value whenever evaluated at $f = f_i$, one of the sinusoidal signal frequencies. However, estimation errors will cause the function of Eq. (3.22) to be finite, but with very sharp peaks at the sinusoidal frequencies.

Two specific frequency estimators have been proposed in the literature. Choosing $\beta_k = 1$ for all k in Eq. (3.22) yields the MUSIC algorithm [21] frequency estimator

$$P_{\text{MUSIC}}(f) = \frac{1}{\mathbf{e}^H(f) \left(\sum_{k=m+1}^p \mathbf{v}_k \mathbf{v}_k^H \right) \mathbf{e}(f)}, \quad (3.23)$$

based strictly on the noise subspace eigenvectors with uniform weighting. Letting $\beta_k = 1/\lambda_k$ yields the EV algorithm [22] frequency estimator

$$P_{\text{EV}}(f) = \frac{1}{\mathbf{e}^H(f) \left(\sum_{k=m+1}^p \frac{1}{\lambda_k} \mathbf{v}_k \mathbf{v}_k^H \right) \mathbf{e}(f)}, \quad (3.24)$$

which weights each noise subspace eigenvector by the inverse of its associated eigenvalue.

Johnson and DeGraaf [22] have shown that the EV method and the MUSIC method produce quite similar results. They differ in at least two respects, however. The first is that the EV method produces fewer spurious peaks than MUSIC for a given choice of order p due to the use of inverse eigenvalue weighting. Second, the

EV method tends to shape the noise spectrum better than MUSIC. The ultimate performance of any of these frequency estimators will be a function of how well the selection of signal and noise subspace is made. As a result, the order selection which determines how the signal and noise eigenvalues are being separated from the noise-only eigenvalues, is the most important aspect of how well these methods work.

As stated earlier, singular value decomposition will be used to determine the eigenvectors and the eigenvalues, which are discussed in the following section.

3.5 Singular Value Decomposition

Singular value decomposition is a powerful computational tool for analyzing matrices and problems involving matrices, which has applications in many fields. A brief discussion of the SVD will be presented here. For complete details of this algorithm, the reader is referred to [23].

Let A be a real $n \times p$ matrix with $n \geq p$. It is well known [23] that

$$A = U\Sigma V^H = \sum_{i=1}^p \sigma_i u_i v_i^H \quad (3.25)$$

where U is an $n \times n$ orthogonal matrix, V is an $p \times p$ orthogonal matrix, and Σ is $p \times n$ diagonal matrix with $\sigma_{ij} = 0$ if $i \neq j$ and $\sigma_{ii} = \sigma_i \geq 0$. The quantities σ_i are called the singular values of A , and the columns of U and V are called the left and right singular

vectors. The σ_i 's are non-negative real numbers arranged in decreasing order ($\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_k$), where k is the rank of matrix A . The decomposition (3.25) is called the *singular value decomposition*.

To compute the SVD of a given matrix A , one can use either the plane rotations method [24] or the QR method [25]. The algorithm used in this work first uses Householder transformations to reduce A to bidiagonal form, and then the QR algorithm to find the singular values of the bidiagonal matrix. The two phases properly combined produce the SVD of matrix A .

3.6 Input Parameters Selection

The simple idea of separating eigenvectors into signal and noise subspace based on an examination of either the eigenvalues of the autocorrelation matrix or the singular values of the data matrix does not work well in practice, especially for short data. But for long data, it is possible to use the eigenvalues of the autocorrelation matrix to indicate the number of sinusoidal signals. Since for m sinusoids the eigenvalues are $\{\lambda_1 + \rho_w, \lambda_2 + \rho_w, \dots, \lambda_m + \rho_w, \rho_w, \rho_w, \dots, \rho_w\}$, the number of sinusoids can be found as the number of the smallest, approximately equal, eigenvalues. For the best performance, the dimension for the autocorrelation matrix should be large so that the majority of the eigenvalues are equal to ρ_w . Doing so runs the risk of statistically unstable eigenvalue estimates. It is not known how best to choose the dimension of the autocorrelation matrix, since the statistics of the estimated eigenvalues are unknown.

The AIC order-selection criterion which was introduced in the previous chapter has been extended by [26] to handle the subspace separation problem. Assuming $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_p$ are the eigenvalues of the sample autocorrelation $\hat{\mathbf{R}}_p$ and assuming $m < p$, where m is the number of sinusoidal signals in the data, and N data samples, then

$$\text{AIC}[m] = (p - m) \ln \left(\frac{\frac{1}{p - m} \sum_{i=m+1}^p \lambda_i}{\prod_{i=m+1}^p \lambda_i^{-(p-m)}} \right) + m(2p - m). \quad (3.26)$$

The number signal components is determined by selecting the minimum value of $\text{AIC}[m]$.

3.7 Implementation of the Eigenanalysis Spectral Estimator

Figure 3.1 shows a flowchart of the processing steps needed to generate two types of eigenanalysis estimators: the EV and MUSIC algorithms. Both algorithms require as inputs the autocorrelation matrix order IP and an estimate of the number of signal components $NSIG$ which is less than IP .

Unlike other PSD estimation techniques in which the order of IP can be selected as it was shown in the previous chapter, no such technique exists to find the dimension of the autocorrelation matrix, IP , for eigenanalysis methods. The order of IP can only be found by the trial and error method, keeping in mind that the dimension for the autocorrelation matrix should be large enough so that the

majority of the eigenvalues are equal to the noise variance. If the IP is too large, it runs the risk of statistically unstable eigenvalues.

- DATA ACQUISITION
 - Number of samples/channel
 - Sample frequency
- SELECT INPUT PARAMETERS
 - Order of data matrix, IP
 - Number of signal components, $NSIG$
- COMPUTE INPUT DATA MATRIX
 - Use modified covariance matrix
- SVD CALCULATION
 - Calculate eigenvalues and eigenvectors of the data matrix
- COMPUTE EIGENANALYSIS ESTIMATOR
 - Make one of the following selections:
 - EV algorithm
 - MUSIC algorithm

Figure 3.1 Flowchart of Two Eigenanalysis Estimators

For this work, the IP was chosen to be 51, which offers the best result for detecting a peak in high frequencies.

Choosing the number of signal components $NSIG$ is a very important factor in this method. The procedure which was described in the previous section will not give the correct number of signal components. A probable explanation is that eigenanalysis methods work best for processes consisting of narrow-band signals in white noise [9]. Therefore, broad-band noise can easily be

detected by these methods. In this research, however, background noise is of a broad-band nature while the leak is broad-band as well. The attempt here, then, is to detect a leak from a process which is also broad-band. Consequently, the number of signal components, *NSIG*, found through trial and error, was chosen to be 41.

Since the autocorrelation sequence normally is not known, the properties of Section 3.2 are mostly of theoretical, rather than practical, interest. The concepts, however, can be extended to the modified covariance data matrices [9]. It is shown in [9] that the data matrices have eigendecomposition properties similar to the autocorrelation matrix. The principal eigenvectors of the data matrix predominantly span the signal subspace, and the singular values of these principal eigenvectors tend to be larger than the noise subspace singular values. Thus, the singular values determined by an SVD of the data matrix are the basis for the separation of the eigenvectors into mostly signal subspace and a mostly noise subspace.

The modified covariance data matrix can be written as

$$\begin{bmatrix} \mathbf{T}_p \\ \mathbf{T}_p^* \mathbf{J} \end{bmatrix}, \quad (3.27)$$

in which $\mathbf{T}_p$ is the order p data matrix of the covariance method of linear prediction

$$\mathbf{T}_p = \begin{bmatrix} x[p+1] & \dots & x[1] \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ x[N] & \dots & x[N-p] \end{bmatrix}, \quad (2.38)$$

where N is the size of input array and p is number of input matrix $\mathbf{T}_p$. The $\mathbf{J}$ matrix is the reflection matrix and is defined as

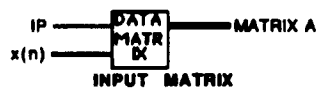
$$\mathbf{J} = \begin{bmatrix} 0 & \dots & 0 & 1 \\ 0 & \dots & 1 & 0 \\ \cdot & \cdot & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 1 & \dots & 0 & 0 \end{bmatrix}, \quad (2.39)$$

which has unity elements along the cross diagonal. The $\mathbf{J}$ matrix reverses, or exchanges, the order of the rows or columns of a matrix.

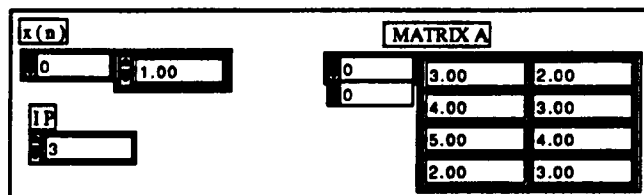
Figure 3.2 shows the connector pane, front panel, and block diagram of the data matrix VI. This VI finds the modified covariance matrix from an array of data. The input to this VI is an array of data $x(n)$ and the data matrix order, IP , and the output is the modified covariance matrix A .

The output of this VI is then passed to the icon labeled "SVD." This VI performs the singular value decomposition on the input matrix and finds eigenvectors and eigenvalues of the input matrix. The output of this VI is input to another icon called "PSD MUSIC."

Connector Pane



Front Panel



Block Diagram

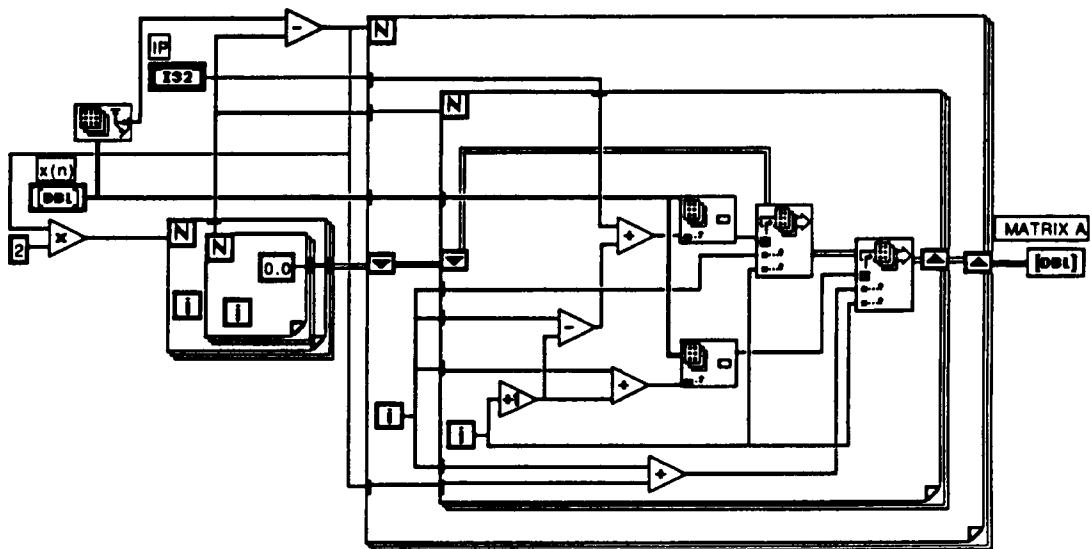


Figure 3.2 Connector Pane, Front Panel, and Block Diagram of Data Matrix VI

This VI finds the PSD estimation for the MUSIC method using Eq. (3.23).

Figure 3.3 shows the front panel and block diagram of the "MUSIC" VI created to find the solution to the Eq. (3.23) using SVD algorithms. All other VI's and subVI's are given in Appendix A.

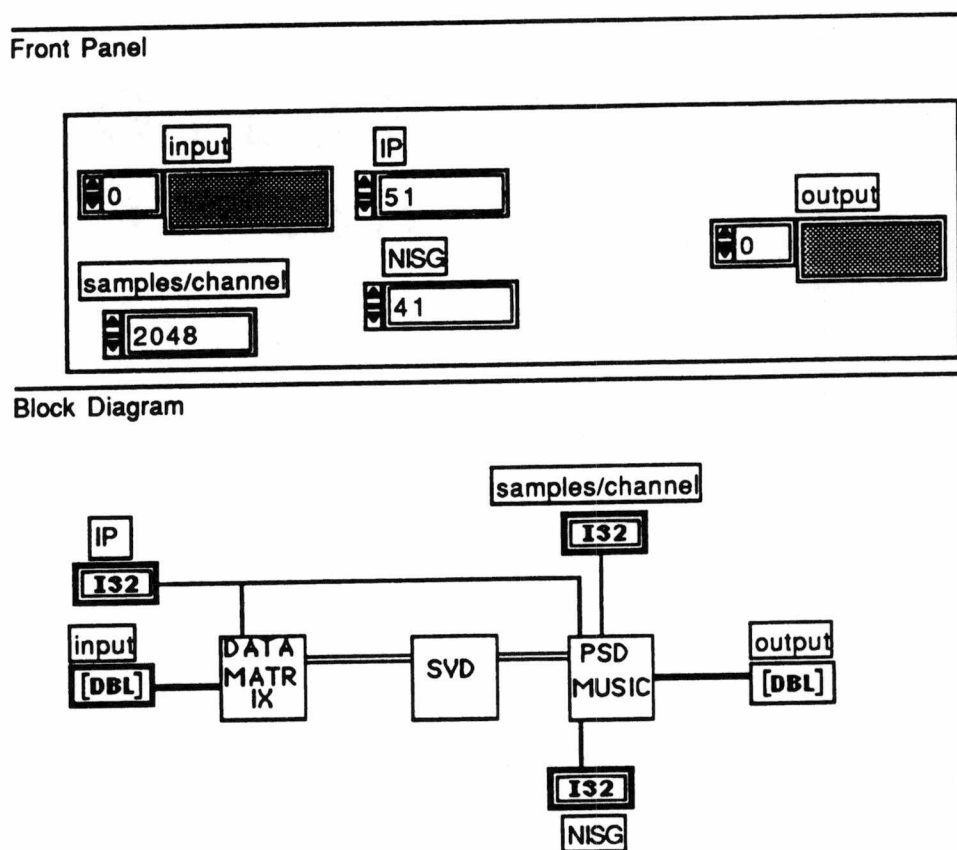


Figure 3.3 Front Panel and Block Diagram of "MUSIC" VI

Next, Chapter IV will compare and discuss the experimental results of various PSD techniques utilized in this research and other research.

CHAPTER IV

EXPERIMENTAL ANALYSIS

4.1 Introduction

The primary goal of this research was to investigate non-parametric methods of PSD estimation, to determine which methods produce the most reliable leak detection system, and also to compare the results of this work with parametric methods (AR, ARMA). The MV PSD method, which uses both Yule Walker and Burg algorithms to find the autocorrelation parameters, was employed. The two eigenanalysis methods used were MUSIC and EV, both utilizing the SVD algorithm to find the necessary eigenvectors and eigenvalues. Acoustic data which were used in this research were obtained from the TVA Kingston Fossil Fuel Power Plant.

This chapter discusses the software developed for this research which includes the noise background analysis, baseline average development, leak detection, and method comparison.

4.2 Noise Background Analysis

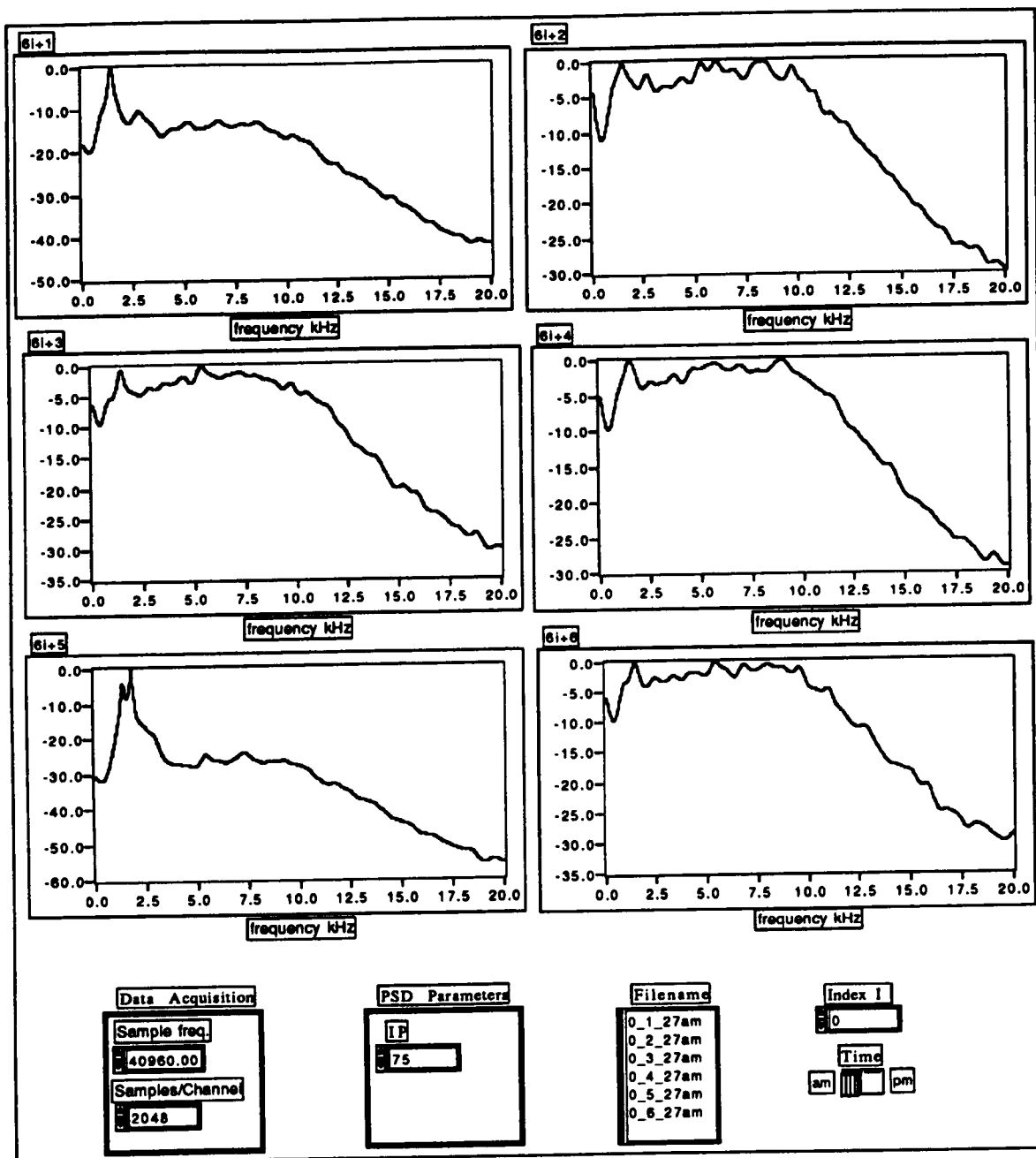
As mentioned in Chapter I, background noise is primarily caused by the combustion process in the boiler furnace, which includes combustion roar and combustion-driven oscillation. Its spectrum has peaks at low frequencies and is quite flat at high frequencies.

One program that has been written to observe the changes of the spectrum of background noise over the period of one day is called background noise PSD. "Twenty-four sensor signals have been acquired once each hour from one channel and saved to data files. These data files are given names related to the channel number and the time when the data are obtained, such as 0-1-27 a.m. The first number 0 means that the data are acquired from channel 0. The second number 1 and the third number 27 indicate the hour and minute when the data are acquired, respectively. The usage of a.m. is to indicate the time of data acquisition" [5]. These data have been retrieved to analyze the spectrum of background noise by the MV method. The order of the autocorrelation matrix for the MV process, IP , can be controlled from the front panel.

Figures 4.1(a) through 4.1(d) show the front panel of the program. In each figure there are six graphs to display the spectrum for data. The index, I , which takes values of 0 and 1, is used to construct file names. The graph with the name of $6I + k$ ($k = 1, 2, 3, 4, 5, 6$), shows the spectrum of the signal retrieved from the data file with the name in the k th element in the array of Filename.

It can be seen from Figures 4.1(a) through 4.1(d), retrieved from 24 different data files, that the spectrum of background noise is quite smooth at high frequencies and has peaks at low frequencies. The spectral levels at high frequencies do not change much at different times, so the curves are very similar at high frequencies. The block diagram of background noise PSD VI is shown in Figure 4.2.

