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Investigations on Hydrocyclones for the Spallation Neutron Source

A Thesis
Presented for the
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Lee Henry Hosack
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Abstract

The hydrocyclone is commonly used to separate oil from water, and particulates from fluid streams in various process industries. Two-fluid hydrodynamic theory is used here to develop a model for hydrocyclone performance in the application of separation of helium bubbles larger than 30 micron in diameter from liquid mercury at system pressure near 1 bar. The application is related to high power liquid metal target development for proton beams used in spallation neutron sources.

Table of Contents

LIST OF FIGURES	V
1 INTRODUCTION.....	1
1.1 BACKGROUND.....	1
1.2 PURPOSE AND ORGANIZATION	2
1.3 INTRODUCTION TO CYCLONE SEPARATION	2
1.4 SCOPE OF WORK	3
2 FLOW CHARACTERIZATION	7
2.1 BASIC MODELS	7
2.2 BOUNDARY LAYER EXAMINATION.....	10
2.3 FORCED VORTEX FLUID THICKNESS	12
2.4 UNDERSTANDING THE GAS CORE.....	15
3 BUBBLE SEPARATION	21
4 CONCLUSIONS AND FUTURE WORK.....	24
LIST OF REFERENCES	25
LIST OF REFERENCES.....	26
APPENDIX.....	27
APPENDIX A: MATLAB CODE	28
APPENDIX B: FREE VORTEX SOLUTION	34
VITA.....	37

List of Figures

Figure 1-1: Conical Hydrocyclone.....	3
Figure 1-2: Hydrocyclone Schematic with Flow Field.....	4
Figure 1-3: Pressure Field on a Bubble in a Hydrocyclone	5
Figure 1-4: Examination of Forces on a Single Bubble.....	6
Figure 2-1: Conical and Cylindrical Hydrocyclone Schematic	7
Figure 2-2: Examination of the Basic Shape of the Velocity Profile	9
Figure 2-3: The Velocity Profile after the Smoothing Process.....	9
Figure 2-4: Boundary Layer Examination	12
Figure 2-5: Hydrocyclone Gas Core	16
Figure 2-6: 2-D Representation of Hydrocyclone Inner Fluid Boundary.....	16
Figure 2-7: 2-D Representation of Hydrocyclone Inner Fluid Boundary.....	17
Figure 2-8: Velocity Profile for Hydrocyclone.....	18
Figure 2-9: Velocity Profile at $k=0.25$	19
Figure 2-10: Velocity Profile at $k=0.5$	20
Figure 2-11: Velocity Profile at $k=0.75$	20
Figure 3-1: Relative Velocity for 30 micron-sized Bubbles.....	22

1 Introduction

1.1 Background

The Spallation Neutron Source (SNS), has completed construction in the Oak Ridge National Laboratory (ORNL), and has recently broken the world record as the most powerful accelerator-based source for neutrons. The neutron source will have a 1-GeV energy proton beam at 1.4 megawatts power delivered to the target. The beam energy is delivered at sixty pulses per second with each pulse lasting one microsecond.

The SNS uses a mercury target for the spallation process. Mercury was chosen because the liquid metal will not have the same radiation damage that is common among solid targets. Mercury also has a high atomic number with an average of 120 neutrons per atom. The proton beam energy raises the temperature and subsequently the pressure of the mercury target during the one microsecond pulse. The short pulse duration does not allow for significant movement of the pressure field. These spatially varying pressure distributions cause high-amplitude pressure waves to propagate in the target and flow delivery system. These large magnitude pressure waves cause cavitation damage to the stainless steel target body. This will reduce target body lifetime, which will reduce beam availability.

One method to mitigate cavitation to the target body is to inject 30 micron diameter gas bubbles into the mercury. A 30 micron diameter helium bubble, with a system pressure of 0.2 MPa has a resonant frequency near 0.5MHz using an isothermal gas process line assumption [1]. Thus, the micro-bubbles can respond to the pressure rise caused by the one-microsecond beam pulse. The desired gas volume fraction is 0.5% to accommodate anticipated mercury expansion. A single bubble of nominal diameter 30 microns has volume equal to 1.13×10^{-13} cubic meters. A sample of one cubic millimeter, or 1.00×10^{-9} cubic meters, must have 44 bubbles to achieve the nominal 0.5% gas volume fraction. The nominal distance between bubble centers will be 350 microns.

1.2 Purpose and Organization

The 30 micron bubbles will protect the target from cavitation by adding compressibility to the mercury. The 30 micron diameter bubbles will protect the target, but if they coalesce and become too large they cannot respond to the one microsecond beam pulse. The larger bubbles may also stratify and cause other problems to the SNS facility. The hydrocyclone is a potential method of removing the larger bubbles.

Bubbles with a diameter less than 100 microns have been shown to dwell in the flow field and do not stratify [1] due to the action of turbulent diffusion. This was shown by using models discussed later in this thesis. By introducing centrifugal acceleration to the fluid, the bubble rise velocity can be increased beyond those associated with buoyant forces to efficiently separate smaller bubbles.

The fluid physics of a hydrocyclone is developed to allow modeling of the separation performance. This thesis assembles fluid physics models and builds tools for designing a hydrocyclone for the SNS facility. When more details of the bubble injection apparatus and the size distribution of the bubble population are known, these tools will be used to design a hydrocyclone for the SNS facility.

1.3 Introduction to Cyclone Separation

Hydrocyclones were originally developed for use in the paper industry to remove debris from liquid paper stock [2]. Cyclone separators have been used in many industries, including paper production, oil/gas separation, mining, and wood processing to remove a discontinuous phase from a continuous phase.

In Figure 1-1 we see an illustration of a conical shaped hydrocyclone where a feed line enters into the cyclone tangent to the inner wall and creates the swirling flow pattern that drives the hydrocyclone.

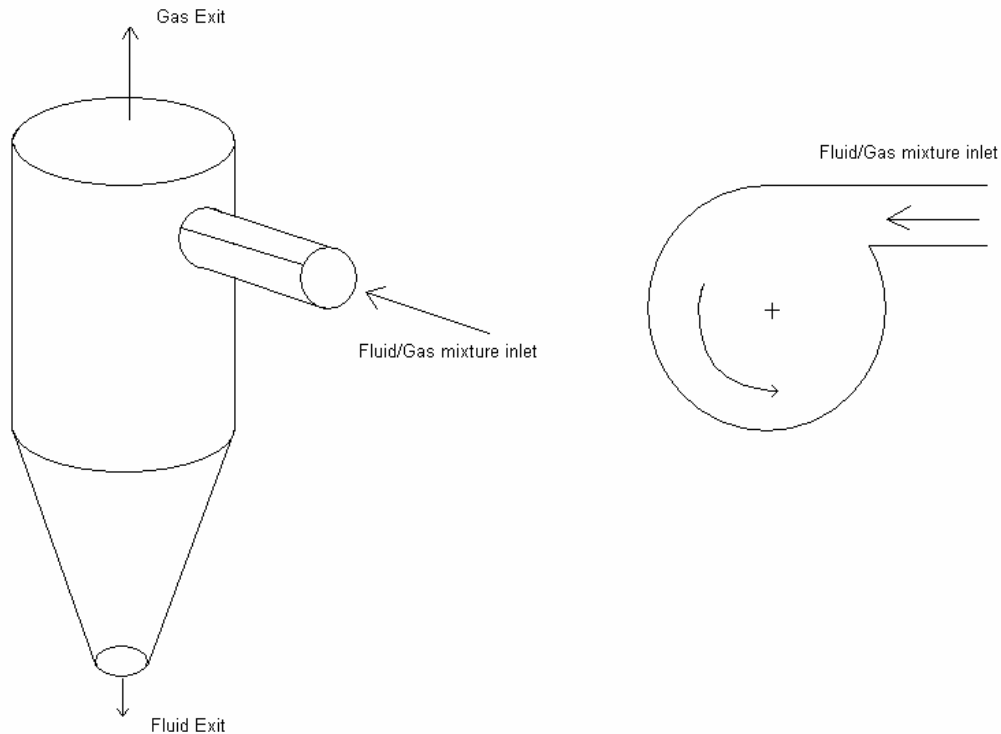


Figure 1-1: Conical Hydrocyclone

The swirling flow pattern in the hydrocyclone is simulated here as the continuous mercury phase. This swirling flow creates increasing pressure with radius within the continuous phase. The gas bubbles experience a drag force due to velocity relative to the fluid as they move toward regions of low fluid pressure. The drag force is due to local velocity fields due to fluid diverting around the bubble. The body forces on the gas are negligible due to the very low helium density relative to the mercury density.

1.4 Scope of Work

The fluid in the cyclone will move due to local acceleration and wall drag. The main accelerations are vertical and radial. The vertical acceleration is due to gravity, and the radial acceleration is centrifugal. An example of a hydrocyclone fluid flow is shown in Figure 1-2.

