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**DEVELOPMENT OF A MEASUREMENT SYSTEM
OPTIMIZED FOR DETECTING PREBREAKDOWN
CURRENTS IN LIQUID DIELECTRICS**

A Dissertation

Presented for the

Doctor of Philosophy

Degree

The University of Tennessee, Knoxville

Alan Lee Wintenberg

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DEDICATION

This dissertation is dedicated to my wife, Kimberly, and to my parents, Richard and Henrietta Wintenberg who have always supported me with love and understanding.

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ABSTRACT

When an insulating fluid is electrically stressed to near its breakdown level, prebreakdown phenomena occur. Currents flowing under these conditions are referred to as prebreakdown currents, and their measurement is important to the understanding of the physical processes leading up to breakdown. Preliminary results indicated that for dc-stressed hexane the currents were pulse-like in nature and that the accuracy with which they could be measured was limited by the bandwidth and noise level of the measurement system. This research comprises a study of fundamental measurement limitations and the development of an optimized measurement system.

A typical system used to measure prebreakdown currents consists of a high voltage source, a test cell and electrodes (to contain and electrically stress the fluid), a detection resistor (to convert the currents to a voltage) and a recorder (to capture a record of the currents). Sometimes a preamplifier is included to allow viewing of lower level currents. This research began with a study of the factors limiting the bandwidth and noise level of such a measurement system. A simple circuit model for the test cell was developed and used. The model consists of a current source shunted by a small capacitance. Two potential preamplifier configurations (voltage-sensitive and transimpedance) were considered for use. For the voltage-sensitive configuration, the bandwidth is limited by the test cell capacitance shunting the detection resistor, and the thermal noise of the resistor

places a lower bound on the system noise level. Analysis revealed that for the same bandwidth, the transimpedance preamplifier could provide a lower noise level and therefore a lower minimum detectable current.

A model describing the transimpedance preamplifier in terms of an open-loop gain function, feedback impedances, source impedances and noise power spectral densities was developed. A simplified version of the model was analyzed algebraically to give equations describing the frequency response and noise performance of the preamplifier. A computer-aided-design tool was developed to perform similar analyses of the complete model numerically. The noise analysis revealed that the input device for the transimpedance preamplifier should have low input noise power spectral density and low input capacity and an FET would be preferred over a bipolar device. A number of different types of FETs were characterized, and it was found that the use of GaAs MESFETs would produce a transimpedance preamplifier with the lowest noise level in a dc to 100 MHz bandwidth.

A 90-MHz bandwidth transimpedance preamplifier using a GaAs MESFET input and a 5 k Ω feedback resistor was developed and incorporated into a system consisting of a cell, the preamplifier, two wideband buffer amplifiers, a 200-Msample/s transient digitizer and waveform reconstruction software. The system has an overall bandwidth of 65 MHz, including the digitizer, and a minimum detectable current of less than 250 nA rms for a signal-to-noise ratio of five. Waveform reconstruction software was used to quadruple the effective sampling

rate from 5 ns per point to 1.25 ns per point. This does not increase the bandwidth, but provides a better representation of the high-frequency components of the digitized waveform than simple linear interpolation does.

The measurement system was tested both with artificially generated current pulses and by measuring prebreakdown currents in hexane with either positive or negative high voltage. Pulses with extremely short durations (7 ns fwhm) and amplitudes ranging from tens of μA down to near the noise level of the measurement system were observed. The usual pulse shape appeared to be the system impulse response, which implies that a still greater bandwidth would be required to see the actual shape of the current pulses.

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CHAPTER 1

INTRODUCTION

1.1 An Overview

Liquid dielectrics are widely used in many types of electrical equipment, including power transformers and capacitors. The fluids serve primarily as electrical insulation and secondarily as coolants for the equipment. A fluid's value as insulation is usually thought of in terms of its *breakdown strength*. The breakdown strength is the electric field strength at which leakage current densities exceed some limiting value – effectively the point where the insulator stops insulating. These breakdown currents can be so large that the insulation and the equipment may be damaged or destroyed. Although various standards which specify how to measure the breakdown strength of a fluid are available, the understanding of the physical mechanisms leading up to breakdown is limited at present.

When a fluid is electrically stressed at or near its breakdown level, what are commonly called prebreakdown phenomena or events occur. One manifestation is the presence of prebreakdown currents. Others include light and sound emission, and bubble formation. These phenomena are important, because at

a sufficiently high stress, breakdown will occur, apparently always following prebreakdown events. Thus prebreakdown phenomena are important as signs warning of impending breakdown. Detection of prebreakdown phenomena could be used to signal the need for shutdown and repair of fluid-filled electrical equipment before significant damage is done.

Prebreakdown phenomena are important in ways other than as precursors to breakdown. Prebreakdown currents in transformer oil can produce hydrogen gas which can eventually accumulate to a concentration where a destructive explosion is a possibility. These currents may also *age* the fluid. Aging refers to a gradual loss of breakdown strength by the fluid due to decomposition by partial discharges. Unchecked aging would eventually lead to a complete breakdown.

A typical experimental arrangement for researching prebreakdown and breakdown phenomena is shown in Figure 1.1. Arrangements like this have been used over the past thirty years and more with many variations on the basic theme. AC, DC and pulsed high voltage supplies have been used, and many shapes (needle, plane, sphere, etc.) and materials (brass, stainless steel, rhodium, etc.) have seen service as electrodes. Many fluids have been investigated, with transformer oil and hexane perhaps being the favorites due to practical usage and molecular simplicity, respectively. Different workers have used different detection impedances Z_D and types of displays, with the earlier researchers mainly measuring dc currents with microammeters and electrometers. Relatively little work has been done in applying high-speed, low-noise

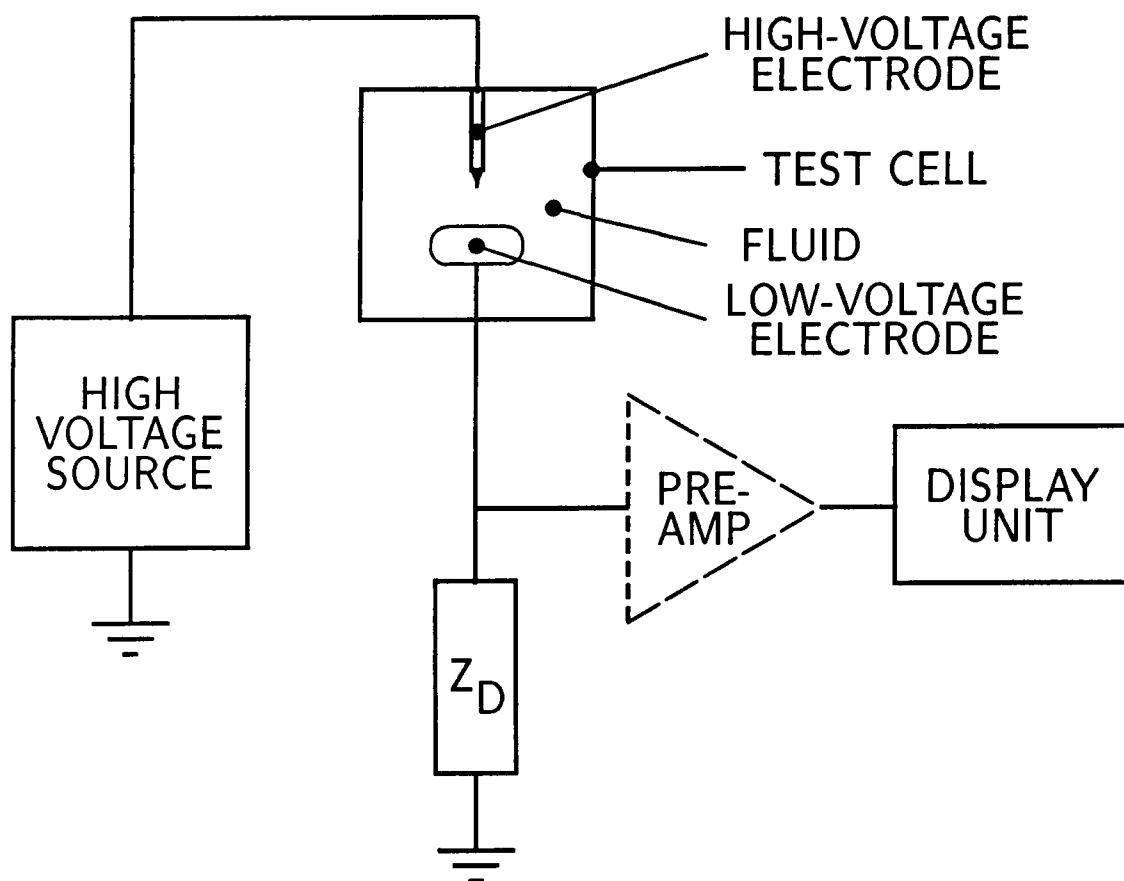


Figure 1.1: Typical experimental arrangement for measuring prebreakdown currents.

electronics to this measurement problem, although several researchers have used wide-bandwidth systems.

Researchers investigating prebreakdown phenomena need an improved system to measure currents and to correlate these measurements with results from more well advanced methods for detecting and measuring prebreakdown events such as fast photography and photomultiplier detection of light emission. This research is important since when the breakdown process and the physical factors affecting it are understood, it may be possible to develop improved fluids. An improved measurement system or parts of it may also be used in tests more sensitive than the simple breakdown voltage tests currently used to screen fluids for their suitability. Another potential use of an improved measurement system or its derivative would be that of on-line monitoring for impending breakdown of large, expensive equipment.

1.2 A Survey of Prebreakdown Current Measurement Techniques

In developing a system to measure prebreakdown currents, it is necessary to consider how they have been measured and what is known about their characteristics. This earlier work serves as a guide and a benchmark for new efforts. Measurements already made suggest which characteristics merit further investigation and dictate some specifications for new measuring equipment.

To appreciate the need for the development of instrumentation for the mea-

surement of prebreakdown currents in liquid dielectrics, it is very helpful to consider the representative approaches used by various researchers. A typical approach prior to the late fifties was that used by Green [1], who, in 1956 used a vibrating-reed electrometer to measure conduction currents in hexane ranging down to picoamperes. The following year, House [2] used high megohm resistors and an electrometer to measure the dc component of conduction currents in stressed hexane, but also used a 100-k Ω detection resistor and a preamp to view current pulses down to the nanoamp range on a oscilloscope. Later, Sharbaugh and Watson [3] applied high voltage pulses to n-hexane and measured the average current passing through the fluid during the application of voltage. They developed a technique where a capacitor was used to store the charge transferred through the fluid during the pulse. After the pulse the capacitor voltage was measured with an electrometer, and the charge and average current were calculated.

In 1961, Darvenzia and Tropper [4] examined highly filtered and distilled transformer oil under high voltage dc stress using, for the first time, a photomultiplier to detect light emission. The photomultiplier output and the conduction currents were both measured using high resistances and electrometers. Meanwhile, in the Soviet Union Komel'kov [5] was photographing pulse discharges in liquids using a camera with rotating film. The discharge currents were viewed on an oscilloscope used in the single shot mode and then photographed. These traces show a series of pulses increasing in amplitude from about 10 mA to

several hundred mA in the approximately 50 μ s before breakdown. From the oscillographs, the bandwidth of the current detection system can be estimated to be at least several MHz. Another advance was made by Nosseir [6] who used an audio bandwidth amplifier and a tape recorder to record prebreakdown current pulses in transformer oil and performed pulse height analysis on them.

Huq and Tropper [7] used a similar (perhaps even the same) setup to measure current pulses in liquids. Pulses sensed in a 500-k Ω resistor were amplified with a preamplifier with a bandwidth extending from 15 Hz to 30 kHz. The preamp output was displayed on an oscilloscope and also sent to an electronic square-law circuit which was in turn connected to a voltmeter. This allowed mean-square values of the current to be recorded. A tape recorder was used in a novel way to perform autocorrelation on the current waveforms. Megahed and Nosseir [8,9] used the same equipment to measure autocorrelation functions and power spectral densities and perform pulse height analysis for several liquids under various conditions.

Rhodes and Brignell [10] reported the first use of a charge-sensitive preamplifier and a multichannel pulse height analyzer, apparently both of which were originally designed as nuclear instrumentation. They reported that pulses of the order of 5 fC are always present in dc stressed n-hexane and that 1 pC pulses occur in intermittent bursts. In further work [11,12,13], Buffam and Brignell used a transient digitizer to capture the output of the charge-sensitive preamplifier. They were able to correlate the low level charge pulses with particle motion by

observing light scattered (by a particle) from the high voltage electrode. Arrival or departure of a particle produced a change in the amount of scattered light and a simultaneous charge pulse. They found no correlation between particle activity and the larger charge pulses.

Nelson and McGrath [14] measured currents in dc-stressed transformer oil using a 10-M Ω resistor and both an electrometer and a wideband amplifier. The input capacitance of the amplifier in conjunction with the 10-M Ω resistor probably limited the bandwidth to a few kHz. Khalifa and Nosseir [15] used a similar setup, but with a 100-k Ω resistor and a 2.5-MHz oscilloscope to observe currents. Assuming a 30-pF oscilloscope input capacitance shunting the detection resistor, a bandwidth of about 50 kHz would result for these measurements.

Fast (50-ns risetime and 1- to 50- μ s duration) high voltage pulses were applied to n-hexane by Heiman et al. [16] in 1976. A 1000-pF capacitor used as the detection impedance stored the charge conducted through the cell during the pulse until it could be read with an electrometer. A 15-MHz oscilloscope was also used to display the voltage developed on the capacitor. By the 1980's when Yamashita et al. [17] worked with pulsed HV, higher frequency techniques and equipment was more readily available. An LED placed in series with the high voltage electrode emitted light when conduction currents flowed. By detecting the light emitted from the LED with a photomultiplier, the current could be measured and displayed on a storage oscilloscope. Currents down to 50 μ A were detectable, and with a negative needle, the current waveform was a series of

increasing pulses (up to 700 μA in 8 μs) followed by breakdown. Later, the response time of the LED detector was decreased to a few nanoseconds, and a 400-MHz storage oscilloscope was used [18]. A very similar setup was used by Yamada and Sato [19] to examine prebreakdown currents in many liquids under very fast (15 ns risetime) high voltage pulses. An LED response time of 10 ns and linearity up to 300 mA was reported.

Hanna et al. [20] applied high voltage pulses to heptane and toluene and observed prebreakdown current pulses ranging from 100 μA to 8 mA using a 50- Ω detection impedance connected to a storage oscilloscope by a shielded cable. It should be noted that a 5-mV pulse is perhaps the smallest pulse that can be observed on a fast trace of a storage oscilloscope and that is also the voltage developed by 100 μA flowing in 50 Ω . Light emitted from the fluid was detected simultaneously with a photomultiplier, and it was found that the light and current pulses were coincident. All pulses had durations of 20 to 30 ns.

Pulsed high voltage was again used by Beroual and Tobazeon [21] who measured currents across 100 Ω and displayed them on a storage oscilloscope. Other workers at the same facility [22] used ac and measured currents with a 50- Ω resistor with sensitivity down to 1 mA or with an FET current probe built into the test cell. The current still developed a voltage across a resistor, but the FET probe buffered the resistor from the cable and oscilloscope capacitance. With this arrangement the total capacitance shunting the resistor was 4 pF, and a 1-k Ω resistor could be used to give a bandwidth of 40 MHz (the oscilloscope

bandwidth was 100 MHz) and a sensitivity of a few μA . In more recent papers, Lesaint et al. [23,24] reported observing current pulses with durations of a few nanoseconds for ac-stressed transformer oil and dc-stressed cyclohexane. In their latest work, the system bandwidth was increased to 300 MHz by using a coaxial test cell impedance matched ($50\ \Omega$) to the cable connecting the cell to a fast preamplifier. A 400-MHz storage scope was used to capture the amplified current signal. The risetimes of the pulses were apparently limited by the measurement system bandwidth. From the oscillographs, one can estimate the rms noise current to be approximately $1\ \mu\text{A}$.

1.3 Preliminary Experiments and Preamplifier Development

The prebreakdown current pulses seen by several researchers using wideband systems ($>1\ \text{MHz}$) are very fast and have risetimes that are apparently limited by the measurement system. The smaller pulses seem to disappear into the noise, leading one to suspect that there are even smaller pulses that would be visible if the noise level were lower. To provide a better view of these pulses, a special preamp was developed [25] with the aim of providing a wider bandwidth and lower noise level. With an input resistance of $50\ \Omega$ in parallel with $10\ \text{pF}$ and an overall current-to-voltage gain of $3850\ \text{A/V}$, the preamplifier provided a 58-MHz bandwidth and had an rms input noise current of $0.24\ \mu\text{A}$. The preamplifier development was influenced by a desire to have approximately a 50-MHz

bandwidth, which was larger than previously used, but not so large as to make development extremely difficult, and to have a low-noise design to give a small minimum detectable signal. The $50\ \Omega$ value was chosen since the preamp and test cell capacitance in parallel with that resistor would then have little effect in limiting the bandwidth. $50\ \Omega$ is also a convenient value to terminate a coaxial cable.

A 200-Megasample/s digitizer was used along with the preamplifier in a setup like Figure 1.2 [26,27,28]. This version of the preamp had a $200\text{-}\Omega$ input resistor, an overall gain of 20,000 A/V and a bandwidth of approximately 25 MHz. The system bandwidth was limited to approximately 4 MHz by the interaction of the protection network with the test cell, feedthrough and preamp input capacitances (a total of 15 to 20 pF). This system was used to observe prebreakdown currents in hexane, trimethylpentane and parafinic transformer oil under a variety of conditions including various voltage levels, impurity concentrations and pressures [26,27,28,29].

For negative dc stresses, prebreakdown current pulses like those in Figure 1.3 were observed in n-hexane. For these conditions, current pulses generally occur in bursts, and the pulse amplitude increases linearly with time during a burst. The spacing between pulses is not a constant – it increases with time. Although pulses as small as $3\ \mu\text{A}$ are clearly visible, at the earliest part of the burst the pulses ($<1\ \mu\text{A}$) seem to disappear into the noise, indicating that a lower-noise measurement system is needed. Figure 1.4 is an expanded view of a single pulse,

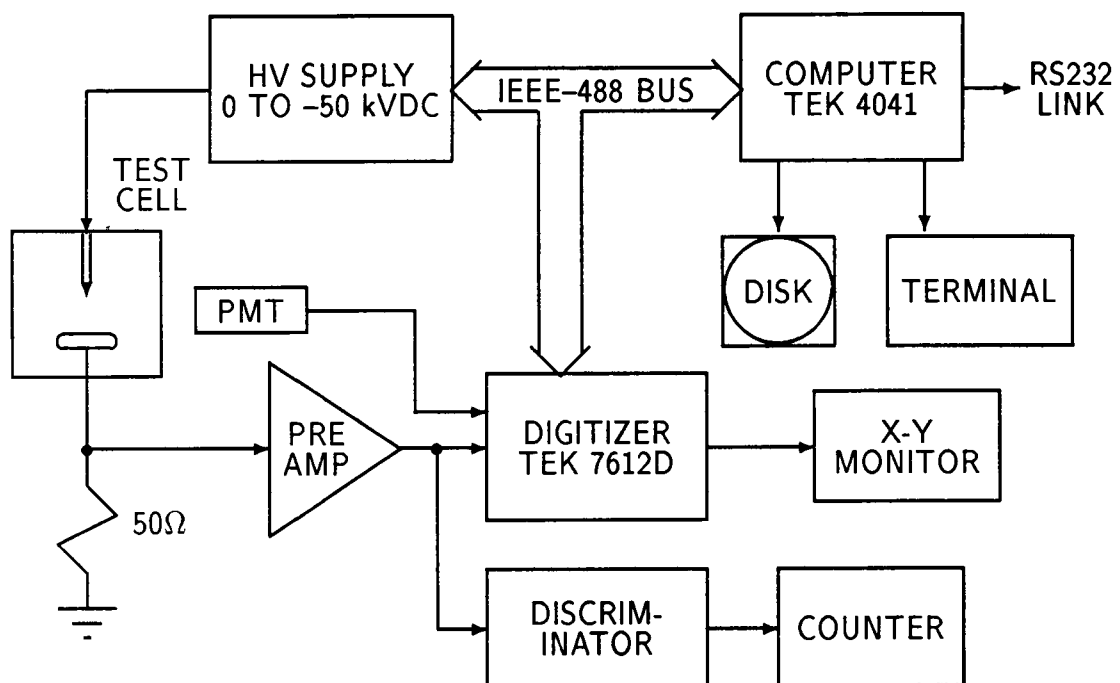


Figure 1.2: Experimental arrangement used to measure prebreakdown currents.

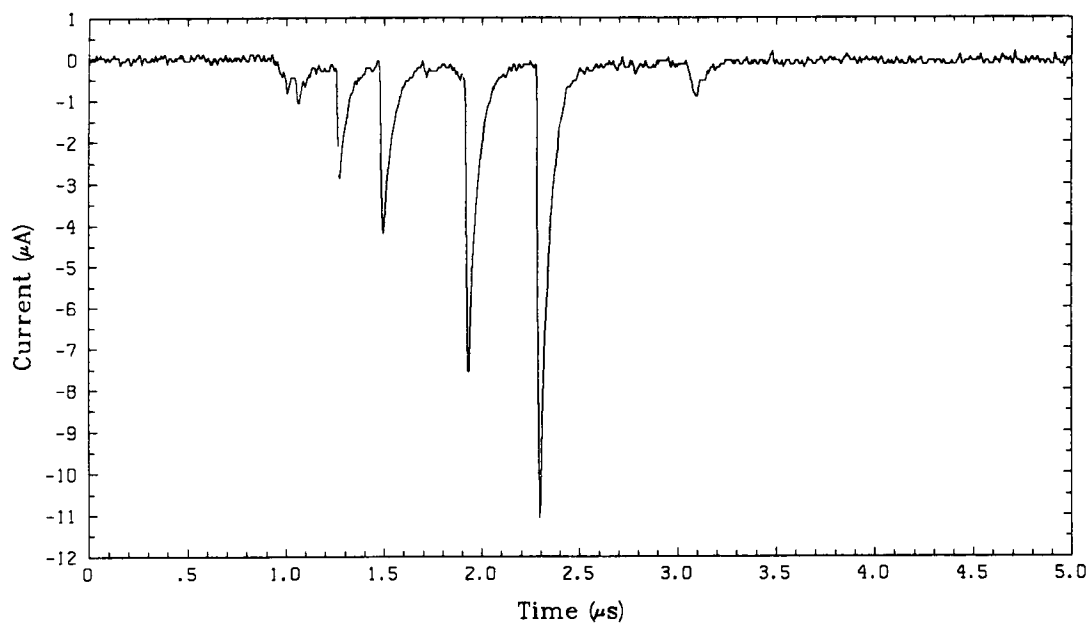


Figure 1.3: Prebreakdown currents in negatively-stressed n-hexane.

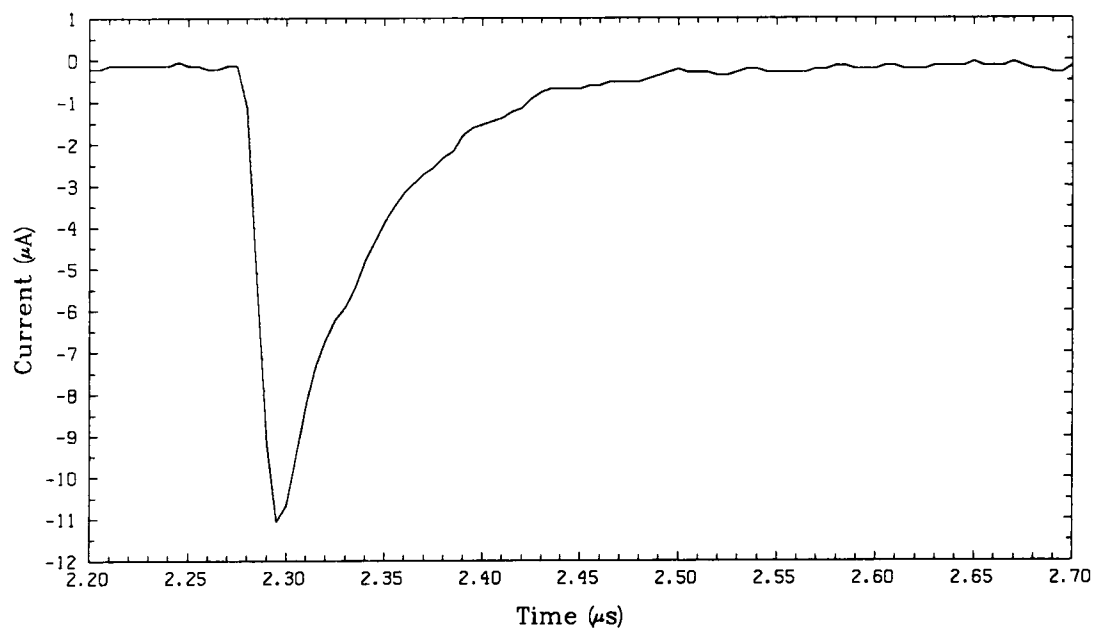


Figure 1.4: A single prebreakdown current pulse in negatively-stressed n-hexane.

and it reveals that the pulse shape is approximately the current impulse response of the system (which consists of the test cell, protection network and preamplifier). This indicates that a wider band measurement system is needed. Use of this system did allow the viewing of smaller current pulses than was previously possible, and the clear view of the increasing sequence of pulses sparked the development of a new theory on the mechanisms producing them [30,31,32].

1.4 Objectives of the Dissertation

The purpose of this work is to develop a system for the detection and measurement of prebreakdown currents in liquid dielectrics subjected to high dc stresses created by a point-plane electrode geometry. Specifically, the goals are to view the currents with as low a noise level as possible while using a bandwidth adequate to distinguish individual pulses and to determine the pulse shape as accurately as possible.

This dissertation describes the development of a measurement system to achieve the above goals. The system includes a test cell, preamplifier, amplifier, digitizer, digital signal processing software and test circuitry. Algebraic and numerical methods to analyze the noise performance of several preamplifier configurations are described. These techniques are then used to produce a design methodology which yields a preamplifier with optimized noise performance. Since the test cell capacitance affects the preamplifier performance, it

is included in the noise analysis, and the test cell is designed to be compatible with optimized noise performance. The actual implementation of the preamplifier and buffering amplifiers are described along with a current pulse generator developed to test the preamplifier. The characterization of the measurement system frequency response and noise performance is discussed. Development of the software needed to reconstruct a continuous waveform from the digitized data is presented, and finally, representative prebreakdown current waveforms from liquid dielectrics are given.

CHAPTER 2

MEASUREMENT SYSTEM CONFIGURATION

2.1 Measurement Goals

The main purpose of the measurement system is to allow the observation of prebreakdown currents with sufficient time and amplitude resolution so the true shapes of individual prebreakdown current pulses are discernible to the lowest level possible. Secondary purposes of the system include simultaneous observation of prebreakdown current pulses and light emission, and capture of current waveforms in a way suitable for integration to give charge data. In order to develop the measurement system, the known characteristics of prebreakdown currents were considered, and the facts and goals were then reconciled with practical limitations.

2.1.1 An Analysis of Available Data On Prebreakdown Currents

Most of the data available on prebreakdown currents indicates a pulse-like nature with pulses of short duration and low amplitudes during the early stages of prebreakdown activity, although the amplitudes increase as breakdown becomes imminent. In fact, some of the most up-to-date information indicates that the

pulses may be impulse-like. Lesaint reported pulses with durations of only a few nanoseconds in transformer oil and cyclohexane [23,24] and felt that these pulses were limited by the system bandwidth. Measurements made using the system of Figure 1.2, which was designed to have a lower noise level than any then available, showed that the pulse amplitudes could be less than $3\ \mu\text{A}$ and hinted that lower level pulses were not visible because of random noise.

With the same type preamplifier described in Section 1.3, but using an improved protection circuit which increased the system bandwidth to about 30 MHz, prebreakdown current pulses like those in Figure 2.1 were observed in negative dc stressed n-hexane. From this figure it is clear that the pulses range in size from $> 30\ \mu\text{A}$ to less than $2\ \mu\text{A}$. In other observations, pulses as large as 0.5 mA were seen. A noise level considerably less than $0.5\ \mu\text{A}$ rms and a considerable amount of amplification is apparently needed to view the lower level pulses that are not clearly seen with this system.

Figure 2.1 uses a microsecond time scale in order to show a typical burst of pulses, but on that time scale, the pulses are very narrow and can be separated by less than 100 ns. An expanded view of one pulse (Figure 2.2), shows that the pulse width is approximately 30 ns, and the pulse shape is essentially the same as the current impulse response of the preamplifier including the protection circuit (Figure 2.3.) From this and Lesaint's results, the conclusion must be drawn that a wideband system with frequency response considerably greater than 30 MHz is needed to view the shape of the current pulses.

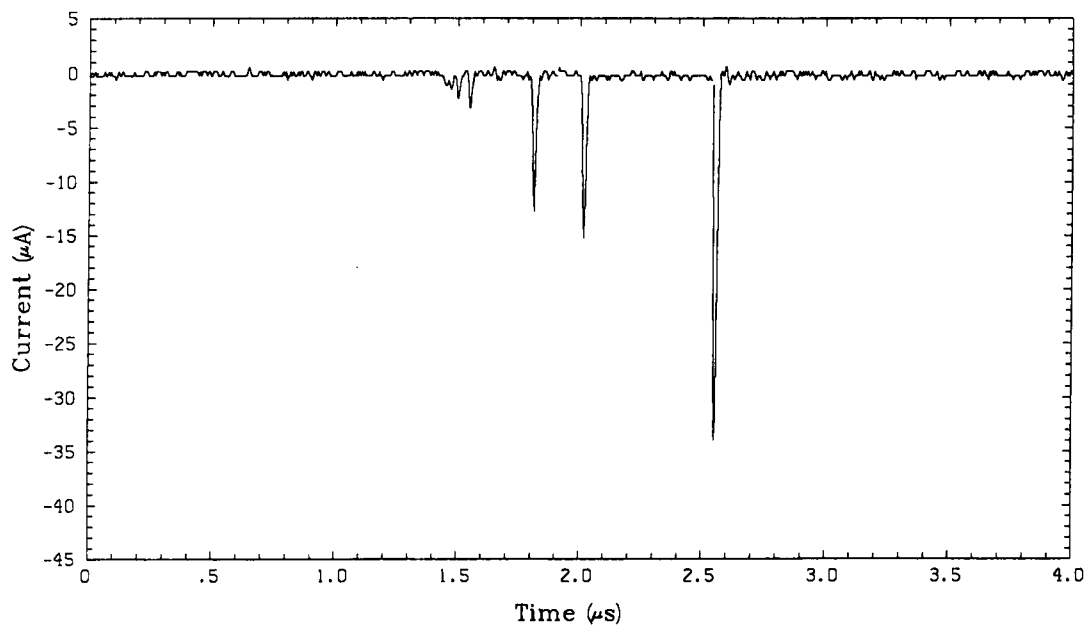


Figure 2.1: Prebreakdown currents in negatively-stressed n-hexane.

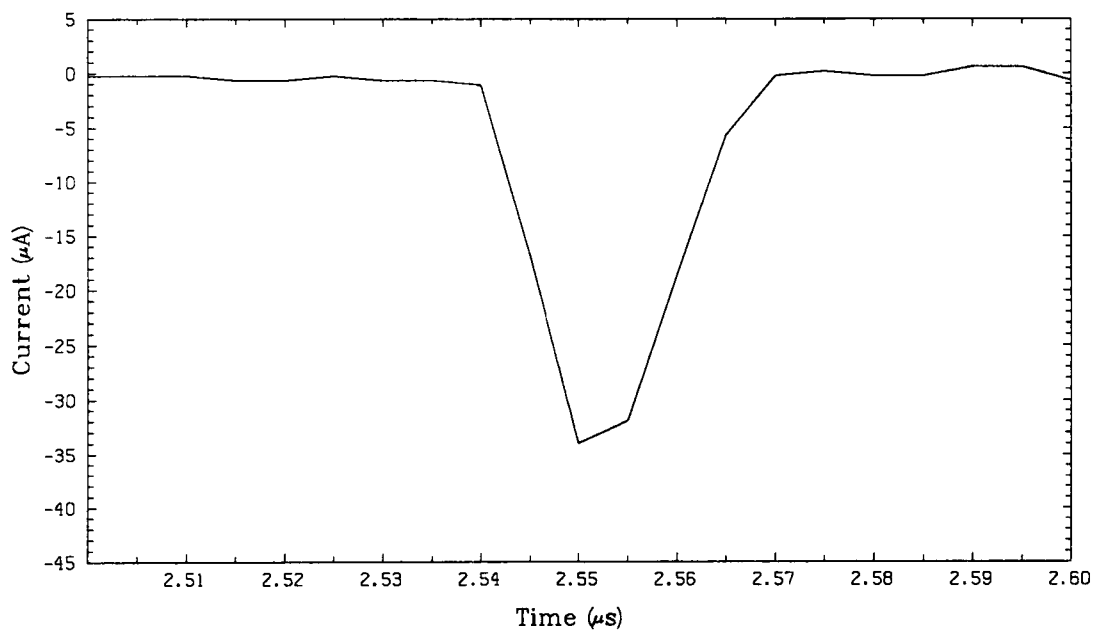


Figure 2.2: A typical prebreakdown current pulse from negatively-stressed n-hexane.

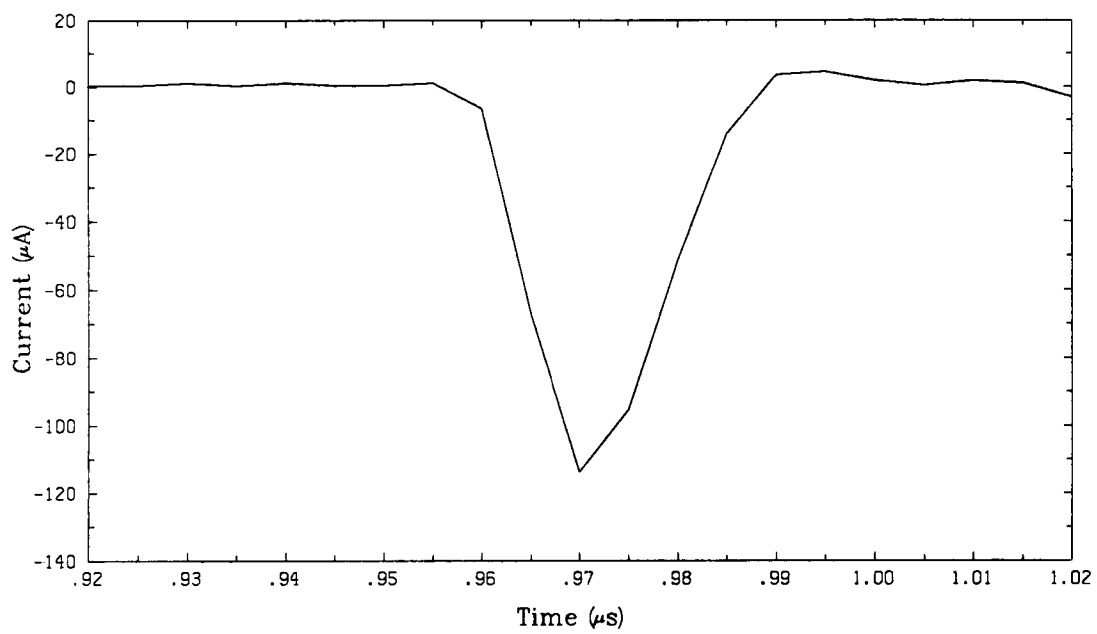


Figure 2.3: Current impulse response of the preamplifier.

Figures 2.1 and 2.2 illustrate some advantages of a digitizer over an analog storage oscilloscope. One obvious advantage is that since the waveform is digitized, it can be readily plotted using different scales as desired to better show different features. Another is that data stored in digital form can be processed after the fact. Processing useful for this application could be some type of filtering or integration to give charge information. A less obvious advantage of the digitizer is its superior horizontal resolution. The digitizer used to capture Figures 2.1 and 2.2 stores 2048 samples while a typical fast analog storage scope stores 10 divisions with resolution to perhaps one tenth of a division. (This depends on the spot size and storage characteristics of the tube.) This might be considered roughly equivalent to 100 samples, so the digitizer stores considerably more samples. In practice this means that the digitizer can capture a burst of pulses like Figure 2.1 and details like that shown in Figure 2.2 can be seen, but with the storage scope, use of a time scale suitable to see the details of one pulse precludes seeing the rest of the burst. The vertical resolution is approximately the same for most transient digitizers and analog storage oscilloscopes: 6 to 8 bits and approximately 7 bits (one part in one hundred), respectively.

2.1.2 Practical Limitations

Considering the nature of the prebreakdown currents (fast pulses arranged in bursts) and the advantages of waveform capture via digitizer, it was decided that a digitizer should be used in the measurement system. The fastest digitizer

available for this research was a 200 Msample/s unit and that specification was accepted as a practical limitation to the measurement system, with the added requirements that the system should also be compatible with analog storage oscilloscopes and faster digitizers. 200 Msample/s corresponds to a Nyquist frequency of 100 MHz, and that is an upper limit for waveform frequency components which can be digitized and reconstructed.

To keep the developmental time and cost to a reasonable level, it was also decided that any electronics developed for the system would employ discrete components and commercially available integrated circuits, employ printed circuit board technology and operate at room temperature. Such electronics would be relatively low cost, easy to use and readily duplicated by other researchers. Better performance might be obtained by using hybrid or custom integrated technology and perhaps a cooled preamplifier, but at much greater expense and less accessibility for other researchers.

2.1.3 Definition of an Optimized System

The measurement goals were refined by analyzing the data available on the nature of prebreakdown current pulses and further limited by practical considerations. In light of those requirements, an optimized system was defined as one providing the best view possible of prebreakdown current pulses with approximately a dc to 100 MHz bandwidth and the lowest noise level achievable in that bandwidth.

2.2 Development of a Workable System Configuration

The measurement system shown in Figure 1.2 was used as a basis around which an improved configuration was developed. The current measurement part of that system consists of a programmable high-voltage dc supply, test cell, preamplifier, digitizer, computer and display software. The supply, test cell and its electrodes are necessary to electrically stress the fluid sample and collect the prebreakdown currents. The preamplifier is needed to amplify the current to a level where it can be captured and displayed by the digitizer and computer, respectively. To the system of Figure 1.2, an additional amplifier, an anti-aliasing filter and digital waveform reconstruction software were added to produce the system shown in Figure 2.4. The amplifier was added to provide additional gain so very small pulses could be examined in greater detail. (This was often found necessary with the system of Figure 1.2.) The digital waveform reconstruction software has the advantage of recreating a representation of the signal that more closely resembles the original continuous signal than does the simple 'connect the dots' plots often used with digitized data. Using waveform reconstruction techniques requires the use of an anti-aliasing filter, so one was included.

The measurement system shown in Figure 2.4 is very general, but at this stage of development, it was not practical to more or less arbitrarily assign specifications for each part and design each separately, as there are too many interdependencies to hope that such a design process would yield an optimized

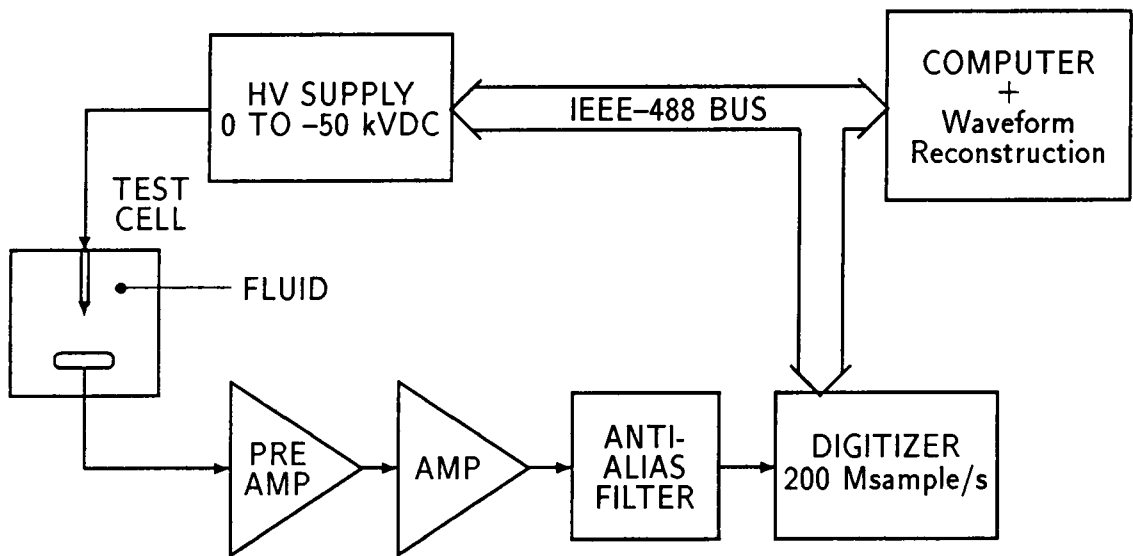


Figure 2.4: General prebreakdown current measurement system.

system. For example, the preamplifier gain and the digitizer signal level requirements determine the amplifier gain, and the preamplifier gain would not be known until the preamp was designed with consideration for its noise level, which in turn depends upon the test cell capacitance and many other factors. An approach that would allow consideration of those interdependencies was chosen. Each part of the system would be designed by considering its function and how its characteristics would affect the rest of the system. Once a part was designed, its characteristics would be used to firm up the requirements for the remaining parts. This process would begin with the design of the test cell and preamplifier, whose characteristics were felt to have the greatest impact on the system noise level.

CHAPTER 3

TEST CELL MODEL AND DEVELOPMENT

Before attempting to design a measurement system, one must consider not only the characteristics of the signal to be measured, but the source impedance as well. For the case of prebreakdown currents, the source is a poorly understood process in a highly stressed region inside the test cell. In this chapter, a simple physical model of prebreakdown current activity is developed and then translated into a circuit model which includes the test cell impedances. These models suggested an improved test cell configuration. Finally, the development of a test cell using the improved configuration is described.

