

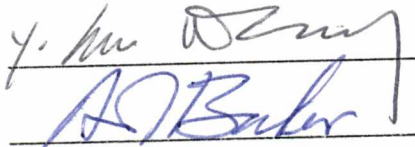
To the Graduate Council:

I am submitting herewith a thesis written by Lumin Jiang entitled "Effectiveness of Bending-Torsional Vibration Damping Using a Piezoactuator." I have examined the final copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Mechanical Engineering.



Grzegorz Kawiecki, Major Professor

We have read this thesis
and recommend its acceptance:



Accepted for the Council:



Associate Vice Chancellor and
Dean of The Graduate School

**EFFECTIVENESS OF BENDING-TORSIONAL VIBRATION
DAMPING USING A PIEZOACTUATOR**

**A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville**

Lumin Jiang

December 1996

Abstract

This study investigates the effectiveness of bending and torsional vibration damping in thin walled beams using piezoactuators.

This project consist of three major parts: an experimental study, a numerical study and a theoretical study. A closed cross-section composite beam with induced piezoelements was fabricated. The effectiveness of piezoelements in bending and torsional vibration damping was tested. Flat beam response to piezoactuation was studied using COSMOS/M finite element software. That study made it possible to understand better the behavior of beams piezoactuated in bending and torsion. Application of Heaviside and Delta functions to model simple piezoactuated beam response is presented.

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NOMENCLATURE

E	Young's modulus
G	shear modulus
M	bending moment
Q_{ij}	reduced stiffness in (1,2,3) composite principal coordinates
$\overline{Q}_{ij}$	reduced stiffness in (x,y,z) structure coordinates
V_k	voltage applied to the k-layer actuator patch
$(d_{ij})_k$	piezoelectric strain coefficient for the k-th layer actuator patch
u	axial displacement
v	transverse displacement
w	bending displacement
β	actuator orientation relative to beam axis
γ	shear strain
ϵ	axial strain
ϕ	twist displacement
χ	midplane mechanical curvature
Λ	piezoceramic actuation strain
ν	Poisson's ratio
σ	axial stress
τ	shear stress

$R_k(x, y)$	generalized location function of the k-th actuator patch
$H(x - x_0)$	Heaviside function
$\delta(x - x_0)$	delta function
$\delta'(x - x_0)$	first derivative of a delta function

Superscripts

m	mechanical
a	actuator

Subscripts

o	mid-plane reference
k	k-th layer
1, 2, 3,	principal material coordinates
x, y, z,	arbitrary structure coordinates

CHAPTER ONE

INTRODUCTION

Induced strain actuators, such as piezoceramic patches are a promising alternative for noise and vibration control. The strain produced by piezoelements can alter the strain in a structure and therefore modify its displacements or damping characteristics. Piezoceramic material can be formed into very thin sheets and adhered to many kinds of surfaces, such as metal, fiber reinforced plastic etc. It also can generate almost linear response to an applied voltage. It has been found^{1,2,3} that piezoelements can generate bending, torsion and extension in isotropic and composite beams. The possibility of using piezoelectric ceramics as sensors for monitoring and as actuators for controlling structural response has recently attracted a significant attention within the research community. The resulting smart structures are usually built of composite materials integrated with distributed microelectric sensors and actuators. The mechanical behavior of those structures can be controlled and monitored actively by external mechanical or electrical signals. This technology seems to be able to play an important role in structural dynamics control.

Crawley and de Luis¹ presented an analytical study on modeling of composite beams with solid rectangular cross sections activated in bending by surface mounted or embedded distributed piezoelectric actuators. The mechanism of inducing torsion using distributed piezoelements is not as well understood as the mechanism of inducing

bending. There were only a few partially successful attempts to develop analytical models capable of predicting magnitudes of piezo-induced torsional-bending response.

It has been shown^{3,4} that piezoactuators are capable of generating bending and torsional structural displacements. Usually, bending and torsional displacements generated in a structure by piezoelectric elements are relatively small. However, they are sufficient to affect aerodynamic loads on structural elements subjected to an airflow. This is the reason why this study will consider the effectiveness of bending-torsional vibration damping using piezoactuators. The ability to alter and control the characteristics of structures may be used to, i.e., reduce stresses and to improve the performance of a helicopter rotor. This can be achieved by both modifying individual blade pitch to reduce the amplitude of vibrations generated by a turbulent flow and by absorbing the vibration energy in the blade by piezo actuators.

CHAPTER TWO

LITERATURE REVIEW

There exist very few publications^{4,5} dealing with the application of piezoelectric elements to simultaneously modify the bending and torsional behavior of structures, even though the applications of piezoelectric elements as actuating and sensing devices in active control system have been investigated extensively. Therefore, the following literature review will be focused on publications that are most closely related to the present research.

2.1 Vlasov's Theory Applications

An analytical formulation based upon Vlasov Theory⁶ for open cross-section beams with bending-torsion couplings was developed by Chandra and Chopra⁷. The analysis was used to predict the static structural response of an open cross-section beam under bending and torsional loads. The predicted results correlated satisfactorily with experimental data for investigated specimens⁷.

2.2 Virtual Work Principle Applications

Models described in Ref. 5 were based on the principle of Virtual Work. To maintain a One - Dimensional (1-D) characterization of the problem, the actuator was

modeled as a line element and only assumed to induce strain along its lengthwise axis. Transverse actuator stiffness was neglected. Hence, the bending slope and twist angle predicted by the model were both expected to approach zero at $\beta = 90^\circ$, where β was the angle between the longitudinal axis of the actuator and the axis of the specimen. But the actual bending slope was not equal zero at $\beta = 90^\circ$ even though it was small and the twist angle was zero.

2.3 Macromechanical Behavior of Laminate

The classical laminate plate theory [CLPT] presented by Jones⁸ [1975] has been widely used for composite material analysis. Following are some of the major formulas that are used in the present study.

The stress-strain relationships in principal material coordinates for a lamina of an isotropic material under plane stress are:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{Bmatrix} \quad (2.1)$$

Matrix elements Q_{ij} (reduced stiffness) are defined in terms of engineering constants as follows:

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}} \quad (2.2)$$

$$Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}} \quad (2.3)$$

$$Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}} \quad (2.4)$$

$$Q_{66} = G_{12} \quad (2.5)$$

Stress-strains relationship in a coordinate system inclined at an angle of θ to the original system can be expressed as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (2.6)$$

where:

$$\overline{Q}_{11} = \cos^4 \theta + 2(Q_{12} + 2Q_{66})\sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta$$

$$\overline{Q}_{12} = (Q_{11} + Q_{22} - Q_{66})\sin^2 \theta \cos^2 \theta + Q_{12}(\sin^4 \theta + \cos^4 \theta)$$

$$\overline{Q}_{22} = Q_{11} \sin^4 \theta + 2(Q_{12} + 2Q_{66})\sin^2 \theta \cos^2 \theta + Q_{22} \cos^4 \theta$$

$$\overline{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})\sin \theta \cos^3 \theta + (Q_{12} - Q_{22} - 2Q_{66})\sin^3 \theta \cos \theta$$

$$\overline{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})\sin^3 \theta \cos \theta + (Q_{12} - Q_{22} - 2Q_{66})\sin \theta \cos^3 \theta$$

$$\overline{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\sin^2 \theta \cos^2 \theta + Q_{66}(\sin^2 \theta + \cos^2 \theta)$$

where θ is the angle between the principal material coordinate system 1-2 and arbitrary x-y axes as shown in Figure 2.1 :

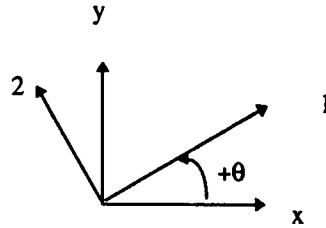


Figure 2.1 Relationship between principal material axes and arbitrary x-y axes

CHAPTER THREE

FABRICATION OF A CLOSED CROSS-SECTION COMPOSITE TUBE

Closed cross-section composite tubes were fabricated of fiberglass cloth and epoxy resin. Piezoelements were attached to the tube for testing the effectiveness of piezoactuators in bending-torsional vibration damping. The modulus of elasticity [E] and shear modulus [G] were determined by bending and torsion tests.

3.1 Composite Tube Manufacturing

Tube surface should be uniform and smooth to prevent problems with piezoelements attachment and mechanical testing⁹. Most important steps of the procedure are listed below:

1. Preparation of a rectangular cross-section polystyrene core.
2. Saturation of a fiberglass cloth with epoxy resin and fabrication of a rectangular cross-section tube.
3. Applying an external vacuum pressure to the assembled tube.
4. Removal of the core from the cured composite tube.

3.1.1 Core Fabrication

A rectangular cross-section tube was used as the base for the piezoelements. A rectangular cross-section core was used to give the required shape to the tube. The core was dissolved using Acetone after the tube was formed.

The polystyrene core was fabricated using a numerically controlled milling machine. The dimensions of the core were 6 mm x 15 mm x 610 mm. The most important step in the milling procedure was to attach well the long, thin, narrow and soft polystyrene core. The machined horizontal surface had to be kept flat during cutting. A flat holder was used to support the core. The core surface and the holder surface were connected by a two-sided adhesive tape. Figure 3.1 shows the details.

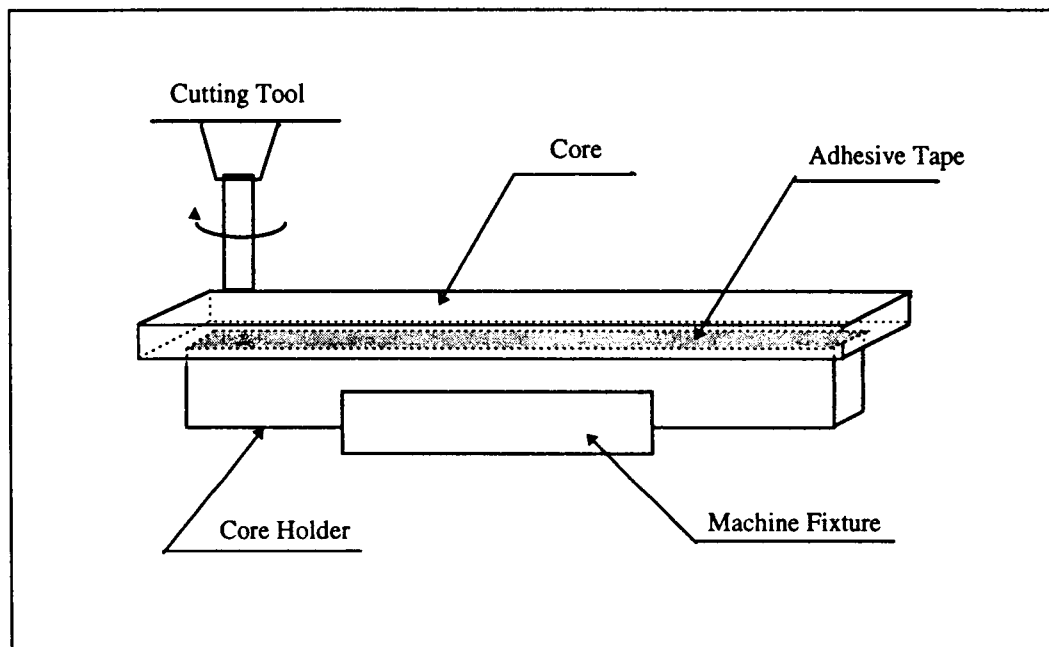


Figure 3.1 Core Cutting Set-up

A proper selection of a cutting tool, as well as spinning and feeding speeds were very important for obtaining a smooth core surface. However, even with the most careful operation some imperfections still remained on the surface of the core. These imperfections were corrected by glossing the surface with sandpaper. Furthermore, in order to avoid creating sharp corners on the four edges, sandpaper was used to round the corners of the core. After that, the core was wrapped longitudinally with a plastic packaging tape to separate the core from epoxy resin. Contacts between epoxy and polystyrene material would cause a chemical reaction eroding the core surface.

3.1.2 Resin and Fiberglass Preparation

Low toxicity lamination epoxy resin (88/87-A), from Fibre Glast, Dayton, OH and the fiberglass (9 oz Fabric, 0.008" thick, style 7781 8 Harness Satin weave), were chosen for fabrication of the composite tube. The resin was used by mixing 5 parts of resin 88 with 1 part of hardener 87 (by volume). About 175 ml of mixture resin could completely saturate one square meter of fiberglass¹⁰. The mixture would gel in about two hours at room temperature and would become 90% cured in about two days. It took almost one week to complete the curing process.

3.1.3 Tube Formation

The entire tube fabrication process had to be finished in two hours to avoid gellation of the epoxy resin. During the fabrication, much care should be taken to ensure a good quality of the tube. Because the harnesses of fiberglass cloth were very smooth

and loosely weaved, the harnesses slid over each other, making the edges uneven. The overall shape of the fiberglass cloth was also not stable during the operation. This made the operation difficult and affected the quality of the tube. Experience showed that the most important factors were patience and the attention to details. The general steps are shown in Figure 3.2.

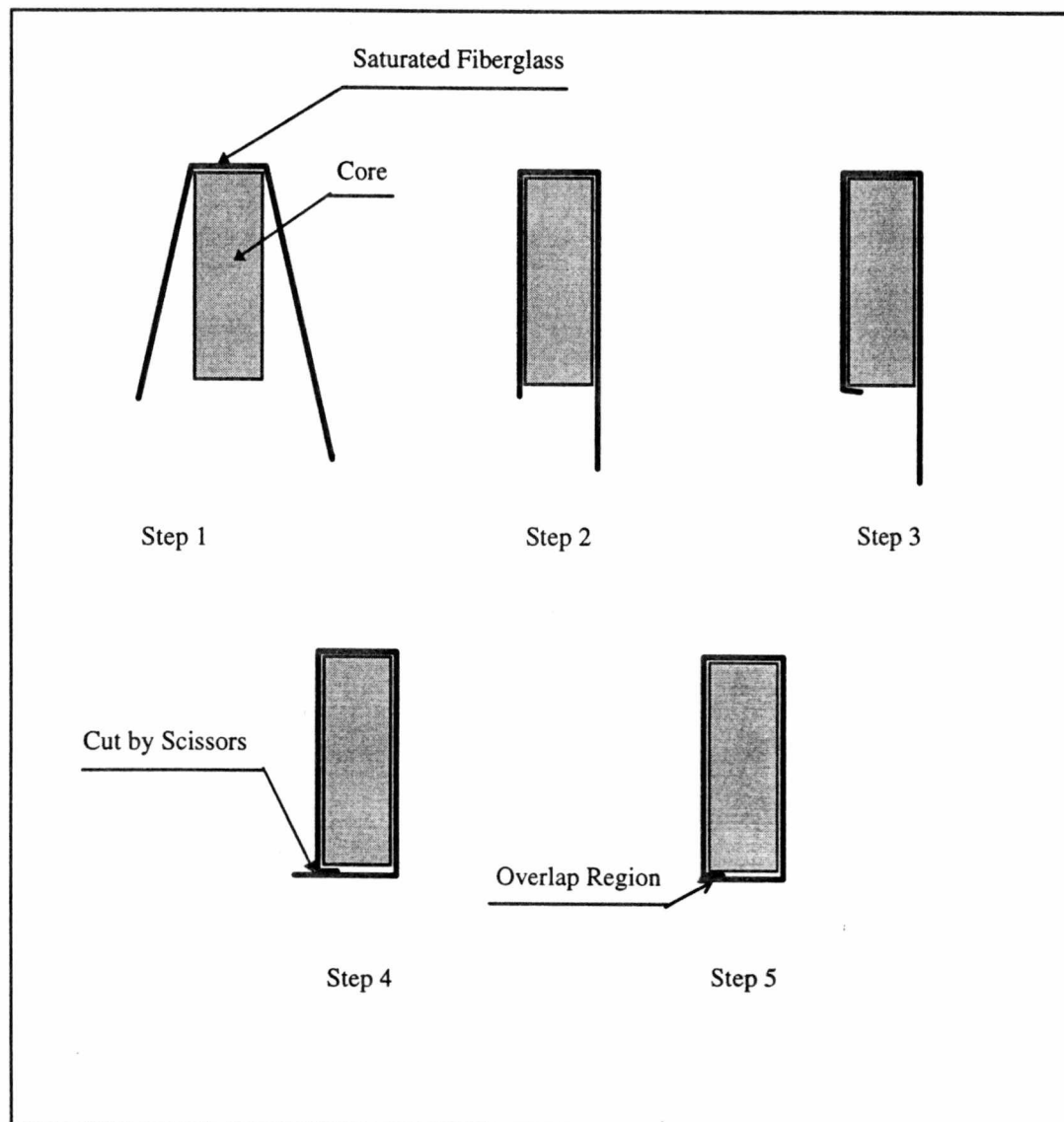


Figure 3.2 General steps for manufacturing the composite beam

In step 4, scissors were used to cut the excess of the fiberglass cloth to make the overlap edge much more smooth. The seam of the overlapped composite was positioned along a corner of beam. This was done to avoid creating bumps on those walls where the piezoelements were to be mounted. After completing step 5, a peel ply was tightly wrapped around the specimen. The peel ply served as a semi-permeable membrane to allow excess resin to flow out the fiberglass cloth. Then a layer of bleeder cloth was tightly wrapped around the peel ply. The bleeder cloth served to remove the excess resin content in the fiberglass.

3.1.4 Vacuum pressuring during the curing process

In order to eliminate bubbles and shape imperfections, vacuum pressuring was applied during the curing process. The uncured tube was placed in a plastic bag from which the air and the volatile gases, which occur during the curing process, were eliminated by means of vacuum. The atmospheric pressure applied to the tube forced the formed shell to conform to the shape of the core. The amount of vacuum pressure was important in controlling the quality of the final product. Too much vacuum would cause an excessive resin flow and result in a resin-starved porous laminate. A vacuum pressure of about -15 to -20 KPa was a proper choice. The vacuuming process should be stopped in about one to one and half an hour. After that period, the resin would gel and since the resin had not completely cured the stressed part of the tube would have a chance to bounce back, preventing the formation of wrinkles.

3.1.5 Core separating

Two days later, when the curing process was completed, the core could be removed from the tube. Two ends of the tube were cut off with a band saw to obtain the required beam length. Acetone was used to dissolve the polystyrene. The packaging tape was separated from inside the tube and removed as well. After a few days, the stresses generated in the tube by the fabrication process would relax and the tube could be used for the experiment.

3.2 Mechanical Characteristics Testing

The knowledge of material properties, such as Young's modulus, shear modulus, size and density, etc., of the composite tube material was necessary to analyze tube behavior.

3.2.1 Geometrical parameters of the tube

After the composite tube fabrication, its geometrical parameters were measured. Tube dimensions are shown in Figure 3.3. The weight of the beam was 50.08 g. The moment of inertia about the A-A axis was $I_{AA} = 3212.53 \text{ mm}^4$, about the B-B axis was $I_{BB}=15305.29 \text{ mm}^4$, and about the longitudinal axis was $J_D = 9932 \text{ mm}^4$. The calculations is shown Appendices I, II and IX.

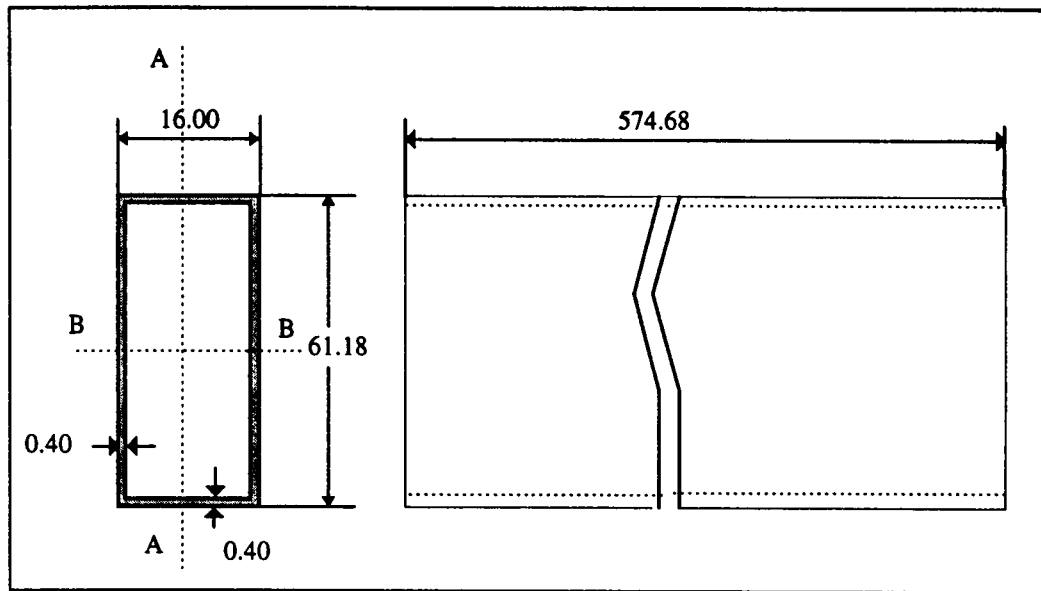


Figure 3.3 Geometry of the composite beam (dimensions in millimeters)

3.2.2 Determination of a modulus of elasticity

To determine the Young's Modulus of the material, a static bending test was carried out on the tube in a test set-up shown in Figure 3.4. The clamp attached to the top of the tube could be used for force application and bending slope measurement. As a load was applied at the top, the mirror attached to the tip of the beam would reflect the laser beam. The amount of bending displacement was calculated by measuring the distance of two laser reflection points on the screen while the beam was loaded. The experimental data showed that the bending displacement and the applied load had a linear relationship. Young's Modulus of the composite beam could be calculated using the following expression⁸:

$$E = \frac{-W}{6uI_{AA}}(L^3 - 3L_wL^2) \quad (3.1)$$

where W is the tip load; L is the length of the beam, L_w is the distance from the load position to the clamped base of the beam; u is the deflection at the point of B. Figure 3.4 shows the details. The resulting average Young's modulus is $E = 6318 \text{ MPa}$.

Figure 3.5 presents the results. Appendix I shows calculations.

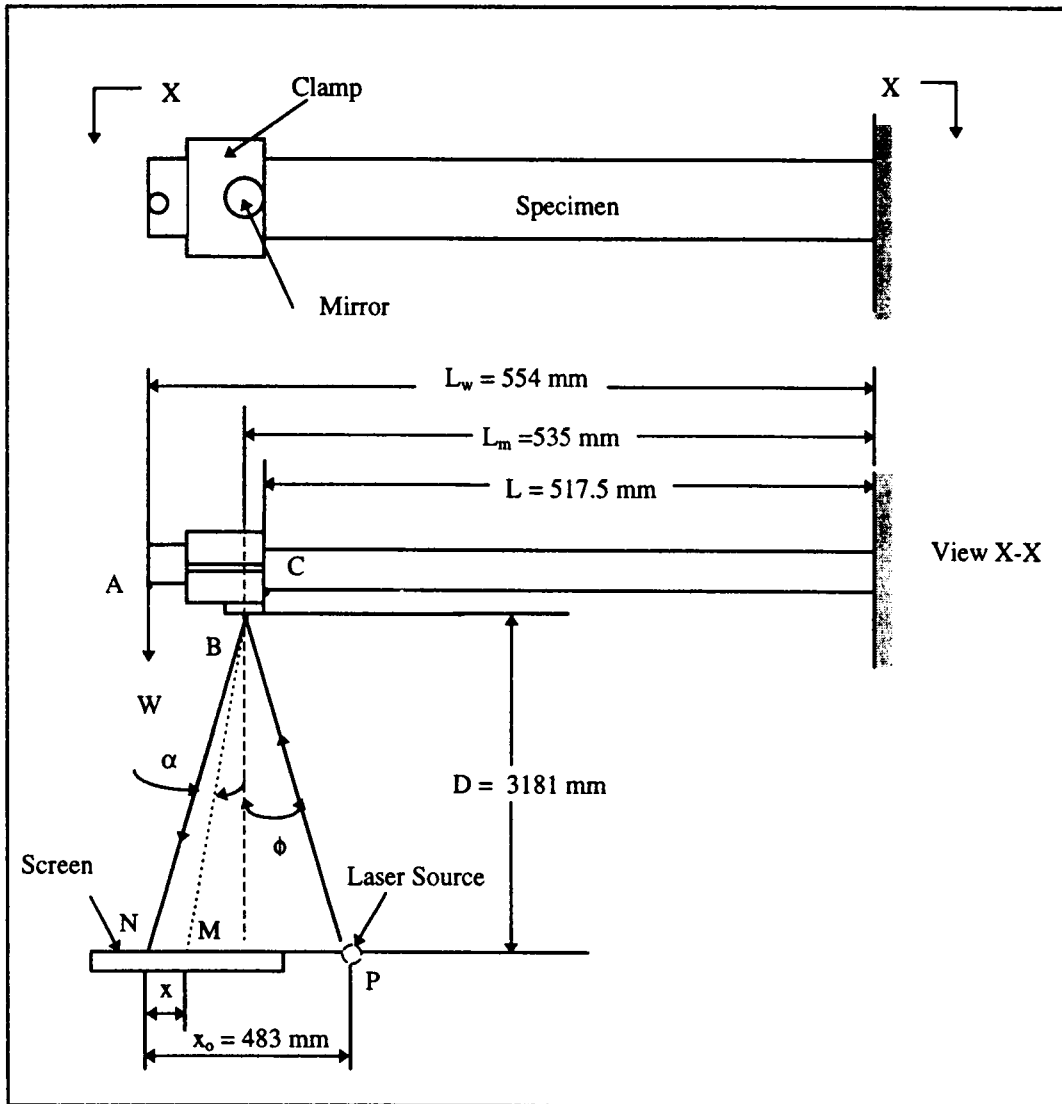


Figure 3.4 Bending test set-up for measuring Young's modulus (E)

Point B was assumed to deflect the same amount as point C. That meant that L_m was considered the length of the entire beam ($L = L_m$).

In Figure 3.4 x_0 is the distance between source laser beam (point P) and laser deflection point on the screen (point M), when weight W is equal to zero, and x denotes the distance between two laser deflection points on the screen, point M with $W=0$ and point N with $W \neq 0$.

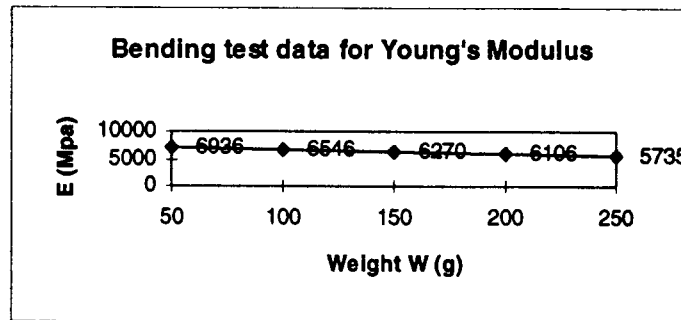


Figure 3.5 Young's modulus determined using bending test results

3.2.3 Determination of a shear modulus

To determine the shear modulus of the composite material, a static torsional test was carried out on the composite tube. The testing device was quite similar to the set-up used for the bending test. However, as shown in Figure 3.6, the method to apply a load was different. The bar attached to the top of the clamp was used for torque application. Two equal forces were applied at both ends of the bar. The resulting twist displacement was determined by measuring the deflection of the laser beam. The actual forces were applied by standard weights through two strings and a pulley assembly. The experimental data showed that the rotational displacement and the applied torque had a linear

relationship; the shear modulus of the composite tube could be calculated from⁸:

$$\frac{d\theta}{dz} = \frac{T}{GJ} \quad (3.2)$$

where T is the applied torque $T = WR$; $J = J_D$ is the torsion constant; θ is the tip rotation of the beam; z is the longitudinal axis of the beam. The resulting average shear modulus is $G = 775 \text{ MPa}$ for the tube. Figure 3.7 shows the results obtained for different weights. Detailed calculation of G and J is given in Appendix II.

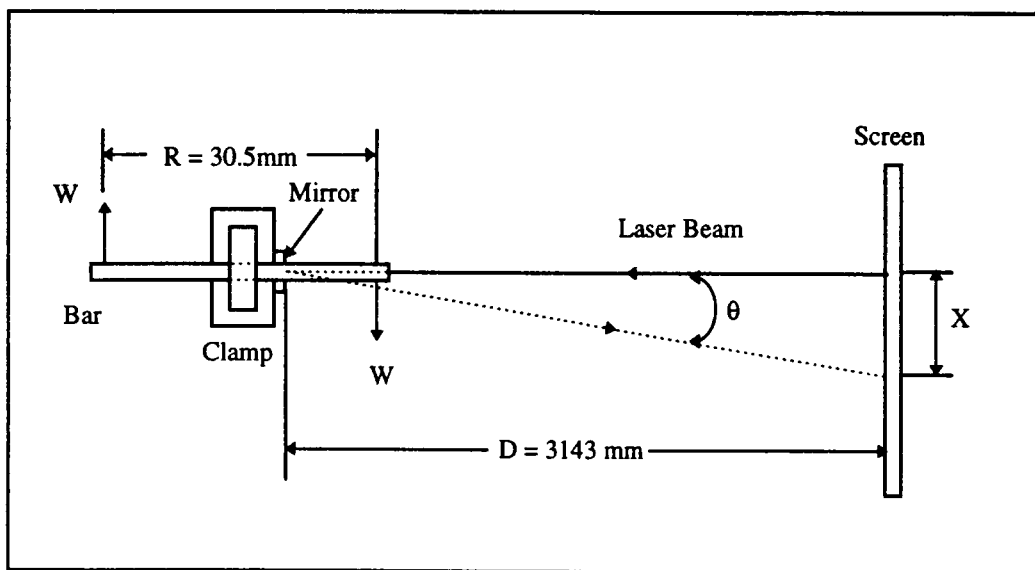


Figure 3.6 Torsion test set-up for measuring shear modulus (G)

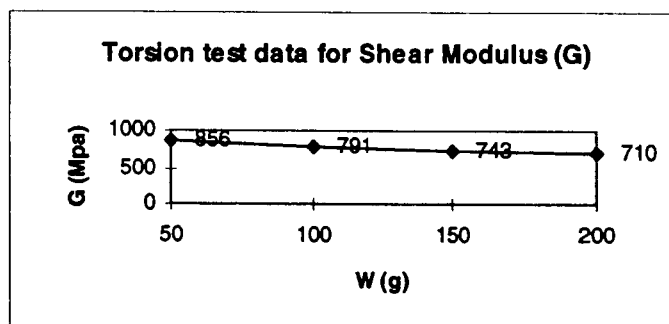


Figure 3.7 Shear modulus determined using torsion test results

CHAPTER FOUR

PIEZOACTUATED SPECIMEN FABRICATION

A fiberglass tube with surface mounted piezoelectric elements was fabricated. Piezoelements were attached to the composite tube. The bending and torsional damping performance of the piezoelements was tested.

4.1 Piezoelements Characteristics

Thirty-two piezoelements were bonded to the composite tube. The layout of the resulting piezoactuators is shown in Figure 4.1. Free strain induced in the material was evaluated for comparing the strain of attached piezoelements.

4.1.1 Piezoceramic material properties

Piezoceramic elements used in this study were PSI-5A-S4-ENH lead zirconate titanate, pre-poled, with nickel full coating on the top and bottom faces of the piezoelements. They were purchased from Piezo Systems Inc., Cambridge, Massachusetts. The purchased plates had a size of 0.07239 x 0.07139 meters and were 2.667×10^{-4} meters thick. The high voltage poling surface of the elements was marked with a green stripe by the manufacturer. The electro-mechanical properties of the PSI-5A-S4-ENH are listed in Table 4.1. The responses of a piezoelement to an activation

potential (V) through its thickness with respect to the elements poling direction are shown in Figure 4.1.

Table 4.1 Electro-mechanical properties of PSI-5A-S4-ENH piezoceramic material

Property	Symbol	Value	Unit
Density	ρ	7800	Kg/m ³
Curie Temperature	t_c	350	°C
Piezoelectric Strain Coefficient	d_{31}	190×10^{-12}	m/V
Piezoelectric Voltage Coefficient	g_{31}	11.6×10^{-3}	V-m/N
Elastic Modulus	E	6.6×10^{10}	N/m ²

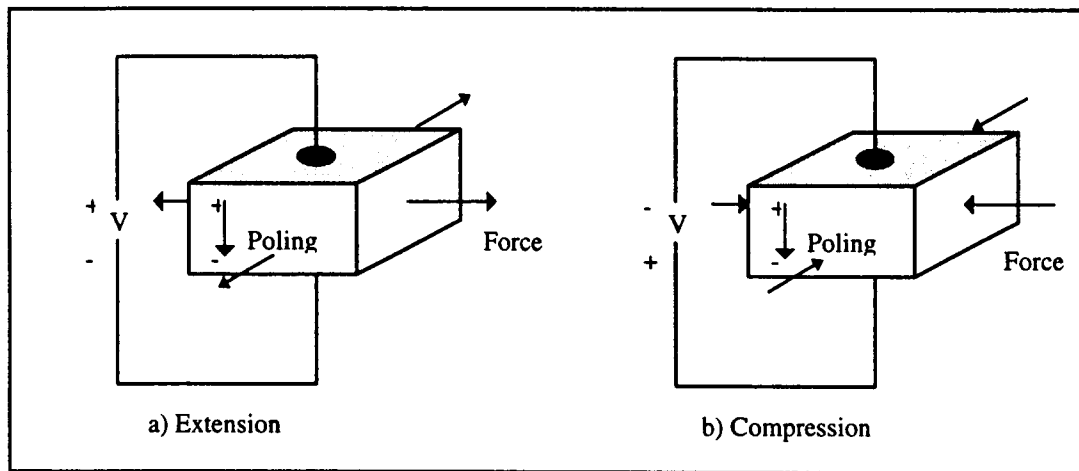


Figure 4.1 Electro-Mechanical response of a piezoelement

4.1.2 Piezoelements cutting procedure

The piezo actuators were cut to the size of $0.062\text{m} \times 0.01\text{m} \times 2.667 \times 10^{-4}\text{m}$. The size of the actuators was designed to fit the size of the selected composite tube. There were 32 pieces of piezoelement attached to the tube, eight to each surface. These actuators were mounted in pairs to both surfaces of the wider walls of the tube. A special

device was manufactured for improving the efficiency of the cutting process. The used set-up is shown in Figure 4.2. Mr. Steven Jesse helped to design and fabricate this device.