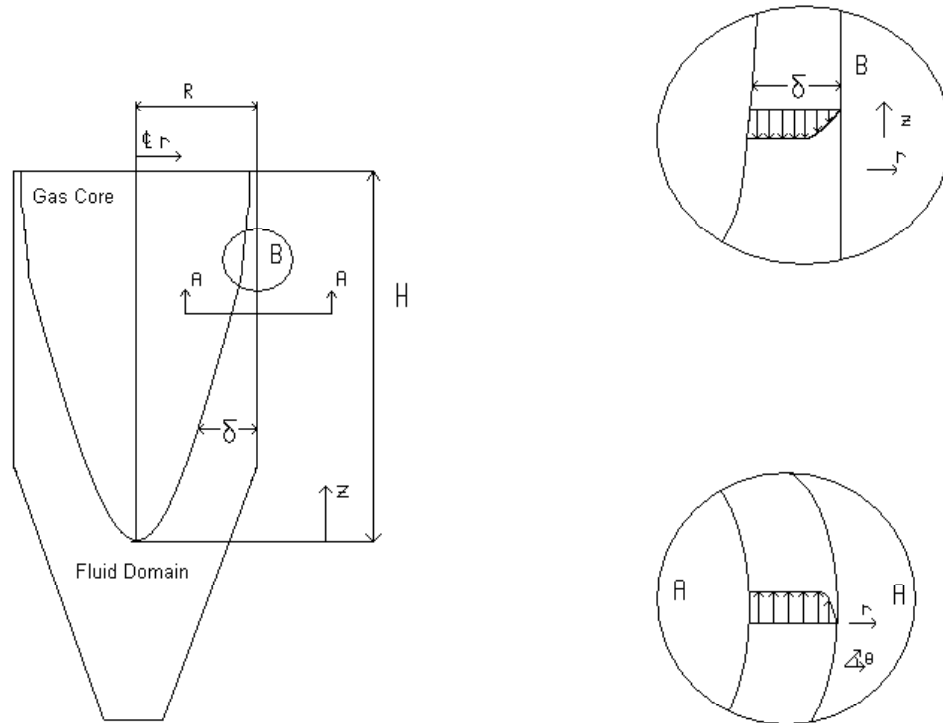


Figure 1-2: Hydrocyclone Schematic with Flow Field

The bubbles in the cyclone will move due to local pressure gradients and inter-phase drag. The local accelerations of the mercury are given as,

Vertical

$$a_z = g \quad (1.1)$$

Radial

$$a_r = \frac{V_{azm}^2}{R} \quad (1.2)$$

These accelerations in the fluid continuous phase cause a pressure gradient, which provides a motivating force on the bubble as shown in Figure 1-3.

The motivating force is balanced by drag between the bubble and the liquid. The balance between motivating force and drag allows the relative velocity to be determined. The

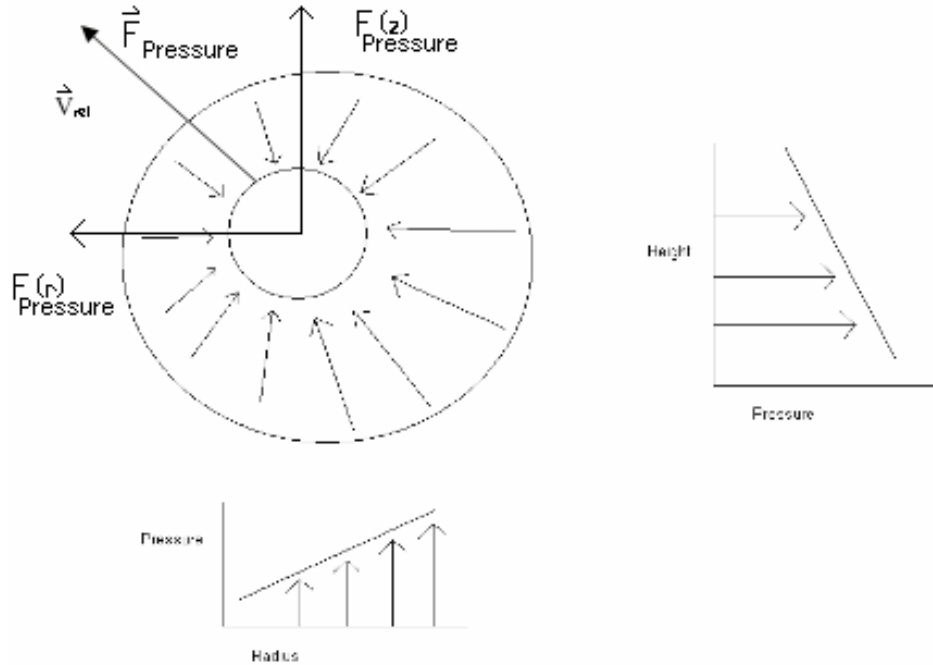


Figure 1-3: Pressure Field on a Bubble in a Hydrocyclone

drag models used are of conventional engineering form, and given as,

Radial

$$F_{R_{drag}} = C_D a_F \frac{1}{2} \rho_l v_R^2 \quad (1.3)$$

Vertical

$$F_{z_{drag}} = C_D a_F \frac{1}{2} \rho_l v_z^2 \quad (1.4)$$

where a_F is the frontal area.

The drag coefficient models used are taken from Wallis [3] and are given as,

For $Re_{bub} > 1000$

$$C_D = \frac{24}{Re_{bub}} (1 + 0.15 * Re_{bub}^{0.687}) \quad (1.5)$$

For $Re_{bub} < 1000$

$$C_D = 0.44 \quad (1.6)$$

These components can be seen in Figure 1-4.

The forces can be rearranged to give a relative velocity due to the drag forces. Where the

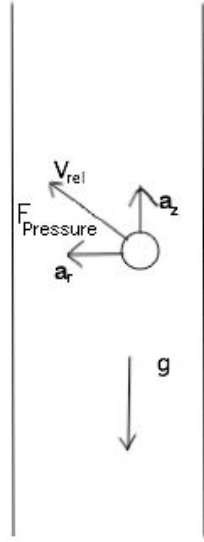


Figure 1-4: Examination of Forces on a Single Bubble

relative velocity in the radial is given as:

$$V_{rr} = \sqrt{\left| \frac{4}{3} \left(\frac{\rho_l - \rho_g}{\rho_l} \right) \frac{a_r D_b}{C_D} \right|} \quad (1.7)$$

and the relative velocity in the vertical direction is given as:

$$V_{rz} = \sqrt{\left| \frac{4}{3} \left(\frac{\rho_l - \rho_g}{\rho_l} \right) \frac{a_z D_b}{C_D} \right|} \quad (1.8)$$

The relative velocity scalar of the bubble from these components is defined as:

$$V_{rel} = \left[V_{rz}^2 + V_{rr}^2 \right]^{1/2} \quad (1.9)$$

This scalar is used to determine the bubble Reynolds number required for selection of drag coefficient models,

$$\text{Re}_{\text{bub}} = \frac{V_{rel} D_{\text{bub}}}{\nu_{Hg}} \quad (1.10)$$

2 Flow Characterization

2.1 Basic Models

Typical hydrocyclones have two characteristic shapes, conical and cylindrical. These shapes can be seen in Figure 2-1. These diagrams show the differences in the hydrocyclone geometries. The cone shape has a bounding radius that changes with height, so R_0 is introduced as a reference radius for the conical hydrocyclone and as the constant radius for the cylindrical hydrocyclone. There are two basic models for hydrocyclone modeling, the forced vortex and the free vortex. The forced vortex model responds as a solid where the velocity is found by the angular frequency multiplied by the radius. The free vortex represents a pressure driven flow with constant vorticity with respect to radius. This free vortex has a velocity profile that is inversely proportional to the radius. A more accurate representation of a hydrocyclone can be found by implementing the two vortex solutions together. The center of the hydrocyclone demonstrates the shape similar to the forced vortex while the outer portion is shaped like the free vortex.[4] The equations that describe the forced vortex and the free vortex are shown in Equations 2.1 and 2.2, respectively.