3.1 Physical Model of Prebreakdown Activity in the Test Cell

There are many theories describing the prebreakdown process in a liquid dielectric, but the important point here is that charge is transferred from the stress-producing electrode to the surrounding fluid. This injection of charge forms a so-called low density region (LDR) [33,21,34,35], and it is the motion of this charge into the LDR that can be detected as a current pulse. For the data taken and presented in Chapters 1 and 2, a needle with high voltage applied was used to stress the fluid, and prebreakdown currents were sensed as image

currents in a plane electrode facing the needle tip. Charge motion near the needle tip distorts the field lines that are intercepted by the plane and this produces the image charge. This process is illustrated in Figure 3.1.

Figure 3.1 is obviously not a scale drawing. For the experiments described in [26,27,28] and in Chapters 1 and 2, the gap between the needle tip and the plane (5 mm) was much less than the distance to the cell wall (about 40 mm.) The tip radius was nominally 3 μm , and the plane was 50 mm in diameter. Based on other observations [36,37], a typical LDR (0 to 100 μm) was also much smaller than the gap.

This physical model for the origin of prebreakdown currents is considerably less detailed than others that have been proposed [38,32]. Its purpose is not to describe the origin, nature, timing or size of prebreakdown current pulses, but only to allow development of a circuit model useful for describing their detection.

3.2 Circuit Model of Prebreakdown Activity in the Test Cell

A relatively simple lumped-element circuit model can be developed to describe the process shown in Figure 3.1 by replacing the various surfaces by nodes and dielectric-filled regions by capacitors. The following nodes can be identified: HV, needle, LDR (surface), plane and ground. If the needle and the HV node are connected with a low-impedance path, then they can be modeled as being shorted together. This reduces the number of nodes to four, and the model then

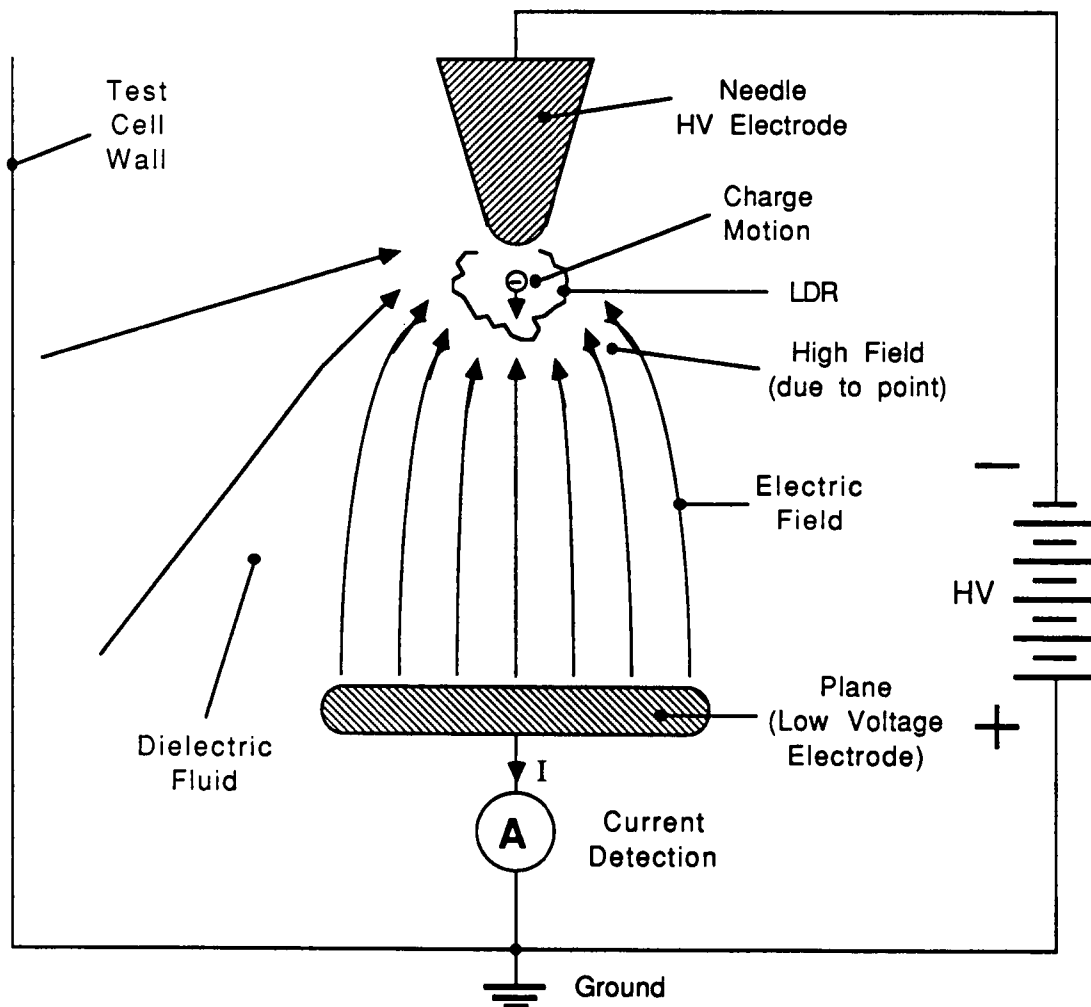


Figure 3.1: Physical model of prebreakdown current generation and detection with high voltage applied to needle.

requires a total of only six capacitors to connect each node to every other node. C_{NG} , C_{NL} and C_{NP} represent the capacitance between the needle and ground, the LDR and the plane, respectively. In addition, the LDR has capacitance C_{LP} to the plane and C_{LG} to ground, and the plane has capacitance C_{PG} to ground. Charge injection from the needle to the LDR is represented by a current source I . It is important to note that I flows in the same direction the charge (which is negative) moves, so I is negative. The current detection (preamplifier) can be represented by its input impedance Z_D (R_D in parallel with C_D), and this results in the circuit model shown in Figure 3.2.

Since the prebreakdown current pulses are mainly high-frequency in nature, the circuit model can be further simplified by being reduced to an ac model. For ac analysis, ground, the needle and the high-voltage terminal are all the same node since the dc supply is an ac short. This shorts C_{NG} , puts C_{NL} and C_{LG} in parallel with the current source, and puts C_{PG} and C_{NP} in parallel with Z_D . This circuit is shown in Figure 3.3. Since C_{LP} , C_{LG} and probably C_{NL} are very small capacitors (possibly on the order of fF) due to the very small size of the LDR [36,37], the impedance of C_{LP} is very large compared to Z_D , which probably consists of a resistor of a few k Ω or less and at least several pF. Thus the current I splits between the three branches containing C_{LP} , C_{LG} and C_{NL} with the portion passing through C_{LP} given by

$$kI = \left(\frac{C_{LP}}{C_{LP} + C_{LG} + C_{NL}} \right) I. \quad (3.1)$$

This is the current available for detection (detectable current), and the portion of

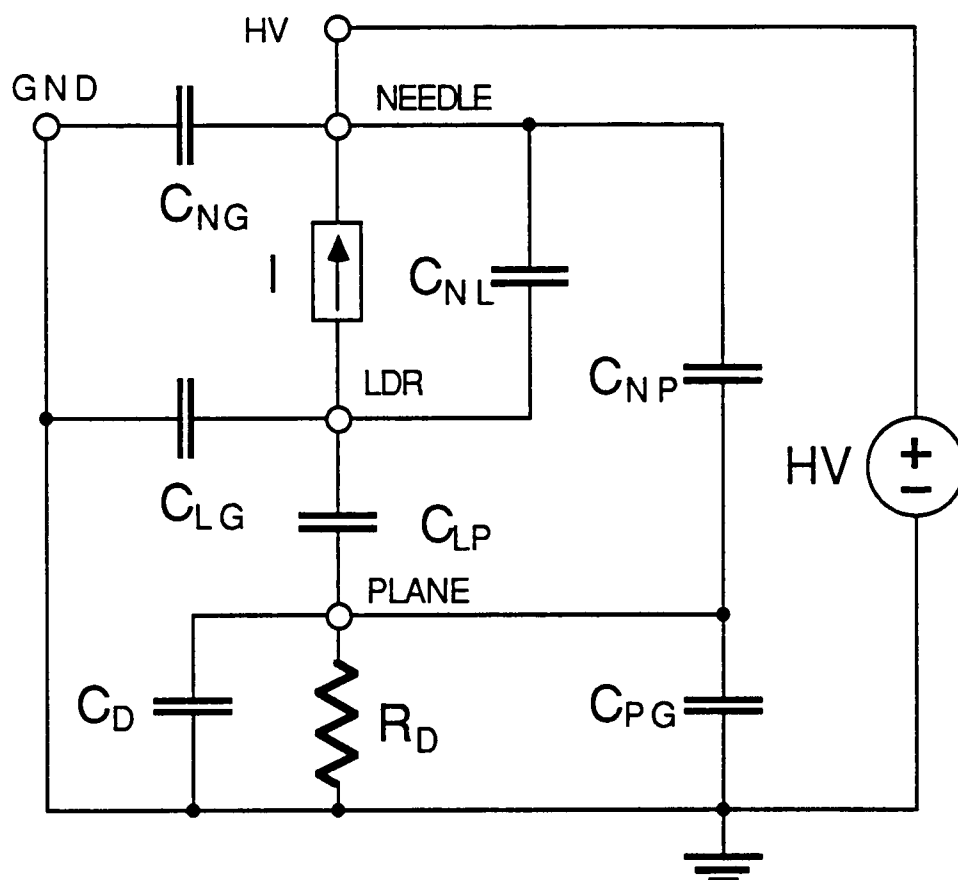


Figure 3.2: Circuit model of prebreakdown current generation and detection with high voltage applied to needle.

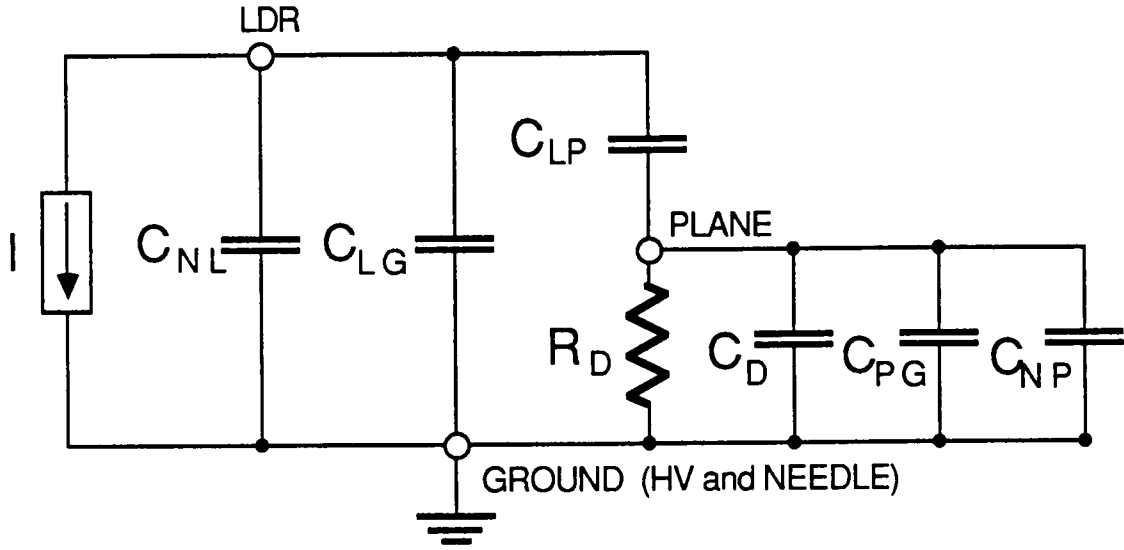


Figure 3.3: AC circuit model of prebreakdown current generation and detection with high voltage applied to needle.

the detectable current passing through Z_D is the detected current. The detected current is approximately the same as the detectable current, depending upon the impedances of C_{PG} and C_{NP} . Thus the detected current is a fraction of the current I injected into the LDR, at least up to frequencies where the stray capacitances become important. A model for the current flowing in Z_D , C_{PG} and C_{NP} is given in Figure 3.4.

Unfortunately, the values of C_{LP} , C_{LG} and C_{NL} are not known, and in fact, change as the LDR grows. The result is that the measured current is an unknown fraction of the actual current and not even a constant fraction at that. However, the magnitudes of these capacitors can be estimated by approximating the LDR as a spherical shell surrounding a spherical electrode with the same radius as

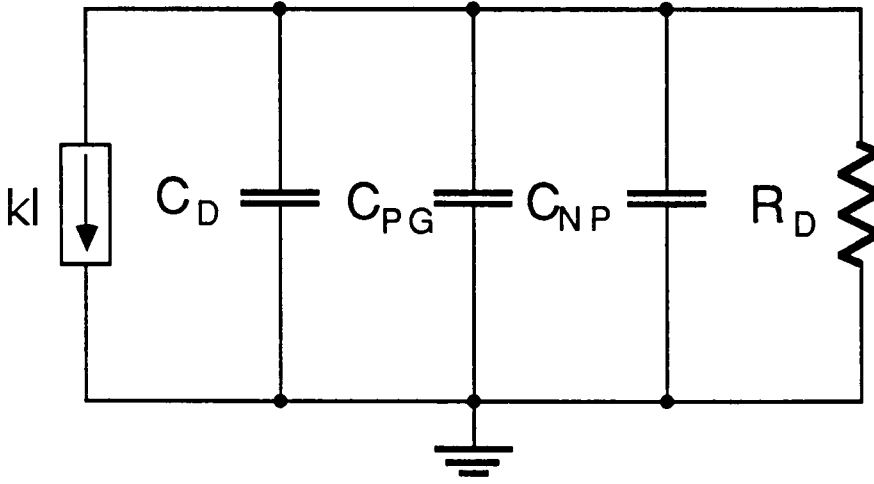


Figure 3.4: AC circuit model of prebreakdown current available for detection.

the needle tip. This results in C_{NL} being described by a spherical capacitor with value

$$C_{NL} = \frac{4\pi\epsilon}{1/r_1 - 1/r_2}, \quad (3.2)$$

where ϵ is the dielectric permittivity inside the LDR, r_1 is the tip radius and r_2 is the LDR radius. This capacitance approaches infinity as r_2 approaches r_1 and drops to $4\pi\epsilon r_1$ as r_2 becomes large. For typical tip radius of $3 \mu\text{m}$ and an assumed dielectric constant of $2\epsilon_0$ ($\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$) inside the LDR and a LDR radius of only $4 \mu\text{m}$, C_{NL} is only 2.6 fF , and this drops to 0.67 fF as the radius r_2 becomes large.

Although neither the plane and the LDR nor the cell and the LDR really form a spherical capacitor, that model is still useful for estimating those capacitances. In the case of the cell wall, which is ground, it nearly surrounds the LDR at a

distance much larger than the LDR radius. Thus, the capacitance C_{LG} would be approximately $4\pi\epsilon r_2$ if the plane were not present. The plane is much closer to the LDR, but still far away compared to r_2 . The plane does not completely surround the LDR, so the capacitance C_{LP} is less than $4\pi\epsilon r_2$, and since the plane shields the cell wall from the LDR, the capacitance C_{LG} is also less than $4\pi\epsilon r_2$. The magnitudes of C_{LG} and C_{LP} depend upon r_2 , but their relative sizes depend only on the exact geometry of the test cell as long as the LDR is small compared to the gap. For a relatively large LDR with radius of $100\text{ }\mu\text{m}$ and the dielectric constant of hexane, which is $1.9\epsilon_0$, C_{LP} and C_{LG} are bounded by 21 fF and would be less for a smaller LDR.

Several conclusions can be drawn from the estimates of the values of C_{LP} , C_{LG} and C_{NL} . First, Eq. 3.1 is valid since the impedances of C_{LP} , C_{LG} and C_{NL} are much greater than Z_D . Secondly, C_{LP} and C_{LG} are greater than C_{NL} except for the case of very small LDR's ($>10\text{ }\mu\text{m}$), and therefore C_{NL} has a significant effect on k when the LDR is very small. Finally, since the plane is much closer than the cell wall is to the LDR, but the wall is much larger, the capacitances C_{LP} and C_{LG} must be roughly equivalent, and therefore k must be on the order of $1/2$ and constant during LDR growth except when the LDR is very small.

3.3 Test Cell with Improved Current Detection

Re-examination of Figure 3.1 reveals that only part of field lines passing through the LDR link the plane. This translates to the current division between C_{LP} and C_{LG} in the ac circuit model (Figure 3.3.) Basically part of the current flows through the plane and part through the cell wall. This observation led to the idea of detecting the current flowing through the needle, in which case there would be no current division. This approach is also more appealing since the measured current is the current flowing through the stressing electrode and not an image current. One small drawback is that the needle potential should be near ground to avoid floating the preamplifier. If the needle is grounded (through Z_D , the preamp input,) the same electric field can be produced near the needle tip by requiring the plane to be a positive high voltage with respect to ground. (The needle is still negative with respect to the plane.) A physical picture of this arrangement is shown in Figure 3.5. The electric field is not the same everywhere, but it is the same in the high-field region that produces the LDR.

The same approach used in Section 3.2 can be used again to develop a circuit model describing the arrangement shown in Figure 3.5. The same five nodes (HV, needle, LDR, plane and ground) are used, but this time the HV node and the plane are connected together. The resulting model is shown in Figure 3.6. No new capacitors are needed, since the HV node was previously connected to

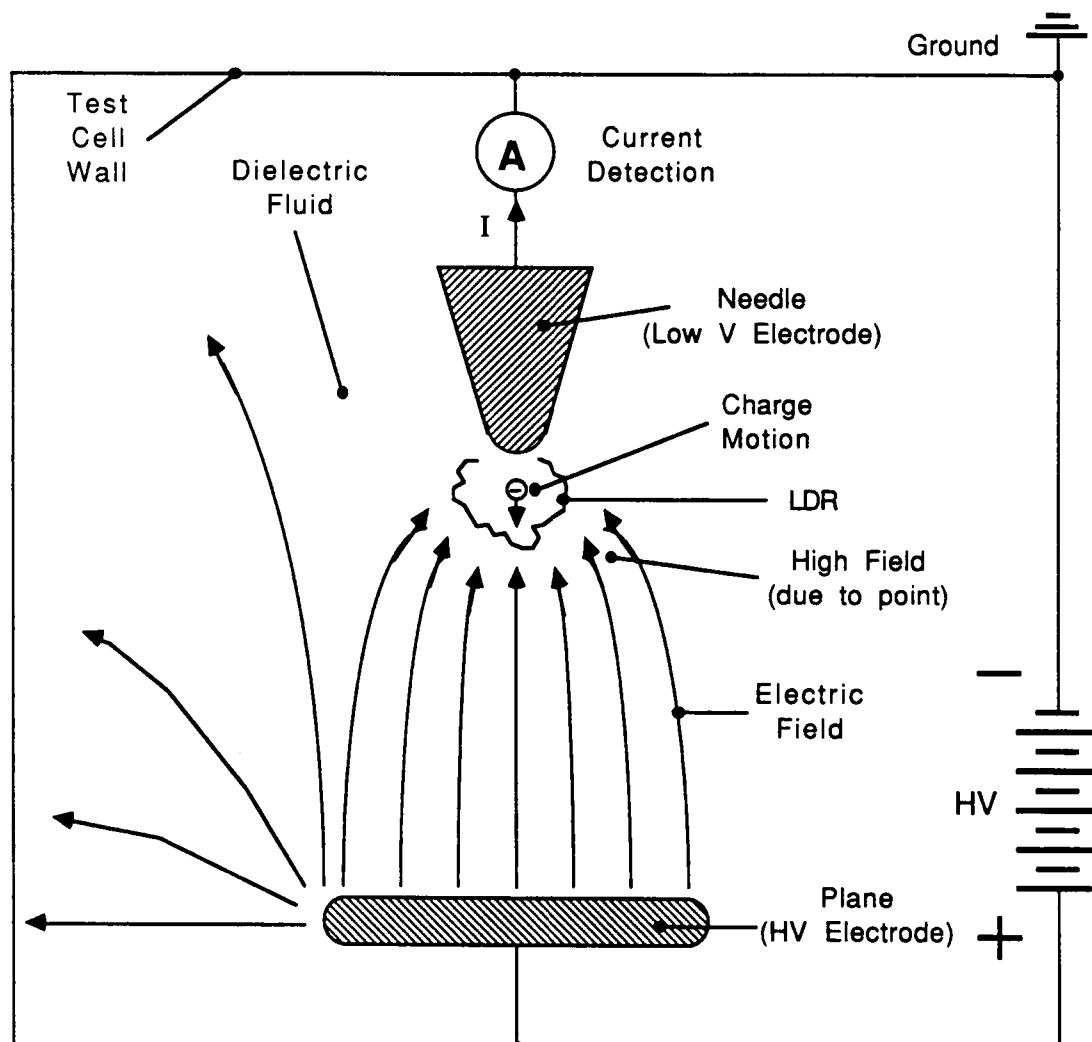


Figure 3.5: Physical model of prebreakdown current generation and detection with high voltage applied to plane.

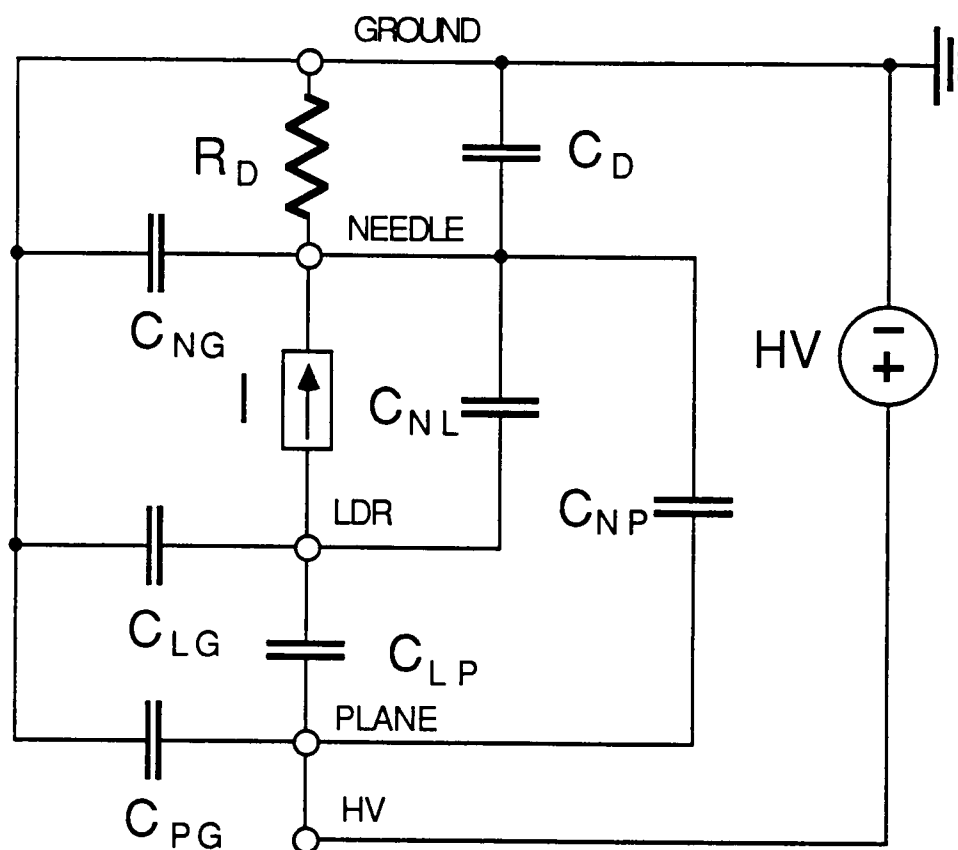


Figure 3.6: Circuit model of prebreakdown current generation and detection with high voltage applied to plane.

the ground node and had no capacitors connected. Once again an ac model can be developed by shorting the dc supply and rearranging the circuit so that the ground, HV and plane nodes are all the same point. The resulting circuit (Figure 3.7) is very similar to Figure 3.3, except that C_{LP} and C_{LG} are in series with I and C_{NL} instead of in parallel and C_{NG} has replaced C_{PG} .

Since C_{LP} , C_{LG} and C_{NL} are very small capacitors, their impedances are much greater than that due to Z_D , C_{NG} and C_{NP} . Therefore a simple model for the current available to be detected can be developed by noting that the current splits between the branches containing C_{LP} , C_{LG} and C_{NL} . The portion of the current that flows through C_{LP} and C_{LG} also flows through Z_D , C_{NG} and C_{NP} , and is given by

$$pI = \left(\frac{C_{LP} + C_{LG}}{C_{LP} + C_{LG} + C_{NL}} \right) I. \quad (3.3)$$

This is approximately the current passing through Z_D , depending upon the impedance due to C_{NG} and C_{NP} . The simplified model is shown in Figure 3.8.

Equation 3.3 and Figure 3.8 reveal the advantages of this configuration for the test cell. The factor p is considerably greater than the factor k of Equation 3.1, and in fact since C_{NL} is much less than the sum of C_{LP} and C_{LG} except for the smallest LDR's, p is nearly unity for most cases, which means that most all of the current is collected with this arrangement. Another advantage is the lower capacitance in parallel with R_D . C_{NG} is probably smaller than C_{PG} since the needle is much smaller than the plane. Low capacitance in parallel with R_D is needed to obtain a wide bandwidth and a high signal-to-noise ratio [39].

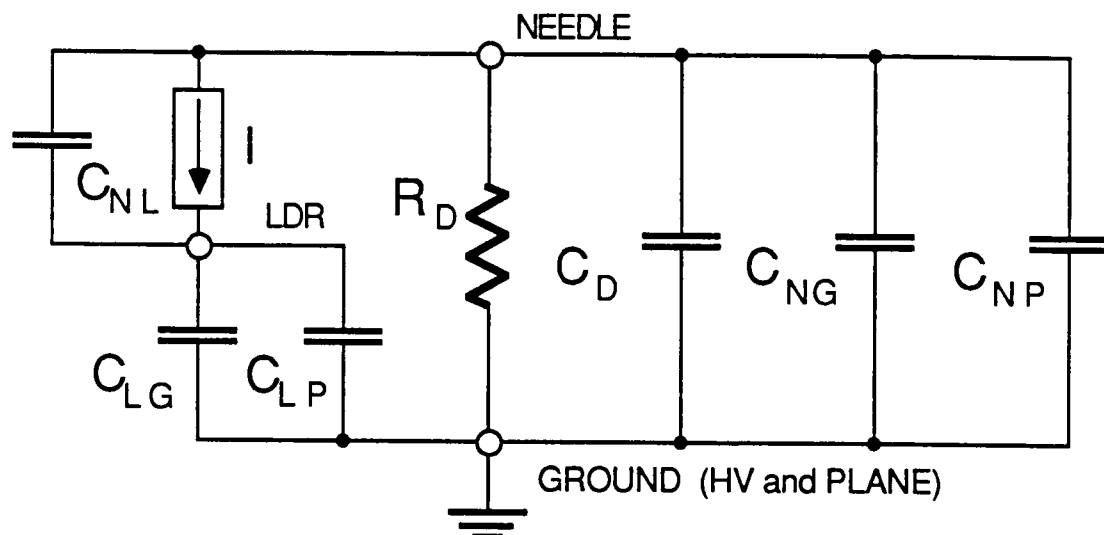


Figure 3.7: AC circuit model of prebreakdown current generation and detection with high voltage applied to plane.

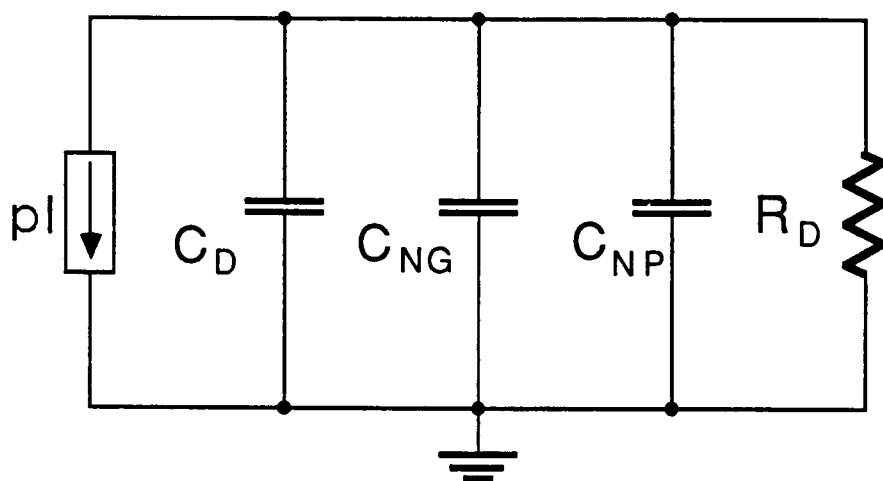


Figure 3.8: AC circuit model of prebreakdown current available for detection using improved configuration.

3.4 Test Cell Design

A test cell was designed using the improved configuration illustrated in Figure 3.5. The main thrust of the design was to produce a cell similar to that used in earlier experiments [26,27,28,36], but with very low stray capacitance (C_{NG} and C_{NP}). It was decided that constructing the cell from nonconductive plastic instead of metal would result in lower capacitance to ground since ground would be somewhere beyond the cell wall. A plastic cell has other advantages. First, feedthroughs do not need to be insulated since the cell wall serves as an insulator. It was found in earlier metal cells that BNC-type feedthroughs could contribute 5 pF or more, and even small feedthroughs with ceramic bushings had 2 pF or more. Of course the feedthrough used with the plastic cell will have some capacitance to ground, but it should be small due to the larger distance involved. Another advantage of plastic is its ease of machining.

The cell's mechanical layout was basically the same as one designed by E. F. Kelley at the National Institute of Standards and Technology (NIST) [40], with the exception of the signal feedthrough, which was redesigned to occupy less space making direct connection of the signal feedthrough to the preamp possible. This eliminates cabling between the preamp and the cell, thus avoiding another source of capacitance shunting the signal. The cell is shown in cross section in Figure 3.9. It provides HV and signal feedthroughs and four viewports which are useful for optical observations using photomultipliers, microscopes, etc. The

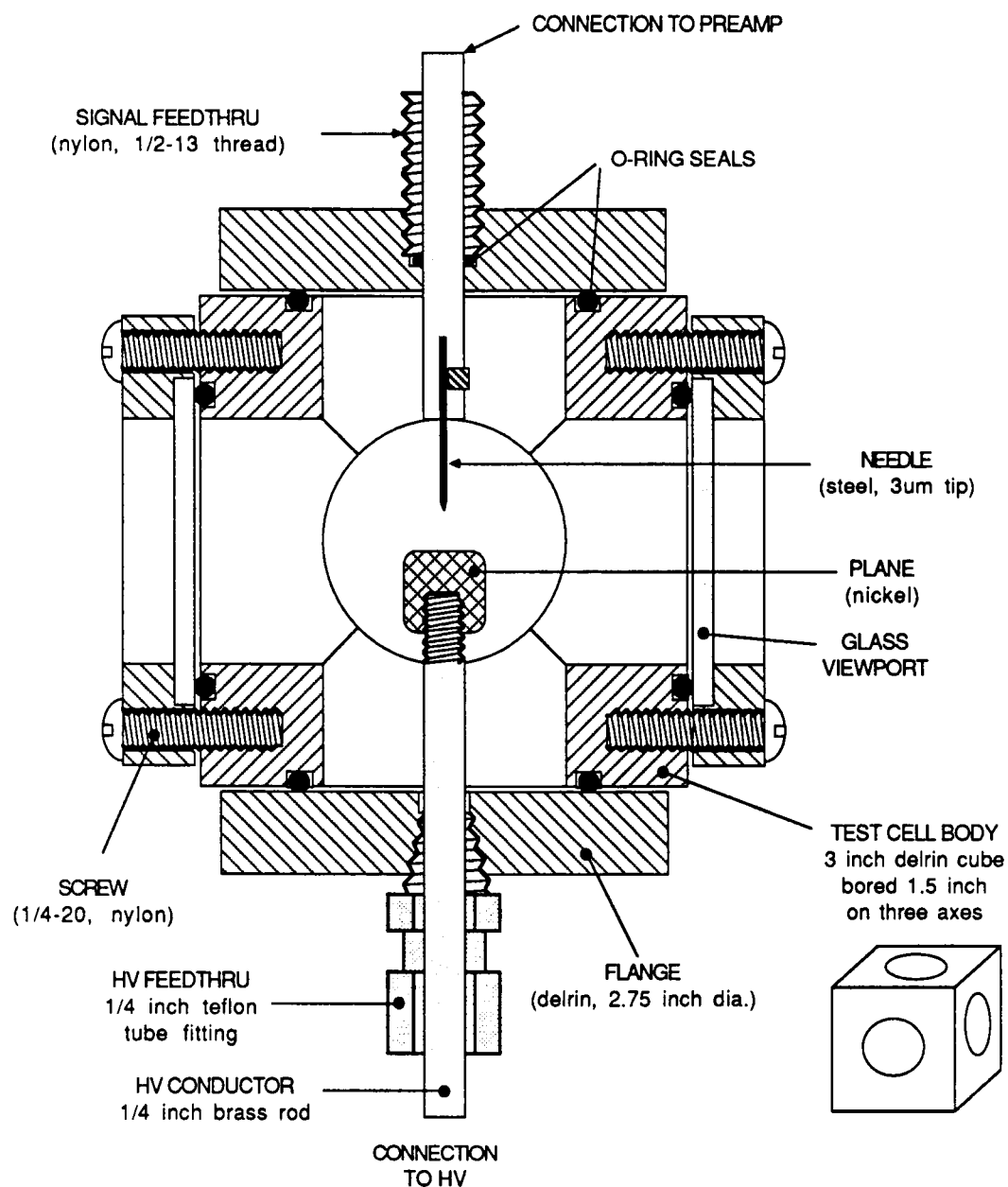


Figure 3.9: Cross section of test cell.

sum of the capacitances C_{NG} and C_{NP} was measured to be 4 pF, when the cell was filled with hexane.

For the circuit models given in Figures 3.6, 3.7 and 3.8 to apply, the high voltage supply needs to be an ac short at the signal frequencies of interest. In reality, it is a high-quality dc supply fed through an RC filter [41]. The series resistance is very high (50 M Ω) to limit the current in the event of a breakdown, but the capacitance is large enough (2350 pF) to supply the charge of a typical prebreakdown burst of pulses without the voltage applied to the HV electrode changing significantly. For example, a burst lasting 10 μ s with an average current of 100 μ A would reduce the HV by less than 0.5 V out of 10 kV or more. The impedance of the HV filter capacitor is only 7 Ω at 10 MHz and less at higher frequencies, and although its impedance is greater at low frequencies, it is a much larger capacitor (and lower impedance at a given frequency) than any of the others in the model. Therefore, the replacement of the HV supply with a short circuit in developing Figure 3.7 from Figure 3.6 was justified.

The various circuit models assume that the plane and the high-voltage electrode are shorted together. In actuality, there is some impedance between these nodes. In order to keep the inductance as low as possible, the plane electrode is connected to the power supply filter capacitor with a 1/4 inch diameter brass rod. Low inductance paths are also used to connect the power supply filter and preamp grounds together. These considerations were necessary so that the model for current detection (a current source in parallel with stray capacitance

and the detection impedance) would be a valid one.

CHAPTER 4

SELECTION OF A PREAMPLIFIER CONFIGURATION

4.1 Possible Preamplifier Configurations

Many types of preamplifiers could conceivably be used to amplify prebreak-down currents, however, a feedback preamplifier is preferred over a non-feedback type due to greater gain stability and linearity over a wide range of signal amplitudes. Also, an amplifier which directly outputs a voltage proportional to the input current is preferred for its simplicity over one which integrates or otherwise shapes the current signal and then requires additional circuitry to obtain a signal proportional to the input current. Therefore, the possibilities for the current preamplifier are narrowed to the two general feedback amplifier configurations which are considered in this dissertation. One configuration is voltage-sensitive and actually amplifies the voltage developed across a detection impedance (actually a resistance) by the input current. The other configuration is current-sensitive and functions as an active detection impedance. The merits and drawbacks of each are considered, and the one considered most suitable is selected for further development.

The two feedback amplifier configurations, voltage-sensitive and current-sensitive, are illustrated in Figure 4.1. An operational amplifier (op-amp) symbol represents the bulk of each amplifier excluding the detection impedance and feedback components. An actual preamplifier would probably not use an ordinary op-amp as shown here due to noise and bandwidth considerations, but the op-amp concept is very useful in understanding the operation of each configuration. The current source labeled I_{signal} represents the prebreakdown currents, and the sources i_n^+ , i_n^- and e_n represent the equivalent input noise current and voltage of the portion of the preamp represented by the op-amp.

Using an ideal op-amp model (infinite gain, infinite input impedance and zero output impedance), the midband gain of the voltage sensitive preamp is found to be

$$\frac{V_o}{I_{signal}} = R_D \left(1 + \frac{R_F}{R_1} \right). \quad (4.1)$$

The bandwidth of this circuit may be limited by R_D and C_S or by a small capacitor in parallel with the feedback resistor. This design has several advantages. It has been widely used for low-noise nuclear instrumentation and the results of many design efforts are available. It can be designed to give a very large bandwidth, if that is needed. Its main disadvantage for this application is that the overall bandwidth is limited by the parallel combination of C_S and R_D , and the low value of R_D required for a large bandwidth may limit the noise performance.

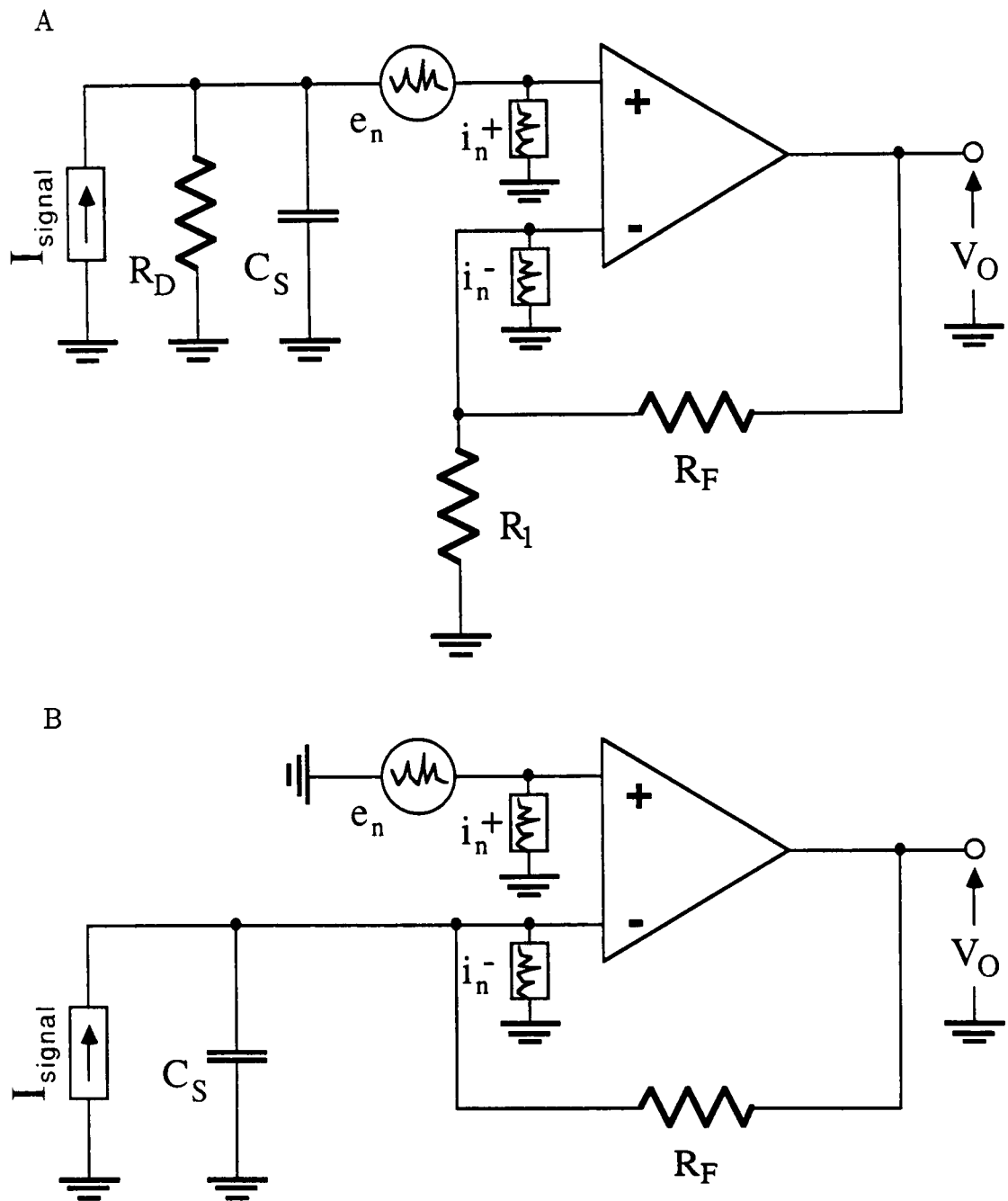


Figure 4.1: Preamplifier configurations. (A) Voltage-sensitive. (B) Current-sensitive.

The current-sensitive or transimpedance preamp has midband gain given by

$$\frac{v_o}{i_{signal}} = -R_F, \quad (4.2)$$

so it is obvious that R_F functions as a detection impedance. Thus for the transimpedance preamp, $R_F = R_D$. The major advantage of the transimpedance preamp over the voltage-sensitive preamp is that the bandwidth is not limited by C_S and R_D because of the location of R_D . A larger value of R_D could be used to give the same bandwidth and lower noise. A major disadvantage of this design is that it is difficult to produce a large bandwidth, stable feedback amplifier due to the pole introduced into the loop transmission by R_D and C_S . A stable design would probably require a capacitor in parallel with R_D which would limit the bandwidth. Another problem with this design is that the magnitude of the transfer function for the e_n source increases at high frequencies and produces a significant amount of high frequency noise.

In choosing a preamplifier configuration, the designer must consider the stability of the feedback loop, the closed-loop system characteristics such as bandwidth, risetime and overshoot, and the total noise characteristics of the preamplifier. The noise performance of each configuration depends upon e_{ni} and i_{ni} as well as R_D . An expression for the total noise power output is complicated because e_{ni} , i_{ni} and the noise power gain are all functions of frequency and depend upon the implementation of the gain block represented here by an op-amp. A direct comparison of the two configurations is not possible since the gain block would be optimized differently for each case.