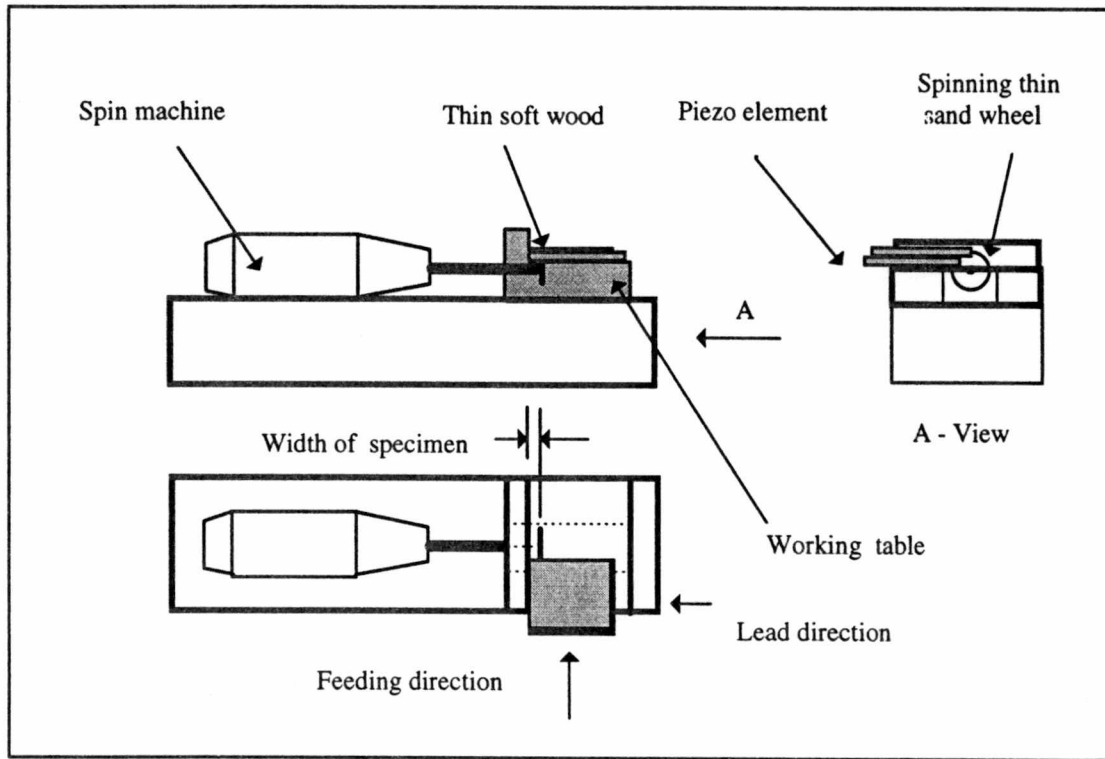


Figure 4.2 Piezoelement cutting device

During cutting, the piezoelement piece should be kept in a stable contact with the surface of the cutting table and with the side wall of the table for guiding. That was assured using a soft wood sheet. The wood sheet was put on the surface the piezoelement plate. A slight pressure was applied by fingers to the top of the soft wood sheet.

4.1.3 Free strain induced by a piezoelement

Experiments were conducted to verify the key data supplied by the manufacturer. A strain gage was bonded to the center position of a piece of piezoceramic material. The adhesive material used to bound the strain gage to the piezo element was Super Glue, from Loctite, with 200 catalyst M-Bond Adhesive, from Measurements Group, Inc. Figure 4.3 shows the sketch and the circuit. Direct current voltages ranging from -120 volts to +120 volts were applied sequentially across the element, with a step size of 20/-20 volts, the induced free strains of the piezoelement were measured using a strain indicator. The sequence of applying the selected voltage was the following: (1) a positive voltage from 0 to 120 volts with a step size of 20 volts, five times; (2) a negative voltage from 0 to -120 volts with step size of -20 volts, five times. The measured strains were plotted in Figure 4.4.

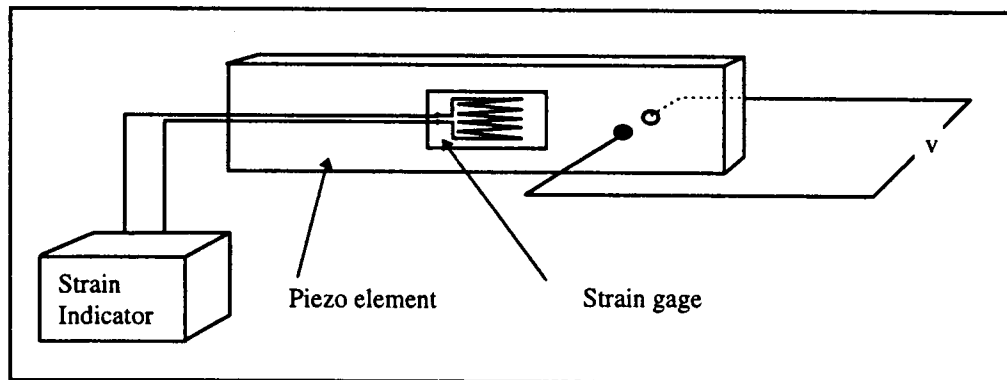


Figure 4.3 Strain gage position for free induced strain test

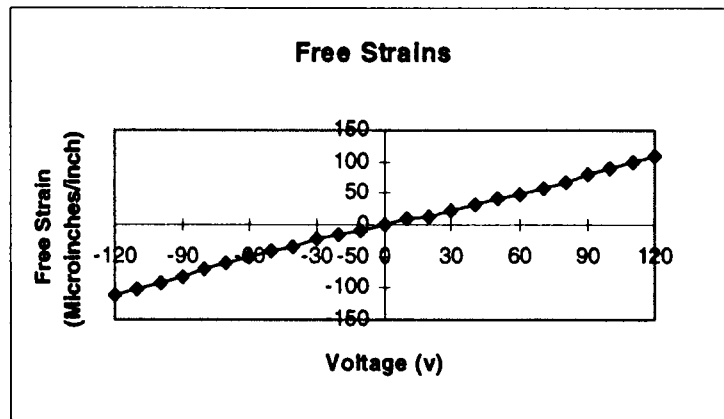


Figure 4.4 Induced free strains versus applied voltages

4.2 Piezoelements Attachment

All 32 - piezoelements were mounted to the composite tube at 45° with respect to the longitudinal axis of the tube. Eight piezoelements were attached to each of the four surfaces of the wider walls of the beam. Figure 4.5 illustrates the attachment positions of the piezoelements.

Front wall wire connections are shown in Figure 4.5. Back wall wire connections are arranged identically.

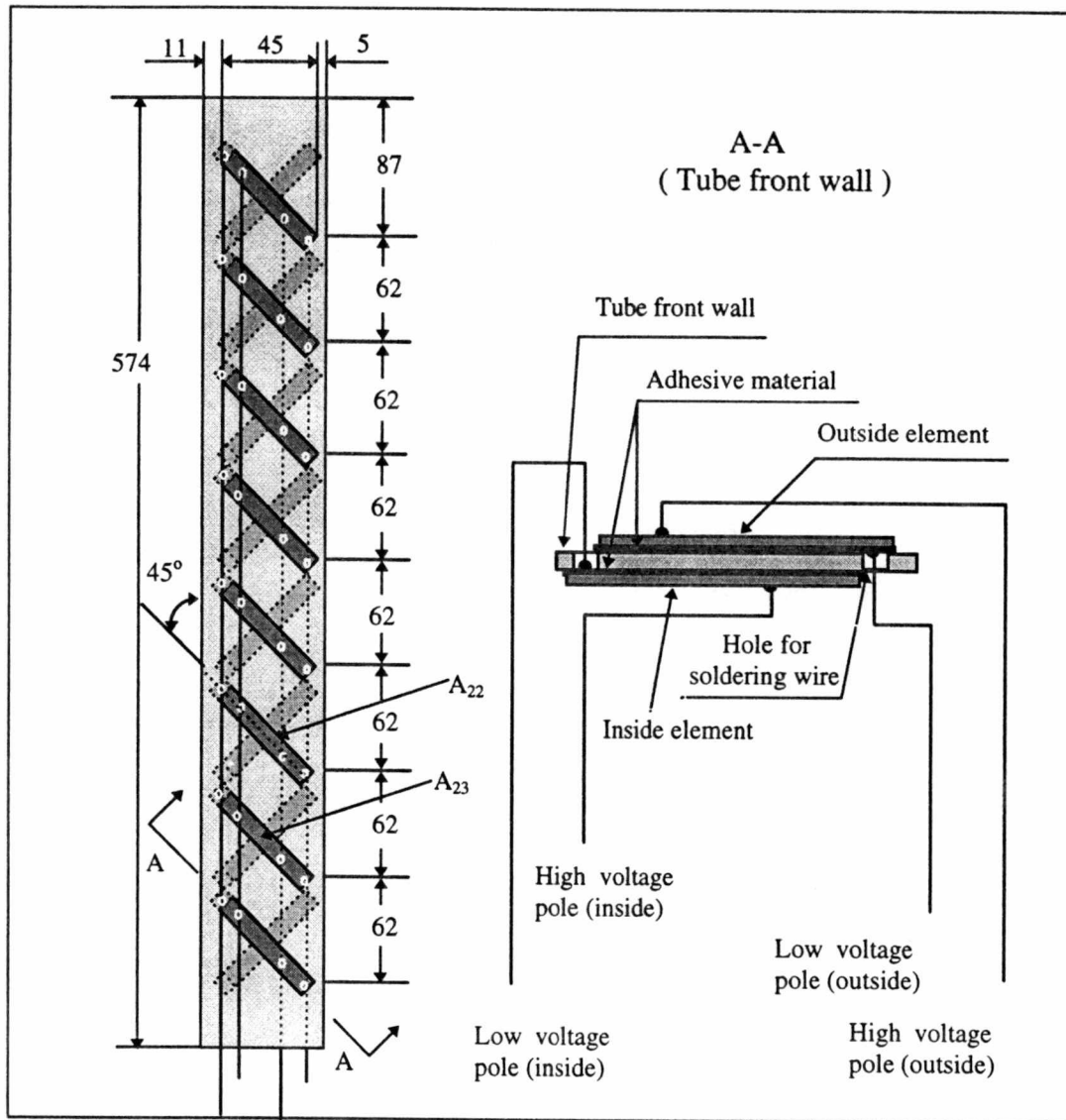


Figure 4.5 Sketch of the specimen (dimensions in millimeters)

4.2.1 Single piezoelement attachment procedure

The detailed procedure of attaching all thirty-two piezoceramic elements to the closed cross-section composite beam walls was quite complicated. Illustrating the method of single piezoelement attachment will make it easier to understand the basic steps. A description of a single piezoelement installation follows.

Piezoelements were bonded to the composite plate surface using Eccobond LV45 epoxy resin and Catalyst LV 15 epoxy resin hardener, purchased from Emerson & Cuming, Inc.. This epoxy exhibits a high shear strength recommended for joining dissimilar materials. The mixture ratio of LV 45 and LV 15 was 4 : 1 by weight. This composition of epoxy resins resulted in a rigid bonding layer.

The first step was to prepare the element for being attached. The low voltage pole face of the element was selected to be bonded to the surface of the tube. This low pole face was cleaned with alcohol (70% Isopropyl, Rubbing Alcohol, from Cumberland-SWAN Inc. Smyrna, TN 37167). Next, a solder dot was made at an appropriate position on the piezoelement. This position was to match the position of a hole in the composite plate. The hole in the composite plate was made to allow for a wire to align with the piezoelement. The wire was subsequently soldered to the low pole surface of the piezoelement with a 25 watt soldering iron and nickel solder was used for soldering. An appropriate length of 32 gage Teflon coated thermocouple wire, bought from OMEGA Engineering, Inc., was tined and soldered to the element. The solder connection was checked by slightly pulling the connected wire without separating it. Prior to bonding the prepared element, guide lines were marked on the surface of the composite plate. The

element was to be mounted at the position marked by the guide lines. Prior to bonding, the bond area on the composite plate and on the prepared element were cleaned and degreased. Electric tape (Vinyl Electrical Tape LR 48769, from 3M Inc.) was used to tape around the attachment position of the composite plate for avoiding the overflow epoxy attaching the composite plate. A schematic demonstrating the piezoelement attachment procedure is shown in Figure 4.6.

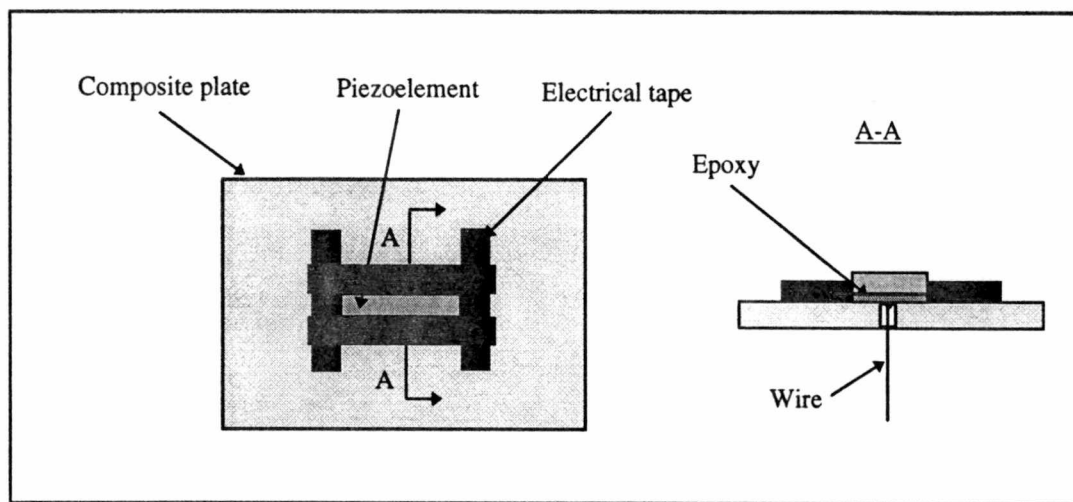


Figure 4.6 Piezoelement attachment procedure

The epoxy was then applied to the element on the low voltage face. Next, the element was placed in the proper position surrounded by the electrical tape. After that, another electrical tape was attached to cover the high voltage face of the element and to the surrounding tape with gentle pressure. Next, two flat polystyrene plates were to be used to cover the two sides of the whole specimen. The final step was to use a clamp to clamp both sides of the polystyrene. The clamp provided an appropriate pressure at the position

of bonded element. Then, the assembly was cured at room temperature of about 20° C to 30° C. The unused epoxy was put on a piece of paper nearby. When the mixture epoxy on this paper could be cracked by hands, that meant the bonding epoxy layer on the composite plate was hard enough. Then, the clamp was released. The electrical tapes were separated one by one. The final step was to solder another wire to the high voltage surface of the element.

4.2.2 Multiple piezoelement attachment procedure

The steps for the preparation of the piezoelements for being attached were the same as for a single element (section 4.2.1). Piezoelement layout on the composite beam is shown in Figure 4.5.

The first step to prepare the composite beam for the attachment of piezoelements was to drill holes through the composite beam to accommodate the solder dots on the low voltage poled faces of prepared elements. Any fibers frayed during drilling procedure were trimmed off. Then, the composite beam was cleaned with alcohol.

Bonding of the piezoelements to the beam was done in four steps. Two steps were required for the elements bonded to both the inside and the outside surfaces.

The details of the first step of the procedure of attaching piezoelements to the inside surface of the composite tube are as follows. All eight piezoelements were bonded simultaneously. In order to accomplish the simultaneous bonding of eight piezoelements, a mounting guide was manufactured of polystyrene. A schematic explaining the procedure is shown in Figure 4.7. This mounting guide was fabricated by the

departmental workshop. The polystyrene had layout guide lines etched into its surface to aid in the proper alignment of the elements. During the bonding of the elements to the beam, channels were routed into the mounting guide to accommodate the lead wires from the elements. Figure 4.7 illustrates the geometry of the mounting guide.

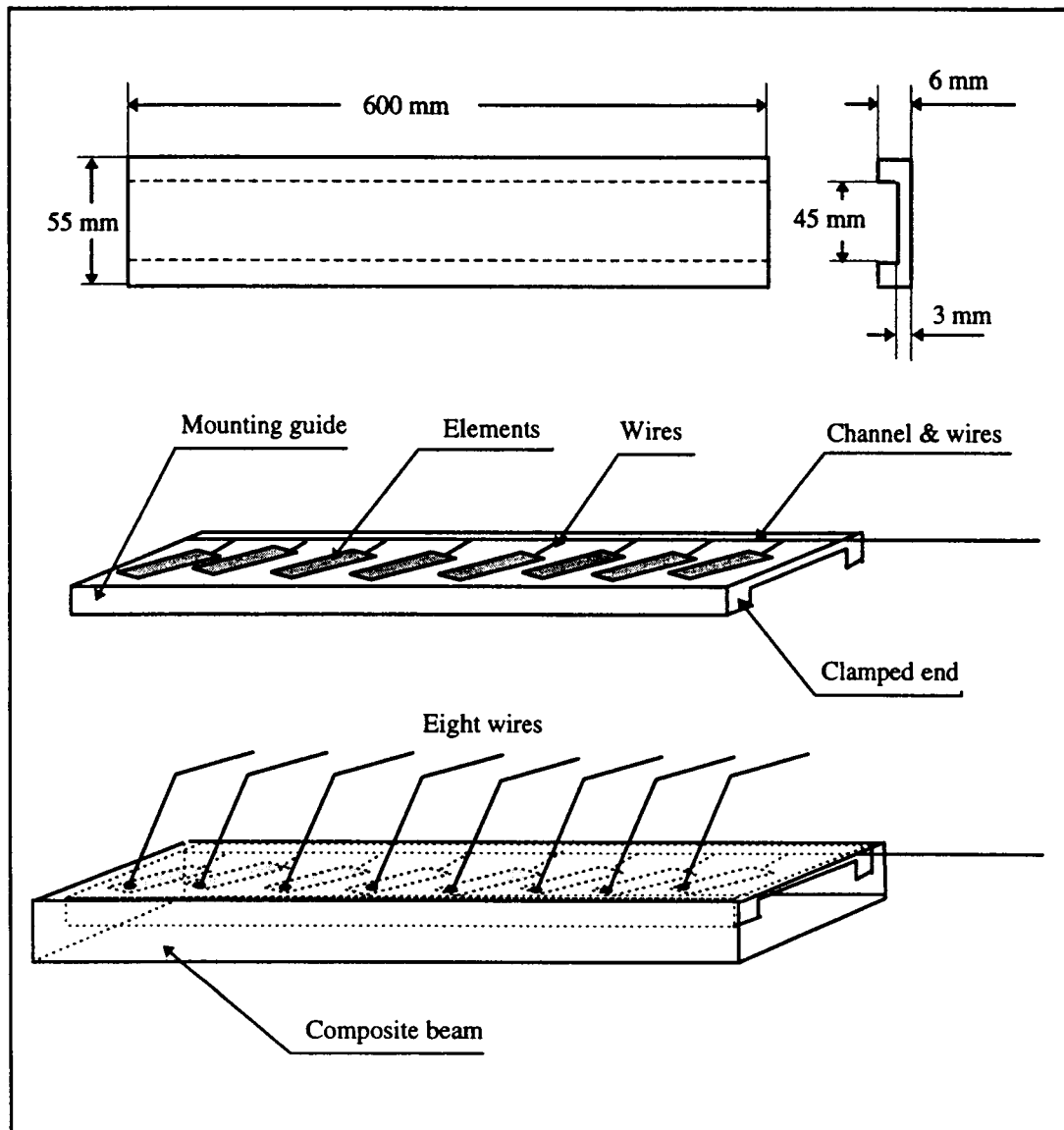


Figure 4.7 The procedure of attaching piezoelements to the inside surface of the tube

The top surface of the mounting guide was then covered with a layer of 3M Scotch tape to prevent any bonding between the mounting guide and the composite beam due to excess epoxy. Piezoelements were secured lightly to the mounting guide with two small pieces of tape. So that they were oriented at 45° with respect to the longitudinal axis of the guide. For this procedure, the tape was looped back onto itself to produce the effect of double sided tape. At this point, all of the elements on the polystyrene mounting guide and their respective lead wires were labeled for identification. The prepared elements secured on the mounting guide were then cleaned with a cleaner to assure a good bonding surface. The polystyrene mounting guide was then secured horizontally with the prepared low voltage faces of the piezoelements facing up. Then, the whole mounting guide was carefully pushed into the composite beam, and the small holes in the beam for solder dots and lead wires could be aligned with their respective piezoelements. Lead wires on the low poled voltage side were soldered now. The soldering procedure was described in section 4.2.1. As the solder spaces were much smaller than section 4.2.1 mentioned, so much more care had to be taken in this part of work. Then, the mounting guide with all 8 piezoelements and the 8 high and 8 low poled voltage lead wires was pulled out of the composite beam. Piezoelements low voltage surfaces were cleaned with a cleaner again. Then, the bonding epoxy was applied to each of the elements low voltage faces. The same mixture ratio of the Eccobond LV45 epoxy resin and Catalyst LV15 epoxy resin hardener was used for bonding the prepared piezoelements to the composite beam as mentioned in section 4.2.1. When all of the elements on the mounting guide had the epoxy applied to them, the composite beam was carefully positioned on the mounting guide with all the

lead wires. Another polystyrene beam was put on the outside surface of the respective wall, clamped and held in place for the 24 to 48 hours room temperature curing period. After that, the clamp was released. The outside polystyrene was taken off. The polystyrene mounting guide was carefully separated from the bonded piezoelements using a thin long and narrow steel beam. The finished product of the above procedure was a set of bonded elements mounted to the inside surface of the tube at 45° with respect to the longitudinal axis of the composite tube.

The technique developed for mounting the piezoelements to the inside wall of the composite beam was repeated the other inside wall of the composite beam. Then, the entire process was repeated for the two outside walls of the composite beam. All the lead wires were attached to the composite walls with an electrical tape to protect them from being destroyed. This completed the fabrication of the composite beam with surface mounted piezoelements. The photograph of the completed composite beam with attached piezoelements is shown in Figure 4.8.

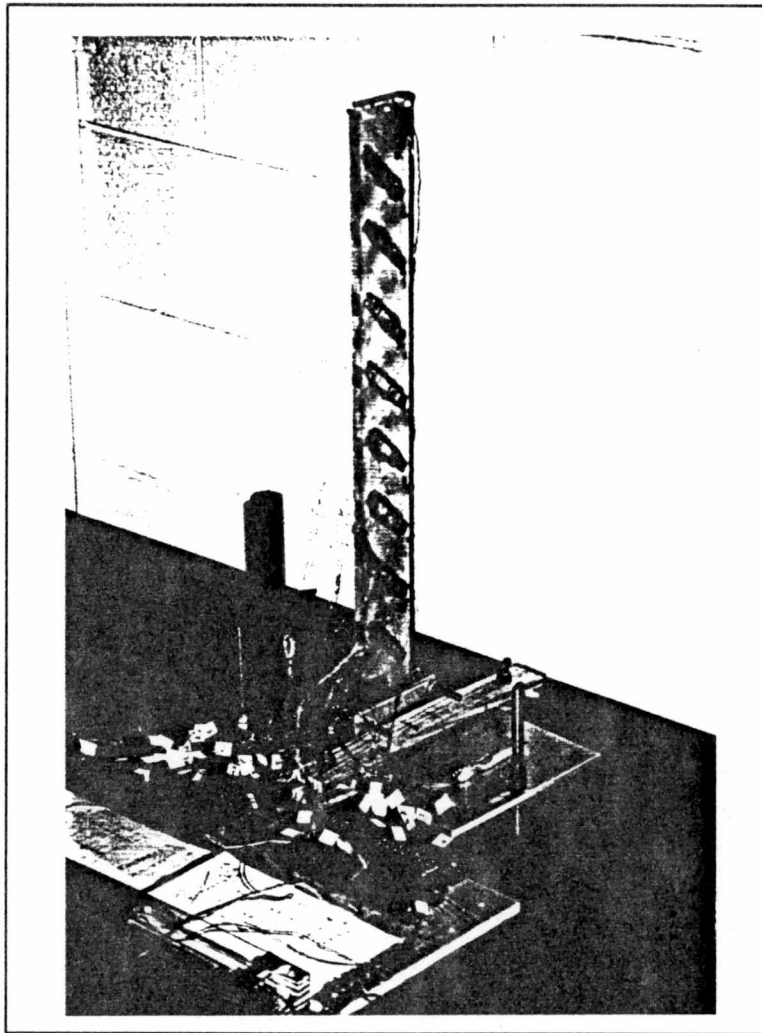


Figure 4.8 Photograph of a composite beam with surface-bonded piezoelements

CHAPTER FIVE

PIEZOACTUATED SPECIMEN TESTING

In order to study the effectiveness of bending-torsional vibration damping using piezoactuators, a closed cross-section fiberglass composite beam with surface mounted piezoelements was tested for its vibration damping performance.

5.1 Determination of Flexural and Torsional Rigidity of the Specimen with Piezoelements

Experiments were performed to test the rigidity of the specimen with piezoelements. The results were compared with those obtained for the beam without piezoelements attached, as described in section 3.2.

5.1.1 Specimen weight parameters

The following table showing the difference between the characteristics of beams with and without surface-bonded piezoelements will be very helpful for discussing the effect of attached piezoactuators.

Table 5.1 Selected specimen parameters

	Length (mm)	Quantity (pcs)	Weight (gm)	Weight %
Specimen	574.7	1	108.75	100
Fiber Tube	574.7	1	50.08	46.10
Piezoelement		32	48.00	44.10
Epoxy Layer		32	10.67	9.80

5.1.2 Experimental investigation of the flexural rigidity of the specimen

The set-up for testing the flexural rigidity of the specimen is shown in Figure 5.1. This set-up is different from the set-up used for the determination of the Young's modulus of the composite tube without piezoelements (compare with Fig. 3.4). Figure 5.1 shows a direct vibration test set-up used for calculating the Young's modulus of the specimen.

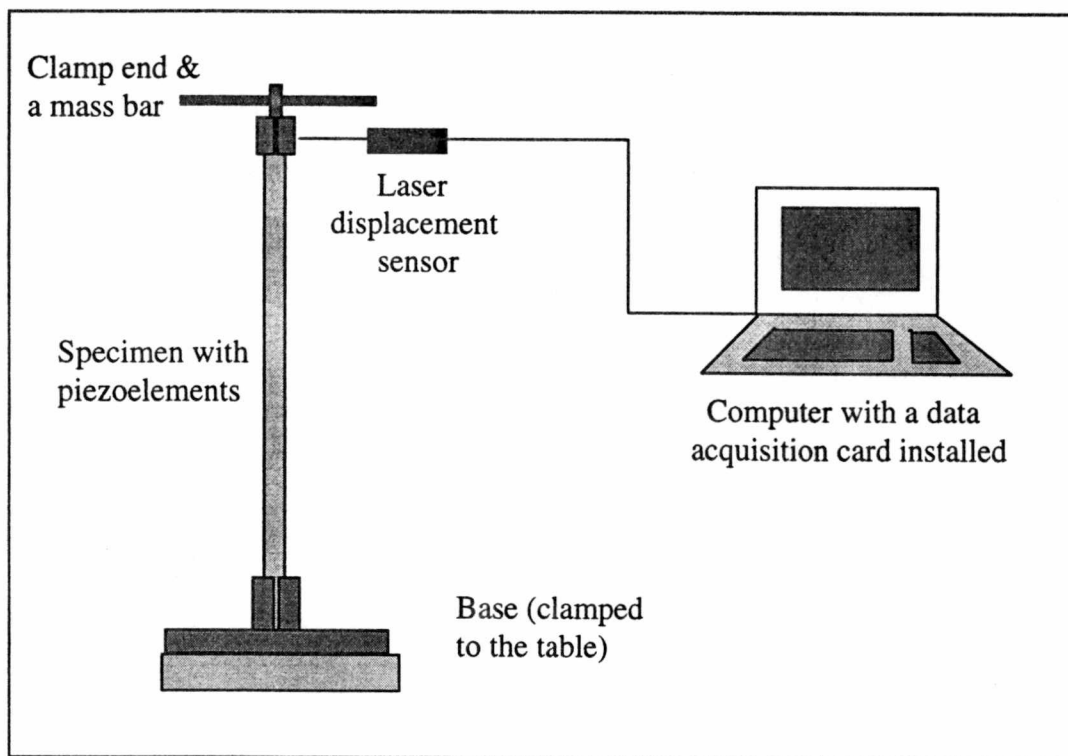


Figure 5.1 Vibration test set-up for measuring Young's modulus

The specimen was clamped at one end and free at the other end. Masses were attached to the tip of the beam to bring the natural frequency into a range that could be easily tested.

The first step in determining the Young's modulus of the specimen was to calculate the beam spring constant using the following formula¹¹:

$$\omega_n^2 = \frac{3.04 E I}{l^3 (M_L + 0.266 M_B)} \quad (5.1)$$

in which $M_L = 0.6875$ Kg is the attached mass and $M_B = 0.10875$ Kg is the mass of the specimen; I is a second area moment of inertia which is equal to $I_b = 4180 \text{ mm}^4$ (see Appendix IX) and $l = 574.6$ mm is the length from the center of the tip attached mass to the clamped end of the beam; $\omega_n = 2 \pi f$, $f = 5.273$ Hz is the measured natural frequency. Bending transient vibrations of the specimen were excited by striking the bar attached at the tip of the specimen. Tip deflections were measured using a laser displacement sensor (Model LM200 from Aromat Corporation, New Providence, New Jersey). Signals from the displacement sensor were fed to the data acquisition card installed in a 486 PC. Then, the natural frequency was obtained using a Fast Fourier Transform. The magnitude of E_a was obtained using Eq. 5.1.

Obtained result, $E_a = 7813$ MPa, was higher than the result obtained for the composite tube without attached piezoelements, which was $E_u = 6318$ Mpa. The increment was calculated using the following formula .

$$\text{Increment \%} = (E_a - E_u) / E_u \times 100\% \quad (5.2)$$

The resulting increment was 23.7%. That change was attributed to the mechanical properties of piezoelements and the epoxy. The weight increment was computed using the following formula .

$$\text{Increment \%} = (W_a - W_u) / W_u \times 100\% \quad (5.3)$$

in which $W_a = 108.75$ gm was the weight of the specimen with attached piezoelements; $W_u = 50.08$ gm was the weight of the composite tube without attached piezoelements. The resulting weight increment was 114.1% that was higher than the Young's modulus increment.

5.1.3 Experimental investigation of the torsional rigidity of the specimen

In order to determine the shear modulus of the specimen with attached piezoelements, a static torsional test was carried out on the specimen. The same method was used as in section 3.2.3. The experimental test-up is shown in Figure 5.2. A clamp was set at the tip of the specimen. A bearing mounted in the clamp was used to constrain the top of the beam. A laser displacement sensor was also installed at the tip of the specimen. The torsional deflections were measured by this laser displacement sensor. Figure 5.2 illustrates the experimental set-up. Steven Jesse, a senior student in the mechanical engineering department significantly contributed to this set-up design and machining. Results are given in Figure 5.3. The average shear modulus magnitude was determined as $G = 1985$ MPa.

This result was higher than the result obtained for the composite tube without attached piezoelements, which was $G_u = 775$ MPa. The increment was calculated using the following formula.

$$\text{Increment \%} = (G_a - G_u) / G_u \times 100\% \quad (5.4)$$

The resulting increment was 156.1%. This change was attributed to the mechanical properties of piezoelements and the epoxy and to the effect of friction in the bearing

installed at the tip of the specimen. The shear modulus increment equal to 156.1% was much higher than the Young's modulus increment, which was equal to 23.7%.

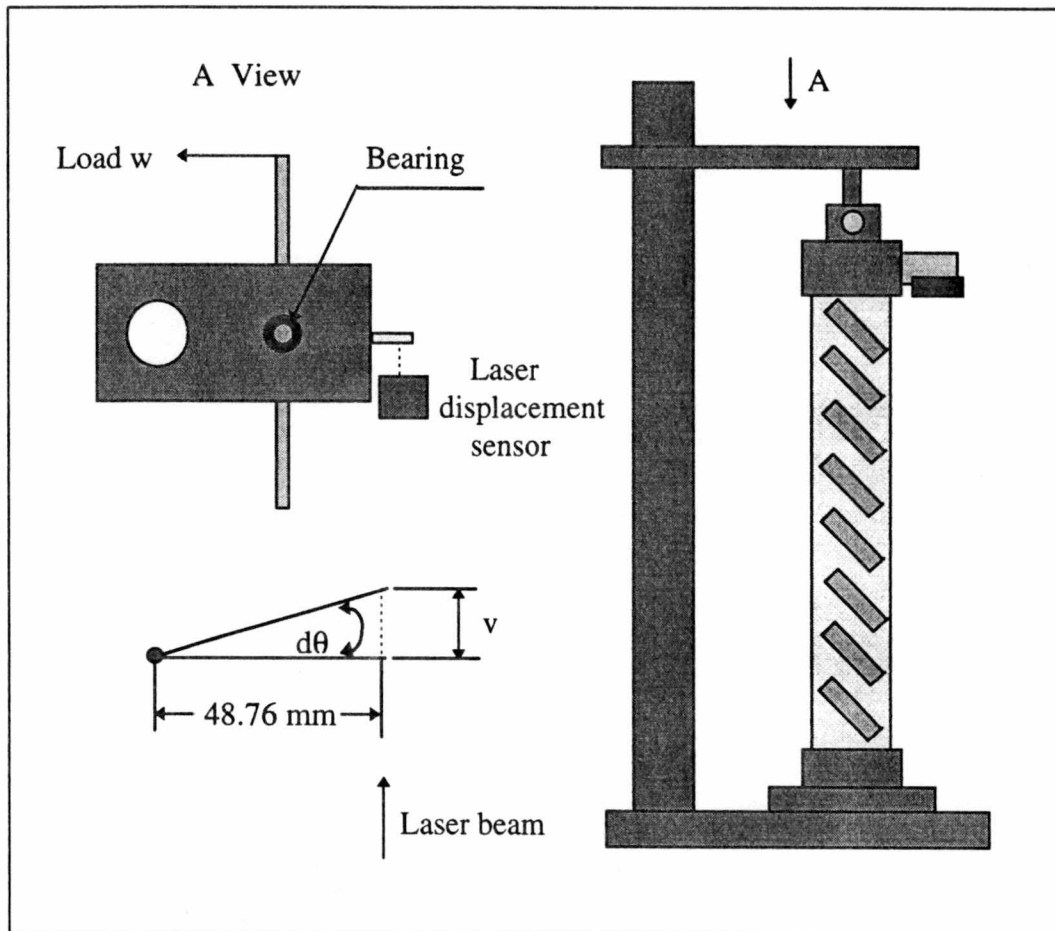


Figure 5.2 Test set-up for measuring shear modulus magnitude

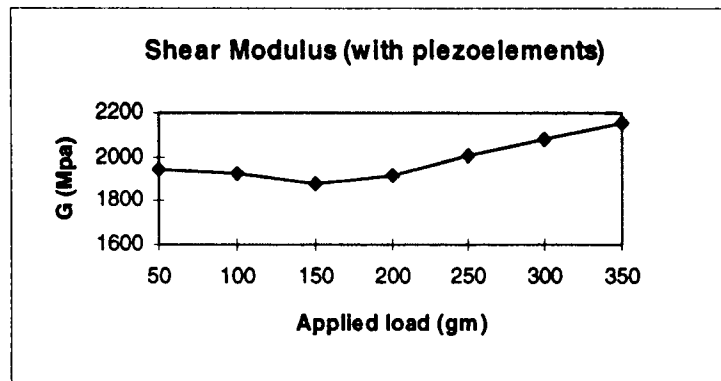


Figure 5.3 Test results for shear modulus of specimen with piezoelements

5.2 Investigation of Piezo Induced Static Bending and Torsional Displacements

The experimental set-up for measuring piezo-induced bending displacements was similar to the set-up for measuring the flexural rigidity, but the set-up for measuring torsional displacement was different than the set-up for measuring the torsional rigidity. Static bending and torsional displacements of the composite beam were induced by piezoelements.