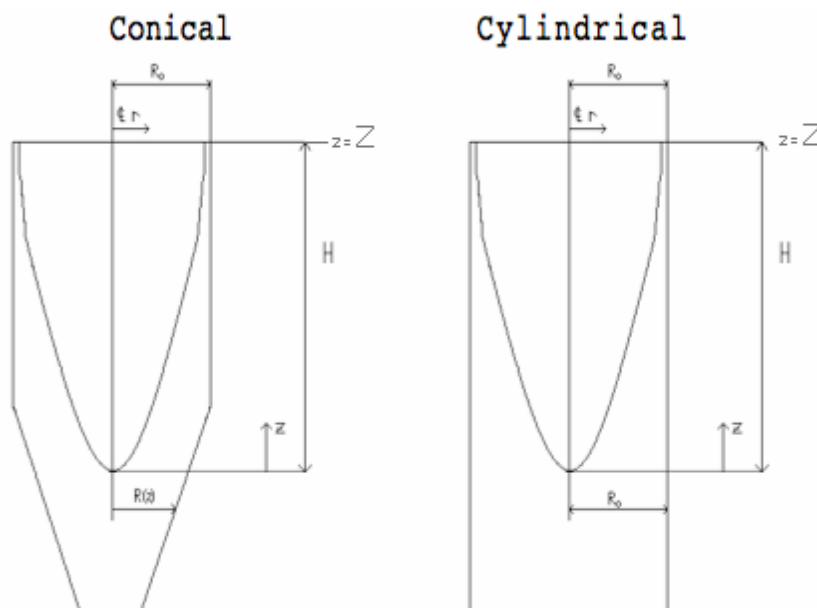


Figure 2-1: Conical and Cylindrical Hydrocyclone Schematic

$$v_{\theta} = \omega r \quad (2.1)$$

$$v_{\theta} = \frac{C_2}{r} \quad (2.2)$$

The solutions can be matched together at a specified transition radius. The value and slope can be matched together to find the appropriate representation of the fluid behavior. A Hydrocyclone literature review showed velocity measurements that indicate the transition radius behaves as a function of r/R , defined as the non-dimensional radius [4,5,6,7]. The non-dimensional radius values for transition range from 0.2 to 0.35. The value of 0.25 is used for this evaluation. The outermost velocity is kept constant for these evaluations. This is done by having a constant ωR_0 , such that:

$$\omega = \frac{C_3}{R_0} \quad (2.3)$$

This gives a profile where the velocity magnitude does not change with the cyclone outer radius. The value of 30 is chosen for C_3 , and the cyclone radius for these examples will be 0.01 m. The plots show the units in millimeters and the basic velocity profile is featured in Figure 2-2. This velocity profile is consistent with a cylindrical hydrocyclone at the bottom where the gas cone ends.

Data from flows in hydrocyclones reveal that a discontinuity in slope does not exist at the transition between characteristic vortex shapes. Therefore a smoothing function is introduced to better characterize the flow in the neighborhood of the discontinuity in slope. The smoothing function is a 2nd order polynomial fit that uses neighboring values to fit the data. Figure 2-3 demonstrates the velocity profile after the smoothing process.

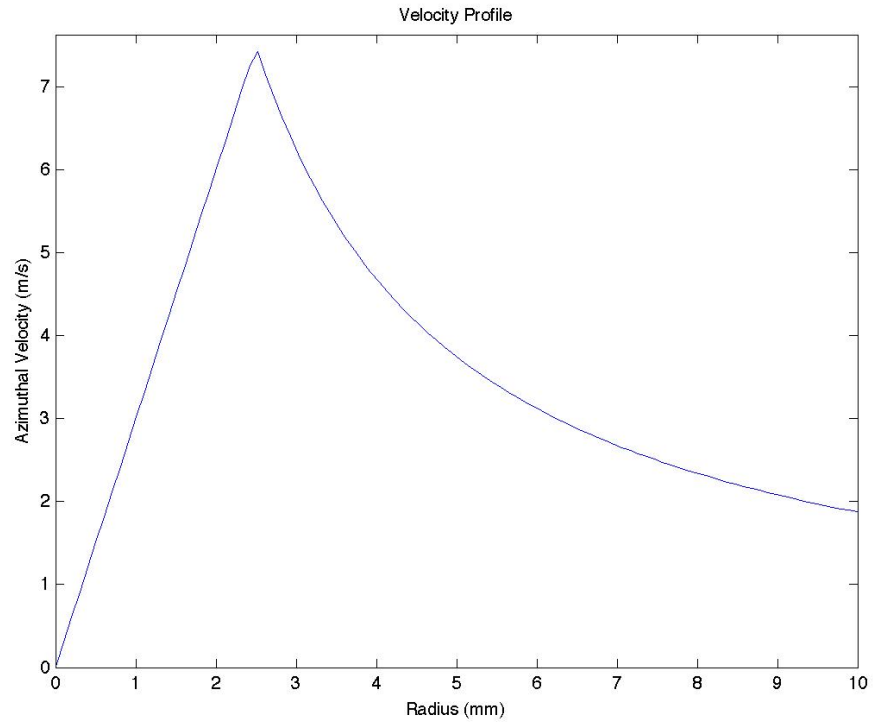


Figure 2-2: Examination of the Basic Shape of the Velocity Profile

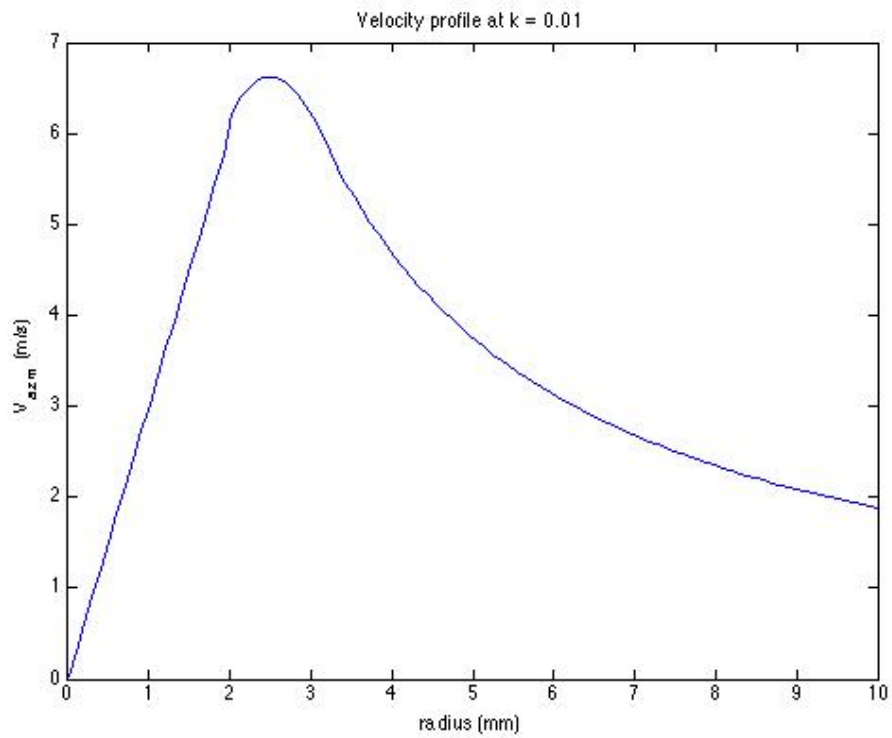


Figure 2-3: The Velocity Profile after the Smoothing Process

2.2 Boundary Layer Examination

The next step is to model the boundary layer. This is accomplished by implementing the distribution described by Martinelli.[8] Martinelli uses a dimensionless turbulent velocity based on the shear velocity to characterize the velocity profile for fully developed turbulent flow. The shear velocity is defined as,

$$u^* = \sqrt{\left(\frac{\tau_0}{\rho}\right)} \quad (2.4)$$

The Reynolds number for the flow can be approximated using an open channel model where the film thickness would represent the open channel characteristic length, such that the hydraulic diameter is given as,

$$D_H = \frac{4A_{xsFlow}}{P_{wet}} = 4\delta \quad (2.5)$$

allowing the Reynolds number to be expressed as,

$$Re = \frac{V_{azm} 4\delta}{\nu} \quad (2.6)$$

The average circumferential fluid velocity of the cyclone is used to calculate the Reynolds number. The Reynolds number model allows approximation of the wall friction factor and wall shear stress in Equation 8. The wall friction factor is modeled as a piecewise function of Reynolds number and is shown in Equations 2.7 and 2.8. Equation 2.8 uses the Moody approximation for the implicit Colebrook Equation. The value of 1.5 microns is assumed for the surface roughness height. This assumption must be re-evaluated during experimentation for better results.

For $Re < 2000$

$$f = \frac{64}{Re} \quad (2.7)$$

For $Re > 2000$

$$f = 5.5(10^{-3}) * \left[1 + \left(2(10^4) * \frac{h}{D_H} + \frac{10^6}{\text{Re}} \right)^{1/3} \right]$$

(2.8)

The wall shear can be related to friction factor from a steady momentum balance for flow between parallel plates, with space between plates taken as 2δ .

$$\tau_o = \frac{\rho V_{azm}^2 f}{8\delta}$$

(2.9)

The dimensionless velocity is found in Equation 2.10.

$$v^+ = \frac{v}{u^*}$$

(2.10)

The distance from the wall can be nondimensionalized as shown in Equation 2.11.

$$y^+ = (R - r) \frac{u^*}{\nu}$$

(2.11)

Where ν is the kinematic viscosity. The dimensionless velocity is shown as:

For $y^+ < 5$

$$v_z^+ = y^+$$

For $5 < y^+ < 30$

$$v_z^+ = -3.05 + 5.0 * \ln(y^+)$$

(2.12)

Solving these equations for the velocity and normalizing to the flow velocity results in the velocity profile shown in Figure 2-4.