4.2 The Transimpedance Amplifier

The transimpedance amplifier was presented and its operation was briefly described in Section 4.1. In this section, a general model for the transimpedance amplifier is developed and then used in two types of analysis. An algebraic analysis is used to obtain some general results, and then computer-aided-design (CAD) tools (whose development is described here) is used for more detailed analysis. References describing similar preamplifiers are reviewed, and this information is used along with results from the analysis to estimate the performance of a transimpedance amplifier optimized for this application.

4.2.1 Development of a General Model for the Transimpedance Amplifier

A model for the transimpedance amplifier is shown in Figure 4.2. This model was chosen because it contains enough features to simulate the most important characteristics of an actual amplifier (frequency response and stability, mainly), but still be simple enough to allow easy exploration of a wide range of parameters. The signal source is modeled as an ideal current source with capacitance C_S in parallel. The amplifier is modeled as a gain block, $A_{OL}(s)$, with a feedback network connected. The gain block has input resistance R_{IN} , input capacitance C_{IN} , zero output impedance and gain $A_{OL}(s)$ given by

$$A_{OL}(s) = \frac{A_0(1 + s/\omega_{Z1})(1 + s/\omega_{Z2})}{(1 + s/\omega_{P1})(1 + s/\omega_{P2})(1 + s/\omega_{P3})(1 + s/\omega_{P4})}. \quad (4.3)$$

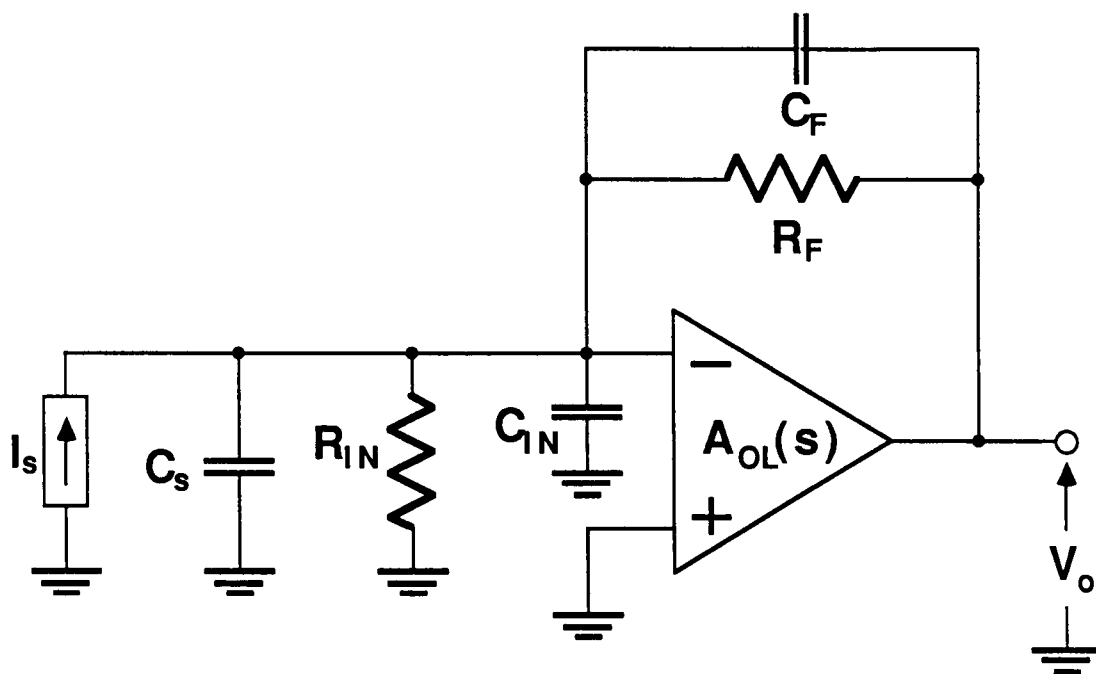


Figure 4.2: Simple transimpedance amplifier circuit model.

In this expression, A_0 is the dc gain, ω_{Z1} and ω_{Z2} are zeroes, and ω_{P1} , ω_{P2} , ω_{P3} and ω_{P4} are poles.

4.2.2 Frequency Response of the Transimpedance Amplifier General Model

For an ideal gain block, the closed-loop or transimpedance gain is

$$A_{CL}^{ideal}(s) = \frac{V_o}{I_s} = \frac{-R_F}{1 + R_F C_F s}. \quad (4.4)$$

Letting $\omega_F = 1/(R_F C_F)$ results in

$$A_{CL}^{ideal}(s) = \frac{-R_F}{1 + s/\omega_F}. \quad (4.5)$$

Thus the dc gain is simply R_F , and the closed-loop response has a single pole which can be set by adjusting C_F (once the gain is set.) The derivation of the gain equation is given in Appendix A.

The actual closed-loop gain can be found from $A_{CL}^{ideal}(s)$ and the loop transmission, $T(s)$, which is given by

$$T(s) = -A_{OL}(s) \left(\frac{R_{IF}}{R_F} \right) \left(\frac{1 + s/\omega_F}{1 + s/\omega_{IF}} \right), \quad (4.6)$$

where $\omega_F = 1/(R_F C_F)$,

$$R_{IF} = R_{IN} \parallel R_F = \frac{R_{IN} R_F}{R_{IN} + R_F}, \quad (4.7)$$

$C_{IF} = C_F + C_S + C_{IN}$, and $\omega_{IF} = 1/(R_{IF} C_{IF})$

Once $T(s)$ is known, the actual closed loop gain is found from

$$A_{CL}(s) = A_{CL}^{ideal}(s) \left(\frac{-T(s)}{1 - T(s)} \right). \quad (4.8)$$

By evaluating this expression for $s = j\omega$, the frequency response is found.

4.2.3 Noise Analysis of the Transimpedance Amplifier General Model

The model for the transimpedance amplifier (Figure 4.2) can be extended so the noise behavior is simulated as well as the frequency response. Since the amplifier was modeled as a gain block and several passive components, it would be reasonable to extend the model by adding sources to represent the random noise due to each component. The contributions of all noise sources inside the gain block can be referred to the input and represented by three sources as shown in Figure 4.3 – this is patterned after a general noise model for an op-amp [42]. As a result, the noise due to the gain block is represented by a voltage source and two current sources. The thermal noise of the feedback resistor, R_F , is represented by another source, and as no noise is generated by capacitors, no additional sources are needed. Thus, only four sources are needed to represent the random noise generated by the transimpedance amplifier. The noise model for the transimpedance amplifier is shown in Figure 4.4.

The noise sources are described by their power spectral densities. The symbol $S(f)$ is used to indicate a power spectral density, and subscripts will be used to indicate the type and location. The subscripts i and o will be used to indicate that a power spectral density is referred to the input or the output, respectively. The term power spectral density is used here in the sense that the noise source

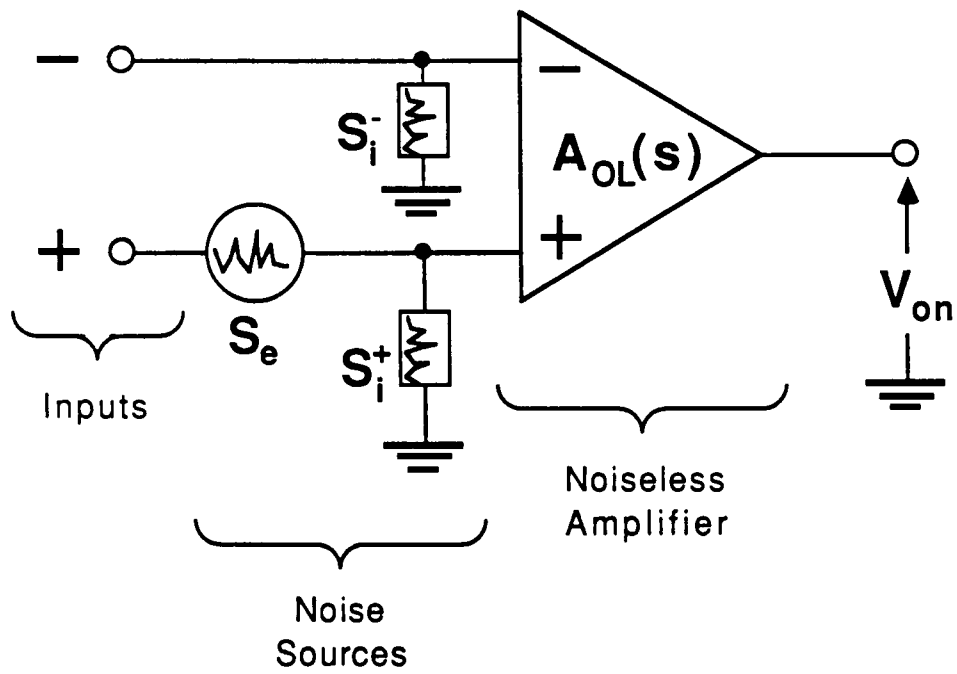


Figure 4.3: Noise model for gain block.

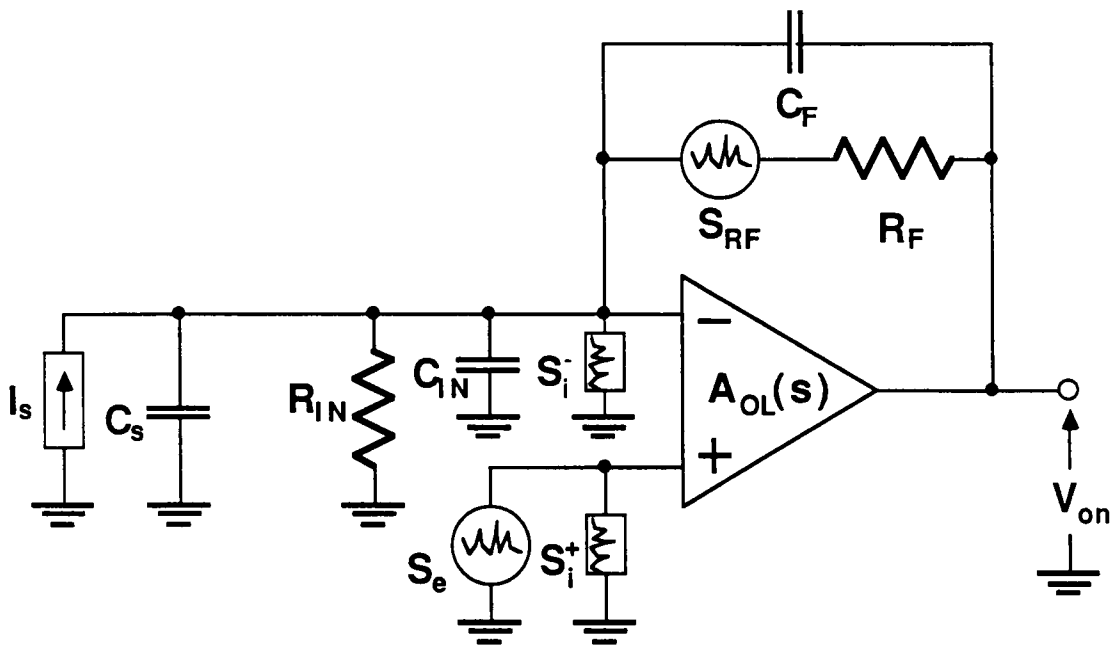


Figure 4.4: Simple transimpedance amplifier noise model.

described by $S(f)$ would produce that power when loaded by 1Ω .

$S_{RF}(f)$ is the power spectral density due to R_F and is given by

$$S_{RF}(f) = 4kTR_F, \quad (4.9)$$

where k is Boltzmann's constant and T is the absolute temperature in Kelvins.

This source has units of V^2/Hz when R_F is given in ohms.

$S_e(f)$ is the power spectral density of the equivalent input noise voltage source for the gain block. The frequency dependence of this source is possibly quite complicated, but for the sake of obtaining a simple model, only the most probable and significant terms will be included. The model for this density is

$$S_e(f) = k_{e1} + \frac{k_{e2}}{f} + \frac{k_{e3}}{f^2}. \quad (4.10)$$

This source also has units of V^2/Hz , and k_{e1} , k_{e2} and k_{e3} are constants which depend upon the gain block characteristics.

$S_i^-(f)$ is the power spectral density of the equivalent input noise current source for the gain block inverting input. Like S_e , the frequency dependence of this source may be quite complicated, but for the sake of obtaining a simple model, only the most probable and significant terms will be included. The model for this density is

$$S_i^-(f) = k_{i1}^- + \frac{k_{i2}}{f} + \frac{k_{i3}}{f^2}. \quad (4.11)$$

This source has units of A^2/Hz , and k_{i1} , k_{i2} and k_{i3} are constants which depend upon the gain block characteristics.

$S_i^+(f)$ is the power spectral density of the equivalent input noise current source for the gain block non-inverting input. The model for this density is the same as for S_i^- , and is given by

$$S_i^+(f) = k_{i4}^- + \frac{k_{i5}}{f} + \frac{k_{i6}}{f^2}. \quad (4.12)$$

This source also has units of A^2/Hz , and k_{i4} , k_{i5} and k_{i6} are constants which depend upon the gain block characteristics.

The output noise power spectral density of an amplifier fed by a noise source X with power spectral density $S_X(\omega)$ is

$$S_{noX}(\omega) = S_X(\omega)A_{CL}(\omega)A_{CL}^*(\omega) = S_X(\omega)|A_{CL}(\omega)|^2, \quad (4.13)$$

where $A_{CL}(\omega)$ is the actual closed-loop gain *for the noise source* and $A_{CL}^*(\omega)$ is its complex conjugate. For simplicity, the notation $A_{CL}(\omega)$ is used instead of $A_{CL}(j\omega)$ where $j\omega$ has been substituted for s . $A_{CL}(\omega)$ is related to the ideal closed-loop gain and the loop transmission (for the source) by

$$A_{CL}(\omega) = A_{CL}^{ideal}(\omega) \left(\frac{-T(\omega)}{1 - T(\omega)} \right). \quad (4.14)$$

It is important to note that $A_{CL}(\omega)$ may be different for each noise source and that these equations could be written in terms of f since $\omega = 2\pi f$.

The total output noise power spectral density of an amplifier is the sum of the output noise power spectral densities due to all the noise sources if the sources are not correlated. For all derivations in this dissertation, it is assumed that the noise sources are not correlated. Thus for the transimpedance amplifier

the total output noise power spectral density is

$$S_{no}(\omega) = S_{noe}(\omega) + S_{noi-}(\omega) + S_{noRF}(\omega), \quad (4.15)$$

since the noise source with spectrum S_i^+ is shorted out and contributes no noise power. Since only S_i^- has any effect on the preamplifier, only i and not $i-$ will be used as a subscript referring to this source.

To find the output noise power in a bandwidth ranging from f_1 to f_2 , the output power spectral density is integrated to give

$$\overline{e_{no}^2} = \int_{f_1}^{f_2} S_{no}(\omega) df. \quad (4.16)$$

$\overline{e_{no}^2}$ denotes the mean-squared output voltage, and therefore the root-mean-squared (rms) output noise voltage is given by $\sqrt{\overline{e_{no}^2}}$. For simplicity, the symbol e_{no} will be used instead of $\sqrt{\overline{e_{no}^2}}$. The rms output noise voltage e_{no} is a useful quantity as it allows the output noise level to be compared directly to the amplified signal.

To perform the integral given as Eq. 4.16, it is necessary to make the frequency units consistent. (ω has rad/s units, and f has units of Hz.) One way to do this is to convert df , f_1 and f_2 by dividing by 2π (rad/s)/Hz to give $\frac{d\omega}{2\pi}$, ω_1 and ω_2 , respectively, and then

$$\overline{e_{no}^2} = \int_{\omega_1}^{\omega_2} S_{no}(\omega) \frac{d\omega}{2\pi}. \quad (4.17)$$

Another way is to convert $S_{no}(\omega)$ to $S_{no}(f)$, which is accomplished by substituting $2\pi f$ for ω and converting all constants given in rad/s to their equivalent

in Hz. Although $S_{no}(f)$ and $S_{no}(\omega)$ are different functions, they represent the same physical phenomena, but just describe it in terms of different units.

Sometimes it is preferable to compare the strength of the noise to that of the signal which is to be amplified. If the noise of the amplifier is thought of as being due to a single noise source connected to the input (like the signal), a direct comparison is possible. The process is known as *referring* the noise to the input, and such a source is called an equivalent input noise source. This source is described in terms of its power spectral density which is given by

$$S_{ni}(\omega) = \frac{S_{no}(\omega)}{|A_{CL}(\omega)|^2}, \quad (4.18)$$

where $A_{CL}(\omega)$ is the actual closed-loop gain for the signal and $S_{no}(\omega)$ is the total output noise power spectral density. For the case of current amplification, the equivalent input noise power spectral density would have units of $[A^2/\text{Hz}]$.

For the purpose of comparing the amplifier noise to an in-band signal, it is more useful to think of the rms output noise voltage as being due to an input current rather than a power spectral density. Thus e_{no} can be referred to the input as an rms input noise current by dividing by the magnitude of the closed-loop gain, which in the case of a transimpedance amplifier is approximately R_F . Thus the rms input noise current is

$$i_{ni} = \frac{e_{no}}{R_F}. \quad (4.19)$$

Occasionally it is useful to think of the rms input noise current due only to one source. In that case, an additional subscript will be added to i_{ni} to indicate the

source of the noise.

For the noise source with spectrum S_e , the ideal closed-loop gain is

$$A_{CLe}^{ideal}(s) = 1 + \left(\frac{R_F}{R_{IF}} \right) \left(\frac{1 + s/\omega_{IN}}{1 + s/\omega_F} \right), \quad (4.20)$$

and the loop transmission is

$$T_e(s) = -A_{OL}(s) \left(\frac{R_{IF}}{R_F} \right) \left(\frac{1 + s/\omega_F}{1 + s/\omega_{IF}} \right), \quad (4.21)$$

which is the same as for the frequency response and stability calculations. The derivation for these equations is given in Appendix A. From these equations, the output power spectral density $S_{noe}(\omega)$ is calculated using the method just outlined and then integrated to give the mean-squared output voltage $\overline{e_{noe}^2}$. The rms input noise current due to S_e is then

$$i_{nie} = \frac{e_{noe}}{R_F}. \quad (4.22)$$

For the noise source with spectrum S_i^- , the ideal closed-loop gain is

$$A_{CLi}^{ideal}(s) = \frac{-R_F}{1 + s/\omega_F}, \quad (4.23)$$

and the loop transmission is the same as for S_e (Eq. 4.21.) These equations are also derived in Appendix A. As before, the output power spectral density $S_{noi}(\omega)$ is calculated and then integrated to give the mean-squared output voltage $\overline{e_{noi}^2}$, and then referred back to the input as an rms input noise current i_{nii} .

For the feedback resistor R_F , the noise power density spectrum is S_{RF} , and the ideal closed-loop gain is

$$A_{CLRF}^{ideal}(s) = \frac{1}{1 + s/\omega_F}. \quad (4.24)$$

The loop transmission is the same as for S_e (Eq. 4.21.) These equations are also derived in Appendix A. The output power spectral density $S_{noRF}(\omega)$ is calculated and integrated to give the mean-squared output voltage $\overline{e_{noRF}^2}$, and then referred back to the input as an rms input noise current i_{niRF} .

4.2.4 Algebraic Analysis of Transimpedance Amplifier Stability, Frequency and Transient Response

In this section, a simplified version of the transimpedance amplifier is analyzed algebraically. The stability of the amplifier against oscillations is considered, and the frequency and transient responses are examined in terms of the amplifier parameters (R_F , C_S , S_e , etc.). Simplifications and approximations are made and justified in order to produce results which are simple enough so useful features and trends are clear. As an example, an equation which stated that a bandwidth was inversely proportional to a capacitance would be very useful since it would be very obvious what would happen should that capacitance change, but another equation in which the bandwidth depended in a complicated way upon several poles, resistances and capacitances would be much less useful since it would not be obvious at all what would happen if one capacitance changed. The beauty of algebraic analysis is that a form which is readily understandable can usually be obtained if it is acceptable to reduce the exactness of the result. Many of the tradeoffs involved in designing an amplifier can be seen and decisions can be made by using the results obtained using simplifying approximations as design

guidelines.

The ideal closed-loop gain for the transimpedance amplifier is given by

$$A_{CL}^{ideal}(s) = \frac{-R_F}{1 + s/\omega_F}. \quad (4.25)$$

Although the actual closed-loop gain is somewhat different due to the effects of the loop transmission, examination of $A_{CL}(s)$ reveals that the midband current-to-voltage gain is R_F and that the bandwidth is ω_F (in rad/s). This frequency is an approximate upper limit for the bandwidth of the preamplifier, since the effect of the loop transmission will be to decrease the actual bandwidth or at most to increase it slightly. As a practical matter, any stray capacitance in parallel with R_F will limit the preamplifier bandwidth. For a 100-MHz bandwidth, even a value of C_F as low as 0.1 pF limits R_F to 16 k Ω .

To simplify the analysis of the transimpedance amplifier, the gain block can be considered more nearly ideal by restricting $A_{OL}(s)$ to have just one pole. Thus

$$A_{OL}(s) = \frac{A_0}{1 + s/\omega_{p1}}. \quad (4.26)$$

The loop transmission $T(s)$ (which was given by Eq. 4.6) simplifies to

$$T(s) = \left(\frac{R_{IF}}{R_F} \right) \left(\frac{-A_0}{1 + s/\omega_{p1}} \right) \left(\frac{1 + s/\omega_F}{1 + s/\omega_{IF}} \right), \quad (4.27)$$

where $\omega_F = 1/(R_F C_F)$ and $\omega_{IF} = 1/(R_{IF} C_{IF})$. The capacitance C_{IF} represents total capacitance at the input ($C_F + C_S + C_{IN}$) and R_{IF} is $R_F \parallel R_{IN}$. From this form, it is clear that the loop transmission has two poles and one zero and that the system is stable and may have ample phase margin depending upon

the location of the poles and the zero. Unfortunately, the only zero of $T(s)$ occurs at ω_F which is the (dominant) pole of the ideal closed-loop gain and is therefore greater than or approximately equal to the closed-loop bandwidth. Since ω_{IF} is less than ω_F (unless $R_{IN} \ll R_F$) and since it is unlikely that the gain block open-loop pole, ω_{p1} , would be greater than the closed-loop bandwidth (this would require a very large gain-bandwidth product), both poles likely occur at frequencies less than the zero. An example of a loop transmission with these constraints is shown in Figure 4.5. From this plot, it is clear that the phase margin can be reasonably large if the zero partially cancels one of the poles. The same would hold true for an amplifier with additional poles providing they occur at frequencies well above the loop transmission unity gain crossover frequency.

The actual closed-loop gain is found by combining $A_{CL}^{ideal}(s)$ and $T(s)$ according to Eq. 4.6. This gives

$$A_{CL}(s) = \left(\frac{-R_F}{1 + s/\omega_F} \right) \frac{\left(\frac{R_{IF}}{R_F} \right) \left(\frac{A_0}{1 + s/\omega_{p1}} \right) \left(\frac{1 + s/\omega_F}{1 + s/\omega_{IF}} \right)}{1 + \left(\frac{R_{IF}}{R_F} \right) \left(\frac{A_0}{1 + s/\omega_{p1}} \right) \left(\frac{1 + s/\omega_F}{1 + s/\omega_{IF}} \right)}. \quad (4.28)$$

Simplifying this expression gives

$$A_{CL}(s) = \frac{-R_F}{\frac{R_F(1 + s/\omega_{p1})(1 + s/\omega_{IF})}{A_0 R_{IF}} + (1 + s/\omega_F)}. \quad (4.29)$$

Expanding the denominator and letting

$$k = \frac{R_{IF}}{R_F} = \frac{R_{IN}}{R_{IN} + R_F}, \quad (4.30)$$

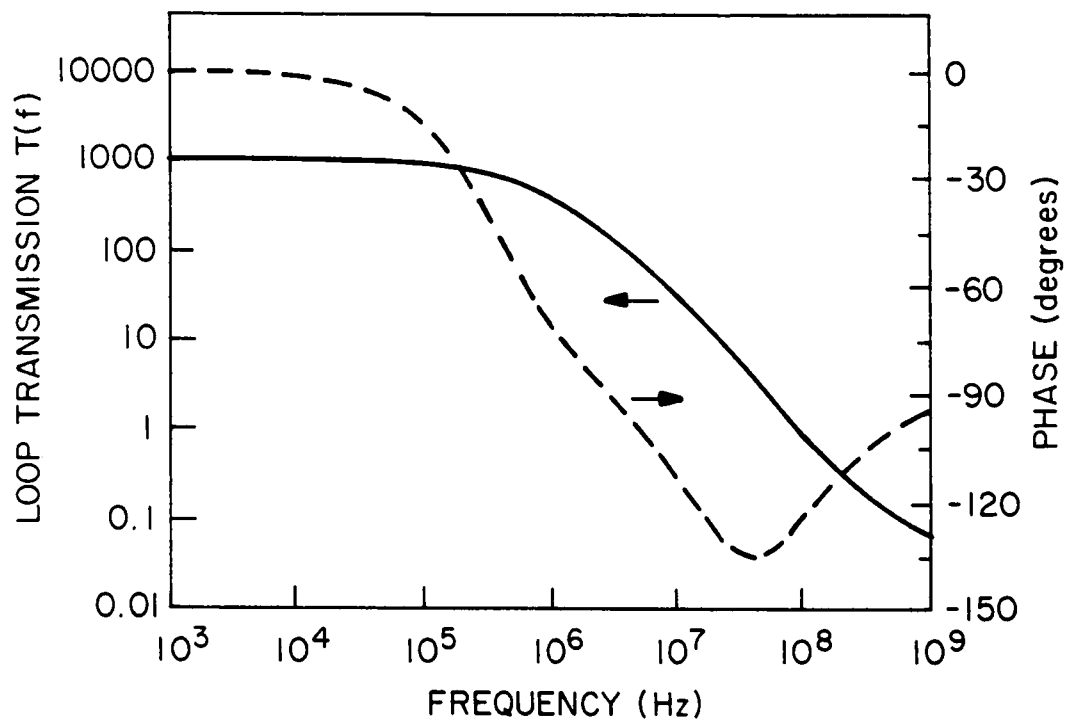


Figure 4.5: Loop transmission example.

gives

$$A_{CL}(s) = \frac{-R_f}{1 + \frac{1}{kA_0} + \frac{s/\omega_{p1}}{kA_0} + \frac{s/\omega_{IF}}{kA_0} + s/\omega_F + \frac{s^2/(\omega_{p1}\omega_{IF})}{A_0}}. \quad (4.31)$$

If kA_0 is assumed to be much greater than one (which is usually necessary for a reasonable midband loop transmission,) then the expression reduces to

$$A_{CL}(s) = \frac{-R_f}{1 + s \left(\frac{1}{kA_0\omega_{p1}} + \frac{1}{kA_0\omega_{IF}} + \frac{1}{\omega_F} \right) + s^2 \left(\frac{1}{kA_0\omega_{p1}\omega_{IF}} \right)}. \quad (4.32)$$

By letting $\omega_I = 1/(R_F C_{IF})$ and then substituting ω_I for $k\omega_{IF}$, $A_{CL}(s)$ becomes

$$A_{CL}(s) = \frac{-R_f}{1 + s \left(\frac{1}{kA_0\omega_{p1}} + \frac{1}{A_0\omega_I} + \frac{1}{\omega_F} \right) + s^2 \left(\frac{1}{A_0\omega_{p1}\omega_I} \right)}. \quad (4.33)$$

This closed-loop gain (Eq. 4.33) has the same form as the general second order system described by

$$A(s) = \frac{A_{dc}}{s^2/\omega_n^2 + 2\xi s/\omega_n + 1}, \quad (4.34)$$

which has dc gain A_{dc} , damping factor ξ and natural frequency ω_n . The frequency and step responses of this system are described in most texts on control systems and a good description applied to feedback amplifiers with second order closed-loop responses is given by Pierce and Paulus [43]. These analyses provide some very useful approximations for the closed-loop bandwidth, f_{BW} and the risetime, t_r in terms of ω_n and ξ , and although those results really only apply to second order systems, they are useful since many higher order systems can be approximated as second order. The bandwidth is given by

$$f_{BW} = \frac{\omega_n}{2\pi} \sqrt{1 - 2\xi^2 + \sqrt{2(2\xi^4 - 2\xi^2 + 1)}} \simeq \frac{\omega_n}{2\pi}. \quad (4.35)$$

The approximation is accurate to $\sim 25\%$ for systems that are slightly underdamped, that is those having $0.9 > \xi > 0.5$. This corresponds to a situation where the value of ω_F is high enough so the loop transmission has a significant effect on the closed-loop. For lower values of ω_F , the system is overdamped, the closed-loop response is approximately the same as the ideal response, and the bandwidth is approximately $\omega_F/(2\pi)$.

For many wideband amplifier applications, a slightly underdamped response is a good compromise that gives fast response yet low overshoot. The risetime in response to a step input is approximately

$$t_r \simeq \frac{0.35}{f_{BW}}. \quad (4.36)$$

This expression is accurate to a few percent for the same slightly underdamped systems and for single pole systems as well. Less accurate ($\pm 25\%$) is the expression

$$t_r \simeq \frac{2.2}{\omega_n}. \quad (4.37)$$

Comparing $A_{CL}(s)$ (Eq. 4.33) to $A(s)$ (Eq. 4.34) results in

$$\omega_n = \sqrt{A_0 \omega_{p1} \omega_I}. \quad (4.38)$$

Thus the closed-loop bandwidth for the slightly-underdamped amplifier is

$$f_{BW} \simeq \frac{1}{2\pi} \sqrt{A_0 \omega_{p1} \omega_I}, \quad (4.39)$$

and the risetime is

$$t_r \simeq \frac{2.2}{\sqrt{A_0 \omega_{p1} \omega_I}}. \quad (4.40)$$

The risetime equation is a more general form of an equation derived by Millard [44] who analyzed a simple shunt-feedback amplifier with an FET cascode gain stage and obtained

$$t_r = 2.2 \sqrt{\frac{R_F C_{IF} C_c}{g_m}}, \quad (4.41)$$

where g_m is the FET transconductance and C_c is the capacitance loading the cascode output. That these two expressions are equivalent can be seen by taking the amplifier pole to be $\omega_{p1} = 1/(R_c C_c)$, where R_c is the resistance loading the cascode output, and substituting for C_c and for $R_F C_{IF}$ (which equals $1/\omega_I$ when R_{IN} is large) to give

$$t_r = 2.2 \sqrt{\frac{1}{g_m R_c \omega_{p1} \omega_I}}. \quad (4.42)$$

Since the gain of this stage is approximately $g_m R_c = A_0$, the equations for t_r are equivalent.

The expression for the risetime (Eq. 4.40) can be rewritten in terms of the gain-bandwidth product ($GBW = A_0 \omega_{p1}$, in rad/s) for the gain block, R_F and C_{IF} . This result is

$$t_r \simeq 2.2 \sqrt{\frac{R_F C_{IF}}{GBW}}. \quad (4.43)$$

This shows that a low risetime requires high GBW and low C_{IF} and R_F . Of course, sufficiently low C_F is required as well. Figure 4.6 is a plot of the gain-bandwidth required for $t_r = 3.5$ ns (100 MHz bandwidth) as a function of R_F and C_{IF} . For values of R_F greater than about 1 k Ω , C_F is small and $C_{IF} \simeq C_S + C_{IN}$.

For example, a high-input impedance amplifier with $R_F = 10$ k Ω , $C_S + C_{IN} = 20$ pF and $C_F = 0.1$ pF (thus $C_{IF} = 20.1$ pF) requires $GBW = 7.9 \times 10^{10}$ rad/s

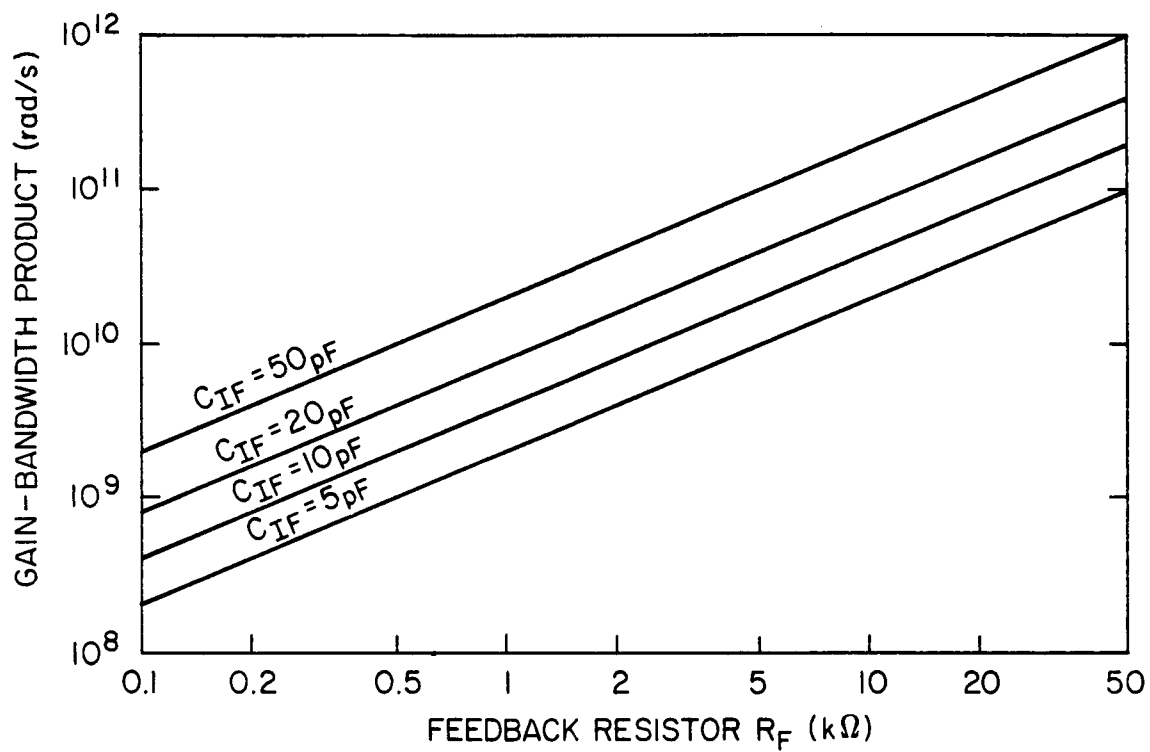


Figure 4.6: Gain-bandwidth required for $t_r = 3.5$ ns.

(12.6 GHz). This gain-bandwidth product could be realized with $A_0 = 500$ and a single 25-MHz pole, but other combinations of gain and open-loop bandwidth could satisfy that equation but not satisfy the requirement $A_0 \gg 1$. A much lower bandwidth and longer risetime results from $A_0 = 10$ and a 1.26-GHz pole, for example. For this case ($R_F = 10 \text{ k}\Omega$, $C_{IF} = 20.1 \text{ pF}$ and $A_0 = 10$) the dominant pole in the loop transmission is only 800 kHz (due to R_F and C_{IF}), and the loop transmission unity gain crossover frequency, f_{co} is only 8 MHz (due to the low value of A_0). This results in a closed-loop bandwidth not much greater than 8 MHz. This demonstrates that Eq. 4.43 does not apply if A_0 is not large enough.

The risetime is only part of the transient response. For fast signals, the next most important characteristic is probably the percent overshoot. (Overshoot in the step response corresponds to peaking in the frequency domain, so restricting the overshoot is analogous to requiring a flat frequency response for a second order system.) Analysis of the general second order system [43] reveals that the percent overshoot, M_{pt} , is

$$M_{pt} = 100 \exp \frac{-\pi\xi}{\sqrt{1-\xi^2}}. \quad (4.44)$$

To keep the overshoot less than 10%, the damping ratio needs to be greater than 0.6. The damping ratio is found by comparing $A_{CL}(s)$ (Eq. 4.33) to $A(s)$ (Eq. 4.34) once more. This results in

$$\xi = \frac{1}{2} \left(\frac{1}{kA_0\omega_{p1}} + \frac{1}{A_0\omega_I} + \frac{1}{\omega_F} \right) \sqrt{A_0\omega_{p1}\omega_I}. \quad (4.45)$$

This equation has too many terms for any relationships to be clear, and further simplification requires some knowledge of the relative sizes of $kA_0\omega_{p1}$, $A_0\omega_I$ and ω_F . It does indicate that if $\omega_f \gg A_0\omega_I$ and $\omega_f \gg kA_0\omega_{p1}$, it has little effect on ξ . On the other hand, if $\omega_f \ll A_0\omega_I$ and $\omega_f \ll kA_0\omega_{p1}$, ξ is large and the system is overdamped.

Another way to estimate the overshoot is to examine the loop transmission. Pierce and Paulus [43] give a useful table relating percent overshoot to the phase margin for a second order system, but these values were derived for an amplifier with resistive feedback, and the relations between the closed-loop and open-loop characteristics are modified by the addition of C_F . The transimpedance amplifier under consideration also has a second-order closed-loop response, so the equations and tables given by Pierce and involving closed-loop characteristics (ξ , ω_n , etc.) are correct for any C_F , but the relations involving open-loop characteristics (f_{co} and phase margin) are correct only for $C_F = 0$.

The effects of C_F on the relation between the open-loop and the closed-loop characteristics were examined by calculating values of the damping factor, phase margin, crossover frequency and closed-loop bandwidth for various combinations of A , ω_{p1} , ω_I and ω_F . To simplify the calculations and limit the possibilities somewhat, only the case of $R_{IN} \gg R_F$ was considered. For that case $k \simeq 1$ and $\omega_{IF} \simeq \omega_F$, and Eq. 4.45 simplifies to

$$\xi = \frac{1}{2} \left(\frac{1}{A_0\omega_{p1}} + \frac{1}{A_0\omega_I} + \frac{1}{\omega_F} \right) \sqrt{A_0\omega_{p1}\omega_I}, \quad (4.46)$$

while Eq. 4.33 reduces to

$$T(s) = \left(\frac{-A_0}{1 + s/\omega_{p1}} \right) \left(\frac{1 + s/\omega_F}{1 + s/\omega_I} \right). \quad (4.47)$$

Open-loop data were generated by solving $|T(f)| = 1$ to get the crossover frequency f_{co} , evaluating $T(f_{co})$ to get the phase and calculating the phase margin by subtracting the phase of $T(f_{co})$ from 180° . Closed-loop data were generated by using Eq. 4.46 to get ξ and then by using Equations 4.44 and 4.35 to calculate the overshoot and the closed-loop bandwidth, respectively. The results for the different cases plotted as a function of phase margin and ω_F normalized with respect to ω_n .

The damping factor ξ as a function of the phase margin is shown in Figure 4.7. The case of $\omega_F/\omega_n = \infty$, which corresponds to $C_F = 0$, is shown along with two other cases. For the case of $\omega_F/\omega_n = \infty$, the loop transmission dominates the closed-loop response, while for the case of $\omega_F/\omega_n = 1$, ω_F should dominate. The behavior of $\omega_F/\omega_n = 2$ should lie between the other cases. For all three cases, ξ varies in the same way with the phase margin when the phase margin is less than $\sim 60^\circ$. For phase margins greater than 60° , the cases $\omega_F/\omega_n = 2$ and $\omega_F/\omega_n = 1$ require a larger phase margin than the $\omega_F/\omega_n = \infty$ case to achieve the same damping factor.

The damping factor data were used to calculate the percent overshoot for the same three cases, and the results are shown in Figure 4.8. Once again, there is very little difference in the three cases for phase margins less than 60° . Above 60° , the overshoot is less than 10% for all cases, but a larger ($\sim 10^\circ$) phase

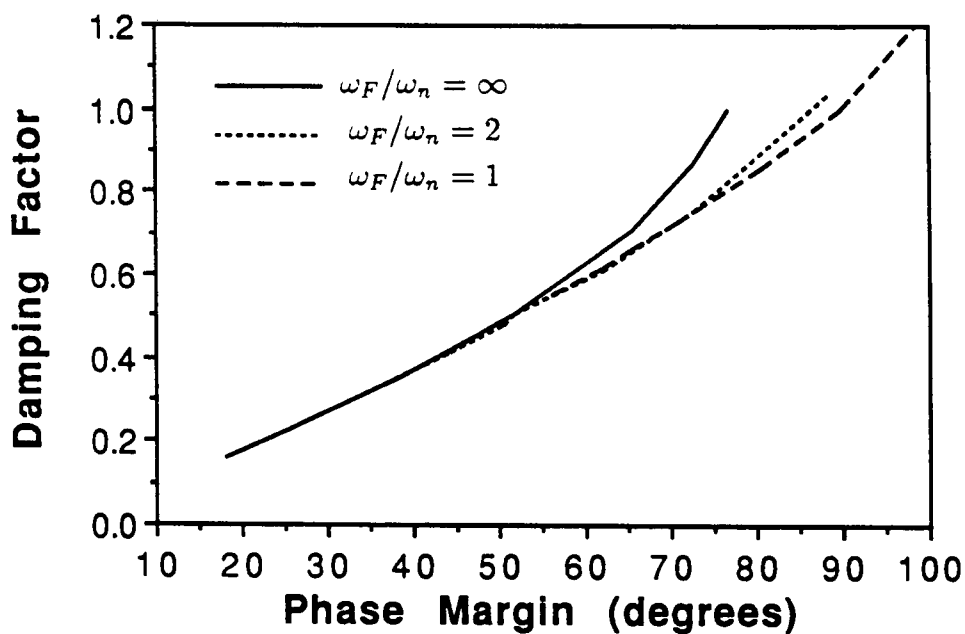


Figure 4.7: Damping factor as a function of phase margin.