5.2.1 Piezo induced static bending displacements

The set-up for measuring the piezoelement induced static bending deflections of the composite beam is shown in Figure 5.4. In order to induce beam bending, all 16 piezoelements bonded to one wall were activated in extension by applied voltages, and all 16 piezoelements bonded to the other wall were activated in compression. To induce extension, high pole voltage surfaces of piezoelements were connected to a positive terminal, and low pole voltage surfaces were connected to a negative terminal. Polarities

were reversed to induce compression. Then, tip deflections of the composite beam were measured over a range of applied voltages from +0/-0 volts to 120/-120 volts with a step size of 20/-20 volts. Three small mirrors were attached at the tip of the specimen, at both corners and at the axis of symmetry. Resulting bending slopes are plotted vs. the applied voltage in Figure 5.5, where L denotes the results obtained using the left mirror, R indicates the results obtained using right mirror, M demonstrates the results obtained using the middle mirror. Also, B means bending deflection, and T indicates torsion deflection.

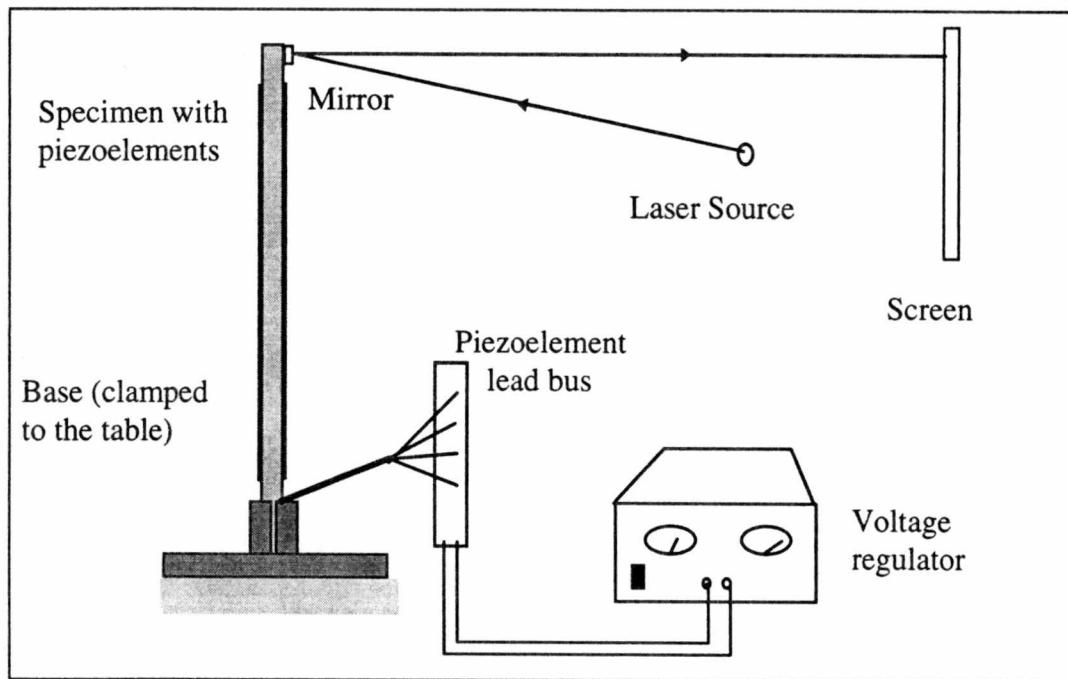


Figure 5.4 Test set-up for composite beam static bending deflections

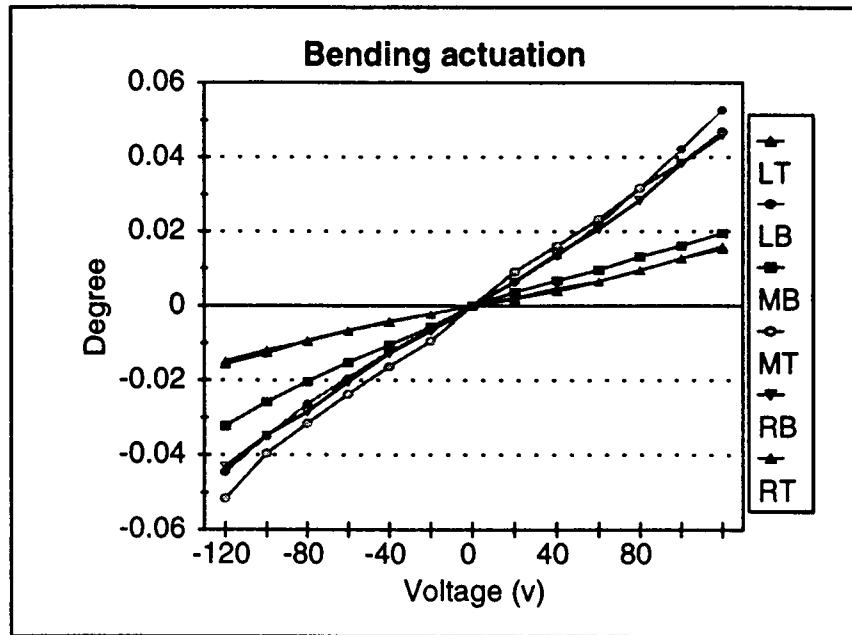


Figure 5.5 Displacements for composite beam static bending actuation

5.2.2 Piezo induced static torsional displacements

The test set-up for measuring the piezoelement induced static torsional deflections of the composite beam is shown in Figure 5.6. In order to induce the twisting of the beam, all 32 piezoelements were activated in the same direction, i.e., either extension or compression, by applied voltages. Torsional displacements were also measured over a range of applied voltages from +0/-0 volts to 120/-120 volts with a step size of 20/-20 volts. Results are plotted in Figure 5.7.

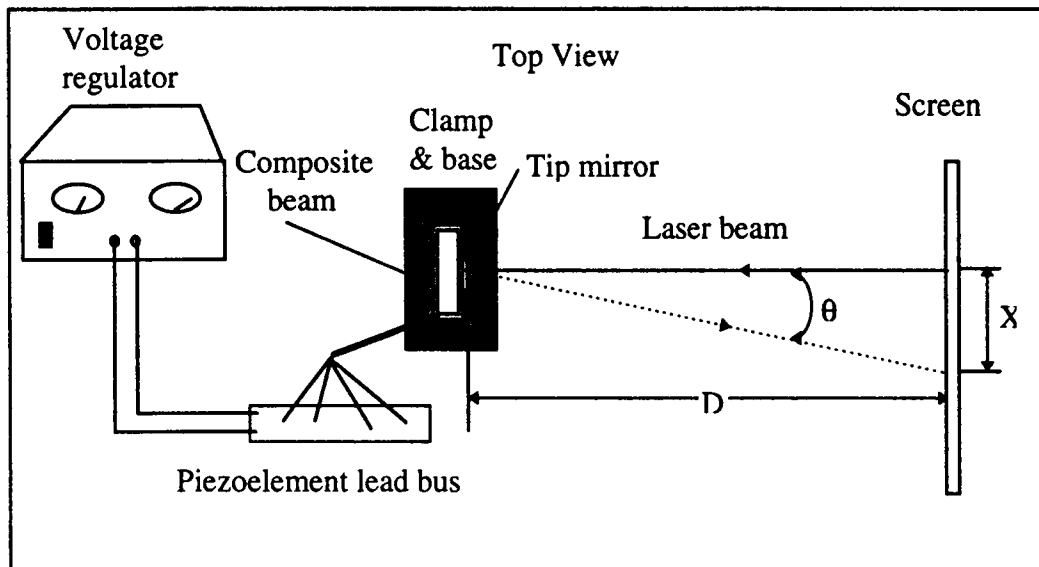


Figure 5.6 Test set-up for measuring composite beam static torsional displacements

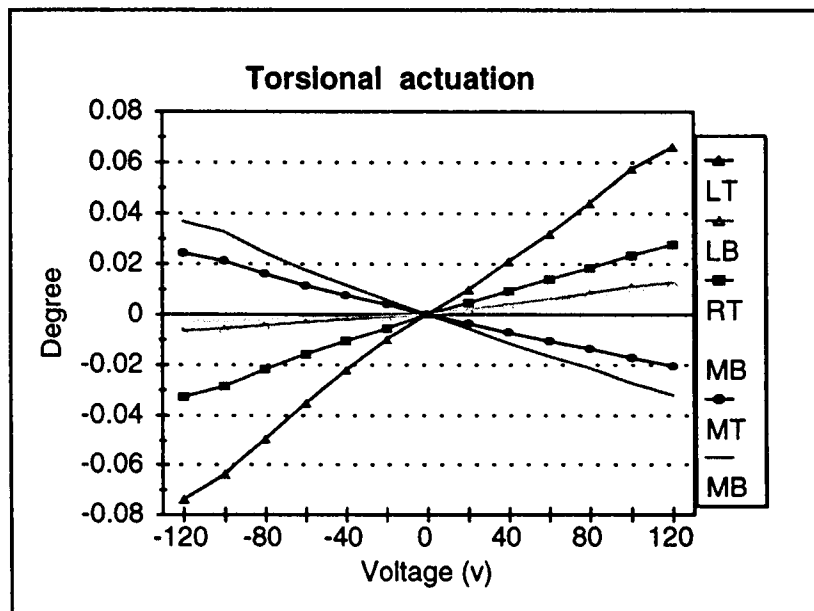


Figure 5.7 Composite beam static torsional displacements test results

We can note that the piezoelements actuated in bending generated not only bending displacements but also torsional displacements, although the torsional displacements were relatively small. Activated in torsion, the piezoelements also generated both torsional and bending displacements, but the bending displacements were also relatively small. That meant that the piezoelements were not capable of generating pure bending or pure torsional displacements when activated in bending or in torsion. This result might be caused by following reasons: the composite tube fabrication was not perfect, bonding layer thicknesses were not uniform, orientations and sizes of attached piezoelements had some scatter, and some of the piezoelements could have failed. Results obtained using the left and right mirrors were very similar, but both differed significantly from the result obtained using the middle mirror. During the measurement the warping of the tip cross section was observed. Therefore, it seems that warping is responsible for the above discrepancy.

5.2.3 Measurement of Young's modulus of the composite material

Testing described was carried out in this section following Dr. Y. Jack Weitsman's advice. Dr. Alex. Selvarathinam and Mr. Jeffrey Howley, Dr. Weitsman's student, helped with the experimental work.

This test was performed using two composite coupons. The coupons were fabricated using the same fiber glass and epoxy as described in section 3.1.2. The coupons had three layers of fiberglass cloth with the same orientation. There were two tabs attached

symmetrically on both ends of each coupon as shown in Figure 5.7. These tabs were attached by Mr. Jeffrey Howley.

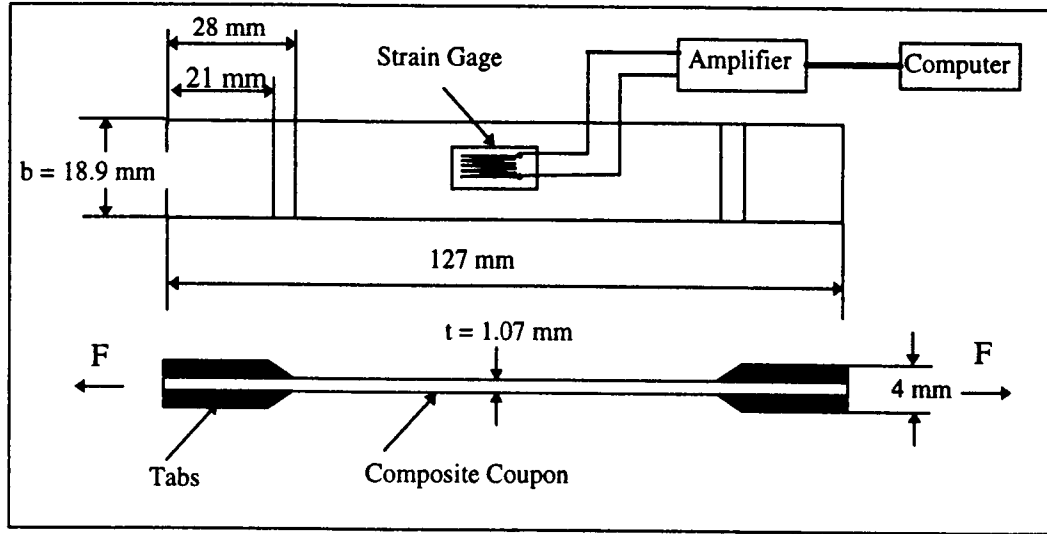


Figure 5.8 Sketch of a tested composite coupon

A strain gage was attached to the middle ply of the tested coupon. The external loads F were applied at the tabs in the longitudinal direction of the coupon using Test Frame 300, Hydraulic Grip Control (from Interlaken Technology Corp.). The strains generated in the coupon were measured using a Signal Conditioning Amplifier- Model 2310 (from Measurement Group, Instruments Division) and stored in a computer. Both loads and strains data were obtained by Dr. Alex. Selvarathinam, The Young's modulus of the composite material was calculated using the following formula.

$$E = \frac{\Delta F}{\Delta \epsilon \cdot A} = \frac{\Delta F}{\Delta \epsilon \cdot b \cdot t} \quad (5.5)$$

where ΔF and $\Delta \epsilon$ were obtained experimentally. The collected data is shown in Appendix VII. The average modulus of elasticity of the composite material determined in this experiment was $E_{ec} = 1.59$ GPa. The thickness of three layers of fiberglass cloth was 0.61mm (the thickness of one layer cloth is 0.008" given by manufacturer specifications). If we assume that only the fiberglass layers carried the force applied to the coupon, then the modulus of composite was $E_{fc} = 2.8$ GPa. These two results were much higher than the result ($E = 0.6318$ GPa) obtained using beam theory calculation (Eq. 3.1). Out of these three results: E may not be very accurate, because the shear flow and warping effect were neglected in the calculation although both of them were important factors for this thin wall structure; E_{ec} may be too low, because the fiber glass cloth and epoxy do not contribute equally; and if assume that only the fiberglass cloth carry the load, that may cause the result of E_{fc} to be too large. The actual modulus could have a value between E_{ec} and E_{fc} .

5.2.4 Strain induced by a piezoelement attached to the composite specimen

This measurement was made to evaluate the actual induced strain of actuators attached to the composite tube.

Strain gages were attached to those two of the actuators surface bonded to the composite tube. The result is shown in Figure 5.9. where A-23 and A-22 are the two tested actuators, their position is shown in Figure 4.5. We can see that the induced strain almost doesn't differ from the free strain.

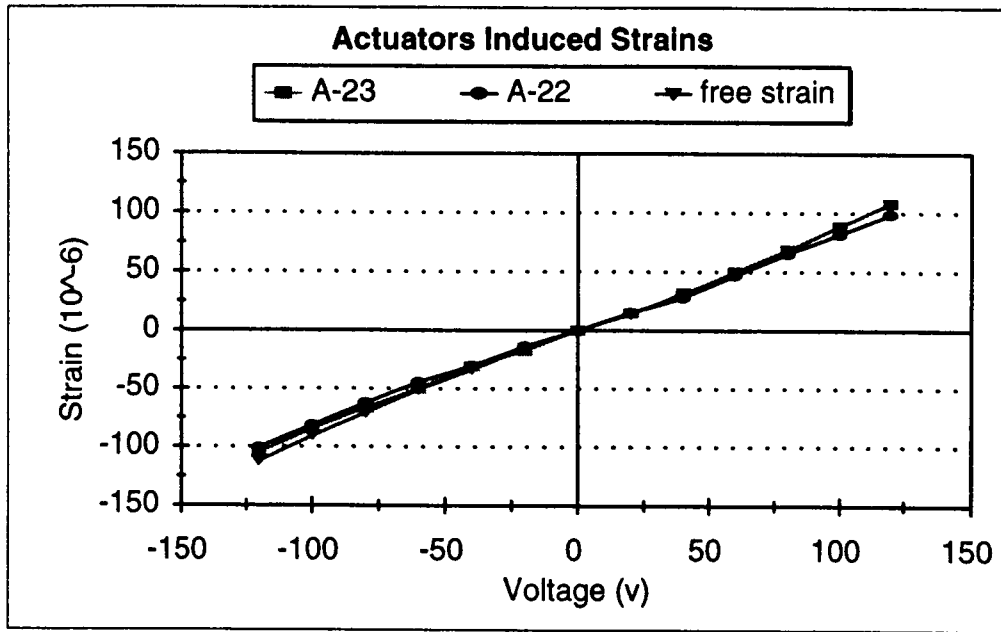


Figure 5.9 Induced strains of piezoactuators bonded to the composite tube

5.2.5 Generated forces analysis

The deformations of the composite beams are attributable to forces induced by the piezoelements. The following calculations were used to compute the magnitudes of forces generated by the piezoelements. For bending deflections, consider a pair of forces $\pm F_b$ applied at the tip of the beam specimen of length L_b , the forces being separated by the distance of $(h-t)$, which equals the height of the beam. Then , $M_b = (h-t) F_b = - E I w''$, where the tip slope is given by

$$w'(L_b) = \theta_b = (h-t) F_b L_b / (E_b I_b) \quad (5.6)$$

Thus,
$$F_b = -(E I w') / [(h-t) L^b] = - E I \theta_b / [(h-t) L^b] \quad (5.7)$$

$$F_{sb} = F_b / 16 \quad (5.8)$$

$$F_{ba} = F_{sb} / \cos\beta \quad (5.9)$$

where: $E = 7813 \text{ MPa}$; $I = 4180 \text{ mm}^4$, $h = 0.016 \text{ m}$ is the thickness of the specimen, $t = 0.4 \text{ mm}$ is the thickness of composite thin wall; $L^b = 0.554 \text{ m}$ is the length of the beam, $w' = \theta_b$ is the tip slope of the beam, θ_b is the beam tip slope obtained experimentally using left mirror, F_b is the computed bending force generated by all piezoelements, F_{sb} is the bending force generated by single piezoelement, F_{ba} is the force generated by a single piezoelement in the piezoactuator's longitudinal direction. For twisting:

$$T = (\theta_t J G) / L^b = F_t (h-t) \quad (5.10)$$

$$F_t = (\theta_t J G) / [L^b (h-t)] \quad (5.11)$$

$$F_{st} = F_t / 16 \quad (5.12)$$

$$F_{ta} = F_{st} / \sin\beta \quad (5.13)$$

where: $J = 9932 \text{ mm}^4$, $G = 1985 \text{ MPa}$, θ_t is the tip twist obtained using left mirror, T is the torque generated by all piezoelements, F_t is the twist force generated by all piezoelements, F_{st} is the twisting force generated by a single piezoelement, F_{ta} is the force generated by a single piezoelement in the piezoactuator's longitudinal direction..

Forces generated by one piezoelement without attached to the composite tube are computed as following:

$$\sigma = \epsilon E_a \quad (5.14)$$

$$F_a = \epsilon E_a b^3 t^a \quad (5.15)$$

where: ϵ is the induced free strain along the longitudinal axis of the piezoelement, σ is the induced stress along the longitudinal axis of the piezoelement, F_a is the induced force

along the longitudinal axis of the actuators, $E_a = 6.6 \times 10^{10}$ MPa is the elastic modulus of the piezoelement, $b^a = 0.01$ m is the width of the piezoelement, $t^a = 2.667 \times 10^{-4}$ m is the thickness of the piezoelement, $\beta = 45^\circ$ is the angle between longitudinal axes of a piezoelement and the composite beam.

In bending actuation, not only bending displacements were generated by piezoelements, but also the twisting displacements. So, both developed forces were added up in the longitudinal direction of the actuators to compare with the free induced forces.

Table 5.2 shows the comparison between forces developed in a single piezoelement generating bending. Table 5.3 shows the torsional actuation case. Figure 5.10 shows the comparison results.

Table 5.2 Comparison of forces developed in a single piezoelement generating bending

Voltage(V)	Ben. force(N)	Tor. force(N)	Total force(N)	Free force (N)
-120	-14.39	-3.14	-17.53	-19.78
-100	-11.62	-2.49	-14.11	-16.01
-80	-9.50	-2.04	-11.54	-12.53
-60	-6.84	-1.41	-8.25	-8.98
-40	-4.30	-0.90	-5.20	-5.95
-20	-2.21	-0.42	-2.63	-2.36
20	2.12	0.45	2.57	2.46
40	4.61	0.96	5.57	5.63
60	6.84	1.39	8.23	8.41
80	9.45	1.98	11.43	11.94
100	12.75	2.66	15.41	15.51
120	15.36	2.56	17.92	18.96

Table 5.3 Comparison of forces developed in a single piezoelement generating torsion

Voltage (V)	Ben. force (N)	Tor. force (N)	Total force (N)	Free force (N)
-120	-2.15	-15.55	-17.7	-19.78
-100	-1.87	-13.40	-15.27	-16.01
-80	-1.44	-10.35	-11.79	-12.53
-60	-1.02	-7.41	-8.43	-8.98
-40	-0.65	-4.67	-5.32	-5.95
-20	-0.03	-2.09	-2.12	-2.36
20	0.62	2.09	2.71	2.46
40	1.36	4.41	5.77	5.63
60	2.06	6.70	8.76	8.41
80	2.86	9.22	12.08	11.94
100	3.73	12.08	15.81	15.51
120	4.27	13.89	18.16	18.96

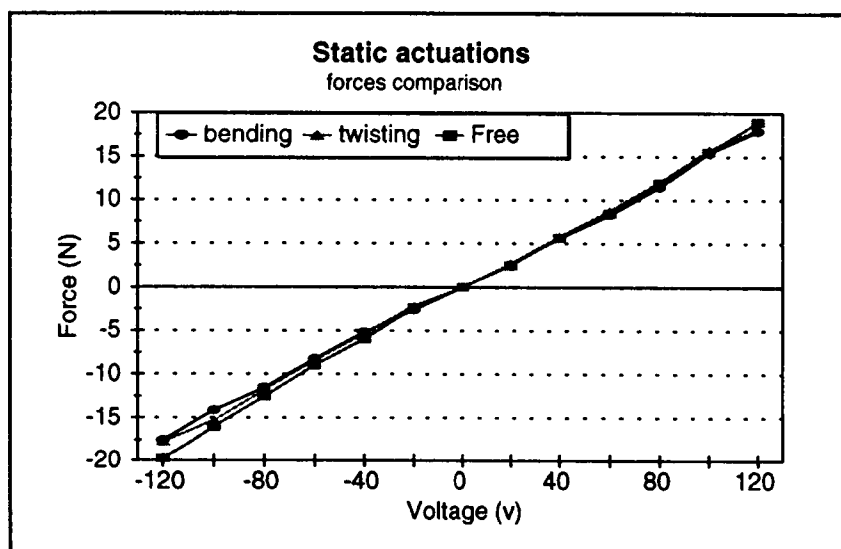


Figure 5.10 Comparison of forces developed in a single piezoelement generating bending and torsion

The result shows that in static bending and twisting actuation, the forces generated by the attached piezoelement to the beam were almost same as the forces expected from free strain measurement throughout the full range of the applied activation voltages. Those small discrepancies were due to the displacements and free strains measurement limitations. This result correlate well with the induced strain measurement result, showing that the induced strain almost does not differ from the free strain.

5.3 Investigation of Bending and Torsional Transient Vibration Damping

An investigation was carried out to evaluate the capability of the piezoelements to reduce bending and torsional transient vibration of the tube.

5.3.1 Transient bending vibration damping

Figure 5.11 shows the schematic of the experimental set-up and the circuit for bending vibration damping testing.

As shown in Figure 5.11, tip displacement signals were sensed by a laser displacement sensor and recorded on a hard disk. A proportional feedback circuit was used for vibration control. A strain gage (from Micro-Measurements, Measurements Group, Inc. Raleigh, North Carolina) was used for recording the feedback signal. This strain gage was attached to the beam about 50 mm from the clamped end.

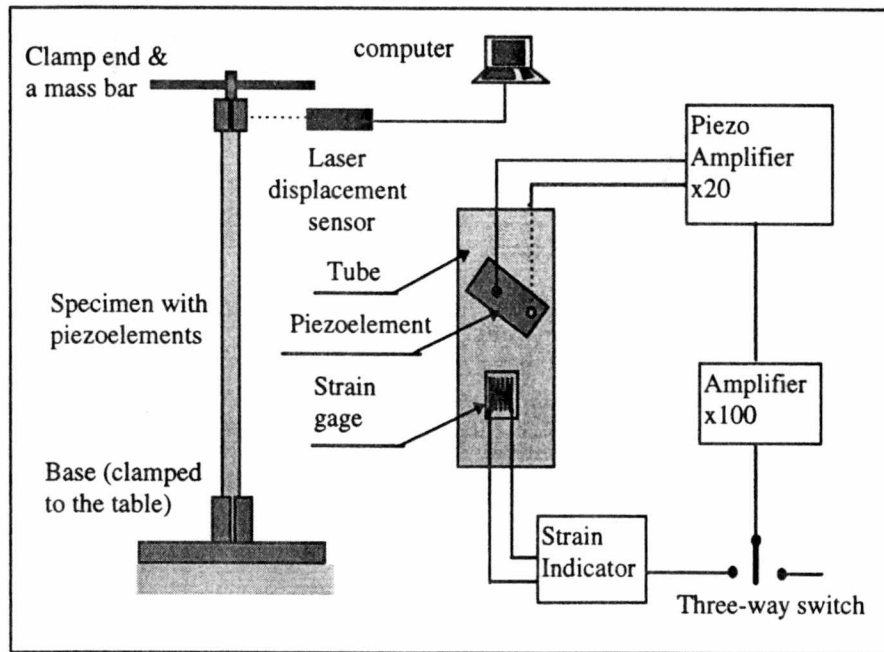


Figure 5.11 Transient bending vibration damping test set-up

A strain indicator (Automations Industries, Inc., Instruments Division, Model-350), which was connected to the strain gage, transferred a voltage signal from the strain gage resistance signal. A three-way switch was used to turn on or off the applied voltage. The off and on switch positions corresponded to passive and active piezoelement states. The voltage signals were routed through two amplifiers. The first stage voltage amplifier, designed and built by the departmental electrician Mr. Dennis Higdon ¹², provided a gain of 100 and could produce a maximum output of 8 volts. This amplifier obtained the input signal from the strain indicator and output the signal to the second stage amplifier. The second stage voltage amplifier was a Piezo Amplifier (Model 60.102 from Piezo Systems, Inc., Cambridge, MA), provide a 20 fold gain and could produce a maximal output of

± 200 volts. The amplified signal was inverted in phase and fed back to the piezoelements. The polarity of the signal fed to the active piezoelements was such that it could induce a strain of an opposite sign to that produced mechanically. Thus, if the treated surface of the beam was subjected to mechanical contraction, the piezoelements were activated to generate extensional strains and vice versa. Bending vibration were induced in the clamped beam upon the release of a load applied to its free end. Beam tip deflections were measured using a laser displacement sensor, recording the natural transient damping response. In the active vibration damping test, the three-way switch was turned on, activating the feedback circuit. The strain indicator and the two amplifiers were also turned on. A sample vibration time history is shown in Figure 5.12.

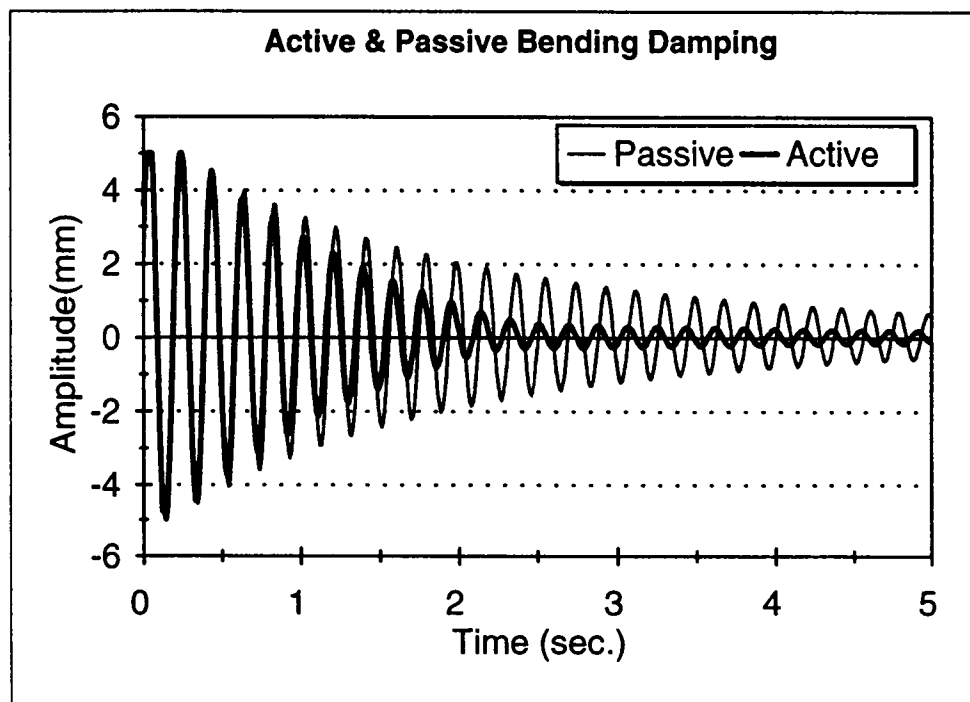


Figure 5.12 Passive versus active transient bending vibration damping

A thin line represents the passive damping history and a thick line represents the active damping history, respectively. The damping coefficient was computed using the expressions (Reference [13], Eqs. (1.69) and (1.73)):

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \quad (5.16)$$

where $\delta = \text{Ln} (x_n / x_{n+1})$ (5.17)

is the logarithmic decrement and x_n / x_{n+1} denotes the ration between two consequent peak displacements. x_n and x_{n+1} can be obtained from the time history graph in Figure 5.11. Appendix IX shows their magnitudes.

The energy dissipated, W_d , is given by (Reference [14], Eq.(3.7-3)):

$$W_d = 2 \zeta \pi k X^2 \quad (5.18)$$

$$k = \omega^2 J \quad (5.19)$$

$$J = \int_m L_m^2 dm = (\frac{M_b}{3} + M_L) L_m^2 \quad (5.20)$$

where: X is the first cycle displacement amplitude; k is the bending stiffness of the beam; J is the inertia of the vibrating mass; $\omega = 2 \pi f$, where f is the fundamental bending frequency of the specimen; L_m denotes the distance from center of the mass attached to the tip of the beam to the clamped end and M_b , M_L denote the weights of the beam and the tip mass, respectively. In the present case, we have $f = 5.237$ Hz, $L_m = 0.535$ m, $M_b=0.10875$ Kg, and $M_L = 0.6875$ Kg .

In the passive case (inactive piezoelements), the specimen had an average damping coefficient of 0.014848 Ns/m during the first eleven cycles, and the energy dissipated per cycle was 5.3×10^{-4} Joules. When the feedback control was used and the piezoelements were active, the first eleven cycles average damping coefficient of the beam was 0.036 Ns/m; the energy absorbed by the piezoelements was 1.29×10^{-3} (Joules) per cycle. The improvement in damping coefficient was 142.5%. It can be seen from the Figure 5.11 that the active damping was not very effective after the vibration amplitudes dropped to about half maximum amplitude. This happened because of the low feedback voltage applied to the piezoelements.

The proportional feedback control reduced the amplitude of vibration very fast during first few cycles, but it was not very effective after the amplitude decreased. The reason for this decrease in the damping effectiveness was the fact that the feedback voltage was proportional to the amplitude of the vibration. The feedback voltage became smaller as the vibrations amplitude decreased. Further decrease in damping can be achieved by a more sophisticated circuit, employing a variable gain feedback. Higher gain could increase the feedback current even though the vibration amplitude was small. Figure 5.13 shows the transient bending vibration test setup. Figure 5.14 shows the equipment used for this test.

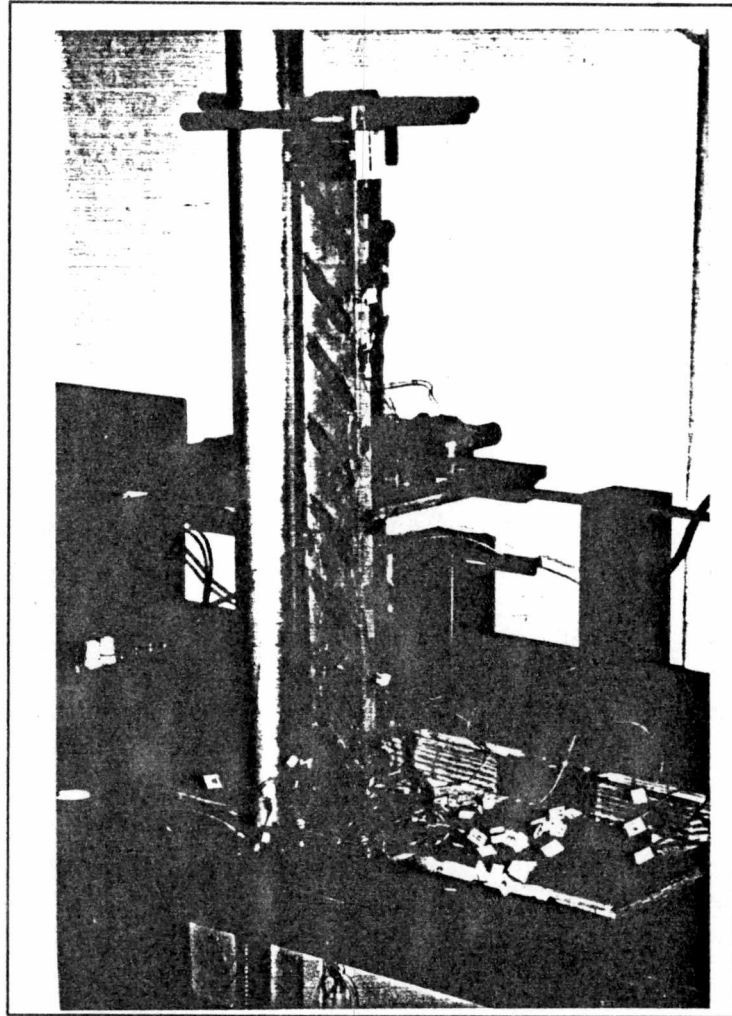


Figure 5.13 Transient bending vibration set up

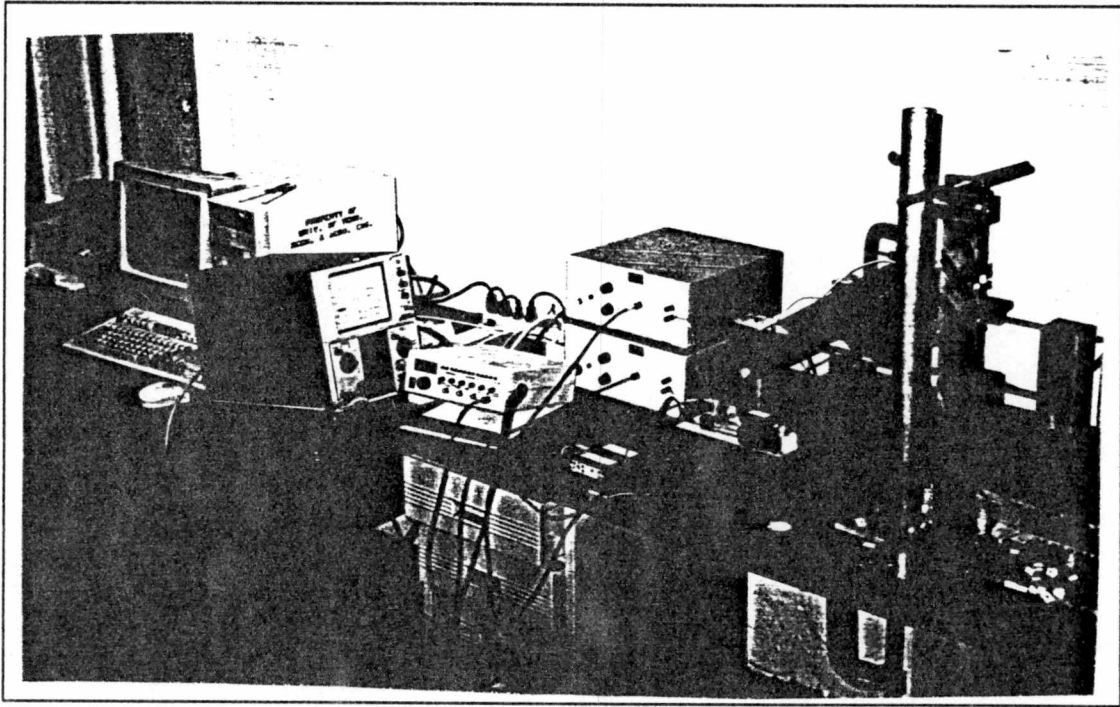


Figure 5.14 Equipment used for the transient bending vibration damping test

5.3.2 Transient torsional vibration damping

The method used to test the transient torsional vibration damping was similar to the one used for testing transient bending vibration damping, but the test set-up was different. Figure 5.15 shows the set-up and Figure 5.16 shows the feedback circuit.