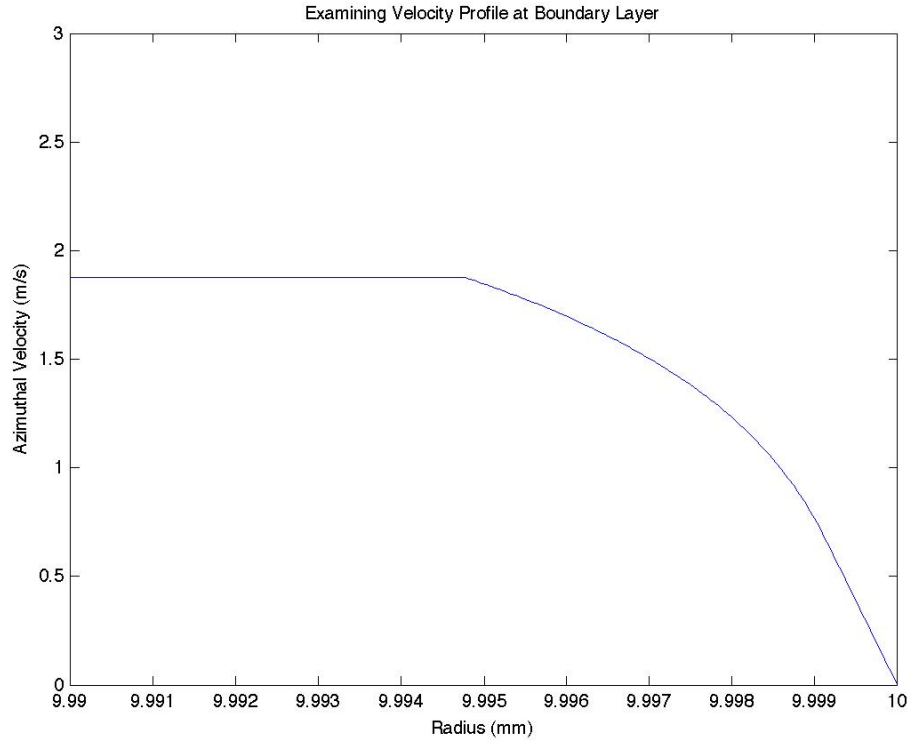


Figure 2-4: Boundary Layer Examination

2.3 Forced Vortex Fluid Thickness

This section will focus on the derivation of the forced vortex solution and implementing it to find the fluid thickness as a function of height. Starting with the Continuity Equation for Incompressible Fluids,

$$\nabla \cdot \vec{v} = 0 \quad (2.13)$$

To maintain a general form, this will be evaluated for a cylindrical hydrocyclone. The Continuity Equation in cylindrical coordinates,

$$\frac{1}{r} \frac{\partial}{\partial r} r v_r + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0 \quad (2.14)$$

The forced vortex solution assumes a steady state condition, where v_r and v_z are zero and that v_θ is only a function of radius.

Next the Motion Equation for Incompressible Fluids is introduced,

$$\rho \frac{D\vec{v}}{dt} = -\nabla P + \mu \nabla^2 \vec{v} + \rho \vec{f} \quad (2.15)$$

R component:

$$\rho \left[\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r} \right] = -\frac{\partial P}{\partial r} + \mu \left[\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} + \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} - \frac{v_r}{r^2} \right] + \rho f_r \quad (2.16)$$

for radially symmetric inviscid flow this simplifies to:

$$\rho \frac{v_\theta^2}{r} = \frac{\partial P}{\partial r} \quad (2.17)$$

θ component:

$$\rho \left[\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + v_z \frac{\partial v_\theta}{\partial z} - \frac{v_\theta v_r}{r} \right] = -\frac{1}{r} \frac{\partial P}{\partial \theta} + \mu \left[\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial v_\theta}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2} \right] + \rho f_\theta \quad (2.18)$$

and for radially symmetric inviscid flow this simplifies to:

$$0 = \frac{d}{dr} \left(\frac{1}{r} \frac{d}{dr} (v_\theta r) \right) \quad (2.19)$$

Z component:

$$\rho \left[\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right] = -\frac{1}{r} \frac{\partial P}{\partial z} + \mu \left[\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right] + \rho f_z \quad (2.20)$$

which for radially symmetric inviscid flow this simplifies to:

$$\frac{\partial P}{\partial z} = -\rho g \quad (2.21)$$

Integrating Equation 2-19:

$$v_\theta = C_1 r + \frac{C_2}{r} \quad (2.22)$$

For forced vortex conditions, the velocity at $r=0$ is not infinite. Therefore the assumption that $C_2=0$ is introduced. From this let $C_1=\omega$ as found in solid mechanics. Where ω is the angular frequency in radians per second. This results in the following:

$$v_{\theta} = \omega r \quad (2.23)$$

Substituting Equation 2-22 into Equation 2-17 yields:

$$\frac{\partial P}{\partial r} = \rho \omega^2 r \quad (2.24)$$

which integrates to

$$P = \frac{\rho \omega^2 r^2}{2} + f(z) \quad (2.25)$$

$$f(z) \text{ is found by integrating } \frac{\partial P}{\partial z}. \quad (2.26)$$

$$\frac{\partial P}{\partial z} = -\rho g \quad (2.27)$$

$$f(z) = -\rho g z + C_3 \quad (2.28)$$

Substituting 2-28 into 2-25:

$$P = \frac{\rho \omega^2 r^2}{2} - \rho g z + C_3 \quad (2.29)$$

The free surface boundary condition is used to find C_3 .

At $z=Z$ and $r=R$ then $p=p_{atm}$. Where R is the outer radius of the cyclone and Z is the maximum liquid level height.

$$P_{atm} = \frac{\rho \omega^2 R^2}{2} - \rho g Z + C_3 \quad (2.30)$$

Taking the difference between P and P_{atm} or Equations 29 and 30,

$$P - P_{atm} = \frac{\rho \omega^2 (r^2 - R^2)}{2} + \rho g (Z - z) \quad (2.31)$$

The free surface can be found where $P=P_{atm}$.

$$\frac{\rho \omega^2 (r^2 - R^2)}{2} = \rho g (z - Z) \quad (2.32)$$

Solving for the height of free surface ($Z-z$), when $r=0$:

$$(Z - z) = \frac{\omega^2 R^2}{2g} \quad (2.33)$$

This radial film thickness, δ , can be solved as a function of vertical position, z .

$$\delta(z) = R - \sqrt{R^2 + \frac{2g}{\omega^2}(z - Z)} \quad (2.34)$$

Using the definition for k :

$$k = z/H \quad (2.35)$$

where:

$$H = Z - z \quad (2.36)$$

Such that the film thickness can be expressed as,

$$\delta(k) = R - \sqrt{R^2 - R^2(1 - k)} \quad (2.37)$$

2.4 Understanding the Gas Core

One of the key characteristics of a hydrocyclone is the gas core. The fluid thickness as described by the forced vortex can be seen in Equation 2-39 as derived in Section 2.3.

$$\delta(k) = R - \sqrt{R^2 - R^2(1 - k)} \quad (2.39)$$

where $k = \frac{z}{H}$

Figure 2-5 shows the hydrocyclone gas core. Figure 2-6 represents the outline of the liquid boundary with the wall in blue and the free surface in red.