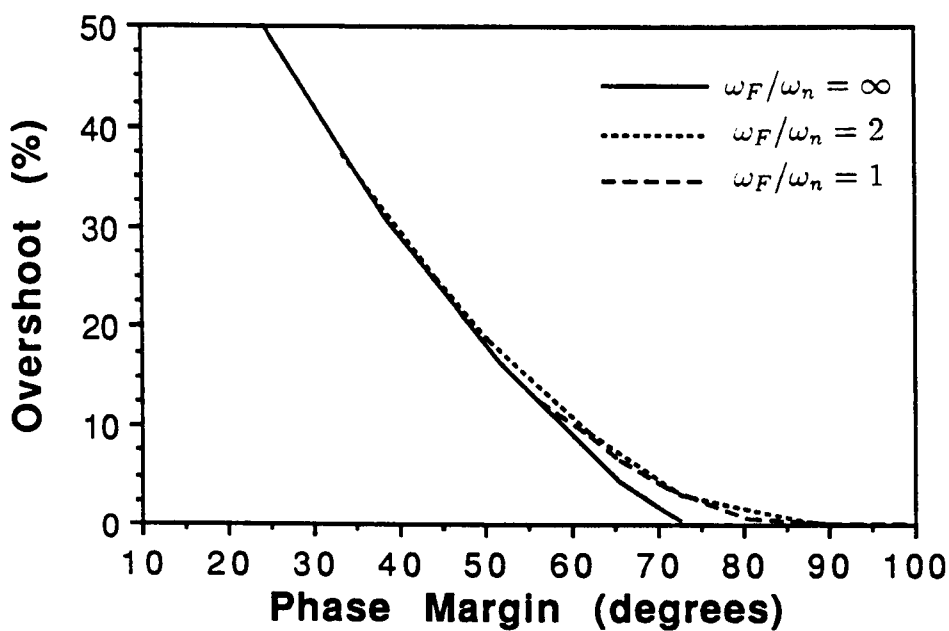


Figure 4.8: Overshoot as a function of phase margin.

margin is required to obtain the same overshoot for the cases of $\omega_F/\omega_n = 2$ and $\omega_F/\omega_n = 1$ compared to the case of $\omega_F/\omega_n = \infty$.

Just as the overshoot can be found from the phase margin, the closed-loop bandwidth can be found from the phase margin and f_{co} . A plot of the ω_{BW}/ω_{co} as a function of the phase margin and ω_F/ω_n is shown in Figure 4.9. For the case of $\omega_F/\omega_n = \infty$, which corresponds to $C_F = 0$, the closed-loop bandwidth can be approximated by

$$\omega_{BW} \simeq \frac{\pi}{2} \omega_{co}, \quad (4.48)$$

but for the other cases of ω_F/ω_n , the factor multiplying the bandwidth is less.

From Figure 4.9, one might conclude that increasing ω_F simply decreases ω_{BW} , but increasing ω_F also increases the phase margin and ω_{co} . An example of this is shown in Figure 4.10 for the case of $A_0 = 10^5$, $\omega_{p1} = 10$ and $\omega_I = 10^4$, which corresponds to $\omega_n = 10^5$. The figure shows that decreasing ω_f/ω_n from 100 to 1 only decreases the closed-loop bandwidth by 22% while increasing the ξ from 0.05 to 0.55. This lowers the overshoot from 85% to 13%, which is a considerable improvement and probably well worth the loss of bandwidth.

Several conclusions can be drawn from the algebraic analysis of the transimpedance amplifier. Stability of the amplifier requires any poles of the gain block in addition ω_{p1} to occur at very high frequencies. Additional phase shift due to these extra poles is critical since there are two poles (ω_{p1} due to the gain block and ω_{IF} due to R_F and $C_{IN} + C_S$) below the zero due to R_F and C_F , and the phase margin is less than 90° due to the zero and those two poles alone.

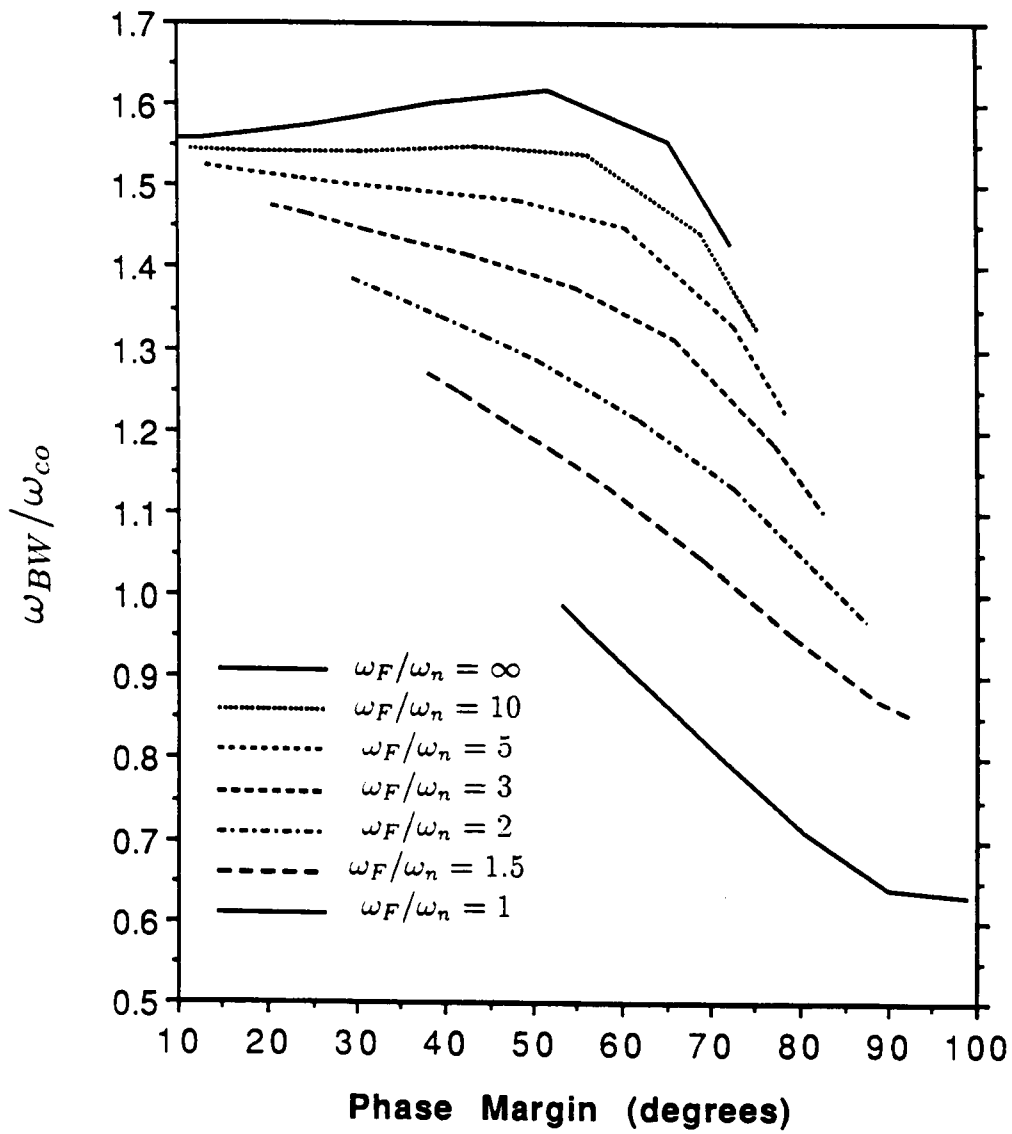


Figure 4.9: Closed-loop bandwidth as a function of phase margin and ω_F/ω_n .

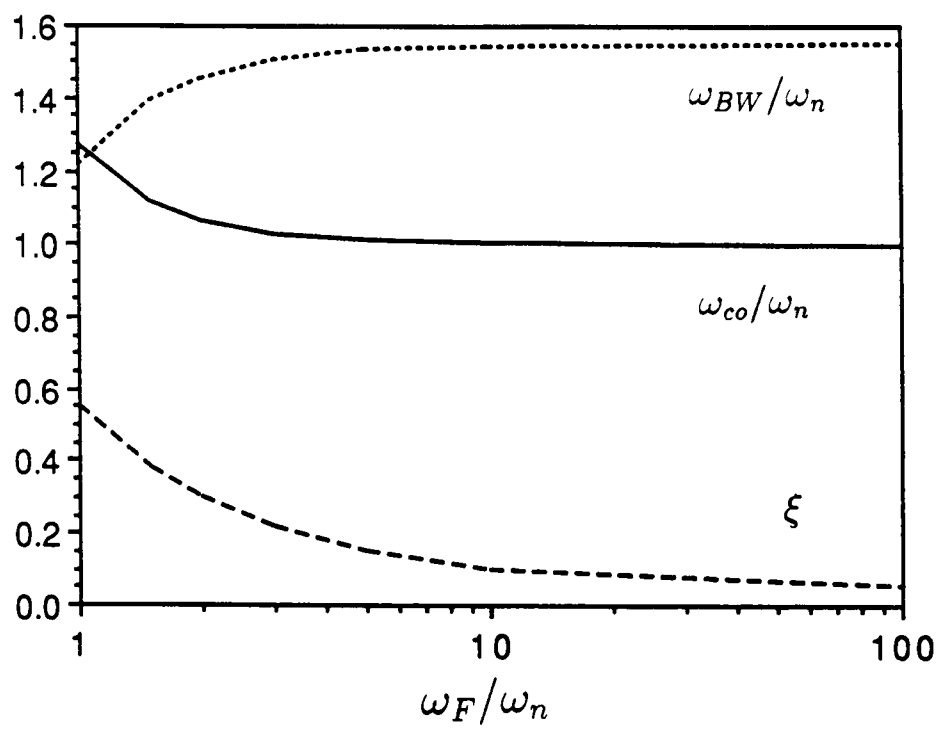


Figure 4.10: Bandwidth, crossover frequency and damping factor as a function of ω_F/ω_n .

Obtaining an acceptable phase margin and therefore low overshoot requires adjusting ω_F as low as possible while still obtaining sufficient bandwidth. A high bandwidth and therefore low t_r requires a high gain-bandwidth product, low C_{IF} , low R_F , yet since the lower limit on C_{IF} is set by the test cell and since noise considerations [39] indicate that high R_F is desired, even higher GBW is required.

One amplifier characteristic not considered so far is the settling time. The settling time is important when pulses are amplified. If the settling time is larger than the interval between pulses, then the response due to a pulse can distort succeeding pulses. Analysis of this effect was not attempted, but it was considered when the actual preamplifier was developed and tested.

4.2.5 Algebraic Analysis of Transimpedance Amplifier Noise Performance

To analyze the noise performance of the transimpedance amplifier without undue complexity, some more simplifications to the general model are in order. First, only the constant term of the input noise power spectral densities $S_e(f)$ and $S_i(f)$ will be considered. This is a reasonable restriction since the wideband noise performance is most important for the amplifiers under consideration while the $1/f$ noise contribution is usually only significant in the low frequency noise spectrum, at least for silicon transistors. Although the noise power density can be very great at low frequencies, the bandwidth involved is small and the

integrated contribution to the total noise power is probably small, too. Second, only the contributions of noise within and slightly above the amplifier bandwidth will be considered. Noise considerably above the amplifier bandwidth will be removed by the anti-aliasing filter before the signal is processed. Third, the ideal closed-loop noise gains will be used in order to give equations that are integrable. The actual spectral densities could be calculated from the actual closed-loop gains, but the form of the densities would probably preclude integration (except by numerical methods) to give the total noise. Also, the ideal closed-loop gain should be a good approximation of the actual gain for frequencies up to the bandwidth of the amplifier. This would probably overestimate the noise for higher frequencies, but that noise should be greatly reduced by the anti-aliasing filter. Several additional assumptions on amplifier parameters will be needed, but these are described and justified as they are made.

To find the total rms input noise current, the contributions to the output noise power due to S_e , S_i^- and S_{RF} are found and summed, and the total is referred back to the input using the method outlined in Section 4.2.3.

The noise source with spectrum S_e , has ideal closed-loop gain

$$A_{CLe}(s) = \left(1 + \frac{R_F}{R_{IN}}\right) \left(\frac{1 + s/\omega_{IF}}{1 + s/\omega_F}\right), \quad (4.49)$$

where $1/\omega_{IF} = R_{IF}C_{IF} = (R_F \parallel R_{IN})(C_F + C_S + C_{IN})$. (The superscript *ideal* is not used with A_{CL} since the gain is assumed to be the same as the ideal gain

for this analysis.) In terms of ω , the closed-loop gain is

$$A_{CL\epsilon}(\omega) = \left(1 + \frac{R_F}{R_{IN}}\right) \left(\frac{1 + j\omega/\omega_{IF}}{1 + j\omega/\omega_F}\right). \quad (4.50)$$

Using this gain in Eq. 4.13 gives the output noise power spectral density

$$S_{noe}(\omega) = S_e A_{CL\epsilon}(\omega) A_{CL\epsilon}^*(\omega) = S_e \left(1 + \frac{R_F}{R_{IN}}\right)^2 \left(\frac{1 + \omega^2/\omega_{IF}^2}{1 + \omega^2/\omega_F^2}\right). \quad (4.51)$$

For many cases of interest, $\omega_F \gg \omega_{IF}$ and the power spectral density increases as ω^2 for frequencies above ω_{IF} before leveling out at ω_F . This is true for cases where R_{IN} is on the order of or greater than R_F and $C_{IF} \gg C_F$. To demonstrate that these cases represent many of the amplifier designs worth considering for this application requires some examination of the gain block input device (which is responsible for the input impedance and probably most of S_e .) For the case of an FET (MOS or junction) input device, R_{IN} is very high and C_{IN} is probably at least a few picofarads. For a reasonably high current gain, an R_F of at least a few k Ω is desired, and this in turn requires $C_F < 1$ pF to achieve a 100 MHz bandwidth. Since C_S for the dielectric fluid test cell is estimated to be a few pF, C_I is considerably greater than C_F , and therefore $\omega_F \gg \omega_{IF}$. For related applications, such as optical detection or radiation detection, C_S may be considerably larger than a few pF, and the same conclusions hold.

For the case of a bipolar input device, $R_{IN} > \beta r_e$ (assuming a common source or collector configuration), where $r_e \simeq 26\text{mV}/I_c$. In order for the transistor to have the high f_T needed for high frequency operation, I_C must be on the order of 1 mA or greater, and a low value of $S_e \simeq 4kT(r_e/2 + r_b)$ requires I_C as large as

possible, but a large I_C results in a large $S_i \simeq 2qI_C/\beta$. A reasonable compromise is $I_C \simeq 1\text{mA}$, and this results in $r_e \simeq 26\Omega$. β values of 50 to 200 are typical for high frequency bipolar transistors with the result that R_{IN} is probably $1\text{ k}\Omega$ or more. This satisfies the requirement that $R_{IN} \sim R_F$. The comments about the relative magnitudes of the capacitances of the FET input amplifier apply here as well, and therefore the assumption that $\omega_F \gg \omega_{IF}$ is reasonable for most designs suitable for this application.

The output noise power due to the power spectrum $S_e(\omega)$ is

$$\overline{e_{noe}^2} = \int_{\omega_{min}}^{\omega_{max}} S_{noe}(\omega) \frac{d\omega}{2\pi}, \quad (4.52)$$

where ω_{max} and ω_{min} are the upper and lower limits of the noise passband. Since the amplifier may be dc coupled or at least have a very wideband characteristic with $\omega_{max} \gg \omega_{min}$, the lower limit can be taken to be zero. The upper limit is approximately the preamplifier bandwidth since the anti-aliasing filter will strongly attenuate noise (and signal) frequencies above that bandwidth. Thus

$$\overline{e_{noe}^2} = \int_0^{\omega_{max}} S_{noe}(\omega) \frac{d\omega}{2\pi}, \quad (4.53)$$

and substituting for $S_{noe}(\omega)$ gives

$$\overline{e_{noe}^2} = \frac{S_e}{2\pi} \left(1 + \frac{R_F}{R_{IN}}\right)^2 \int_0^{\omega_{max}} \frac{1 + \omega^2/\omega_{IF}^2}{1 + \omega^2/\omega_F^2} d\omega. \quad (4.54)$$

Rearranging gives

$$\overline{e_{noe}^2} = \frac{S_e}{2\pi} \left(1 + \frac{R_F}{R_{IN}}\right)^2 \int_0^{\omega_{max}} \frac{\omega_F^2 + \omega^2 \omega_F^2/\omega_{IF}^2}{\omega_F^2 + \omega^2} d\omega. \quad (4.55)$$

This integral can be evaluated [45] by noting that

$$\int \frac{dx}{\alpha^2 + x^2} = \frac{1}{\alpha} \tan^{-1} \left(\frac{x}{\alpha} \right), \quad (4.56)$$

and

$$\int \frac{x^2 dx}{\alpha^2 + x^2} = x - \alpha \tan^{-1} \left(\frac{x}{\alpha} \right). \quad (4.57)$$

Thus

$$\overline{e_{noe}^2} = \frac{S_e}{2\pi} \left(1 + \frac{R_F}{R_{IN}} \right)^2 \left\{ \omega_F \tan^{-1} \left(\frac{\omega_{max}}{\omega_F} \right) + \frac{\omega_F^3}{\omega_{IF}^2} \left[\frac{\omega_{max}}{\omega_F} - \tan^{-1} \left(\frac{\omega_{max}}{\omega_F} \right) \right] \right\}, \quad (4.58)$$

which can be simplified to

$$\overline{e_{noe}^2} = \frac{S_e \omega_F}{2\pi} \left(1 + \frac{R_F}{R_{IN}} \right)^2 \left(a + \frac{\omega_F^2}{\omega_{IF}^2} b \right), \quad (4.59)$$

by letting $m = \omega_{max}/\omega_F$, $a = \tan^{-1}(m)$ and $b = m - \tan^{-1}(m)$. For $\omega_{max} \simeq \omega_F$ (or $m \simeq 1$), a is approximately unity and b is somewhat less than one. A list of values for a and b is given in Table 4.1.

The output noise power $\overline{e_{noe}^2}$ (actually V^2) can be referred to the input as an rms input noise current i_{nie} by finding its root-mean-square value e_{noe} and

Table 4.1: Values of a and b for various $m = \omega_{max}/\omega_F$.

m	a	b
1	0.785	0.215
1.2	0.876	0.324
1.5	0.983	0.517
2.0	1.107	0.893

dividing by the current gain of the amplifier:

$$i_{nie} = \frac{\sqrt{e_{noe}^2}}{R_F}. \quad (4.60)$$

Substituting for $\overline{e_{noe}^2}$ gives

$$i_{nie} = \sqrt{S_e \frac{\omega_F}{2\pi}} \left(\frac{1}{R_F} + \frac{1}{R_{IN}} \right) \left(a + \frac{\omega_F^2}{\omega_{IF}^2} b \right)^{\frac{1}{2}}. \quad (4.61)$$

This can be simplified by making several assumptions. First, assume that the bandwidth f_{BW} is set by ω_F , and therefore $f_{BW} \simeq \omega_f/(2\pi)$. This is reasonable since ω_F does set the ideal closed-loop bandwidth and the actual bandwidth should not be drastically different. Second, take $\omega_{max} = \omega_F$ (making $a = 1$). This is also a reasonable assumption since the anti-aliasing filter incorporated in the measurement system cuts off frequencies above f_{BW} (and therefore ω_F). The anti-aliasing filter actually will modify the frequency response somewhat, but it should have a very sharp cut-off characteristic so the bandwidth is not reduced significantly. Now i_{nie} is given by

$$i_{nie} \simeq \sqrt{S_e f_{BW}} \left(\frac{1}{R_F} + \frac{1}{R_{IN}} \right) \left(0.79 + 0.22 \frac{\omega_F^2}{\omega_{IF}^2} \right)^{\frac{1}{2}}. \quad (4.62)$$

For the case $R_{IN} \gg R_F$, R_{IF} is approximately R_F and

$$\frac{\omega_F^2}{\omega_{IF}^2} \simeq \frac{C_{IF}^2}{C_F^2}. \quad (4.63)$$

Thus

$$i_{nie} \simeq \frac{\sqrt{S_e f_{BW}}}{R_F} \left(0.79 + 0.22 \frac{C_{IF}^2}{C_F^2} \right)^{\frac{1}{2}}. \quad (4.64)$$

For the case $C_{IF} = C_F + C_S + C_{IN} \gg C_F$,

$$0.22 \frac{C_{IF}^2}{C_F^2} \gg 0.79, \quad (4.65)$$

and the expression for i_{nie} simplifies to

$$i_{nie} \simeq 0.5 \frac{C_{IF}}{R_F C_F} \sqrt{S_e f_{BW}}. \quad (4.66)$$

By noting that $2\pi f_{BW} \simeq \omega_F = 1/(R_F C_F)$,

$$i_{nie} \simeq \pi f_{BW}^{\frac{3}{2}} C_{IF} S_e^{\frac{1}{2}} \quad (4.67)$$

is obtained. This is an important result since it states that the rms input noise current due to the S_e source depends directly on the total capacitance connected to the input and on the square root of S_e . Good low noise performance therefore requires an amplifier with low input capacitance and low S_e .

For the noise source with spectrum S_i^- , the closed-loop gain can be approximated as

$$A_{CLi}(\omega) = \frac{R_F}{1 + j\omega/\omega_F}, \quad (4.68)$$

where ω_F is $1/(R_F C_F)$. Using this gain in Eq. 4.13 gives the output noise power spectral density

$$S_{noi}(\omega) = S_i A_{CLi}(\omega) A_{CLi}^*(\omega) = S_i \left(\frac{R_F^2}{1 + \omega^2/\omega_F^2} \right). \quad (4.69)$$

This spectrum has units of V^2/Hz and is essentially white up to ω_F .

The output noise power due to this power spectrum is

$$\overline{e_{noi}^2} = \int_0^{\omega_{max}} S_{noi}(\omega) \frac{d\omega}{2\pi}, \quad (4.70)$$

where the upper and lower limits of the noise passband are the same as for $\overline{e_{noe}^2}$ (ω_{max} and 0). Substituting for $S_{noi}(\omega)$ and rearranging gives

$$\overline{e_{noi}^2} = \frac{S_i R_F^2}{2\pi} \int_0^{\omega_{max}} \frac{\omega_F^2 d\omega}{\omega_F^2 + \omega^2}. \quad (4.71)$$

Evaluating this integral by using

$$\int \frac{dx}{\alpha^2 + x^2} = \frac{1}{\alpha} \tan^{-1} \left(\frac{x}{\alpha} \right), \quad (4.72)$$

results in

$$\overline{e_{noi}^2} = \frac{S_i R_F^2 \omega_F}{2\pi} \tan^{-1} \left(\frac{\omega_{max}}{\omega_F} \right). \quad (4.73)$$

This can be simplified by again letting $m = \omega_{max}/\omega_F$ and $a = \tan^{-1}(m)$ with the result

$$\overline{e_{noi}^2} = \frac{a S_i R_F^2 \omega_F}{2\pi}. \quad (4.74)$$

For $\omega_{max} \simeq \omega_F$ (or $m \simeq 1$), a is approximately one (actual values of a are given in Table 4.1.)

This output noise power ($\overline{e_{noi}^2}$, actually V²) can be referred back as an rms input noise current i_{nii} by finding its rms value e_{noi} and dividing by the current gain of the amplifier:

$$i_{nii} = \frac{\sqrt{\overline{e_{noi}^2}}}{R_F} = \sqrt{\frac{a S_i^2 \omega_F}{2\pi}}. \quad (4.75)$$

Making the same assumptions as for i_{nie} ($f_{BW} = \omega_f/(2\pi)$ and $\omega_{max} = \omega_F$ or $a = 1$) gives

$$i_{nii} \simeq 0.9 \sqrt{S_i f_{BW}}. \quad (4.76)$$

This can be written in a form similar to Eq. 4.67 by using $2\pi f_{BW} \simeq 1/(R_F C_F)$, thus

$$i_{nii} \simeq 2\pi f_{BW}^{\frac{3}{2}} C_F R_F S_i^{\frac{1}{2}}. \quad (4.77)$$

The form of Eq. 4.76 may be the most useful since it shows that the rms input noise current is directly proportional to the roots of S_i and f_{BW} .

In the transimpedance configuration, the noise source with spectrum S_i^+ is shorted out and contributes no noise power.

For the feedback resistor R_F , the noise power density spectrum is S_{RF} , and the closed-loop gain can be approximated by

$$A_{CLRF}(\omega) = \frac{1}{1 + j\omega/\omega_F}. \quad (4.78)$$

The output noise power spectral density is

$$S_{noRF}(\omega) = S_{RF} A_{CLRF}(\omega) A_{CLRF}^*(\omega) = \frac{S_{RF}}{1 + \omega^2/\omega_F^2}. \quad (4.79)$$

This spectrum has units of V^2/Hz and is essentially white up to ω_F .

This power spectrum produces output noise power

$$\overline{e_{noRF}^2} = \int_0^{\omega_{max}} S_{noRF}(\omega) \frac{d\omega}{2\pi}, \quad (4.80)$$

where the upper and lower limits are the same as for $\overline{e_{noe}^2}$. Substituting for $S_{noRF}(\omega)$ and rearranging gives

$$\overline{e_{noRF}^2} = \frac{S_{RF}}{2\pi} \int_0^{\omega_{max}} \frac{\omega_F^2 d\omega}{\omega_F^2 + \omega^2}. \quad (4.81)$$

Evaluating this integral results in

$$\overline{e_{noRF}^2} = \frac{S_{RF}\omega_F}{2\pi} \tan^{-1} \left(\frac{\omega_{max}}{\omega_F} \right). \quad (4.82)$$

Using the same assumptions as for e_{noe} and e_{noi} ($m = \omega_{max}/\omega_F$ and $a = \tan^{-1} m$) results in

$$\overline{e_{noRF}^2} = \frac{aS_{RF}\omega_F}{2\pi}, \quad (4.83)$$

where a is approximately unity.

The output noise power $\overline{e_{noRF}^2}$, can be referred back as an rms input noise current:

$$i_{niRF} = \frac{\sqrt{\overline{e_{noRF}^2}}}{R_F} = \sqrt{\frac{aS_{RF}\omega_F}{2\pi R_F^2}}. \quad (4.84)$$

Using the same assumptions as for i_{nie} ($f_{BW} = \omega_f/(2\pi)$ and $\omega_{max} = \omega_F$ or $a = 1$) gives

$$i_{niRF} \simeq \frac{0.9}{R_F} \sqrt{S_{RF}f_{BW}}. \quad (4.85)$$

This can be written in a form similar to Eq. 4.67 by using $2\pi f_{BW} \simeq 1/(R_F C_F)$, thus

$$i_{niRF} \simeq 2\pi f_{BW}^{\frac{3}{2}} C_F S_{RF}^{\frac{1}{2}}. \quad (4.86)$$

The rms input noise currents due to S_e , S_i and S_{RF} can be combined into a total rms input noise current i_{ni} by

$$i_{ni} = \sqrt{i_{nie}^2 + i_{ni}^2 + i_{niRF}^2}. \quad (4.87)$$

Substituting for the individual noise currents gives

$$i_{ni} \simeq 2\pi f_{BW}^{\frac{3}{2}} \sqrt{\frac{1}{4} C_{IF}^2 S_e + C_F^2 S_{RF} + C_F^2 R_F^2 S_i}, \quad (4.88)$$

or

$$i_{ni} \simeq f_{BW}^{\frac{1}{2}} \sqrt{\frac{C_{IF}^2 S_e}{4R_F^2 C_F^2} + \frac{S_{RF}}{R_F^2} + S_i}. \quad (4.89)$$

By substituting $S_{RF} = 4kTR_F$, the expression for i_{ni} reduces to

$$i_{ni} \simeq f_{BW}^{\frac{1}{2}} \sqrt{\frac{C_{IF}^2 S_e}{4C_F^2 R_F^2} + \frac{4kT}{R_F} + S_i}. \quad (4.90)$$

Equation 4.90 is a reasonably simple expression for the rms input noise current in terms of the bandwidth, R_F , C_F , C_{IF} , S_e and S_i . To obtain low i_{ni} , several requirements are clear from the expression. S_e and S_i should be as low as possible. For a fixed bandwidth, the product $R_F C_F$ is fixed, and since the noise term due to R_F goes as $1/R_F$, R_F should be as high as possible, and C_F should be correspondingly low. Since the S_e noise term depends upon C_{IF}^2 , special attention should be paid to keeping it low. Although Eq. 4.90 is not an exact expression for the rms input noise current of the transimpedance amplifier, the resulting requirements on the amplifier parameters serve as very useful design guidelines.

4.3 The Voltage-Sensitive Amplifier

The voltage-sensitive amplifier was introduced and its operation was briefly described in Section 4.1. In this section, a general model for the voltage-sensitive amplifier is developed and then used in algebraic and computer-aided analysis. The model development and analysis parallel that used for the transimpedance amplifier. Literature describing similar preamplifiers is reviewed, and that infor-

mation is used along with results from the analysis to estimate the performance of a voltage-sensitive preamplifier optimized for prebreakdown currents.

4.3.1 Development of a General Model for the Voltage-Sensitive Amplifier

A model for the voltage-sensitive amplifier is shown in Figure 4.11. This model is similar to the transimpedance amplifier model and was chosen to provide reasonably accurate simulation without undue complexity. The same model for the signal source is used: an ideal current source with capacitance C_S in parallel. Like the transimpedance amplifier, the amplifier is modeled as a gain block, $A_{OL}(s)$, with a feedback network and a current detection resistor, R_D added. The gain block model differs slightly since both inputs are used. The input impedance is Z_{IN}^+ for the non-inverting terminal and Z_{IN}^- for the inverting terminal. Both input impedances are modeled by a parallel RC network connected to ground (R_{IN}^+ , C_{IN}^+ and R_{IN}^- , C_{IN}^- .) The gain block has zero output impedance and, as before, gain $A_{OL}(s)$ given by

$$A_{OL}(s) = \frac{A_0(1 + s/\omega_{Z1})(1 + s/\omega_{Z2})}{(1 + s/\omega_{P1})(1 + s/\omega_{P2})(1 + s/\omega_{P3})(1 + s/\omega_{P4})}. \quad (4.91)$$

In this expression, A_0 is the dc gain, ω_{Z1} and ω_{Z2} are zeroes, and ω_{P1} , ω_{P2} , ω_{P3} and ω_{P4} are poles.

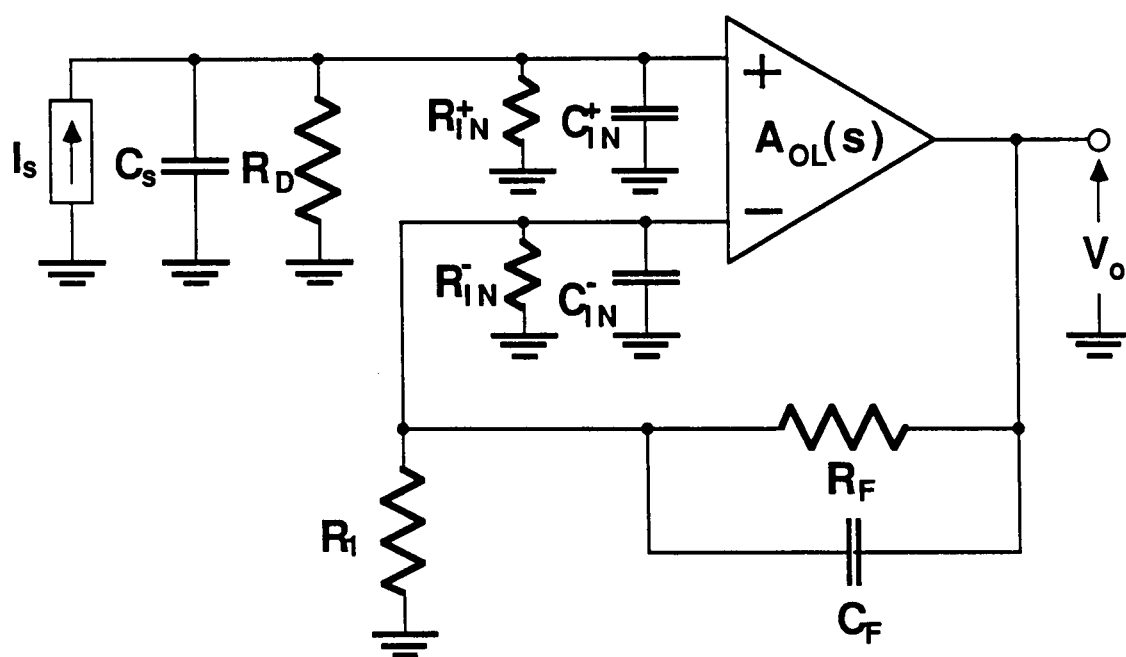


Figure 4.11: Simple voltage-sensitive amplifier circuit model.

4.3.2 Frequency Response of the Voltage-Sensitive Amplifier General Model

For an ideal gain block, the closed-loop gain (current-to-voltage) depends upon the impedance seen by the current source (across which a voltage is developed) and the voltage gain of the feedback amplifier. The impedance Z_D seen by the source is the parallel combination of R_D , $1/(sC_S)$, and Z_{IN}^+ , which can be expressed as

$$Z_D = \frac{R_I}{1 + sR_IC_I} = \frac{R_I}{1 + s/\omega_I}, \quad (4.92)$$

by letting $R_I = R_D \parallel R_{IN}^+$, $C_I = C_{IN}^+ + C_S$, and $\omega_I = 1/(R_IC_I)$. The details of this calculation are given in Appendix B.

The ideal voltage gain of the amplifier is

$$A_v(s) = 1 + \frac{Z_F}{Z_X}, \quad (4.93)$$

where

$$Z_F = R_F \parallel \frac{1}{C_F s}, \quad (4.94)$$

$$Z_X = R_1 \parallel R_{IN}^- \parallel \frac{1}{C_{IN}^- s} = R_X \parallel \frac{1}{C_{IN}^- s}, \quad (4.95)$$

and R_X is the parallel combination of R_{IN}^- and R_1 . The voltage gain can be written as

$$A_v(s) = \left(1 + \frac{R_F}{R_X}\right) \left(\frac{1 + s/\omega_{XF}}{1 + s/\omega_F}\right), \quad (4.96)$$

by using

$$R_{XF} = R_X \parallel R_F = R_1 \parallel R_{IN}^- \parallel R_F, \quad (4.97)$$

$$C_{XF} = C_F + C_{IN}^-, \quad (4.98)$$

$$\omega_F = \frac{1}{R_F C_F}, \quad (4.99)$$

and

$$\omega_{XF} = \frac{1}{R_{XF} C_{XF}}. \quad (4.100)$$

The development of the equation for the gain is also given in Appendix B.

The current-to-voltage gain is the product of the detection impedance and the voltage gain, therefore

$$A_{CL}^{ideal}(s) = \frac{V_o}{I_s} = Z_D A_v(s) = Z_D \left(1 + \frac{Z_F}{Z_X}\right). \quad (4.101)$$

Substituting for Z_D and $A_v(s)$ gives the ideal closed-loop gain

$$A_{CL}^{ideal}(s) = \left(\frac{R_I}{1 + s/\omega_I}\right) \left(1 + \frac{R_F}{R_X}\right) \left(\frac{1 + s/\omega_{XF}}{1 + s/\omega_F}\right). \quad (4.102)$$

The actual closed-loop gain is somewhat different and can be found from $A_{CL}^{ideal}(s)$ and the loop transmission, $T(s)$, which is given by

$$T(s) = -A_{OL}(s) \left(\frac{R_X}{R_X + R_F}\right) \left(\frac{1 + s/\omega_F}{1 + s/\omega_{XF}}\right), \quad (4.103)$$

where $R_{XF} = R_X \parallel R_F$, $C_{XF} = C_F + C_{IN}^-$, $\omega_F = 1/(R_F C_F)$ and $\omega_{XF} = 1/(R_{XF} C_{XF})$ (as before). The loop transmission is derived in detail in Appendix B. In the usual way, the actual closed loop gain is found from

$$A_{CL}(s) = A_{CL}^{ideal}(s) \left(\frac{-T(s)}{1 - T(s)}\right), \quad (4.104)$$

and the frequency response is found by evaluating this expression for $s = j\omega$.

4.3.3 Noise Analysis of the Voltage-Sensitive Amplifier General Model

A model for the noise behavior was developed by adding six sources of noise to the simple voltage-sensitive amplifier (as compared to four sources for the transimpedance amplifier.) The contributions of all noise sources inside the gain block are again referred to the input and represented by a voltage source and two current sources. The thermal noise of the feedback resistor, R_F , the detection resistor, R_D , and the gain-setting resistor, R_1 are each represented by a source. Any noise due to Z_{IN}^+ and Z_{IN}^- is included in the gain block noise model. The noise model for the transimpedance amplifier is shown in Figure 4.12.

Once again, the sources are described by their power spectral densities. S_{RF} is the power spectral density due to R_F and is given by

$$S_{RF} = 4kTR_F, \quad (4.105)$$

where k is Boltzmann's constant and T is the absolute temperature in Kelvins. This source has units of V^2/Hz when R_F is given in ohms. Similarly, the power spectral densities due to a R_D and R_1 are

$$S_{RD} = 4kTR_D, \quad (4.106)$$

and

$$S_{R1} = 4kTR_1, \quad (4.107)$$

respectively. The gain block equivalent input noise voltage and current sources

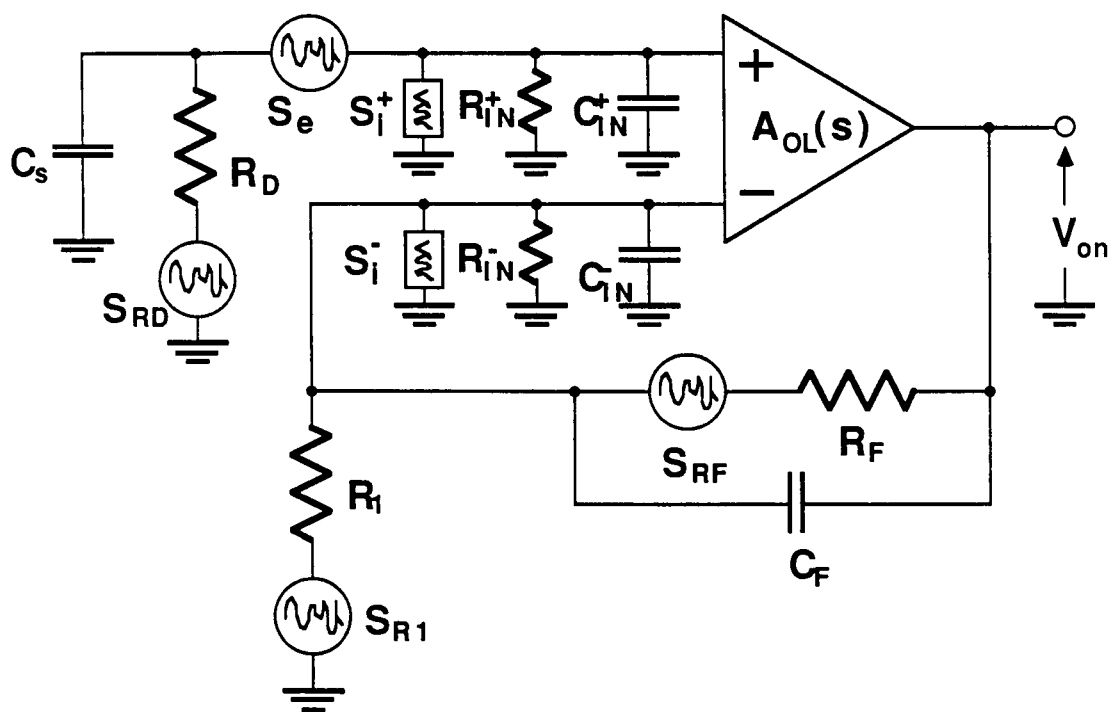


Figure 4.12: Simple voltage-sensitive amplifier noise model.

are modeled as they were for the transimpedance amplifier, and the power spectral densities S_e , S_i^- and S_i^+ are given by Eqs. 4.10, 4.11 and 4.12, respectively.

The voltage-sensitive amplifier noise characteristics are found in the same general way as for the transimpedance amplifier. The basic procedure and equations are repeated here. The output noise power spectral density due to a noise source with power spectral density $S(\omega)$ is found from

$$S_{no}(\omega) = S(\omega)A_{CL}(\omega)A_{CL}^*(\omega) = S(\omega)|A_{CL}(\omega)|^2, \quad (4.108)$$

where $A_{CL}(\omega)$ is the actual closed-loop gain and $A_{CL}^*(\omega)$ is its complex conjugate. $A_{CL}(\omega)$ is found from the ideal closed-loop gain and the loop transmission using

$$A_{CL}(\omega) = A_{CL}^{ideal}(\omega) \left(\frac{-T(\omega)}{1 - T(\omega)} \right). \quad (4.109)$$

The output power spectral density is integrated to give the total output noise power:

$$\overline{e_{no}^2} = \int_{f_1}^{f_2} S_{no}(\omega) df, \quad (4.110)$$

and the rms value of the output noise power is referred to the input as an rms input noise current i_{ni} by dividing by the midband current-to-voltage gain, A_{CL}^{mid} . For the voltage-sensitive amplifier, the midband gain is approximately $R_D(1 + R_F/R_1)$ for the case where the loop transmission is reasonably high, $R_D \ll R_{IN}^+$ and $R_1 \ll R_{IN}^-$. These are reasonable restrictions for most amplifiers. Thus the rms input noise current is

$$i_{ni} = \frac{e_{no}}{A_{CL}^{mid}} = \frac{e_{no}}{R_D(1 + R_F/R_1)}. \quad (4.111)$$

For the noise source with spectrum S_e , the ideal closed-loop gain is

$$A_{CLe}^{ideal}(s) = \left(\frac{R_{IN}^+}{R_D + R_{IN}^+} \right) \left(\frac{1 + s/\omega_D}{1 + s/\omega_I} \right) \left(1 + \frac{R_F}{R_{IN}^- \parallel R_1} \right) \left(\frac{1 + s/\omega_{XF}}{1 + s/\omega_F} \right), \quad (4.112)$$

where ω_{XF} and ω_F are defined by Eqs. 4.99 and 4.100, and $\omega_D = 1/(R_D C_S)$ and $\omega_I = 1/(R_I C_I)$. The derivation for this equation is given in Appendix B along with derivations for each of the other noise sources. The actual closed-loop gain can be found from the ideal gain and the loop transmission found for the frequency response and stability calculations (Eq. 4.103). The actual closed-loop gain can be used to find the output power spectral density $S_{noe}(\omega)$ which can in turn be used to find e_{noe} and i_{nie} .