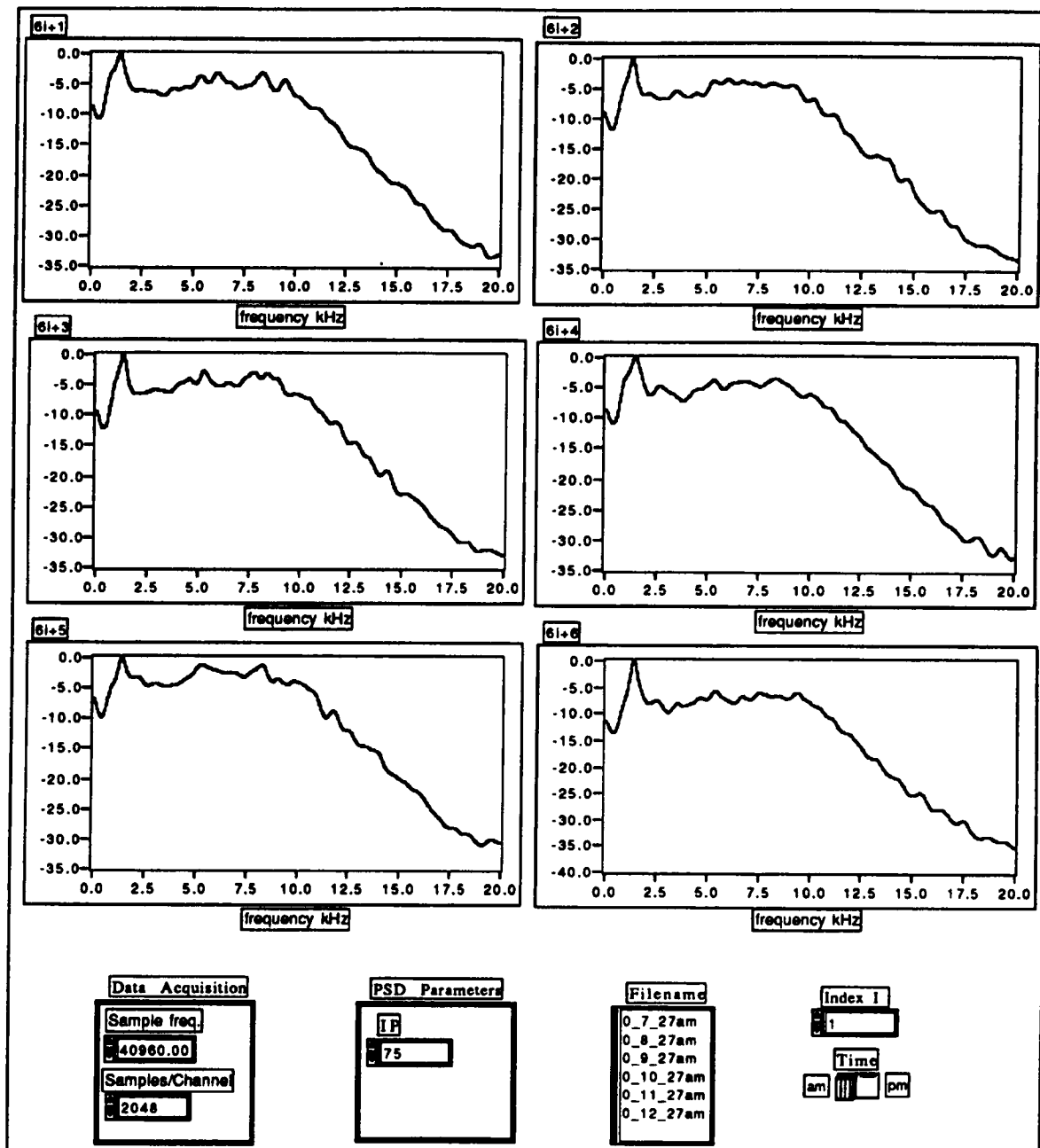
Front Panel



a

Figure 4.1 Front Panel of Background Noise PSD VI

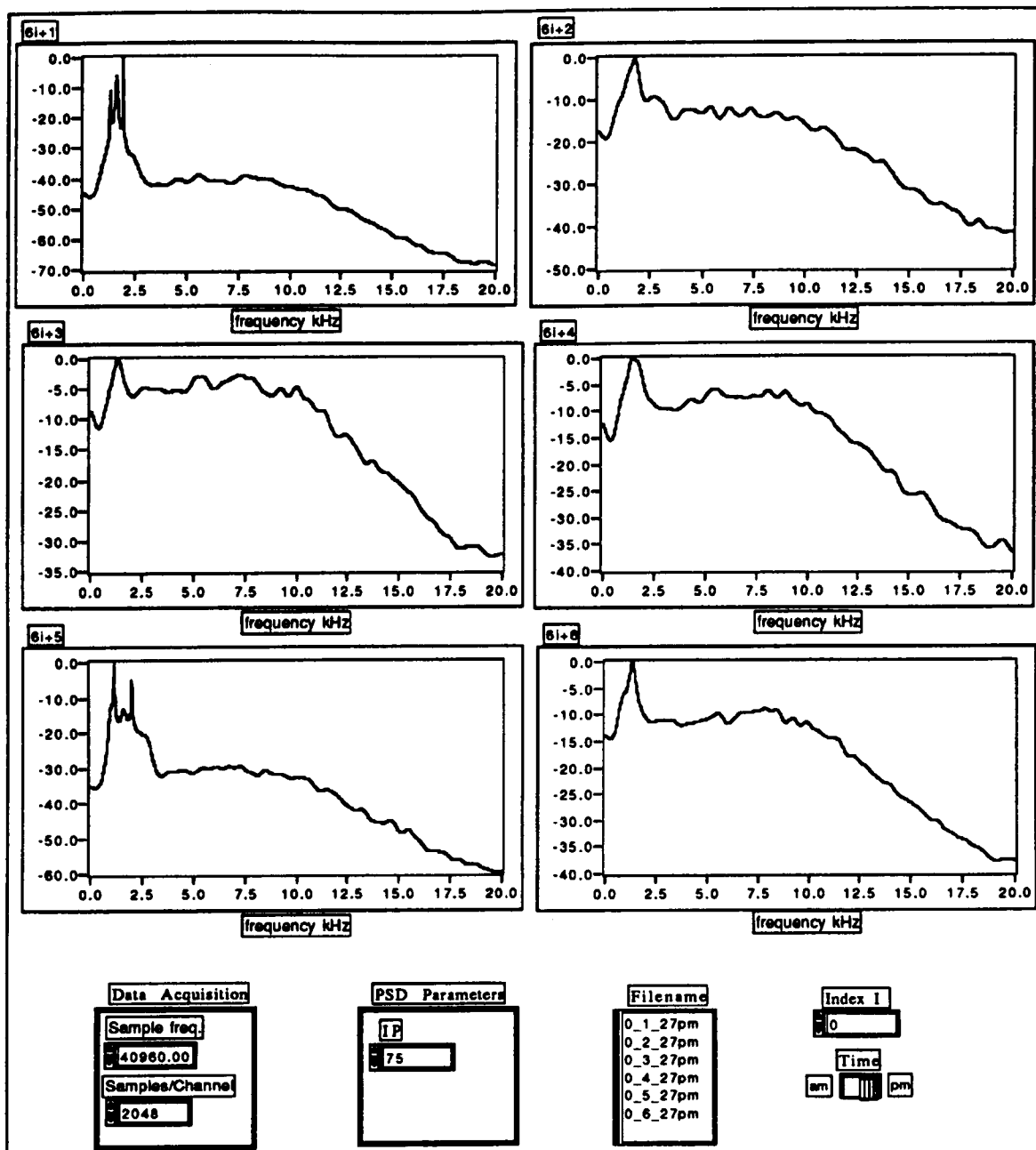
Front Panel



b

Figure 4.1 (Continued)

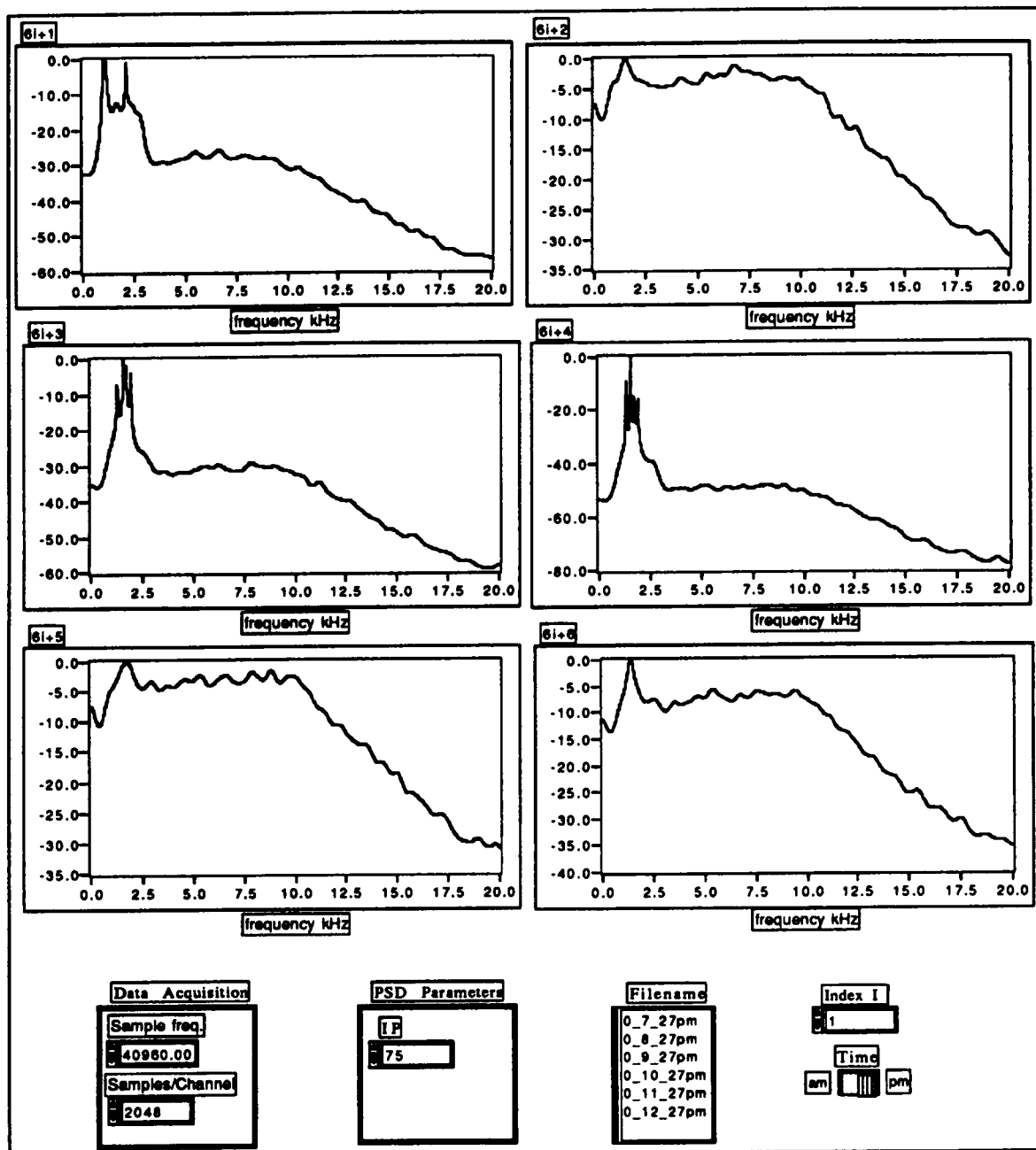
Front Panel



c

Figure 4.1 (Continued)

Front Panel



d

Figure 4.1 (Continued)

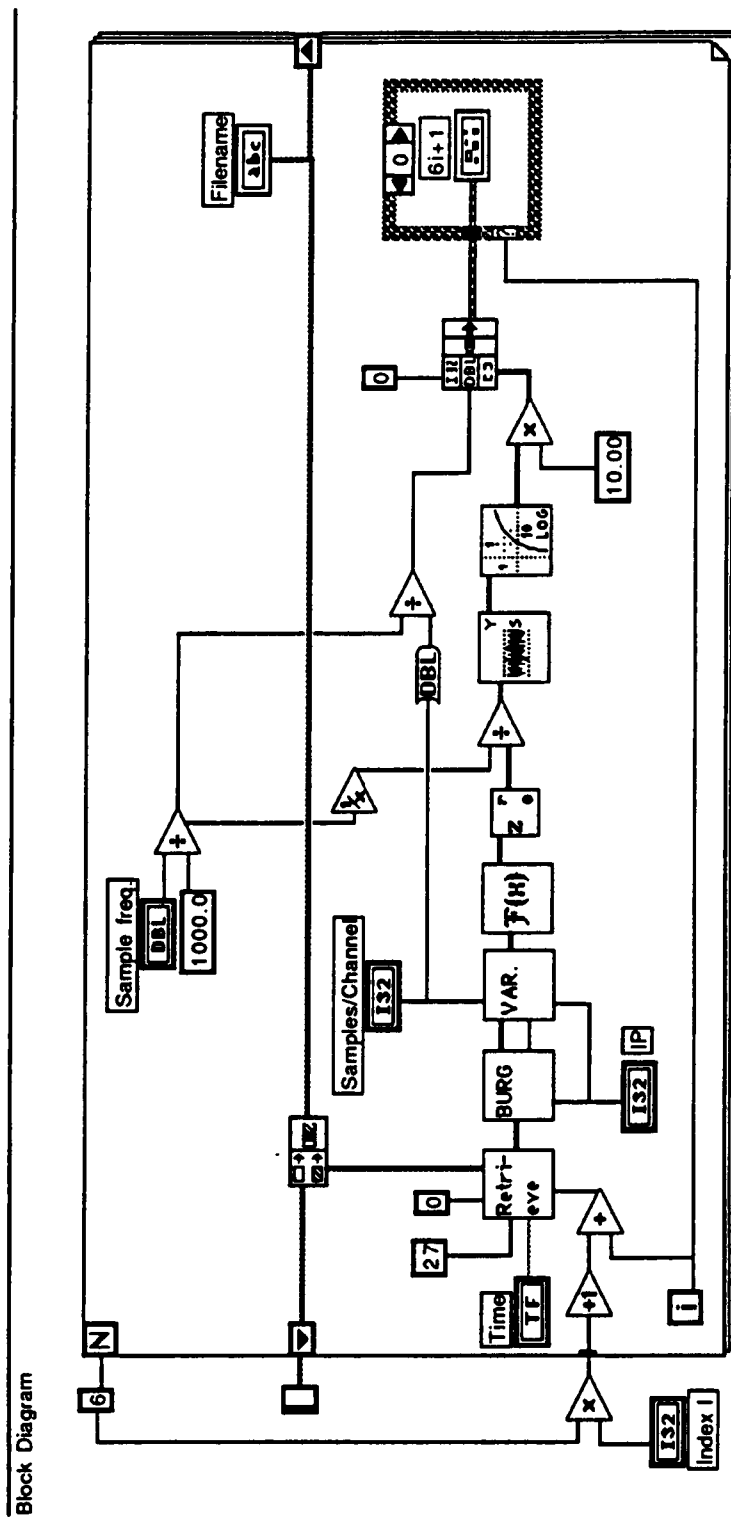


Figure 4.2 Block Diagram of Background Noise PSD VI

4.3 Baseline Average Development

It was shown in the previous section that background noise has a smooth spectrum and does not change much at high frequencies. Since most of the leaks occur at high frequencies [5], it is beneficial to compare a suspected leak in a channel to its background noise.

Figures 4.3 and 4.4 show the PSD of one file that contains only background noise and a second file that shows that a leak has occurred. The legends at the bottom of the graphs list the data file's name, the front panel controls for choosing the desired PSD method, and the PSD parameters. First impressions of the two plots indicate that a leak's spectral signature is generally elevated 15 to 20 decibel-Watts (dBW) between 1000 and 2000 Hz. This conclusion, however, does not take into account the variation that might normally occur in the PSD estimate from one day to the next. In other words, it may be that this channel normally undergoes 15 to 20 dBW swings in the frequency bands between 1000 and 2000 Hz, regardless of the leak condition.

Any meaningful analysis therefore, must account for normal fluctuations in the PSD estimate. Since each sensor channel's PSD has its own characteristic fluctuations, developing an average PSD estimate for a particular sensor would serve as a baseline PSD plot to overlay on subsequent PSD plots of sensor data from that channel. This baseline plot would indicate with a higher degree of

Front Panel

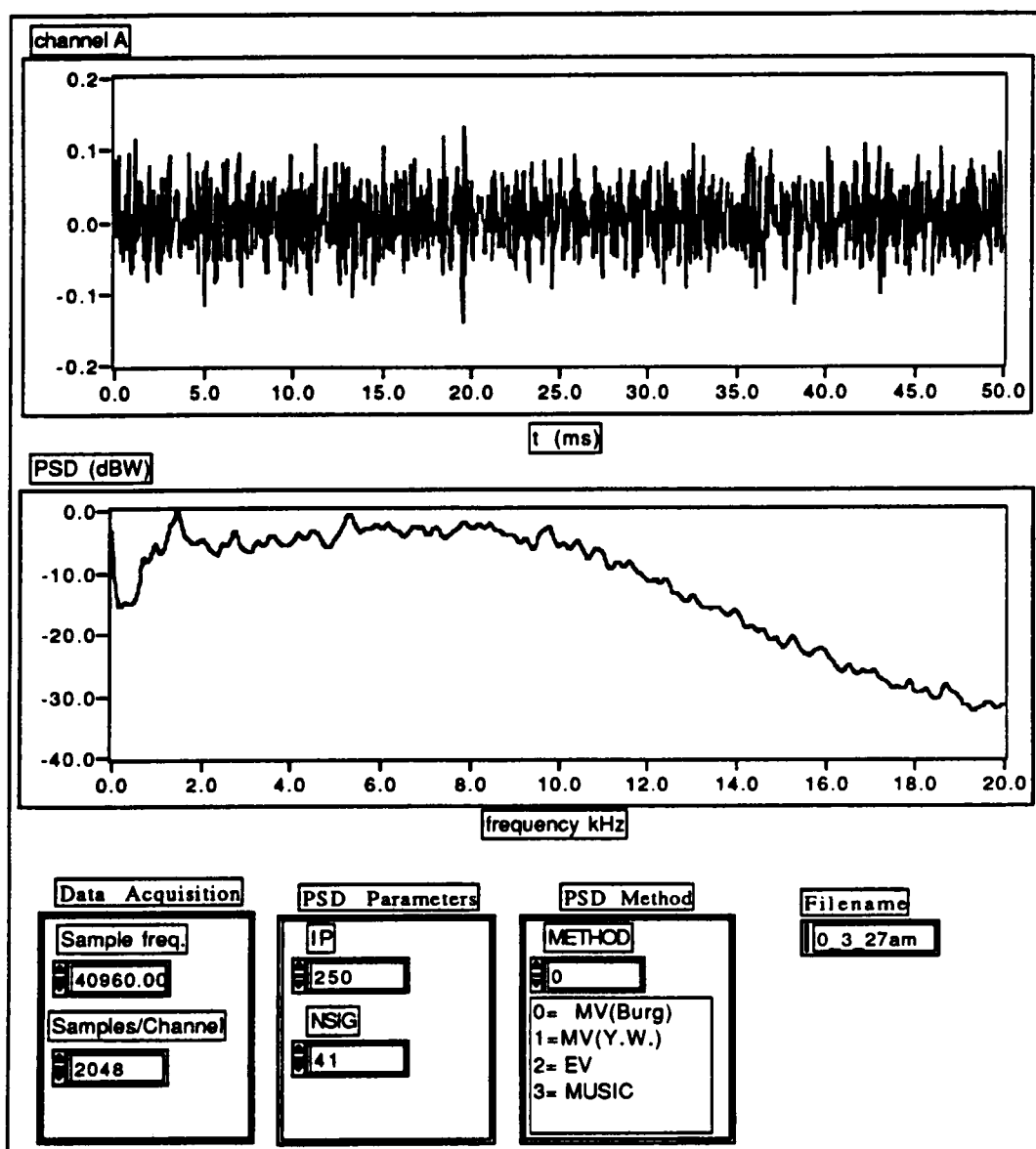


Figure 4.3 Non-Leak PSD Plot

Front Panel

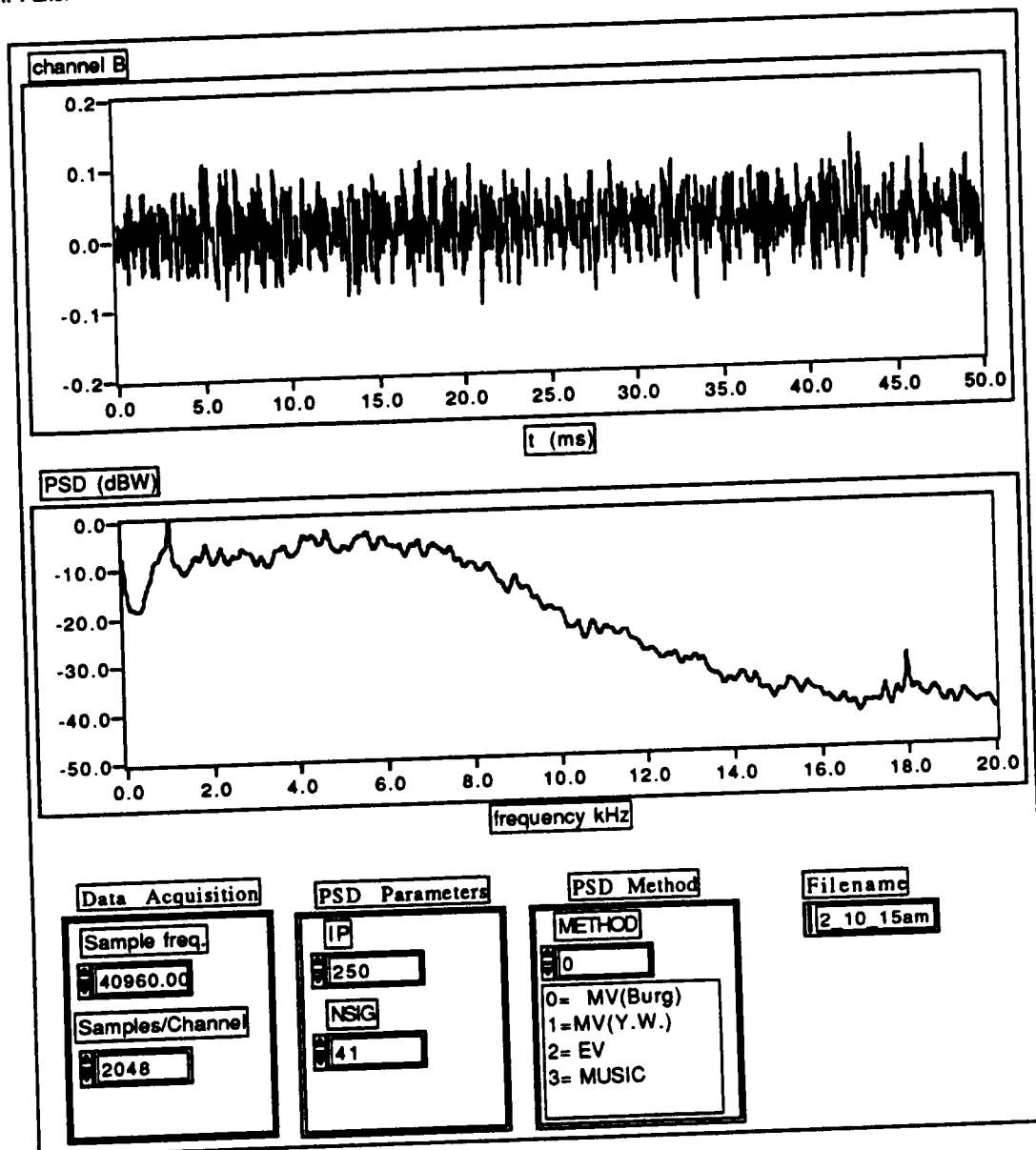


Figure 4.4 Leak PSD Plot

confidence whether suspected leak PSD estimates actually deviated from the norm in a frequency band.

The baseline average PSD estimate for each method and each sensor becomes the standard ingredient for the individual file or files comparison in this research. Figures 4.5 and 4.6 show the front panel and the block diagram of the "Base Average" VI created to compute average baselines. The values of average baselines can be saved for use in subsequent PSD plot comparisons. For each sensor channel being analyzed, baselines should be computed for each PSD method, since every PSD method will have its own spectral peculiarities given the same acoustic data. The bottom of Figure 4.5 shows the front panel control for choosing the desired PSD method and lists the data files used in computing the baseline.