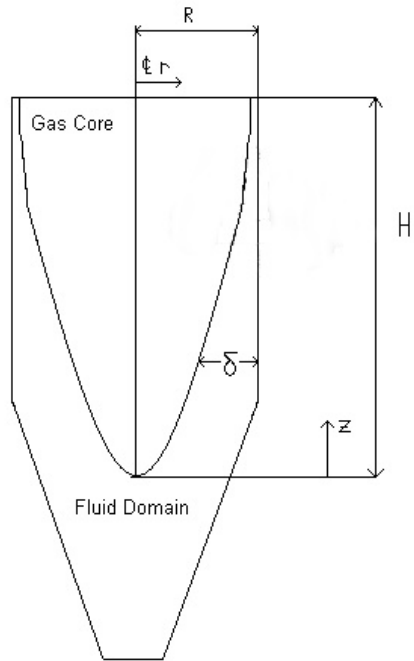


Figure 2-5: Hydrocyclone Gas Core

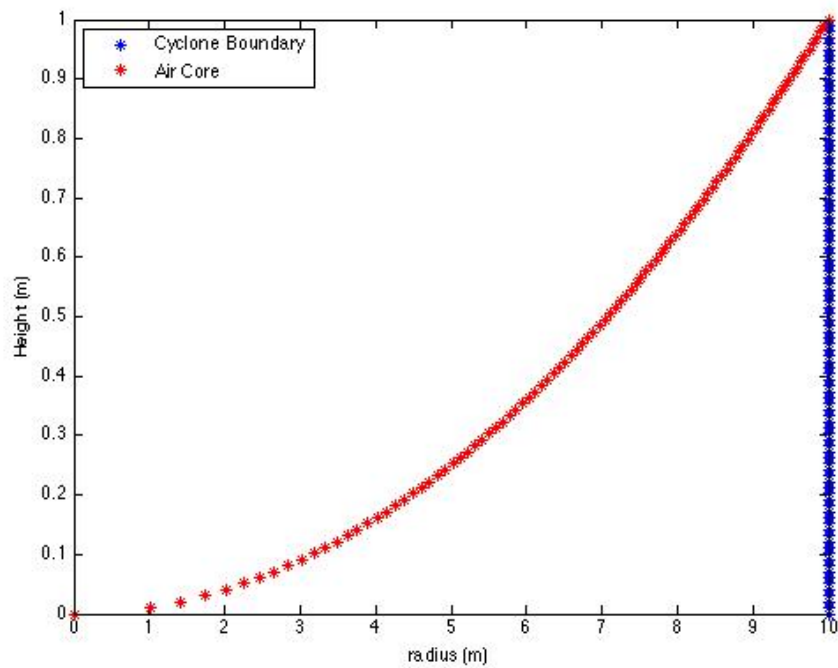


Figure 2-6: 2-D Representation of Hydrocyclone Inner Fluid Boundary.

For this application the gas core boundary represents the free surface between mercury and the helium gas core. This free surface is characterized by the isobar corresponding to the gas core pressure. A cone-shape is often added to the cyclone boundary shape to reduce liquid thickness near the bottom, which helps improve separation, by reducing the distance the gas bubble must travel to the free surface. Figure 2-7 shows the conical shape of the cyclone body and how this changes the gas core solution.

This body shape gives a more uniform fluid thickness than the uniform radius. The velocity can be found for this cyclone body shape as shown in Figure 2-8. The non-dimensional radius value of 0.25 is maintained for the transition between the forced and free vortex solutions. The angular velocity, ω , is kept constant throughout the height of the hydrocyclone for these models. These assumptions will be reviewed and tuned during planned experimentation to better understand flow conditions in the hydrocyclone.

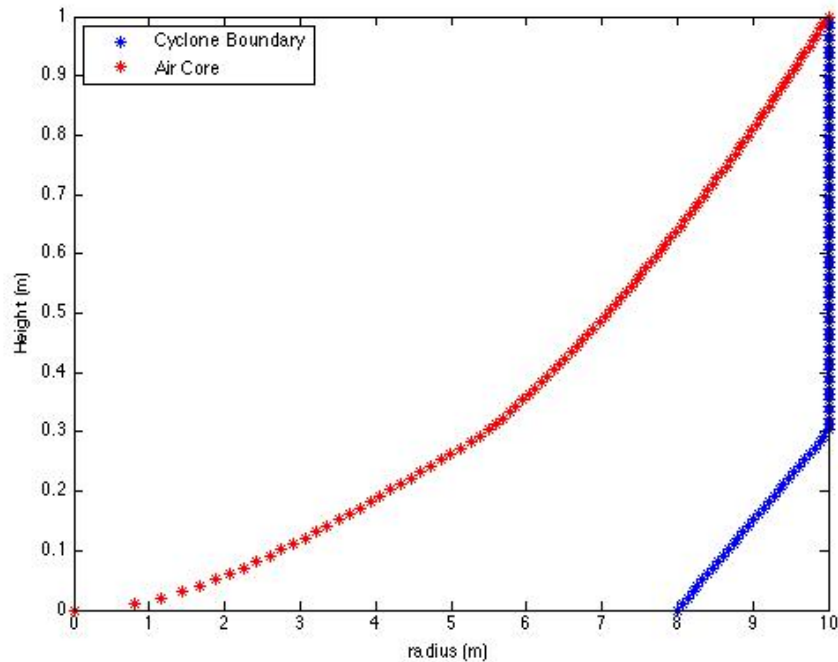


Figure 2-7: 2-D Representation of Hydrocyclone Inner Fluid Boundary.

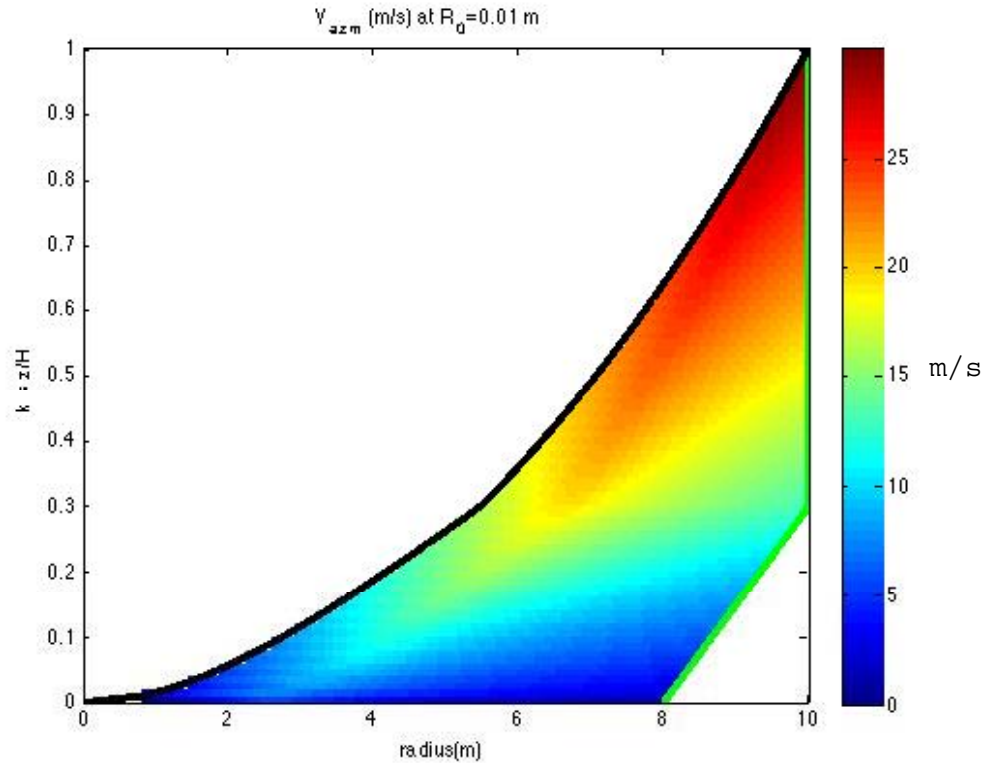


Figure 2-8: Velocity Profile for Hydrocyclone

Figure 2-8 shows the general velocity profile in the radial and axial directions. Figures 2-9, 2-10 and 2-11 show the velocity profile for fixed axial positions to better display information for comparison. The red stars on the graphs represents the liquid volume average azimuthal velocity of the axial slice. These average velocities can also be averaged for the various axial positions to give a single scalar value to represent the hydrocyclone performance. A MATLAB code has been used to generate these plots and is provided in Appendix A.

Literature supports that these characteristic velocity profiles roughly represent the velocity distributions in a hydrocyclone.[5,6] These experiments were performed with Laser Doppler Velocimeters (LDV). These predictions lie within the range of data collected and could be modified to more accurately represent a single experiment. The goal of this portion of the effort is to generate models suited to design that can be first

tuned in a water experiment designed to emulate SNS gas separation challenges, and then use the tuned models to design a hydrocyclone for the SNS.