For the noise source with spectrum S_i^- , the ideal closed-loop gain is

$$A_{CLi-}^{ideal}(s) = Z_F = \frac{R_F}{1 + s/\omega_F}, \quad (4.113)$$

where $\omega_F = 1/(R_F C_F)$. For this source, the output power spectral density is denoted by $S_{noi-}(\omega)$, the mean-squared output voltage by $\overline{e_{noi-}^2}$, and the rms input noise current by i_{nii-} .

For the noise source with spectrum S_i^+ , the ideal closed-loop gain is the same as for the signal source and is given by Eq. 4.101, which is adapted for use here as

$$A_{CLi+}^{ideal}(s) = \left(\frac{R_I}{1 + s/\omega_I} \right) \left(1 + \frac{R_F}{R_X} \right) \left(\frac{1 + s/\omega_{XF}}{1 + s/\omega_F} \right). \quad (4.114)$$

For this source, the output power spectral density is denoted by $S_{noi+}(\omega)$, the mean-squared output voltage by $\overline{e_{noi+}^2}$, and the rms input noise current by i_{nii+} .

For the feedback resistor R_F , the noise power density spectrum is S_{RF} , and the ideal closed-loop gain is

$$A_{CLRF}^{ideal}(s) = \frac{1}{1 + s/\omega_F}. \quad (4.115)$$

The output power spectral density $S_{noRF}(\omega)$ is calculated and integrated to give the mean-squared output voltage $\overline{e_{noRF}^2}$, and then referred back to the input as an rms input noise current i_{niRF} .

For the detection resistor R_D , the noise power density spectrum is S_{RD} , and the ideal closed-loop gain is

$$A_{CLRD}^{ideal}(s) = \left(\frac{R_{IN}^+}{R_D + R_{IN}^+} \right) \left(\frac{1}{1 + s/\omega_I} \right) \left(1 + \frac{R_F}{R_{IN}^- \parallel R_1} \right) \left(\frac{1 + s/\omega_{XF}}{1 + s/\omega_F} \right). \quad (4.116)$$

For this source, the output power spectral density is denoted by $S_{noRD}(\omega)$, the mean-squared output voltage is $\overline{e_{noRD}^2}$, and the rms input noise current i_{niRD} .

For the gain-setting resistor R_1 , the noise power density spectrum is S_{R1} , the ideal closed-loop gain is

$$A_{CLR1}^{ideal}(s) = \frac{R_F/R_1}{1 + s/\omega_F}. \quad (4.117)$$

For this source, the output power spectral density is denoted by $S_{noR1}(\omega)$, the mean-squared output voltage is $\overline{e_{noR1}^2}$, and the rms input noise current i_{niR1} .

4.3.4 Algebraic Analysis of Voltage-Sensitive Amplifier Stability and Frequency Response

In this section, a simplified version of the voltage-sensitive amplifier is analyzed algebraically. The stability and frequency response of the amplifier are examined in terms of the amplifier parameters (R_D , C_{IN} , S_e , etc.). A number of simplifications and approximations are made in order to produce equations which are simple enough so useful features can be easily seen. Although not exact solutions, these equations should be useful as a basis for comparing the voltage-sensitive amplifier to the transimpedance amplifier.

To simplify the analysis of the voltage-sensitive amplifier, the same approach that was used in Section 4.2.4 for the transimpedance amplifier is used. Once again, the gain block can be considered more nearly ideal by restricting $A_{OL}(s)$ to have just one pole:

$$A_{OL}(s) = \frac{A_0}{1 + s/\omega_{p1}}. \quad (4.118)$$

The voltage-sensitive amplifier model is considerably more complicated, so to reduce the complexity to a level more like that used with the transimpedance amplifier, more simplifications are needed. Since it would be desirable that the current-to-voltage gain would be set by R_D , it will be assumed that $R_{IN+} \gg R_D$. Similarly, R_1 should set the gain, so it will be assumed that $R_{IN-} \gg R_1$, and in order to simplify the frequency characteristics, it will be assumed that C_{IN-} is zero or at least negligible.

Having made these assumptions, the ideal closed-loop voltage gain simplifies

to

$$A_v(s) \simeq \left(1 + \frac{R_F}{R_1}\right) \left(\frac{1 + s/\omega_{XF}}{1 + s/\omega_F}\right), \quad (4.119)$$

where

$$\omega_{XF} \simeq \frac{1}{(R_F \parallel R_1)C_F}. \quad (4.120)$$

By letting

$$A_1 = 1 + \frac{R_F}{R_1} = \frac{R_F}{R_F \parallel R_1}, \quad (4.121)$$

and substituting for $R_F \parallel R_1$, the expression for ω_{XF} simplifies to $\omega_{XF} \simeq A_1\omega_F$, and therefore

$$A_v(s) \simeq A_1 \left(\frac{1 + s/A_1\omega_F}{1 + s/\omega_F}\right). \quad (4.122)$$

Thus the ideal closed-loop voltage gain has one pole at ω_F and one zero at a higher frequency. If the dc gain A_1 is high (> 10), the response is approximately the same as for a single pole system up to frequencies well above the -3dB bandwidth, which is given by $f_{-3\text{dB}} \simeq \omega_f/(2\pi)$.

The loop transmission $T(s)$ is given by

$$T(s) = \frac{-A_{OL}(s)}{A_v(s)} = \left(\frac{-A_0}{1 + s/\omega_{p1}}\right) \left(\frac{1}{A_1}\right) \left(\frac{1 + s/\omega_F}{1 + s/A_1\omega_F}\right). \quad (4.123)$$

From this expression, it is seen that $T(s)$ has poles ω_{p1} and $A_1\omega_F$, and zero ω_F , and is therefore stable. Since the frequency of the zero is lower than at least one of the poles, stability should be considerably less of a problem than for the transimpedance amplifier even with additional high frequency poles.

Since a low phase margin may not be a problem, the actual closed-loop gain may be approximately the same as the ideal gain, but even if $A_v(s)$ can be

adjusted to give sufficient bandwidth and a flat response, the frequency response of the current-to-voltage gain due to Z_D , which is given by

$$Z_D = R_D \parallel Z_{IN+} \parallel 1/(sC_s), \quad (4.124)$$

may be the limiting factor. The ideal closed-loop current gain was given by Eq. 4.102, and using the simplifying assumptions, that reduces to

$$A_{CL}(s) \simeq Z_D A_v(s) = \left(\frac{R_D}{1 + s/\omega_I} \right) (A_1) \left(\frac{1 + s/A_1\omega_F}{1 + s/\omega_F} \right), \quad (4.125)$$

where $\omega_I \simeq 1/(R_D C_I)$ and $C_I = C_{IN+}^+ + C_s$. Even if $A_v(s)$ has no frequency dependence, $A_{CL}(s)$ still has a pole due to Z_D , and the bandwidth (f_{BW}) would be limited to

$$f_{BW} \simeq \frac{1}{2\pi R_D (C_s + C_{IN+}^+)}. \quad (4.126)$$

This would be approximately true for cases with f_{-3dB} somewhat greater than f_{BW} .

The equation for f_{BW} probably reveals the most important design constraints for the voltage-sensitive amplifier when used to amplify prebreakdown currents. It shows that C_s and C_{IN+} need to be as low as possible since R_D needs to be as high as possible for gain and low-noise performance. For optimum performance, R_D should be high enough that it (and C_s and C_{IN+}) limit the bandwidth. Of course, the amplifier must be stable and have f_{-3dB} greater than f_{BW} as well.

4.3.5 Algebraic Analysis of Voltage-Sensitive Amplifier Noise Performance

To analyze the noise performance of the voltage-sensitive amplifier algebraically without undue complexity, the same simplifications that were made to the general model in the frequency analysis of the voltage-sensitive amplifier (Section 4.3.4) will be made along with simplifications to the noise model used in the analysis of the transimpedance amplifier (Section 4.2.5). For the noise model, the first simplification was to consider only the constant term of the input noise power spectral densities, which are $S_e(f)$, $S_i^+(f)$ and $S_i^-(f)$ for the voltage-sensitive amplifier. The white noise sources were deemed most important for a wideband amplifier. The second simplification was to consider only the contributions of noise within and slightly above the amplifier bandwidth. This was deemed a reasonable assumption due to the presence of the anti-aliasing filter. The third simplification was to use the ideal closed-loop noise gains in order to give integrable equations. The ideal closed-loop gains should be reasonable approximations of the actual gains up to the noise bandwidth. Any additional assumptions will be made and justified as needed.

The total output noise power is found by calculating the output noise power due to S_e , S_i^+ , S_i^- , S_{RD} , S_{R1} , and S_{RF} , and summing these contributions. The individual and total output noise powers are referred back to the input as rms input noise currents using the method outlined in Section 4.3.3.

By taking into consideration the various assumptions about the amplifier

parameters, the ideal closed-loop gain for the noise source with spectrum S_e becomes

$$A_{CLe}(s) = \left(\frac{1 + s/\omega_D}{1 + s/\omega_I} \right) (A_1) \left(\frac{1 + s/(A_1\omega_F)}{1 + s/\omega_F} \right), \quad (4.127)$$

where $A_1 = 1 + R_F/R_1$, $\omega_F = 1/(R_F C_F)$, $\omega_D = 1/(R_D C_S)$ and $\omega_I = 1/(R_D C_I)$. This is still a complicated expression, but it can be simplified further by considering the noise bandwidth. Since frequencies well above ω_F will not be passed by the anti-aliasing filter, the zero at $A_1\omega_F$ will have negligible effect on the total noise power. Thus an approximation for the gain is

$$A_{CLe}(s) \simeq \left(\frac{1 + s/\omega_D}{1 + s/\omega_I} \right) \left(\frac{A_1}{1 + s/\omega_F} \right). \quad (4.128)$$

Also since $C_I = C_{IN}^+ + C_S$, ω_D is a higher frequency than ω_I , and therefore the magnitude of the term involving those two frequencies is always less than one. An upper bound on the gain for the S_e noise source is obtained by setting that term to one, and therefore

$$A_{CLe}(s) \simeq \left(\frac{A_1}{1 + s/\omega_F} \right), \quad (4.129)$$

or in terms of ω ,

$$A_{CLe}(\omega) \simeq \left(\frac{A_1}{1 + j\omega/\omega_F} \right). \quad (4.130)$$

The output noise power spectral density is found from this and S_e by using Eq. 4.108, giving

$$S_{noe}(\omega) = S_e A_{CLe}(\omega) A_{CLe}^*(\omega) = S_e A_1^2 \left(\frac{1}{1 + \omega^2/\omega_F^2} \right). \quad (4.131)$$

The output noise power spectral density is integrated over a range of frequencies to give the output noise power

$$\overline{e_{noe}^2} = \int_{\omega_{min}}^{\omega_{max}} S_{noe}(\omega) \frac{d\omega}{2\pi}. \quad (4.132)$$

For a dc-coupled amplifier, the lower limit ω_{min} can be taken to be zero. If the frequency ω_F is used to control the amplifier bandwidth, and an anti-aliasing filter is used to greatly attenuate voltages at frequencies above the amplifier bandwidth, then the upper limit ω_{max} can be taken to be ω_F . Substituting for $S_{noe}(\omega)$ and applying those limits then gives

$$\overline{e_{noe}^2} = \int_0^{\omega_F} S_e A_1^2 \left(\frac{1}{1 + \omega^2/\omega_F^2} \right) \frac{d\omega}{2\pi}. \quad (4.133)$$

Rearranging gives

$$\overline{e_{noe}^2} = \frac{S_e A_1^2 \omega_F^2}{2\pi} \int_0^{\omega_F} \frac{d\omega}{\omega_F^2 + \omega^2}, \quad (4.134)$$

which can be evaluated using

$$\int \frac{dx}{\alpha^2 + x^2} = \frac{1}{\alpha} \tan^{-1} \left(\frac{x}{\alpha} \right), \quad (4.135)$$

to give

$$\overline{e_{noe}^2} = \frac{S_e A_1^2 \omega_F^2}{2\pi} \left(\frac{1}{\omega_F} \tan^{-1}(1) \right). \quad (4.136)$$

Evaluating $\tan^{-1} 1$ and rearranging gives

$$\overline{e_{noe}^2} = S_e A_1^2 \left(\frac{\pi}{4} \right) \left(\frac{\omega_F}{2\pi} \right). \quad (4.137)$$

Using the assumption that $f_{BW} \simeq \frac{\omega_F}{2\pi}$ results in

$$\overline{e_{noe}^2} = S_e A_1^2 \left(\frac{\pi}{4} \right) f_{BW}. \quad (4.138)$$

A noise bandwidth f_{NBW} can be defined by taking $f_{NBW} = \frac{\pi}{4}f_{BW}$, and therefore

$$\overline{e_{noe}^2} = S_e A_1^2 f_{NBW}. \quad (4.139)$$

If $\omega = \infty$ had been used for ω_{max} , then f_{NBW} would have equaled $\frac{\pi}{2}f_{BW}$, and the noise power would be twice as large.

The output noise voltage $e_{noe} = \sqrt{\overline{e_{noe}^2}}$ is referred back to the input as an rms input noise current by dividing by the amplifier current gain. Thus

$$i_{nie} = \frac{e_{noe}}{A_1 R_D} = \frac{\sqrt{S_e f_{NBW}}}{R_D}. \quad (4.140)$$

For the S_i^- noise source, the ideal closed-loop gain is

$$A_{CLi-}(s) = \frac{R_F}{1 + s/\omega_F}, \quad (4.141)$$

which results in an output noise power spectral density given by

$$S_{noi-}(\omega) = S_{noi-} \left(\frac{R_F^2}{1 + \omega^2/\omega_F^2} \right). \quad (4.142)$$

This is the same as the approximation for $S_{noe}(\omega)$, except for the constant term, so the integration procedure gives a mean-squared output noise voltage expression that is the same except for the constant, and it is given by

$$\overline{e_{noi-}^2} = S_{i-} R_F^2 f_{NBW}. \quad (4.143)$$

The rms input noise current is therefore

$$i_{nii-} = \frac{\sqrt{S_{i-} R_F^2 f_{NBW}}}{A_1 R_D}. \quad (4.144)$$

The noise source with spectrum S_i^+ has ideal closed-loop gain given by

$$A_{CLi+}(s) = \left(\frac{R_I}{1 + s/\omega_I} \right) (A_1) \left(\frac{1 + s/(A_1\omega_F)}{1 + s/\omega_F} \right), \quad (4.145)$$

but by using the simplifying assumptions developed for the S_e noise source, the expression becomes

$$A_{CLi+}(s) \simeq \left(\frac{R_D}{1 + s/\omega_I} \right) \left(\frac{A_1}{1 + s/\omega_F} \right). \quad (4.146)$$

Since the factor $1/(1 + s/\omega_I)$ is also part of the signal gain expression, it is clear that ω_I must be $\geq 2\pi f_{BW}$ in order not to limit the bandwidth. If ω_F limits the bandwidth to f_{BW} , then $\omega_I \gg 2\pi f_{BW}$, and the expression for A_{CLi+} simplifies to

$$A_{CLi+}(s) \simeq \frac{R_D A_1}{1 + s/\omega_F}, \quad (4.147)$$

where $\omega_F \simeq 2\pi f_{BW}$. Similarly, if ω_I limits the bandwidth to f_{BW} , then $\omega_F \gg 2\pi f_{BW}$, and the expression for A_{CLi+} simplifies to

$$A_{CLi+}(s) \simeq \frac{R_D A_1}{1 + s/\omega_I}, \quad (4.148)$$

where $\omega_I \simeq 2\pi f_{BW}$, which is really the same expression obtained for the case where ω_F limit the bandwidth. Either gain should be a useful approximation for the purpose of calculating the mean-squared output voltage even if the bandwidth is determined by both ω_F and ω_I as long as either $\omega_F \simeq 2\pi f_{BW}$ or $\omega_I \simeq 2\pi f_{BW}$ is appropriately used in the evaluation, even though neither is strictly true.

This noise gain results in an output noise power spectral density given by

$$S_{noi+}(\omega) = S_{i+} \left(\frac{R_D^2 A_1^2}{1 + \omega^2/\omega_F^2} \right). \quad (4.149)$$

The mean-squared output voltage can be calculated as

$$\overline{e_{noi+}^2} = S_{i+} R_D^2 A_1^2 f_{NBW}, \quad (4.150)$$

and the resulting rms input noise current is

$$i_{ni+} = \sqrt{S_{i+} f_{NBW}}. \quad (4.151)$$

The feedback resistor has noise power density spectrum $S_{RF} = 4kTR_F$, and ideal closed-loop gain

$$A_{CLRF}(s) = \frac{1}{1 + s/\omega_F}. \quad (4.152)$$

This results in an output power spectral density of

$$S_{noRF}(\omega) = \frac{4kTR_F}{1 + \omega^2/\omega_F^2}, \quad (4.153)$$

and integrating this gives the mean-squared output voltage

$$\overline{e_{noRF}^2} = 4kTR_F f_{NBW}. \quad (4.154)$$

The rms input noise current is

$$i_{niRF} = \frac{\sqrt{4kTR_F f_{NBW}}}{A_1 R_D}. \quad (4.155)$$

The gain-setting resistor R_1 calculations are very similar to the ones for the feedback resistor. R_1 has noise power density spectrum $S_{RF} = 4kTR_1$, and ideal closed-loop gain

$$A_{CLR1}(s) = \frac{R_F/R_1}{1 + s/\omega_F}. \quad (4.156)$$

The resulting output power spectral density is

$$S_{noR1}(\omega) = \frac{4kTR_1(R_F/R_1)^2}{1 + \omega^2/\omega_F^2}, \quad (4.157)$$

the mean-squared output voltage is

$$\overline{e_{noR1}^2} = 4kT \frac{R_F^2}{R_1} f_{NBW}, \quad (4.158)$$

and the rms input noise current is

$$i_{niR1} = \frac{\sqrt{4kT \frac{R_F^2}{R_1} f_{NBW}}}{A_1 R_D}. \quad (4.159)$$

For the S_{RD} source,

$$A_{CLRD}(s) = \left(\frac{1}{1 + s/\omega_I} \right) (A_1) \left(\frac{1 + s/(A_1\omega_F)}{1 + s/\omega_F} \right), \quad (4.160)$$

which is the same as for the S_{i+} source, except for a constant, so the same simplifying assumptions lead to

$$A_{CLRD}(s) \simeq \frac{A_1}{1 + s/\omega_F}, \quad (4.161)$$

which can be used for calculating the in-band noise power. This noise gain results in an output noise power spectral density given by

$$S_{noRD}(\omega) = 4kTR_D \left(\frac{A_1^2}{1 + \omega^2/\omega_F^2} \right), \quad (4.162)$$

and a mean-squared output voltage of

$$\overline{e_{noRD}^2} = 4kTR_D A_1^2 f_{NBW}, \quad (4.163)$$

The rms input noise current is given by

$$i_{niRD} = \sqrt{\frac{4kT}{R_D} f_{NBW}}. \quad (4.164)$$

The mean-squared output noise voltages can be summed to give the total mean-squared output noise voltage

$$\overline{e_{no}^2} = [S_e A_1^2 + S_{i-} R_F^2 + S_{i+} R_D^2 A_1^2 + 4kT(R_F + R_F^2/R_1 + R_D A_1^2)] f_{NBW}. \quad (4.165)$$

This expression applies even if the noise bandwidth is somewhat different from that estimated earlier. A total rms input noise current is found from

$$i_{ni} = \frac{\sqrt{\overline{e_{no}^2}}}{A_1 R_D}. \quad (4.166)$$

Substituting for $\overline{e_{no}^2}$ and simplifying the result gives

$$i_{ni} = \left[\frac{S_e}{R_D^2} + \frac{S_{i-} R_F^2}{A_1^2 R_D^2} + S_{i+} + 4kT \left(\frac{R_F}{A_1 R_D} + 1 \right) \frac{1}{R_D} \right]^{\frac{1}{2}} \sqrt{f_{NBW}}. \quad (4.167)$$

Examining Eq. 4.167 reveals several approaches that are useful in reducing the total rms input noise current. If a gain block that has negligible noise current sources (one using FET inputs, for example) is used, then the terms involving S_{i-} and S_{i+} are not important. Otherwise, S_{i-} and S_{i+} should be minimized. All the remaining terms have R_D in the denominator, so it should be made as large as possible. The factor $R_F/(A_1 R_D)$ equals $(R_F \parallel R_1)/R_D$, so $R_F \parallel R_1$ should be small compared to R_D . Interestingly, the gain A_1 is not important to the noise current, as long as it is reasonably high so the various approximations made earlier apply. The bandwidth was assumed to be a constant for this application,

but if it can be adjusted for others, it should be as small as possible. The remaining factor, S_e , should also be as small as possible to obtain the lowest rms input noise current.

4.4 Computer-Aided Analysis and Design of Preamplifiers

The closed-loop characteristics of an amplifier (gain, bandwidth, etc.) may be adjusted by changing one or more of the amplifier parameters (open-loop gain, feedback impedance, open-loop poles, etc.). Analysis involves taking the *parameters* and using them to determine the amplifier *characteristics*, whereas design is a matter of choosing realistic parameter values which result in the desired characteristics. For some types of networks, RLC filters, for example, synthesis procedures are available which allow determination of all components from the desired transfer function. For a high performance amplifier, design is probably not a straightforward procedure, and a combination of design aids and procedures are generally used.

The following example will point out some of the difficulties involved in the design process. Consider that the bandwidth of a hypothetical amplifier is to be set by adjusting the value of the feedback capacitor. As a design aid, a plot (Figure 4.13) of bandwidth as a function of feedback resistance for a particular set of amplifier parameters would be useful. An experienced designer probably would know the general shape of this plot and could get the value needed with-

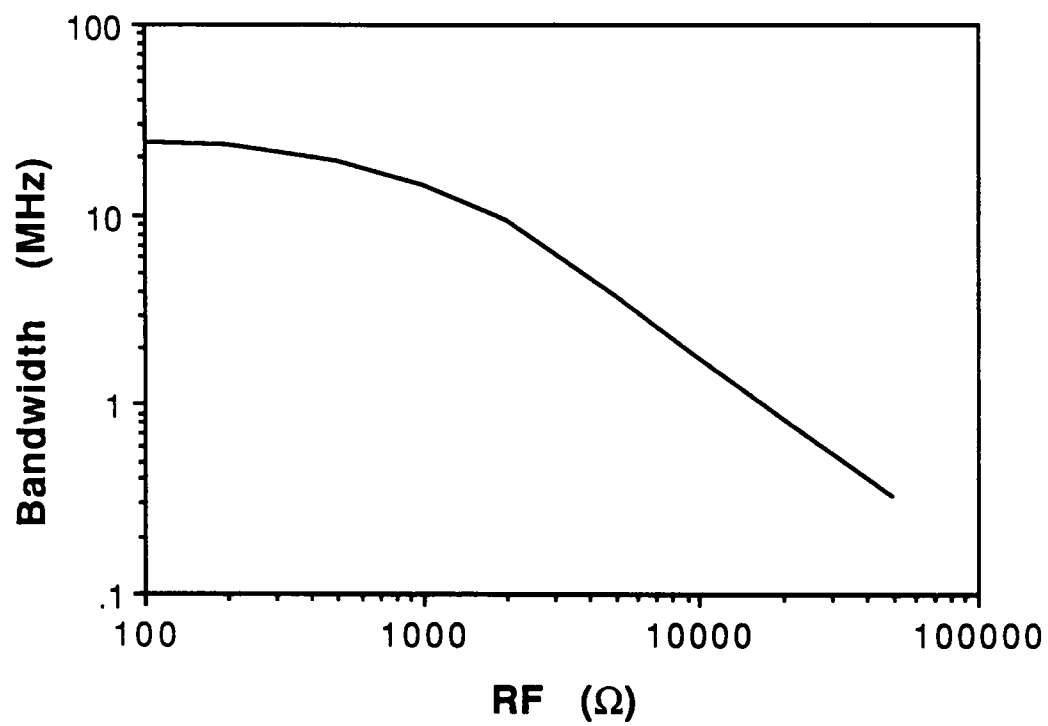


Figure 4.13: Bandwidth for hypothetical amplifier.

out the plot by using algebra or by finding it empirically. However, this is a fairly simple design problem, and usually the designer needs the flexibility to change any or all of the parameters to obtain a desired set of characteristics. What approach could be used if two (or more) parameters could be adjusted at once and two (or more) characteristics are important? Consider that both the feedback resistance and capacitance may be adjusted and that both the phase margin and bandwidth are important. The effects of varying both parameters are hard to visualize for just one characteristic, let alone both. In this case, a plot such as Figure 4.14 would help. For a particular bandwidth, Figure 4.14 shows that several combinations of R_F and C_F give the desired bandwidth, but the phase margin is poorer for some cases. Which case provides the best phase margin would probably not be obvious without the plot.

How could such a set of curves be generated ? If the amplifier is fairly simple, (one or two real poles at most) and the frequency dependence of the noise sources are very simple (a constant or one zero), algebraic equations for the bandwidth and the total input noise power can probably be found with a great deal of labor. Once found, the number of terms in the expression may preclude any conclusions from being drawn. (Conclusions like those which could be drawn from Figure 4.14.) Of course a plot could be obtained by numerically evaluating the equations.

A powerful program such as SPICE [46] could be used to generate data for plots. For a particular amplifier with most of the components already deter-

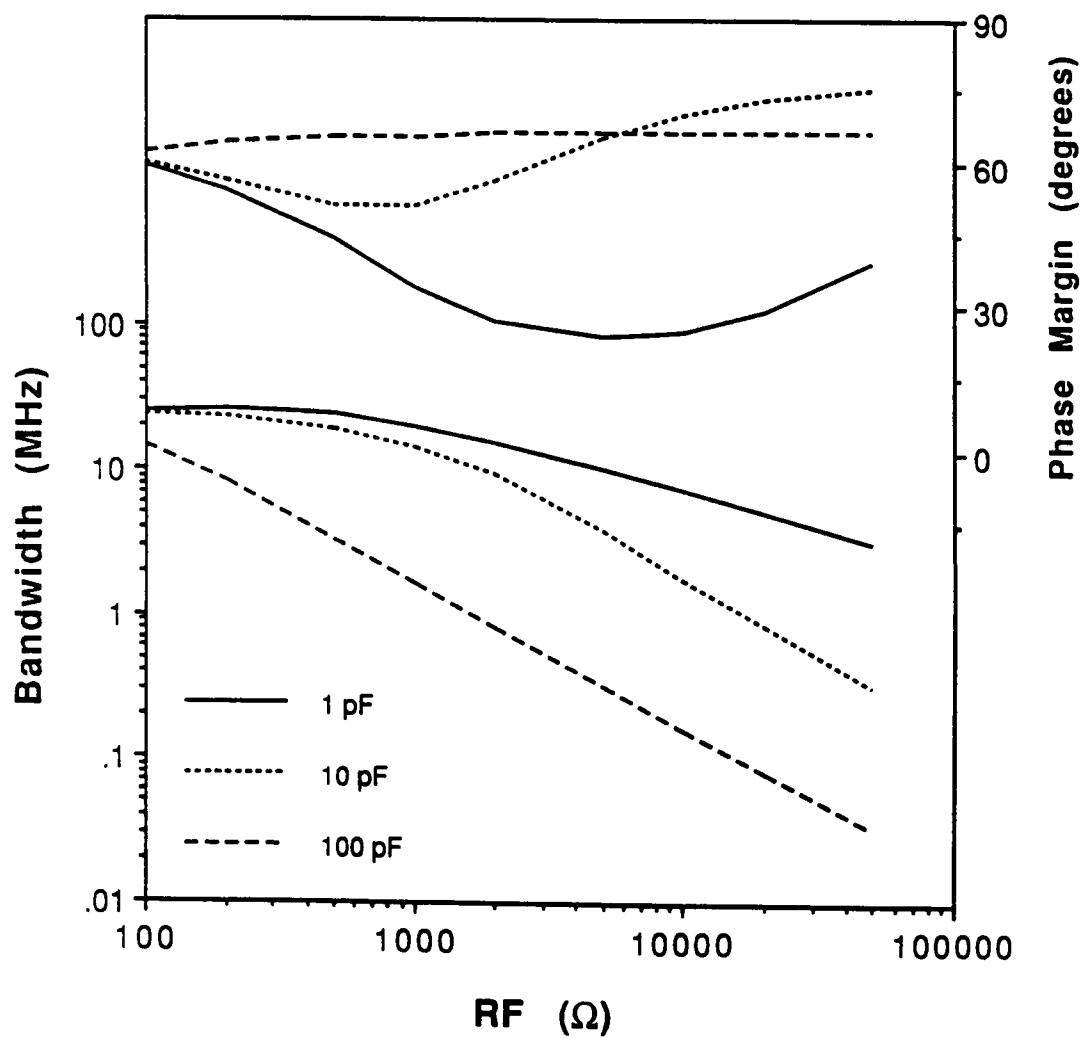


Figure 4.14: Bandwidth and phase margin as a function of feedback capacitance and resistance for hypothetical amplifier.

mined, SPICE is probably the preferred design tool. As one (or two) components are varied over a wide range, SPICE will give excellent results fairly rapidly. There are several drawbacks that become serious problems as the number of cases to be evaluated increases.

1. An amplifier described in a simple algebraic format (dc gain and poles) must be converted to RC networks and dependent generators.
2. The amount of computer time needed mounts rapidly as the number of cases to evaluate increases.
3. The output does not give characteristics such as the bandwidth or phase margin directly – these must be found manually from a listing of the frequency response or a plot in the case of newer versions such as PSpice [47].
4. The result from each case must be tabulated manually and then plotted to give a design curve (of bandwidth vs. resistance, for example).
5. Noise is given as a power spectral density and not as total noise power over a given bandwidth. (PSpice will plot the integral of the noise power, and the total noise power can be read from this plot.)

A more reasonable approach to generating amplifier design curves with widely ranging parameters is to describe the amplifier in terms of the most important parameters such as the dc gain, poles, zeroes, feedback resistance, etc., and use circuit analysis to obtain the loop transmission and ideal closed-loop gain as algebraic expressions. These are fairly simple expressions, even

with all the poles, zeroes, resistances and capacitances expressed as variables. With the aid of a simple computer program, the loop transmission $T(s)$ can be generated easily for any frequency. Such a program can also solve for the actual closed-loop gain A_{CL} from

$$A_{CL}(s) = A_{CL}^{ideal}(s) \left(\frac{-T(s)}{1 - T(s)} \right). \quad (4.168)$$

The program could also iteratively solve $|T(s)| = 1$ for the phase margin and $A_{CL}(j2\pi f) = A_{CL}(0)/\sqrt{2}$ for the bandwidth. A relatively small amount of algebra, programming and computer time could yield results for a wide range of parameters.

4.4.1 TRAMP: A CAD Tool for the Transimpedance Amplifier

A program like that described at the end of Section 4.4 was developed as an aid in analyzing and designing transimpedance amplifiers. The program, called TRAMP (for TRansimpedance AMPlifier) uses the general model developed in Sections 4.2.1 through 4.2.3 as a basis to calculate the frequency response and noise performance of a transimpedance amplifier with parameters chosen by the user. A functional description of TRAMP and an example of its use follow. Details of the program operation and organization and a complete listing are given in Appendix C.

Functional Description of TRAMP

To use the program TRAMP, the user must first enter the amplifier parameters which include the source capacitance, feedback impedance, and the gain block poles, zeroes, midband gain and noise source coefficients. Once the parameters are entered, the program can calculate the phase margin, frequency response, noise power spectral density and the rms input noise current for a user-specified bandwidth.

The frequency response is found by evaluating $A_{CL}^{ideal}(s)$ (Eq. 4.4) for $s = j\omega = j2\pi f$ to get the ideal closed-loop gain and similarly evaluating $T(s)$ (Eq. 4.6) to get the loop transmission. From these two quantities, the actual closed-loop gain, $A_{CL}(f)$ is found using Eq. 4.8. These calculations are repeated for a number of frequencies sufficient to produce a plot of the loop transmission and frequency response.

The phase margin is found by using the bisection method to solve iteratively for the frequency where $|T(f)| = 1$. This frequency is called the unity-gain crossover frequency, f_{co} , and the phase margin is 180° plus the phase of $T(f_{co})$. The user must supply two frequencies estimated to bound f_{co} in order to start the iterative process. The process is fairly forgiving, and initial frequency estimates within an order of magnitude or so of f_{co} usually lead rapidly to a solution.

The closed-loop bandwidth, f_{BW} , is found in much the same way as the phase margin. The actual closed-loop gain is iteratively solved for the frequency where $A_{CL}(f) = A_{CL}/\sqrt{2}$. This frequency is the -3 -dB corner frequency and is

also the bandwidth for a dc-coupled or very wideband amplifier.

To calculate the output noise power spectral densities, the program uses the noise source power spectral densities and the gain equations given in Section 4.2.3. The program can find and print the source power spectral density $S(f)$, the loop transmission $T(f)$, the actual closed-loop gain $A_{CL}(f)$ and the output power spectral density $S_{no}(f)$ due to each source for any given frequency. It is important to note that $S(\omega)$ and $S(f)$ represent the same function, only using different frequency units. The program can also produce a list of the output noise power spectral density at each frequency in a range. The user can select both the range of frequencies to be covered and the frequency spacing which may be linear or logarithmic. These lists are useful for viewing functional trends and for plotting the density functions.

The total output noise power for a given frequency band can be calculated from the noise power spectral density by using Eq. 4.16. The program carries out this integration numerically using a technique called adaptive quadrature [48]. Basically this is a technique which uses the trapezoidal method to approximate integration, but varies the step size depending upon the amplitude of the function. In regions where the function's value is consistently small, and the contribution to the integral is therefore small, large steps are used. In regions where the amplitude is large, smaller steps are used. This process minimizes computation time by using large steps where possible, but maintains high accuracy by using small steps where needed.

The user can specify lower and upper frequency limits and TRAMP will calculate the output noise power due to each source and also the total noise power. It will also refer the rms output noise voltage back to the input as an rms input noise current by dividing by the dc current gain (Eq. 4.19).

TRAMP Example

To illustrate the use of TRAMP, a simple amplifier design was developed using a hypothetical operational amplifier. The main goal of the design was to produce an amplifier that would have reasonably realistic parameters and characteristics, yet be simple enough that the frequency response and noise performance could be easily calculated by hand and compared with the results from TRAMP. For the op-amp, a single pole characteristic was assumed along with an open-loop gain of 10^6 and a gain-bandwidth product (GBW) of 100 MHz. The input noise voltage and current of the op-amp were assumed to have white spectra and to have values of $10 \text{ nV/Hz}^{1/2}$ and $10 \text{ pA/Hz}^{1/2}$ respectively. The other op-amp parameters were set high or low enough to be negligible. The source capacitance was also taken to be negligible, and the feedback resistor value was set to $1 \text{ k}\Omega$. The feedback capacitance (159 pF) was used to set the closed-loop bandwidth to 1 MHz. All of the amplifier parameters are given in Table 4.2.

This amplifier was analyzed using the techniques developed in Section 4.2.4 along with several additional approximations. The loop transmission has a pole due to the op-amp, a zero due to the feedback network and another pole due to

Table 4.2: Amplifier parameters for TRAMP example.

Parameter	Value	Units
A_0	1×10^6	V/V
R_{IN}	100	M Ω
C_{IN}	1	pF
ω_{p1}	628	rad/s
ω_{p2}	1×10^{14}	rad/s
ω_{p3}	1×10^{14}	rad/s
ω_{p4}	1×10^{14}	rad/s
ω_{z1}	1×10^{14}	rad/s
ω_{z2}	1×10^{14}	rad/s
R_F	1	k Ω
C_F	159	pF
C_S	0.1	pF
τ	0	s

the input impedance in parallel with the feedback impedance. Since the input impedance is very high, the zero and the second pole nearly cancel, and the loop transmission is nearly the same as the open-loop gain of the op-amp. Thus the crossover frequency is very nearly the gain-bandwidth of the op-amp, and the phase margin is 90°. Since the loop transmission is large at frequencies up to and above the ideal closed-loop bandwidth, the actual closed-loop bandwidth is controlled by R_F and C_F . These frequency response results are summarized in Table 4.3.

The noise performance of the amplifier was calculated as well. For the S_e noise source, the amplifier functions as unity gain stage with a bandwidth approximately the gain-bandwidth of the op-amp. Approximating the closed-loop

Table 4.3: Example amplifier characteristics calculated by hand.

Source	Source PSD ($f = 0$)	$S_{no}(0)$ (V^2/Hz)	Bandwidth (MHz)	e_{no} (μV rms)	i_{ni} (nA rms)
S_e	$10^{-16} V^2/Hz$	10^{-16}	100	125	125
S_i	$10^{-16} A^2/Hz$	10^{-16}	1	12.5	12.5
S_{RF}	$1.7 \times 10^{-17} V^2/Hz$	1.7×10^{-17}	1	5.1	5.1

response as having a single pole characteristic therefore gives a noise bandwidth of about $\frac{\pi}{2}GBW$ and the total output noise power is then $\overline{e_{noe}^2} \simeq \frac{\pi}{2}S_eGBW$, which can be easily calculated. For the S_i noise source the gain is R_F^2 and the noise bandwidth is approximately $\frac{\pi}{2}f_{BW}$, where f_{BW} is the closed-loop bandwidth of the amplifier. Thus $\overline{e_{noi}^2} \simeq \frac{\pi}{2}S_iR_{IN}^2f_{BW}$. Similarly the gain of the S_{RF} source is unity up to f_{BW} and $\overline{e_{noRF}^2} \simeq \frac{\pi}{2}S_{RF}f_{BW}$. The results of these calculations are also given in Table 4.3.

The same amplifier was analyzed using TRAMP, and as expected almost exactly the same values were found (Table 4.4). The noise was integrated from 1 Hz to 1000 MHz (instead of zero to infinity) to give the total noise, and that

Table 4.4: Example amplifier characteristics calculated by TRAMP.

Source	Input PSD ($f = 0$)	$S_{no}(0)$ (V^2/Hz)	Bandwidth (MHz)	e_{no} (μV rms)	i_{ni} (nA rms)
S_e	$10^{-16} V^2/Hz$	10^{-16}	101	122	122
S_i	$10^{-16} A^2/Hz$	10^{-16}	1	12.5	12.5
S_{RF}	$1.7 \times 10^{-17} V^2/Hz$	1.7×10^{-17}	1	5.1	5.1

probably accounts for the small discrepancy in e_{no} and i_{ni} . The advantages of TRAMP are not clear from these results, but even for this simple example more useful information is easily available from TRAMP. Loop transmission data obtained from TRAMP were plotted to give Figure 4.15, and from this figure, the single pole characteristic and 90° phase margin are clearly visible. Similarly, frequency response data were plotted to give Figure 4.16. This shows that frequency response is flat as expected and gives the complete phase response which might be important for some applications.

Figure 4.17 is a plot of the output noise power spectral density due to each noise source alone and also their total. This plot clearly shows the relative contributions and the frequency roll-offs of each source. The noise due to the S_i and S_{RF} sources begins to drop at 1 MHz while the noise due to the S_e source does not begin to drop until 100 MHz. This type of plot would be more useful for a more complicated density resulting from non-white noise sources or a more complicated frequency response. The value of Figure 4.17 is mainly that it shows the same noise power spectral densities described by the manual calculations thus verifying the operation of TRAMP.

4.4.2 VAMP: A CAD Tool for the Voltage-Sensitive Amplifier

A program very similar to TRAMP was developed as an aid in analyzing and designing voltage-sensitive amplifiers. The program, called VAMP (for Voltage-sensitive AMPlifier) uses the general model developed in Sections 4.3.1 through

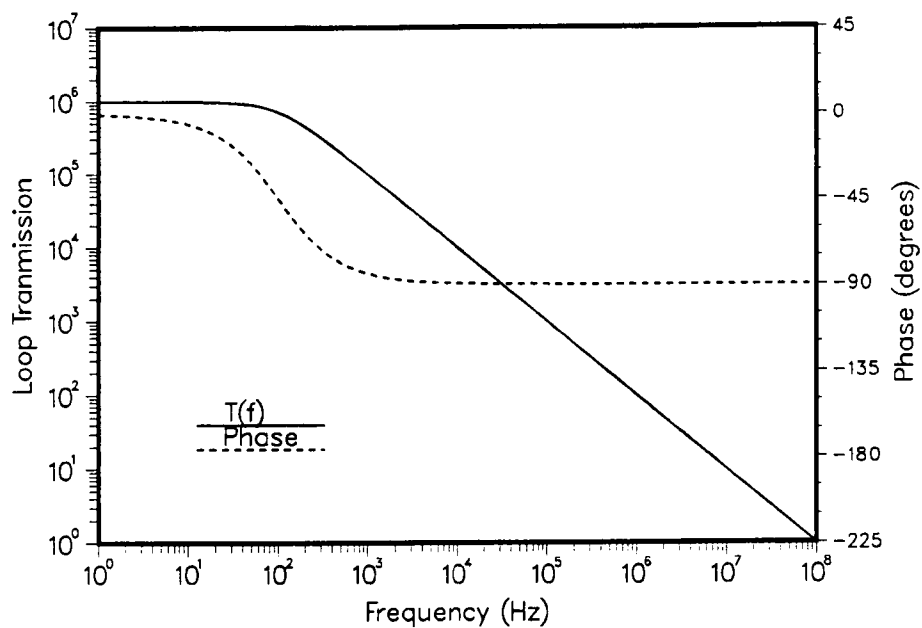


Figure 4.15: Loop transmission of example amplifier.