The main structure in the block diagram of Figure 4.5 is the LabVIEW while loop. Shift registers along the sides of the while loop store the baseline array and the list of file names used. The VI labeled "retrieve" in the left of the loop allows the user to retrieve data files and pass them into the program. This VI returns the file name and the data. The data is passed to the right into the case structure that contains the selected PSD method, and then the computed PSD estimate is passed out to the right of the PSD case structure and into another case structure. When the loop execution count is zero (indicating the first loop execution), the "false" case is selected, and the PSD data is simply loaded into the shift register. On every subsequent loop execution, the "true" case is selected, and the PSD is averaged. Once all of the desired files have been processed, the while loop is terminated and the baseline array

Front Panel

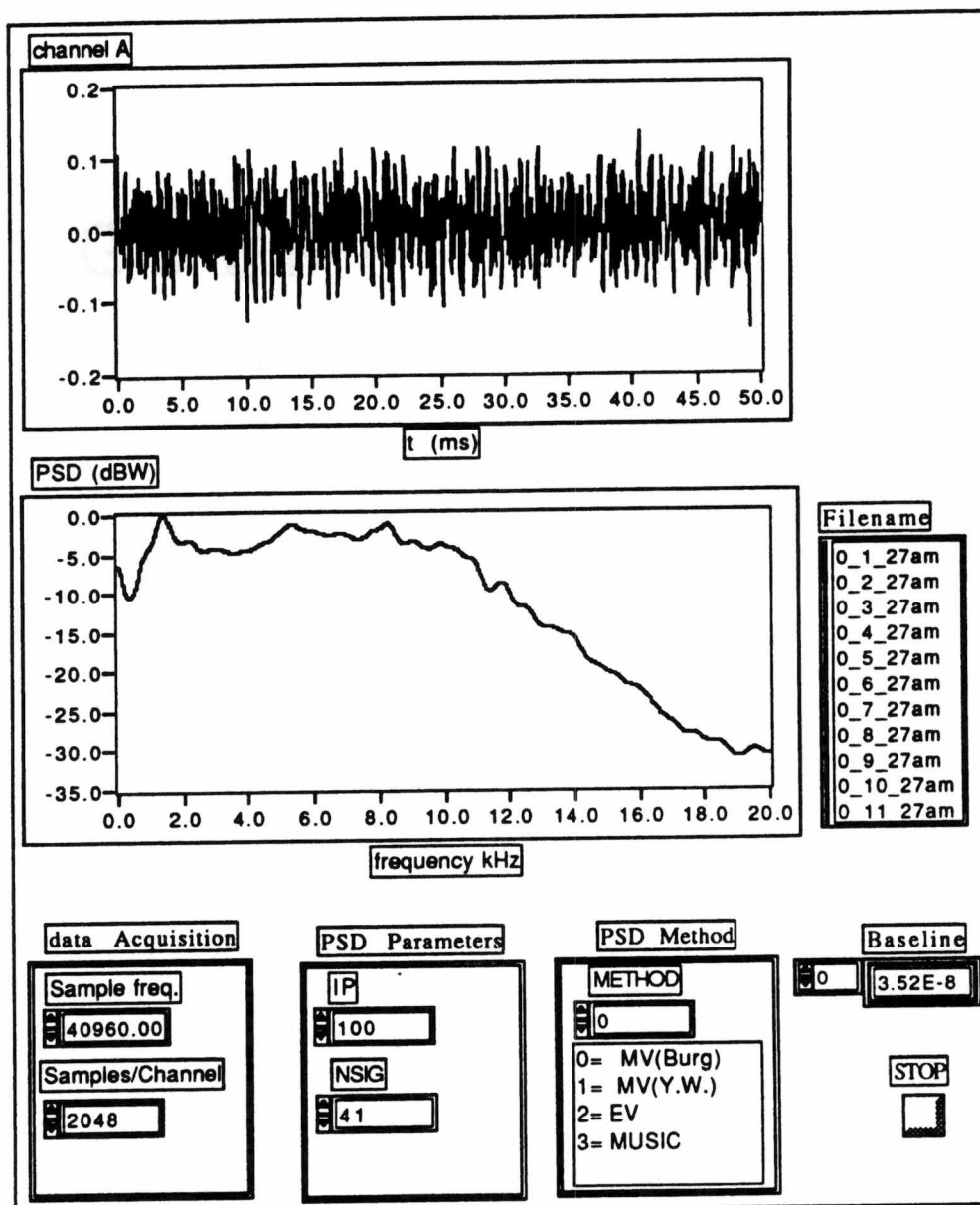


Figure 4.5 Front Panel of "Base Average" VI

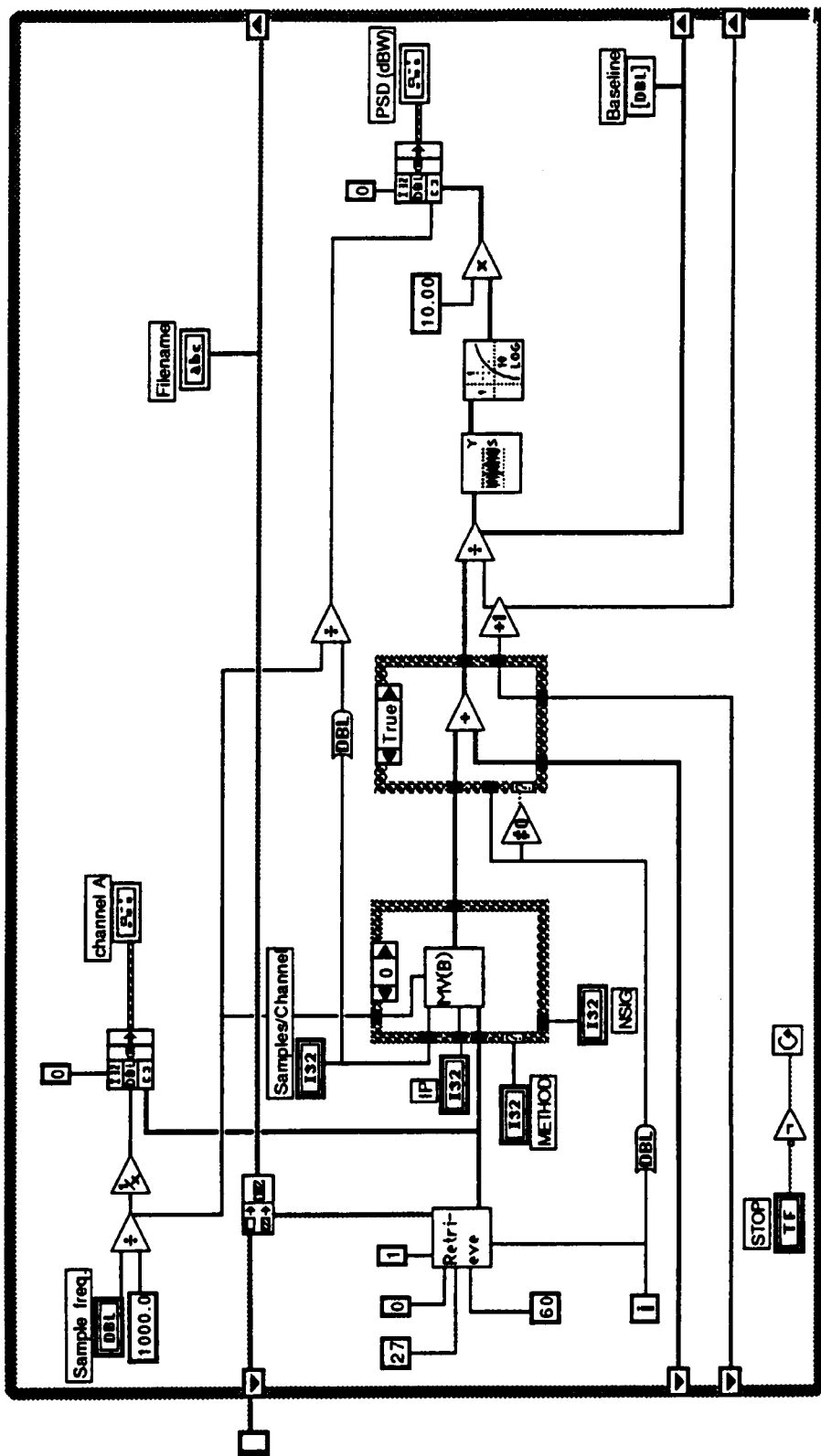


Figure 4.6 Block Diagram of "Base Average" VI

is saved on the right part of the figure.

Characterizing a sensor channel with a baseline only has value if the files used in computing them are carefully chosen. "Using data files recorded when the furnace is experiencing any brief, abnormal conditions will introduce unwanted bias in the baseline" [4]. Consequently special consideration should be taken for choosing data files for a baseline curve.

Another method for finding a baseline curve, which was used in [5] for spectral estimation, uses the Daniell periodogram. If the spectrum has been estimated for a sensor signal, the Daniell periodogram can be used to smooth the rapid fluctuation of the spectrum. The modified spectrum estimate at frequency f_i could be obtained by averaging the P points on either side of this frequency

$$\hat{P}_D(f_i) = \frac{1}{2P+1} \sum_{n=i-P}^{i+P} \hat{P}_{xx}(f_n), \quad (4.1)$$

where i is the index of the frequency points in the spectral curve, and it takes values from 0 to $k - 1$.

Figure 4.7 shows the connector pane, front panel, and block diagram of the "Base Curve" VI created to compute the baselines. Since most leaks occur at high frequencies (usually over 5 kHz), it is reasonable to assume that the base curve and the signal spectrum are the same at low frequencies (such as below 4 kHz). At high frequencies, the base curve is very smooth and follows the shape of background noise spectrum.

In this VI, the baseline is the same as the spectrum below--a frequency called "Start Freq." This frequency is given by an operator from the front panel. In order to make the baseline continuous near Start Freq., those frequencies that are k ($k < p$) points away from Start Freq. are modified by taking an average of $(2k+1)$ points with k points on each side [5].

As mentioned earlier in this section, it is best to compare the non-leak data from a sensor channel to the data from the same sensor channel when a leak has been suspected. Since non-leak data were unavailable for the sensor channel in which the leak had been suspected, only the second method was employed to find a baseline curve.

4.4 Leak Detection Development

The purpose of this research is to detect tube leaks in the fossil-fueled power plant using spectral analysis; that is, to detect a peak in the spectrum of sensor signal at high frequencies. Figure 4.8 shows the block diagram of "PSD Compare," a VI designed to compare the average spectrum of the data files with the baseline curve. If the maximum difference between the spectrum and its baseline curve exceeds a threshold set by an operator, an alarm is set to warn an operator that a leak might have occurred.

In the block diagram of Figure 4.8, at each step in the loop, the data are retrieved from data files for the specified channel. The data are passed to the right into the case structure that

Block Diagram

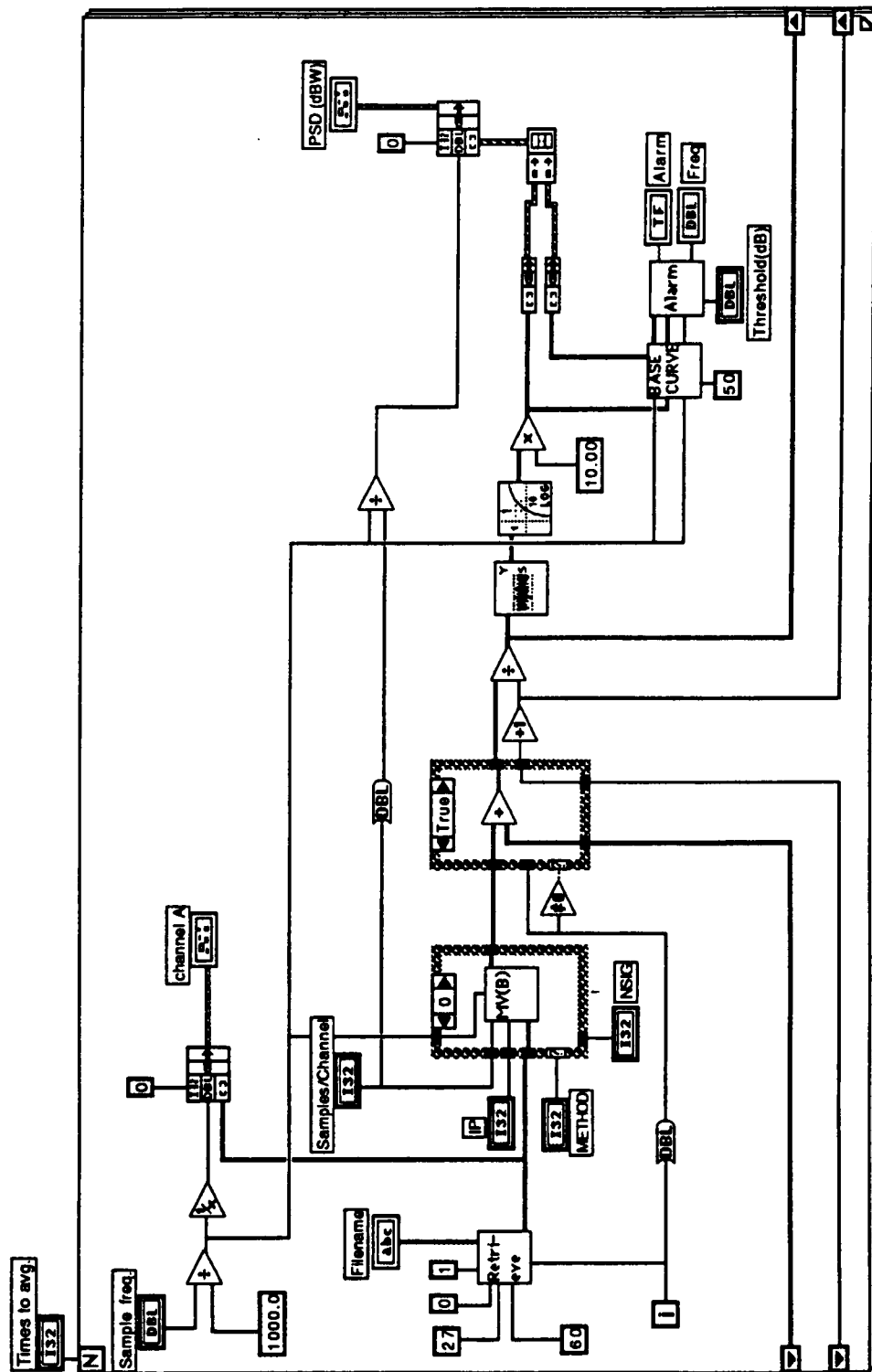


Figure 4.8 Block Diagram of "PSD Compare" VI

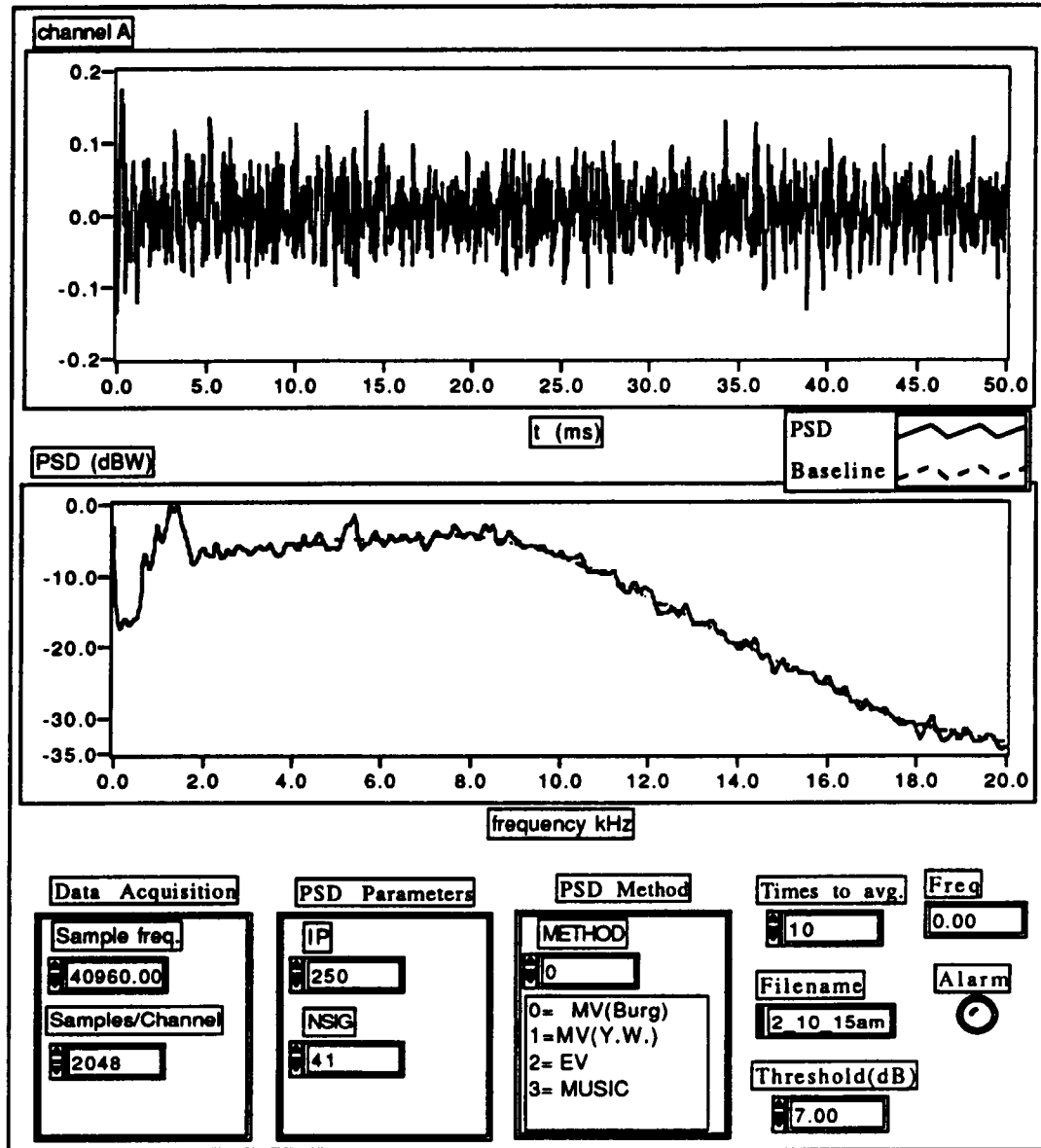
contains the selected PSD method. The computed PSD estimate is normalized, passed for display, and then passed to the "Base-Curve" VI to compute the baseline array for the average spectrum. Both baseline and spectrum arrays are then passed into "alarm" VI. If the maximum difference between the average spectrum and its baseline exceeds the threshold, an alarm will sound, and the frequency where the maximum difference occurs will be displayed in the front panel. This warns an operator that a leak might have occurred near the sensor. The following section compares the abilities of different PSD methods for finding a peak or peaks at high frequencies.

4.5 Method Comparison

The main interest in using different PSD methods is to determine which method can detect a higher peak or peaks in high frequencies. Two different sets of data were chosen to check for any peak in high frequencies. The first includes only background noise which was recorded when no official leak was declared. These data which contain ten different data files are labeled as channel A and are presented as "0-1-27 a.m." The second data set was recorded from a different sensor channel at a different time of the day. These data files are labeled as channel B and are presented as "2-1-15 a.m." Each of these data files contains ten different files which were recorded at different times throughout the day.

Figure 4.9 shows the spectrum of channel A with different PSD methods. From this figures, one can note that the

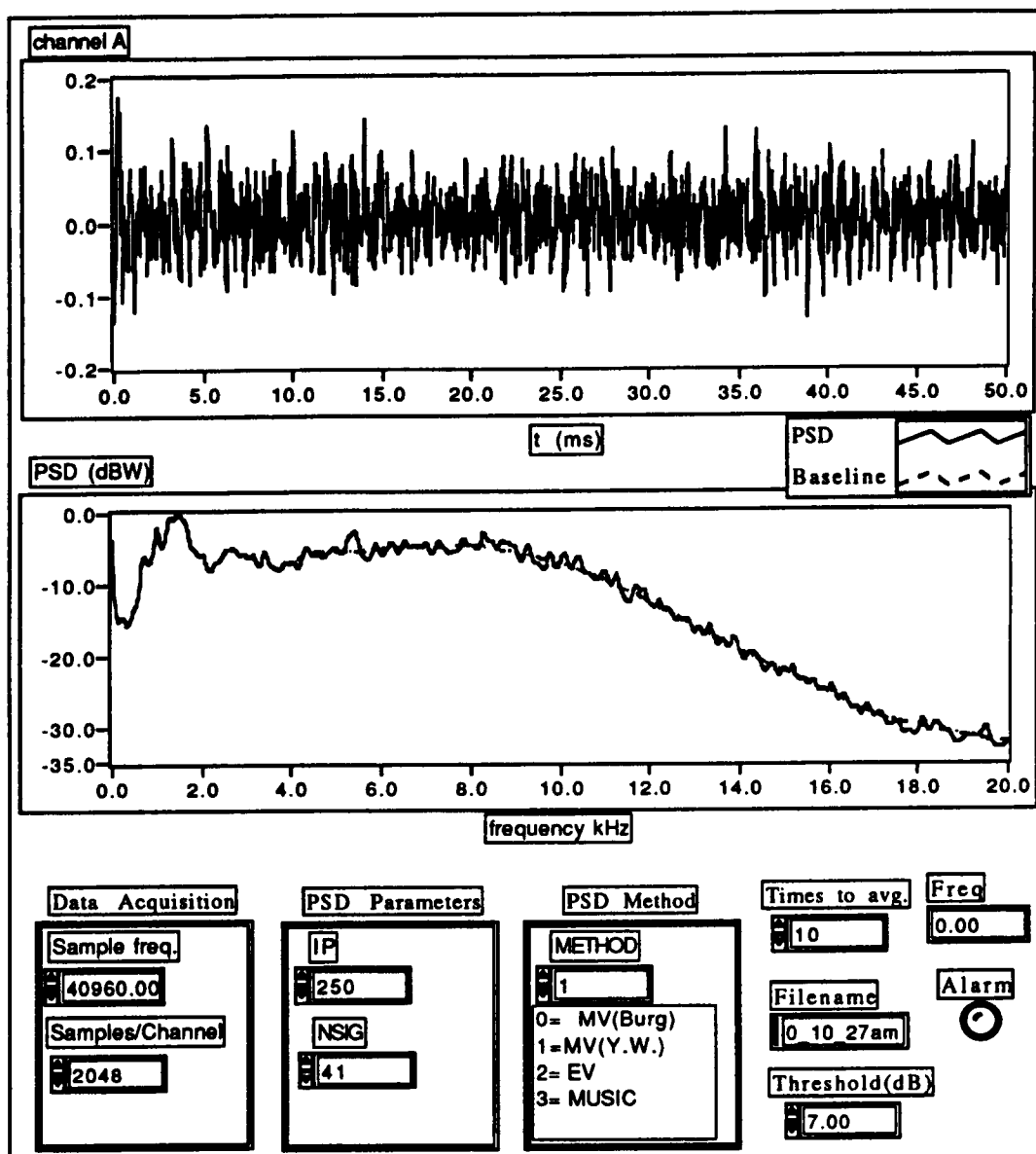
Front Panel



a

Figure 4.9 Front Panel of "PSD Compare " VI (No Leak)

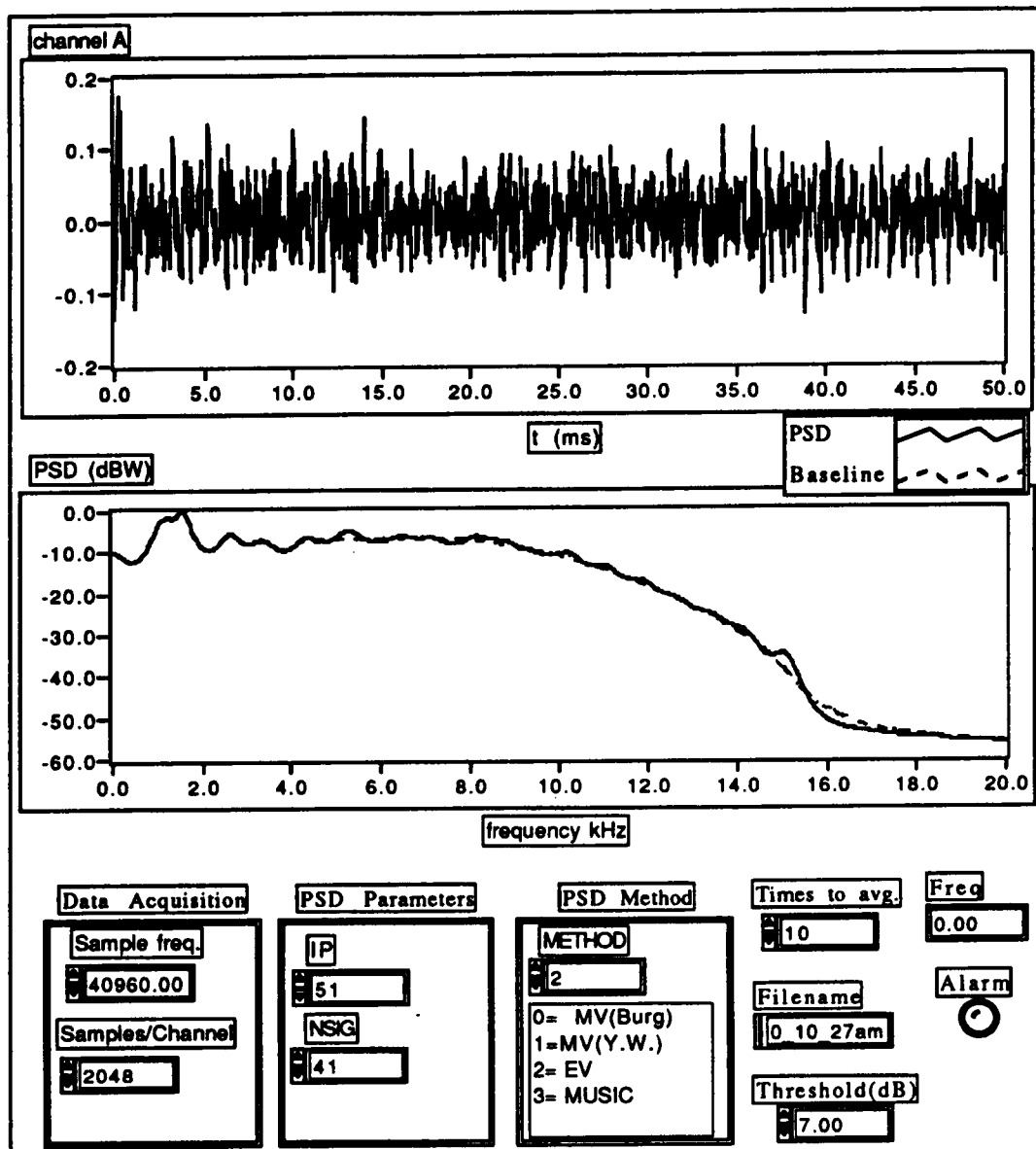
Front Panel



b

Figure 4.9 (Continued)