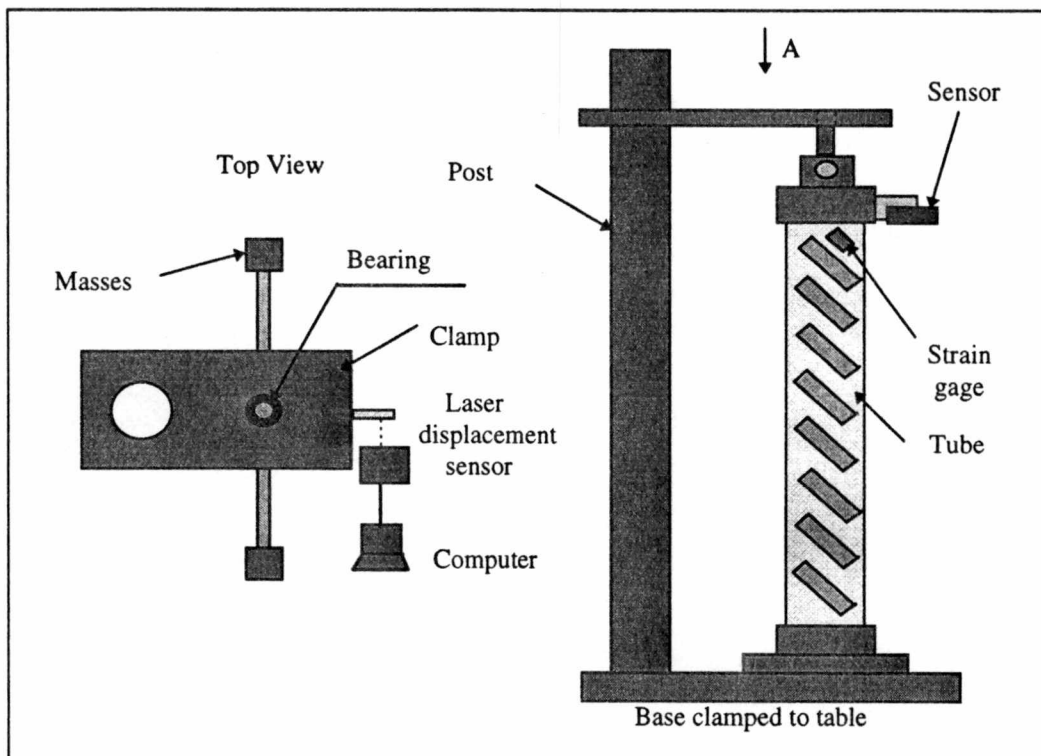


Figure 5.15 Sketch of the setup of torsional vibration damping test

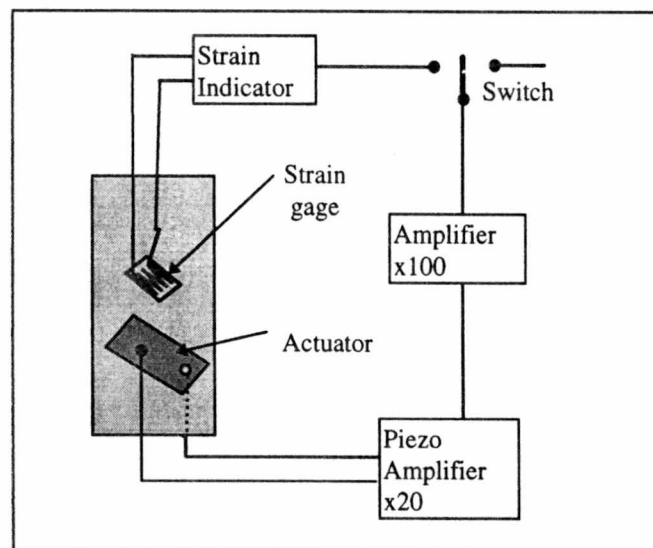


Figure 5.16 Feedback circuit for torsional vibration damping testing

A clamp with a transverse bar was installed at the top of the specimen. Two equal masses were installed at both ends of the bar, see Figure (5.17). A laser displacement sensor was used to obtain the displacement data. The strain gage was attached at a distance of 20 mm from the tip of the beam and at the same longitudinal direction as the piezoelement. For passive damping testing, the feedback circuit connection was switched off. Torsional vibrations were excited rotating the masses about the axis of the specimen and releasing. Tip torsional deflections were measured using the laser displacement sensor. The signal from the laser displacement sensor was stored on a disc. For active damping testing, the feedback circuit connection was switched on and data recorded in a similar manner. Figure 5.18 shows a sample of time history of passively and actively damped tip torsional deflections. The thick line represents active damping test results, and the thin line represents passive damping test results. It can be seen that the improvement due to active damping was negligible. The damping coefficient and the dissipated energy were computed using equations (5.16) - (5.19) and the formulas (see Appendix IX):

$$J = (M_b R^2 / 3 + 2 M_m R_m^2 + M_c b_c^2 / 3) \quad (5.21)$$

where: we had $\omega = 2 \pi f$, $f = 12.45$ Hz the natural torsional frequency of the specimen; $M_b = 0.3205$ Kg is the mass of the attached bar; $M_m = 0.7935$ Kg is the mass of one of the attached tip masses; $R = 0.15$ m is the half length of the bar; $R_m = 0.12$ m is the length from the tip rotation center to the attached mass center. M_c is the mass of the clamp, b_c is the width of the clamp, and x_n and x_{n+1} can be obtained from the Figure (5.18). The data is shown in Appendix X.

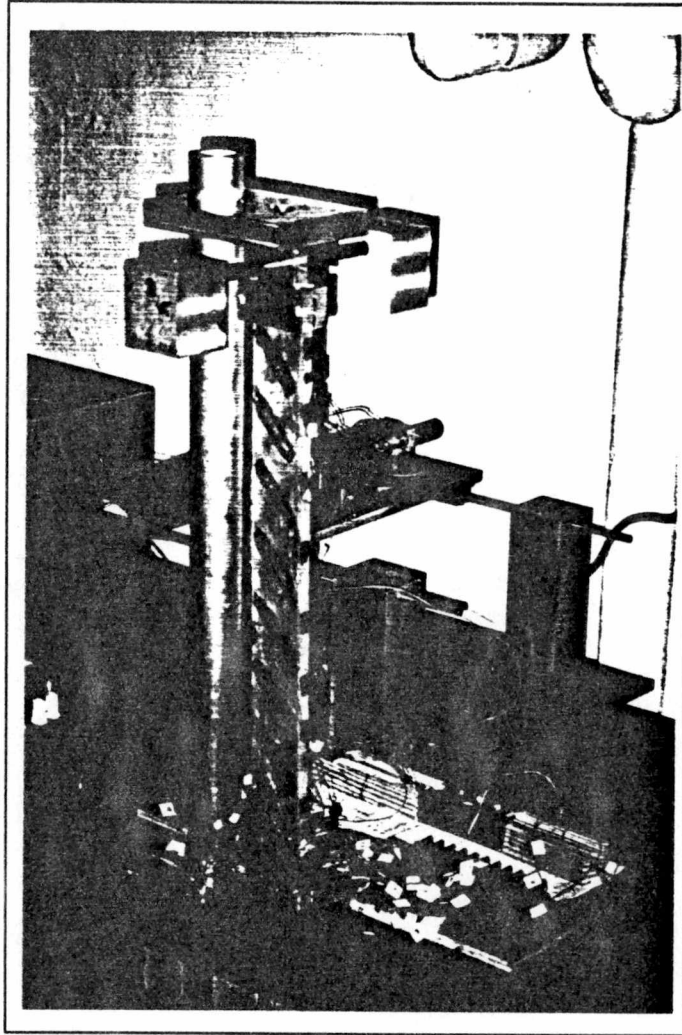


Figure 5.17 Transient torsional vibration damping test set-up.

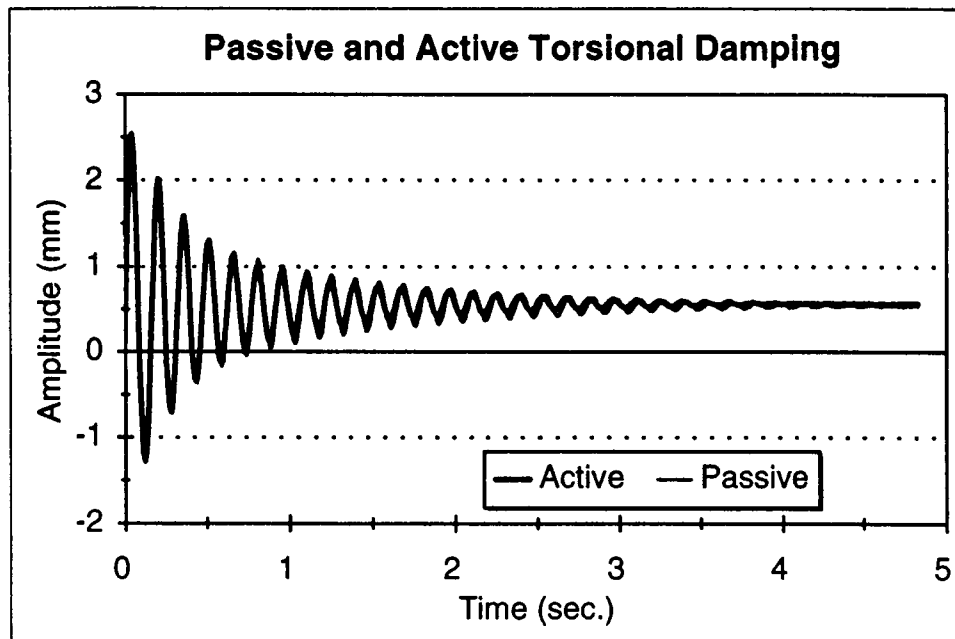


Figure 5.18 Transient torsional damping results

When the piezoelements were inactive, for the first nine cycles, the specimen had an average damping coefficient of 0.0205328 Ns/m. The energy dissipated per cycle for passively damped torsional vibration was 1.29×10^{-4} (Joules). When the feedback control was used and the piezoelements were active, the first nine cycles average damping coefficient of the beam was 0.021706 Ns/m; the energy absorbed by the piezoelements was 1.36×10^{-4} (Joules) per cycle. The improvement in damping coefficient was 5.4%. Comparing with the case of bending damping, the present improvement was very small. One of the reasons was that the specimen torsional rigidity was increased by 156.1% after the piezoelements were attached. This increment was much higher than the increment in

the flexural rigidity, which was 23.7%. The other reason was that the current method to induce torsional vibration was not very effective. Beam displacements were limited to torsion only by a bearing attached at the tip, which caused an increase in the torsional friction. Furthermore, large masses attached to the tip of the specimen highly increased the mass torsional moment of inertia of the system. It seems reasonable to expect that an improvement in torsional damping may occur if the attached end masses do not cause a substantial increase in the inertia of the system. An improved feedback circuit may not affect the results substantially, since it may be noted that even in first several cycles, when the feedback currents were at the maximal level, there was still no obvious improvement in the torsional damping. Figure 5.17 shows the transient torsional vibration damping test set-up.

5.4 Investigation of Bending and Torsional Steady State Vibration Damping

This investigation was carried out to evaluate the damping effectiveness of the piezoelements in steady state bending and torsional vibration.

5.4.1 Steady state bending vibration damping

A shaker table was used to excite steady state bending vibrations of the beam. The shaker table was connected to the Vibration Excitor System (TTI, 423 S. Brookhurst St. Anaheim, Calif. 92804). This system was able to excite steady state vibrations at different frequencies. The specimen was clamped to the shaker table. Figure (5.19) shows

a schematic top view of this set-up. The remaining portions of the mechanism were the same as those in the transient bending vibration test.

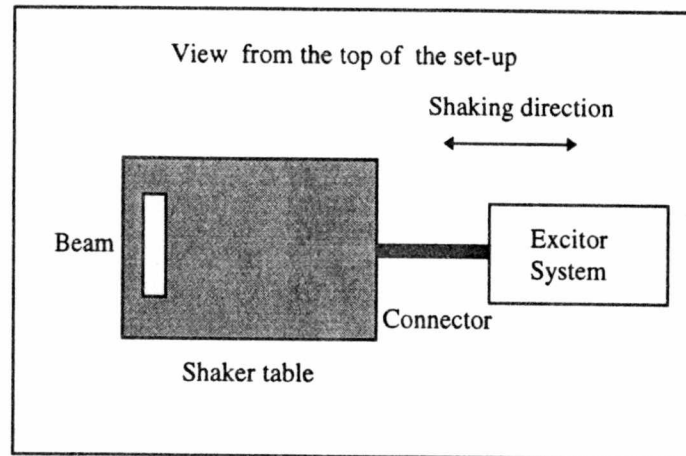


Figure 5.19 Schematic of a set-up for testing the steady state bending vibration damping

About ten frequencies around the fundamental bending frequency of the beam were preselected for testing. Upon turning on the excitor system, tip deflections were measured using the same method as used in the transient bending vibration test. Three levels of actuation were employed with the piezoelements for each frequency. At zero level, the feedback circuit connection was turned off and the piezoelements were passive. At the highest level, the sensitivity of the strain indicator was set at full gain maximum magnitude and the piezoelements were fully active. At the intermediate level, the gain was halved. The results are shown in Figure 5.20, where average amplitudes are plotted vs. frequency.

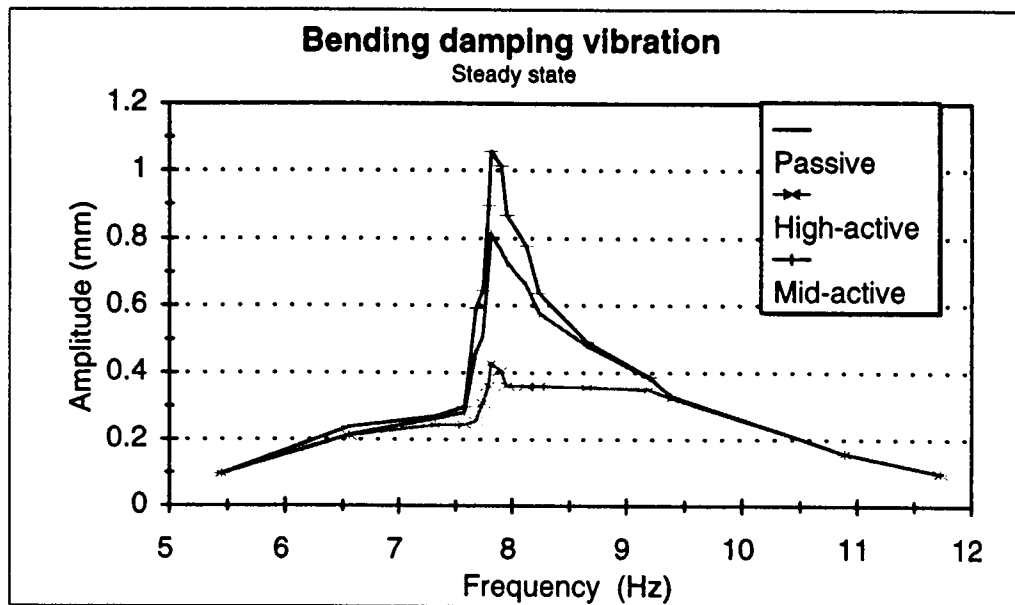


Figure 5.20 Steady state bending vibration damping results

The tip bending displacement amplitude reduction was computed using the following formula:

$$\% \text{ Improvement} = (A_{\text{passive}} - A_{\text{active}}) / A_{\text{active}} \times 100\% \quad (5.22)$$

where A is peak amplitude of tip deflections.

For maximum gain $k = 100$,

$$\begin{aligned} \% \text{ Improvement} &= (1.058 - 0.428) / 0.428 \times 100\% \\ &= 147\% \end{aligned}$$

For $k=50$:

$$\begin{aligned} \% \text{ Improvement} &= (1.058 - 0.813) / 0.813 \times 100\% \\ &= 30\% \end{aligned}$$

The above results show that higher tip bending displacement amplitude reduction was obtained using a higher gain in the feedback circuit.

The damping coefficient of steady state bending vibration was computed using the half power point method. This method is often used in a steady-state harmonic response. The following formula ¹⁵ was used:

$$\zeta = \frac{\omega_2 - \omega_1}{2 \omega_n} \quad (5.23)$$

where $\omega_n = 2\pi f_n$, f_n being the fundamental frequency at which the tip bending displacement amplitude A_n is maximum (resonance frequency); $\omega_1 = 2\pi f_1$, f_1 is the first frequency at which tip displacement amplitude is $A_1 = \frac{\sqrt{2}}{2} A_n$; $\omega_2 = 2\pi f_2$, f_2 is the second frequency at which tip displacement amplitude is $A_2 = \frac{\sqrt{2}}{2} A_n$. In the present case we have $f_n = 7.813$ Hz. The half power method validation in the case is shown in Appendix XII.

The following damping coefficients were calculated using the data shown in Figure 5.19.

For passive damping: $A_n = 1.058$ mm, $A_1 = A_2 = 0.748$ mm,
 $f_1 = 7.76$ Hz, $f_2 = 8.16$ Hz,
 $\zeta_p = (8.16 - 7.76) / (2 \times 7.813) = 0.0256$

For active damping with $k=100$:

$A_n = 0.428$ mm, $A_1 = A_2 = 0.303$ mm,

$$f_1 = 7.75 \text{ Hz}, \quad f_2 = 9.59 \text{ Hz},$$

$$\zeta_{fa} = (9.59 - 7.75) / (2 \times 7.813) = 0.118$$

$$\text{improvement\%} = (\zeta_{fa} - \zeta_p) / \zeta_p \times 100\% = 363.3 \%$$

For active damping with $k=50$:

$$A_n = 0.813 \text{ mm}, \quad A_1 = A_2 = 0.575 \text{ mm},$$

$$f_1 = 7.76 \text{ Hz}, \quad f_2 = 8.22 \text{ Hz},$$

$$\zeta_{ma} = (8.22 - 7.76) / (2 \times 7.813) = 0.029$$

$$\text{improvement \%} = (\zeta_{ma} - \zeta_p) / \zeta_p \times 100\% = 13.3 \%$$

These results show that we can expect higher damping efficiency for higher gains. The gain in the feedback circuit had a great effect on the damping coefficient in steady-state bending vibration. Using a smaller frequency step would be necessary to obtain accurate results using the half-power method. The above calculations give only a rough approximation of damping magnitude due to limitations of the currently available excitor system.

5.4.2 Steady state torsional vibration damping

The shaker table was also used to perform the steady state torsional vibration damping test. The specimen was clamped to the shaker table as shown schematically in Figure 5.21. The remaining portion of the set-up was identical to that used in the transient torsional vibration test.

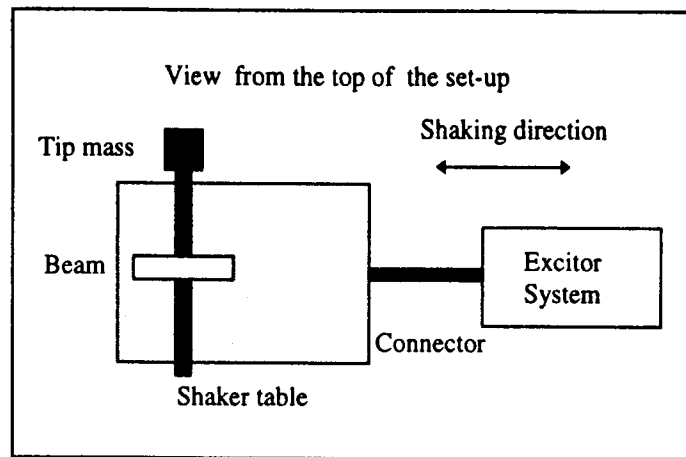


Figure 5.21 Schematic of a set-up for testing steady-state torsional vibration damping.

Similarly to the case of steady-state bending vibration test, about ten frequency magnitudes around the fundamental torsional frequency of the beam were preselected. Tip deflections were measured using the same method as used in the transient bending vibration test. The results are shown in Figure 5.22.

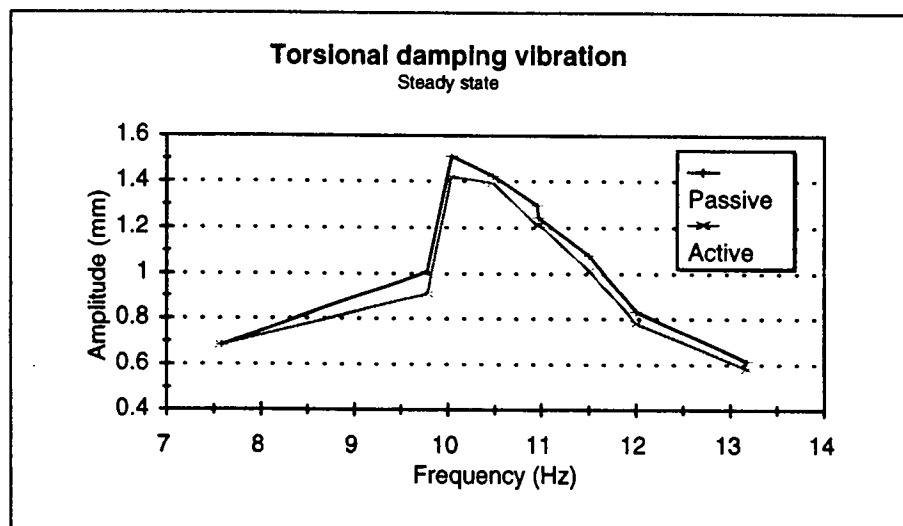


Figure 5.22 Steady state torsional vibration damping results

The tip torsional displacement amplitude reduction was computed using the formula (5.22) at the natural frequency $f = 10.04$ (Hz).

For active damping with $k=100$:

$$\begin{aligned}\% \text{ Improvement} &= (1.51 - 1.42) / 1.42 \times 100\% \\ &= 6.3 \%\end{aligned}$$

This result shows that the tip torsional displacement amplitude reduction was very small.

The damping coefficient for steady state torsional vibration was computed using the same method as that for steady state bending vibration, yielding the following results:

For passive damping: $A_n = 1.51(\text{mm})$, $A_1 = A_2 = 1.068 (\text{mm})$,

$$f_1 = 9.81 (\text{Hz}), f_2 = 11.55(\text{Hz}),$$

$$\zeta_{tp} = (11.55 - 9.81) / (2 \times 10.04) = 0.0867$$

For full active damping: $A_n = 1.42 (\text{mm})$, $A_1 = A_2 = 1.01 (\text{mm})$,

$$f_1 = 9.78 (\text{Hz}), f_2 = 11.66 (\text{Hz}),$$

$$\zeta_{ta} = (11.66 - 9.78) / (2 \times 10.04) = 0.0936$$

$$\text{improvement\%} = (\zeta_{ta} - \zeta_{tp}) / \zeta_{tp} \times 100\% = 7.96 \%$$

This result shows only a small improvement (8%) has been achieved even under the maximum gain for the active torsional vibration damping. This improvement is much smaller than the one obtained from fully active bending vibration damping (363%).

These results show that the effectiveness of piezoelements for steady state torsional vibration damping is similar as in the case of transient damping vibration. That is, the

active bending damping performance is much better than active damping torsional performance for both transient and steady state damping vibrations. In bending damping vibration, increasing the gain of the feedback circuit proved to be very effective for improving performance. On the other hand, a more sophisticated mechanism must be devised to improve the damping of torsional vibrations.

CHAPTER SIX

NUMERICAL STUDIES OF OPEN CROSS-SECTION BEAMS

Theoretical models capable of predicting of bending and twisting deflections of piezoactuated closed cross-section structures, are not yet well developed. Therefore, numerical studies were performed using a finite element software COSMOS/M to obtain insight into the behavior of an open cross-section beam actuated by skewed piezoelements. Shear center locations along the actuated portion of the beam and deflections of unactuated portions of a beam were investigated.

6.1 Shear Center Locations

In most of analytical models, the coordinate systems are based on the position of the shear center locations. The concept of shear center is very important in the thin-walled beam theory, and it is necessary to know those methods to identify shear center positions.

6.1.1 Definitions of shear center

Shear center is also called a flexural center. A literature review shows that there are several different approaches to define the shear center.

According to Reference [8], a shear center is a point through which a transverse end load of arbitrary inclination in the yz plane must pass if the beam is to bend without

twisting. Here, y, z are the principal coordinates of the cross-section of the beam. The shear center location is given by:

$$e_y = \frac{I_z S_{wz} - I_{yz} S_{wy}}{I_y I_z - I_{yz}^2}, \quad e_z = \frac{I_{yz} S_{wz} - I_y S_{wy}}{I_y I_z - I_{yz}^2} \quad (6.1)$$

where, I 's are moments of inertia, S_{wy} and S_{wz} are sectorial moments.

According to Reference [16], the center of shear (called also flexural center) is the point at which an applied shearing force would produce a flexure without torsion. It is the same definition as used in Reference [8] mentioned. However, the computation of the shear center is different. Here, the coordinates x_F and y_F of the center of shear are given by:

$$x_F = -\frac{1}{I_x} \iint_S y \varphi(x, y) dx dy, \quad y_F = -\frac{1}{I_y} \iint_S x \varphi(x, y) dx dy \quad (6.2)$$

where $\varphi(x, y)$ denotes the warping function for the cross section and I_x and I_y are the moments of inertia with respect to the axes x and y .

According to Reference [17], the center of flexure is a point through which the resultant of a terminal shearing force on a cantilevered beam must pass in order to produce "torsion-free bending." In this case, the coordinates of a center of flexure are given as:

$$x_F = \frac{1}{2(1+\nu)I_x} \iint \left(x \frac{\partial \phi_2}{\partial y} - y \frac{\partial \phi_2}{\partial x} - (1+\nu)xy^2 + \nu x^3 \right) dx dy \quad (6.3)$$

$$y_F = \frac{1}{2(1+\nu)I_y} \iint \left(x \frac{\partial \phi_1}{\partial y} - y \frac{\partial \phi_1}{\partial x} - (1+\nu)xy^2 + \nu x^3 \right) dx dy \quad (6.4)$$

where ϕ_1 and ϕ_2 are harmonic functions which satisfy the conditions

$$\frac{d\phi_1}{dn} = [(1+\nu)x^2 - \nu y^2] \cos(x, n) \quad (6.5)$$

$$\frac{d\phi_2}{dn} = [(1+\nu)y^2 - \nu x^2] \cos(y, n) \quad (6.6)$$

on the boundary C, n being the exterior normal, and x and y are the principal axes.

Another reference [18] shows that shear center positions can be obtained as

$$y^* = \frac{1}{V_x} \iint (y \tau'_{zx} - x \tau'_{zy}) dx dy \quad (6.7)$$

$$x^* = -\frac{1}{V_y} \iint (y \tau''_{zx} - x \tau''_{zy}) dx dy \quad (6.8)$$

where V_x and V_y are the x and y components of the shear force; shear stresses τ'_{zx} and τ'_{zy} are obtained corresponding to the case in which $V_y = 0$; similarly, for the case in which $V_x = 0$, stresses τ''_{zx} and τ''_{zy} are obtained.

6.1.2 The determination of the shear center position for a piezoactuated portion of the beam

A typical cross-section of a piezoactuated beam is asymmetrical, as shown in Figure 6.1. It is very difficult to find theoretically the precise location of the shear center for that kind of cross-section. Therefore, the cross-section shown in Figure 6.2 is considered to be approximately with a symmetrical structure. The shear center position for such a cross-section can be expressed¹⁹ as:

$$x_{sc} = \frac{I_{x1} r_1 + I_{x2} r_2}{I_x}, \quad y_{sc} = 0 \quad (6.9)$$

Eq (6.9) has the following form¹⁹ for the cross-section shown in Figure 6.1.

$$X_{sc} = \frac{I_{xs} b_s + (I_{xe} + I_{xa})(x_1 + x_2)}{2I_x}, \quad Y_{sc} = 0 \quad (6.10)$$

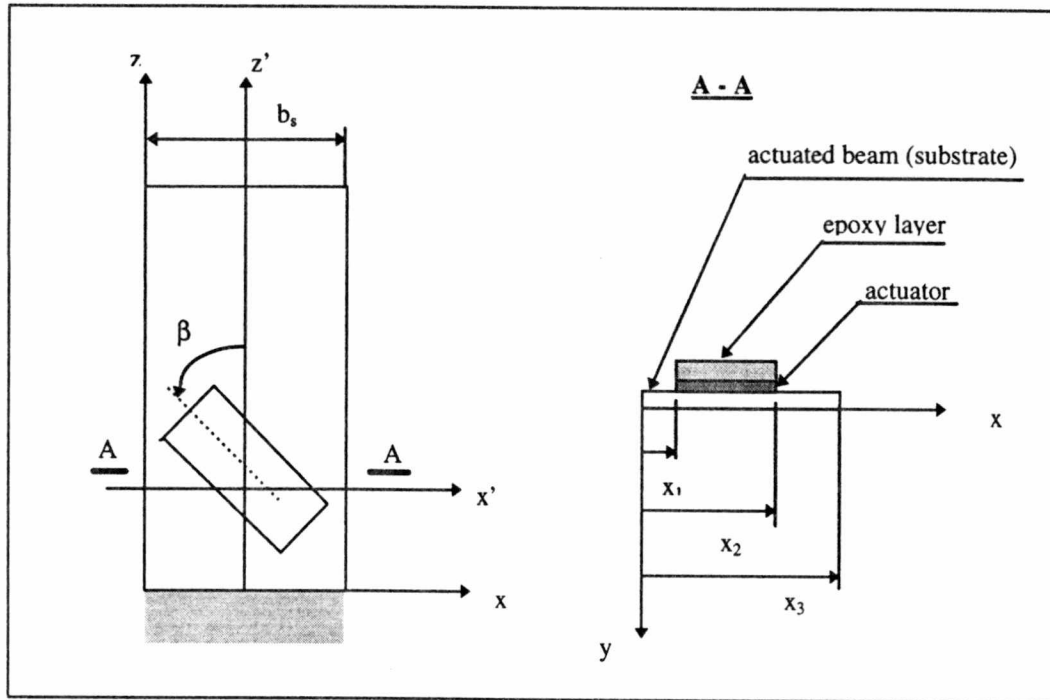


Figure 6.1 Investigated specimen

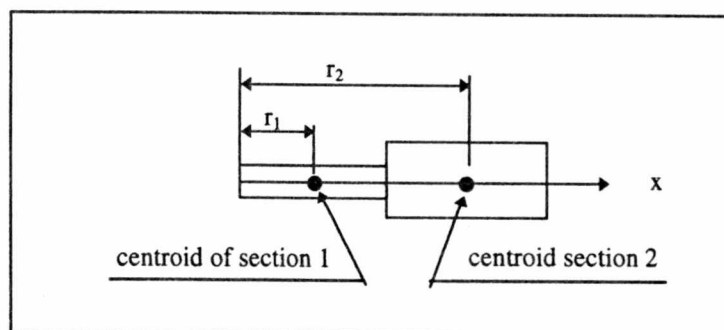


Figure 6.2 Cross-section with one axis of symmetry

The procedure to determine experimentally the shear center locations is the following¹⁹. A transverse force is applied at tip corner A of the beam (see Figure 6.3), and the beam twist angle ϕ_{za} with respect to z axis is obtained. Then, the same force is applied to tip B and the twist angle ϕ_{zb} is determined. The position of a shear center X_{sc} is computed using the following equation¹⁹:

$$a X_{sc} + b = 0 \quad (6.11)$$

where the coefficients **a** and **b** are obtained from the following system of equations.

$$a b^s + b = \phi_{za} \quad (6.12)$$

$$a 0 + b = \phi_{zb} \quad (6.13)$$

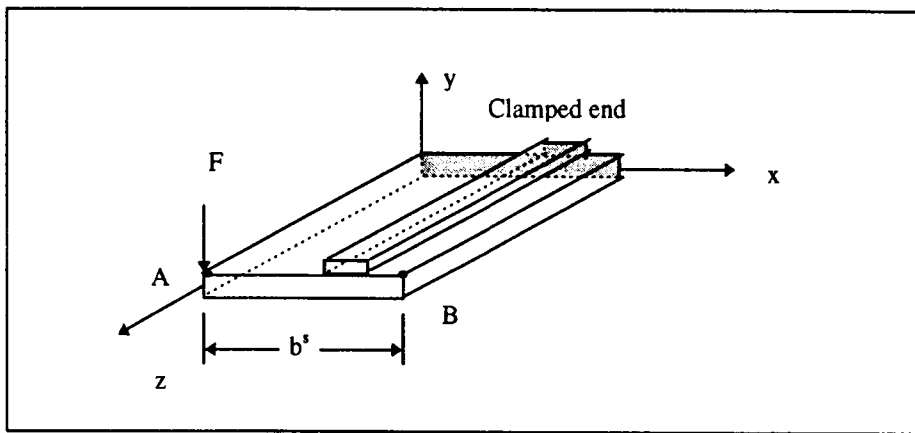


Figure 6.3 Passive beam with an onlay

The onlay is put on the beam to simulate a cross section similar to the one as shown in Figure(6.1).

6.1.3 The finite element analysis of shear center position

The accuracy of the approximation method using equation (6.10) was verified using the finite element analysis package COSMOS/M. The finite element analysis was carried out to determine elastic axes locations in beams with onlay strips bonded at an angle to the longitudinal axis of the beam. Those models simulated the structure of a beam with a skewed piezoactuator. Figure 6.4 shows a sketch of such a beam.