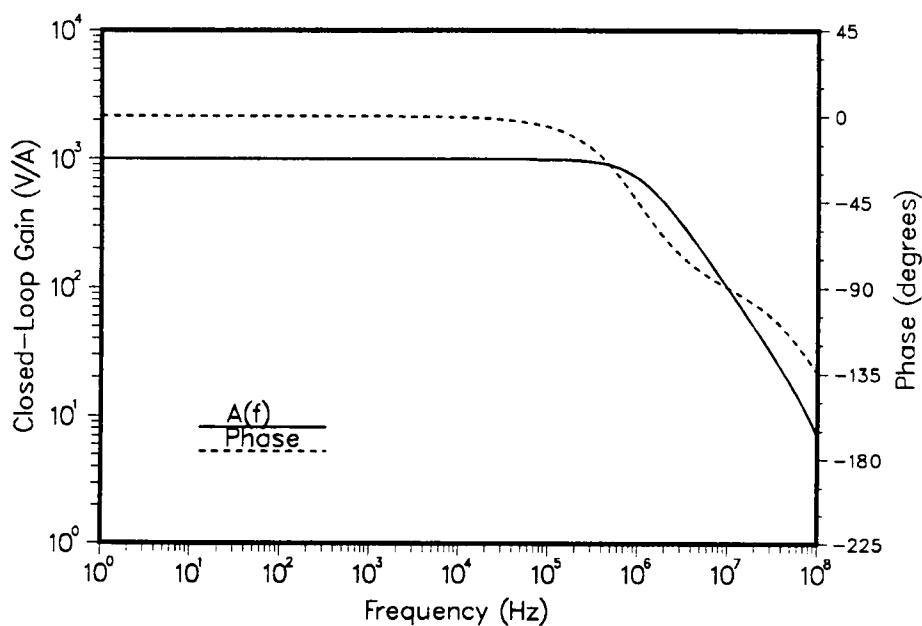


Figure 4.16: Closed-loop frequency response of example amplifier.

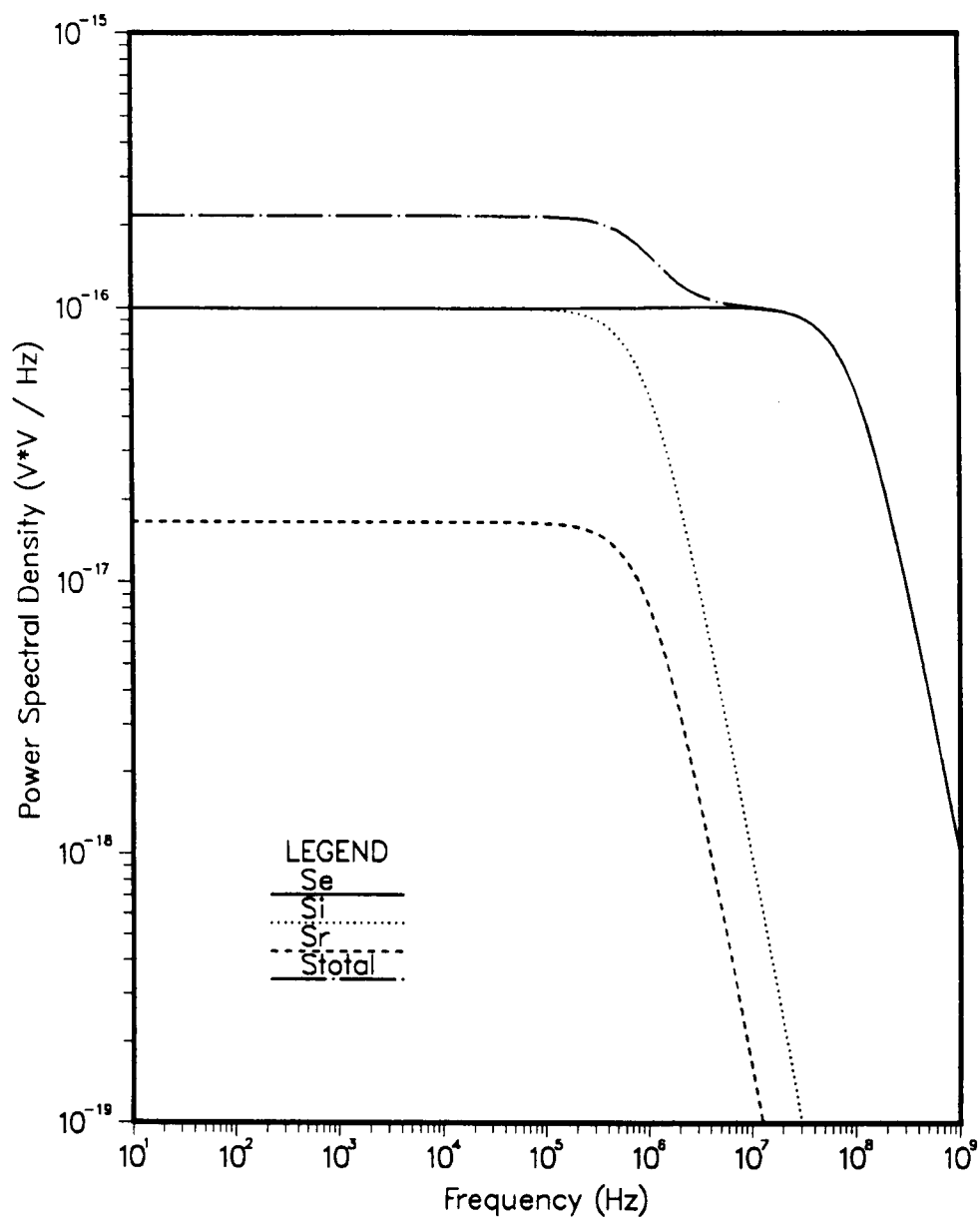


Figure 4.17: Noise power spectral densities for example amplifier.

4.3.3 as a basis to calculate the frequency response and noise performance of a transimpedance amplifier with parameters chosen by the user. The program has the same functions as TRAMP, and its use is very similar. An example of the use TRAMP was given in Section 4.4.1 Details of the program operation and organization and a complete listing are given in Appendix D.

4.5 Preamplifier Performance Estimation, Comparisons and Configuration Selection

A general comparison of the noise performance of the transimpedance and voltage-sensitive preamplifiers can be made by comparing Eqs. 4.90 and 4.167 for the case of equal bandwidths. For the transimpedance preamplifier, the rms input noise current was given by Eq. 4.90, which stated that

$$i_{ni} \simeq f_{BW}^{\frac{1}{2}} \sqrt{\frac{C_{IF}^2 S_e}{4C_F^2 R_F^2} + \frac{4kT}{R_F} + S_i}. \quad (4.169)$$

The rms input noise current for the voltage sensitive preamplifier was given by Eq. 4.167, but that equation can be simplified by assuming that $R_F \parallel R_1 \ll R_D$ and that $f_{NBW} \simeq f_{BW}$. R_1 would typically be low to reduce its noise and to give a reasonably high gain, and as f_{NBW} actually equals $\frac{\pi}{4}f_{BW}$, the assumptions are reasonable. Thus

$$i_{ni} \simeq f_{BW}^{\frac{1}{2}} \sqrt{\frac{S_e}{R_D^2} + \frac{4kT}{R_D} + S_{i+}}. \quad (4.170)$$

These equations are suprisingly similar and reveal that the voltage-sensitive preamplifier has lower i_{ni} for the same values of S_e and $S_i = S_{i+}$ if R_D can be

as large as R_F for the transimpedance preamplifier. (It should be noted that in the derivation of Eq. 4.90 that it was assumed that $C_{IF} \gg C_F$, so the factor multiplying S_e/R_F^2 must be greater than unity.) If R_D must be less than R_F (due to bandwidth considerations, for example), then the relative importance of the S_e , S_i , R_D and R_F noise sources determine which preamplifier has lower i_{ni} .

In the remainder of this section, results from the algebraic analyses of the two preamplifier types are used to estimate the noise performance of each. It will be assumed that either will be capable of the frequency response desired. For both types, the capacitance at the input should be minimized, so as a reasonable estimation of a minimum practical value, the sum $C_{IN} + C_S$ will be assumed to be 10 pF. Also for both amplifiers, the system bandwidth will be assumed to be 100 MHz with an anti-aliasing filter completely removing any noise contribution from frequencies above 100 MHz.

4.5.1 Transimpedance Amplifier Performance Estimation

For the transimpedance amplifier, a feedback resistor of 2 k Ω will be taken as a very conservative maximum value. Millard and Blalock [49] reported on a transimpedance amplifier achieving a 50-MHz bandwidth while using a 51.1-k Ω feedback resistor, so it seems reasonable to assume that a 100-MHz bandwidth could be achieved with a much smaller resistor. A 2-k Ω resistor will require $C_F = 1.6$ pF to set the bandwidth to 100 MHz and a gain-bandwidth of about 1.3 GHz, which is large, but not unreasonably so. Adding C_F to the 10 pF

assumed for C_S and C_{IN} gives $C_{IF} = 11.6$ pF. If a FET is used as an input device, then the noise current power spectral density S_i can be approximated as zero. With a single JFET like those used in the voltage sensitive preamp developed by Blalock [25], the noise voltage power spectral density S_e could be less than 10^{-18} V²/Hz.

With these values for C_{IF} , f_{BW} , R_F , S_e and S_i , the total rms input noise current for the transimpedance amplifier can be calculated from Eq. 4.90, which simplifies to

$$i_{ni} \simeq \sqrt{f_{BW}} \sqrt{\frac{C_{IF}^2 S_e}{4C_F^2 R_F^2} + \frac{4kT}{R_F}}, \quad (4.171)$$

when S_i is zero. Evaluating the expression gives $i_{ni} = 34$ nA. This value is not by any means the minimum value achievable, only a low value that seems readily obtainable.

4.5.2 Voltage-Sensitive Amplifier Performance Estimation

A very simple estimate of the minimum rms input noise current for the voltage-sensitive amplifier can be made by considering the extreme case where the noise due to the amplifier is insignificant compared to the detection resistor R_D . The noise of R_D is minimum when R_D is the maximum allowed by bandwidth considerations. For the case of a 100-MHz bandwidth and 10-pF capacitance shunting R_D , the maximum value of R_D is 159 Ω . The rms input noise current is given by Eq. 4.167 with all the amplifier noise sources set to zero. For that case, the

expression reduces to

$$i_{ni} \simeq \sqrt{\frac{4kT}{R_D}} \sqrt{f_{BW}}, \quad (4.172)$$

and for $R_D = 159 \, \Omega$, $i_{ni} = 102 \, \text{nA}$. If $S_e = 10^{-18} \, \text{V}^2/\text{Hz}$ were assumed as for the transimpedance amplifier, i_{ni} would increase to $120 \, \text{nA}$.

4.5.3 Selection of a Configuration

From the performance estimates, it seems clear that the transimpedance amplifier has a clear edge in noise performance for this application. For a 100-MHz bandwidth and 10 pF at the amplifier input, the minimum rms input noise current was 102 nA for the voltage-sensitive amplifier, while a reasonably conservative estimate for the transimpedance amplifier produced a figure of 34 nA. Even if an actual transimpedance amplifier achieves a noise current twice as large as that estimated, it would still be less than that possible with the voltage-sensitive amplifier. Therefore, the transimpedance configuration was selected for the preamplifier.

CHAPTER 5

AMPLIFIER DEVELOPMENT

In this chapter, the development of the preamplifier and two associated buffer amplifiers is described. A discussion of the modeling and characterization of transistors useful as preamplifier input devices is also included.

5.1 Preamplifier Input Device Characterization and Selection

For a well designed low-noise preamplifier, the total noise produced by the preamplifier can be little more than the noise produced by the input device alone. For this reason, selection of the input device and its operating region is very critical to the success of a low-noise preamplifier design. In this section, simple noise models for bipolar and field effect transistors are presented and adapted for use in the transimpedance amplifier noise model given in Chapter 4. This allows comparison of the devices on the basis of the rms input noise current that would be generated by a particular device when used in a transimpedance preamplifier. The remainder of the section describes the development of instruments used to characterize the devices and the results of these characterizations.

5.1.1 Simple Transistor Noise Models

The general noise model of the transimpedance amplifier that was developed in Chapter 4 can be applied very easily to the analysis of an actual amplifier if it is assumed that the majority of the preamplifier noise excepting R_F is due to the input device and that the noise model for the transistor can be simplified to give expressions for $S_e(\omega)$ and $S_i(\omega)$. There are a number of noise models available for different types of transistors, and those models are examined here, with the discussion being limited to silicon bipolar transistors and several types of field-effect transistors (FET's). The simple models are limited in their accuracy compared to more complex models or experimental data, but their usefulness lies in the ease with which different types of transistors and operating points can be compared.

Bipolar Transistor Noise Model

Chenette [50] developed a noise model based on the hybrid pi model for the bipolar transistor. In that model, sources were added to the hybrid pi model to generate noise. Motchenbacher [51] took Chenette's model and used it to calculate the equivalent noise voltage squared (ENV^2) and equivalent noise current squared (ENI^2) of the transistor. These two noise generators replace all of the internal noise generators of the model and are really power spectral densities as they have units of V^2/Hz and A^2/Hz , respectively. Thus, from

Motchenbacher,

$$ENV^2 = 4kTr'_b + 2qI_Cr_e^2 + 2qI_Br_b'^2 + \left\{ \frac{1}{f} \text{ terms} \right\}, \quad (5.1)$$

and

$$ENI^2 = 2qI_B + 2qI_C \left(\frac{f}{f_T} \right)^2 + \left\{ \frac{1}{f} \text{ terms} \right\}, \quad (5.2)$$

where r'_b is the base ohmic resistance, $r_e = kT/(qI_E)$, I_B is the base current, I_C is the collector current, I_E is the emitter current and f_T is the current gain-bandwidth product. The $\frac{1}{f}$ terms are not repeated here since they are important only below ~ 1 kHz and are not important for the frequency range of concern. These expressions can be simplified [52] to

$$ENV^2 \simeq 4kT \left(r'_b + \frac{r_e}{2} \right), \quad (5.3)$$

and

$$ENI^2 \simeq 2qI_B, \quad (5.4)$$

for the midband region when the $\frac{1}{f}$ terms are not important.

FET Noise Model

A basic discussion of amplifier noise analysis and a very useful summary of a noise model for the FET developed by van der Ziel [53,54] are given by Kennedy [52]. In that formulation, which is somewhat rearranged here, the FET has

$$ENV^2 = 4kT \left(\eta \frac{2}{g_m} \right) \left\{ 1 + \left(\frac{f_b}{f} \right)^x \right\}, \quad (5.5)$$

and

$$ENI^2 = 2qI_G + \kappa kT \left(\frac{\omega^2 C_{gs}^2}{g_m} \right), \quad (5.6)$$

where g_m is the transconductance, f_b is a break frequency between low-frequency and white noise, x is the exponent describing the behavior of the low-frequency noise, I_G is the gate current, κ is a factor approximately equal to one, and C_{gs} is the gate-to-source capacitance. The values of η , f_b and x depend upon the device, but for a JFET, $\eta \approx 1$ and f_b may be on the order of a few kHz or less [55,56]. For most FET devices, $0.5 < x < 2$ [52]. The low-frequency noise is commonly referred to as 1/f noise, since x is often unity. The part of the ENV^2 expression given by η^2/g_m is often called the noise resistance, R_n , of the FET since it has dimensions of ohms and a resistor equal to R_n would produce an equivalent amount of thermal noise. For a MOSFET, $1 < \eta < 3$ [52], and f_b may be on the order of one MHz [56]. The noise characteristics of GaAs MESFET's are somewhat similar to those of MOSFET's, except that f_b may be even greater [57]. It should be noted that ENV^2 and ENI^2 are partially correlated, and calculations ignoring that fact will overestimate the total noise.

5.1.2 Input Device Comparisons

For either the bipolar transistor or the FET, ENV^2 and ENI^2 are equivalent to S_e and S_i , respectively, if the preamplifier is designed so the input device generates essentially all of the noise. In Chapter 4, an approximation for the

rms input noise current was found to be

$$i_{ni} \simeq f_{BW}^{\frac{1}{2}} \sqrt{\frac{C_{IF}^2 S_e}{4C_F^2 R_F^2} + \frac{4kT}{R_F}} + S_i, \quad (5.7)$$

where C_{IF} was the sum of the source, preamp input and feedback capacitances. For this application, a good transistor is one that minimizes i_{ni} , and therefore S_e , S_i and C_{IN} should all be as low as possible. Unfortunately, since i_{ni} also depends on C_S , R_F , etc., the relative importance of minimizing S_e , S_i or C_{IN} is not clear.

Even if the parameters C_S , R_F , etc. are fixed by other considerations, the choice between an FET or a bipolar transistor as the input device of the transimpedance preamplifier is not a very clear one as it still depends strongly upon the device parameters that are assumed as being the best available for the particular type of transistor. This choice has been studied by engineers developing amplifiers for nuclear instrumentation and for fiber optic receivers. Although neither problem has exactly the same constraints as the prebreakdown current measurement problem, there are some striking similarities and some pertinent work has been done.

In nuclear instrumentation, the measurement process that bears the closest resemblance to process of measuring prebreakdown currents is the one applied to measure radiation energy. A good overview and description of this process is given by Knoll [58]. In this type of experiment, radiation incident on a detector produces an impulse of charge which is amplified by a charge-sensitive preamplifier. The detector impedance is mainly capacitive, and the charge collected can

be modeled as being due to a current source. The charge-sensitive amplifier is a current integrator, and its output is essentially a step in response to the impulse. This pulse is shaped by repeated differentiations and an RC integration to give a quasi-gaussian shaped pulse whose amplitude can be easily measured. The pulse amplitude is proportional to the charge deposited, which was proportional to the radiation energy. Thus the pulse amplitude is proportional to the energy.

An important figure of merit for the charge-sensitive preamplifier is the equivalent noise charge (ENC) which is given by the rms noise voltage e_{no} at the output of the shaping filter divided by the charge gain from the detector up to that point. e_{no} is essentially bandpassed preamplifier noise, so low ENC is obtained through the use of a low-noise preamplifier which requires a low-noise input transistor. For this application, like the prebreakdown current measurement, a good transistor is one with low S_e , S_i and C_{IN} .

A good review of transistor technologies and their usefulness as input stages for charge-sensitive preamplifiers is given by Schmitt et al. [56], and a brief summary of those comments follows. For longer shaping filter time constants (> 100 ns), JFETs are preferred due to their lower $1/f$ noise and low gate current. For medium time constants (between 30 and ~ 500 ns, MOSFETs are preferred due to their lower S_e for a given C_{IN} compared to JFETs, despite having more $1/f$ noise. For shorter time constants (< 20 ns), bipolar transistors are preferred due to their low S_e and high bandwidth capability, despite significant current noise.

The short time constant case probably is the one most relevant to the design of a 100-MHz bandwidth transimpedance amplifier. A very detailed study of the choice between FETs and bipolar transistors was made by Gatti et al. [59], who found paralleled bipolar transistors superior to paralleled JFETs for time constants less than about 30 ns. However, their conclusions are very sensitive to the assumed device parameters, and since that time, the characteristics of both FETs and bipolar transistors have been improved.

Wideband data transmission using optical receivers require low-noise preamplifiers with many of the same characteristics needed for amplifying prebreakdown currents. An optical receiver basically consists of a photodetector, a preamplifier, a shaping filter and circuits to regenerate the (transmitted) digital data [60]. Light pulses incident on the photodetector (usually a PIN or avalanche photodiode) produce currents which are amplified by the preamplifier. The two main types of preamplifiers used are the high-impedance and the transimpedance designs. These two types are the same as the voltage-sensitive and transimpedance amplifiers considered for prebreakdown current measurements. The photodiode replaces the test cell and electrodes, but the circuit model of a current source in parallel with source capacitance is the same.

Minimization of the receiver noise gives requirements much like those for measuring prebreakdown currents. For either type of amplifier, use of a bipolar transistor means r'_b and C_{IN} should be low and β should be high to give low I_B . When an FET is used, C_{IN} should be low and g_m should be high. In any

case, R_D or R_F should be as high as possible [61,60]. For wideband receivers, the high-impedance amplifier with an FET input stage was found to give the best noise performance [60], but once again the conclusions are strictly correct only for an amplifier constructed using devices with the assumed parameters.

Since the bipolar transistor was considered superior for some charge-sensitive applications, it should be seriously considered for use in the prebreakdown current transimpedance amplifier input stage. A cascode configuration for the first stage gives good high frequency performance and minimum input capacitance. A bipolar transistor used as the input device of a cascode has C_{IN} given by

$$C_{IN} = \frac{1}{2\pi r_e f_T}. \quad (5.8)$$

If a low-noise transistor has $r'_b = 15\Omega$, $\beta = 100$ and $f_T = 1$ GHz for $I_C = 1$ mA, then $C_{IN} = 6.1$ pF. These were the parameters assumed by Gatti et al. [59] to represent the best devices, and they are very similar to those found by Mori et al. [62] in a newer study except that higher f_T values are available. The ENV^2 is given by Eq. 5.3 and is 4.65×10^{-19} V²/Hz. The ENI^2 is given by Eq. 5.4, and a value for i_{ni} due to ENI^2 alone can be found from

$$i_{ni} = \sqrt{ENI^2 f_N} = \sqrt{2qI_B f_N}, \quad (5.9)$$

where f_N is the noise bandwidth. For a noise bandwidth of 100 MHz, i_{ni} is 18 nA rms for $I_C = 1$ mA. A lower I_C reduces I_B and consequently C_{IN} and ENI^2 , but it increases ENV^2 . A higher I_C increases C_{IN} and ENI^2 , but decreases ENV^2 .

An optimal operating point for the bipolar transistor depends not only on the device, but on R_F , C_S , etc. as well. However, the calculated values of ENV^2 and i_{ni} can serve to compare bipolar and FET performance for this application. FETs are available with ENV^2 and C_{IN} values comparable to those of the bipolar transistor biased at 1 mA and without the drawback of non-negligible ENI^2 . Biasing the bipolar transistor at lower collector currents does not improve the situation as ENV^2 is larger than for the FET and ENI^2 is still not as small. Biasing the bipolar transistor at higher collector currents does not improve the situation either as i_{ni} due to ENI^2 approaches the level predicted in Section 4.5.1 (34 nA) for an entire transimpedance amplifier with an FET input. Thus it appears that FETs offer better noise performance than bipolar transistors for this application.

When an FET is used for the input stage, S_i is probably negligible, and this allows Eq. 5.7 to be simplified to

$$i_{ni} \simeq f_{BW}^{\frac{1}{2}} \sqrt{\frac{C_{IF}^2 S_e}{4C_F^2 R_F^2} + \frac{4kT}{R_F}}. \quad (5.10)$$

The factors in this expression that can be affected by the choice of an FET are C_{IF} and S_e , and for a given bandwidth, C_F and R_F , i_{ni} is reduced by lowering the product $S_e C_{IF}^2$. Substituting for S_e and C_{IF} gives

$$S_e C_{IF}^2 = 4kT \left(\eta \frac{\frac{2}{3}}{g_m} \right) (C_S + C_{IN} + C_F)^2. \quad (5.11)$$

A “goodness” factor λ can be defined by dividing $S_e C_{IF}^2$ by $\frac{8}{3}kT$ to give

$$\lambda = \frac{\eta}{g_m} (C_S + C_{IN} + C_F)^2. \quad (5.12)$$

λ serves as a basis for comparing FETs, with lower λ corresponding to lower noise.

Often an improvement in the noise performance can be made by paralleling input devices. Higher g_m and therefore lower R_n results from paralleled devices. Unfortunately, C_{IN} also increases, so decreases in λ depend upon the relative sizes of C_{IN} and $C_S + C_F$. For charge-sensitive preamplifiers used in radiation measurements, the optimum performance is obtained when n identical FETs are used with $nC'_{IN} = C_S$, where C'_{IN} denotes the input capacitance of a single FET and $nC'_{IN} = C_{IN}$. g_m increases as ng'_m , where g'_m is the transconductance of one FET.

Similar results were found for the wideband transimpedance amplifier when paralleled transistors are used. Substituting nC'_{IN} for C_{IN} and ng'_m for g_m in the expression for λ and dropping the “ $\prime$ ” superscript gives

$$\lambda = \frac{\eta}{ng_m}(C_S + nC_{IN} + C_F)^2. \quad (5.13)$$

Henceforth g_m and C_{IN} will apply to single or multiple transistors depending upon the context. Minimizing λ requires $nC_{IN} = C_S + C_F$, or $nC_{IN} \simeq C_S$ when C_F is small.

This expression was evaluated for $\eta = 1$ using g_m and C_{IN} values for several different low-noise FETs, and the results are given in Table 5.1. As expected, λ is smallest for a given device when $n \approx C_S/C_{IN}$, however, the minimum is fairly broad, and there may be several values of n which give essentially the same noise performance. The g_m , C_{IN} and I_D values given are believed to be typical

Table 5.1: λ values for low-noise FETs.

Device		J309	SK152	BF964s	MRF966
type		JFET	JFET	MOSFET	MESFET
material		silicon	silicon	silicon	GaAs
C_{IN} (pF)		6.3	8	2.2	1
g_m (mS)		14	24	18	22
I_D (mA)		20	12	10	25
λ	$n = 1$	9.1	7.0	2.9	1.64
	$n = 2$	9.5	9.2	2.5	1.11
	$n = 3$	10.8	11.7	2.5	0.97
	$n = 4$			2.7	0.92
	$n = 5$				0.91
	$n = 6$				0.92

values and are based on manufacturer's specification and various experimental measurements. For the JFETs, the I_D values are actually I_{DSS} values since g_m is maximum for I_{DSS} . For the MOSFET and GaAs MESFET, the I_D values are not maximum, but correspond to approximately maximum g_m . The MOSFET and GaAs MESFET are actually dual gate devices and are biased so the gate 1-to-source voltage controls the drain current when the device is to be used as an amplifier and not as a mixer or automatic gain control stage.

Based on this study, the GaAs MESFET appears to be the best device due mainly to its extremely low input capacitance. Although the study assumed $\eta = 1$, the λ values for the GaAs MESFET are so much lower that η could be as large as 2.5, and the MESFET would still provide superior noise performance. Another potential drawback to the use of GaAs MESFETs is the $1/f$ noise, which

was not accounted for in this simple study.

5.1.3 FET Characterization

Certain characteristics of the input device need to be measured as they are critical to the design and analysis of the transimpedance preamplifier. In order to apply the FET noise model, C_{IN} , g_m and $S_e(\omega)$ need to be known. C_{IN} and g_m are needed as well for the frequency and stability analyses.

Measurement of C_{IN} and g_m

To measure C_{IN} and g_m , a simple instrument was developed which could test both enhancement- or depletion-mode FETs under typical operating conditions. The instrument can accomodate both single and dual gate devices. Gate-to-source bias voltages can be varied over a wide range with $-3\text{ V} < V_{G1S} < 3\text{ V}$ and $0\text{ V} < V_{G2S} < 5\text{ V}$. The drain current can be adjusted from 0 to 30 mA in 2-mA steps, while V_{DS} is fixed at 10 V. The FET to be tested forms part of a folded cascode stage along with a MPSH81 bipolar transistor biased with $I_E = 10\text{ mA}$. The instrument is shown in Figure 5.1.

To use the instrument, the FET I_D is selected by closing some combination of switches 1 to 4 and then by adjusting V_{G1S} to give $V_{DS} = 10\text{ V}$. A low-amplitude ($\sim 50\text{ mV}$) squarewave is applied to the input, and the response is viewed on an oscilloscope. The step response of the cascode amplifier circuit is normally very fast, but for this instrument, it is deliberately slowed by placing

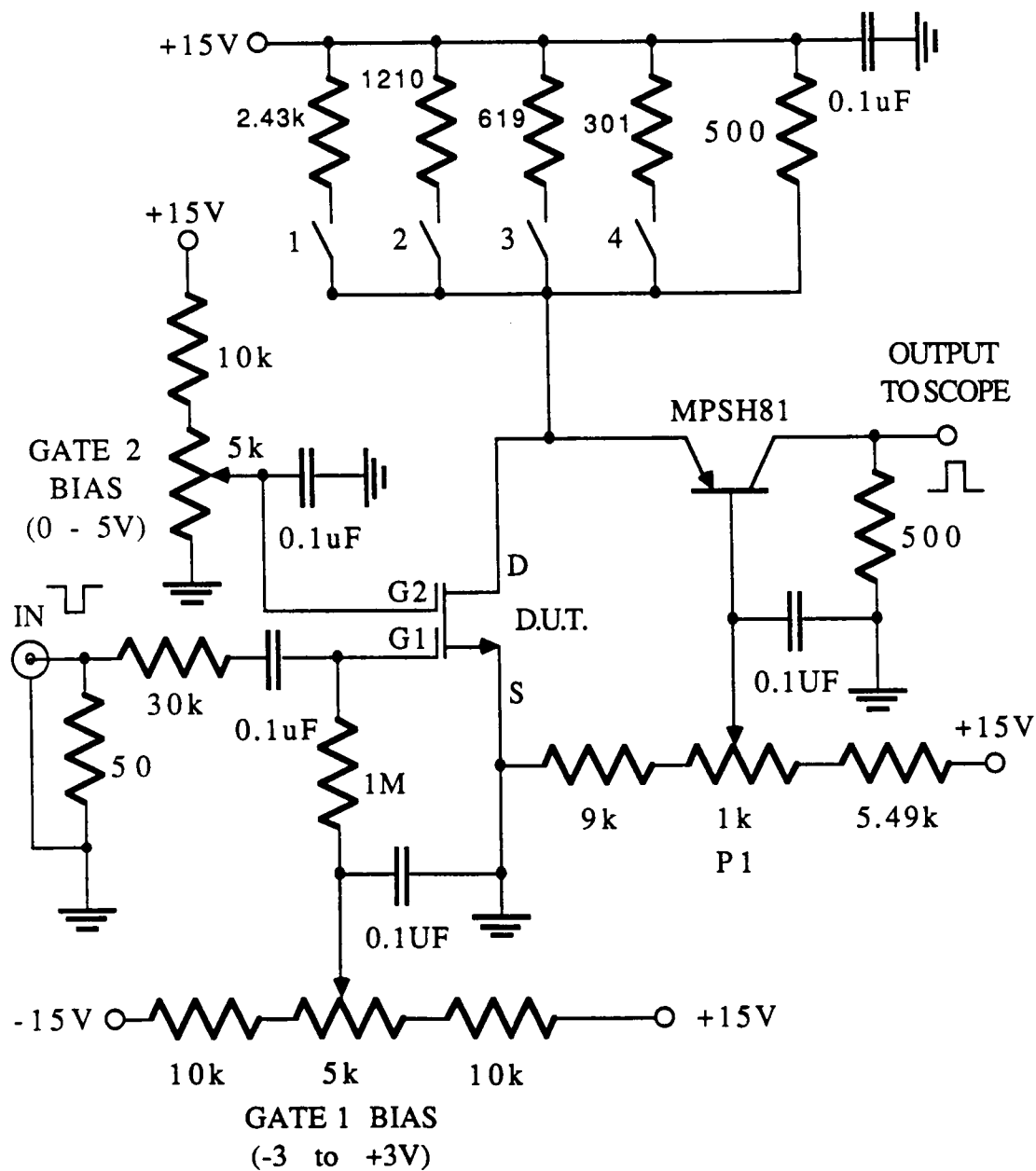


Figure 5.1: FET input capacitance measurement circuit.

$R_G = 30 \text{ k}\Omega$ in series with the gate. The output risetime is dominated by the $R_G C_{IN}$ time constant since it is much greater than the generator, amplifier or oscilloscope time constants. The input capacitance can then be calculated from the measured risetime by

$$C_{IN} = \frac{t_r}{2.2R_G} - C_{stray}, \quad (5.14)$$

where C_{stray} is the fixture capacitance and is 1 pF.

The same setup can be used to measure the g_m of the FET. For a cascode amplifier, the voltage gain is

$$A_v = \frac{V_{out}}{V_{in}} \simeq -g_m R_L, \quad (5.15)$$

where R_L is the parallel combination of $500 \text{ }\Omega$ and the output impedance of the bipolar transistor and is approximately $460 \text{ }\Omega$. The g_m of the FET can be calculated from the measured gain and R_L .

Measurement of FET Noise Voltage Spectrum

Another instrument was developed in order to measure $S_e(\omega)$ for FETs. Since $S_e(\omega)$ is very small for a low noise transistor, a large amount of amplification is needed to boost the noise signal so it can be analyzed. The FET being tested becomes part of a common source amplifier which is followed by a low-noise amplifier (LNA). The amplified noise of the FET is digitized, and spectral analysis is performed on the noise waveform. The measured noise power spectral density is then referred back to the FET input by dividing by the system voltage

gain squared. A block diagram of the instrument and the associated equipment is shown in Figure 5.2.

The circuit diagram of the noise measurement instrument is shown in Figure 5.3. It can test FETs with g_m above 5 mS at drain currents ranging from 0 to 30 mA for $V_{DS} = 5$ V. V_{DS} is adjustable from approximately 2 to 20 V. The maximum drain current available depends on V_{DS} and is less for higher V_{DS} . The instrument can be used with either enhancement- or depletion-mode devices with one or two gates. The gate 1-to-source voltage (V_{G1S}) can range from -5 V to 5 V while gate 2-to-source voltage (V_{G2S}) can range from 0 to 5 V.

The biasing portions of the instrument are basically three simple power supplies. The gates are biased using potentiometers and are ac-grounded with capacitors, while the drain is biased with a regulated supply. Buffered monitors allow the measurement of the reference voltage V_{ref} , I_D and V_{DS} . Three feedback loops allow the user to set any one of the three monitored quantities, while one of others (only two of the three are independent) must be set by manually adjusting the gate voltages V_{G1S} and V_{G2S} . The reference voltage loop uses the U3-based amplifiers to maintain V_{ref} at desired level by controlling the voltage at the base of the D41E5 transistor which in turn controls the emitter voltage (V_{ref}). The I_D loop monitors the current by measuring the voltage developed across the $500\ \Omega$ resistor and uses the U2-based amplifiers to maintain the current supplied by the D41E5 transistor. The V_{DS} loop is very similar to the V_{ref} loop except that it monitors and controls V_{DS} .

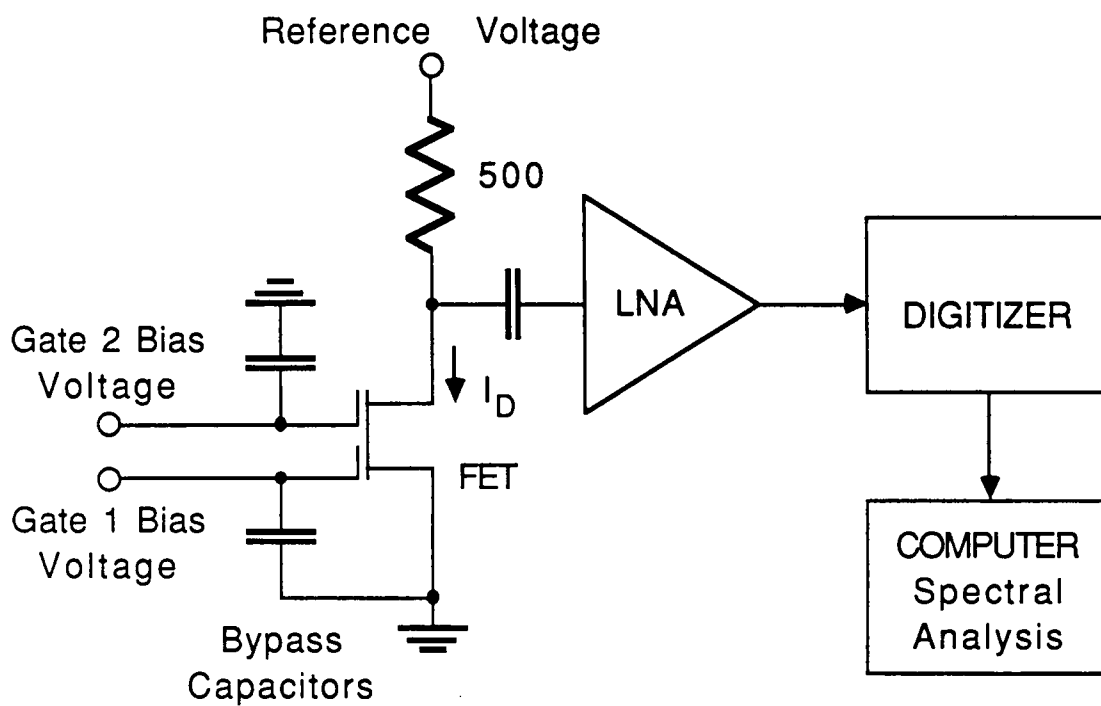


Figure 5.2: FET noise measurement block diagram.

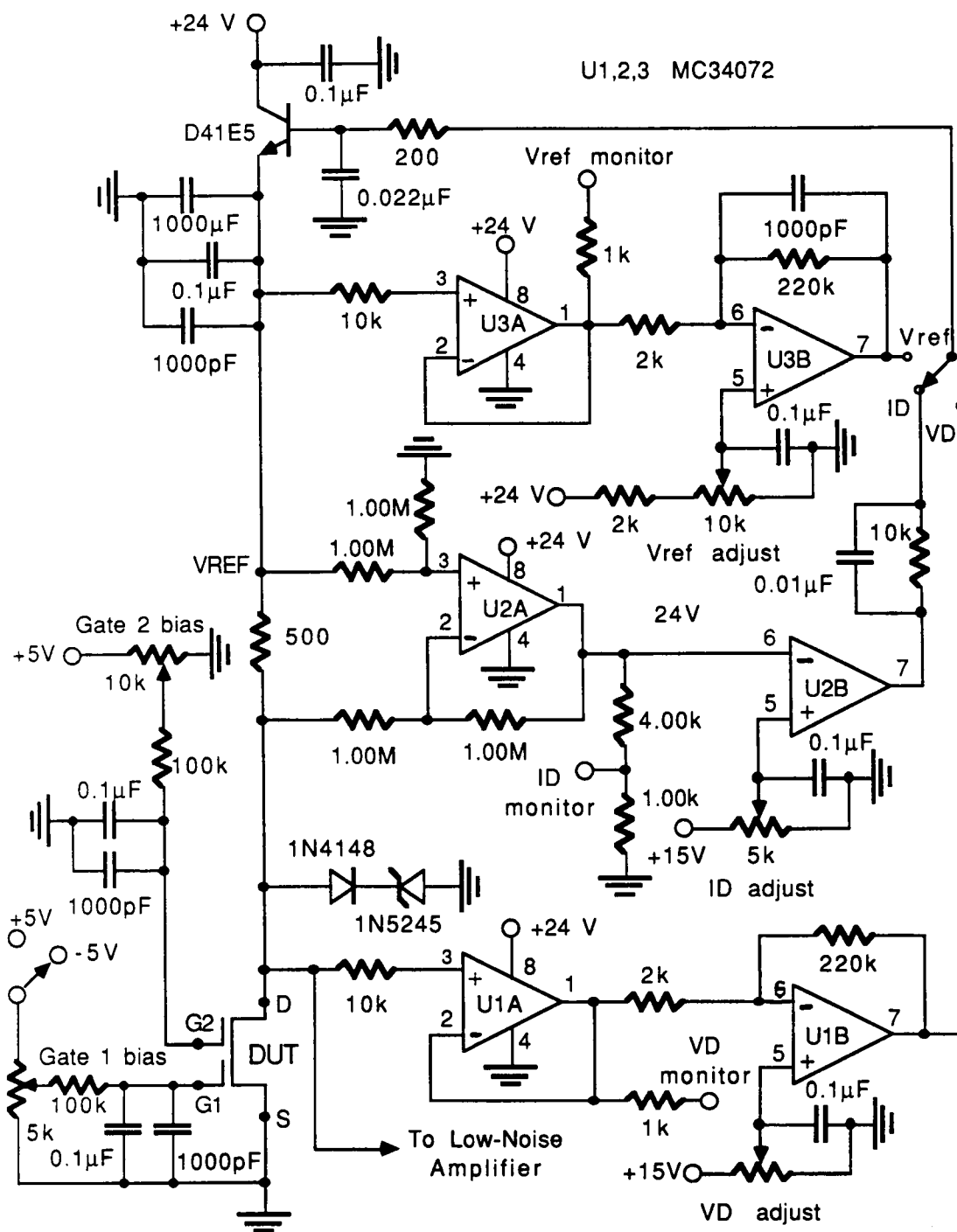


Figure 5.3: FET noise measurement instrument circuit diagram.

The $500\ \Omega$ load of the common source (CS) stage (the device under test) was chosen for several reasons. First, the largest resistor possible is needed as that provides the maximum amplification. The more amplification that is provided by the CS stage, the less important are the noise contributions by the load resistor and the LNA. For an FET with minimum g_m (5 mS), the $500\ \Omega$ resistor contributes about one third of the total thermal noise power, but for an FET with $g_m = 15$ mS, it contributes only one fifth of the total. If greater accuracy is needed, the noise due to the instrument alone can be measured and subtracted from the total noise. The second reason to choose $500\ \Omega$ was the frequency response of the instrument. A 20-MHz bandwidth was chosen as it was felt that would be sufficient for most measurements and not require excessive development time. With 10 to 15 pF estimated for the LNA input capacitance, $500\ \Omega$ was the maximum value that would not jeopardize the frequency response.

The instrument is normally used by first adjusting the bias points to obtain the desired I_D and V_{DS} . The I_D mode is useful for setting up. In this mode, the drain current can be set as needed, and V_{G1S} can then be adjusted to give the desired V_{DS} without changing I_D . To make a noise measurement, the instrument needs to be in the V_{ref} mode, so V_{ref} should be measured before leaving the I_D mode, the V_{ref} control set to the measured value and the mode control switched to V_{ref} . This should maintain the same I_D and V_{DS} . Once the FET is correctly biased, the amplified noise can be analyzed.

The LNA is a modified version of the voltage-sensitive amplifier developed

by Blalock et al. [25]. This particular version has an unloaded gain of 200 and approximately a 25-MHz bandwidth. The amplifier has noise resistance $R_n = 44 \Omega$, so the noise generated by the amplifier is small compared to that generated by the 500Ω resistor and insignificant compared to that generated by the FET. Since the amplifier is used in this application to measure the noise voltage developed across the 500Ω resistor, the protection circuit and input resistor were removed.

Spectral analysis of the digitized noise waveforms is performed by taking the fast Fourier transform (FFT) of each waveform and then squaring the magnitudes of the Fourier coefficients to give the power (actually V^2) in each frequency increment. The average power in each frequency increment is found by summing the contributions of all the waveforms at that frequency and dividing by the number of waveforms. Taking a number of noise waveforms and averaging the noise power improves the statistical accuracy of the measurement. The average power is then divided by the width in Hz of the frequency increment to give the output power spectral density. When the digitizer is used with a sampling rate of 100 Msamples/s, the Nyquist frequency is 50 MHz. For 2048 samples, that results in a frequency increment of 48.8 kHz, so the resulting spectrum ranges from about 50 kHz to 50 MHz with 48.8 kHz steps. $S_e(\omega)$ for the FET is finally obtained by dividing by the voltage gain squared which is given by

$$A_v^2 = (500g_m A_{LNA})^2, \quad (5.16)$$

where g_m is the FET transconductance at the I_D value for the test and A_{LNA}

is the voltage gain of the low-noise amplifier from its input to the input of the digitizer.