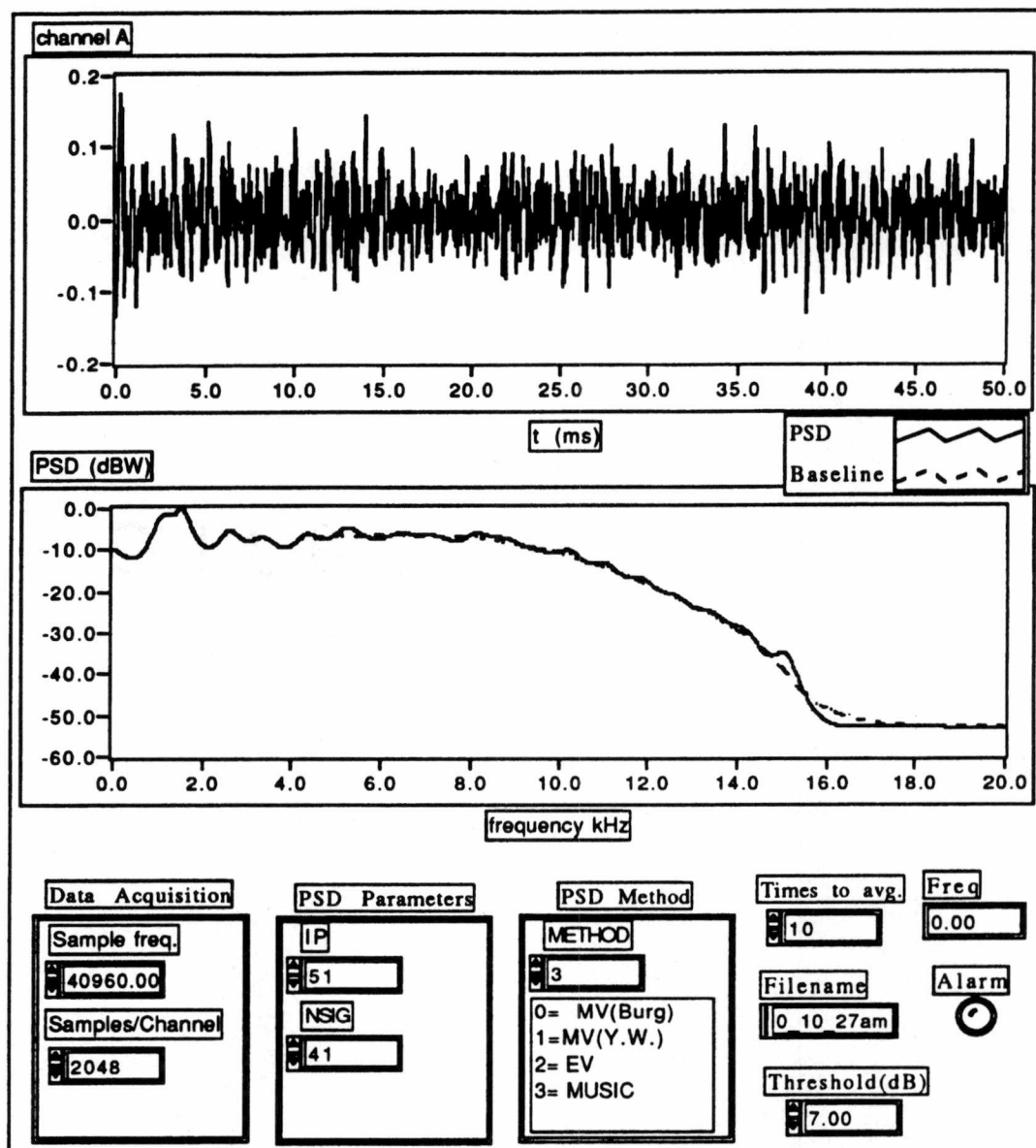
Front Panel



c

Figure 4.9 (Continued)

Front Panel



d

Figure 4.9 (Continued)

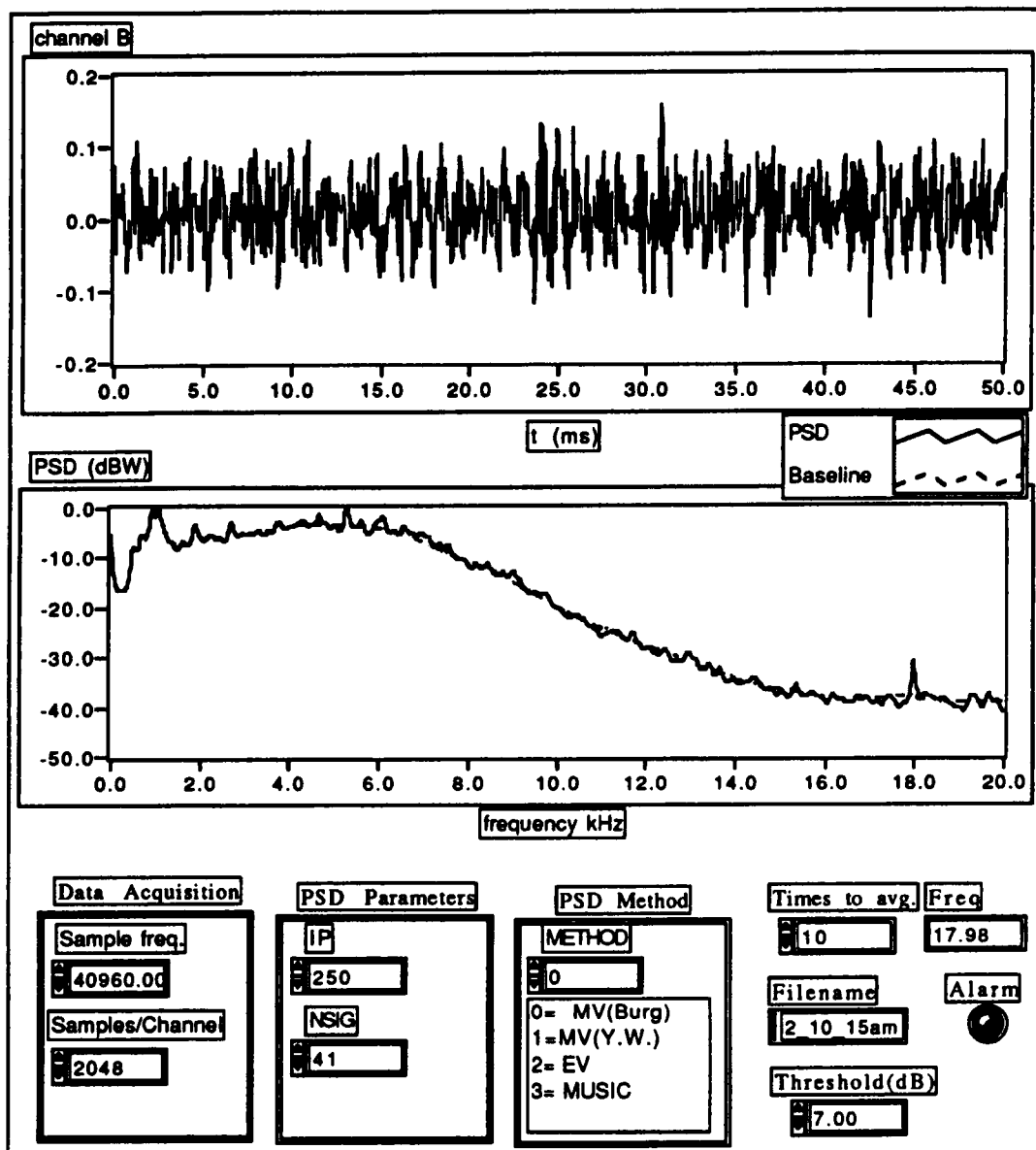
spectrum of channel A is very smooth at high frequencies. There is no peak that exceeds the threshold (7 dB). However, the spectrum of channel B, which has been shown in Figures 4.10(a) through 4.10(d), has a peak near 18 kHz that exceeds the threshold in Figures 4.9(a) through 4.9(d).

As was discussed in Chapter III, another way for finding the estimated autocorrelation matrix for MV spectral estimation is to use eigenvalues and eigenvectors. Eigenvalues and eigenvectors can then be used to evaluate Eq. (3.20). If these values are substituted into Eq. (3.17), the created spectral estimation will have reduced noise contribution due to the omission of the noise subspace eigenvectors [20]. Figure 4.11 shows the graph for the above procedure. Comparing Figure 4.11 to Figures 4.10(a) and 4.10(b), it is clear that noise variation has been reduced dramatically; however, it did not give a larger peak around 18 kHz.

As one can see, all five different methods can detect the same peak around 18 kHz. In Figures 4.10(a), 4.10(b), and 4.11, which use minimum variance methods, a small peak near 18 kHz is evident. Peaks in Figures 4.10(c) and 4.10(d) are much larger than peaks in Figures 4.10(a), 4.10(b), and 4.11, which indicates that eigenanalysis PSD estimation methods are to be the most reliable for leak detection. Table 4.1 lists the maximum difference between the average PSD and its baseline, ΔP , and the frequency where the largest peak happens, f_p , for the minimum variance and eigenanalysis methods.

As mentioned in Chapter I, both classical and parametric spectral analyses have been implemented for boiler tube leak

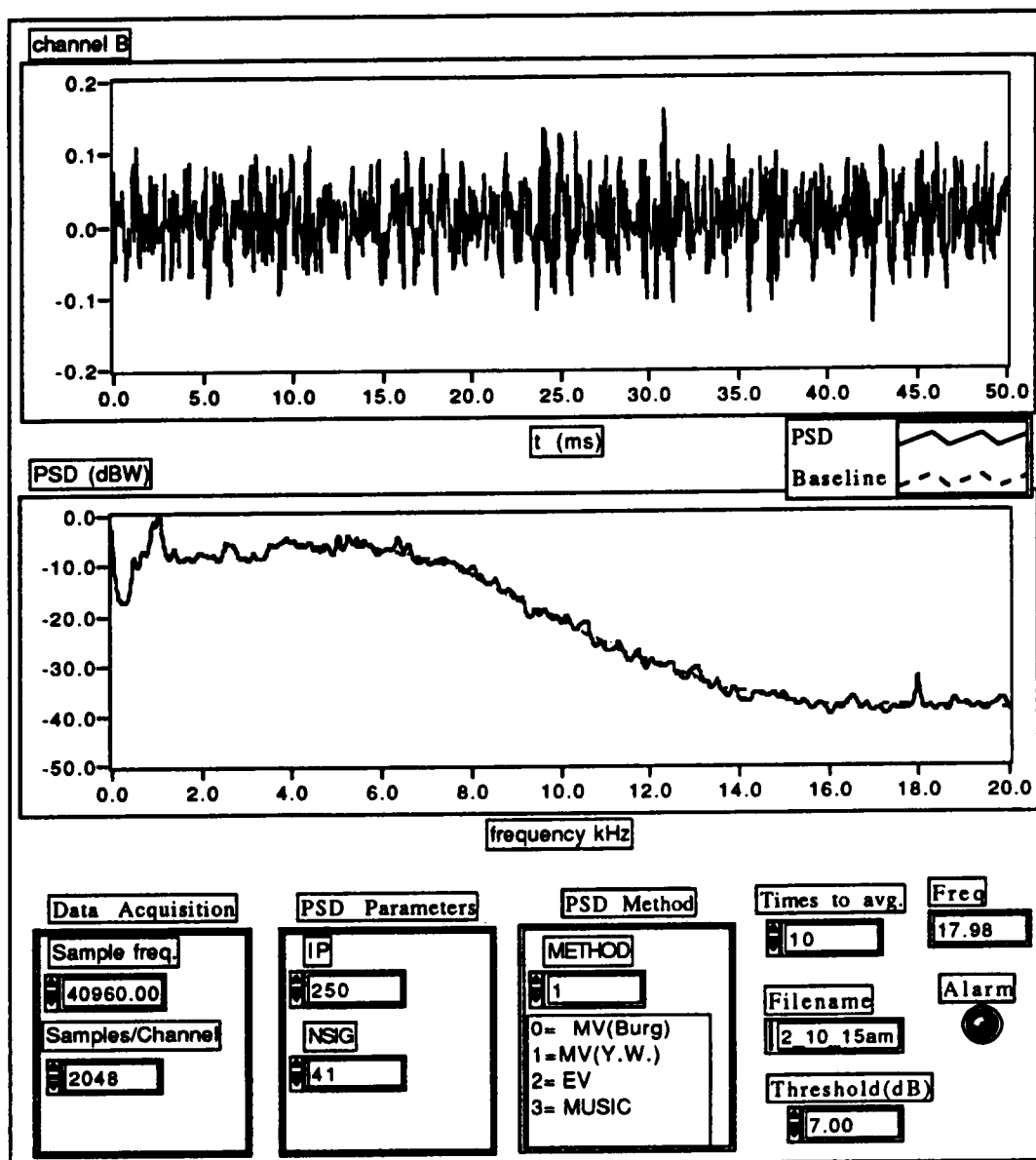
Front Panel



a

Figure 4.10 Front Panel of "PSD Compare " VI (Leak)

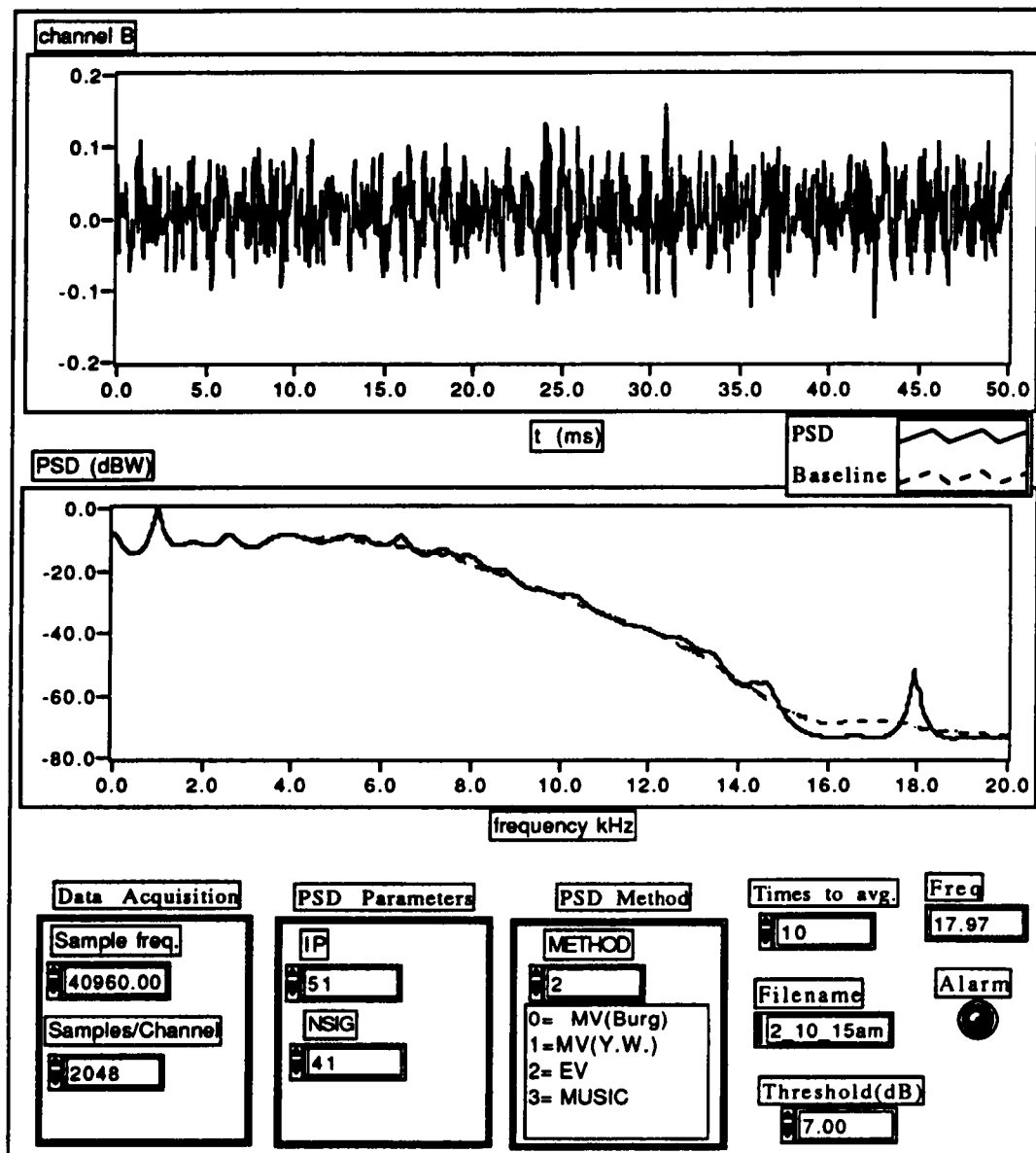
Front Panel



b

Figure 4.10 (Continued)

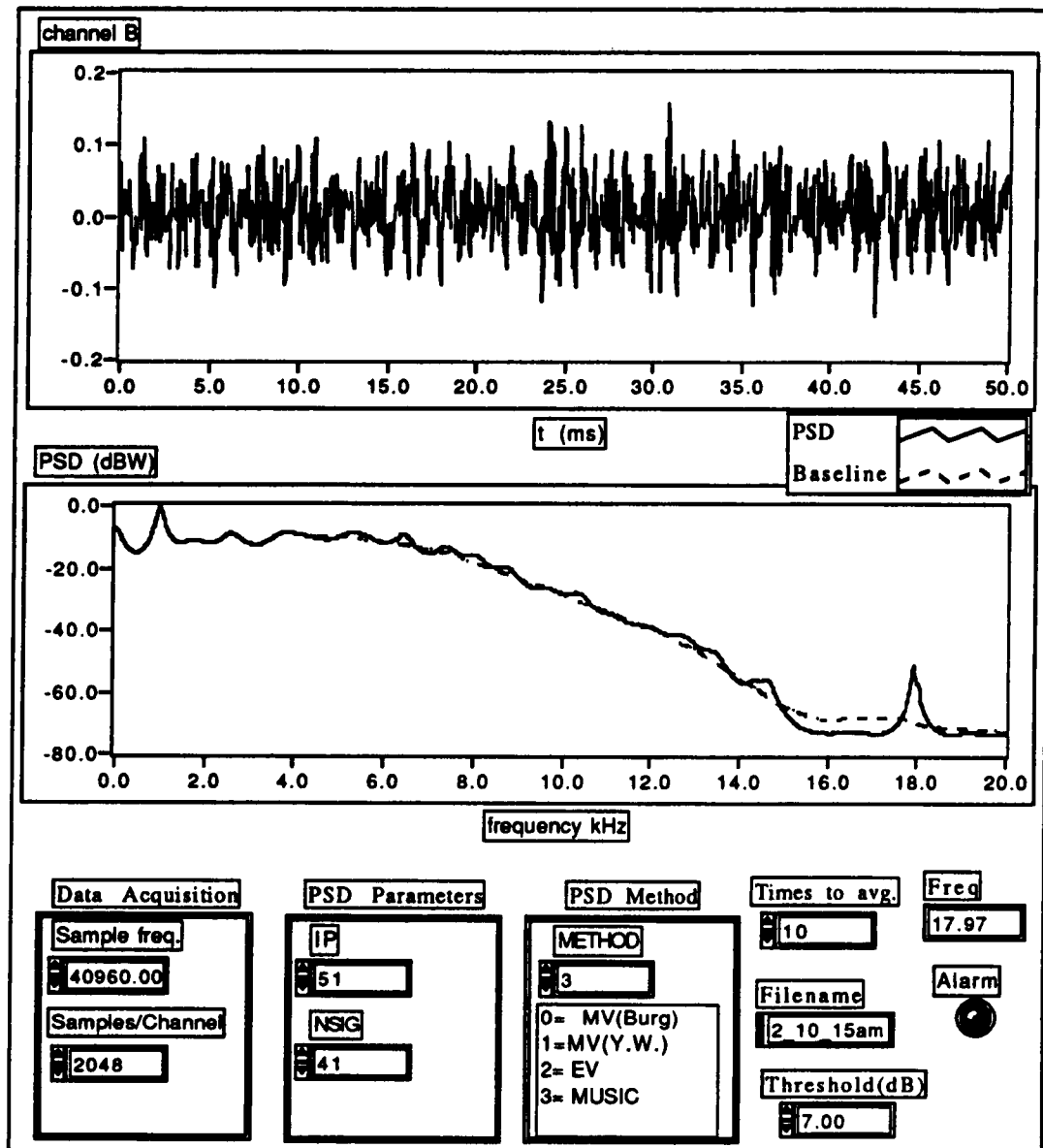
Front Panel



c

Figure 4.10 (Continued)

Front Panel



d

Figure 4.10 (Continued)

Front Panel

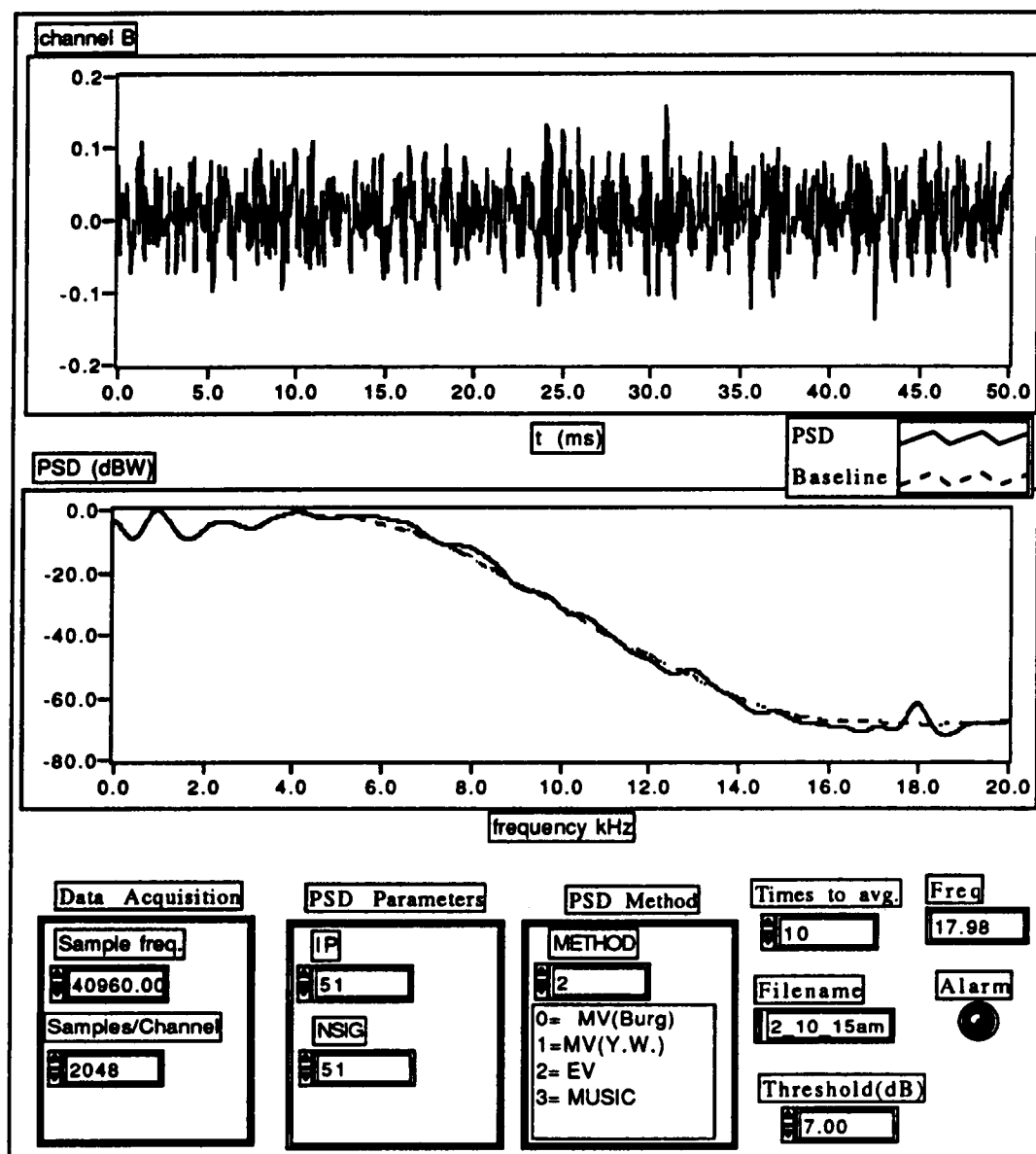


Figure 4.11 Front Panel of "PSD Compare " VI (MV)

detection by Wan [5]. It was shown in that study that the AR and ARMA, particularly the latter, are capable of detecting any peak at high frequencies. This result is shown in table 4.2.