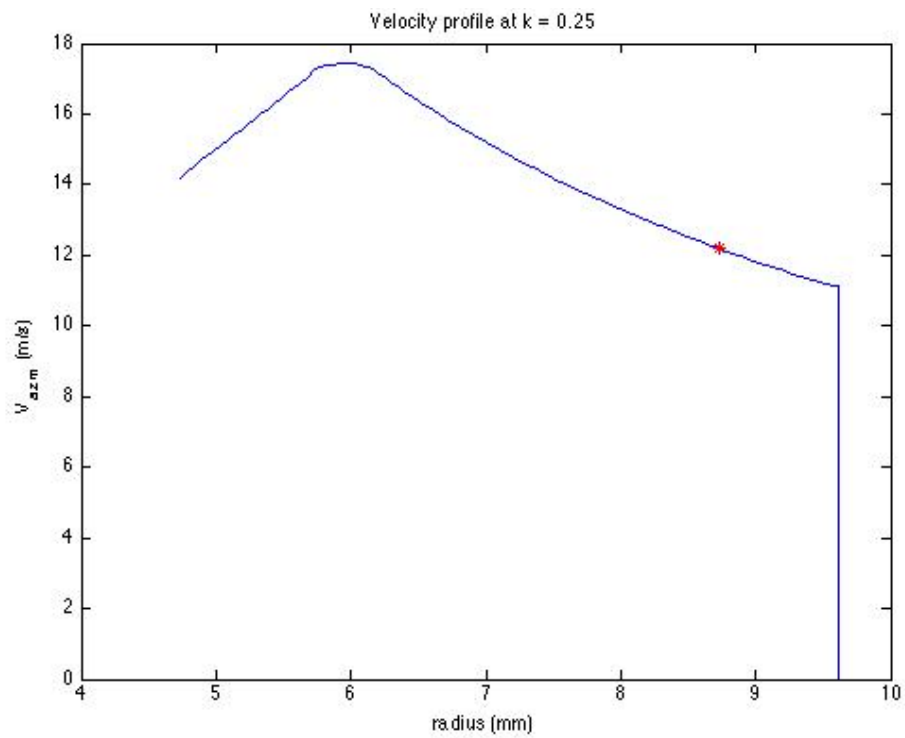


Figure 2-9: Velocity Profile at $k=0.25$

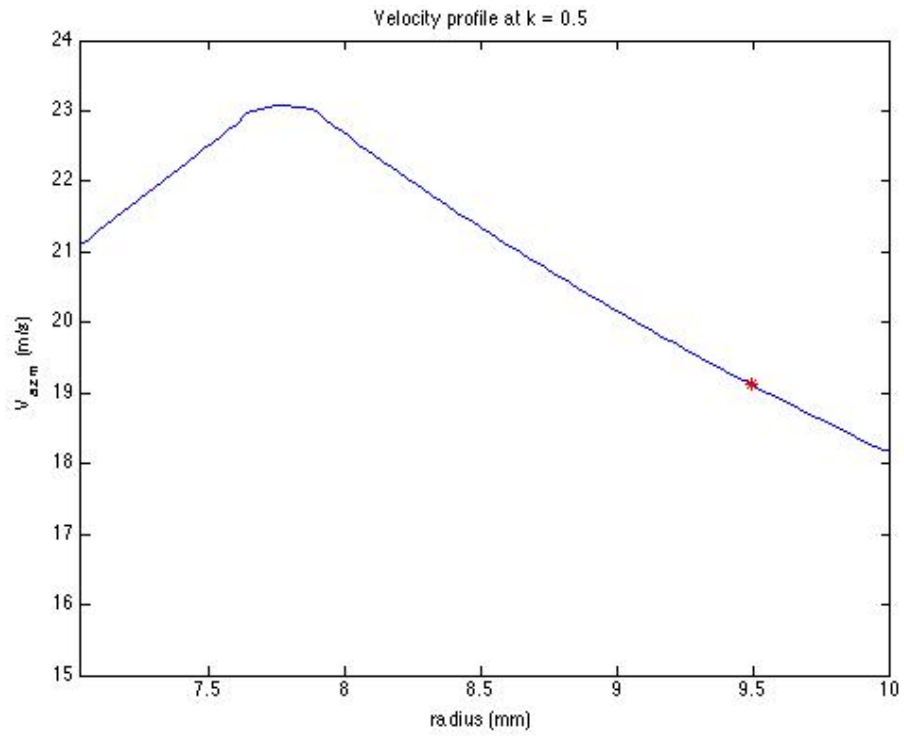


Figure 2-10: Velocity Profile at $k=0.5$

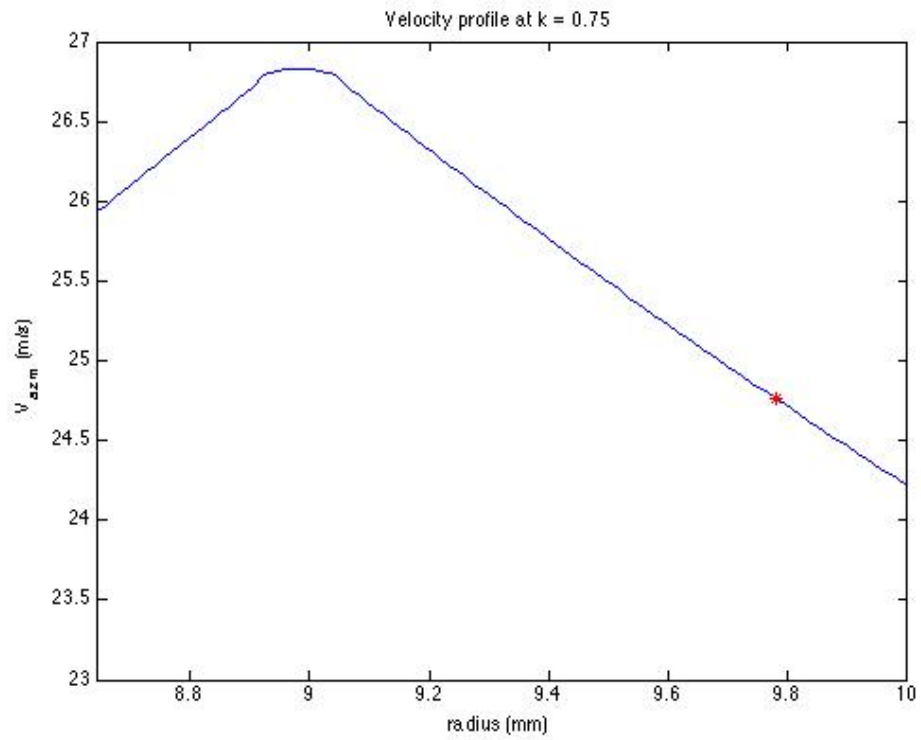


Figure 2-11: Velocity Profile at $k=0.75$

3 Bubble Separation

The velocity components of the bubble relative to the liquid for the radial and azimuthal directions are given as:

$$V_{rr} = \sqrt{\left| \frac{4}{3} \left(\frac{\rho_l - \rho_g}{\rho_l} \right) \frac{a_r D_b}{C_D} \right|} \quad (3.1)$$

$$V_{rz} = \sqrt{\left| \frac{4}{3} \left(\frac{\rho_l - \rho_g}{\rho_l} \right) \frac{a_z D_b}{C_D} \right|} \quad (3.2)$$

These relative velocity components can be combined into a scalar value as shown in Equation 3.3.

$$V_{rel} = \left[V_{rz}^2 + V_{rr}^2 \right]^{1/2} \quad (3.3)$$

The turbulent diffusion in the cyclonic flow will tend to disperse the bubbles. This would counter the separation and must be modeled. To characterize this, the relative velocity scalar, which represents the separation, is compared with the shear velocity, which represents the turbulent diffusion. Thomas discusses similar physics for stratification in open channel flows, pipes, and mixing tanks.[9] Equation 3.4 shows the ratio of the relative velocity over the shear velocity. Thomas reports a value of 0.2 as the limiting value for separation. This would mean that design conditions where the ratio has a value lower than 0.2 do not separate. The equation for the separation ratio is given as:

$$q = \frac{V_{rel}}{u_*} \quad (3.4)$$

The relative velocity is plotted in Figure 3-1.