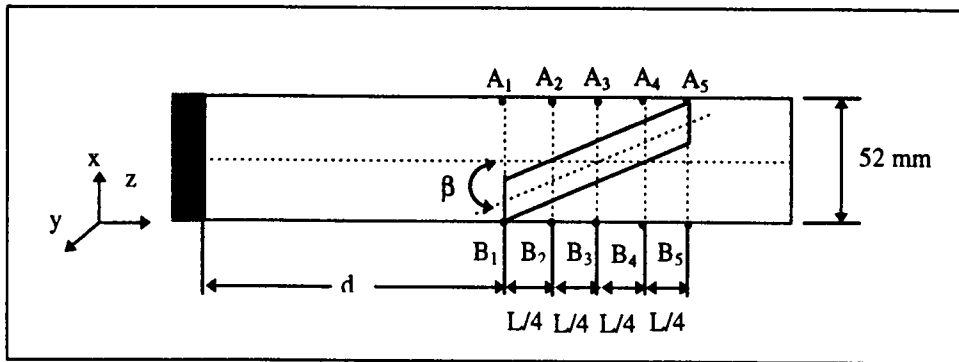


Figure 6.4 Beam with a skewed onlay

The substrate beam was 52mm wide and 2.5mm thick. The onlays were 10mm wide and 2.5mm thick. It was assumed that both the substrate and the onlays had the same material characteristics, i.e. $E = 6.1 \cdot 10^{10}$ Pa, $G = 2.41 \cdot 10^{10}$ Pa and $\nu = 0.33$. For convenience, the shorter edges of the skewed onlays were selected to be perpendicular to the longitudinal axis of the beam. The substrate was meshed using 5mm x 5mm finite elements, and the onlays were meshed using 3mm x 3mm finite elements. These selections were made within the software element edges ratio range. Eight - node solid finite elements were used to in the model. A $F_y = -5$ N force was consecutively applied to the following key points, $A_1 \dots A_5$ and $B_1 \dots B_5$. A sample finite element analysis program is presented in

appendix XV. Vertical displacements due to the forces applied at key points were used to compute the resulting twist angles about z axis.

For a transverse force applied at key points A_1 and then B_1 :

$$\phi_{A1} = (V_{AA1} - V_{AB1})/b^s \quad (6.14)$$

$$\phi_{B1} = (V_{BA1} - V_{BB1})/b^s \quad (6.15)$$

in which V_{AA1} was the vertical displacement at A_1 due to the force applied at A_1 ; V_{AB1} was the vertical displacement at B_1 due to the force applied at A_1 ; V_{BA1} was the vertical displacement at A_1 due to the force applied at B_1 ; V_{BB1} was the vertical displacement at B_1 due to the force applied at B_1 ; $b^s = 52$ mm was the width of the substrate beam. Then, the shear center location of the cross-section A_1 - B_1 was obtained by substituting ϕ_{A1} and ϕ_{B1} to Equations (6.11) - (6.13). An identical procedure was used to determine shear center locations at other cross-sections A_2 - B_2 to A_5 - B_5 .

Several geometry cases were considered. As shown in Figure 6.4, the cases were: onlays bonded at distances d of 80mm and 160mm from the root of the beam, at angles $\beta = 30^\circ, 45^\circ$ and 60° to the longitudinal axis of the beam. Shear center locations along the portion of the beam with the onlay were determined at intervals corresponding to the quarter of the length of the piezoelement projected on the longitudinal axis of the beam. The results obtained for the passive beams with skewed onlays were shown in Figure 6.5-6.7. The neutral axis is in the position of vertical coordinate at 0.026m. Some of the data points shown were obtained by Dr. Grzegorz Kawiecki and Mr. Tony Jopling.

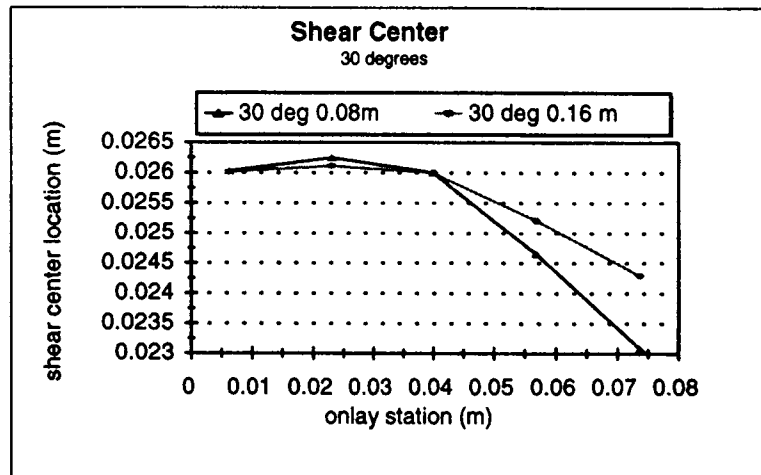


Figure 6.5 Shear center location for a passive beam with an onlay bonded at 30°

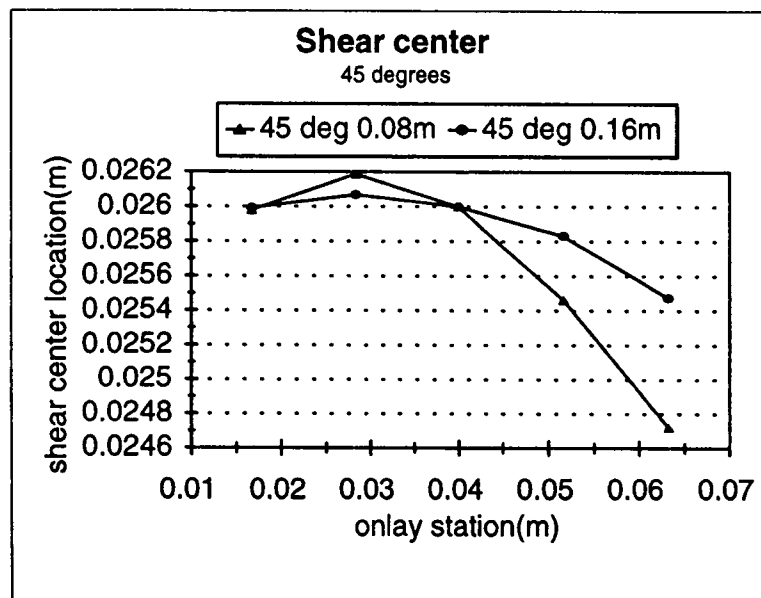


Figure 6.6 Shear center location for a passive beam with an onlay bonded at 45°

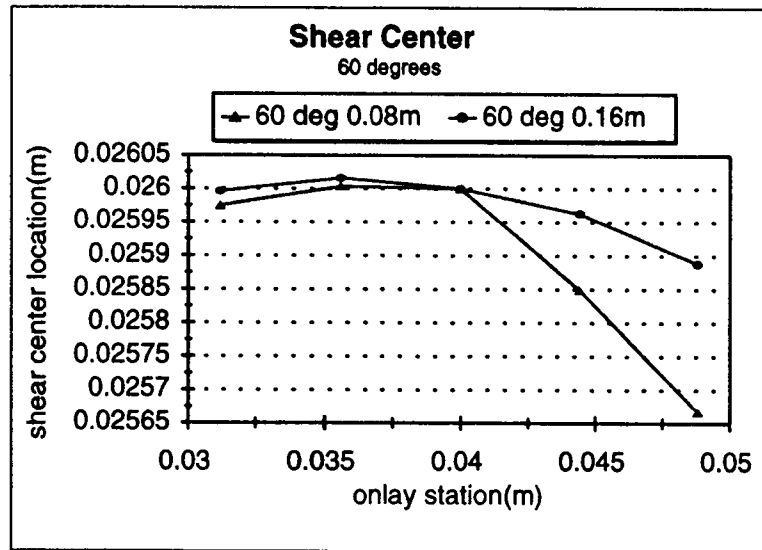


Figure 6.7 Shear center location for a passive beam with an onlay bonded at 60°

The finite element analysis results show that shear center positions along the portion of beam with a skewed onlay have a nonlinear distribution, instead of the expected linear one. The location of the elastic axis is also strongly dependent on the skew angle β and on the distance d between the base of the beam and the onlay. In general, elastic axes are located much closer to the longitudinal axis of the beam than could be expected from Equation (6.10). However, some of data points by Figure 6.5 -6.7 shows that the deviations of the shear center position from the longitudinal axis of the beam are sufficiently large, particularly for low skew angles β . That means that the assumption for the elastic axis of the piezoactuated portion of the beam coinciding with the axis of the beam is not very accurate.

6.2 Deflections Along the Unactuated Portions of a Beam

Most of the analytical models^{5,6} developed so far for modeling the behavior of beams actuated by skewed piezoelements are based on the assumption that the slopes and torsional displacements along the free portions of piezoactuated beams do not vary. However, the discrepancies between theoretical results and experimental results were usually significant. A recent finite element analysis, using COSMOS finite element analysis software, shows significant differences between magnitudes of slopes and twist angles measured along free, not subjected to any external loads, portions of a beam. Several passive (unactuated) and actuated plate models were investigated.

6.2.1 Passive plate model analysis

In order to find out the better method used to calculate the tip twist, three measurement methods were used for a passive plate analysis. An 80mm x 52mm x 2.5mm aluminum plate model was selected for the study. A 5N transverse force was applied interchangeably at mid-lengths of its longer edges as shown in Figure (6.8).

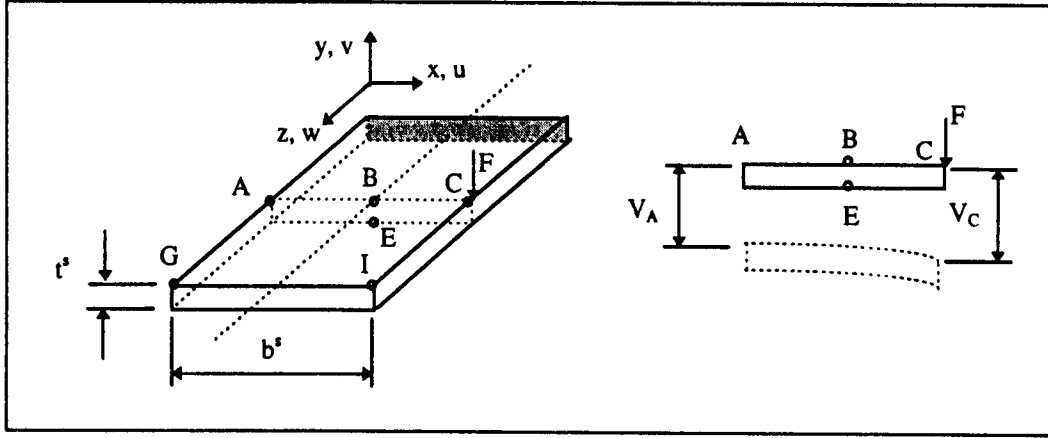


Figure 6.8 Investigated cantilevered plate

The following material characteristics were chosen: $E = 6.1 \cdot 10^{10}$ Pa, $G = 2.41 \cdot 10^{10}$ Pa, $\nu = 0.33$. The specimen was meshed using 2mm x 2mm finite elements. Eight - node solid finite elements were used to obtain the results. In order to decide which method should be used to compute the rotation of a cross-section with a significant transverse curvature, two different methods were used to calculate the twist at the loaded cross-section.

$$\phi_{z\text{-chord}} = \arcsin ((V_C - V_A) / b^s) \quad (6.16)$$

$$\phi_{zc} = \arcsin ((U_B - U_E) / t^s) \quad (6.17)$$

$$\phi_{z\text{-tip}} = \arcsin ((V_G - V_I) / b^s) \quad (6.18)$$

where V_C , V_A , V_G , and V_I are vertical displacements along y axis at C, A, G and I; U_B and U_E are the horizontal displacements along x axis at points B and E; b^s and t^s are the width and the thickness of the specimen, respectively.

In the method based on Eq.(6.17), shear center positions of cross-section were identified by the method shown in section 6.1.3. The results are shown on the Table 6.1

Table 6.1 Tip rotational displacements of a cantilevered plate

Displacement	F_A	F_C
$\phi_{z\text{-chord}} [^\circ]$	0.0279	-0.0279
$\phi_{z\text{-sc}} [^\circ]$	0.024796	-0.02426
$\phi_{z\text{-tip}} [^\circ]$	0.03416	-0.03416

Although both methods based on Eqs. (6.16) and (6.17) gave similar results in this case, further experiments have shown that the method based on calculating vertical displacement at the ends of a cross-section (Eqs.(6.16) & (6.18)) was more reliable. The result of this study shows that even in a passive (unactuated) plate there is a significant difference between the magnitudes of twist at the loaded cross-section and at the tip of the plate.

6.2.2 Piezoactuated specimen analysis

In this finite element analysis, an aluminum, 226mm x 50mm x 0.77mm plate with two 47mm x 13mm x 0.24mm piezoelements bonded at an angle β with respect to the longitudinal axis of the beam and at a distance of 100mm one from the other as shown in Figure 6.9, was modeled. In order to simplify modeling, it was assumed that both the piezoelements and the substrate had the same material characteristics, $E = 7.23 \cdot 10^{10}$ Pa, G

$= 2.72 \cdot 10^{10}$ Pa and $\nu = 0.33$. The bonding layer thickness was included in the thickness of the piezoelements. A sketch of the model with surface-bonded piezoelements is shown in Figure (6.9)

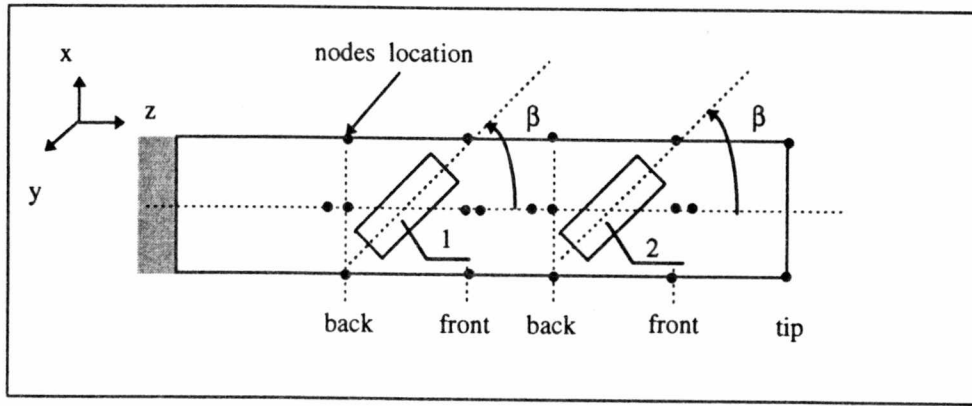


Figure (6.9) Schematic of the piezoactuated specimen

Since the modeled specimen was very thin, fine meshes(3mm x 3mm for the substrate, 1mm x 1mm for the piezoelements) were used in order to generate finite elements with a satisfactory aspect ratio. Experiments showed that the most accurate results could be obtained using 8-node solid finite elements. However, the nodes of this type of a finite element do not have rotational degrees of freedom. Slopes ϕ_y generated by piezoelements were determined from the difference in vertical displacements of two neighboring nodes at a known distance one from the other. Torsional ϕ_z generated by piezoelements were determined from the difference in vertical displacements of two edge nodes at the same cross section. The locations of those nodes are also shown in Figure (6.9). The model was analyzed for seven load cases presented in Table (6.2). Beam slopes right in front of the piezoelement ($\phi_{y-front}$) and the beam slope right behind the piezoelement (ϕ_{y-back}) were

calculated for each of the first six load cases. Then, the difference between the two slopes was computed as:

$$\phi_y = \phi_{y\text{-front}} - \phi_{y\text{-back}} \quad (6.19)$$

Same method was used for the torsional displacement.

$$\phi_z = \phi_{z\text{-front}} - \phi_{z\text{-back}} \quad (6.20)$$

The piezo-induced loads were modeled as negative pressure applied to the side walls of piezoelements. Pressure magnitude was defined as:

$$p = F_1 / t^a \quad (6.21)$$

where the magnitude of line load F_1 was determined using the induced strain data presented in Ref. [5], which give $F_1 = E \epsilon t^a$, where F_1 is force per unit length.

Slopes and torsional displacements at several positions along a piezoactuated beam were determined using the finite element method. Locations of those positions are shown in Figure (6.9) as black spots. Specimens with piezoactuators bonded at $\beta=0^\circ$, $\beta=45^\circ$ and $\beta=90^\circ$ were modeled. Data for $\beta=0^\circ$ and $\beta=90^\circ$ were obtained and computed by Dr. Grzegorz Kawiecki. Difference between slopes and torsional displacements in front of a piezoactuator and at the tip of the beam were calculated by:

$$\Delta\phi_y = ((\phi_{y\text{-front}} - \phi_{y\text{-tip}}) 100) / \phi_{y\text{-front}} \quad (6.22)$$

$$\Delta\phi_z = ((\phi_{z\text{-front}} - \phi_{z\text{-tip}}) 100) / \phi_{z\text{-front}} \quad (6.23)$$

The results of the analysis of slope and torsional displacement variations along a piezoactuated beam are shown in Table (6.2).

Table 6.2 Displacements along piezoactuated beams (prepared by Dr. Grzegorz Kawiecki)

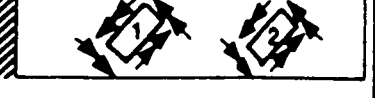
Load Case		Bending [°]			Twist [°]	
		$\beta = 0^\circ$	$\beta = 45^\circ$	$\beta = 90^\circ$	$\beta = 45^\circ$	
	ϕ_{y1back}	0.005445	0.01670	0.01291	ϕ_{z1back}	-0.02126
	$\phi_{y1front}$	0.3357	0.1222	0.1863	$\phi_{z1front}$	-0.2145
	ϕ_{y1}	0.3303	0.1389	0.1733	ϕ_{z1}	-0.1819
	ϕ_{y1tip}	0.3432	0.1049	0.1992	ϕ_{z1tip}	-0.2353
	ϕ_{y1back}	-0.04140	-0.00126	-0.04222	ϕ_{z1back}	0.01571
	$\phi_{y1front}$	-0.09177	0.05481	-0.05130	$\phi_{z1front}$	0.1643
	ϕ_{y1}	-0.05038	0.05607	-0.00908	ϕ_{z1}	0.1449
	ϕ_{y1tip}	-0.13264	0.05340	-0.09500	ϕ_{z1tip}	0.1765
	ϕ_{y1back}	-0.03595	-0.01796	-0.02930	ϕ_{z1back}	-0.01133
	$\phi_{y1front}$	0.2440	0.1770	0.1350	$\phi_{z1front}$	-0.04840
	ϕ_{y1}	0.2800	0.1950	0.1643	ϕ_{z1}	-0.03707
	ϕ_{y1tip}	0.2105	0.1583	0.1042	ϕ_{z1tip}	-0.05879
	ϕ_{y2back}	0.006082	-0.01646	0.01066	ϕ_{z2back}	-0.02731
	$\phi_{y2front}$	0.3459	0.1255	0.1618	$\phi_{z2front}$	-0.2092
	ϕ_{y2}	0.3398	0.1419	0.1511	ϕ_{z2}	-0.1819
	ϕ_{y2back}	-0.04245	-0.00157	-0.04613	ϕ_{z2back}	0.0165
	$\phi_{y2front}$	-0.1067	0.05234	-0.06404	$\phi_{z2front}$	0.1613
	ϕ_{y2}	-0.06424	0.05390	-0.01791	ϕ_{z2}	0.1448
	ϕ_{y2back}	-0.03637	-0.01802	-0.03547	ϕ_{z2back}	-0.01092
	$\phi_{y2front}$	0.2392	0.1778	0.09771	$\phi_{z2front}$	-0.04789
	ϕ_{y2}	0.2755	0.1958	0.1332	ϕ_{z2}	-0.03708
	ϕ_{y2back}	-0.03637	-0.01802	-0.03547	ϕ_{z2back}	-0.01092
	ϕ_x	0.5555	0.3906	0.296	ϕ_z	-0.07415
	ϕ_{ytip}	0.4497	0.3361	0.202	ϕ_{ztip}	-0.1088
	$\phi_{y_{exper}}$	0.356	0.2850	0.189	$\phi_{z_{exper}}$	-0.104
	$\phi_{y_{theory}}$	0.4890	0.3212	0.268	$\phi_{z_{theory}}$	-0.0983

Figure (6.10) and Figure (6.11) highlight those differences for a beam actuated by the piezoelement closer to the base of the beam. Those discrepancies vary from $\sim 60\%$ for torsional displacements to $\sim 80\%$ for slopes.

The results of this numerical study indicate that there is a problem with some of currently used models. The main issue is to account properly for the boundary condition introduced by the clamped end of the beam and for the deflections of the unloaded portions of the beam. It is important to account for slopes and torsional displacements at the back of the piezoactuators. Meanwhile, it is apparent that expressions describing the exact position of elastic axis should be used in theoretical models to improve the accuracy of those models.

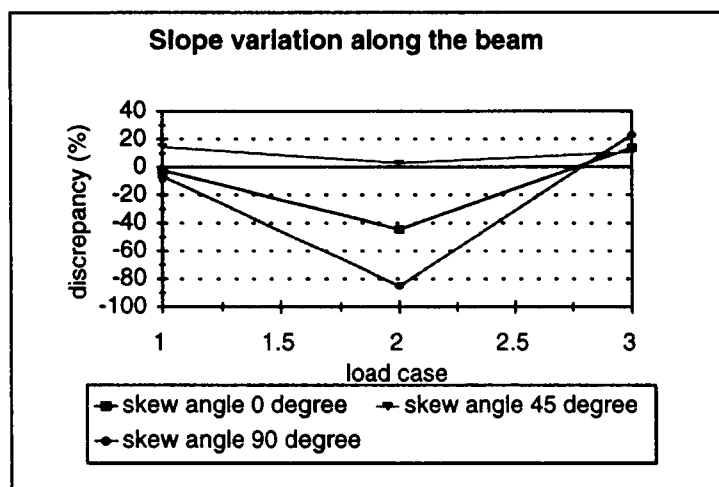


Figure (6.10) Slope variation along the beam

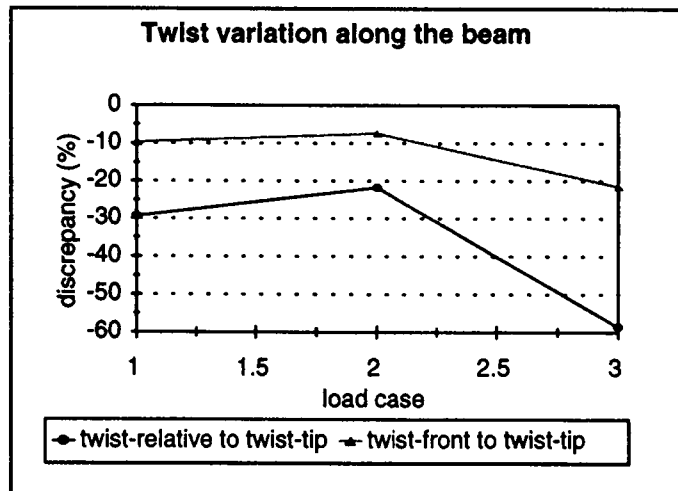


Figure (6.11) Twist variation along the beam

CHAPTER SEVEN

APPLICATION OF HEAVISIDE AND DELTA FUNCTIONS TO MODEL BENDING-TORSIONAL RESPONSE OF PIEZOACTUATED BEAMS

A generalized location function expressed using the Heaviside function was introduced to represent the location of a piezoactuator. The equivalent loads induced by the actuator could be determined using Heaviside and Delta functions so that the responses of the generating loads might be investigated. The study of a plate approximated as a beam subjected to a non-skewed line moment modeled using Heaviside and Delta function will be outlined. The derivation and the resulting governing equations will be presented

7.1 Heaviside and Delta Function Concept

In this section, the basic concept of Heaviside unit function and its derivative, the Dirac delta function, will be introduced. Also, the generalized location function and its derivative will be discussed.

7.1.1 The Heaviside unit function

The Heaviside unit function is defined²⁰ as:

$$\begin{aligned} H(x) &= 0, & x &\leq 0; \\ H(x) &= 1, & x &> 0. \end{aligned} \tag{7.1}$$

It has an ordinary discontinuity at point $x = 0$ and was defined to facilitate the representation of functions which have such discontinuities. The graph of $H(x-a)$ is shown in Figure (7.1a). The function $H(x) - H(x-a)$ has a value of unity over the interval $0 < x \leq a$ and a value of zero for $x \leq 0$ and $x > a$, as Figure (7.1b) shows.

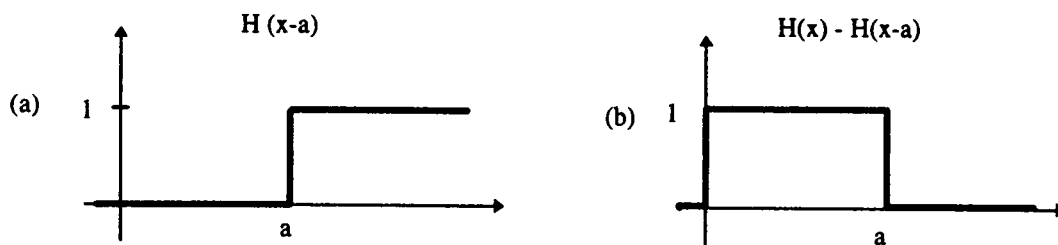


Figure 7.1 Heaviside function

7.1.2 The Dirac delta function

This is not a mathematical function at all in the usual sense of the term. Its use is to represent symbolically an ideally concentrated quantity (such as a concentrated load on a beam, or an impulsive force in dynamics) in the same way that the Heaviside's unit function was used to represent a discontinuous quantity.

The delta function $\delta(x)$ is defined as the limit as $\epsilon \rightarrow 0$ of the function $\Delta(x)$ shown in Figure 7.2 defined by

$$\begin{aligned} \Delta(x) &= 0, & x &\leq 0 \\ \Delta(x) &= 1/\epsilon, & 0 < x &\leq \epsilon \\ \Delta(x) &= 0, & x &> \epsilon \end{aligned} \quad (7.2)$$

or in terms of the unit function,

$$\delta(x) = \lim_{\varepsilon \rightarrow 0} \frac{H(x) - H(x-a)}{\varepsilon} \quad (7.3)$$



Figure 7.2 The Dirac delta function

7.1.3 The generalized location function and its derivatives

The generalized location function is defined as²⁰:

$$\begin{aligned} R(x, y) &= 1, & x_1 \leq x \leq x_2, & \quad y_1 \leq y \leq y_2; \\ &= 0, & \text{elsewhere} \end{aligned} \quad (7.4)$$

$R(x, y)$ can be expressed in form of the Heaviside function:

$$R(x, y) = [H(x-x_1) - H(x-x_2)] [H(y-y_1) - H(y-y_2)] \quad (7.5)$$

Then the derivatives of generalized location function are expressed as²⁰:

$$\frac{\partial R}{\partial x} = [\delta(x-x_1) - \delta(x-x_2)] [H(y-y_1) - H(y-y_2)] \quad (7.6)$$

$$\frac{\partial R}{\partial y} = [H(x-x_1) - H(x-x_2)] [\delta(y-y_1) - \delta(y-y_2)] \quad (7.7)$$

$$\frac{\partial^2 R}{\partial x \partial y} = [\delta(x-x_1) - \delta(x-x_2)] [\delta(y-y_1) - \delta(y-y_2)] \quad (7.8)$$

$$\frac{\partial^2 R}{\partial^2 x} = [\delta'(x-x_1) - \delta'(x-x_2)] [H(y-y_1) - H(y-y_2)] \quad (7.9)$$

$$\frac{\partial^2 R}{\partial^2 y} = [H(x-x_1) - H(x-x_2)] [\delta'(y-y_1) - \delta'(y-y_2)] \quad (7.10)$$

The mathematical interpretation of the Heaviside and Delta functions is illustrated in Figure (7.3). We can note that $[H(x-x_1) - H(x-x_2)]$ represents a uniform distribution between x_1 and x_2 , $[\delta(x-x_1) - \delta(x-x_2)]$ represents two concentrated sources at x_1 and x_2 respectively, and $[\delta'(x-x_1) - \delta'(x-x_2)]$ represents two moment sources at x_1 and x_2 , respectively²¹.

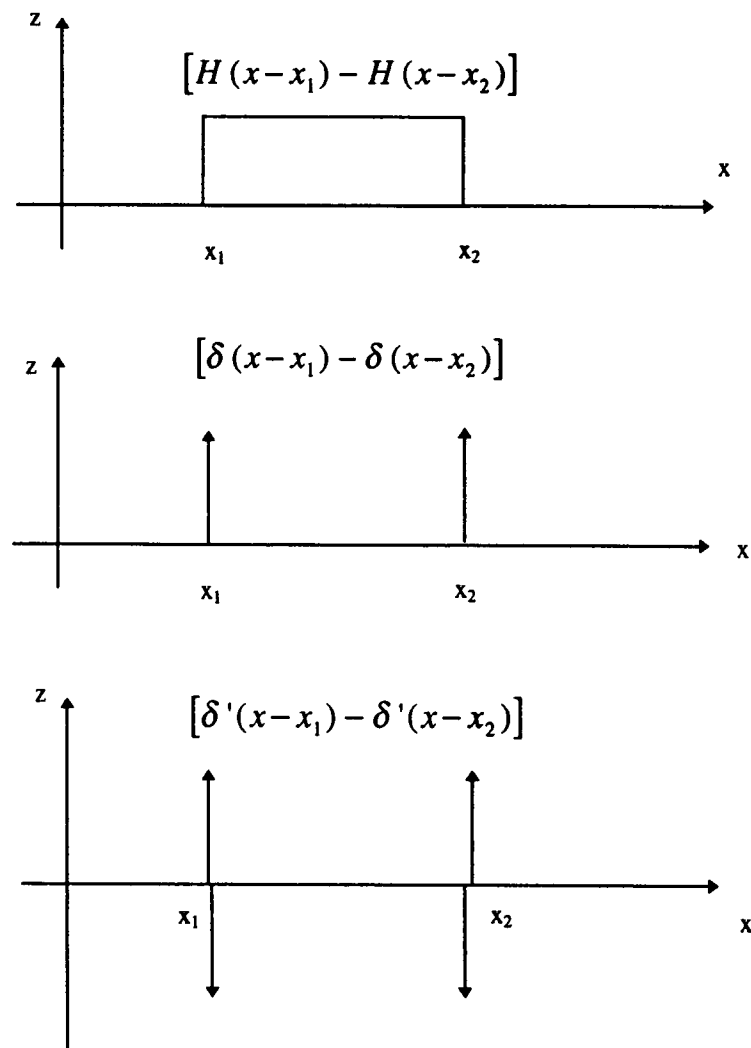
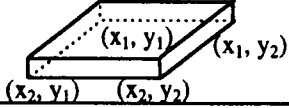
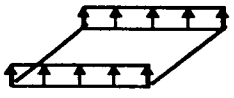
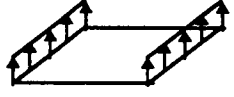
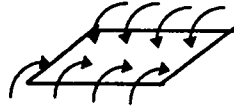
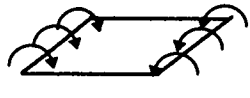
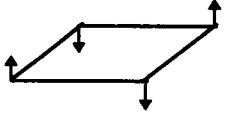


Figure 7.3 Illustrations of Heaviside and Delta functions

So the derivatives of a generalized location function can be graphically shown in Table(7.1).

Table 7.1 Generalized location function and its derivative²¹

$R(x, y) = [H(x-x_1) - H(x-x_2)] [H(y-y_1) - H(y-y_2)]$	
$\frac{\partial R}{\partial x} = [\delta(x-x_1) - \delta(x-x_2)] [H(y-y_1) - H(y-y_2)]$	
$\frac{\partial R}{\partial y} = [H(x-x_1) - H(x-x_2)] [\delta(y-y_1) - \delta(y-y_2)]$	
$\frac{\partial^2 R}{\partial^2 y} = [H(x-x_1) - H(x-x_2)] [\delta'(y-y_1) - \delta'(y-y_2)]$	
$\frac{\partial^2 R}{\partial^2 x} = [\delta'(x-x_1) - \delta'(x-x_2)] [H(y-y_1) - H(y-y_2)]$	
$\frac{\partial^2 R}{\partial x \partial y} = [\delta(x-x_1) - \delta(x-x_2)] [\delta(y-y_1) - \delta(y-y_2)]$	

The physical meanings of the derivatives of the generalized location function are²¹ :

$\frac{\partial R}{\partial x}$ and $\frac{\partial R}{\partial y}$ vertical line forces.

$\frac{\partial^2 R}{\partial^2 x}$ and $\frac{\partial^2 R}{\partial^2 y}$ line moments along the edges of the location.

$\frac{\partial^2 R}{\partial x \partial y}$ concentrated forces at the corners of the location.

7.2 Modeling of a Concentrated Load Applied to a Plate Using Heaviside

Functions

Heaviside function was used to model vertical displacement behaviors of a long cantilever plate. The result was verified by the finite element analysis using COSMOS software.

7.2.1 Load modeling using a Heaviside function

Figure (7.4) shows the modeled specimen, a cantilever plate $L = 200$ mm long, $d = 50$ mm wide and $t = 5$ mm thick, subjected to a line moment $M_y = 0.275$ Nm applied at $a = 150$ mm from the clamped end. The cross-sectional moment of inertia is $I_{xx} = (bt^3)/12$, and the Young's modulus is $E = 6.1 \cdot 10^{10}$ Pa.

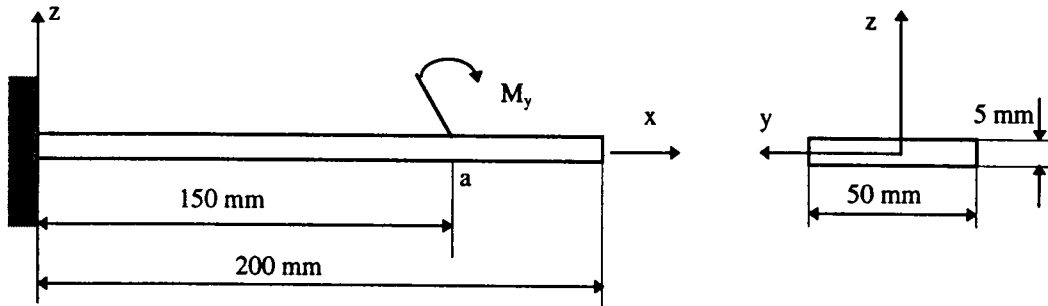


Figure 7.4 Modeled cantilever plate

Using beam theory and the Heaviside function

$$-M_y [H(x) - H(x-a)] = E I z'' \quad (7.11)$$

$$\text{then} \quad z'' = -\frac{M_y}{E I} H(x) + \frac{M_y}{E I} H(x-a) \quad (7.12)$$

$$z' = -\frac{M_y}{EI} x H(x) + \frac{M_y}{EI} (x-a) H(x-a) + C_1 \quad (7.13)$$

Since the boundary conditions at $x = 0$ are: $z'(x = 0) = 0$, $H(x = 0) = 0$, $H(x - a) = 0$,

then $C_1 = 0$.

$$\text{So, } z = -\frac{M_y}{EI} \frac{x^2}{2} H(x) + \frac{M_y}{EI} \frac{1}{2} (x-a)^2 H(x-a) + C_2 \quad (7.14)$$

Since the boundary conditions at $x = 0$ are: $z = 0$ then $C_2 = 0$.

$$\text{so, } z = -\frac{M_y}{EI} \frac{x^2}{2} H(x) + \frac{M_y}{EI} \frac{1}{2} (x-a)^2 H(x-a) \quad (7.15)$$

Since the Heaviside functions are:

$$H(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

$$H(x-a) = \begin{cases} 0 & x < a \\ 1 & x \geq a \end{cases}$$

$$\text{thus, } z = \begin{cases} -\frac{M_y}{EI} \frac{x^2}{2} & 0 \leq x < a \\ -\frac{M_y}{EI} x a + \frac{M_y}{EI} \frac{a^2}{2} & x \geq a \end{cases} \quad (7.16)$$

Substitute the magnitudes of M_y , a , E , and I to Eq (7.16), then deflection equation

becomes:

$$z = \begin{cases} -4.328 \cdot 10^{-3} x^2 & 0 \leq x < 0.15 \\ -1.298 \cdot 10^{-3} x + 9.738 \cdot 10^{-5} & x \geq 0.15 \end{cases} \quad (7.17)$$

Figure (7.6) shows the plot of bending deflections along z axes.