Table 4.1 Spectrum Peak Detection

	MV(Y.W.)	MV(Burg)	EV	MUSIC
$\Delta P(\text{dBW})$	7.03	7.36	17.96	17.80
$f_p(\text{kHz})$	17.98	17.98	17.97	17.97

Table 4.2 Spectrum Peak Detection

	Periodogram	AR	ARMA
$\Delta P(\text{dBW})$	7.24	7.58	8.97
$f_p(\text{kHz})$	17.98	17.98	17.96

From these two tables, it can be seen that the eigenanalysis (EV and MUSIC) methods are far superior in peak detection compared with other methods.

The eigenanalysis PSD estimate methods are the most reliable for peak detection. Their unique combination of peak resolution and amplification and relatively smooth and stable spectrum show the most promise in detecting boiler tube leaks.

CHAPTER V

CONCLUSION

5.1 Conclusion

In this thesis, non-parametric spectral estimation methods were developed to investigate their effectiveness for tube leak detection. Such methods included the minimum variance method and eigenanalysis methods (incorporating both the EV and MUSIC techniques). The findings concluded that all of the PSD estimation methods studied performed well enough to detect a leak in the boiler system. But when comparing the non-parametric methods to those of both classical and parametric techniques, eigenanalysis methods proved to be far superior over the latter. However, the EV method did show the most promise with respect to the *earliest* detection for leakage.

5.2 Implications for Future Research

Since it is obvious that further study of spectral estimation methods for tube leak detection is necessary, this researcher suggests the following:

- An attempt should be made for finding an algorithm which can determine the correct data matrix order and the number of signal components utilizing eigenanalysis methods.
- Because the data matrix order must be large enough to detect peaks at high frequencies, but not so large that it causes the

eigenvalues to be unstable, and not so small that it will not resolve any peak at higher frequencies, finding the correct data matrix order for eigenanalysis methods is highly significant.

- Finally, finding an algorithm which determines the number of signal components for eigenanalysis methods for this type of data will help to eliminate the excessive time expended utilizing trial and error.

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LIST OF REFERENCES

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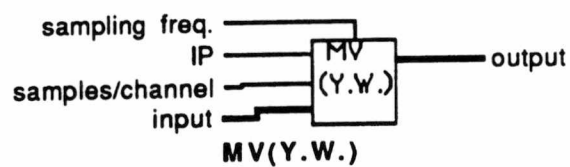
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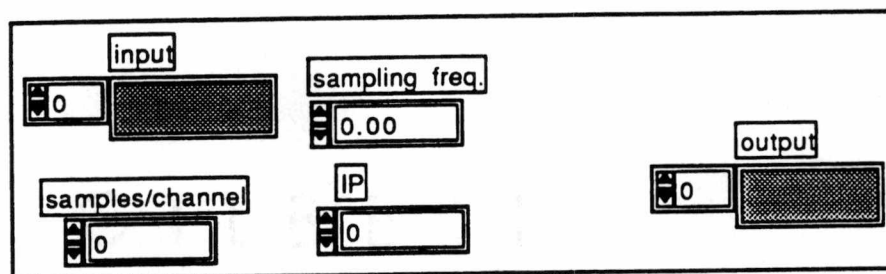
APPENDICES

APPENDIX A

Connector Pane



Front Panel



Block Diagram

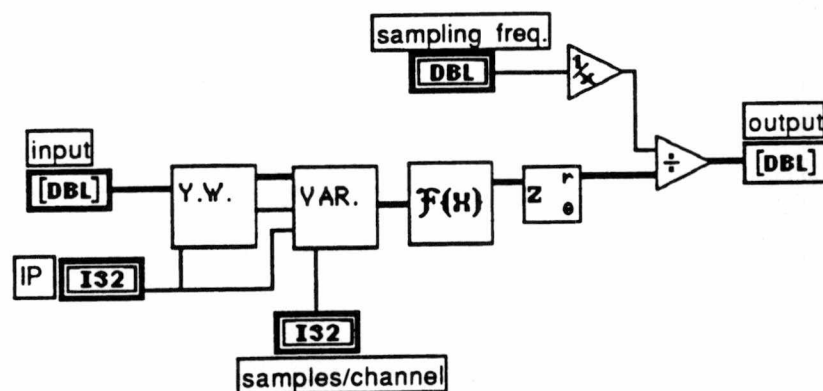
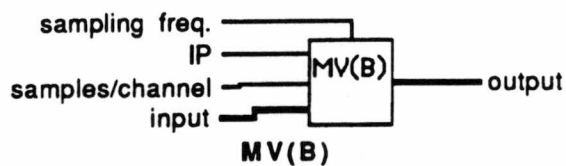
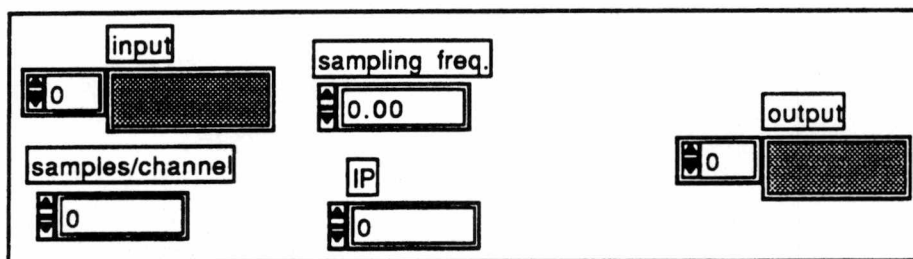


Figure A.1.1 Connector Pane, Front Panel, and Block Diagram of MV(Y.W.) VI

Connector Pane



Front Panel



Block Diagram

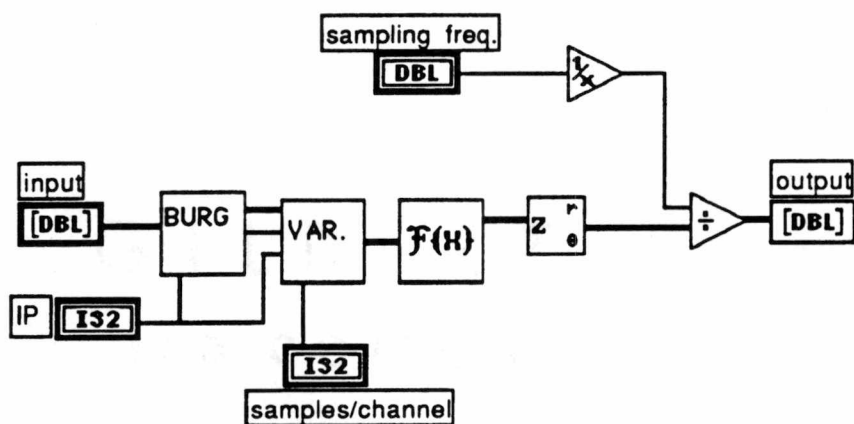


Figure A.1.2 Connector Pane, Front Panel, and Block Diagram of MV(B) VI

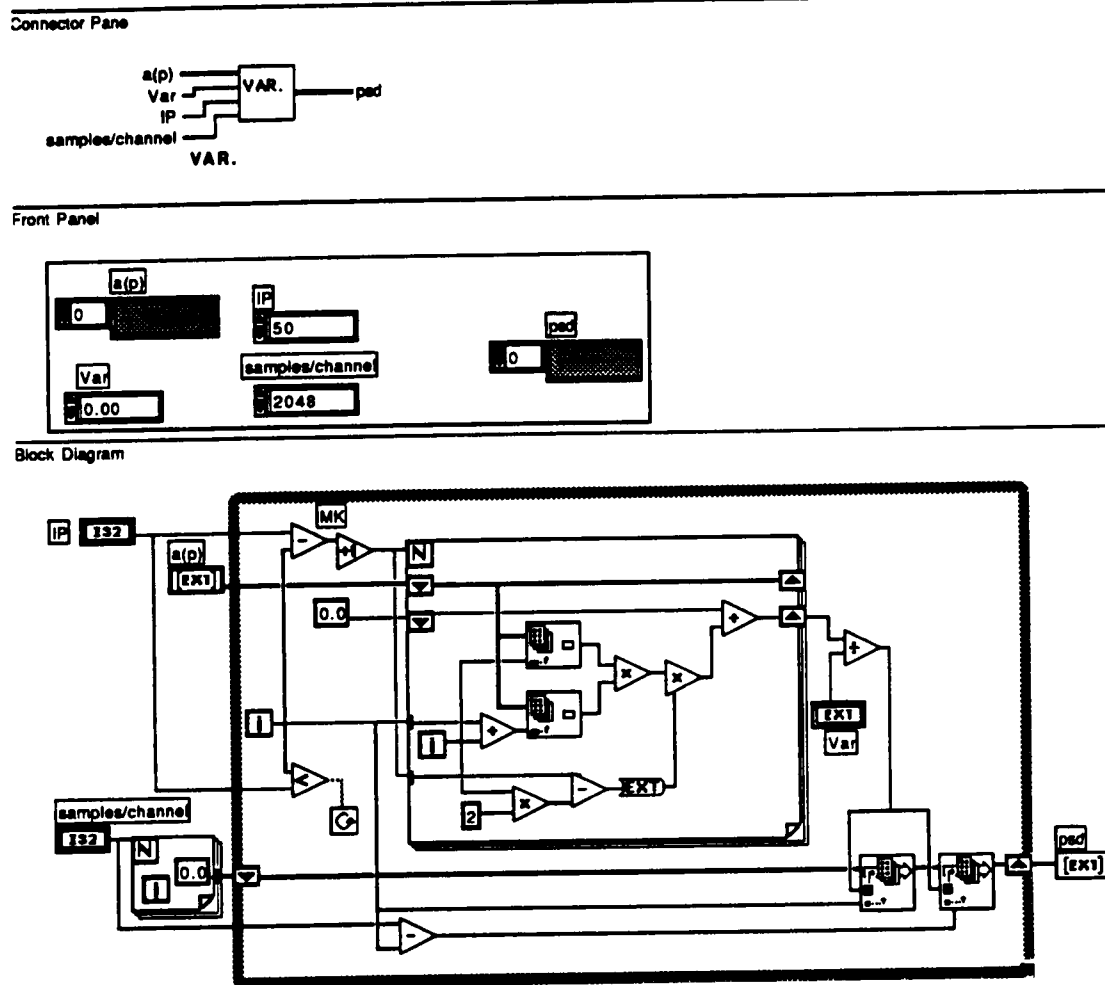
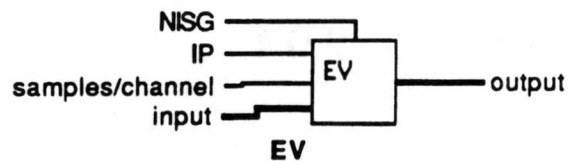
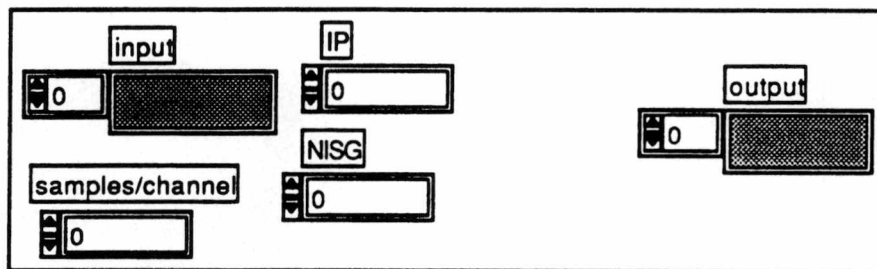


Figure A.1.3 Connector Pane, Front Panel, and Block Diagram of VAR. VI

Connector Pane



Front Panel



Block Diagram

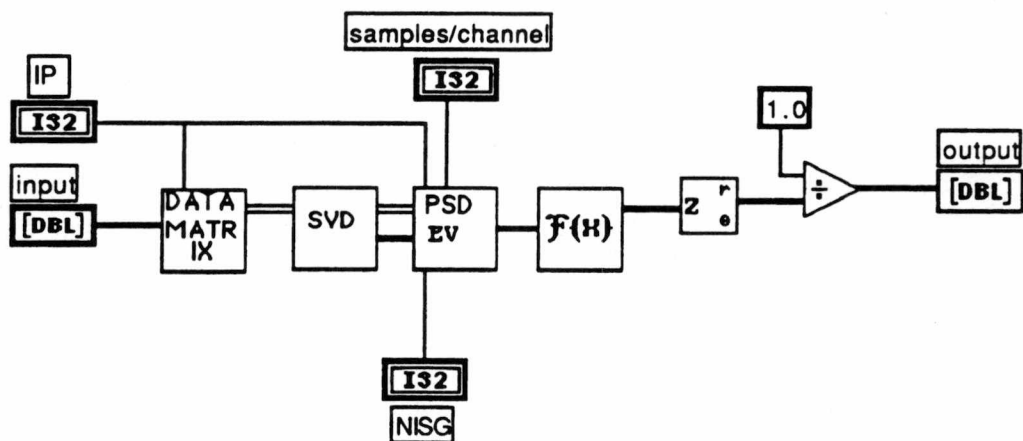


Figure A.2.1 Connector Pane, Front Panel, and Block Diagram of EV VI

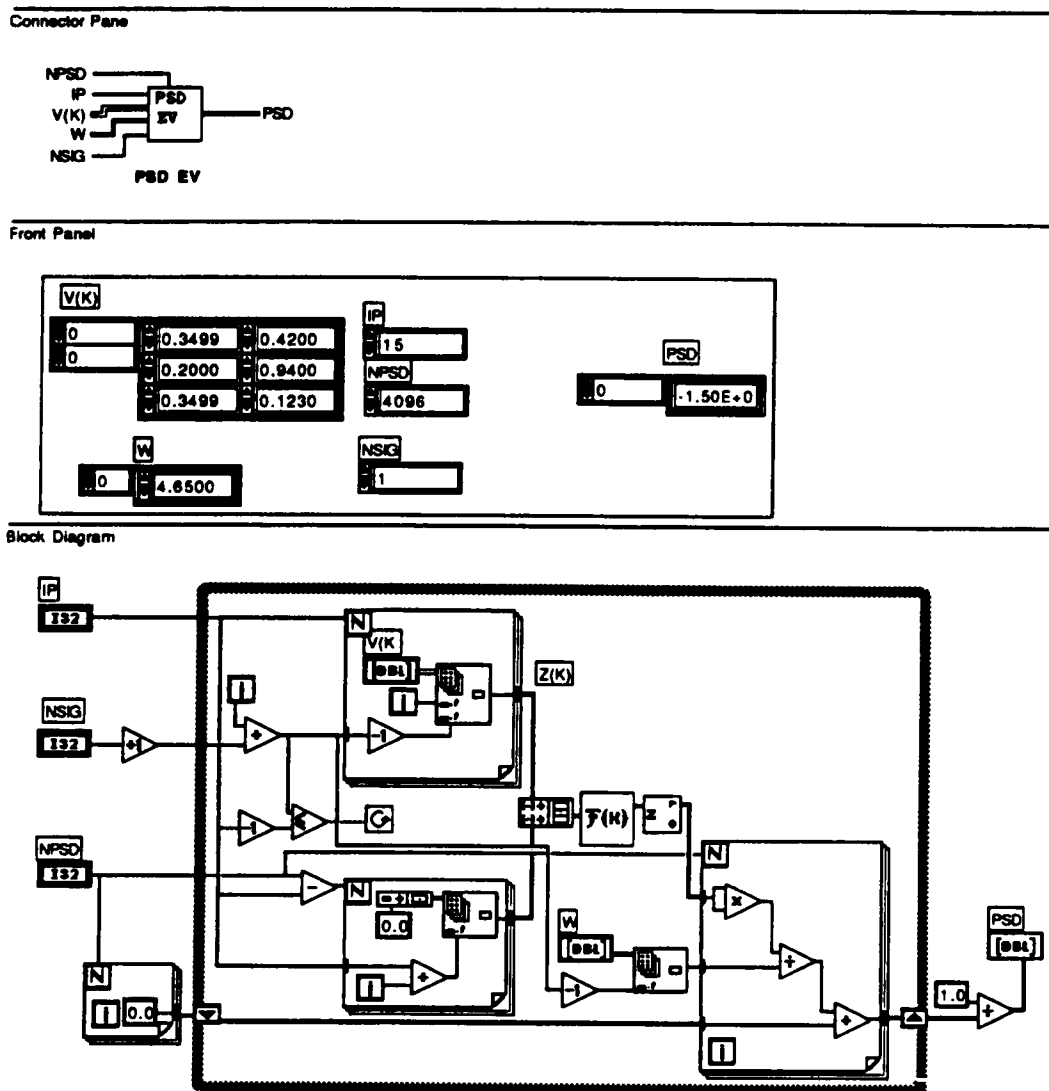
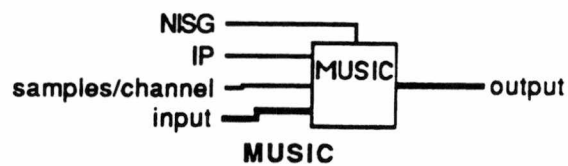
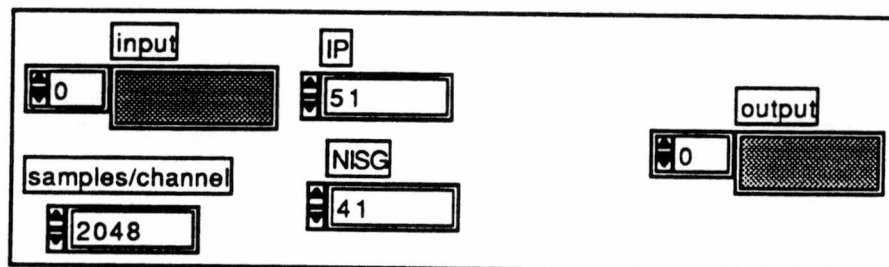


Figure A.2.2 Connector Pane, Front Panel, and Block Diagram of PSD EV VI

Connector Pane



Front Panel



Block Diagram

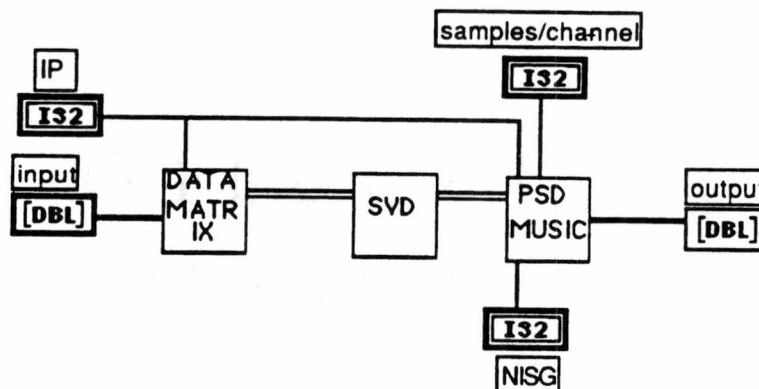
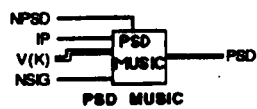
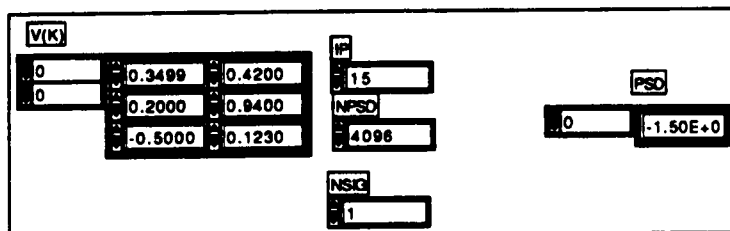