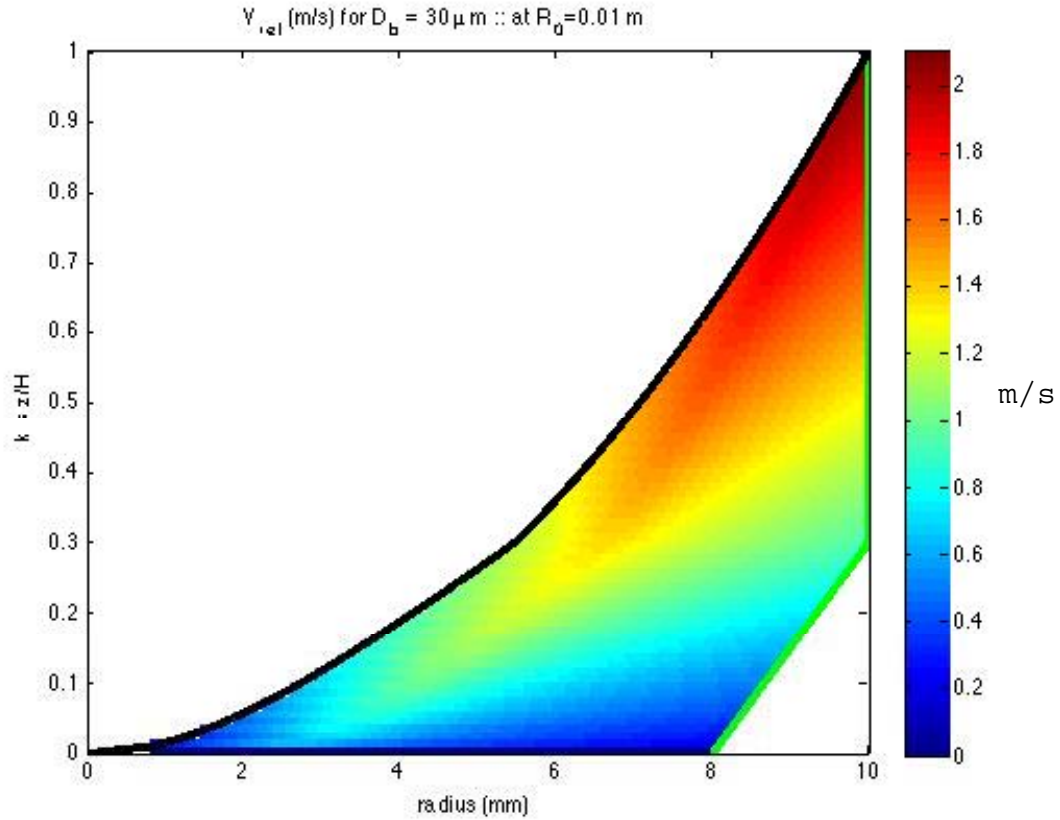


Figure 3-1: Relative Velocity for 30 micron-sized Bubbles

The average velocity for each radial and axial position in the hydrocyclone is presented in Figure 3-1. By comparing the relative velocity and the shear velocity the separation ratio can be evaluated. For the $R_0=0.01$ m geometry, the separation ratio is found to be 0.1755 for a 30 micron diameter bubble. These calculations were repeated for various cyclone radii to find the smallest bubble diameter with a separation ratio equal to the limiting value, 0.2. Table 3-1 shows the results from the calculations.

Table 3-1: Results from simulations

R_0 (m)	Bubble Diameter (μm)
0.01	32
0.1	51
0.5	69
1	75
2	79

4 Conclusions and Future Work

Models are developed for hydrocyclone performance simulation and design. These models include consideration of turbulent diffusion effects on the cyclone separation performance, which is not considered in other closed form modeling approaches sited to design activities. Computational fluid dynamic approaches can also simulate the turbulent diffusion, or so-called counter gradient diffusion, but only if rather specialized modeling approaches are used, such as large eddy simulation. These approaches are computationally cumbersome, and the prediction of the free surface contour, with related turbulence suppression at the free surface, is also very challenging. The modeling approach developed here is better suited to design applications.

These models remain unverified for this application. An experiment has been designed and is in construction to verify the models presented herein first using a water flow. The hydrocyclone is clear to allow optical measurement. The contour of the free surface will be measured, and particle image velocimetry (PIV) will be used to measure velocity profiles. Bubble formation and injection techniques will also be tested in this facility, and these bubbles will serve as optical targets for the PIV. A mercury hydrocyclone will then be designed for installation in the full scale mercury target test facility (TTF) at ORNL. Data from that experimental campaign will then be used to tune the final design for installation in the SNS target flow loop.

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APPENDIX

Appendix A: MATLAB code

```
% Lee Hosack
% 7/10/07

clc
clear

R0=0.01;
% Transition Radius
tf=0.25;
rt=R0.*tf;
% Factor for increasing resolution for boundary layer
Factor=0.9995;
% F1 - Fraction 1: Fraction for smoothing (x direction)
F1=0.05;
% F2 - Fraction 2: Fraction for smoothing (y direction)
F2=0.15;

% Delta calculational Method
% 1 - Forced Vortex Solution
% 2 - Free Vortex Solution
DM=1;

% Save Figures
% print = 0 - Figures Not Saved
% print = 1 - Save Figures
print=0;
plotfigs=1;
% Non-dimensional air coreheight - fraction of distance measured from
the
% bottom of the core (increasing ratio=decreasing height)

%% 'Fixed'
% gravity
g=9.81;
% Density (kg/m^3)
rho=13498.9;
% Kinematic viscosity (m^2/s)
nu=0.0015624./rho;
% Epsilon
e=0.00000001;
% Forced vortex constant - omega (rad/sec)
C2=30./R0;
% Free Vortex Constant
C1=C2.*(rt)^2;

k=linspace(0,1,100);
figure(2)
for cc=1:length(k)

    if k(cc)>0.3
        R1(cc)=R0;
    else
        %R1(cc)=R0;
        R1(cc)=0.666667.*k(cc).*R0+.8.*R0;
    end

    R=R1(cc);
%% Initial Velocity Profile
```

```

r1(:,cc)=[linspace(0,R.*Factor,100) linspace(R.*Factor,R,200)];
RL=length(r1);
for i=1:RL
    if r1(i)>rt
        Vazm(i,cc)=C1./r1(i,cc);
    elseif r1(i)<=rt
        Vazm(i,cc)=r1(i,cc).*C2;
    end
end

VL(:,cc)=Vazm(:,cc);

%% Film Thickness
% Air core height

NDD(cc)=1-k(cc);
if DM==1
    H_z(cc)=C2.^2.*R.^2./(2.*g).*NDD(cc);
    delta1(cc)=R-sqrt(R.^2-H_z(cc).*(2.*g./C2.^2));
    delta(cc)=R-sqrt(R.^2-R.^2.*(1-k(cc)));
elseif DM==2
    H=(1./(g.*2.*e.^2)).*C1.^2-1./(g.*2.*R.^2).*C1.^2).*NDD;
    delta=R-e-sqrt(C1.^2./(2.*g.*(H)+C1.^2./R));
end
% Delta Location for plotting
rz(cc)=R-delta(cc);

DeltaLoc_1=find(rz(cc)>=r1(:,cc));
DeltaLoc(cc)=max(DeltaLoc_1);

%% Radius/Velocity Profile
r(:,cc)=[linspace(rz(cc),R.*Factor,100) linspace(R.*Factor,R,200)];
RL=length(r);
rt1(cc)=tf.*(R-rz(cc))+rz(cc);
rt=rt1(cc);
C1=C2.*(rt)^2;

smoothx(cc)=(R-rz(cc)).*F1;

clear CLee
ccc=0;
for i=1:RL
    if r(i,cc)>rt
        Vazm1(i,cc)=C1./r(i,cc);
    elseif r(i,cc)<=rt
        ccc=ccc+1;
        if ccc==1
            RTL=i;
        end
        Vazm1(i,cc)=r(i,cc).*C2;
    end
    if r(i,cc)<=rt-smoothx(cc)
        CLee(i)=i;
    end
end

end

rstart=r(max(CLee),cc);
Vstart=Vazm1(max(CLee),cc);
Vplus=abs((Vazm1(RTL,cc)-Vstart)).*F2;

```

```

Dr(cc)=r(RL)-r(1);
for i=1:RL
    if r(i,cc)>=rt-smoothx(cc) && r(i,cc)<=rt+smoothx(cc)
        Vazml(i,cc)=Vstart+Vplus-Vplus.*(rt-
r(i,cc)).*.7./smoothx(cc)).^2;
    end
end

%% Reynolds Number

% Wall Shear Stress
if cc>1 && cc<length(k)
    %% Average Velocity

mu(cc)=mean(Vazml(:,cc));

MP_1=find(mu(cc)<=Vazml(:,cc));
MP(cc)=max(MP_1);

%% U-star calculation
% Flow Reynolds Number
Re(cc)=Vazml(MP(cc),cc).*4.*(R-r(2))./nu;
if Re(cc)==0
    f1=0;
elseif Re(cc)<=2000
    f1=64/Re(cc);
elseif Re(cc)>2000
    % f1=0.316.*Re(cc).^-.25;
    f1=10^(0.171168713432893*log10(Re(cc))-3.295091007038305);
end

tau=rho.*Vazml(MP(cc),cc).^2.*f1./(8.*(R-r(2)));

% Shear Velocity
ustar=(tau./rho).^0.5;

%% Separation
% Defining acceleration terms
ar=Vazml(:,cc).*2./R;
az(1:RL)=g;
if cc==2
    az=az';
end

% Bubble diameter (m)
Db=3E-5;

% Drag/Relative velocity calculation
res=1;
jj=1;
clear Cd
Cd=12;
count3=0;
res=1;
while res>.0000000000000001
    cd=Cd;
    % Relative Velocity
    Vrr=(abs(4./3.*(rho-1)./rho.*ar.*Db(jj)./(Cd'))).^0.5;
    Vrz=(abs(4./3.*(rho-1)./rho.*az.*Db(jj)./(Cd'))).^0.5;