5.1.4 Measured Characteristics of MRF966 GaAs MESFET

The transconductance of a typical MRF966 GaAs MESFET was measured as a function of drain current with $V_{G2S} = 0$ V and $V_{DS} = 10$ V. I_D and g_m values were recorded along with the corresponding V_{G1S} values and are plotted in Figure 5.4. The input capacitance C_{IN} (actually C_{iss}) was 0.9 pF. These values agree with the manufacturer's data [63].

The noise voltage spectrum of a typical MRF966 GaAsFET was measured and is shown in Figure 5.5. The bias conditions were: $I_D = 10$ mA, $V_{G1S} = -2.1$ V, $V_{G2S} = 0$ V, $V_{DS} = 5$ V and $g_m \simeq 17$ mS. The spectrum obviously has a 1/f-type component that dominates for frequencies up to at least 20 MHz. The decrease in the spectrum above 20 MHz is due to bandwidth limitations of the noise measurement instrument. For the 1/f region, a power-law fit of the data is $S_e(f) = k_1 f^x$, where $k_1 = 1.05 \times 10^{-8}$ V²/Hz and $x = 1.37$. If it is assumed that the thermal noise of the FET equals 1.5 times the theoretical amount ($\eta = 1.5$ and therefore $R_n = 4kT/g_m$), then putting $S_e(f)$ in the form of Eq. 5.5 gives

$$S_e(f) = 4kT \left(\frac{\frac{2}{3}\eta}{g_m} \right) \left\{ 1 + \left(\frac{f_b}{f} \right)^x \right\} = 9.22 \times 10^{-19} \left\{ 1 + \left(\frac{21.9 \text{ MHz}}{f} \right)^{1.37} \right\} [\text{V}^2/\text{Hz}]. \quad (5.17)$$

Unfortunately, it is not possible to determine the thermal contribution exactly from Figure 5.5, since the white noise is obscured by the instrument frequency

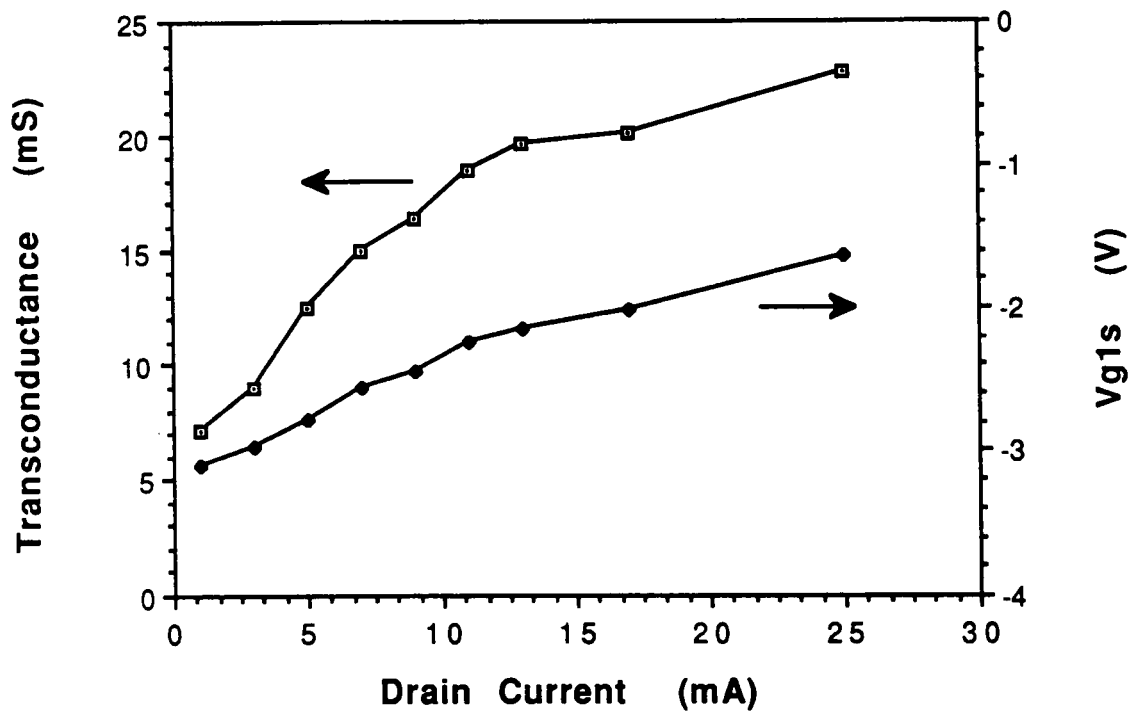


Figure 5.4: Transconductance and V_{G1S} as a function of I_D for MRF966 device.

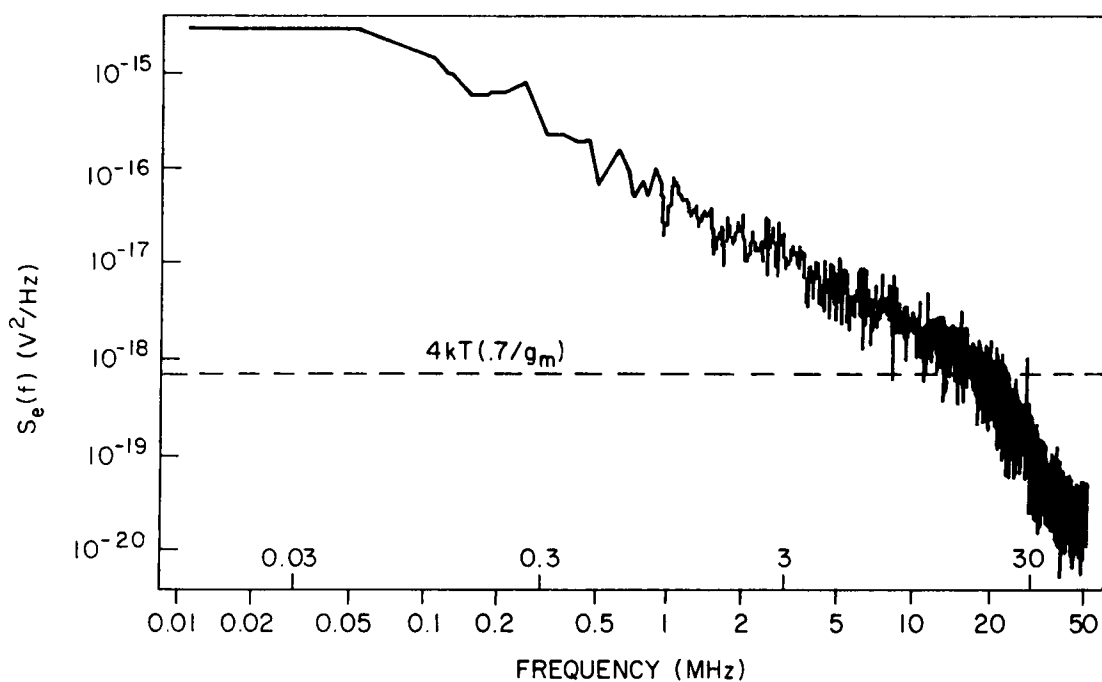


Figure 5.5: Input noise power spectral density for typical MRF966.

roll-off. It is reasonable to assume that $\eta < 3$, since that would require $f_b = 12$ MHz, and it clearly is higher than that.

5.1.5 Simple Model for Dual Gate GaAs MESFET

The drain current for an FET in saturation can be approximated by a simple square law if channel length modulation is neglected:

$$I_D = \beta (|V_{GS}| - |V_T|)^2, \quad (5.18)$$

where β is the transconductance parameter, V_{GS} is the gate-to-source voltage and V_T is the threshold voltage. Slightly different terminology is used for different types of FETs, but the same square law can be used for JFETs, silicon MOSFETs or GaAs MESFETs as well. For JFETs, I_{DSS} (I_D for $V_{GS} = 0$ V) is usually specified instead of β , but they are related by $\beta = I_{DSS}/V_T^2$.

The transconductance for a square law device is given by

$$g_m \equiv \left(\frac{\partial I_D}{\partial V_{GS}} \right)_{V_{DS}=\text{const}}, \quad (5.19)$$

when channel length modulation is neglected. By substituting for I_D (Eq. 5.18), the transconductance becomes

$$g_m = 2\beta(|V_{GS}| - |V_T|) = 2\sqrt{\beta I_D}. \quad (5.20)$$

The same data that produced Figure 5.4 was re-plotted in Figure 5.6 with I_D and g_m as functions of V_{G1S} . It can be seen that $g_m \propto |V_{GS}| - c_1$, where c_1 is a constant, and $I_D \propto (|V_{GS}| - c_2)^2$, where c_2 is a constant, for $I_D < 20$ mA. If the

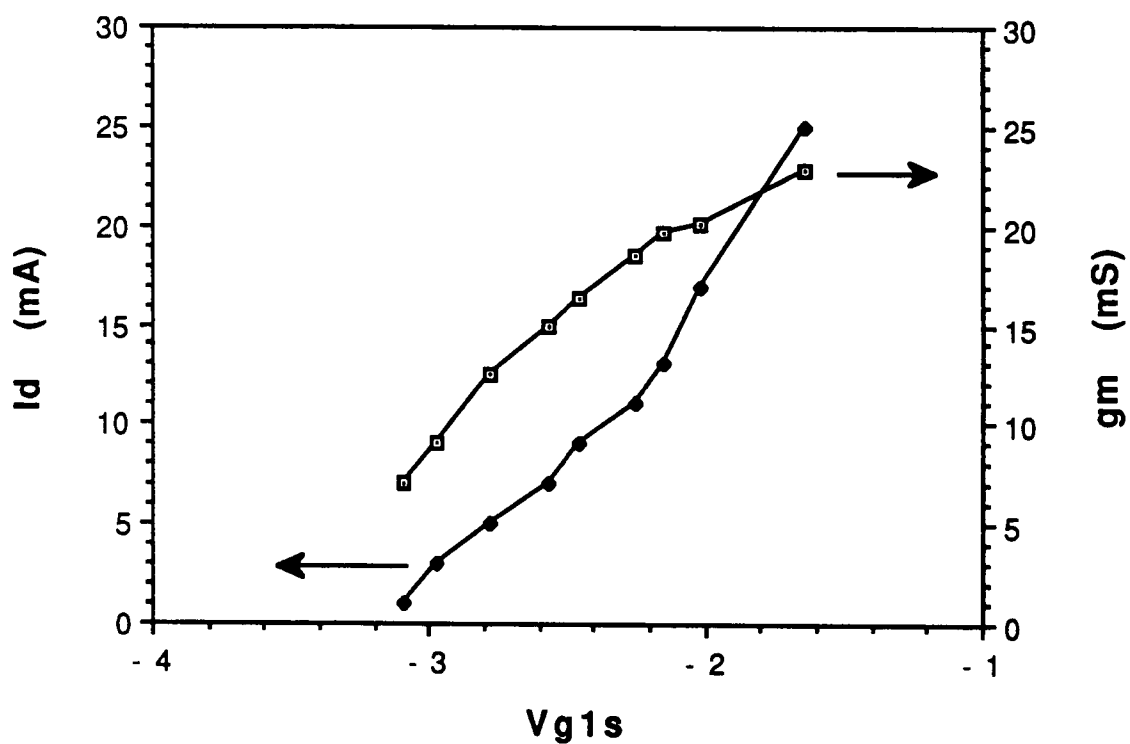


Figure 5.6: Drain current and transconductance as a function of V_{G1S} for MRF966 device.

data of Figure 5.6 is fitted to Eq. 5.18, then $\beta = 6.74 \text{ ma/V}^2$ and $V_T = -3.55 \text{ V}$. The square law FET model applies reasonably well for small currents, but does not fit the experimental or the manufacturer's data for higher currents. For example, the g_m of the device actually decreases for higher currents, and that is not predicted at all by Eq. 5.20.

The MRF966 is not a single MESFET, but a dual-gate device. Ordinarily it is operated as a three-terminal device by connecting gate 2 to the source, but in that mode it does not behave like a square-law device except at low currents. A more accurate model is that of two FETs connected in series as shown in Figure 5.7. This model can explain the behavior of the dual-gate device by viewing the circuit as a cascode amplifier. In the cascode configuration, the first transistor functions as a common-source amplifier with the second transistor serving as a load for the first and as a common-gate amplifier. The cascode has several advantages over a common-source stage alone. It has the same transconductance, but better input-to-output isolation and lower input capacitance due to decreased Miller effect.

To understand the behavior of the dual-gate device, it is necessary to find the region of operation for each of the FETs in the model. The operation of a FET is usually divided into two regions, the *ohmic* or *linear* region and the *saturation* or *constant current* region. Operation in the ohmic region is defined by

$$V_{DS} < |V_T| - |V_{GS}|, \quad (5.21)$$

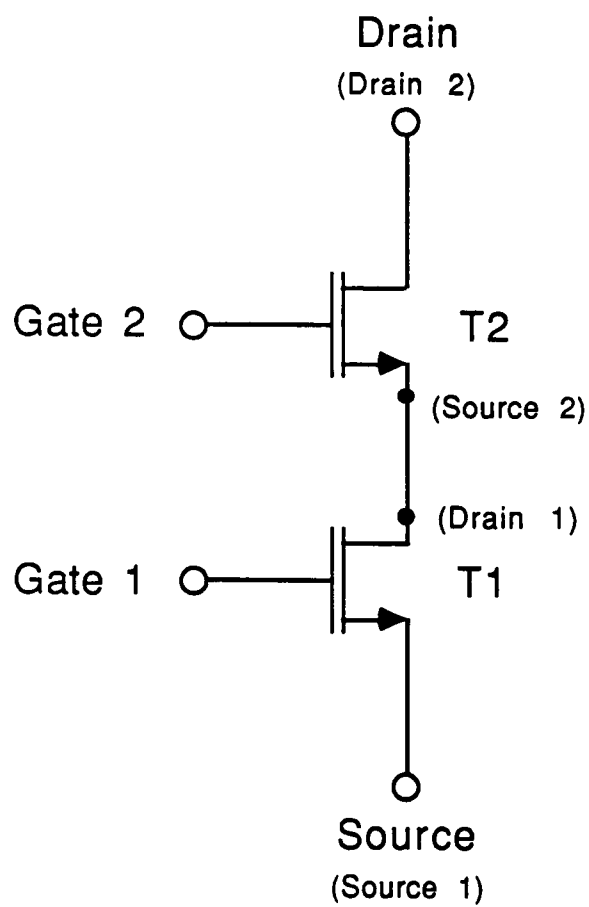


Figure 5.7: Cascode model for dual-gate FET.

while the saturation region is defined by

$$V_{DS} > |V_T| - |V_{GS}|. \quad (5.22)$$

When used as an amplifier, an FET is usually operated in the saturation region since the output impedance is higher and the g_m is much higher for a given I_D .

With the two FETs connected as shown in Figure 5.7, $I_D = I_{D1} = I_{D2}$, and since gate 2 is connected to the source (source 1), $V_{G2S2} = -V_{D1S1}$. For small drain currents, both FETs are saturated and have essentially identical V_{GS} . As the drain current increases, V_{G1S1} and V_{G2S2} decrease, and V_{D1S1} therefore decreases. At some point, T1 is no longer saturated and its g_m begins to decrease, even as the drain current continues to increase. The transition point occurs when

$$V_{D1S1} = |V_T| - |V_{G1S1}|. \quad (5.23)$$

Since the drain current is the same for both FETs, $V_{G1S1} \simeq V_{G2S2}$ and $V_{G2S2} = -V_{D1S1}$, $|V_{G1S1}| \simeq V_{D1S1}$, and therefore

$$|V_{G1S1}| \simeq |V_T| - |V_{G1S1}|, \quad (5.24)$$

which can be rewritten as

$$2|V_{GS}| \simeq |V_T|, \quad (5.25)$$

since $V_{GS} = V_{G1S1}$. Thus the transition point occurs when $V_{GS} = V_T/2$. For a typical MRF966, that point would be $V_{GS} = -1.77$ V.

A simple SPICE model based on the circuit of Figure 5.7 was developed for the MRF966. The β and V_T values used were those found experimentally.

The SPICE n-channel JFET model was used for the FETs since it is a simple square law model and allows simply specifying C_{gs} and C_{gd} without requiring any knowledge of the device geometry like the MOSFET model does. The zero-bias junction capacitances can be adjusted so the junction capacitances with bias applied match the experimentally observed capacitances, and the small-signal model will be mathematically accurate even though not physically accurate. This model was used to generate the common-source characteristic curves shown in Figure 5.8 and the g_m values shown in Table 5.2.

In SPICE, the thermal noise of the FET channel is modeled by a current noise generator connected between the drain and source with noise power spectral density given by

$$S_{ic}(\omega) = 4kT \left(\frac{2}{3}g_m \right) \left[1 + \frac{f_b}{f} \right], \quad (5.26)$$

where

$$f_b = \frac{K_F I_D^{A_F}}{4kT \left(\frac{2}{3}g_m \right)}. \quad (5.27)$$

f_b is the break frequency between $1/f$ -type noise and white noise. The noise model parameters that can be adjusted by the user are K_F and A_F . A_F is usually unity, and for that case K_F is given by

$$K_F = \frac{4kT \left(\frac{2}{3}g_m \right) f_b}{I_D}. \quad (5.28)$$

Thus K_F can be calculated from a measured values of f_b , g_m and I_D .

For an FET used in a common-source configuration, the noise due to the

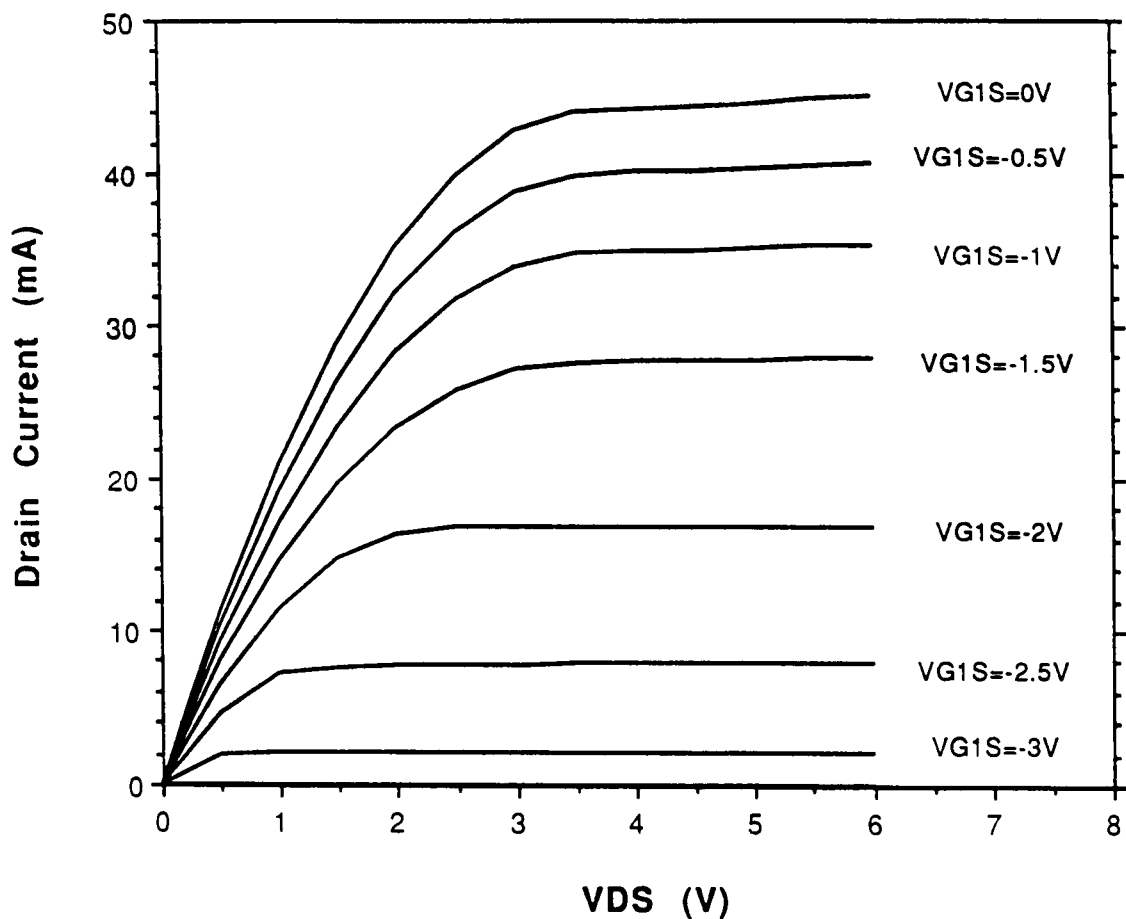


Figure 5.8: Common source characteristic curves for typical MRF966.

Table 5.2: Comparison of experimentally-measured and modeled MRF966 transconductances.

I_D (mA)	Experiment		SPICE model	
	V_{G1S} (V)	g_m (mS)	V_{G1S} (V)	g_m (mS)
7	-2.57	14.9	-2.55	14.2
9	-2.45	16.3	-2.42	15.8
11	-2.25	18.5	-2.30	17.7
17	-2.02	20.1	-2.00	22.0
25	-1.64	22.8	-1.65	23.6

channel noise generator can be referred to the input to give

$$ENV^2 = 4kT \left(\frac{\frac{2}{3}}{g_m} \right) \left\{ 1 + \frac{f_b}{f} \right\}, \quad (5.29)$$

and

$$ENI^2 = 4kT \left(\frac{\frac{2}{3}}{g_m} \right) (\omega^2 C_{gs}^2) \quad (5.30)$$

A comparison of these equations with Equations 5.5 and 5.6 reveals that the SPICE model lacks the shot noise due to the gate current and assumes that $\eta = 1$, $\kappa = 8/3$ and $x = 1$. The lack of the gate current shot noise is not important for an application like a wideband transimpedance amplifier, but the fixed values for η and x limit the accuracy of the model.

From the measured data for the MRF966, it was concluded that η was definitely greater than one, and a value of 1.5 was taken as a reasonable assumption. Also, the frequency dependence of the $1/f$ -type noise was found to be $1/f^{1.37}$. Thus, use of the SPICE model will result in underestimation of the noise generated by the FET, however, the general trends of the noise power spectral density will be correct. For a typical MRF966 with $I_D = 10$ mA and $g_m = 18$ mS, f_b was found to be about 22 MHz which results in $K_F = 4.4 \times 10^{-13}$, when A_F is assumed to be one. These parameters were included in the model of the MRF966 MESFET.

5.1.6 Input Device Selection

At the end of Section 5.1.2, it was concluded that the GaAs MESFET appeared to be the best choice for an input device. This was due mainly to its extremely

low input capacitance and high g_m . For this conclusion to be valid, two requirements had to be met. One was that the η value for the GaAs MESFET be less than 2.5 in order for it to provide superior noise performance compared to a silicon MOSFET. Characterization of the MRF966 device showed that η was much less than three, which indicates that that device would perform as well or better than a MOSFET. The other requirement was for the $1/f$ -type noise not to dominate or significantly impact the noise performance of the preamplifier. Because of the high-frequency peaking of the transimpedance amplifier noise spectrum, it is likely that the $1/f$ -type noise does not contribute a significant amount of power to the total. Preliminary models for a transimpedance amplifier based on the MRF966 device indicated that the $1/f$ contribution was insignificant, and that finding was re-examined with the noise analysis of the final preamplifier design.

The MRF966 GaAs MESFET was selected for use as the preamplifier input device because its noise performance seemed to be comparable to typical GaAs MESFETs [60], and it was commercially available at a relatively low cost (a few dollars per unit.)

5.2 Current Pulse Generator Development

The testing of the prebreakdown current pulse measurement system requires a source of fast current pulses with well known characteristics. Such a generator

should be able to produce pulses similar to those produced by stressed liquids, and thus the pulses should have amplitudes on the order of $100\mu\text{A}$ and durations of about 100 ns. The charge of such a pulse would be about 10 pC, since $Q = i\Delta t$. The pulse risetime and falltime should be much less than 100 ns and preferably short enough so the risetime would be less than that of the preamplifier and other parts of the system. This would allow determination of the system bandwidth. The output impedance of the current pulse generator should be high relative to the input impedance of the preamplifier so it would (electrically) resemble the test cell.

One very simple way to produce a current pulse is to use a differential pair of transistors to switch current off and on. This is the basic idea behind emitter-coupled logic (ECL), which is one of the fastest logic families. The basic circuit for such a current switch is shown in Fig. 5.9. This circuit operates in the following way. When V_{in} is greater than V_{REF} , Q1 conducts and Q2 is cutoff, and the load current is zero. When V_{in} is driven below V_{REF} , Q1 is cutoff and Q2 conducts, and a negative current flows in the load. This load current (I_o) is approximately I_s . The switching action is extremely fast since neither transistor is ever saturated. This requires that V_{REF} is sufficiently negative so that $|V_{REF}| > |I_s R_{load}|$.

To take advantage of the fast switching action of the differential pair and achieve a very low risetime current pulse requires that the load resistance and whatever capacitance is in parallel result in a very small time constant. For

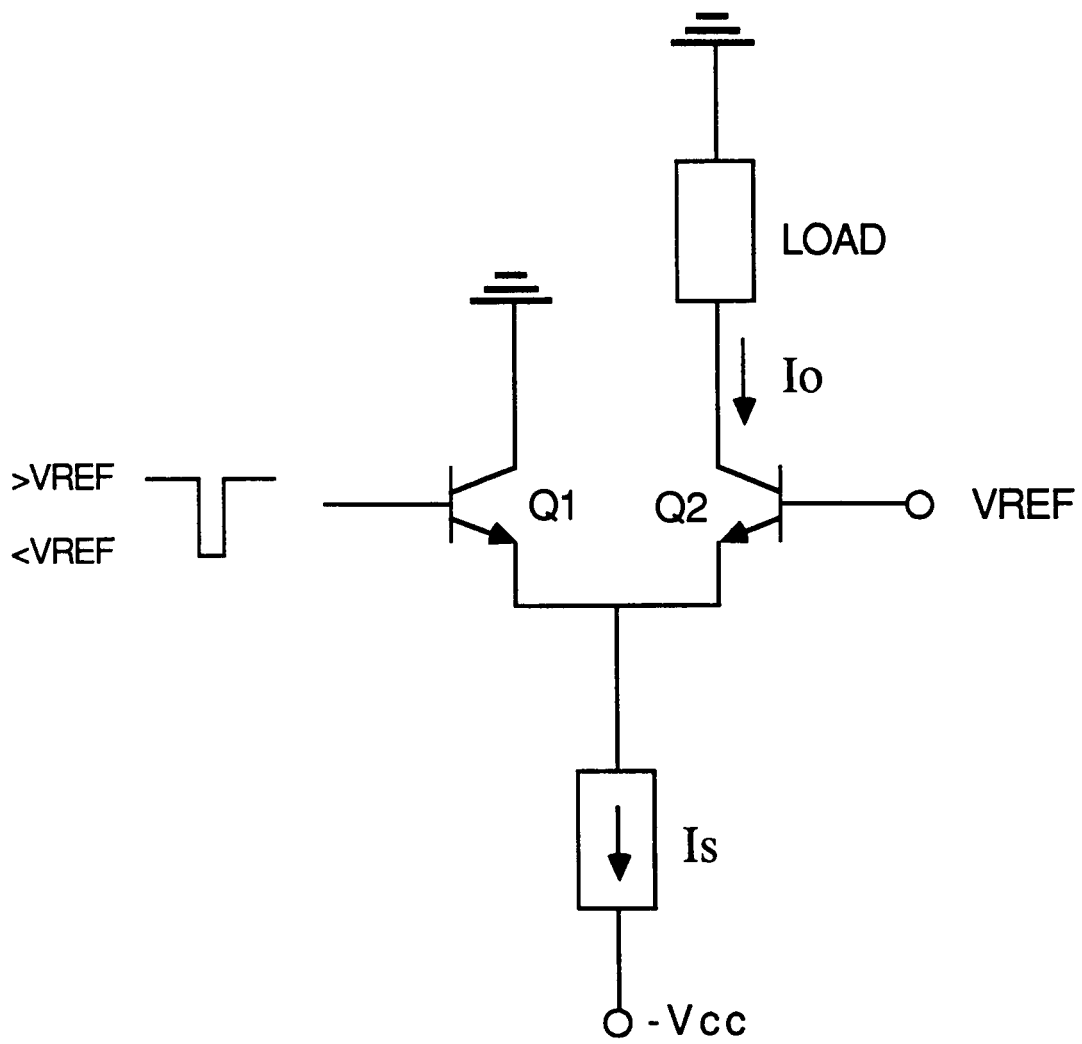


Figure 5.9: Basic current switch.

a parallel RC network driven by a unit step of current, the risetime is about 2.2τ , where $\tau = RC$. For example, $50\ \Omega$ and $100\ \text{pF}$ would result in a risetime of $11\ \text{ns}$. The output capacitance of the differential pair ($\sim C_{cb}$) contributes to this capacitance and therefore should be as low as possible. Fast switching also requires that the transistor used have high f_T at low current levels. This requirement combined with the need for low C_{cb} indicates that small geometry, high frequency transistors are required.

A current pulse generator was developed which used the current switch shown in Fig. 5.9. Several features were added to turn the current switch into a useful instrument. A pulse source was made by using two one-shots (AM26S02) in an astable configuration. This configuration allows independent adjustment of the pulse duration with respect to the pulse repetition rate, and nearly independent adjustment of the pulse repetition rate since the time between pulses is usually long compared to the pulse duration. External gating allows the normally continuous pulse train to be turned on or off electronically, and a synchronization pulse output allows an oscilloscope to be triggered at the same time as a current pulse is output. Finally, the instrument includes two current pulse drivers (which are implementations of the basic current switch). The local driver provides currents in the range 1 to $100\ \mu\text{A}$, while the remote driver delivers a fixed current ($\sim 100\ \mu\text{A}$), but with a much faster risetime. (A modified version of the remote driver was developed with current variable over the range of 90 to $900\ \mu\text{A}$. with some degradation of performance relative to the fixed current ver-

sion.) The circuit diagram for the pulse source, the enable input buffer, the synchronization pulse driver and the trigger for the remote current driver are all shown in Fig. 5.10, and the local and remote current pulse drivers are shown in Fig. 5.11.

For the local driver, an LM3146 was chosen for the differential pair. This transistor array has lower C_{cb} and higher f_T at lower current levels than commonly used switching transistors such as the 2N3904. An additional transistor in the array could also be used in the fixed current source. Risetimes achieved using this circuit were not as low as hoped for, and it was found that a major problem lay in connecting the instrument with the amplifier under test – the connectors added parallel capacitance. This led to the development of the remote driver. In this arrangement, the driver is mounted on the amplifier under test, and the trigger pulse is transmitted through a terminated cable connecting the generator and the driver. By mounting the current pulse driver directly on the input node of the amplifier, unwanted capacitance is minimized. The remote driver also uses higher frequency transistors. The BFR90 has $f_T = 5$ GHz for $I_C = 2$ mA and has $C_{cb} = 0.5$ pF.

Several tests were run to determine the output characteristics of the current pulse generator. The same basic test circuit (Fig. 5.12) was used to determine the risetime, output capacitance and resistance, output current, and the overshoot. The resistor R_1 was changed to allow the measurement of different parameters. To simplify the analysis of the measurements, the current pulse

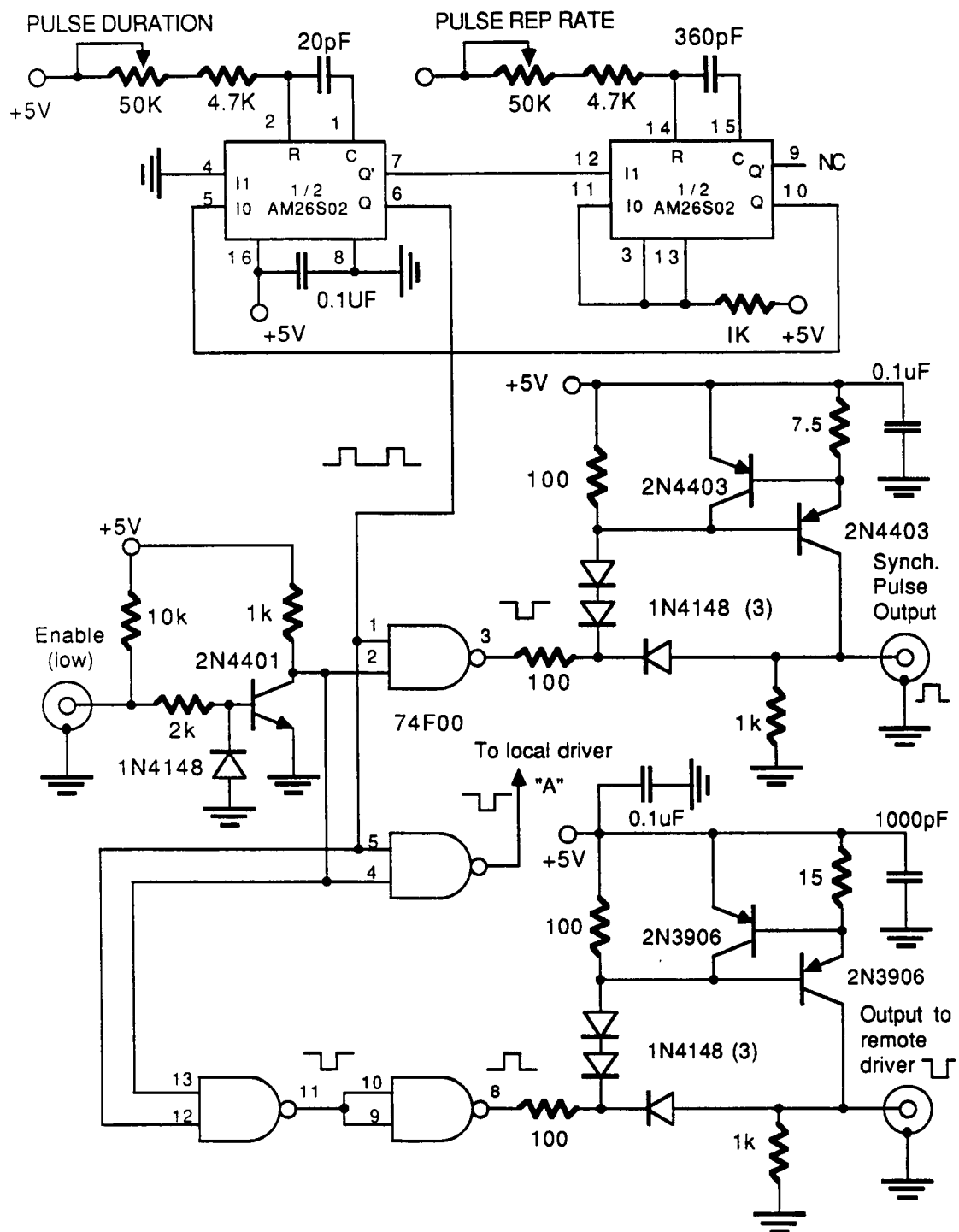


Figure 5.10: Current pulse generator time base and synchronization circuits.

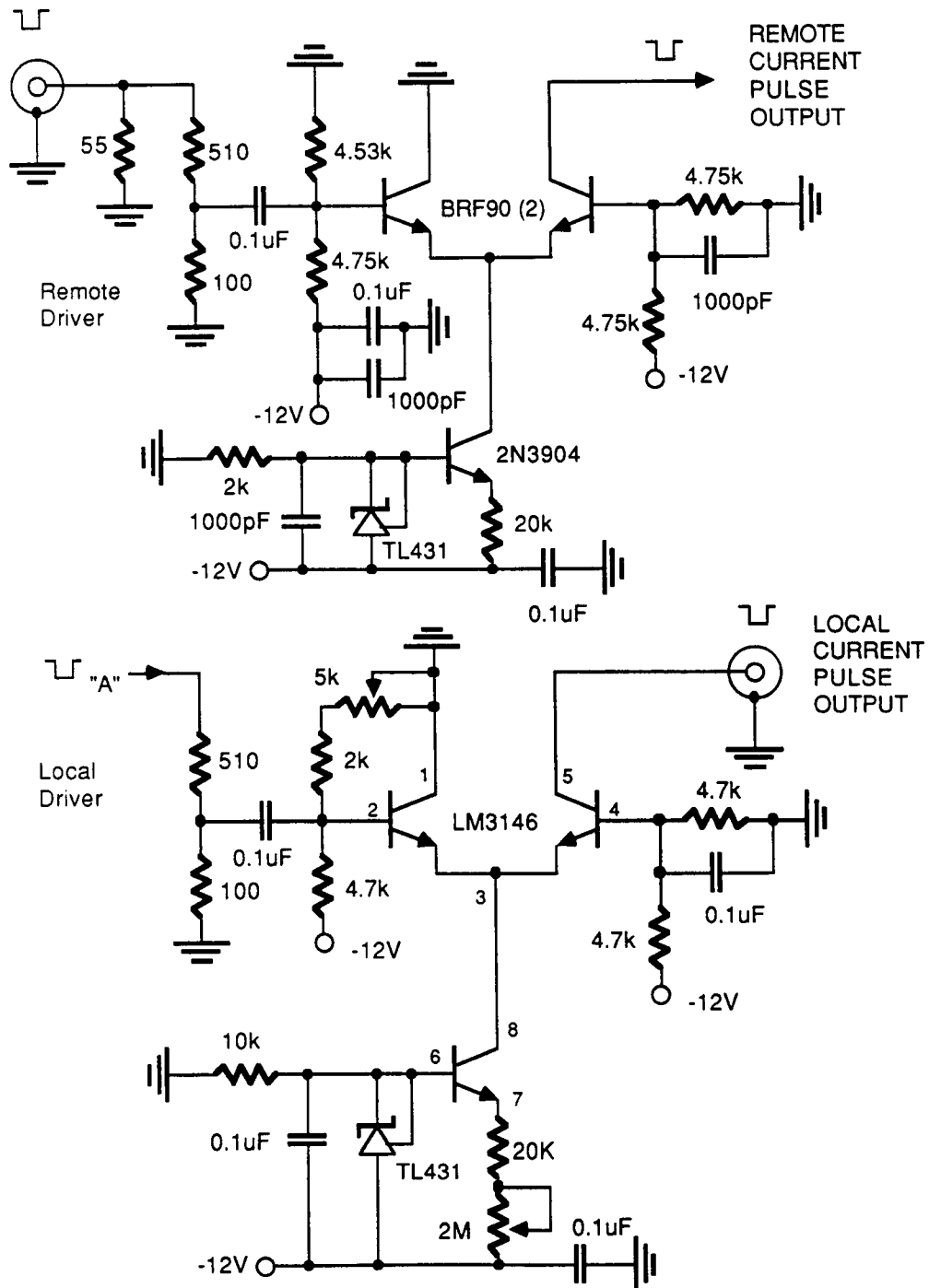


Figure 5.11: Current pulse generator output drivers.

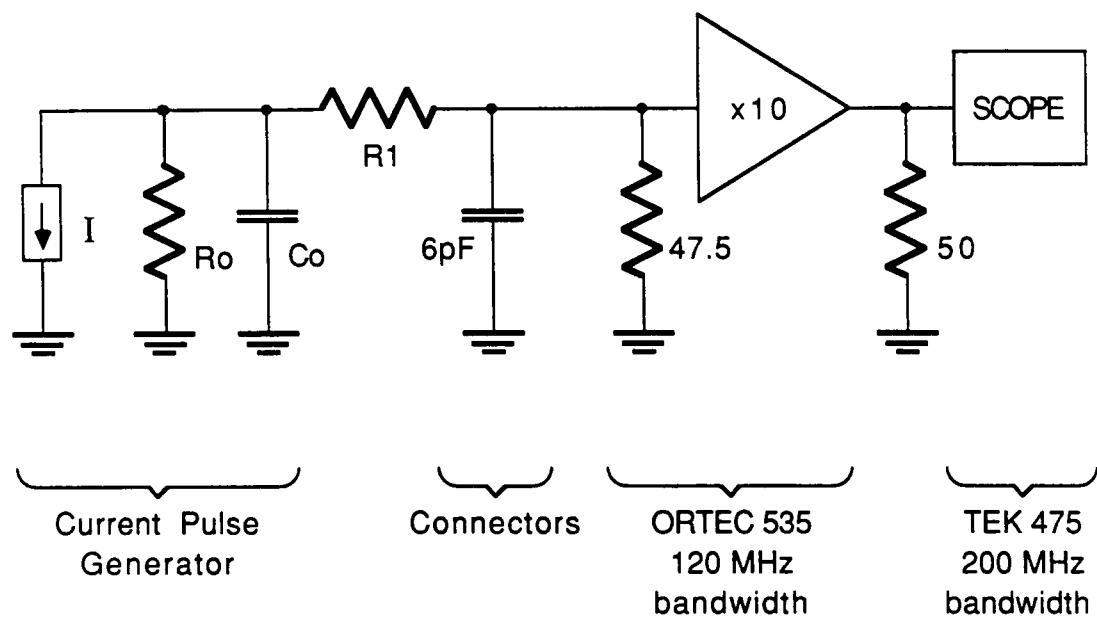


Figure 5.12: Current pulse generator test circuit.

generator is modeled as a pulsed current source with output resistance R_o and output capacitance C_o in parallel. The two BNC bulkhead connectors and the BNC male-to-male adapter used to connect the generator to the amplifier are shown as a lumped capacitor (measured to be 6 pF). The ORTEC model 535 amplifier was adjusted for a 120-MHz bandwidth and a gain of 10 when the output was terminated with 50 Ω .