Figure A.3.1 Connector Pane, Front Panel, and Block Diagram of MUSIC VI

Connector Pane



Front Panel



Block Diagram

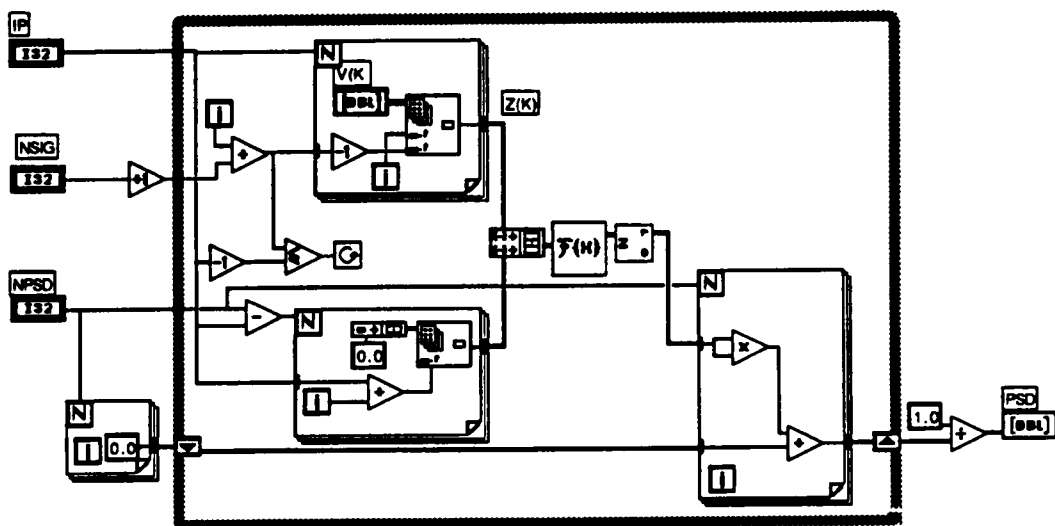


Figure A.3.2 Connector Pane, Front Panel, and Block Diagram of PSD MUSIC VI

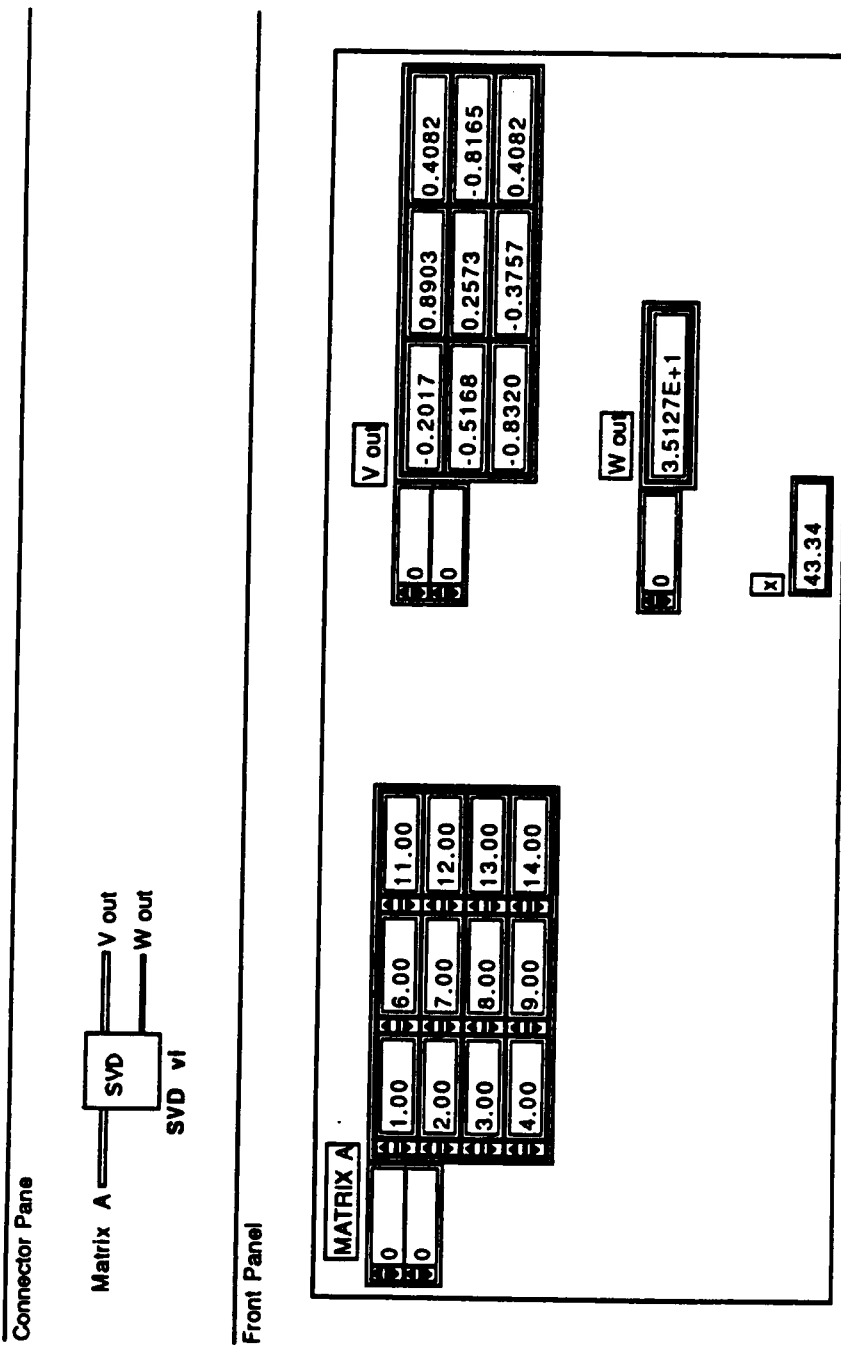


Figure A.4.1 Connector Pane and Front Panel of SVD VI

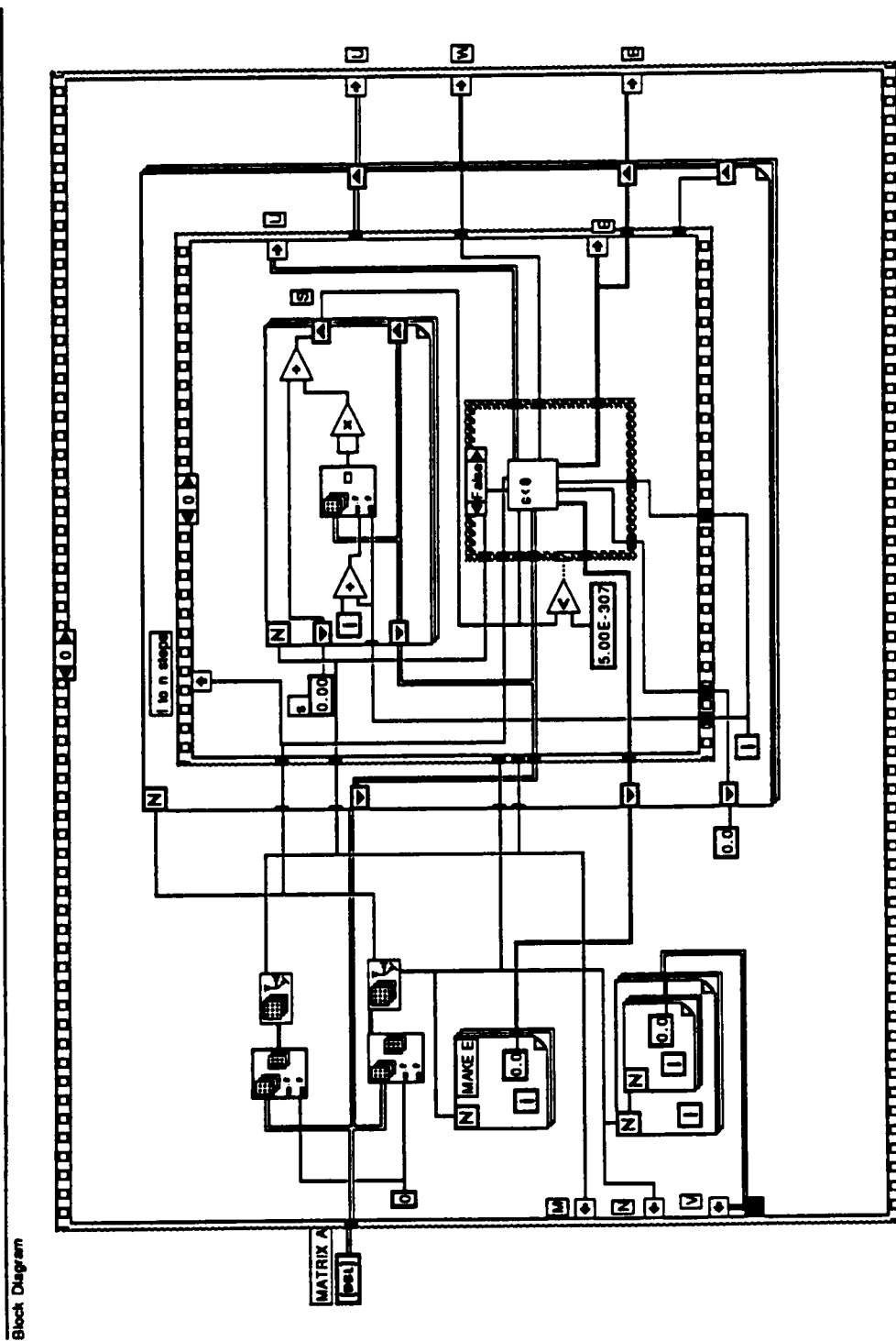


Figure A.4.2 Block Diagram of SVD VI

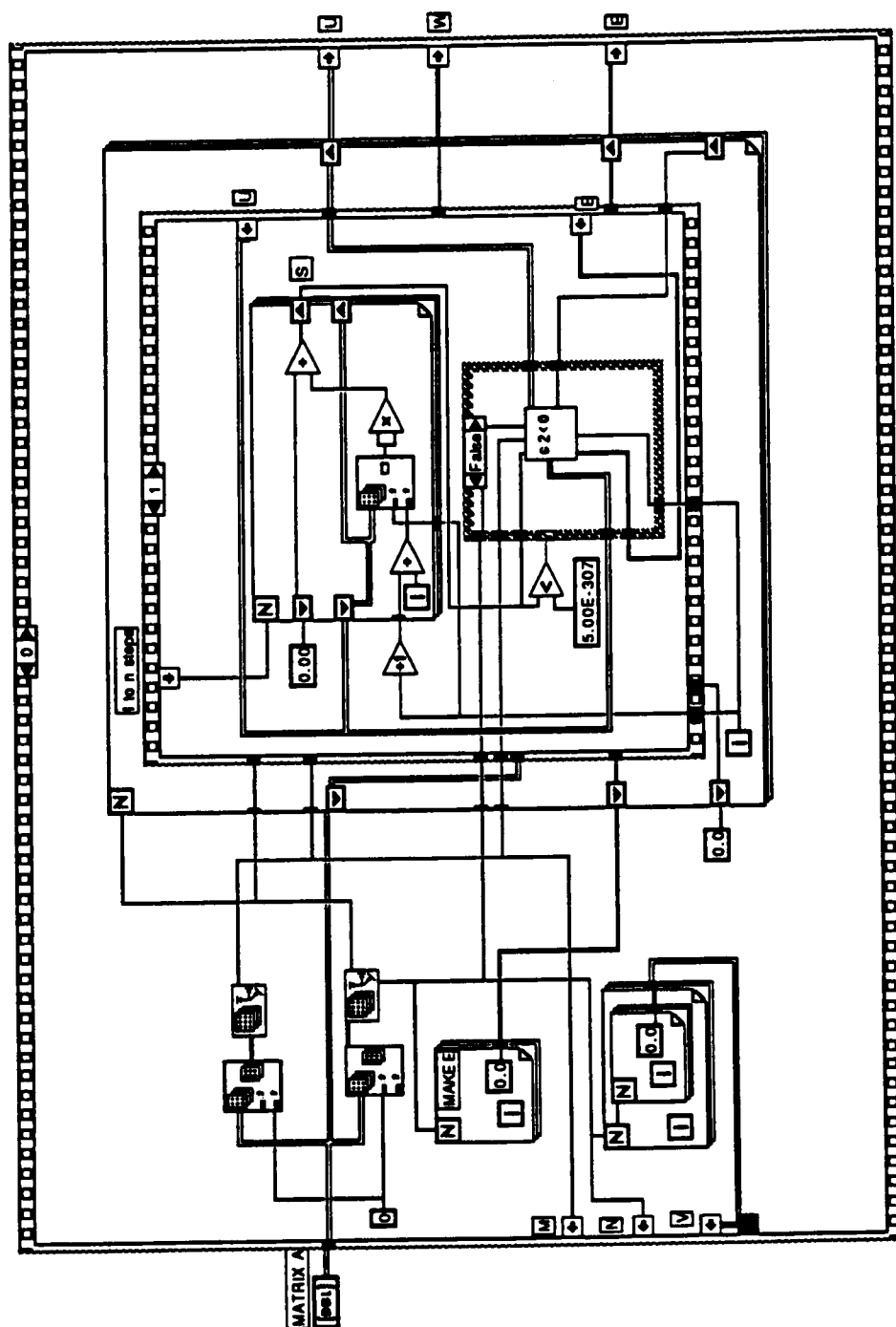
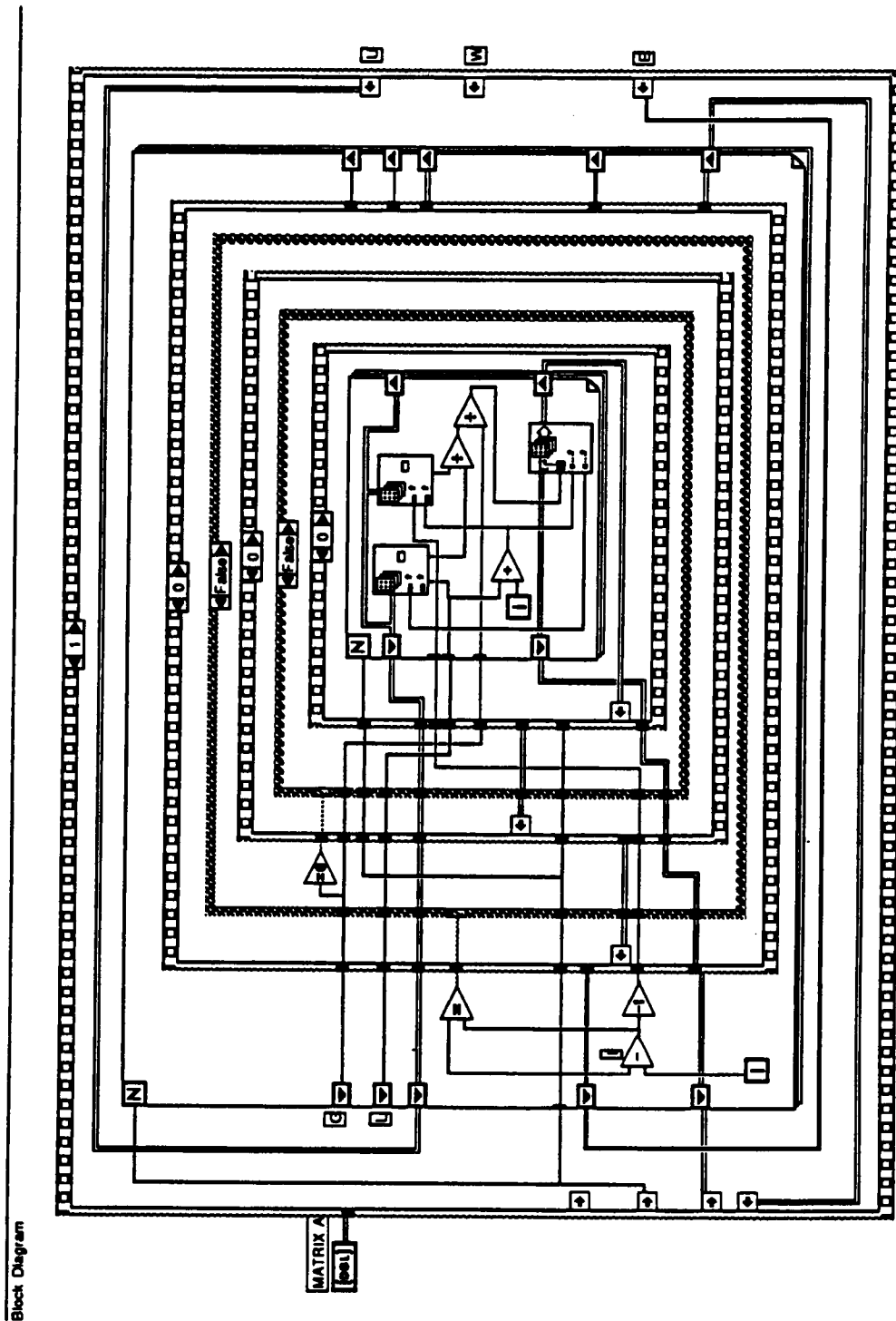


Figure A.4.2 Block Diagram of SVD VI (Continued)



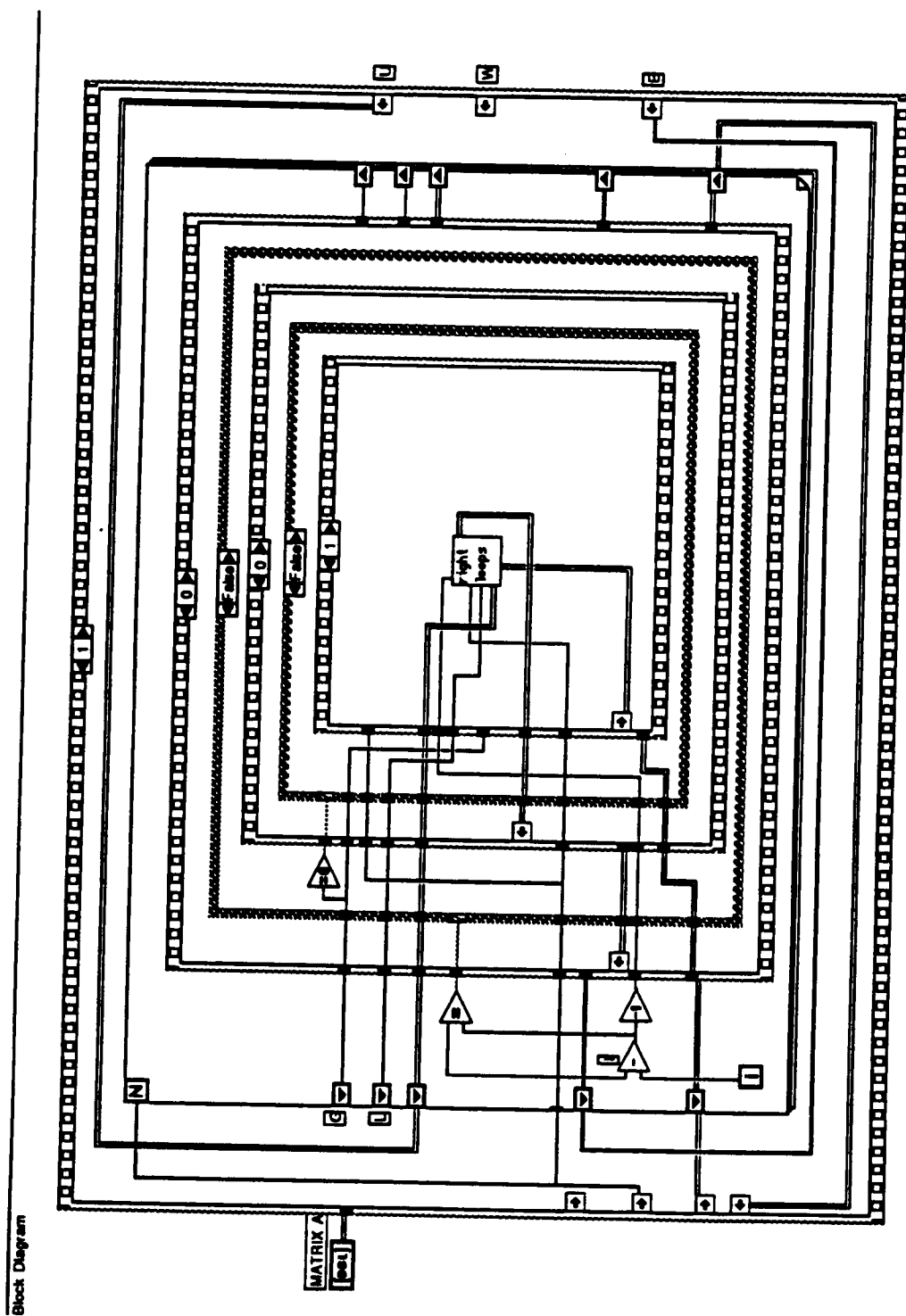


Figure A.4.2 Block Diagram of SVD VI (Continued)

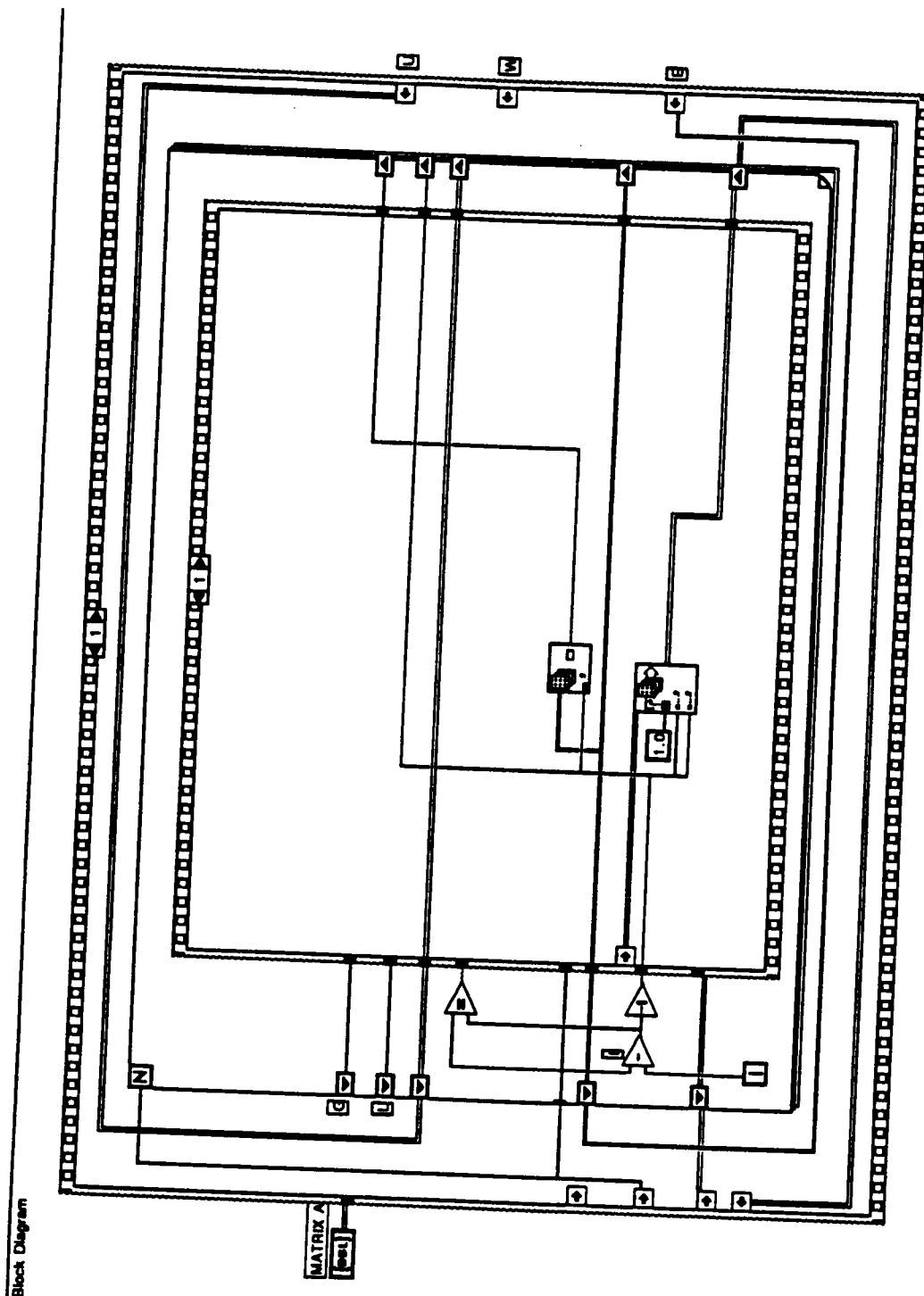


Figure A.4.2 Block Diagram of SVD VI (Continued)