7.2.2 Modeling using the COSMOS finite element package

The utility of Heaviside function modeling was verified using COSMOS finite element software. The modeled specimen had the same size and material characteristics as the one evaluated theoretically. Used finite element size was 5 mm x 5 mm. Eight-node solid element was used. The moment load shown in Figure(7.5) was modeled.

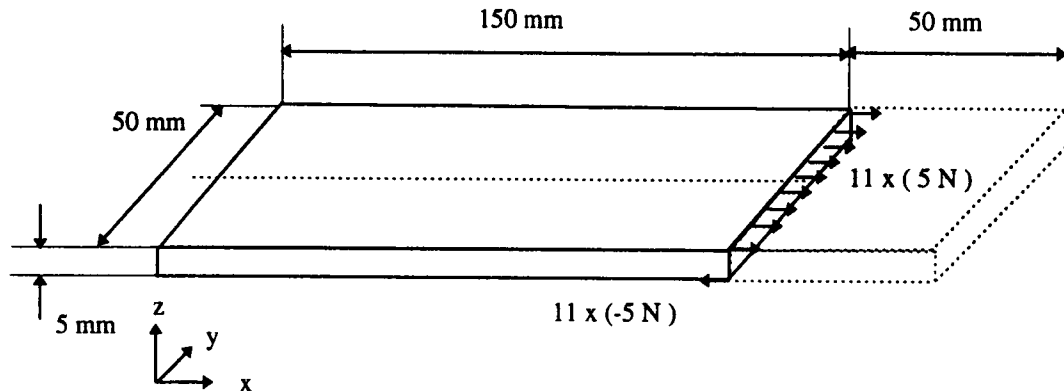


Figure 7.5 Schematic of a COSMOS model

A line moment was selected as the concentrated load. This moment was generated by the forces applied to a specimen along the x axis direction. Those forces were applied to the nodes in the cross-section at 150 mm from the clamped end of the beam. As shown in Figure (7.5), a positive force $F_x = 5 \text{ N}$ was applied to each node in upper surface of the specimen at that cross-section. A negative force $F_x = -5 \text{ N}$ was applied to each node in the lower surface of the specimen. There were 11 upper surface nodes and 11 lower surface nodes. The distance between the upper surface and the lower surface was equal to the thickness of the specimen. So, the magnitude of this moment was $11 \times 5 \times 0.005 = 0.275 \text{ Nm}$. This moment had the same value as the one modeled using the Heaviside function approach. This method ensured the generated line moment was applied at the

exact position of 150 mm from the clamped end of the plate. Vertical displacements of the central line along the x direction of the plate were obtained. Figure (7.6) shows the result, which is in excellent agreement (2.7% discrepancy at the tip) with the result obtained using Heaviside function theory. This means the Heaviside function can be used to model the effect of a concentrated load in a plate.

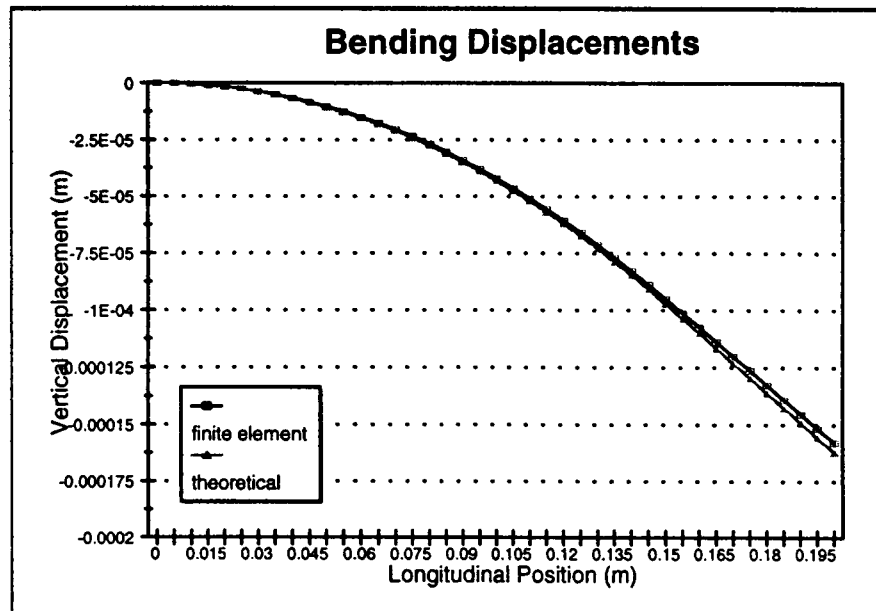


Figure 7.6 Theoretical analysis vs. the finite element results

7.3 Heaviside and Delta Function Application to Skewed Piezoactuated Plate Analysis

This study is based on the concept of Heaviside and Delta functions and the Classical Laminated Plate Theory. This study presents the derivation and the resulting of

the governing equations modeling the responses of a plate actuated by a skewed piezoelement.

7.3.1 The case of a non-skewed piezoactuator

Let us consider a rectangular laminated plate with an induced strain actuator, such as a piezoelement patch. Figure (7.7) shows the actuator and the pertinent coordinates. The derivation presented below is based on Ref.[21].

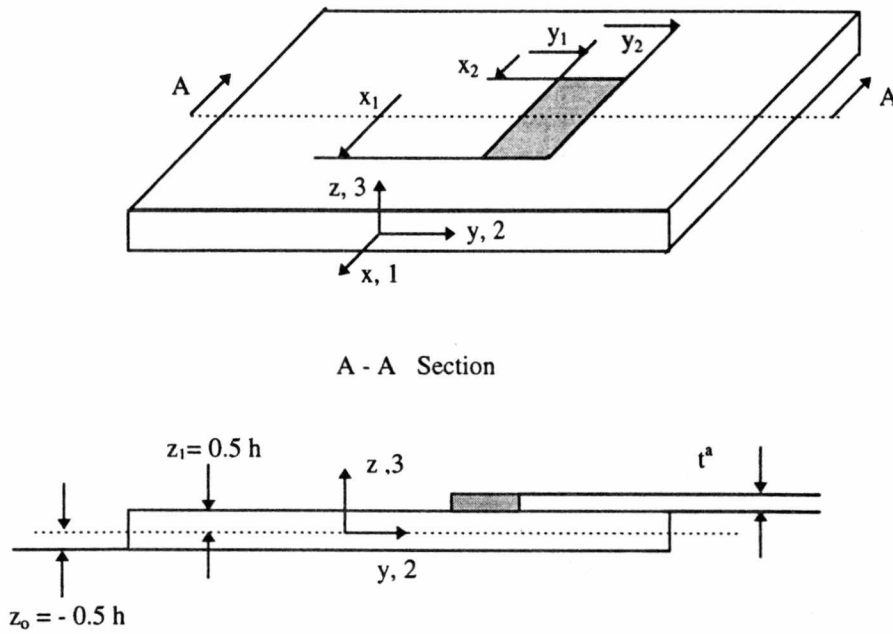


Figure 7.7 Schematic of a plate with a single non-skewed actuator

Recall the stress-strain relationship.

$$\{\sigma\} = [\bar{Q}] \{\epsilon\} \quad (7.18)$$

The total strain can be shown as the sum of mechanical and induced actuator strains.

$$\{\varepsilon\} = \{\varepsilon^m\} + \{\Lambda\} \quad (7.19)$$

under the Kirchhoff assumption, the mechanical strain vector is given by:

$$\{\varepsilon^m\} = \{\varepsilon^0\} + z\{\chi\} \quad (7.20)$$

where the midplane mechanical strains are:

$$\{\varepsilon^0\} = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} = \begin{bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{bmatrix} \quad (7.21)$$

the midplane curvatures can be expressed as:

$$\{\chi\} = \begin{bmatrix} \chi_x \\ \chi_y \\ \chi_{xy} \end{bmatrix} = - \begin{bmatrix} \frac{\partial^2 \omega_0}{\partial x^2} \\ \frac{\partial^2 \omega_0}{\partial y^2} \\ 2 \frac{\partial^2 \omega_0}{\partial x \partial y} \end{bmatrix} \quad (7.22)$$

and the actuator strains can be represented¹⁶ as:

$$\{\Lambda\} = \begin{bmatrix} \Lambda_x \\ \Lambda_y \\ \Lambda_{xy} \end{bmatrix} = \begin{bmatrix} [H(z) - H(z - t^a)] \frac{d_{31}}{t^a} V R(x, y) \\ [H(z) - H(z - t^a)] \frac{d_{32}}{t^a} V R(x, y) \\ [H(z) - H(z - t^a)] \frac{d_{36}}{t^a} V R(x, y) \end{bmatrix} \quad (7.23)$$

where the Heaviside function, $H(z - t^a)$, is defined as follows:

$$\begin{aligned}
H(z - t^a) &= 1, \quad z \geq t^a \\
&= 0, \quad z < t^a
\end{aligned} \tag{7.24}$$

Then the following equations can be obtained²¹.

$$\{N\} = [A]\{\varepsilon^o\} + [B]\{\chi\} - [E]\{d\} \tag{7.25}$$

and

$$\{M\} = [B]\{\varepsilon^o\} + [D]\{\chi\} - [F]\{d\} \tag{7.26}$$

where

$$\{d\} = \begin{bmatrix} d_{31} & R \\ d_{32} & R \\ d_{36} & R \end{bmatrix} \tag{7.27}$$

The laminate extensional stiffnesses are given by²²:

$$A_{ij} = \sum_{k=1}^n (\bar{Q}_{ij})_k (z_k - z_{k-1}) \tag{7.28}$$

Recall that for isotropic material, the laminate coupling stiffnesses are:

$$[B] = 0 \tag{7.29}$$

the laminate bending stiffnesses are:

$$D_{ij} = \frac{1}{3} \sum_{k=1}^n (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3) \tag{7.30}$$

the actuator extensional stiffnesses are²¹:

$$E_{ij} = \sum_{k=1}^m (\bar{Q}_{ij})_k^a V_k \tag{7.31}$$

the actuator bending-twisting stiffnesses are²¹:

$$F_{ij} = \frac{1}{2} \sum_{k=1}^m (\bar{Q}_{ij})_k^a V_k (z_{k+}^a - z_{k-}^a) \quad (7.32)$$

In this model, $n = 1$, $m = 1$, $z_0 = -0.5 h$, $z_1 = 0.5 h$, $z_{k+}^a - z_{k-}^a = t^a$, $V_k = V$, so,

$$A_{ij} = \bar{Q}_{ij} h \quad (7.33)$$

$$D_{ij} = \frac{h^3}{12} \bar{Q}_{ij} \quad (7.34)$$

$$E_{ij} = (\bar{Q}_{ij})^a V \quad (7.35)$$

$$F_{ij} = \frac{t^a}{2} (\bar{Q}_{ij})^a V \quad (7.36)$$

Substitute Eqs. (7.21), (7.22), (7.27) and (7.28) to Eqs. (7.25) and (7.26) and the resulting equations can be expressed as:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \end{Bmatrix} - \begin{bmatrix} E_{11} & E_{12} & E_{16} \\ E_{12} & E_{22} & E_{26} \\ E_{16} & E_{26} & E_{66} \end{bmatrix} \begin{Bmatrix} d_{31}R \\ d_{32}R \\ d_{36}R \end{Bmatrix} \quad (7.37)$$

and

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \chi_x \\ \chi_y \\ \chi_{xy} \end{Bmatrix} - \begin{bmatrix} F_{11} & F_{12} & F_{16} \\ F_{12} & F_{22} & F_{26} \\ F_{16} & F_{26} & F_{66} \end{bmatrix} \begin{Bmatrix} d_{31}R \\ d_{32}R \\ d_{36}R \end{Bmatrix} \quad (7.38)$$

and expanding Eqs. (7.37) and (7.38):

$$N_x = A_{11}\varepsilon_x^o + A_{12}\varepsilon_y^o + A_{16}\gamma_{xy}^o - E_{11}d_{31}R - E_{12}d_{32}R - E_{16}d_{36}R \quad (7.39)$$

$$N_y = A_{12}\epsilon_x^0 + A_{22}\epsilon_y^0 + A_{26}\gamma_{xy}^0 - E_{12}d_{31}R - E_{22}d_{32}R - E_{26}d_{36}R \quad (7.40)$$

$$N_{xy} = A_{16}\epsilon_x^0 + A_{26}\epsilon_y^0 + A_{66}\gamma_{xy}^0 - E_{16}d_{31}R - E_{26}d_{32}R - E_{66}d_{36}R \quad (7.41)$$

$$M_x = D_{11}\chi_x + D_{12}\chi_y + D_{16}\chi_{xy} - F_{11}d_{31}R - F_{12}d_{32}R - F_{16}d_{36}R \quad (7.42)$$

$$M_y = D_{12}\chi_x + D_{22}\chi_y + D_{26}\chi_{xy} - F_{12}d_{31}R - F_{22}d_{32}R - F_{26}d_{36}R \quad (7.43)$$

$$M_{xy} = D_{16}\chi_x + D_{26}\chi_y + D_{66}\chi_{xy} - F_{16}d_{31}R - F_{26}d_{32}R - F_{66}d_{36}R \quad (7.44)$$

There is no external load or dynamic load. So, the equilibrium equations are given as:

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \quad (7.45)$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \quad (7.46)$$

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = 0 \quad (7.47)$$

Equations (7.21), (7.22), and (7.39 - 7.44) were substituted into the equilibrium equations (Eqs. (7.45 - 7.47)) in terms of the midplane displacements u_0 , v_0 and w_0 .

Using symbols u , v and w for brevity, these equations are:

$$N_x = A_{11} \frac{\partial u}{\partial x} + A_{12} \frac{\partial v}{\partial y} + A_{16} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - (E_{11}d_{31} + E_{12}d_{32} + E_{16}d_{36})R \quad (7.48)$$

$$N_y = A_{12} \frac{\partial u}{\partial x} + A_{22} \frac{\partial v}{\partial y} + A_{26} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - (E_{12}d_{31} + E_{22}d_{32} + E_{26}d_{36})R \quad (7.49)$$

$$N_{xy} = A_{16} \frac{\partial u}{\partial x} + A_{26} \frac{\partial v}{\partial y} + A_{66} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - (E_{16}d_{31} + E_{26}d_{32} + E_{66}d_{36})R \quad (7.50)$$

$$M_x = - \left(D_{11} \frac{\partial^2 w}{\partial x^2} + D_{12} \frac{\partial^2 w}{\partial y^2} + 2D_{16} \frac{\partial^2 w}{\partial x \partial y} \right) - (F_{11}d_{31} + F_{12}d_{32} + F_{16}d_{36})R \quad (7.51)$$

$$M_y = - \left(D_{12} \frac{\partial^2 w}{\partial x^2} + D_{22} \frac{\partial^2 w}{\partial y^2} + 2D_{26} \frac{\partial^2 w}{\partial x \partial y} \right) - (F_{12}d_{31} + F_{22}d_{32} + F_{26}d_{36})R \quad (7.52)$$

$$M_{xy} = - \left(D_{16} \frac{\partial^2 w}{\partial x^2} + D_{26} \frac{\partial^2 w}{\partial y^2} + 2D_{66} \frac{\partial^2 w}{\partial x \partial y} \right) - (F_{16}d_{31} + F_{26}d_{32} + F_{66}d_{36})R \quad (7.53)$$

Then, differentiate equations (7.48 ~ 7.53) with respect to ∂x , ∂y , ∂x^2 , ∂y^2 and $\partial x \partial y$ to obtain the following equations.

$$\frac{\partial N_x}{\partial x} = A_{11} \frac{\partial^2 u}{\partial x^2} + A_{12} \frac{\partial^2 v}{\partial x \partial y} + A_{16} \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2} \right) - (E_{11}d_{31} + E_{12}d_{32} + E_{16}d_{36}) \frac{\partial R}{\partial x} \quad (7.54)$$

$$\frac{\partial N_y}{\partial y} = A_{12} \frac{\partial^2 u}{\partial x \partial y} + A_{22} \frac{\partial^2 v}{\partial y^2} + A_{26} \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y} \right) - (E_{12}d_{31} + E_{22}d_{32} + E_{26}d_{36}) \frac{\partial R}{\partial y} \quad (7.55)$$

$$\frac{\partial N_{xy}}{\partial x} = A_{16} \frac{\partial^2 u}{\partial x^2} + A_{26} \frac{\partial^2 v}{\partial x \partial y} + A_{66} \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2} \right) - (E_{16}d_{31} + E_{26}d_{32} + E_{66}d_{36}) \frac{\partial R}{\partial x} \quad (7.56)$$

$$\frac{\partial N_{xy}}{\partial y} = A_{16} \frac{\partial^2 u}{\partial x \partial y} + A_{26} \frac{\partial^2 v}{\partial y^2} + A_{66} \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y} \right) - (E_{16} d_{31} + E_{26} d_{32} + E_{66} d_{36}) \frac{\partial R}{\partial y} \quad (7.57)$$

$$\frac{\partial^2 M_x}{\partial x^2} = - \left(D_{11} \frac{\partial^4 w}{\partial x^4} + D_{12} \frac{\partial^4 w}{\partial x^2 \partial y^2} + 2D_{16} \frac{\partial^4 w}{\partial x^3 \partial y} \right) - (F_{11} d_{31} + F_{12} d_{32} + F_{16} d_{36}) \frac{\partial^2 R}{\partial x^2} \quad (7.58)$$

$$\frac{\partial^2 M_y}{\partial y^2} = - \left(D_{12} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} + 2D_{26} \frac{\partial^4 w}{\partial x \partial y^3} \right) - (F_{12} d_{31} + F_{22} d_{32} + F_{26} d_{36}) \frac{\partial^2 R}{\partial y^2} \quad (7.59)$$

$$\frac{\partial^2 M_{xy}}{\partial x \partial y} = - \left(D_{16} \frac{\partial^4 w}{\partial x^3 \partial y} + D_{26} \frac{\partial^4 w}{\partial y^3 \partial x} + 2D_{66} \frac{\partial^4 w}{\partial x^2 \partial y^2} \right) - (F_{16} d_{31} + F_{26} d_{32} + F_{66} d_{36}) \frac{\partial^2 R}{\partial x \partial y} \quad (7.60)$$

Equations (7.54 ~ 7.60) were substituted into the equilibrium equations (7.45 ~ 7.47), resulting in:

$$\begin{aligned} & A_{11} \frac{\partial^2 u}{\partial x^2} + A_{16} \frac{\partial^2 v}{\partial x^2} + A_{66} \frac{\partial^2 u}{\partial y^2} + A_{26} \frac{\partial^2 v}{\partial y^2} + 2A_{16} \frac{\partial^2 u}{\partial x \partial y} + (A_{12} + A_{66}) \frac{\partial^2 v}{\partial x \partial y} \\ & - (E_{11} d_{31} + E_{12} d_{32} + E_{16} d_{36}) \frac{\partial R}{\partial x} - (E_{16} d_{31} + E_{26} d_{32} + E_{66} d_{36}) \frac{\partial R}{\partial y} = 0 \end{aligned} \quad (7.61)$$

$$\begin{aligned} & A_{16} \frac{\partial^2 u}{\partial x^2} + A_{66} \frac{\partial^2 v}{\partial x^2} + A_{26} \frac{\partial^2 u}{\partial y^2} + A_{22} \frac{\partial^2 v}{\partial y^2} + (A_{12} + A_{66}) \frac{\partial^2 u}{\partial x \partial y} + 2A_{16} \frac{\partial^2 v}{\partial x \partial y} \\ & - (E_{16} d_{31} + E_{26} d_{32} + E_{66} d_{36}) \frac{\partial R}{\partial x} - (E_{12} d_{31} + E_{22} d_{32} + E_{26} d_{36}) \frac{\partial R}{\partial y} = 0 \end{aligned} \quad (7.62)$$

$$\begin{aligned}
& - \left[D_{11} \frac{\partial^4 w}{\partial x^4} + 4 D_{16} \frac{\partial^4 w}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + 4D_{26} \frac{\partial^4 w}{\partial y^3 \partial x} + D_{22} \frac{\partial^4 w}{\partial y^4} \right] \\
& - (F_{11} d_{31} + F_{12} d_{32} + F_{16} d_{36}) \frac{\partial^2 R}{\partial x^2} - (F_{12} d_{31} + F_{22} d_{32} + F_{26} d_{36}) \frac{\partial^2 R}{\partial y^2} \\
& - 2(F_{16} d_{31} + F_{26} d_{32} + F_{66} d_{36}) \frac{\partial^2 R}{\partial x \partial y} = 0
\end{aligned} \tag{7.63}$$

For an isotropic material, which gives $Q_{16} = Q_{26} = 0$, that means:

$$A_{16} = E_{16} = D_{16} = F_{16} = A_{26} = E_{26} = D_{26} = F_{26} = 0.$$

Equations (7.61 ~ 7.63) can be simplified as follows:

$$A_{11} \frac{\partial^2 u}{\partial x^2} + A_{66} \frac{\partial^2 u}{\partial y^2} + (A_{12} + A_{66}) \frac{\partial^2 v}{\partial x \partial y} - (E_{11} d_{31} + E_{12} d_{32}) \frac{\partial R}{\partial x} - E_{66} d_{36} \frac{\partial R}{\partial y} = 0 \tag{7.64}$$

$$A_{66} \frac{\partial^2 v}{\partial x^2} + A_{22} \frac{\partial^2 v}{\partial y^2} + (A_{12} + A_{66}) \frac{\partial^2 u}{\partial x \partial y} - E_{66} d_{36} \frac{\partial R}{\partial x} - (E_{12} d_{31} + E_{22} d_{32}) \frac{\partial R}{\partial y} = 0 \tag{7.65}$$

$$\begin{aligned}
& D_{11} \frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22} \frac{\partial^4 w}{\partial y^4} + (F_{11} d_{31} + F_{12} d_{32}) \frac{\partial^2 R}{\partial x^2} \\
& + (F_{12} d_{31} + F_{22} d_{32}) \frac{\partial^2 R}{\partial y^2} + 2 F_{66} d_{36} \frac{\partial^2 R}{\partial x \partial y} = 0
\end{aligned} \tag{7.66}$$

For the a non-skewed piezoelement patch, equations (7.64) and (7.65) include the $\partial R/\partial x$ and $\partial R/\partial y$ which can be recognized as vertical line forces illustrated in Table (7.1). The $\partial^2 R/\partial x^2$ and $\partial^2 R/\partial y^2$ which are included in equation (7.66) represent line moments along the edges of the actuator patch illustrated in Table (7.1). Additionally, the term $\partial^2 R/\partial x \partial y$ which also is present in equation (7.66) represent the concentrated forces at the corners of the actuator patch also shown in Table (7.1).

Equations (7.64), (7.65) and (7.66) are the governing force-displacement equations for a non-skewed actuator patch. Heaviside and Delta functions in those equations are illustrated in Table (7.1).

7.3.2 The case of a skewed piezoactuator

The step for the derivation of skewed piezoactuator is the same as the steps of non-skewed piezoactuator derivation. Similarly as in the non-skewed piezoactuator case, the governing equations have the same format as equations (7.64), (7.65) and (7.66). However, the generalized location function $R(x,y)$ is much different, as well as its derivative $\partial R/\partial x$, $\partial R/\partial y$, $\partial^2 R/\partial x^2$, $\partial^2 R/\partial y^2$ and $\partial^2 R/\partial x \partial y$. Figure (7.8) shows the sketch of the skewed actuator patch location.

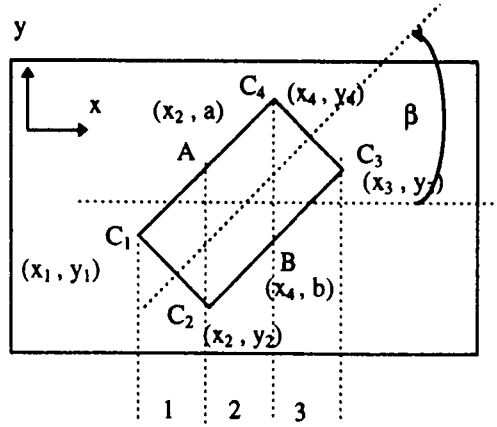


Figure 7.8 Skewed actuator patch location sketch

The rectangular actuator is attached to the plate at an angle β with respect to the x axis of the plate. The four corners (C_1, C_2, C_3, C_4) of the patch have $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and (x_4, y_4) coordinates in the x - y system shown. In order to use the generalized location functions, three sections (C_1C_2A, AC_4BC_2 and C_3C_4B) are divided along the x coordinate direction. Point $A (x_2, a)$ is at the C_1C_4 edge, point $B(x_4, b)$ is at C_2C_3 edge. Both AC_2 and BC_4 are in vertical direction. Figure(7.8) shows those three sections.

From the geometry of the specimen,

$$\beta = \text{Arc tan } \frac{y_4 - y_1}{x_4 - x_1} \quad (7.67)$$

$$a = y_2 + \frac{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}{\cos \beta} \quad (7.68)$$

$$b = y_4 - \frac{\sqrt{(x_3 - x_4)^2 + (y_3 - y_4)^2}}{\cos \beta} \quad (7.69)$$

In section 1 (area C_1C_2A),

$$\text{line } C_1C_2 \text{ equation is: } \tan \beta = \frac{x - x_1}{y_1 - y} \Rightarrow y = y_1 - \text{Ctg}\beta(x - x_1) \quad (7.70)$$

$$\text{line } C_1A \text{ equation is : } \tan \beta = \frac{y - y_1}{x - x_1} \Rightarrow y = y_1 + \tan\beta(x - x_1) \quad (7.71)$$

Then the generalized location function and its derivatives of section 1 are expressed using Heaviside and Delta functions as:

$$R_1(x, y) = [H(x - x_1) - H(x - x_2)] \{H[y - y_1 - \text{Ctg}\beta(x - x_1)] - H[y - y_1 + \tan\beta(x - x_1)]\} \quad (7.72)$$

$$\begin{aligned} \partial R_1 / \partial x = & [\delta(x - x_1) - \delta(x - x_2)] \{H[y - y_1 - \text{Ctg}\beta(x - x_1)] - H[y - y_1 + \tan\beta(x - x_1)]\} \\ & + [H(x - x_1) - H(x - x_2)] \{\delta[y - y_1 - \text{Ctg}\beta(x - x_1)] - \delta[y - y_1 + \tan\beta(x - x_1)]\} \end{aligned} \quad (7.73)$$

$$\partial R_1 / \partial y = [H(x - x_1) - H(x - x_2)] \{\delta[y - y_1 - \text{Ctg}\beta(x - x_1)] - \delta[y - y_1 + \tan\beta(x - x_1)]\} \quad (7.74)$$

$$\begin{aligned} \partial^2 R_1 / \partial x^2 = & [\delta'(x - x_1) - \delta'(x - x_2)] \{H[y - y_1 - \text{Ctg}\beta(x - x_1)] - H[y - y_1 + \tan\beta(x - x_1)]\} \\ & + 2 [\delta(x - x_1) - \delta(x - x_2)] \{\delta[y - y_1 - \text{Ctg}\beta(x - x_1)] - \delta[y - y_1 + \tan\beta(x - x_1)]\} \\ & + [H(x - x_1) - H(x - x_2)] \{\delta'[y - y_1 - \text{Ctg}\beta(x - x_1)] - \delta'[y - y_1 + \tan\beta(x - x_1)]\} \end{aligned} \quad (7.75)$$

$$\partial^2 R_1 / \partial y^2 = [H(x - x_1) - H(x - x_2)] \{\delta'[y - y_1 - \text{Ctg}\beta(x - x_1)] - \delta'[y - y_1 + \tan\beta(x - x_1)]\} \quad (7.76)$$

$$\begin{aligned}\partial^2 R_1 / \partial x \partial y = & [\delta(x - x_1) - \delta(x - x_2)] \{ \delta[y - y_1 - \text{Ctg}\beta(x - x_1)] - \delta[y - y_1 + \text{Tan}\beta(x - x_1)] \} \\ & + [H(x - x_1) - H(x - x_2)] \{ \delta'[y - y_1 - \text{Ctg}\beta(x - x_1)] - \delta'[y - y_1 + \text{Tan}\beta(x - x_1)] \}\end{aligned}\quad (7.77)$$

In section 2 (area AC_4BC_2),

$$\text{line } AC_4 \text{ equation is: } \text{Tan}\beta = \frac{y - a}{x - x_2} \Rightarrow y = a + \text{Tan}\beta(x - x_2) \quad (7.78)$$

$$\text{line } C_2B \text{ equation is: } \text{Tan}\beta = \frac{y - y_2}{x - x_2} \Rightarrow y = y_2 + \text{Tan}\beta(x - x_2) \quad (7.79)$$

Then the generalized location function and its derivatives of section 2 are expressed using Heaviside and Delta functions as:

$$R_2(x, y) = [H(x - x_2) - H(x - x_4)] \{ H[y - a - \text{Tan}\beta(x - x_2)] - H[y - y_2 - \text{Tan}\beta(x - x_2)] \} \quad (7.80)$$

$$\begin{aligned}\partial R_2 / \partial x = & [\delta(x - x_2) - \delta(x - x_4)] \{ H[y - a - \text{Tan}\beta(x - x_2)] - H[y - y_2 - \text{Tan}\beta(x - x_2)] \} \\ & + [H(x - x_2) - H(x - x_4)] \{ \delta[y - a - \text{Tan}\beta(x - x_2)] - \delta[y - y_2 - \text{Tan}\beta(x - x_2)] \}\end{aligned}\quad (7.81)$$

$$\partial R_2 / \partial y = [H(x - x_2) - H(x - x_4)] \{ \delta[y - a - \text{Tan}\beta(x - x_2)] - \delta[y - y_2 - \text{Tan}\beta(x - x_2)] \} \quad (7.82)$$

$$\begin{aligned}\partial^2 R_2 / \partial x^2 = & [\delta'(x - x_2) - \delta'(x - x_4)] \{ H[y - a - \text{Tan}\beta(x - x_2)] - H[y - y_2 - \text{Tan}\beta(x - x_2)] \} \\ & + 2 [\delta(x - x_2) - \delta(x - x_4)] \{ \delta[y - a - \text{Tan}\beta(x - x_2)] - \delta[y - y_2 - \text{Tan}\beta(x - x_2)] \} \\ & + [H(x - x_2) - H(x - x_4)] \{ \delta'[y - a - \text{Tan}\beta(x - x_2)] - \delta'[y - y_2 - \text{Tan}\beta(x - x_2)] \}\end{aligned}\quad (7.83)$$

$$\partial^2 R_2 / \partial y^2 = [H(x - x_2) - H(x - x_4)] \{ \delta' [y - a - \tan \beta (x - x_2)] - \delta' [y - y_2 - \tan \beta (x - x_2)] \}$$

(7.84)

$$\begin{aligned} \partial^2 R_2 / \partial x \partial y = & [\delta(x - x_2) - \delta(x - x_4)] \{ \delta [y - a - \tan \beta (x - x_2)] - \delta [y - y_2 - \tan \beta (x - x_2)] \} \\ & + [H(x - x_2) - H(x - x_4)] \{ \delta' [y - a - \tan \beta (x - x_2)] - \delta' [y - y_2 - \tan \beta (x - x_2)] \} \end{aligned}$$

(7.85)

In section 3 (area C_3C_4B),

$$\text{line } C_4C_3 \text{ equation is: } \tan \beta = \frac{x - x_4}{y_4 - y} \quad \Rightarrow \quad y = y_4 - \text{Ctg} \beta (x - x_4) \quad (7.86)$$

$$\text{line } BC_3 \text{ equation is: } \tan \beta = \frac{y - b}{x - x_4} \quad \Rightarrow \quad y = b + \tan \beta (x - x_4) \quad (7.87)$$

Then the generalized location function and its derivatives of section 3 are expressed using Heaviside and Delta functions as:

$$R_3(x, y) = [H(x - x_4) - H(x - x_3)] \{ H[y - y_4 + \text{Ctg} \beta (x - x_1)] - H[y - b + \tan \beta (x - x_4)] \}$$

(7.88)

$$\begin{aligned} \partial R_3 / \partial x = & [\delta(x - x_4) - \delta(x - x_3)] \{ H[y - y_4 + \text{Ctg} \beta (x - x_1)] - H[y - b + \tan \beta (x - x_4)] \} \\ & + [H(x - x_4) - H(x - x_3)] \{ \delta [y - y_4 + \text{Ctg} \beta (x - x_1)] - \delta [y - b + \tan \beta (x - x_4)] \} \end{aligned}$$

(7.89)

$$\partial R_3/\partial y = [H(x - x_4) - H(x - x_3)] \{ \delta[y - y_4 + \text{Ctg}\beta(x - x_1)] - \delta[y - b + \text{Tan}\beta(x - x_4)] \} \quad (7.90)$$

$$\begin{aligned} \partial^2 R_3/\partial x^2 = & [\delta'(x - x_4) - \delta'(x - x_3)] \{ H[y - y_4 + \text{Ctg}\beta(x - x_1)] - H[y - b + \text{Tan}\beta(x - x_4)] \} \\ & + 2 [\delta(x - x_4) - \delta(x - x_3)] \{ \delta[y - y_4 + \text{Ctg}\beta(x - x_1)] - \delta[y - b + \text{Tan}\beta(x - x_4)] \} \\ & + [H(x - x_4) - H(x - x_3)] \{ \delta'[y - y_4 + \text{Ctg}\beta(x - x_1)] - \delta'[y - b + \text{Tan}\beta(x - x_4)] \} \end{aligned} \quad (7.91)$$

$$\partial^2 R_3/\partial y^2 = [H(x - x_4) - H(x - x_3)] \{ \delta'[y - y_4 + \text{Ctg}\beta(x - x_1)] - \delta'[y - b + \text{Tan}\beta(x - x_4)] \} \quad (7.92)$$

$$\begin{aligned} \partial^2 R_3/\partial x \partial y = & [\delta(x - x_4) - \delta(x - x_3)] \{ \delta[y - y_4 + \text{Ctg}\beta(x - x_1)] - \delta[y - b + \text{Tan}\beta(x - x_4)] \} \\ & + [H(x - x_4) - H(x - x_3)] \{ \delta'[y - y_4 + \text{Ctg}\beta(x - x_1)] - \delta'[y - b + \text{Tan}\beta(x - x_4)] \} \end{aligned} \quad (7.93)$$

Therefore, the generalized location functions and their derivatives are:

$$R(x, y) = R_1(x, y) + R_2(x, y) + R_3(x, y) \quad (7.94)$$

$$\partial R/\partial x = \partial R_1/\partial x + \partial R_2/\partial x + \partial R_3/\partial x \quad (7.95)$$

$$\partial R/\partial y = \partial R_1/\partial y + \partial R_2/\partial y + \partial R_3/\partial y \quad (7.96)$$

$$\partial^2 R/\partial x^2 = \partial^2 R_1/\partial x^2 + \partial^2 R_2/\partial x^2 + \partial^2 R_3/\partial x^2 \quad (7.97)$$

$$\partial^2 R/\partial y^2 = \partial^2 R_1/\partial y^2 + \partial^2 R_2/\partial y^2 + \partial^2 R_3/\partial y^2 \quad (7.98)$$

$$\partial^2 R/\partial x \partial y = \partial^2 R_1/\partial x \partial y + \partial^2 R_2/\partial x \partial y + \partial^2 R_3/\partial x \partial y \quad (7.99)$$

The governing force-displacement equation of actuation for a skewed piezoelement patch are (7.64 ~ 7.66) in which $\partial R/\partial x$, $\partial R/\partial y$, $\partial^2 R/\partial x^2$, $\partial^2 R/\partial y^2$ and $\partial^2 R/\partial x \partial y$ can be found in the equations (7.95 ~ 7.99).