```

```

Vrel(:,jj)=(Vrr.^2+Vrz.^2).^0.5;
% Reynolds number of bubble
Re_b_r=Db(jj).*Vrel./nu;

%Bubble Drag Models
for iii=1:RL
    if Re_b_r(iii)<1000
Cd(iii)=24./Re_b_r(iii).*(1+0.15.*Re_b_r(iii).^0.687);
    else
        Cd(iii)=.44;
    end
    end
    fdiff=(Cd-cd)./Cd;
    count3=count3+1;
    res=mean(abs(fdiff(2:length(fdiff)))));
end
qk=1;

q1(cc)=Vrel(MP(cc))./ustar;
us(cc)=ustar;
Vrelative(:,cc)=Vrel;

yplus=(R-r(:,cc)).*ustar./nu;

%% Boundary layer calculation
Vp=Vazml(:,cc);
count2=0;
clear matrix2
for i=1:RL
    if yplus(i)==0
        Vazml(i,cc)=0;
    elseif yplus(i)<=5
        count2=count2+1;
        Vazml(i,cc)=(yplus(i)).*ustar;
        matrix2(count2)=i;
    elseif yplus(i)<=30
        count2=count2+1;
        matrix2(count2)=i;
        Vazml(i,cc)=ustar.*(5.*log(yplus(i))-3.05);
    end
end

% boundary layer transition position
BLS=min(matrix2);
% ratio for normalizing boundary layer
ratio(cc)=Vazml(BLS-1,cc)./Vazml(BLS,cc);

% Normalizing Boundary layer
for i=1:count2
    j=matrix2(i);
    Vazml(j,cc)=Vazml(j,cc).*ratio(cc);
end

end

if rem(cc,2)==0
    h=pcolor(1E3.*r(:,cc),k(cc-1:cc),(Vazml(:,cc-1:cc)'));
    hold on
end

if rem(cc,2)==1 && cc>1
    h=pcolor(1E3.*r(:,cc-1),k(cc-1:cc),(Vazml(:,cc-1:cc)'));

```

```

        hold on
    end

end

summ=0;
for j=1:RL
    for i=1:length(k)
        summ=summ+Vazm(j,i);
    end
end
averageVazm=summ/(length(k).*RL)

averageRe=mean(Re)
q=mean(q1)

%VolumetricFlowRate=averageRe.*nu.*pi.*R0.*.25

importance=[R0 Db.*1E6 q];

plot(1E3.*R1,k,'g',1E3.*rz,k,'k','LineWidth',3);

caxis;
colorbar;
axis([0 1E3.*R0 0 1]);

shading flat
title(['V_a_z_m (m/s) at R_0=' num2str(R0) ' m'])
colormap jet
xlabel('radius(m)')
ylabel('k \equiv z/H')
hold off

if plotfigs==1

    figure(1)
    % plot(R1,max(H_z)-H_z,'b*',rz,max(H_z)-H_z,'r*')
    plot(1E3.*R1,k,'b*',1E3.*rz,k,'r*')

    xlabel('radius (m)')
    ylabel('Height (m)')
    legend('Cyclone Boundary','Air Core','location','NW')

    figure(3)
    LL=75;
    plot(1E3.*r(:,LL),Vazm1(:,LL));
    hold on
    if LL>1 && LL<length(k)
        plot(1E3.*r(MP(LL),LL),Vazm1(MP(LL),LL),'r*');
    end
    hold off
    ylabel('V_a_z_m (m/s)')
    xlabel('radius (mm)')
    title(['Velocity profile at k = ' num2str(LL/length(k))])
    axis([1E3.*r(1,LL) 1E3.*max(r(:,LL)) 23 27])

    figure(4)

    for cc=2:length(k)-1
        if rem(cc,2)==0
            h=pcolor(r(:,cc).*1E3,k(cc-1:cc),(Vrelative(:,cc-1:cc)));
            hold on

```

```

end

if rem(cc,2)==1 && cc>1
    h=pcolor(r(:,cc-1).*1E3,k(cc-1:cc),(Vrelative(:,cc-1:cc)'));
    hold on
end
end

plot(Rl.*1E3,k,'g',rz.*1E3,k,'k','LineWidth',3);

caxis;
colorbar;
axis([0 R0.*1E3 0 1]);

shading flat
title(['V_r_e_l (m/s) for D_b = ' num2str(Db.*1E6) '\mum :: at R_0='
num2str(R0) ' m'])
colormap jet
xlabel('radius (mm)')
ylabel('k \equiv z/H')
hold off
end

```

Appendix B: Free Vortex Solution

Continuity Equation for Incompressible Fluids

$$\nabla \cdot \vec{v} = 0$$

In cylindrical coordinates

$$\frac{1}{r} \frac{\partial}{\partial r} r v_r + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} = 0$$

Assuming a steady state condition where v_z is zero and that v_θ is only a function of radius. Leaving the radial term.

$$\frac{1}{r} \frac{\partial}{\partial r} r v_r = 0$$

Which integrates to:

$$v_r = \frac{C_1}{r}$$

Motion Equation for Incompressible Fluids

$$\rho \frac{D\vec{v}}{dt} = -\nabla P + \mu \nabla^2 \vec{v} + \rho \vec{f}$$

R component:

$$\rho \left[\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r} \right] = -\frac{\partial P}{\partial r} + \mu \left[\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} + \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} - \frac{v_r}{r^2} \right] + \rho f_r$$

Simplifies to:

$$\rho \left[\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} - \frac{v_\theta^2}{r} \right] = -\frac{\partial P}{\partial r} + \mu \left[\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} - \frac{v_r}{r^2} \right]$$

θ component:

$$\rho \left[\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + v_z \frac{\partial v_\theta}{\partial z} - \frac{v_\theta v_r}{r} \right] = -\frac{1}{r} \frac{\partial P}{\partial \theta} + \mu \left[\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial v_\theta}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2} \right] + \rho f_\theta$$

Simplifies to:

$$0 = \frac{d}{dr} \left(\frac{1}{r} \frac{d}{dr} (v_\theta r) \right)$$

Z component:

$$\rho \left[\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} \right] = -\frac{1}{r} \frac{\partial P}{\partial z} + \mu \left[\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2} \right] + \rho f_z$$

Simplifies to:

$$\frac{\partial P}{\partial z} = -\rho g$$

Solving for v_θ :

$$0 = \frac{d}{dr} \left(\frac{1}{r} \frac{d}{dr} (v_\theta r) \right)$$

Integrating gives:

$$v_\theta = C_3 r + \frac{C_2}{r}$$

For free vortex conditions, the angular momentum is not a function of radius. Therefore we set $C_3 = 0$. This gives the form r^{-1} . We introduce a small distance, ε , so that the velocity does not reach infinity when $r=0$. However, ε will not be carried explicitly in further derivations to maintain the standard form of the equation.

$$v_\theta = \frac{C_2}{r}$$

or

$$v_\theta = \frac{C_2}{r + \varepsilon}$$

Substituting into the previous equation

$$\rho \left[\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} - \frac{v_\theta^2}{r} \right] = -\frac{\partial P}{\partial r} + \mu \left[\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r} \frac{\partial v_r}{\partial r} + \frac{v_r}{r^2} \right]$$

yields:

$$\frac{\partial P}{\partial r} = \rho \left[\frac{C_2^2}{r^3} + \frac{C_1^2}{r^3} \right]$$

which integrates to

$$P = \frac{-\rho}{2r^2} [C_2^2 + C_1^2] + f(z) + C$$

$f(z)$ is found by integrating $\frac{\partial P}{\partial z}$.

$$\frac{\partial P}{\partial z} = -\rho g$$

$$f(z) = -\rho g z + C$$

Substituting:

$$P = \frac{-\rho}{2r^2} [C_2^2 + C_1^2] - \rho g z + C_4$$

The free surface boundary condition is used to find C_4 .

At $z=Z$ and $r=R$ then $p=p_{atm}$. Where R is the outer radius of the cyclone and Z is the liquid level height.

$$P_{atm} = \frac{-\rho}{2R^2} [C_2^2 + C_1^2] - \rho gZ + C_4$$

Taking the difference between P and P_{atm} :

$$P - P_{atm} = -\frac{\rho}{2r^2} [C_2^2 + C_1^2] + \frac{\rho}{2R^2} [C_2^2 + C_1^2] + \rho g(Z - z)$$

The free surface can be found where $P=P_{atm}$. Solving for the height of free surface ($Z-z$), when $r=\varepsilon$:

$$(Z - z) = \frac{1}{g2\varepsilon^2} [C_2^2 + C_1^2] - \frac{1}{g2R^2} [C_2^2 + C_1^2]$$

This radial film thickness, δ , can be solved as a function of vertical position, z .

$$\delta(z) = R - \varepsilon - \sqrt{\frac{C_1^2 + C_2^2}{2g(Z - z) + \frac{1}{R}(C_1^2 + C_2^2)}}$$

Vita

Lee Hosack was born in Athens, Tennessee, on January 25, 1984. From 1989 to 2002, he attended the public schools in Englewood. From August 2002 until May 2006 he attended the University of Tennessee where his major was Nuclear Engineering. From August 2006 he attended the University of Tennessee as a graduate student in Nuclear Engineering where he received his masters degree in December 2007. He worked in the High Power Target Group as a graduate research assistant on a DOE funded project towards investigating the application of hydrocyclones towards the SNS facility.