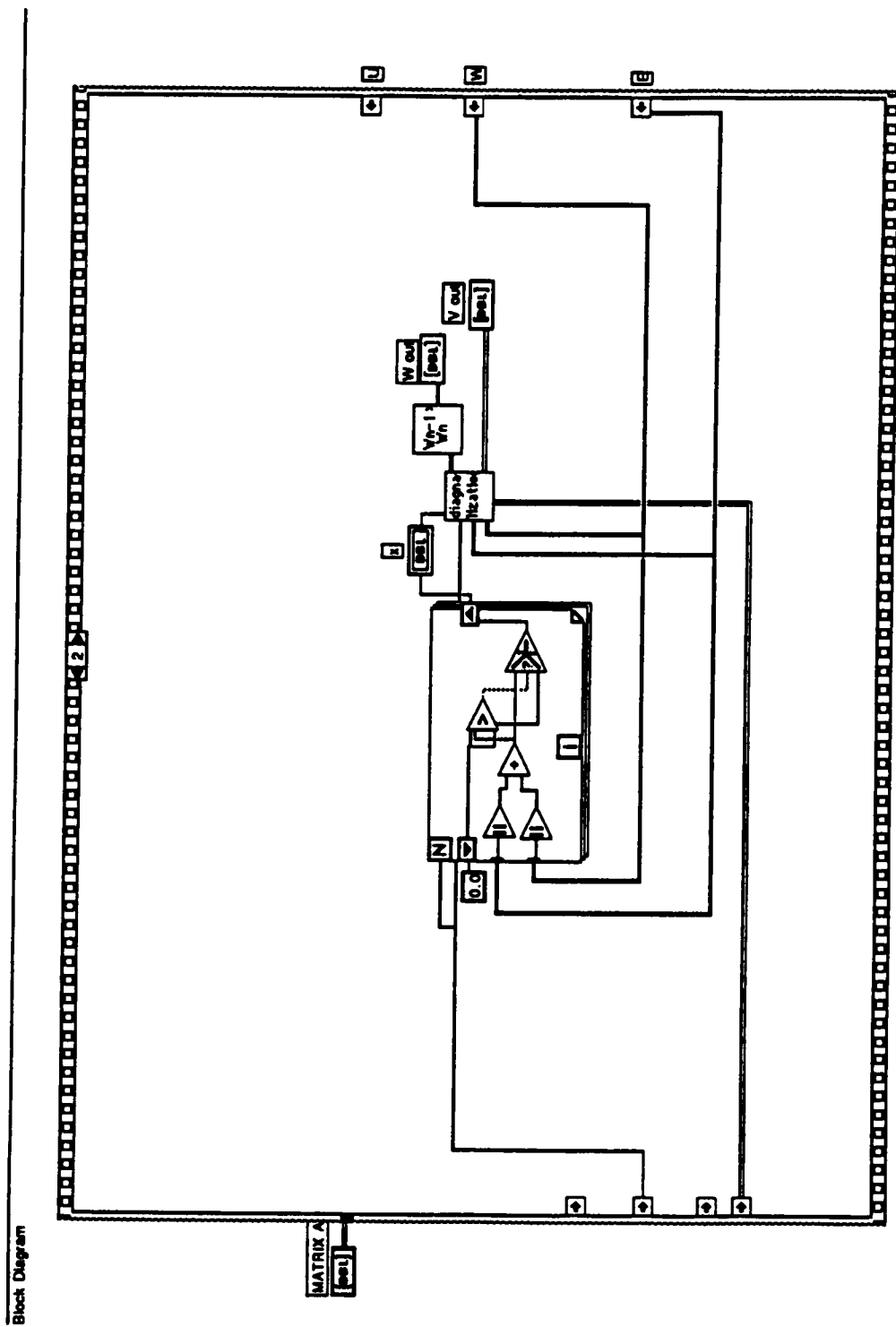


Figure A.4.2 Block Diagram of SVD VI (Continued)

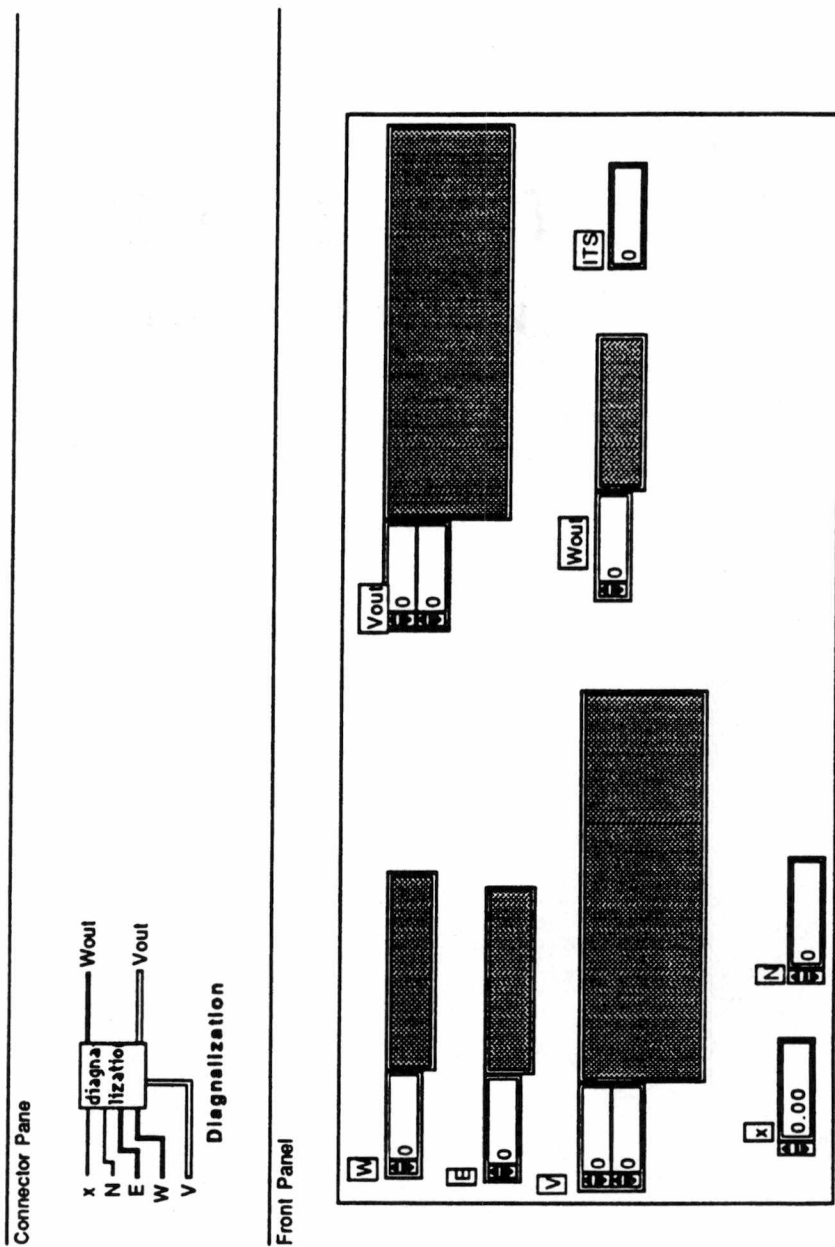


Figure A.4.3 Connector Pane and Front Panel of Diagonalization VI

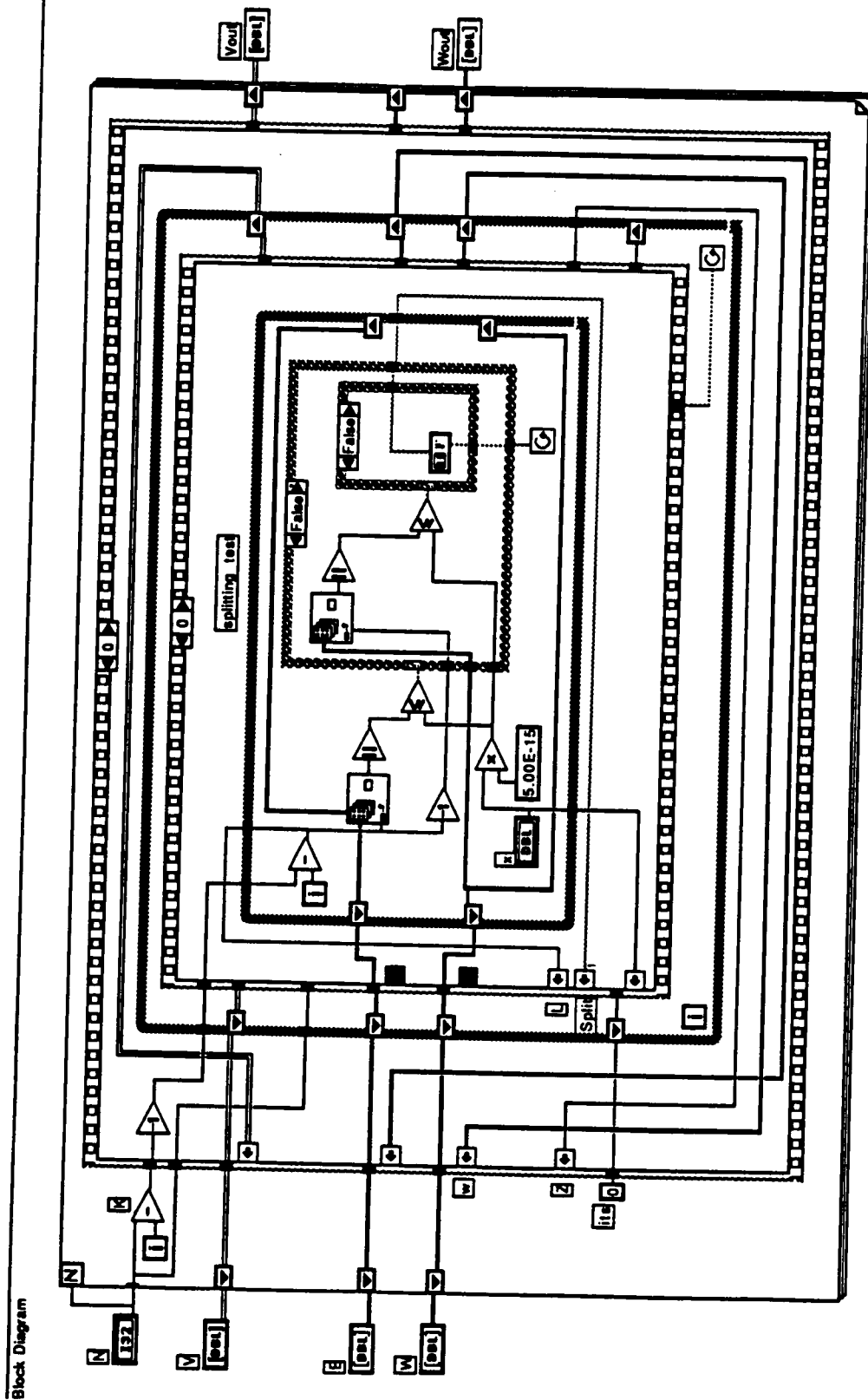


Figure A.4.4 Block Diagram of Diagonalization VI

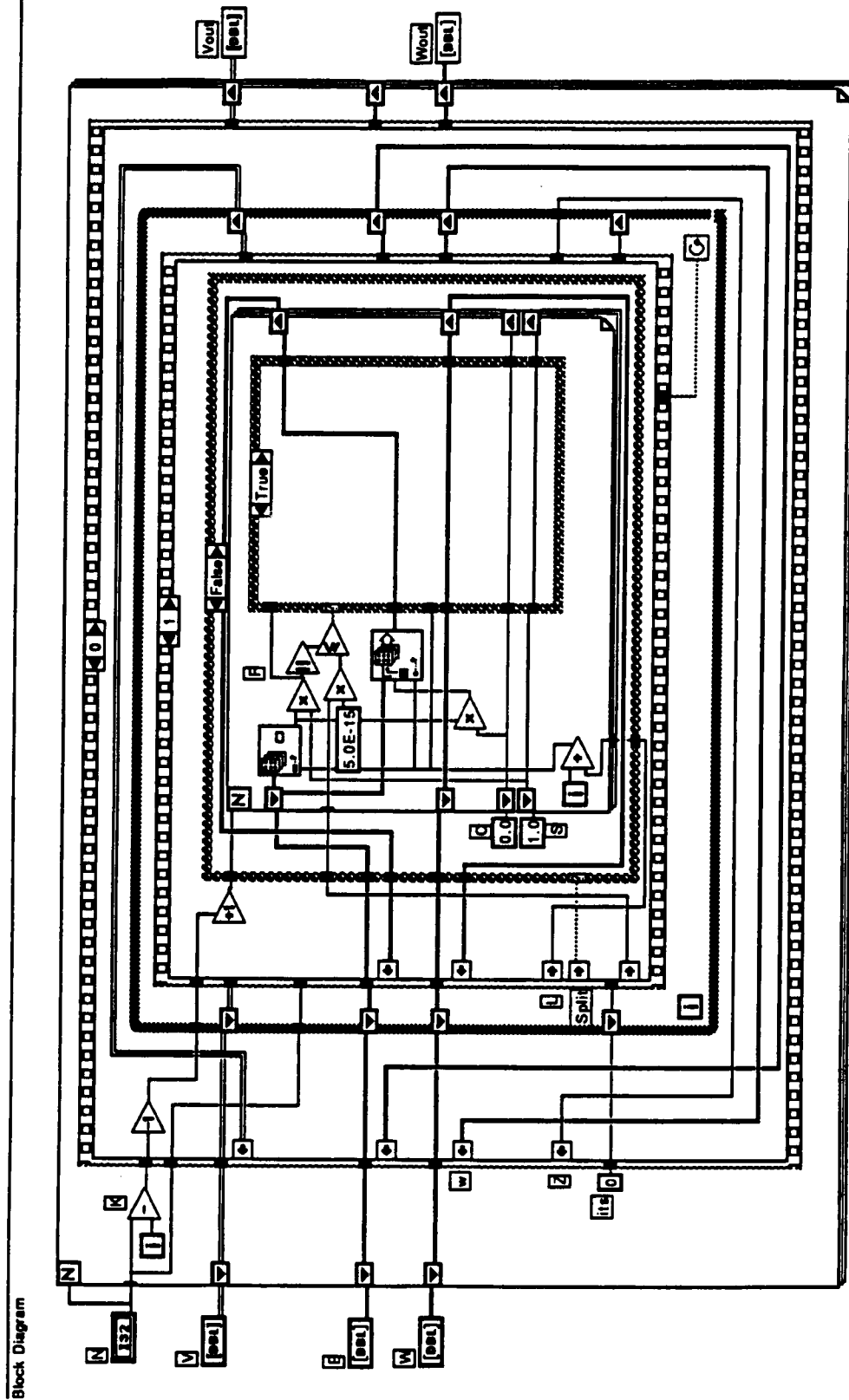


Figure A.4.4 Block Diagram of Diagonalization VI (Continued)

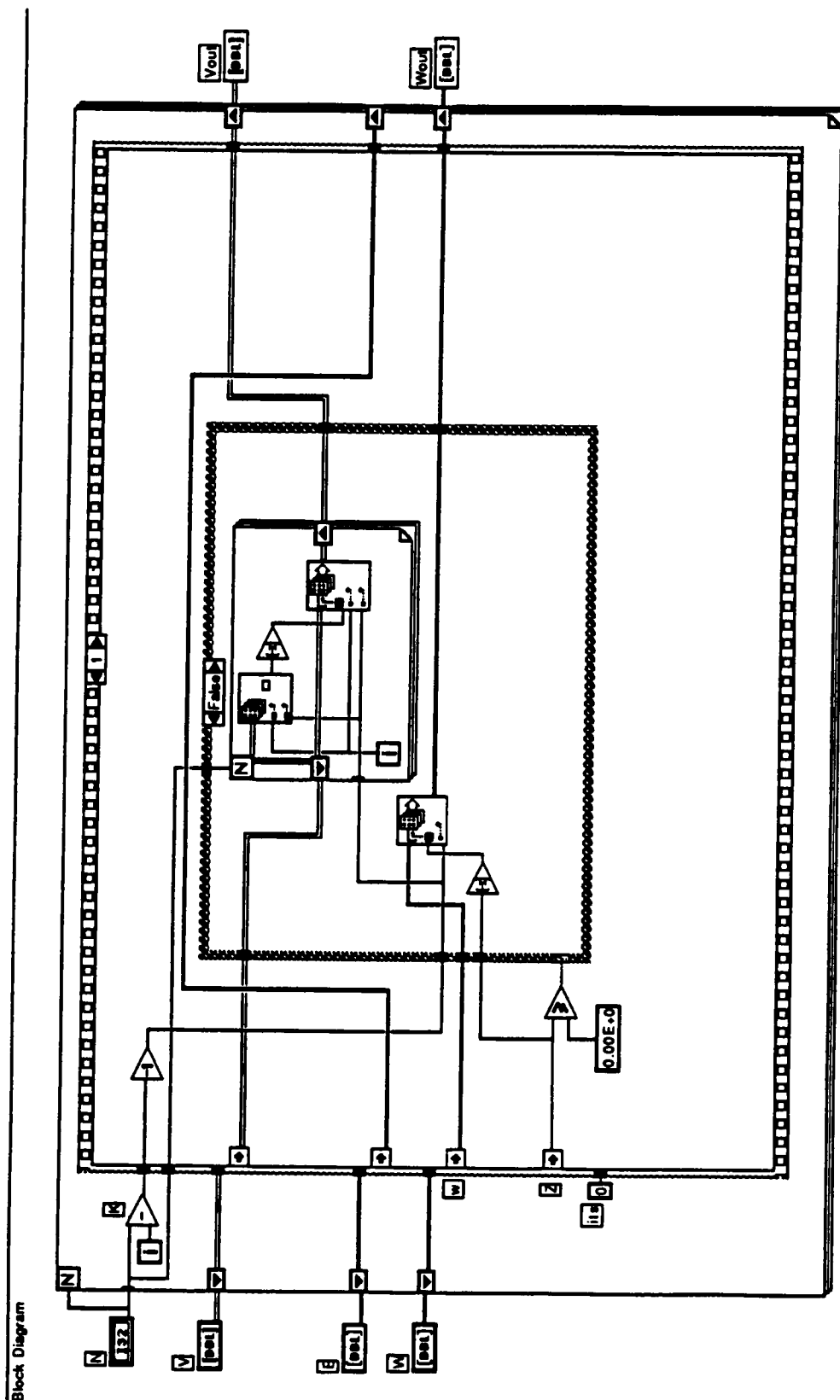


Figure A.4.4 Block Diagram of Diagonalization VI (Continued)

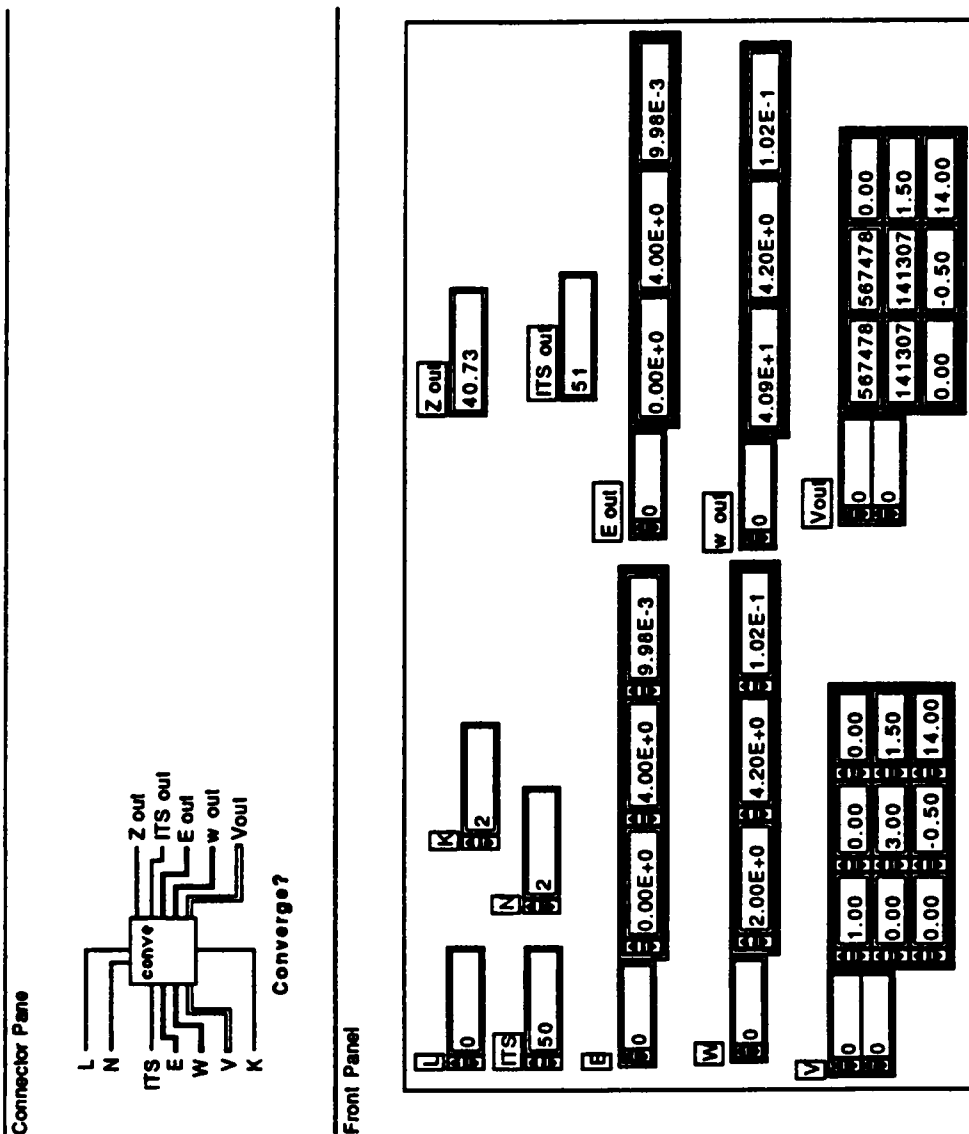


Figure A.4.5 Connector Pane and Front Panel of Converge VI

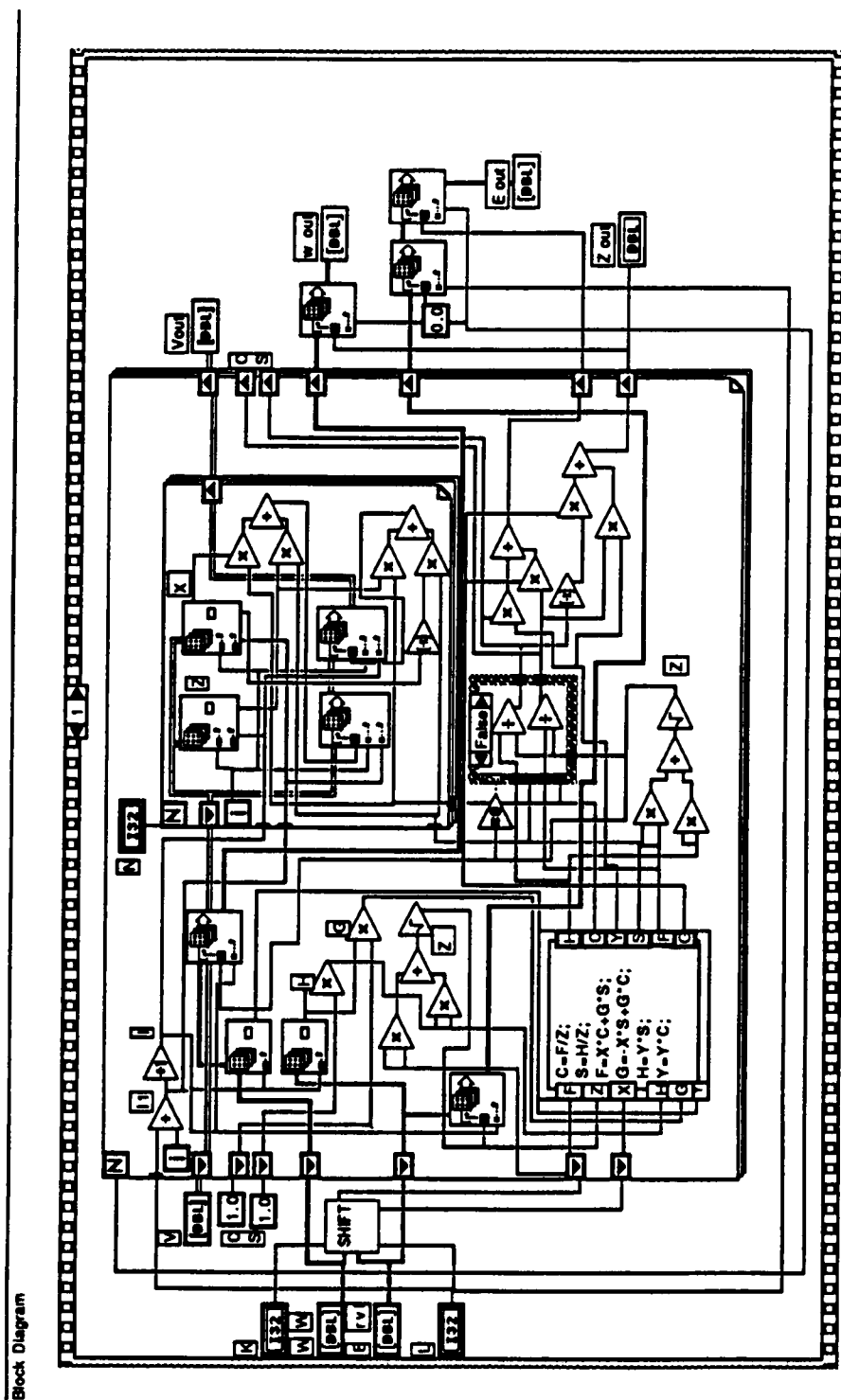
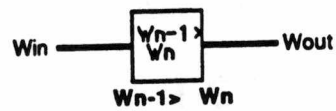