CHAPTER EIGHT

CONCLUSIONS

This investigation has attempted to evaluate the effectiveness of bending and torsional vibration damping in thin walled beams using piezoactuators. The experimental results obtained for a closed cross-section composite beam with surface-bonded skewed piezoelements showed that the activated piezoelements were capable of improving both transient and steady state bending vibration damping performance. However, the torsional vibration damping improvement was small. There were several reasons explaining such results: the specimen fabrication could not be perfect under current conditions, the performance of the proportional feedback control was not optimal, the torsional mass moment of inertia of the tested specimen was too high, the torsional stiffness increment caused by the surface-bonded piezoelements was much larger than bending stiffness increment. A more sophisticated mechanism must be devised to improve the torsional vibration damping performance. A better feedback signal could be obtained to improve the torsional vibration damping, if the strain gage was attached at a better position on the tube. Also, when the signal was picked up directly through the piezoelement itself, which means the piezoelements would work as actuators and sensors at the same time, the improvement might be better.

The numerical study indicated a potential problem with some of the assumptions used in recently developed theoretical models. The result shows that the unloaded

portions of the beam are affected by the loads. The main issue is to account properly for the boundary condition introduced by the clamped end of the beam and for the deflections of unloaded portions of the beam. Also, if the exact position of the elastic axis was identified and used in those theoretical models, the accuracy of those models would have been significantly improved. Moreover, shear lag in the bonding layer has to be accounted for in the theoretical models.

Heaviside and Delta function studies showed that they could be used in modeling the effect of a concentrated load applied to the beam. However, the method to apply them for a two-dimensional, skewed actuator case has not been fully understood.

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APPENDICES

APPENDICES

Appendix I Young's Modulus for the Composite Tube

Formulas for Young's modulus calculation:

$$E = \frac{-W}{6vI_{AA}} (L^3 - 3L_w L^2)$$

$$L_w = 21.8125 \text{ in} = 554 \text{ mm}$$

$$L = 20.375 \text{ in} = 517.5 \text{ mm}$$

$$L_m = 21.1 \text{ in} = 535 \text{ mm}$$

$$\phi = \tan^{-1} [x_o / (2D)]$$

$$\alpha = \tan^{-1} [(x_o - x) / (2D)] - \phi$$

$$v = L_m \times \tan \alpha$$

$$I_{AA} = [b h^3 - (b-2t)(h-2t)^3] / 12 = 3212.53 \text{ mm}^4 \text{ (as given in Appendix IX)}$$

Bending test data for determination of Modulus of Elasticity

Load W (gms)	Test 1 x (in)	Test 2 x (in)	Test 3 x (in)	Test 4 x (in)	Test 5 x (in)	Average x (in)	Def. v (in)	E (MPa)
50	0.5032	0.5132	0.5238	0.5122	0.5543	0.5271	0.0441	11073
100	1.0733	1.1432	1.0996	1.1269	1.1412	1.1168	0.0934	10450
150	1.7333	1.7918	1.7299	1.7618	1.7718	1.7487	0.1463	10009
200	2.3423	2.5006	2.4023	2.3198	2.4033	2.3937	0.2003	9747
250	3.0845	3.3625	3.2403	3.0841	3.1535	3.185	0.2665	9155

Appendix II Shear Modulus for the Composite Tube

Formulas for shear modulus calculation:

$$\theta = \frac{TL}{GJ}$$

$$\theta = v (\text{Pi} / 180)$$

$$v = \text{ArcTan} (x / D)$$

$$D = 3143$$

$$L = 20.165 \text{ in} = 517.5 \text{ mm}$$

$$J = 2 A^2 t / (b + h) = 9932 \text{ mm}^4 \quad (\text{ as given in Appendix IX})$$

Torsion test data for determination of Shear Modulus (G)

Load W (g)	Test 1 x (in)	Test 2 x (in)	Test 3 x (in)	Test 4 x (in)	Test 5 x (in)	Average x (in)	Def. v (Deg.)	G (MPa)
50	1.1517	1.1783	1.1578	1.0661	1.0729	1.1254	0.5214	856
100	2.4928	2.4315	2.5672	2.3568	2.336	2.4369	1.1282	791
150	3.996	3.8423	3.9932	3.8563	3.7545	3.8885	1.8	743
200	5.4675	5.4062	5.4869	5.3408	5.4395	5.4282	2.5118	710

Appendix III**Induced Free Strain in the Piezoelement**

Data was obtained on 03/12/96

Free strain for a piezoelement (From PiezoSystem) 03/12/96

Voltage	1	2	3	4	5	Average
-120	-114	-112	-112	-112	-112	-112.4
-110	-103	-102	-103	-105	-102	-103
-100	-92	-88	-89	-93	-92	-91
-90	-84	-80	-80	-83	-82	-81.8
-80	-73	-70	-70	-72	-71	-71.2
-70	-64	-60	-60	-62	-61	-61.4
-60	-54	-51	-48	-52	-50	-51
-50	-45	-40	-40	-45	-44	-42.8
-40	-34	-30	-34	-36	-35	-33.8
-30	-24	-24	-22	-24	-24	-23.6
-20	-14	-14	-14	-16	-16	-14.8
-10	-6	-8	-8	-10	-9	-8.2
0	0	0	0	0	0	0
10	8	6	7	10	10	8.2
20	15	15	12	14	14	14
30	23	23	20	22	23	22.2
40	32	33	30	32	33	32
50	40	40	40	40	40	40
60	50	48	45	48	48	47.8
70	58	58	54	57	60	57.4
80	68	67	66	70	68	67.8
90	80	78	77	78	79	78.4
100	87	88	88	88	90	88.2
110	102	100	98	98	98	99.2
120	110	110	106	107	106	107.8

Appendix IV

Shear Modulus for a Specimen with Piezoactuators

Load w (gm)	50	100	150	200	250	300	350
Deflection (mm)	0.056	0.115	0.181	0.244	0.3	0.361	0.424
G (MPa)	1946	1920	1874	1911	2007	2081	2158

The calculations are the same as given in Appendix II.

Appendix V

Bending Static Displacements

Left Mirror

Fully clamped

Voltage	Trial 1		Trial 2		Trial 3		Trial 4		Trial 5	
	Dis.(in)	Ang.(deg)	Dis.	Ang.	Dis.	Ang.	Dis.	Ang.	Dis.	Ang.
-120	0.2563	-108	0.2413	-110	0.235	-108	0.2405	-1100	0.236	-108
-100	0.2041	-108	0.1845	-110	0.1935	-108	0.1981	-1100	0.1862	-108
-80	0.1592	-108	0.1515	-110	0.1543	-108	0.1592	-1100	0.1595	-108
-60	0.1121	-108	0.1048	-110	0.1128	-108	0.1182	-110	0.1133	-108
-40	0.0685	-108	0.0732	-110	0.0738	-108	0.0695	-110	0.071	-108
-20	0.038	-108	0.0362	-110	0.0298	-108	0.0383	-110	0.0363	-108
20	0.0411	75	0.0362	71	0.0342	70	0.035	73	0.0348	70
40	0.0862	75	0.0745	71	0.076	70	0.0788	73	0.0732	70
60	0.1197	75	0.01142	71	0.1183	70	0.1121	73	0.1108	70
80	0.1714	75	0.154	71	0.1607	70	0.1553	73	0.1511	70
100	0.2183	75	0.2129	71	0.2108	70	0.21	73	0.2007	70
120	0.2721	75	0.2447	71	0.2693	70	0.2482	73	0.2453	70

Twist Displacements

Voltage	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
-120	-0.079201	-0.082529	-0.072619	-0.082256	-0.072928	-0.078128
-100	-0.06307	-0.063103	-0.059795	-0.067754	-0.057539	-0.061989
-80	-0.049196	-0.051816	-0.047681	-0.05445	-0.049288	-0.0501
-60	-0.034641	-0.035844	-0.034857	-0.040427	-0.035012	-0.035237
-40	-0.021168	-0.025036	-0.022805	-0.02377	-0.02194	-0.022839
-20	-0.011743	-0.012381	-0.009209	-0.013099	-0.011217	-0.01178
20	0.0106375	0.0117856	0.0116971	0.010233	0.0119023	0.0113734
40	0.0223102	0.0242548	0.0259935	0.0230389	0.0250359	0.0241099
60	0.0309806	0.003718	0.040461	0.0327749	0.0378958	0.0338838
80	0.0443616	0.0501375	0.0549626	0.0454053	0.0516792	0.049074
100	0.0565002	0.0693135	0.0720978	0.0613981	0.0686434	0.0664517
120	0.0704247	0.0796665	0.092106	0.0725667	0.0838975	0.0787102

Bending Displacements

Voltage	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
-120	-0.243756	-0.226748	-0.223498	-0.225996	-0.224449	-0.225731
-100	-0.194111	-0.173373	-0.184029	-0.186153	-0.177087	-0.182423
-80	-0.151408	-0.142363	-0.146748	-0.149599	-0.151694	-0.149252
-60	-0.106613	-0.09848	-0.107279	-0.111072	-0.107755	-0.107216
-40	-0.065147	-0.068785	-0.070188	-0.065309	-0.067525	-0.067206
-20	-0.03614	-0.034017	-0.028341	-0.03599	-0.034523	-0.034843
20	0.0396996	0.0342278	0.0321375	0.0334707	0.0327013	0.0334666
40	0.0832628	0.0704411	0.0714166	0.0753568	0.0687855	0.0724049
60	0.1156213	0.0107978	0.1111656	0.1072018	0.1041179	0.1074951

80	0.1655597	0.1456099	0.1510086	0.1485141	0.1419876	0.1483775
100	0.2108616	0.2013009	0.1980872	0.200824	0.1885963	0.2000707
120	0.2628284	0.2313684	0.2530592	0.2373548	0.2305066	0.2405942

Specimen

distance from specimen to screen = 150 in

Fully Clamped

Voltage

	Twist Ang	Bend Ang.
-120	-0.014921	-0.043111
-100	-0.011839	-0.03484
-80	-0.009568	-0.028505
-60	-0.00673	-0.020477
-40	-0.004362	-0.012835
-20	-0.00225	-0.006655
0	0	0
20	0.0021722	0.0063916
40	0.0046046	0.0138283
60	0.0064713	0.0205301
80	0.0093724	0.028338
100	0.0126913	0.0382107
120	0.0150325	0.0459501

Middle Mirror

Fully clamped

Voltage	Trial 1		Trial 2		Trial 3		Trial 4		Trial 5	
	Dis.(in)	Ang.(deg)	Dis.	Ang.	Dis.	Ang.	Dis.	Ang.	Dis.	Ang.
-120	0.3418	-145	0.3082	-145	0.3162	-147	0.3	-155	0.3158	-150
-100	0.2528	-145	0.2441	-145	0.2496	-147	0.2286	-155	0.2595	-150
-80	0.1963	-145	0.1933	-145	0.2018	-147	0.1846	-155	0.1988	-150
-60	0.1426	-145	0.1413	-145	0.1502	-147	0.1432	-155	0.1513	-150
-40	0.0923	-145	0.0978	-145	0.1059	-147	0.0973	-155	0.1118	-150
-20	0.0441	-145	0.0496	-145	0.0608	-147	0.0638	-155	0.0708	-150
20	0.0668	28	0.0615	20	0.0419	28	0.0408	18	0.0493	20
40	0.1008	28	0.1051	20	0.0851	28	0.0838	18	0.0899	20
60	0.1415	28	0.1401	20	0.1152	28	0.1139	18	0.1403	20
80	0.1982	28	0.1972	20	0.1641	28	0.1626	18	0.178	20
100	0.2408	28	0.241	20	0.2022	28	0.2	18	0.2152	20
120	0.2928	28	0.2932	20	0.2492	28	0.2488	18	0.2572	20

Twist Displacements

Voltage	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
-120	-0.279986	-0.252463	-0.265188	-0.271892	-0.273491	-0.27019
-100	-0.207082	-0.199955	-0.209332	-0.207182	-0.224734	-0.207865

-80	-0.1608	-0.158342	-0.169244	-0.167304	-0.172166	-0.165783
-60	-0.116811	-0.115746	-0.125968	-0.129783	-0.13103	-0.124188
-40	-0.075608	-0.080113	-0.088815	-0.088184	-0.096822	-0.085704
-20	-0.036125	-0.04063	-0.050991	-0.057822	-0.061315	-0.049815
20	0.0589809	0.0577911	0.0369955	0.0388031	0.0463268	0.0476403
40	0.0890011	0.0987617	0.0751388	0.0796985	0.0844784	0.0843927
60	0.1249371	0.1316509	0.1017156	0.1083253	0.1318389	0.1216378
80	0.1750002	0.1853074	0.1448917	0.1546418	0.1672653	0.1656358
100	0.2126138	0.2264659	0.178532	0.1902113	0.2022219	0.2016823
120	0.2585271	0.2755179	0.2200305	0.2366229	0.2416889	0.245613

Bending Displacements

Voltage	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
-120	-0.196048	-0.176776	-0.172215	-0.126785	-0.1579	-0.168964
-100	-0.145	-0.14001	-0.135942	-0.096611	-0.12975	-0.135234
-80	-0.112593	-0.110872	-0.109908	-0.078015	-0.0994	-0.106727
-60	-0.081792	-0.081046	-0.081805	-0.060519	-0.07565	-0.079496
-40	-0.052941	-0.056096	-0.057677	-0.041121	-0.0559	-0.054979
-20	-0.025295	-0.028449	-0.033114	-0.026963	-0.0354	-0.029509
20	0.0313607	0.0210342	0.0196709	0.0126079	0.0168616	0.0191889
40	0.0473227	0.0359463	0.039952	0.0258956	0.0307476	0.0355487
60	0.0664302	0.047917	0.0540831	0.035197	0.0479854	0.0499952
80	0.0930493	0.0674464	0.0770403	0.0502462	0.0608796	0.0684554
100	0.1130488	0.0824269	0.0949271	0.0618034	0.0736027	0.0836522
120	0.1374613	0.1002803	0.1169923	0.0768834	0.0879676	0.1017467

Condensed Results

distance from specimen to screen = 150 in

Fully Clamped

Voltage	Twist Ang	Bend Ang.
-120	-0.051602	-0.03227
-100	-0.039699	-0.025828
-80	-0.031662	-0.020383
-60	-0.023718	-0.015183
-40	-0.016368	-0.0105
-20	-0.009514	-0.005636
0	0	0
20	0.0090986	0.0036648
40	0.0161178	0.0067893
60	0.0232311	0.0095484
80	0.0316341	0.013074
100	0.0385185	0.0159764
120	0.0469086	0.0194322

Right Mirror

Fully clamped

Voltage	Trial 1		Trial 2		Trial 3		Trial 4		Trial 5	
	Dis.(in)	Ang.(deg)	Dis.	Ang.	Dis.	Ang.	Dis.	Ang.	Dis.	Ang.
-120	0.2563	-108	0.2413	-110	0.235	-108	0.2405	-110	0.236	-108
-100	0.2041	-108	0.1845	-110	0.1935	-108	0.1981	-110	0.1862	-108
-80	0.1592	-108	0.1515	-110	0.1543	-108	0.1592	-110	0.1595	-108
-60	0.1121	-108	0.1048	-110	0.1128	-108	0.1182	-110	0.1133	-108
-40	0.0685	-108	0.0732	-110	0.0738	-108	0.0695	-110	0.071	-108
-20	0.038	-108	0.0362	-110	0.0298	-108	0.0383	-110	0.0363	-108
20	0.0411	75	0.0362	71	0.0342	70	0.035	73	0.0348	70
40	0.0862	75	0.0745	71	0.076	70	0.0788	73	0.0732	70
60	0.1197	75	0.01142	71	0.1183	70	0.1121	73	0.1108	70
80	0.1714	75	0.154	71	0.1607	70	0.1553	73	0.1511	70
100	0.2183	75	0.2129	71	0.2108	70	0.21	73	0.2007	70
120	0.2721	75	0.2447	71	0.2693	70	0.2482	73	0.2453	70

Twist Displacements

Voltage	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
-120	-0.079201	-0.082529	-0.072619	-0.082256	-0.072928	-0.078128
-100	-0.06307	-0.063103	-0.059795	-0.067754	-0.057539	-0.061989
-80	-0.049196	-0.051816	-0.047681	-0.05445	-0.049288	-0.0501
-60	-0.034641	-0.035844	-0.034857	-0.040427	-0.035012	-0.035237
-40	-0.021168	-0.025036	-0.022805	-0.02377	-0.02194	-0.022839
-20	-0.011743	-0.012381	-0.009209	-0.013099	-0.011217	-0.011178
20	0.0106375	0.0117856	0.0116971	0.010233	0.0119023	0.0113734
40	0.0223102	0.0242548	0.0259935	0.0230389	0.0250359	0.0241099
60	0.0309806	0.003718	0.040461	0.0327749	0.0378958	0.0338838
80	0.0443616	0.0501375	0.0549626	0.0454053	0.0516792	0.049074
100	0.0565002	0.0693135	0.0720978	0.0613981	0.0686434	0.0664517
120	0.0704247	0.0796665	0.092106	0.0725667	0.0838975	0.0787102

Bending Displacements

Voltage	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
-120	-0.243756	-0.226748	-0.223498	-0.225996	-0.224449	-0.225731
-100	-0.194111	-0.173373	-0.184029	-0.186153	-0.177087	-0.182423
-80	-0.151408	-0.142363	-0.146748	-0.149599	-0.151694	-0.149252
-60	-0.106613	-0.09848	-0.107279	-0.111072	-0.107755	-0.107216
-40	-0.065147	-0.068785	-0.070188	-0.065309	-0.067525	-0.067206
-20	-0.03614	-0.034017	-0.028341	-0.03599	-0.034523	-0.034843
20	0.0396996	0.0342278	0.0321375	0.0334707	0.0327013	0.0334666
40	0.0832628	0.0704411	0.0714166	0.0753568	0.0687855	0.0724049
60	0.1156213	0.0107978	0.1111656	0.1072018	0.1041179	0.1074951
80	0.1655597	0.1456099	0.1510086	0.1485141	0.1419876	0.1483775
100	0.2108616	0.2013009	0.1980872	0.200824	0.1885963	0.2000707
120	0.2628284	0.2313684	0.2530592	0.2373548	0.2305066	0.2405942

Condensed Results

distance from specimen to screen= 150 in

Fully Clamped		
Voltage	Twist Ang	Bend Ang.
-120	-0.014921	-0.043111
-100	-0.011839	-0.03484
-80	-0.009568	-0.028505
-60	-0.00673	-0.020477
-40	-0.004362	-0.012835
-20	-0.00225	-0.006655
0	0	0
20	0.0021722	0.0063916
40	0.0046046	0.0138283
60	0.0064713	0.0205301
80	0.0093724	0.028338
100	0.0126913	0.0382107
120	0.0150325	0.0459501

Appendix VI

Static Torsional Displacements

Left mirror

Fully clamped

Voltage	Trial 1		Trial 2		Trial 3		Trial 4		Trial 5	
	Dis.(in)	Ang.(deg)	Dis.	Ang.	Dis.	Ang.	Dis.	Ang.	Dis.	Ang.
-120	0.4091	-176	0.409	-176	0.41	-175	0.4062	-174	0.4172	-174
-100	0.3641	-176	0.3581	-176	0.352	-175	0.3499	-174	0.352	-174
-80	0.2727	-176	0.2727	-176	0.2733	-175	0.2708	-174	0.2781	-174
-60	0.2066	-176	0.1981	-176	0.1933	-175	0.1939	-174	0.1922	-174
-40	0.1174	-176	0.1253	-176	0.1159	-175	0.1279	-174	0.1253	-174
-20	0.0591	-176	0.0596	-176	0.0494	-175	0.0511	-174	0.0555	-174
20	0.0567	11	0.0561	10.5	0.0539	11	0.0559	11.5	0.0559	10.5
40	0.1108	11	0.1017	10.5	0.1265	11	0.1174	11.5	0.1264	10.5
60	0.1853	11	0.1605	10.5	0.1913	11	0.1721	11.5	0.1812	10.5
80	0.2469	11	0.2347	10.5	0.2547	11	0.2422	11.5	0.2535	10.5
100	0.3265	11	0.3147	10.5	0.3288	11	0.3141	11.5	0.33	10.5
120	0.3703	11	0.352	10.5	0.382	11	0.3663	11.5	0.3803	10.5

Twist Displacements

Voltage	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
-120	-0.408103	-0.408004	-0.40844	-0.403975	-0.414915	-0.408182
-100	-0.363213	-0.357228	-0.350661	-0.347983	-0.350072	-0.352653
-80	-0.272036	-0.272036	-0.27226	-0.269317	-0.276577	-0.27211
-60	-0.206097	-0.197617	-0.192564	-0.192838	-0.191147	-0.19434
-40	-0.117114	-0.124995	-0.115459	-0.127199	-0.124614	-0.122241
-20	-0.058956	-0.059455	-0.049212	-0.05082	-0.055196	-0.054991
20	0.0556583	0.0551606	0.0529097	0.0547778	0.0549639	0.0549674
40	0.1087643	0.099997	0.1241758	0.1150432	0.1242834	0.1159944
60	0.1818955	0.1578124	0.1877853	0.168645	0.1781658	0.1762354
80	0.2423638	0.2307699	0.2500204	0.2373378	0.2492551	0.2429855
100	0.3205013	0.3094303	0.322759	0.3077943	0.3244741	0.3175635
120	0.3634965	0.3461057	0.3749816	0.3589464	0.3739318	0.3654583

Bending Displacements

Voltage	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
-120	-0.028537	-0.02853	-0.035734	-0.042459	-0.043609	-0.035577
-100	-0.025398	-0.02498	-0.030679	-0.036575	-0.036794	-0.030884
-80	-0.019023	-0.019023	-0.02382	-0.028306	-0.029069	-0.023716
-60	-0.014412	-0.013819	-0.016847	-0.020268	-0.02009	-0.017116
-40	-0.008189	-0.00874	-0.010101	-0.013369	-0.013097	-0.010646
-20	-0.004123	-0.004157	-0.004305	-0.005341	-0.005801	-0.004601
20	0.0108189	0.0102234	0.0102846	0.0111447	0.010187	0.0104423
40	0.0211416	0.0185334	0.0241373	0.0234058	0.0230346	0.0225273
60	0.0353569	0.0292488	0.0365018	0.0343112	0.0330211	0.0342297

80	0.0471107	0.0427707	0.0485991	0.0482869	0.0461967	0.0471981
100	0.0622991	0.0573495	0.062738	0.0626215	0.0601377	0.0616861
120	0.0706566	0.0641469	0.072889	0.0730285	0.0693042	0.0709499

Specimen

Condensed Results

distance from specimen to the screen= 158.5 in

Fully Clamped		
Voltage	Twist	Ang Bend Ang.
-120	-0.073776	-0.00643
-100	-0.06374	-0.005582
-80	-0.049182	-0.004287
-60	-0.035126	-0.003094
-40	-0.022094	-0.001924
-20	-0.009939	-0.000832
0	0	0
20	0.009935	0.0018874
40	0.0209653	0.0040717
60	0.0318534	0.0061868
80	0.0439181	0.0085308
100	0.0573976	0.0111494
120	0.0660542	0.0128238

Middle Mirror

Fully clamped

Voltage	Trial 1		Trial 2		Trial 3		Trial 4		Trial 5	
	Dis.(in)	Ang.(deg)	Dis.	Ang.	Dis.	Ang.	Dis.	Ang.	Dis.	Ang.
-120	0.2679	59	0.2383	57.5	0.224	52	0.2372	55	0.2512	59
-100	0.2255	59	0.2177	57.5	0.2035	52	0.2075	55	0.2216	59
-80	0.1786	59	0.1589	57.5	0.1483	52	0.1581	55	0.1675	59
-60	0.1269	59	0.1141	57.5	0.1112	52	0.1079	55	0.1239	59
-40	0.081	59	0.0783	57.5	0.0755	52	0.07	55	0.0855	59
-20	0.0263	59	0.0439	57.5	0.0404	52	0.0367	55	0.0462	59
20	0.0274	-122	0.0425	-122	0.0315	-122	0.0364	-124	0.0469	-120
40	0.0572	-122	0.0827	-122	0.0617	-122	0.0792	-124	0.0839	-120
60	0.0909	-122	0.115	-122	0.099	-122	0.1157	-124	0.1233	-120
80	0.1314	-122	0.1477	-122	0.1362	-122	0.1345	-124	0.1692	-120
100	0.173	-122	0.184	-122	0.1753	-122	0.1715	-124	0.2238	-120
120	0.1971	-122	0.2215	-122	0.2043	-122	0.2018	-124	0.3016	-120

Twist Displacements

Voltage	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
-120	0.1379787	0.1280385	0.1379082	0.1360523	0.1293776	0.134446

-100	0.1161411	0.1169701	0.1252871	0.1190171	0.1141324	0.1173761
-80	0.0919858	0.0853769	0.0913026	0.0906824	0.0862689	0.089418
-60	0.0653583	0.0613059	0.0684616	0.0618889	0.0638132	0.0636868
-40	0.0417181	0.0420706	0.0464824	0.0401504	0.0440358	0.0426081
-20	0.0135455	0.0235875	0.0248727	0.0210503	0.0237948	0.0228108
20	-0.01452	-0.022522	-0.016692	-0.020355	-0.02345	-0.019856
40	-0.030311	-0.043824	-0.032696	-0.044288	-0.04195	-0.03949
60	-0.04817	-0.060941	-0.052462	-0.064699	-0.06165	-0.058351
80	-0.069631	-0.078269	-0.072175	-0.075211	-0.0846	-0.075219
100	-0.091676	-0.097505	-0.092895	-0.095902	-0.1119	-0.095434
120	-0.104447	-0.117377	-0.108263	-0.112845	-0.1508	-0.112828

Bending Displacements

Voltage	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
-120	0.2296351	0.2009802	0.1765144	0.1943029	0.2153204	0.2035345
-100	0.1932912	0.1836063	0.1603602	0.169974	0.1899483	0.1811762
-80	0.1530901	0.1340149	0.116862	0.1295079	0.1435755	0.1356995
-60	0.1087745	0.096231	0.0876268	0.0883865	0.106203	0.0969402
-40	0.0694306	0.0660376	0.0594948	0.0573406	0.0732878	0.0649876
-20	0.0225435	0.0370249	0.0318356	0.0300629	0.0396011	0.0329745
20	-0.023237	-0.036042	-0.026714	-0.030177	-0.040617	-0.030978
40	-0.048508	-0.070134	-0.052325	-0.06566	-0.07266	-0.062706
60	-0.077088	-0.097526	-0.083957	-0.09592	-0.106781	-0.092467
80	-0.111434	-0.125257	-0.115504	-0.111506	-0.146531	-0.117422
100	-0.146712	-0.156041	-0.148663	-0.14218	-0.193816	-0.150472
120	-0.16715	-0.187843	-0.173256	-0.1673	-0.261193	-0.176133

Condensed Results

distance from specimen= 158.5 in

Fully Clamped

Voltage	Twist Ang	Bend Ang.
-120	0.0243003	0.0367876
-100	0.021215	0.0327465
-80	0.0161617	0.0245268
-60	0.011511	0.0175213
-40	0.0077012	0.0117461
-20	0.0041229	0.0059599
0	0	0
20	-0.003589	-0.005599
40	-0.007138	-0.011334
60	-0.010547	-0.016713
80	-0.013595	-0.021223
100	-0.017249	-0.027197
120	-0.020393	-0.031835

Right Mirror

Fully clamped

Voltage	Trial 1		Trial 2		Trial 3		Trial 4		Trial 5	
	Dis.(in)	Ang.(deg)	Dis.	Ang.	Dis.	Ang.	Dis.	Ang.	Dis.	Ang.
-120	0.1772	-175	0.1717	-175	0.1998	-175	0.1753	-175	0.19	-175
-100	0.1575	-175	0.1479	-175	0.1669	-175	0.1505	-175	0.1647	-175
-80	0.1181	-175	0.1145	-175	0.1332	-175	0.1169	-175	0.1267	-175
-60	0.0837	-175	0.0819	-175	0.0981	-175	0.0755	-175	0.0979	-175
-40	0.0547	-175	0.0575	-175	0.0615	-175	0.0441	-175	0.0655	-175
-20	0.0304	-175	0.0309	-175	0.0302	-175	0.0208	-175	0.035	-175
20	0.0375	19.5	0.0297	19	0.0262	21	0.0269	20	0.0242	26
40	0.0669	19.5	0.0587	19	0.0532	21	0.053	20	0.047	26
60	0.0961	19.5	0.0847	19	0.0867	21	0.0752	20	0.0737	26
80	0.1257	19.5	0.1061	19	0.1134	21	0.1069	20	0.1027	26
100	0.1609	19.5	0.1369	19	0.1479	21	0.1292	20	0.1312	26
120	0.1886	19.5	0.1592	19	0.1701	21	0.1603	20	0.154	26

Twist Displacements

Voltage	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
-120	-0.176526	-0.171047	-0.19904	-0.174633	-0.189277	-0.180145
-100	-0.156901	-0.147337	-0.166265	-0.149927	-0.164073	-0.156967
-80	-0.117651	-0.114064	-0.132693	-0.116455	-0.126218	-0.120108
-60	-0.083381	-0.081588	-0.097727	-0.075213	-0.097527	-0.087499
-40	-0.054492	-0.057281	-0.061266	-0.043932	-0.065251	-0.05768
-20	-0.030284	-0.030782	-0.030085	-0.020721	-0.034867	-0.030384
20	0.0353491	0.0280819	0.0244598	0.0252777	0.0217508	0.0259398
40	0.0630627	0.0555019	0.0496665	0.0498037	0.0422433	0.0516574
60	0.0905878	0.0800854	0.0809414	0.0706649	0.0662411	0.0772306
80	0.11849	0.1003195	0.105868	0.1004531	0.0923061	0.1022136
100	0.151671	0.1294415	0.1380765	0.1214083	0.1179218	0.1296421
120	0.1777822	0.1505266	0.158802	0.1506327	0.1384143	0.1533204

Bending Displacements

Voltage	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
-120	-0.015444	-0.014965	-0.017414	-0.015278	-0.01656	-0.015761
-100	-0.013727	-0.01289	-0.014546	-0.013117	-0.014355	-0.013733
-80	-0.010293	-0.009979	-0.011609	-0.010189	-0.011043	-0.010508
-60	-0.007295	-0.007138	-0.00855	-0.00658	-0.008533	-0.007655
-40	-0.004767	-0.005011	-0.00536	-0.003844	-0.005709	-0.005046
-20	-0.00265	-0.002693	-0.002632	-0.001813	-0.00305	-0.002658
20	0.0125178	0.0096694	0.0093892	0.0092003	0.0106086	0.0098891
40	0.0223317	0.0191109	0.0190652	0.0181271	0.0206034	0.0195932
60	0.0320788	0.0275756	0.0310705	0.0257199	0.032308	0.0302417
80	0.0419595	0.0345428	0.0406389	0.036562	0.0450207	0.0397201
100	0.0537095	0.0445703	0.0530026	0.044189	0.0575143	0.0504275

120 0.062956 0.0518305 0.0609584 0.0548258 0.0675092 0.0595801

Condensed Results

distance from specimen= 158.5 in

Fully Clamped
Voltage

	Twist Ang	Bend Ang.
-120	-0.03256	-0.002849
-100	-0.028371	-0.002482
-80	-0.021709	-0.001899
-60	-0.015815	-0.001384
-40	-0.010425	-0.000912
-20	-0.005492	-0.00048
0	0	0
20	0.0046885	0.0017874
40	0.0093367	0.0035413
60	0.0139589	0.005466
80	0.0184745	0.0071792
100	0.023432	0.0091145
120	0.0277117	0.0107687

Appendix VII Experiment Data for Composite Material Young's Modulus

This data was obtained by Mr. Alex. Selvarathiram

Test one		Test two		Test three	
Strain	Load	Strain	Load	Strain	Load
me	lbs	me	lbs	me	lbs
-31.43407	2.954192	317.3925	60.896634	114.7496	141.93548
-33.26518	2.551347	316.78213	59.889523	110.477	143.00974
-39.67406	3.088473	313.11991	60.695212	88.503677	143.00974
-120.8533	4.63271	292.36732	61.5009	208.13623	163.68908
129.0933	39.680166	213.62957	74.996185	375.98804	185.84552
258.79696	61.635182	387.27989	101.85247	523.08725	201.28788
318.00287	67.006439	565.81317	121.39042	646.38203	213.37321
357.67693	71.840571	654.62203	133.40861	768.45607	223.84716
403.4547	74.123356	694.90646	138.04132	869.16715	232.17261
477.61467	79.091769	741.90497	141.93548	960.11231	240.63234
628.37611	88.558611	797.75384	145.49394	1044.3434	248.01782
810.57161	102.7253	867.64122	151.60375	1132.8471	255.00046
979.64415	116.95914	950.65157	156.97501	1226.8441	263.05734
1131.3211	129.38017	1059.6026	164.89761	1326.9448	271.65136
1269.2648	140.52553	1206.3967	174.49873	1449.0188	280.51393
1395.0011	149.65667	1450.8499	189.00113	1615.6499	294.07636
1516.7699	158.85495	1710.8676	208.7405	1787.1639	309.25016
1636.4025	168.45607	1921.1402	225.65996	1943.4187	323.61827
1752.678	177.38578	2096.0112	237.94671	2092.9594	336.91214
1864.6809	185.71123	2235.1756	248.95779	2225.4097	346.84896
1981.2616	192.69387	2334.3608	257.28324	2345.6526	357.59148
2164.3727	204.91348	2412.7934	261.24455	2461.623	367.79687
2363.3534	218.54305	2493.057	267.48863	2569.6585	375.98804
2578.2037	233.78399	2573.6259	272.52419	2677.694	384.04492
2756.737	248.08496	2658.7725	277.8283	2808.3132	392.63894
2909.9399	258.82748	2748.1918	284.20667	2989.5932	404.99283
3032.9295	267.75719	2846.4614	290.31648	3167.8213	418.55525
3140.965	276.35121	2953.2762	297.16483	3333.2316	432.11768
3240.4553	282.86386	3082.6746	305.6917	3484.6034	443.26304
3337.199	289.77935	3253.5783	315.82995	3627.43	453.33415
3433.9427	296.56056	3450.7279	329.19095	3751.9456	463.27097
3534.6538	303.07321	3625.5989	341.74627	3878.9026	472.93924
3640.8582	310.52583	3783.99	352.48878	3996.0936	480.86184
3753.1663	317.97845	3928.0374	362.62703	4114.5054	488.3816
3876.7663	325.22965	4062.3188	371.48961	4235.9691	496.16993
4015.0151	334.96506	4197.5158	379.74792	4383.6787	505.16678
4156.0106	345.90899	4325.3883	388.14051	4557.6342	518.19208
4405.0417	359.20286	4445.6313	396.19739	4723.6549	531.75451
4635.7616	376.79373	4580.2179	404.38856	4879.9097	544.37696
4828.0282	391.02756	4706.5645	413.78826	5015.4118	555.25376