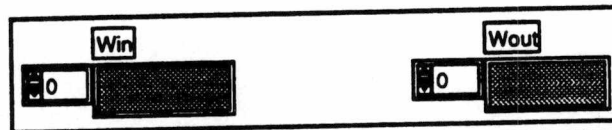
Figure A.4.6 Block Diagram of Converge VI

Figure A.4.6 Block Diagram of Converge VI (Continued)

Connector Pane



Front Panel



Block Diagram

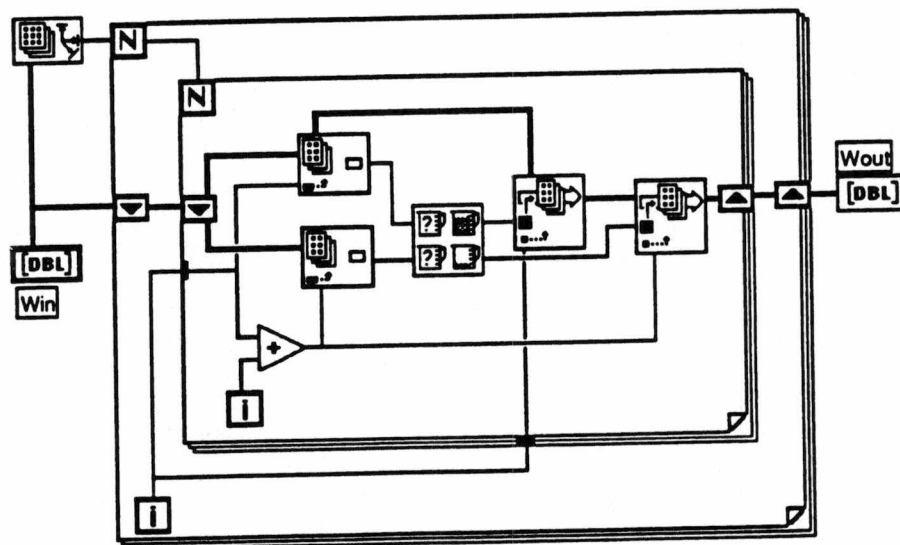


Figure A.4.7 Connector Pane, Front Panel, and Block Diagram of $W_{n-1} > W_n$ VI

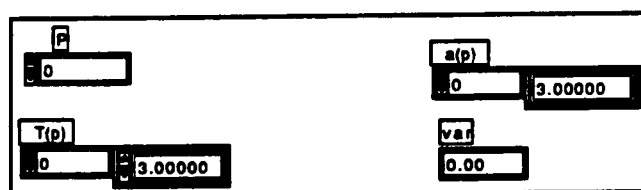
APPENDIX B

This Appendix lists the programs that are used in this thesis and are written by Mr. Curtis S. Jones [4] and Ms. Qin Wan [5].

Connector Pane



Front Panel



Block Diagram

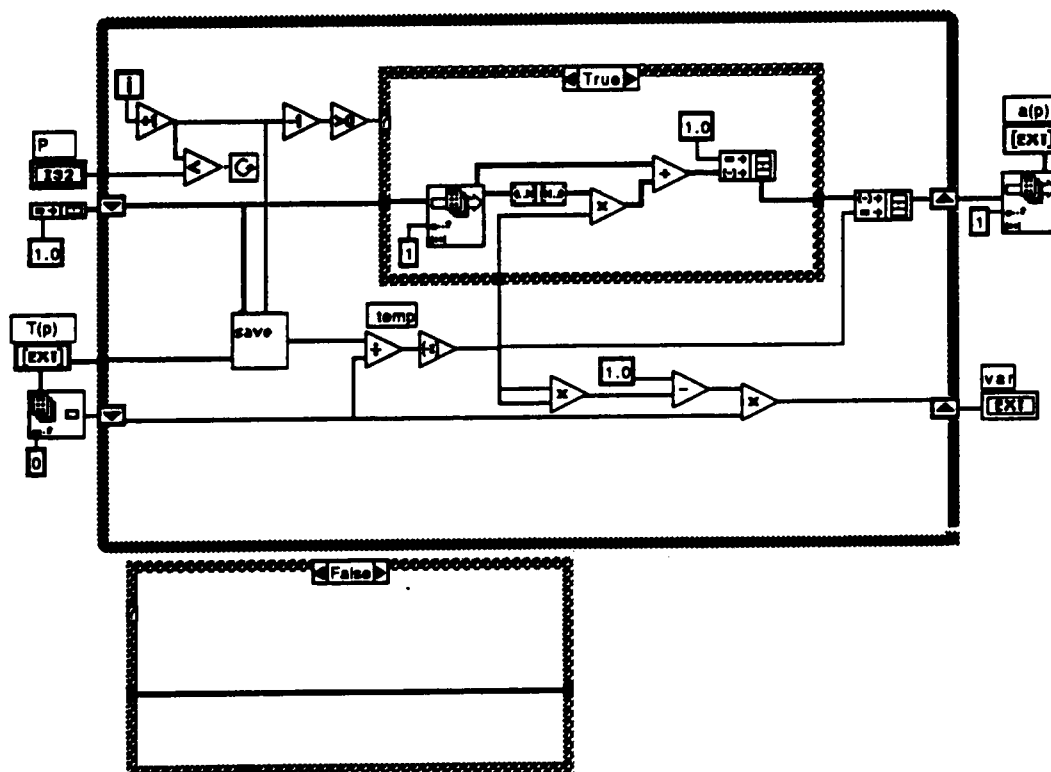
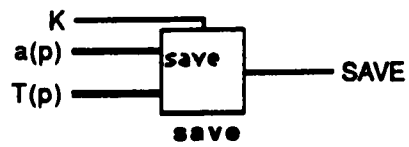
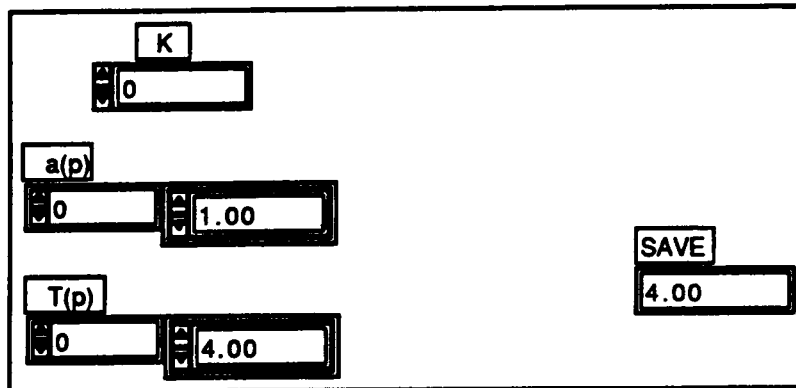


Figure B.1.1 Connector Pane, Front Panel, and Block Diagram of Levinston VI

Connector Pane



Front Panel



Block Diagram

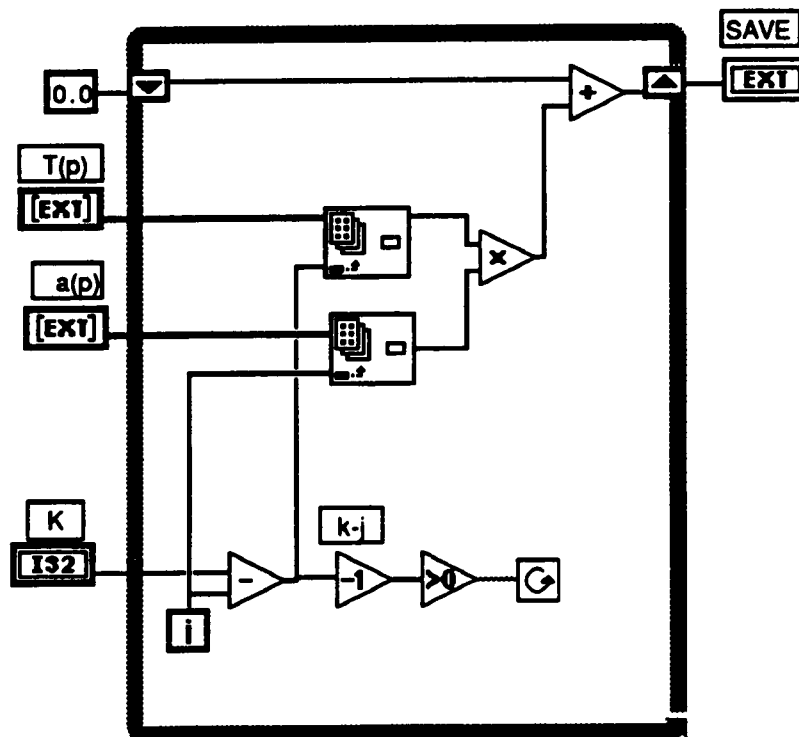
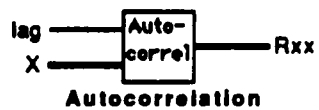
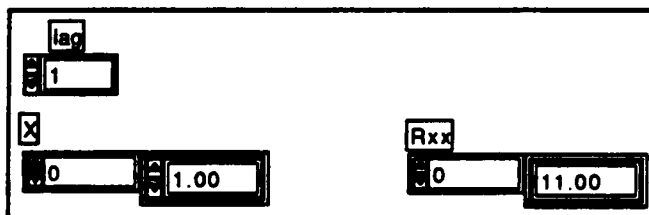


Figure B.1.2 Connector Pane, Front Panel, and Block Diagram of Save VI

Connector Pane



Front Panel



Block Diagram

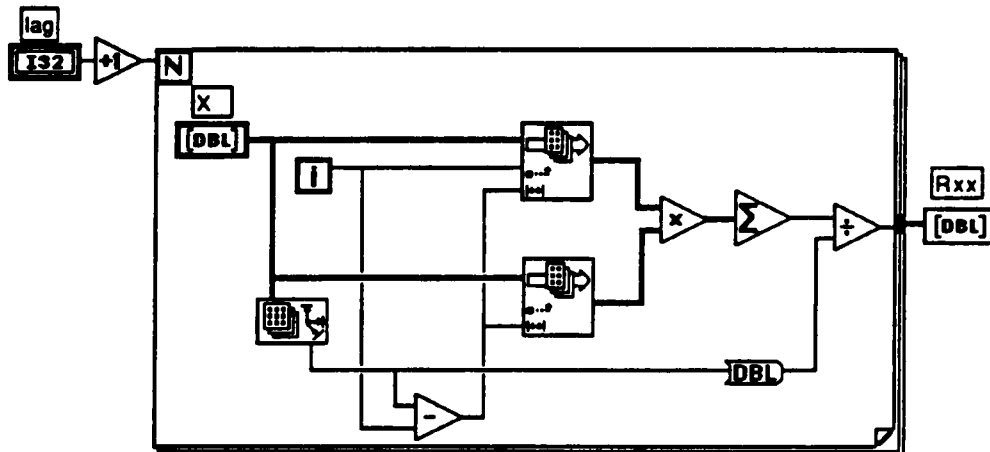
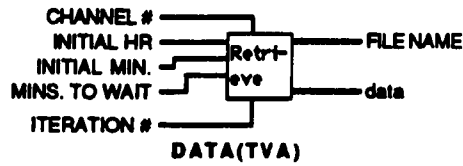
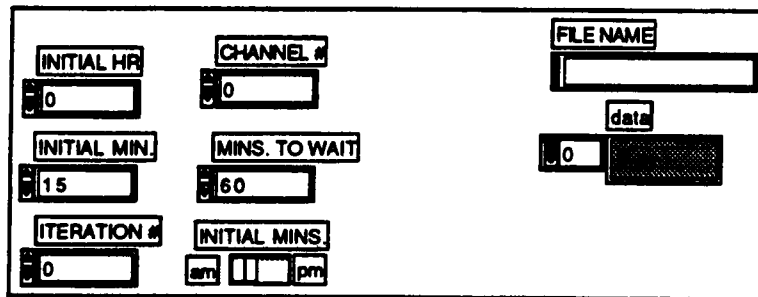


Figure B.2.1 Connector Pane, Front Panel, and Block Diagram of Autocorrelation VI

Connector Pane



Front Panel



Block Diagram

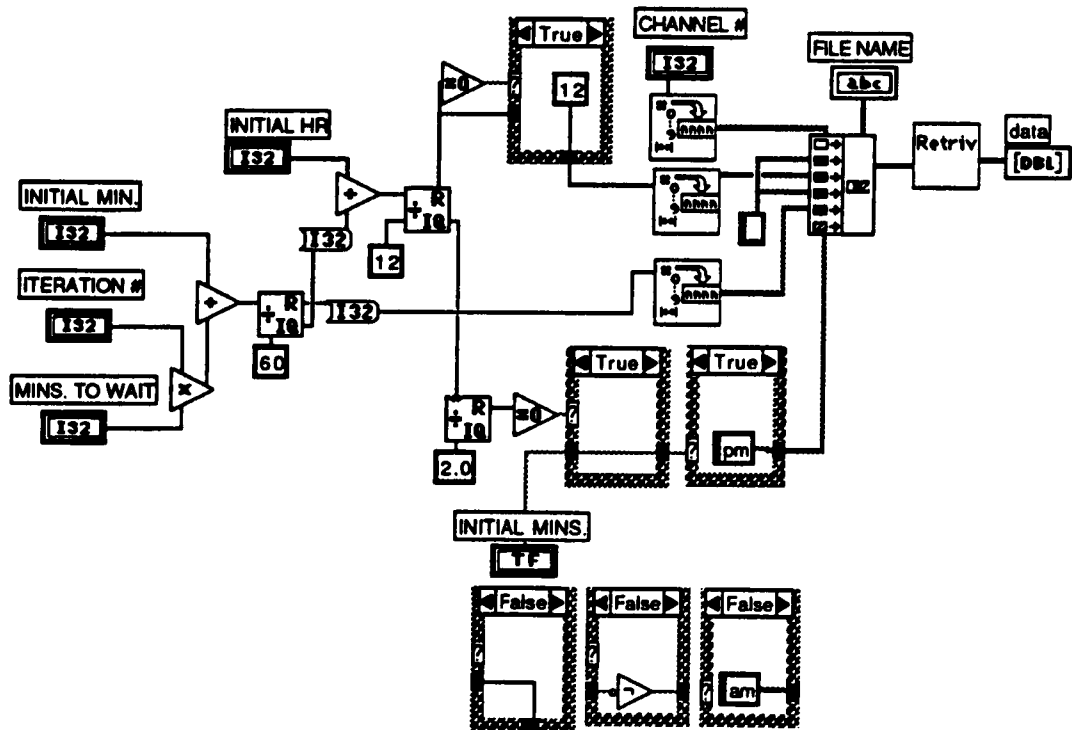
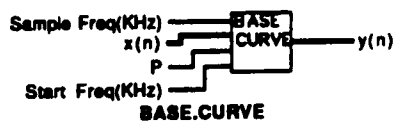
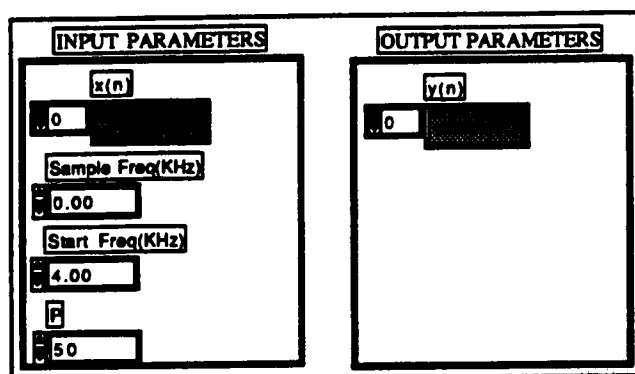


Figure B.3.1 Connector Pane, Front Panel, and Block Diagram of DATA(TVA) VI

Connector Pane



Front Panel



Block Diagram

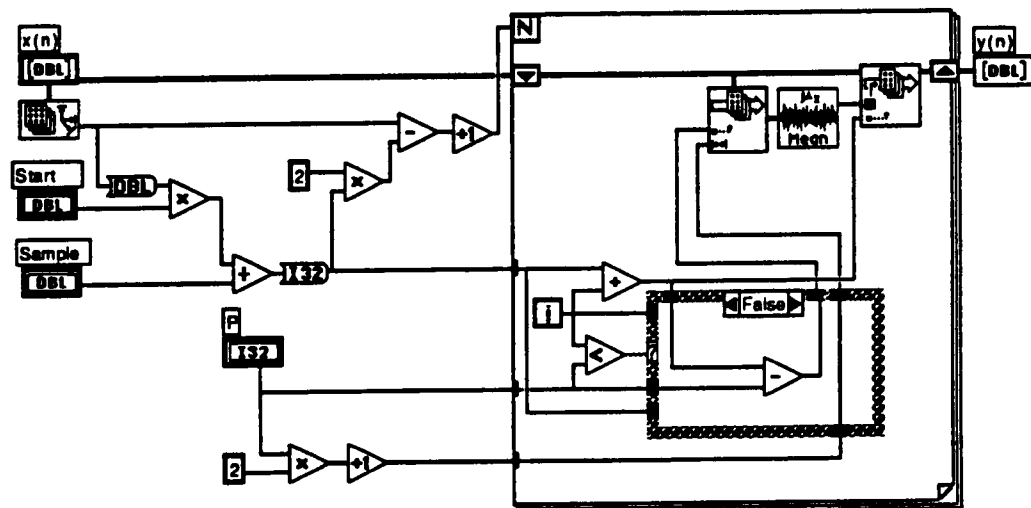
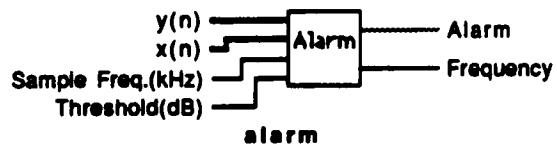
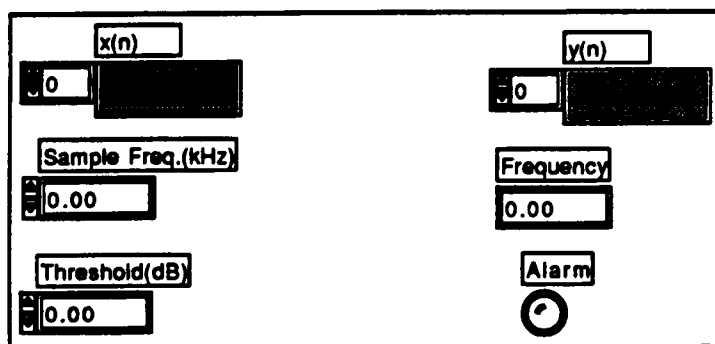


Figure B.3.2 Connector Pane, Front Panel, and Block Diagram of Base Curve VI

Connector Pane



Front Panel



Block Diagram

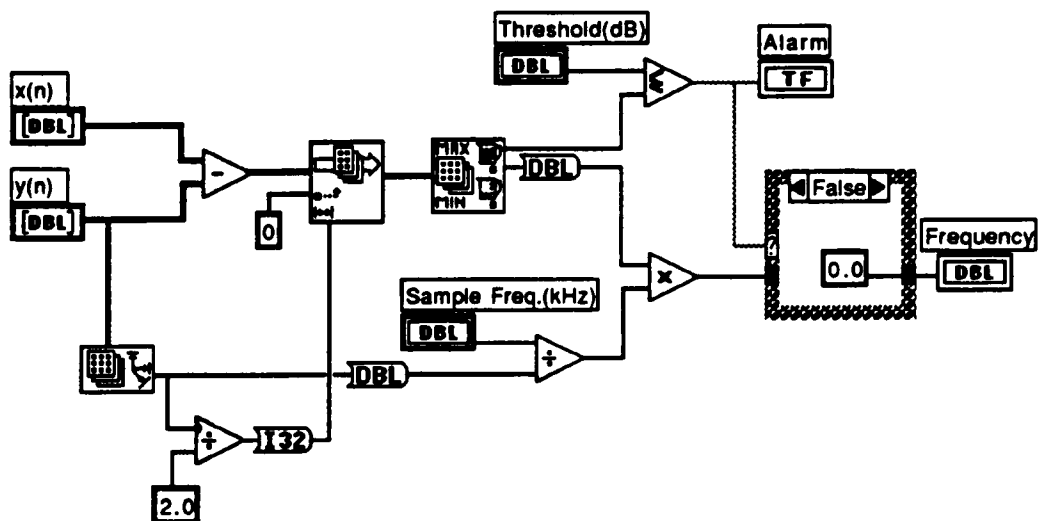


Figure B.3.3 Connector Pane, Front Panel, and Block Diagram of Alarm VI

VITA

Ali Reza Hajialigol was born in Iran on November 27, 1961. He graduated from high school in 1980. In the fall of 1992, he entered the University of Tennessee, Knoxville, with a Bachelor of Science degree in Electrical Engineering from the same university, to pursue a Master of Science degree in Electrical Engineering. He received his M.S. in Electrical Engineering from the University of Tennessee, Knoxville, in May 1995.