Appendix VIII Strains Induced by Attached Piezoelements

This strain data was obtained for two actuators attached to the composite tube

Actuator 23

Voltage(v)	Trial 1 (me)	Trial 2 (me)	Trial 3 (me)	Trial 4 (me)	Trial 5 (me)	Average (me)
-120	-127	-104	-96	-94	-104	-105
-100	-104	-76	-87	-72	-86	-85
-80	-84	-62	-63	-61	-63	-66.6
-60	-62	-48	-43	-47	-50	-50
-40	-33	-32	-28	-28	-33	-30.8
-20	-15	-18	-18	-16	-19	-17.2
0	0	0	0	0	0	0
20	16	15	16	14	15	15.2
40	35	27	30	30	32	30.8
60	58	46	52	44	50	50
80	80	66	64	68	66	68.8
100	110	86	86	76	82	88
120	134	104	106	92	103	107.8

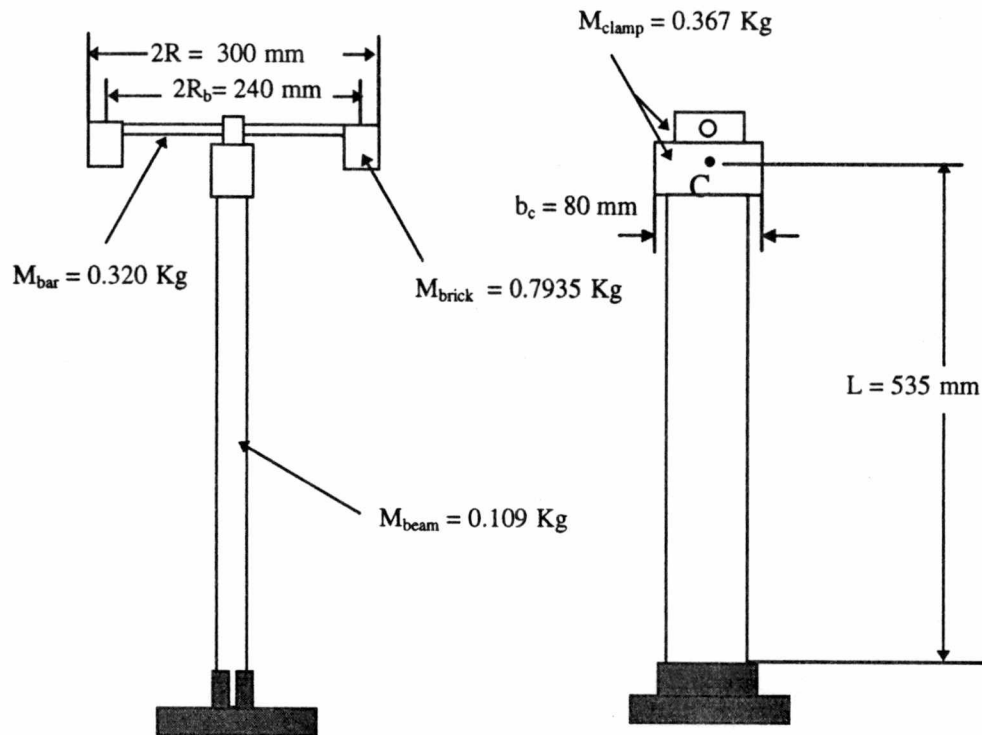
Actuator 22

Voltage	Trial 1 (me)	Trial 2 (me)	Trial 3 (me)	Trial 4 (me)	Trial 5 (me)	Average (me)
-120	-105	-100	-96	-103	-100	-100.8
-100	-85	-88	-82	-76	-78	-81.8
-80	-66	-63	-60	-61	-62	-62.4
-60	-41	-44	-50	-43	-44	-44.4
-40	-32	-24	-34	-30	-33	-30.6
-20	-15	-13	-16	-11	-13	-13.6
0	0	0	0	0	0	0
20	13	12	17	15	18	15
40	28	23	29	27	33	28
60	49	44	48	46	48	47
80	70	63	64	63	66	65.2
100	84	76	82	82	83	81.4
120	100	96	94	98	104	98.4

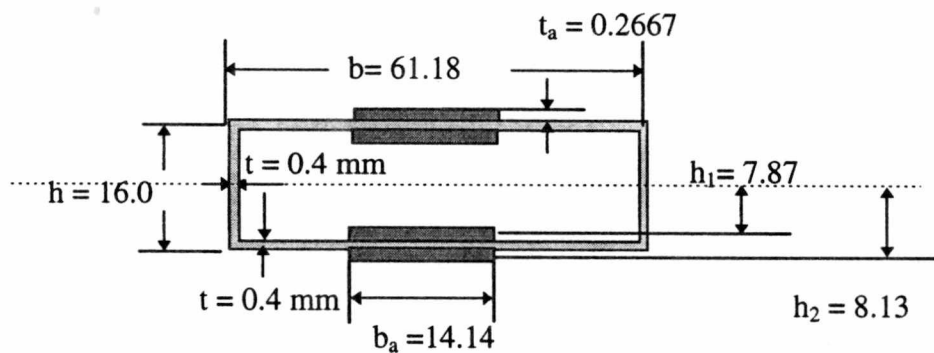
Appendix IX

The Parameters of the Specimen

The schematic of the specimen:



The closed cross-section of the beam:



For static displacements

the bending moment of inertia : $I = I_b + I_a = 4180 \text{ mm}^4$

where $I_b = b h^3 / 12 - (b-2t) (h-2t)^3 / 12 = 3212.53 \text{ mm}^4$

$$I_a = \frac{b_a t_a^3}{3} + 2 b_a t_a (h_1^2 + h_2^2) = 967 \text{ mm}^4$$

the torsional moment of inertia $J = 2 A t / (b+h) = 2 b h t / (b+h) = 9932 \text{ mm}^4$

For dynamic displacements:

the mass bending moment of inertia $J_B = [M_{\text{beam}}/3 + M_{\text{bar}} + M_{\text{clamp}}] L^2$

$$= 0.207 \text{ Kg m}^2$$

the mass torsional moment of inertia $J_T = M_{\text{bar}} R^2/3 + 2 M_{\text{brick}} R_b^2 + (M_{\text{clamp}} b_c^2)/3$

$$= 0.026 \text{ Kg m}^2$$

Weight per unit length of the specimen:

without the attached actuators $\rho_u = 50.08 \text{ g} / 574.7 \text{ mm} = 0.0871 \text{ Kg/m}$

with the attached actuators $\rho_a = 108.75 \text{ g} / 574.7 \text{ mm} = 0.189 \text{ Kg/m}$

Appendix X Transient Damping Coefficients Experimental Data

Actively damped bending transient vibrations

Time	Amplitude	Delta	Coefficients	Coefficient (average)
0.25	500	0.0980539	0.0156118	0.0360084
0.44	453.3	0.1737548	0.0276574	
0.64	381	0.1549822	0.0246712	
0.83	326.3	0.1809128	0.0287958	
1.02	272.3	0.1758059	0.0279836	
1.21	228.4	0.1798727	0.0286304	
1.4	190.8	0.208446	0.0331738	
1.59	154.9	0.2136663	0.0340036	
1.78	125.1	0.2699872	0.0429519	
1.96	95.5	0.2992672	0.0476	
2.15	70.8	0.3907435	0.0621002	
2.33	47.9	0.2421115	0.0385242	
2.52	37.6	0.0602922	0.0096002	
2.71	35.4			

Passively damped bending transient vibrations

Time(sec)	Amplitude	Delta	Coefficients	Average Coefficient
0.24	500	0.1018113	0.0162099	0.014848
0.44	451.6	0.1210823	0.019277	
0.64	400.1	0.09869	0.015713	
0.83	362.5	0.1052073	0.0167504	
1.03	326.3	0.086705	0.0138052	
1.22	299.2	0.1067727	0.0169996	
1.41	268.9	0.0979913	0.0156018	
1.6	243.8	0.0824725	0.0131314	
1.8	224.5	0.0957557	0.015246	
1.99	204	0.0747869	0.0119079	
2.17	189.3	0.0848526	0.0135103	
2.36	173.9	0.0715016	0.0113849	
2.55	161.9	0.0816912	0.013007	
2.74	149.2			

Passively damped torsional transient vibrations

Time	Amplitude	Delta	Coefficient	Coefficient (average)
0.04	254	0.2325379	0.037003	0.0205328
0.2	201.3	0.2371508	0.037736	
0.36	158.8	0.2078331	0.0330763	
0.51	129	0.1341961	0.0213639	
0.66	112.8	0.1194467	0.0190167	
0.8	100.1	0.0778805	0.0124004	
0.95	92.6	0.0681447	0.0108504	
1.1	86.5	0.0583152	0.0092855	
1.24	81.6	0.0400053	0.0063701	
1.39	78.4	0.0350458	0.0055804	
1.53	75.7			

Actively damped torsional transient vibrations

This is for actively damped torsional transient vibration

Time	Amplitude	Delta	Coefficient	Coefficient (average)
0.04	300.2	0.2031907	0.0323383	0.0217106
0.2	245	0.2246753	0.0357535	
0.36	195.7	0.2076787	0.0330518	
0.52	159	0.1883776	0.0299829	
0.67	131.7	0.1226353	0.0195242	
0.82	116.5	0.0739099	0.0117683	
0.97	108.2	0.0748192	0.011913	
1.12	100.4	0.062681	0.0099806	
1.26	94.3	0.0477832	0.0076086	
1.41	89.9	0.0478451	0.0076184	
1.55	85.7			

Appendix XI

Steady State Damping Amplitude Data

For bending damping:

Steady state bending vibration damping amplitudes

Frequency (hz)	5.45	6.56	7.32	7.58	7.68	7.74	7.79	7.813	7.9
Amplitude									
Passive	0.097	0.2375	0.27	0.2985	0.593	0.645	0.89575	1.05775	1.01425
F-active	0.09525	0.21	0.243	0.24425	0.25525	0.3085	0.3605	0.428	0.4065
M-active	0.09655	0.2133	0.263	0.2815	0.4606	0.5105	0.7123	0.8125	0.7615
Frequency (hz)	7.95	8.12	8.23	8.65	9.2	9.39	10.89	11.73	
Amplitude									
Passive	0.86825	0.778	0.63825	0.49225	0.38825	0.32975	0.1576	0.0977	
F-active	0.3605	0.36	0.359	0.355	0.35	0.323	0.1575	0.0978	
M-active	0.7295	0.6605	0.578	0.481	0.386	0.3305	0.1574	0.0979	

Steady state torsional vibration damping amplitudes

Frequency (hz)	7.58	9.78	10.04	10.48	10.96	10.97	11.51	12	13.18
Amplitude									
Passive	0.68275	1.00875	1.51025	1.425	1.29575	1.2445	1.07975	0.83175	0.6155
Active	0.68275	0.908	1.4215	0.3925	1.20725	1.21825	1.0153	0.7805	0.5825

Appendix XII Half-Power Method Validation

Considering the system equation of motion is:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = \frac{F}{m}e^{j\omega t} \quad (1)$$

Assume the steady state solution is:

$$x = Xe^{j\omega t} \quad (2)$$

Substitute (2) in (1) to obtain:

$$(-\omega^2 + j\omega 2\zeta\omega_n + \omega_n^2)Xe^{j\omega t} = \frac{F}{m}e^{j\omega t} \quad (3)$$

The solution of the equation (3) is:

$$|X| = \frac{\frac{F}{k}}{\sqrt{[1 - (\frac{\omega}{\omega_n})^2]^2 + [2\zeta(\frac{\omega}{\omega_n})]^2}} \quad (4)$$

Then

$$|X| = \frac{\frac{F}{k}}{\sqrt{[1 - (\frac{f}{f_n})^2]^2 + [2\zeta(\frac{f}{f_n})]^2}} \quad (5)$$

Assume $a = F/k$, in case of resonance $f = f_n$, $|X| = a / (2\zeta)$

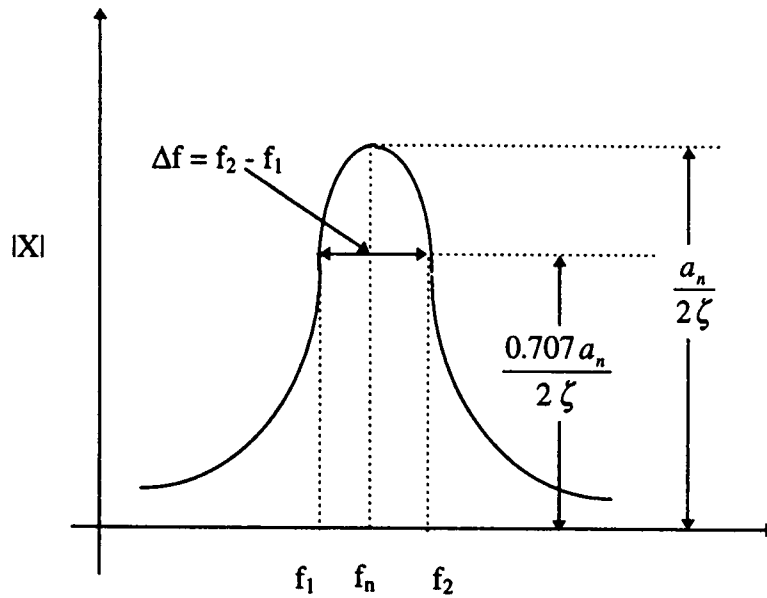


Figure 1. Steady state vibration amplitude vs. frequency

Figure 1. shows the vibration peak amplitude vs. frequency, which correlates with the experimental test results shown in Figure 5.19 and Figure 5.21.

Currently, as we do not have the device to measure the forces at f_1 , f_2 and f_n . Let us assume if they are not equal, and find out what are the relationship between equal force damping coefficients and not equal force damping coefficients obtained using the half-power method.

Assume $a_1 = F_1 / k$, $a_2 = F_2 / k$ and $a_n = F_n / k$ represent three frequencies f_1 , f_2 and f_n status. Substitute them to Eq. (5), resulting in:

$$|X_1| = \frac{a_1}{\sqrt{[1 - (\frac{f}{f_n})^2]^2 + [2\zeta (\frac{f}{f_n})]^2}} \quad (6)$$

$$|X_2| = \frac{a_2}{\sqrt{[1 - (\frac{f}{f_n})^2]^2 + [2\zeta (\frac{f}{f_n})]^2}} \quad (7)$$

Then substitute $|X_1| = |X_2| = 0.707 a_n / (2 \zeta)$ to Eq. (6) and (7), and assume $\zeta \ll 1$ is very small and neglect ζ^2 and resulting in:

$$(\frac{f_1}{f_n})^2 = 1 - 2\zeta^2 \pm 2\zeta \sqrt{\zeta^2 + 2(\frac{a_1}{a_n})^2 - 1} \approx 1 \pm 2\zeta \sqrt{2(\frac{a_1}{a_n})^2 - 1} \quad (8)$$

$$(\frac{f_2}{f_n})^2 = 1 - 2\zeta^2 \pm 2\zeta \sqrt{\zeta^2 + 2(\frac{a_2}{a_n})^2 - 1} \approx 1 \pm 2\zeta \sqrt{2(\frac{a_2}{a_n})^2 - 1} \quad (9)$$

$$\text{Then } (\frac{f_1}{f_n})^2 = 1 - 2\zeta \sqrt{2(\frac{a_1}{a_n})^2 - 1} \quad \text{and} \quad (\frac{f_2}{f_n})^2 = 1 + 2\zeta \sqrt{2(\frac{a_2}{a_n})^2 - 1}$$

are selected to correspond the Figure 1.

As $f_1 + f_2 \approx 2 f_n$, resulting in:

$$(\frac{f_2}{f_n})^2 - (\frac{f_1}{f_n})^2 = \frac{f_2^2 - f_1^2}{f_n^2} \approx \frac{2(f_2 - f_1)}{f_n} \quad (10)$$

Meanwhile,

$$\begin{aligned}
 \left(\frac{f_2}{f_n}\right)^2 - \left(\frac{f_1}{f_n}\right)^2 &= 1 + 2\zeta \sqrt{2\left(\frac{a_2}{a_n}\right)^2 - 1} - \left(1 - 2\zeta \sqrt{2\left(\frac{a_1}{a_n}\right)^2 - 1}\right) \\
 &= 2\zeta \left(\sqrt{2\left(\frac{a_2}{a_n}\right)^2 - 1} + \sqrt{2\left(\frac{a_1}{a_n}\right)^2 - 1} \right)
 \end{aligned} \tag{11}$$

Then,

$$\frac{2(f_2 - f_1)}{f_n} = 2\zeta \left(\sqrt{2\left(\frac{a_2}{a_n}\right)^2 - 1} + \sqrt{2\left(\frac{a_1}{a_n}\right)^2 - 1} \right) \tag{12}$$

So, the damping coefficient is given as:

$$\begin{aligned}
 \zeta &= \frac{2(f_2 - f_1)}{f_n \left(\sqrt{2\left(\frac{a_2}{a_n}\right)^2 - 1} + \sqrt{2\left(\frac{a_1}{a_n}\right)^2 - 1} \right)} \\
 &= \frac{(f_2 - f_1)}{2 f_n} \frac{2}{\left(\sqrt{2\left(\frac{a_2}{a_n}\right)^2 - 1} + \sqrt{2\left(\frac{a_1}{a_n}\right)^2 - 1} \right)}
 \end{aligned} \tag{13}$$

When experiments were performed, the power of the excitor system was kept almost the same level. So, we assumed the driving force to have a constant magnitude, that meant $a_1 \approx a_2 \approx a_n$. Then Eq. 13 becomes:

$$\zeta \approx \frac{(f_2 - f_1)}{2 f_n} = \frac{(\omega_2 - \omega_1)}{2 \omega_n} \tag{5.23}$$

So, the half-power method can be used in this case to approximate the steady state vibration damping coefficients.

Appendix XIII**Shear Center Locations**

The onlay bonded at 30°.

Onlay station(m)	0.006225	0.023113	0.04	0.056887	0.073775
Shear center (0.08m)	0.026273	0.0262465	0.026	0.0246576	0.0230811
Shear center (0.16m)	0.026013	0.026106	0.026	0.0252096	0.0242931

The onlay bonded at 30°.

Onlay station(m)	0.01675	0.02837	0.04	0.05163	0.06325
Shear center (0.08m)	0.0259793	0.0261887	0.026	0.0254652	0.0247232
Shear center (0.16m)	0.0259925	0.0260683	0.026	0.025832	0.0254749

The onlay bonded at 60°.

Onlay station(m)	0.031182	0.035591	0.04	0.0444092	0.0488183
Shear center (0.08m)	0.025975	0.026004	0.026	0.0258493	0.0256674
Shear center (0.16m)	0.025996	0.026016	0.026	0.025963	0.0258883

Appendix XIV Results for the Unactuated Portion of the Beam

Finite element analysis results for unactuated portion
beam

CASE	Twist Degrees					
	Front			Back		
	actuator 1	actuator 2	S-center	actuator 1	actuator 2	S-center
1	0.208914	0.21447	0.199974	0.0270389	0.0212596	0.0340956
2	-0.160555	-0.164338	-0.164902	-0.015709	-0.012045	-0.011191
3	0.208959	0.214865	0.198923	0.0271575	0.021415	0.0374405
4	-0.161273	-0.165053	-0.16595	-0.016446	-0.012613	-0.012861
5.1	0.0483424	0.0501172	0.0351052	0.0113304	0.0092148	0.0229054
5.2	0.106405	0.108531	0.0916904	0.0693909	0.0675325	0.0833197
5						
6	0.0483585	0.0501328	0.0351122	0.0113297	0.0092144	0.0229049
7	0.0476859	0.0498123	0.0329724	0.0107112	0.0088017	0.0245796

	Overall twisting			
	actuator 1	actuator 2	S-center	Tip
1	0.181875	0.193211	0.165878	0.235216
2	-0.144485	-0.152293	-0.153711	-0.176496
3	0.181801	0.19345	0.161482	
4	-0.144827	-0.15244	-0.15309	
5.1	0.037012	0.0409024	0.0121999	
5.2	0.0370136	0.0409984	0.0083707	
5	0.0740257	0.0819007	0.0205705	0.108531
6	0.0370288	0.0409184	0.0122073	0.0587191
7	0.0369747	0.0410106	0.0083928	

Bending Degrees

	Upper	Front Lower	S-center	Upper	Back Lower	S-center
1	-0.14809	-0.055405	-0.121545	-0.049716	0.0466808	0.0166788
2	-0.011683	-0.074781	-0.055014	0.0216484	-0.039465	0.0012643
3	-0.157315	-0.058807	-0.122432	-0.04976	0.0460207	0.0128545
4	-0.009467	-0.072519	-0.053346	0.0222842	-0.041647	0.0026307
5.1	-0.159899	-0.130298	-0.176439	-0.028073	0.0072141	0.0179504
5.2	-0.325211	-0.289745	-0.333519	-0.184913	-0.153916	-0.143068
5						
6	-0.159781	-0.130186	-0.176559	-0.028065	0.007216	0.0179397
7	-0.166794	-0.131326	-0.175778	-0.027476	0.0043741	0.0154851

	Upper	Overall bending Lower	S-center	Tip
1	-0.098377	-0.102086	-0.138224	-0.105061
2	-0.033331	-0.035316	-0.056278	-0.053362
3	-0.107556	-0.104827	-0.135286	
4	-0.031751	-0.030873	-0.055977	
5.1	-0.131826	-0.137512	-0.194389	
5.2	-0.140298	-0.135829	-0.190451	
5	-0.272124	-0.273341	-0.38484	-0.333519
6	-0.131716	-0.137402	-0.194499	-0.158442
7	-0.139318	-0.1357	-0.191263	

Appendix XV**COSMOS Program for Shear Center Analysis**

TITLE, PLATE ON P.633 SHEAR CENTER LOCATION ALONG THE ELEMENT
MPROP 1 EX 6.1E10 NUXY .33 GXY 2.41E10
PARASSIGN ELBE REAL 0.16
PARASSIGN LUSU REAL 0.06755
PARASSIGN BARM REAL 0.0033775
PARASSIGN DISL REAL 0.003
PARASSIGN ENDP REAL 0.03
PARASSIGN BUSU REAL 0.052
PARASSIGN X13 REAL ELBE
PARASSIGN X16 REAL ELBE+LUSU/4.0
PARASSIGN X19 REAL ELBE+LUSU/2.0
PARASSIGN X22 REAL ELBE+LUSU*0.75
PARASSIGN X25 REAL ELBE+LUSU
PARASSIGN X28 REAL ELBE+LUSU+ENDP-DISL
PARASSIGN X3 REAL ELBE+LUSU+ENDP
PARASSIGN X10 REAL X13+BARM
PARASSIGN X11 REAL ELBE+LUSU-BARM
PARASSIGN X30 REAL X13-DISL
VIEW 0 0 1 0
PT 1 0 0 0
PT 2 X3 0 0
PT 3 X3 BUSU 0
PT 4 0 BUSU 0
SCALE 0
SCALE 0.7
PT 5 ELBE 0.000726497 0
PT 6 X25 0.0397265 0
PT 7 X25 0.0512735 0
PT 8 ELBE 0.0122735 0
PT 9 ELBE 0.0065 0
PT 10 X10 0.0065 0
PT 11 X11 0.0455 0
PT 12 X25 0.0455 0
PT 13 X13 BUSU 0
PT 14 X13 BUSU/2 0
PT 15 X13 0.0 0
PT 16 X16 BUSU 0
PT 17 X16 BUSU/2.0 0
PT 18 X16 0.0 0
PT 19 X19 BUSU 0
PT 20 X19 .0 0
PT 21 X19 BUSU/2.0 0
PT 22 X22 BUSU 0
PT 23 X22 BUSU/2 0
PT 24 X22 0 0
PT 25 X25 BUSU 0
PT 26 X25 BUSU/2 0
PT 27 X25 .0 0
PT 28 X28 BUSU/2.0 0
PT 29 X3 BUSU/2.0 0
PT 30 X30 BUSU/2.0 0

CRLINE 1 2 1
CRLINE 2 2 2
CRLINE 2 2 3
CRLINE 1 1 2
CRLINE 3 3 4
CRLINE 4 4 1
CRLINE 5 8 5
CRLINE 6 5 6
CRLINE 7 6 7
CRLINE 8 7 8
PTTOL 1E-5
CT 1 0 .005 1 1 0
CT 2 0 .003 1 6 0
RG 1 2 1 2 0
RG 2 1 2 0
MA_RG 1 2 1 0 0
NPTPUSH 84 13 0
NPTPUSH 80 16 0
NPTPUSH 77 19 0
NPTPUSH 73 22 0
NPTPUSH 70 25 0
NPTPUSH 469 14 0
NPTPUSH 494 17 0
NPTPUSH 689 21 0
NPTPUSH 540 23 0
NPTPUSH 558 26 0
NPTPUSH 33 15 0
NPTPUSH 37 18 0
NPTPUSH 40 20 0
NPTPUSH 44 24 0
NPTPUSH 47 27 0
NPTPUSH 601 28 0
NPTPUSH 58 29 0
MARGCH 1 2 1 Q 4 1 0.4 1
EGROUP 1 SOLID 0 1 0 0 0 0
ACTDMESH PH 1
VIEW 2 1 1 0
PHEXTR RG 2 2 1 Z .0025 1 0 1 0.003 0.0001
PHEXTR RG 1 2 1 Z -.0025 1 1 1 0.005 0.0001
SHOW_MERGE 1 1 0.001
ECOMPRESS 1 10000
NCOMPRESS 1 10000
NMERGE 1 10000 1 0.0001 0 1 0
DCR 4 AL 0 4 1
DCR 20 AL 0 20 1
FND 84 FZ -5 84 1
ACTSET LC 2
FND 469 FZ -5 469 1
ACTSET LC 3
FND 33 FZ -5 33 1
ACTSET LC 4
FND 80 FZ -5 80 1
ACTSET LC 5
FND 494 FZ -5 494 1
ACTSET LC 6
FND 37 FZ -5 37 1
ACTSET LC 7
FND 77 FZ -5 77 1

ACTSET LC 8
FND 689 FZ -5 689 1
ACTSET LC 9
FND 40 FZ -5 40 1
ACTSET LC 10
FND 73 FZ -5 73 1
ACTSET LC 11
FND 540 FZ -5 540 1
ACTSET LC 12
FND 44 FZ -5 44 1
ACTSET LC 13
FND 70 FZ -5 70 1
ACTSET LC 14
FND 558 FZ -5 558 1
ACTSET LC 15
FND 47 FZ -5 47 1
PRINT_NDSET 10 84 84 33 33 80 80 37 37 73 73 44 44 70 70 47 47 58 58 9&
41 941
PRINT_ELSET 1 1 1
ECHECK 1 10000 1 5 0 0
A_STATIC N 0 0 1E-06 1E+10 0 0 0 0 0

Appendix XVI COSMOS Program for Deflections Analysis of the Unactuated Portion of the Beam

MPROP 1 EX 7.23E10 NUXY .33 GXY 2.72E10
PARASSIGN BUS REAL 0.050038
PARASSIGN BU2 REAL 0.025019
PARASSIGN DEL REAL 0.143807
PARASSIGN X5 REAL 0.11058
PARASSIGN X6 REAL DEL
PARASSIGN X7 REAL 0.134827
PARASSIGN X8 REAL 0.1016
PARASSIGN X9 REAL 0.0956
PARASSIGN X10 REAL 0.0986
PARASSIGN X11 REAL 0.146807
PARASSIGN X12 REAL 0.149807
PARASSIGN X17 REAL X9+DEL
PARASSIGN X18 REAL X10+DEL
PARASSIGN X19 REAL X11+DEL
PARASSIGN X20 REAL X12+DEL
PARASSIGN Y5 REAL 0.0039154
PARASSIGN Y6 REAL 0.0371423
PARASSIGN Y7 REAL 0.0461226
PARASSIGN Y8 REAL 0.0128957
VIEW 0 0 1 0
PT 1 0 0 0
PT 2 .340 0 0
PT 3 .340 BUS 0
PT 4 0 BUS 0
SCALE 0
SCALE 0.7
PT 5 X5 Y5 0
PT 6 X6 Y6 0
PT 7 X7 Y7 0
PT 8 X8 Y8 0
PT 9 X9 0.0250248 0
PT 10 X10 0.0250248 0
PT 11 X11 0.0246988 0
PT 12 X12 0.0246988 0
PT 13 X5+DEL Y5 0
PT 14 X6+DEL Y6 0
PT 15 X7+DEL Y7 0
PT 16 X8+DEL Y8 0
PT 17 X9+DEL 0.0262072 0
PT 18 X10+DEL 0.0262072 0
PT 19 X11+DEL 0.0237977 0
PT 20 X12+DEL 0.0237977 0
PT 21 X9 BUS 0
PT 22 X10 BUS 0
PT 23 X11 BUS 0
PT 24 X12 BUS 0
PT 25 X17 BUS 0
PT 26 X18 BUS 0
PT 27 X19 BUS 0
PT 28 X20 BUS 0
PT 29 X9 .0 0

PT 30 X10 .0 0
 PT 31 X11 .0 0
 PT 32 X12 .0 0
 PT 33 X17 .0 0
 PT 34 X18 .0 0
 PT 35 X19 .0 0
 PT 36 X20 .0 0
 CRLINE 1 1 2
 CRLINE 2 2 3
 CRLINE 3 3 4
 CRLINE 4 4 1
 CRLINE 5 5 6
 CRLINE 6 6 7
 CRLINE 7 7 8
 CRLINE 8 8 5
 CRLINE 9 13 14
 CRLINE 10 14 15
 CRLINE 11 15 16
 CRLINE 12 16 13
 CT 1 0 0.0032 1 1 0
 CT 2 0 .001 1 5 0
 CT 3 0 .001 1 9 0
 RG 1 3 1 2 3 0
 RG 2 1 2 0
 RG 3 1 3 0
 MA_RG 1 3 1 0 0
 EGROUP 1 SOLID 0 1 0 0 0 0 0
 MARGCH 1 3 1 Q 4 1 0.4 1
 NPTPUSH 201 21 0
 NPTPUSH 200 22 0
 NPTPUSH 185 23 0
 NPTPUSH 184 24 0
 NPTPUSH 929 9 0
 NPTPUSH 944 10 0
 NPTPUSH 1092 11 0
 NPTPUSH 1107 12 0
 NPTPUSH 31 29 0
 NPTPUSH 32 30 0
 NPTPUSH 47 31 0
 NPTPUSH 48 32 0
 NPTPUSH 156 25 0
 NPTPUSH 155 26 0
 NPTPUSH 140 27 0
 NPTPUSH 139 28 0
 NPTPUSH 1527 17 0
 NPTPUSH 1542 18 0
 NPTPUSH 1690 19 0
 NPTPUSH 1705 20 0
 NPTPUSH 76 33 0
 NPTPUSH 77 34 0
 NPTPUSH 92 35 0
 NPTPUSH 93 36 0
 ACTDMESH PH 1
 VIEW 2 1 1 0
 PHEXTR RG 2 3 1 Z 0.00024384 1 0 1 0.001 0.0001
 PHEXTR RG 1 3 1 Z -0.0007747 1 1 1 0.0032 0.0001
 SHOW_MERGE 1 1 0.0001
 ECOMPRESS 1 30000

NCOMPRESS 1 30000
NMERGE 1 30000 1 0.0001 0 1 0
DCR 32 AL 0 32 1
DCR 4 AL 0 4 1
PSF 2 -1.88982E7 4 2 -1.88982E+07 -1.88982E+07 4
ACTSET LC 2
PSF 1 -1.88982E7 3 2 -1.88982E+07 -1.88982E+07 4
ACTSET LC 3
PSF 6 -1.88982E7 8 2 -1.88982E+07 -1.88982E+07 4
ACTSET LC 4
PSF 5 -1.88982E7 7 2 -1.88982E+07 -1.88982E+07 4
ACTSET LC 5
PSF 1 -1.88982E7 8 1 -1.88982E+07 -1.88982E+07 4
ACTSET LC 6
PSF 1 -1.88982E7 4 1 -1.88982E+07 -1.88982E+07 4
ACTSET LC 7
PSF 5 -1.88982E7 8 1 -1.88982E+07 -1.88982E+07 4
PRINT_NDSET 10 31 48 76 93 139 156 184 201 929 944 1092 1107 1527 1527&
1542 1542 1690 1690 1705 1705
PRINT_ELSET 1 1 1
ECHECK 1 30000 1 5 0 0
VIEW 1 1 1 0
VIEW 2 1 1 0

Appendix XVII COSMOS Program for Verifying Theoretical Models Based on Heaviside Functions

MPROP 1 EX 6.1E10 NUXY 0.33 GXY 2.41E10
PARASSIGN LENGTH REAL 0.2
PARASSIGN WIDTH REAL 0.05
VIEW 0 0 1 0
PT 1 0 0 0
PT 2 0 WIDTH 0
VIEW 0 0 -1 0
VIEW 0 0 1 0
PT 3 LENGTH WIDTH 0
PT 4 LENGTH 0 0
SCALE 0
CRLINE 1 1 2
CRLINE 2 2 3
CRLINE 3 3 4
CRLINE 4 4 1
SF2CR 1 1 3 0
M_SF 1 1 1 4 10 40 1 1
ACTDMESH PH 1
VIEW 2 1 1 0
PHEXTR SF 1 1 1 Z -0.005 1 1 1 0.005 0.0001
ECOMPRESS 1 800
NCOMPRESS 1 902
DCR 1 ALL 0 1 1
DCR 8 ALL 0 8 1
EGROUP 1 SOLID 0 1 0 0 0 0 0
VIEW 1 1 2 0
VIEW 2 1 1 0
PRINT_NDSET 1 6 446
FND 331 FX 50 341 1
FND 782 FX -50 792 1
A_STATIC N 0 0 1E-06 1E+10 0 0 0 0 0

VITA

Lumin Jiang was born on October 26, 1965 in Guangzhou, China. He graduated from Guangzhou No. Six High School in 1983. The same year, he enrolled the First Mechanical Engineering Department in South China University of Technology. He received the degree of Bachelor of Science in Mechanical Manufacturing Engineering in 1987. Between 1987 and 1993, he worked as an engineer at Guangdong Province Technical Consulting Firm. In 1994, he worked as a marketing representative in IBM China Company Limited, South China Branch. In January 1995, he entered the University of Tennessee Graduate School, where he will receive his Master of Science degree in Mechanical Engineering in December 1996.