Initially, the resistor R_1 was shorted, and the test circuit was used to measure the output of the remote driver. For this case, the magnitude of the pulse displayed on the oscilloscope was 45 mV. The output current was then calculated by dividing by the gain of the amplifier and its input resistance to give a value of 95 μ A. The observed risetime was 4 ns, but the risetime of the amplifier was only 3 ns and that of the scope was 2 ns. For a system like this where no one risetime is dominant, the overall risetime is given by [43]

$$t_r = 1.1\sqrt{t_{r_{gen}}^2 + t_{r_{amp}}^2 + t_{r_{scope}}^2}, \quad (5.31)$$

where $t_{r_{gen}}$, $t_{r_{amp}}$ and $t_{r_{scope}}$ are the risetimes of the generator, the amplifier and the oscilloscope, respectively. Solving for $t_{r_{gen}}$ results in a value of 2 ns.

To determine the output resistance, 47 k Ω was used for R_1 . The pulse amplitude (at the oscilloscope) dropped from 45 mV to 44 mV. From this data, R_o was calculated as 2.1 M Ω . This was not a very accurate measurement as it depends on the difference between 45 mV and 44 mV, but R_o does appear to be greater than 1 M Ω .

To determine the output capacitance, 10 k Ω was used for R_1 . For this

case, the dominant time constant was $\tau = C_o R_o \parallel (10\text{k}\Omega + 47.5\Omega)$ since the time constant due to $47.5\ \Omega$ and $6\ \text{pF}$ was $0.3\ \text{ns}$ and negligible. The risetime is given by 2.2τ , so by measuring the risetime, C_o could be calculated. Similar tests were conducted to find parameters for the local current pulse driver. The parameters describing the current pulse generator are summarized in Table 5.3.

Table 5.3: Current pulse generator characteristics.

Parameter	Local Output	Remote Output
Pulse duration	80 to 650 ns	
Pulse interval	600 to 6000 ns	
Synchronization pulse	>3.5 V into $50\ \Omega$	
Trigger for remote	2.5 V into $50\ \Omega$	
Current	1 to $100\ \mu\text{A}$	$95\ \mu\text{A}$
Risetime	$\sim 10\ \text{ns}$ for $100\ \mu\text{A}$ $\sim 30\ \text{ns}$ for $40\ \mu\text{A}$ $\sim 150\ \text{ns}$ for $10\ \mu\text{A}$	<2 ns
Overshoot	varies with current	< 10%
Output capacitance	$6.2\ \text{pF}$	$1\ \text{pF}$
Output resistance	>1 M Ω	>1 M Ω

5.3 Preamplifier Development

A transimpedance preamplifier using a GaAs FET as an input device was developed. The preamp was designed to have approximately a 100-MHz bandwidth and rms input noise current as low as possible. Since very large current

pulses might be encountered under some conditions, the preamp was also designed to clamp and recover quickly from overloading inputs.

5.3.1 Approaching the Design

To begin the design of the preamp, a number of decisions on its basic structure were made after consideration of the specifications and some practical constraints. It was decided that for a high-frequency design, a simple design offered less opportunity for parasitic oscillations and freedom from the cumulative phase shift and limited phase margin that can result from using several stages of amplification inside a feedback loop. The basic preamplifier design chosen is simple indeed and consists of only two stages: a folded cascode and an emitter follower. The cascode provides gain, and the follower buffers the previous stage from the load and provides a low output impedance. Multiple FETs in parallel are used to match the preamp input capacitance to the source capacitance in order to optimize the noise performance. The feedback resistor is only large enough so its noise power approximately equals that due to S_e . At that point, the input noise current is only $\sqrt{2}$ times that due to S_e alone. Further increases in R_F would decrease the preamp noise, but the increased gain-bandwidth that would be required would greatly complicate the amplifier design. Diodes in parallel with the feedback resistor are used to limit the response to overload signals and to serve as C_F under small-signal conditions.

5.3.2 Preliminary Design

The basic design of the preamplifier is shown in Figure 5.13. In the preliminary design stages, the bias points for the transistors and the values of R_F , C_F and R_C were chosen based on simple analyses of the preamp frequency and noise performance. Less critical components were chosen later and assumed to be ideal or negligible initially.

The number of FETs to put in parallel was decided by examining Table 5.1. Since the noise due to four FETs is only slightly greater than the optimum five, it was decided that four would be used to make construction easier. The drain current for each FET was chosen to be 10 mA. At that level, $g_m \approx 18$ mS, which is only 75% of the maximum available, but to reach the maximum g_m would require $I_D \approx 30$ mA. The additional current would increase the operating temperature of the FET considerably (which might negate the noise improvement from the increased g_m), and greatly increase the drain on the power supply. 10 mA per FET is a compromise between power consumption and noise performance with the most weight given to noise performance. Thus, for the four FETs combined, g_{m1} is 72 mS, R_n is 14.6Ω if η is taken as 1.5, and C_{IN} is 5 pF when an additional 1 pF stray capacitance is assumed.

For a 10-k Ω feedback resistor, C_F must be 0.16 pF in order to obtain a 100-MHz bandwidth, and the input noise current due to R_F alone would be 13 nA (from Eq. 4.84). The input noise current due to S_e alone is 14.9 nA from Eq. 4.64, so a 10-k Ω resistor would satisfy the noise requirements. Unfortunately,

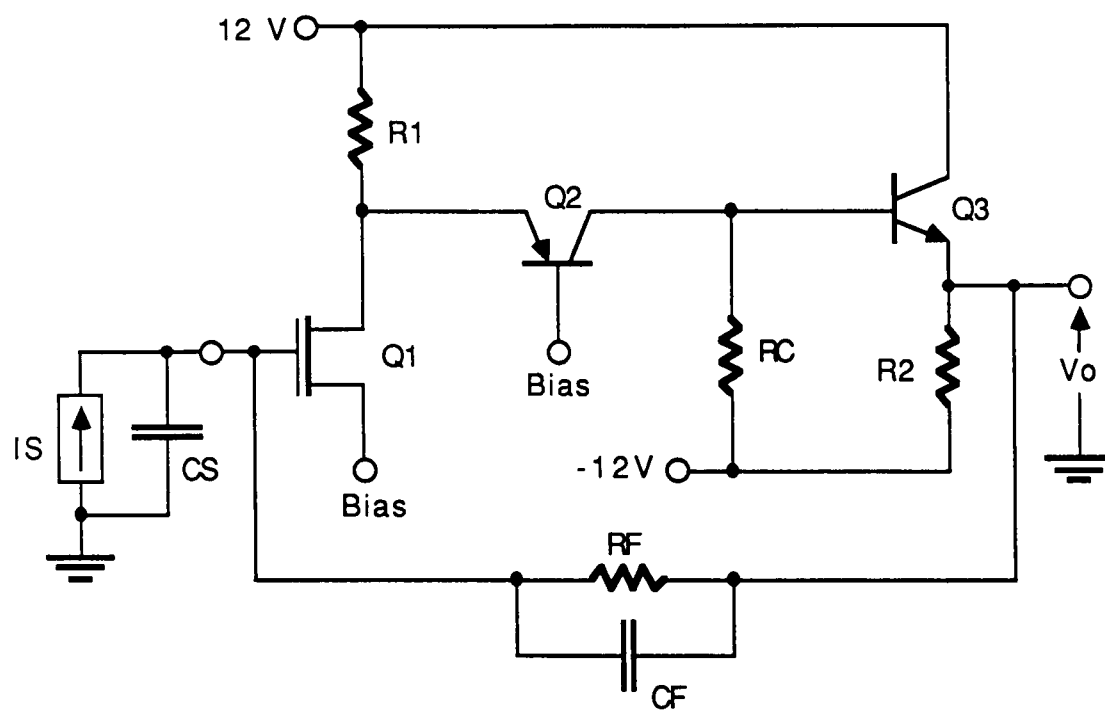


Figure 5.13: Preamplifier basic circuit.

the number of diodes required to produce the small C_F needed is large. Since bi-directional clamping is preferred so the preamplifier can be used with input signals of either polarity, two strings of diodes in parallel with R_F are required. For diodes with junction capacities of 1 pF (1N4148,) each string would contain 12 diodes, and the physical size of the string would probably degrade the preamp high frequency performance. It was decided that $R_F = 5 \text{ k}\Omega$ would be a more practical choice, especially considering that the parasitic (parallel) capacitance of the feedback resistor might be 0.1 pF or more. For the case of $R_F = 5 \text{ k}\Omega$, $C_F = 0.32 \text{ pF}$, only six diodes per string would be required, and the input noise current due to R_F would be 18.2 nA, and the total input noise current would be 23.8 nA.

The number of diodes used in the clamping circuit also determines the dynamic range needed. Assuming a 0.6-V forward voltage for each diode means that voltage across the entire string would be 3.6 V under clamped conditions and that the preamp needs at least a $\pm 3.6 \text{ V}$ dynamic range. The clamping voltage determines the maximum signal that can be amplified linearly. The maximum signal is therefore $\sim 3.6 \text{ V} / 5 \text{ k}\Omega = 720 \text{ }\mu\text{A}$. The minimum visible signal is determined by the noise level and is defined here as $5i_{ni}$ which is 119 nA in this case.

MPSH81 and MPSH10 transistors were selected for Q2 and Q3. These transistors have $f_T \simeq 1 \text{ GHz}$, low C_{cb} ($\sim 0.7 \text{ pF}$) and breakdown voltages large enough so they can easily be used with $\pm 12 \text{ V}$ supply voltages. For several rea-

sons, collector currents of 6 and 12 mA were chosen for Q2 and Q3 respectively. Collector currents of at least a few mA are needed to give the maximum f_T . For Q2, I_{C2} needs to be large enough so its input impedance will be low relative to R_1 , and therefore the current splitting between R_1 and Q2 (which would reduce the cascode gain) will be low. However, I_{C2} cannot be too large without limiting the midband open-loop gain to too low a value. This situation arises because R_C approximately equals the magnitude of the negative supply voltage divided by I_{C2} , and the midband open-loop gain is approximately $g_{m1}R_C$. I_{C3} also needs to be large so the class A output stage of the preamp will be able to drive moderate loads.

With $I_{C2} \simeq 6$ mA, R_C is 2 k Ω and therefore $g_{m1}R_C$ is approximately 144, which is also the midband loop transmission. The midband loop transmission will actually be less due to various losses, but that value is accurate enough for an estimate of the loop transmission and phase margin. Similar accuracy is acceptable for calculating the three poles and zero of the loop transmission. The pole at the input of the amplifier is due to R_F and C_{IF} , and has a frequency of 3.1 MHz. R_C and the capacitance at the base of Q3 produce another pole. The capacitance at that node is approximately $2C_{cb}$, which is 1.4 pF, plus stray capacitance. Assuming a total of 2 pF gives a pole frequency of 40 MHz. There are also poles at the emitters of Q2 and Q3 with frequencies near $f_T \simeq 1$ GHz. The feedback impedance produces a zero with a frequency of 100 MHz. TRAMP was used to find the loop transmission unity gain crossover frequency of 193 MHz

and the phase margin of 54 degrees. These values indicate that the frequency response should be controlled by the feedback impedance and that the step response should have little overshoot.

Frequency Response and Noise Analysis Using TRAMP

The frequency response of the preliminary preamplifier design was also analyzed with TRAMP. As shown in Figure 5.14, there is no peaking in the closed-loop response, and the bandwidth is 132 MHz.

If the input FET is the dominant amplifier noise voltage source, then Eq. 5.18 can be divided by four (the number of FETs in parallel) to give $S_e(f)$ for the preamp. Since TRAMP only allows $1/f$ and $1/f^2$ terms for the noise power spectral densities, the $1/f^{1.37}$ term was approximated as $1/f$. Thus for the preamp

$$S_e(f) \approx 2.31 \times 10^{-19} \left\{ 1 + \frac{21.9 \text{ MHz}}{f} \right\} = \left\{ 2.31 \times 10^{-19} + \frac{5.06 \times 10^{-12} \text{ Hz}}{f} \right\}, \quad (5.32)$$

when $S_e(f)$ has units of V^2/Hz . The second form for $S_e(f)$ can be used in TRAMP to predict the noise behavior shown in Figure 5.15.

The output noise power spectral densities shown in Figure 5.15 illustrate how the different sources contribute to the total. At low frequencies ($< 50 \text{ kHz}$) the $1/f$ noise due to $S_e(f)$ dominates, while in a very wide midband range (50 kHz to 50 MHz), the white contribution due to R_F is dominant. Above 50 MHz, the spectrum has an f^2 dependency as a result of $S_e(f)$ and its transfer function. The noise spectrum continues to increase up to $\sim 150 \text{ MHz}$ and finally begins to

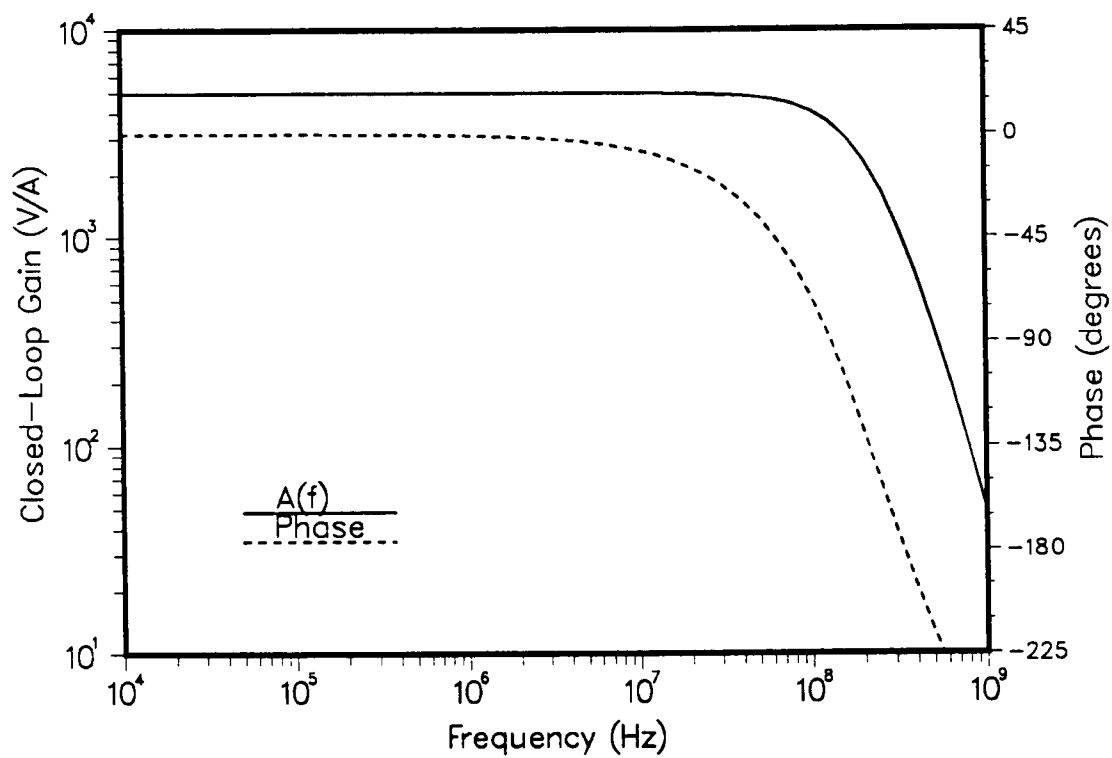


Figure 5.14: Frequency response of preliminary preamplifier circuit.

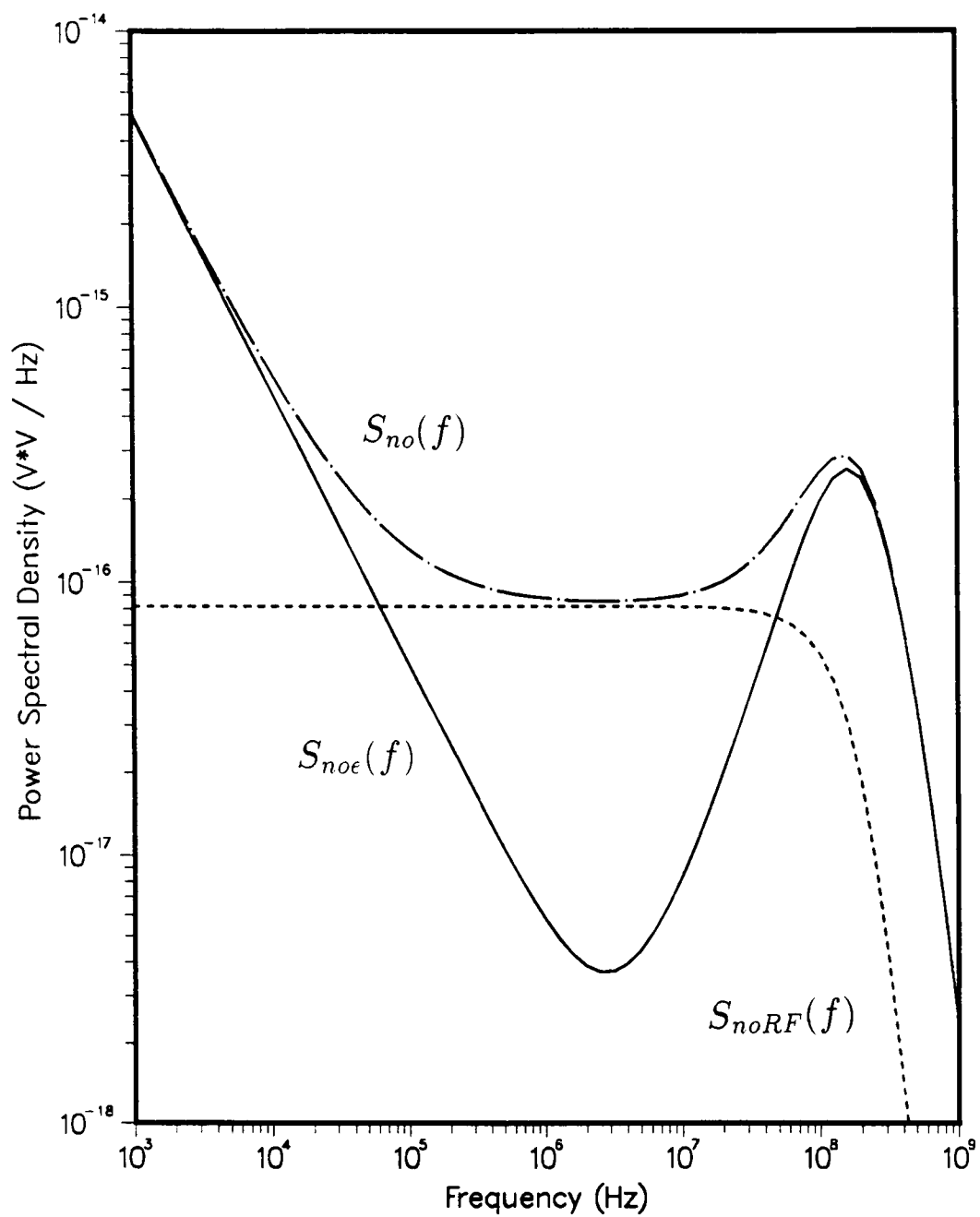


Figure 5.15: Output noise power spectral density of preliminary preamplifier circuit.

decrease when the loop transmission for the S_e source becomes less than unity. This points out one major problem with the use of the transimpedance configuration as a low-noise amplifier: noise produced by the S_e source is not restricted to the signal bandwidth, and therefore the amplifier produces significant noise power above the signal bandwidth.

Although Figure 5.15 is a very good illustration of the noise power spectral densities for the preamplifier, it does not indicate clearly how much each source contributes to the total i_{ni} . Since i_{ni} depends on the square root of the *integral* of the noise power spectral density, and since a visual representation of an integral is the area under the function, a linear plot of the spectral densities (Figure 5.16) is more useful for that purpose. From this plot, it can be seen that the $1/f$ term, although large in amplitude, contributes very little power since it occupies such a small bandwidth (only ~ 100 kHz). (Numerical integration confirms this.) The noise due to R_F is a major contribution, but the bulk of the noise in a 200-MHz bandwidth is due to S_e . If the noise bandwidth is limited to 100 MHz as intended for the signal, the contributions are roughly equal, and the total is much less. This points out the necessity of limiting the bandwidth of the part of the system following the preamplifier to a value only large enough not to restrict the overall signal bandwidth.

A lower frequency limit for the integration of noise power spectral density has not been specified, except to indicate that it is less than 50 kHz. This limit is important because the mathematical contribution of a $1/f$ term is infinite if

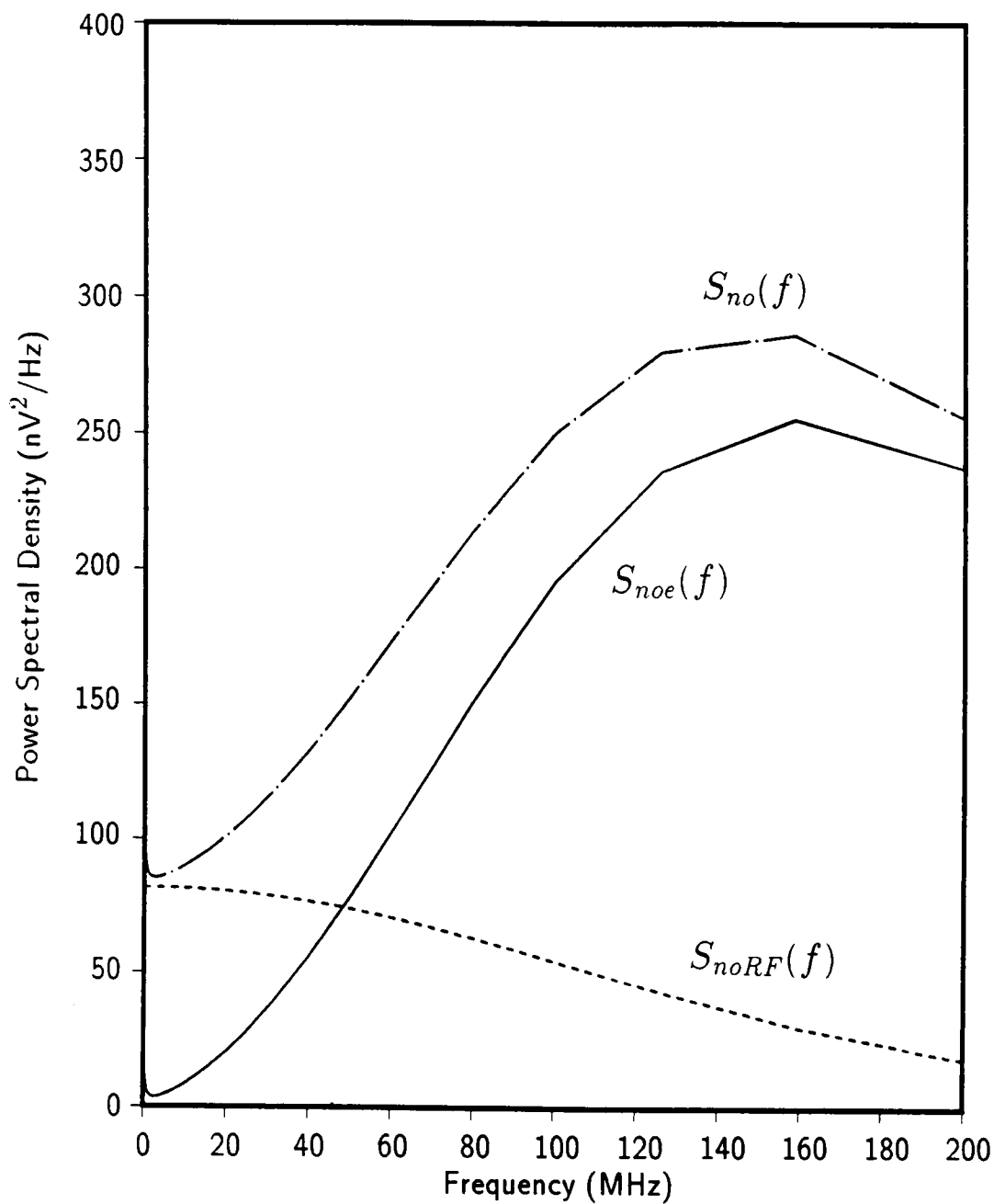


Figure 5.16: Linear plot of preamplifier output noise power spectral density.

the lower limit is zero. Although the preamplifier is dc-coupled, the usefulness of the measurement system is limited at very low frequencies. The lower limit could be estimated as being a fraction of $f_L = 1/T_t$ where T_t is the duration of the captured waveform. For a 200 Msample/s digitizing rate and 2048 samples, $T_t = 10.24 \mu\text{s}$ and $f_L = 98 \text{ kHz}$. Noise at frequencies considerably lower than f_L would appear only as a shift in the dc level and would not effect the observation of fast pulses. It was decided that for simulations of the noise behavior a very conservative lower limit of 1 Hz would be used, and calculation of the total noise power from measurements and FFT analysis would use all terms except the 0 Hz term.

TRAMP was used to find i_{ni} due to S_e , R_F and their sum for a 1-Hz to 100-MHz bandwidth. For R_F , $i_{niRF} = 17.0 \text{ nA rms}$, and for S_e , $i_{nie} = 18.5 \text{ nA rms}$, which results in a total of $i_{ni} = 25.1 \text{ nA rms}$. These values are very nearly the same as those given by the simple algebraic analysis. The values calculated by algebraic means did not include the effects of the finite loop transmission, which is very important at high frequencies, while TRAMP did, so the agreement must be considered quite adequate. The $1/f$ noise of the amplifier contributed less than 0.1 nA to the total i_{ni} , so it is indeed insignificant compared to the total noise power.

5.3.3 Final Design and Analysis

The preliminary preamplifier design gradually evolved to the final version shown in Figure 5.17. Actually, only a few components needed to be added to the basic design. Capacitively bypassed resistors were used to set the bias voltages at the source of Q1, and the emitter and base of Q2. Power supply bypass capacitors were added as well. SPICE was used to calculate most aspects of the preamplifier's behavior. The loop transmission, the closed-loop frequency response and the small-signal step response were simulated, and a noise analysis was performed as well. These simulations were useful not only to verify the basic design, but also to assess the effects of various modifications on the frequency and step responses. The only significant modification found necessary was the reduction of R_C to 1.1 k Ω in order to improve the phase margin. The SPICE listing is given in Appendix E.

The preamplifier loop transmission was found by disconnecting the feedback path from the output and driving that point with a voltage generator. The gain from that point to the output is the loop transmission. The midband loop transmission was 65, the unity gain crossover frequency was 110 MHz and the phase margin was 69 degrees. Lowering R_C from 2 k Ω to 1.1 k Ω reduced the midband loop transmission and crossover frequency and raised the phase margin compared to the case of the preliminary design. The phase margin indicates that there should be very little overshoot in the step response. A plot of the loop transmission is shown in Figure 5.18. The decrease at low frequencies is due to

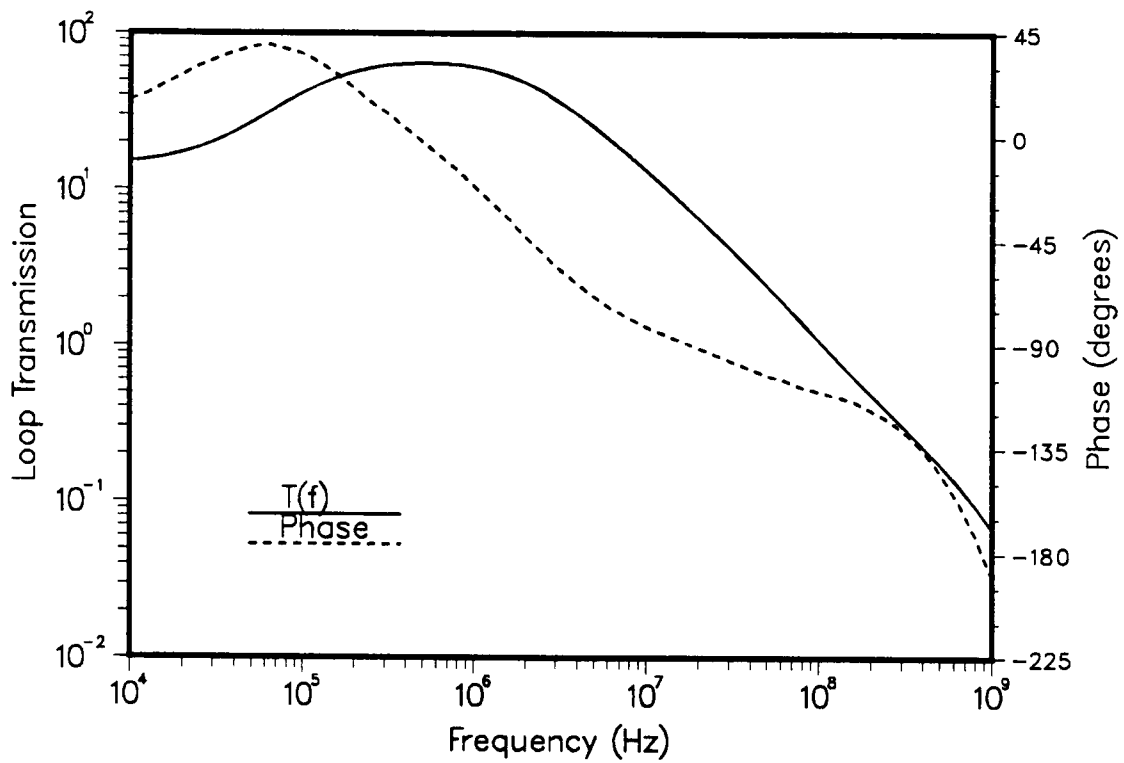


Figure 5.18: Preamplifier loop transmission simulated with SPICE.

the relatively small capacitor bypassing the source resistor.

The closed loop frequency response is shown in Figure 5.19. There is no peaking in the response and the -3dB bandwidth is 87 MHz. This bandwidth satisfies the design goal of an approximately 100-MHz bandwidth. The transient analysis results were equally satisfactory. For a $100\text{-}\mu\text{A}$ pulse, which produces a -0.5 V output signal, the risetime was 4.2 ns, and the overshoot was 1.1 %.

The noise analysis confirmed that the total output noise power spectral density is completely dominated by contributions due to R_F or the input FET except at very high frequencies. At 100 MHz, the contributions due to R_1 , Q2 and Q3 become comparable to that due to the FET, and at 1000 MHz, the noise is dominated by the output transistor Q3. A plot of the output noise power spectral density is given in Figure 5.20. This density is nearly the same as that predicted by TRAMP. The noise density is slightly less in the vicinity of 50 kHz due to the SPICE noise model for the FET which assumes $R_n = 0.667/g_m$ instead of the value of $1/g_m$ assumed for the TRAMP simulation. The high frequency peak and roll-off in the spectrum occurs at lower frequencies than for the TRAMP simulation. This effect is due to the fact that compared to the preliminary design, the final amplifier design has reduced amplifier loop transmission for the S_e noise source. This plot of the noise power spectral density indicates that the earlier estimates for i_{ni} are reasonable.

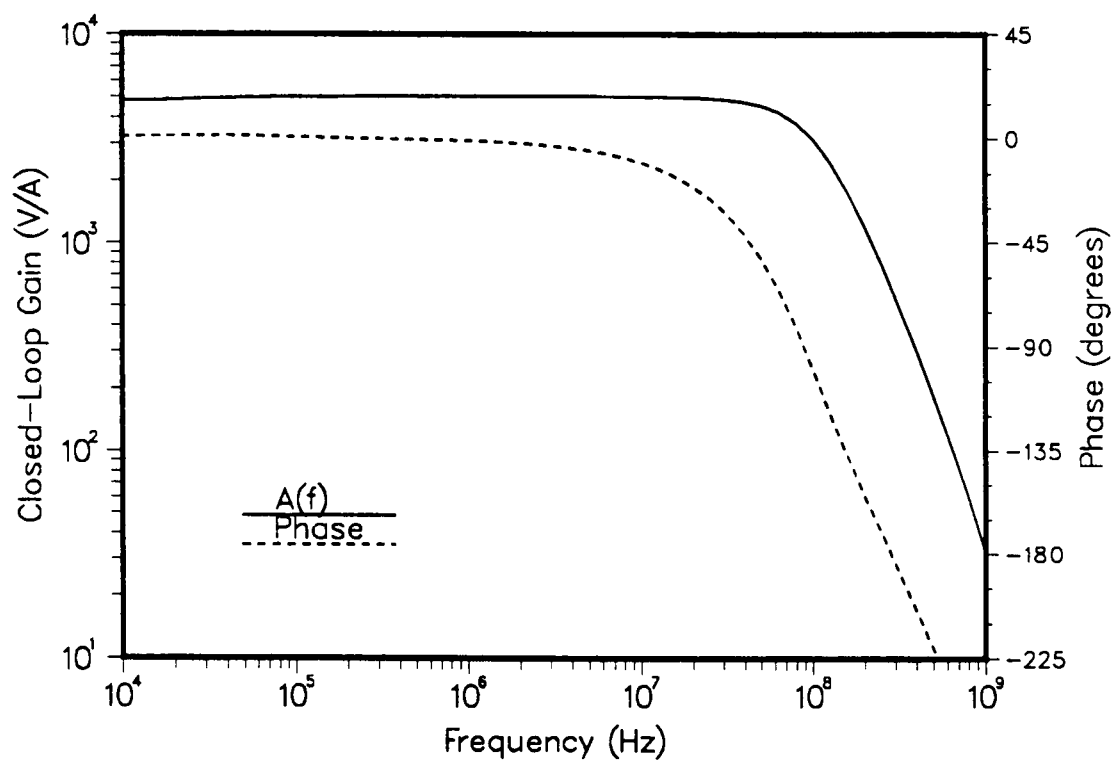


Figure 5.19: Preamplifier closed-loop frequency response simulated with SPICE.

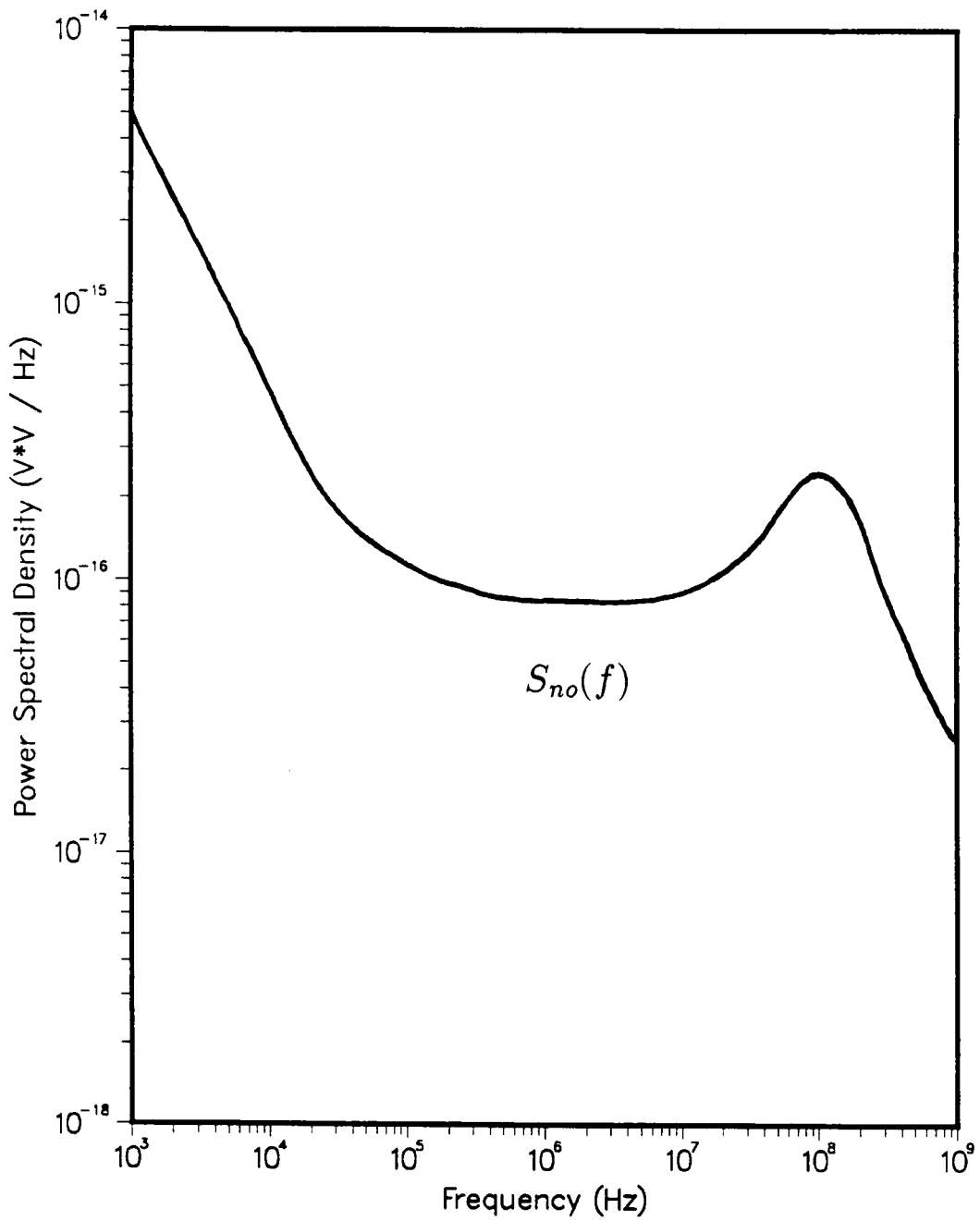


Figure 5.20: Preamplifier total output noise power spectral density simulated with SPICE.

5.3.4 Experimental Results

The preamplifier was constructed on a double-sided printed circuit (pc) board with an extensive ground plane. Bypass capacitors and power supply filters were used as shown in Figure 5.17. These were found crucial to preventing parasitic oscillations. The feedback network (R_F and associated diodes) was constructed as short as possible considering the number of components involved and was elevated approximately 6 mm above the ground plane to minimize capacitance to ground. Excessive overshoot was the result if the last step was not taken. The MRF966 devices were arranged in a stack with the leads of each device folded down and soldered to the corresponding leads of the device below. The bottom device was soldered to the pc board. This arrangement minimizes lead lengths and stray capacitance.

The preamplifier pc board was mounted in a cast aluminum box with a tight-fitting cover. The power supply ground was connected to the box, and in turn, the ground plane was connected to the box at several points. For testing purposes, a BNC bulkhead connector was connected to the preamp input. The uninsulated barrel of the connector was connected directly to the box. A male-to-male BNC connector was used to connect the preamp to the current pulse generator or by itself for noise tests. The capacitance of the BNC connectors (4 pF) is about the same as the test cell (source) capacitance. A 50- Ω resistor connected in series between the BNC bulkhead connector and the preamplifier input was needed to prevent low-level parasitic oscillations, but represents such

a low impedance level compared to the source capacitance, that it does not effect the frequency response or noise characteristics, at least below 100 MHz.

The measured preamplifier bias currents and voltages were very close to the design values. Important bias levels are given in Figure 5.17.

The preamplifier response due to a rectangular, $-100\text{-}\mu\text{A}$ current pulse is shown in Figure 5.21. Most of the overshoot is due to the current pulse generator and the amplifier used as a buffer between the preamplifier and the oscilloscope. The current pulse generator has 10 to 25 % overshoot depending upon the current level, $< 2\text{-ns}$ risetime and is described in Section 5.2. The low-gain buffer amplifier has approximately 10 % overshoot, a 3-ns risetime, gain of -1 and is described in Section 5.4.

The risetimes, falltimes, and overshoots for the same combination of current pulse generator, current preamplifier and low-gain buffer amplifier were measured for a number of different current pulse amplitudes. The output of the buffer amplifier was terminated in $50\ \Omega$ and observed on a 175-MHz oscilloscope. These results are given in Table 5.4. For the purposes of this dissertation, the risetime of a pulse will refer to the 10 to 90 % transition time of the leading edge regardless of the pulse polarity, and the falltime will similarly refer to the transition of the trailing edge. Overshoot is defined in the usual way, but the symbol M_{pt}^+ refers to the overshoot of the leading edge while the symbol M_{pt}^- refers to the overshoot of the trailing edge.

Several conclusions can be drawn from this data. The varying overshoot of

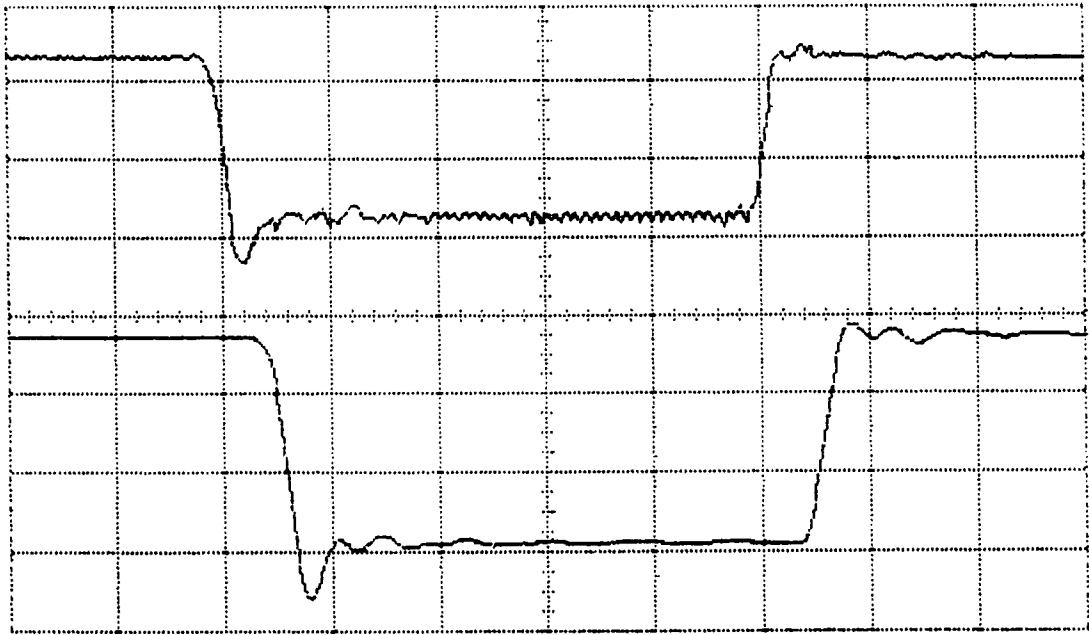


Figure 5.21: Preamplifier current pulse response. The top trace is the output of the current pulse generator driven directly into oscilloscope input ($50\ \Omega$, $\sim 30\ \text{pF}$) through a very short cable. The bottom trace is the signal amplified by the preamplifier and the low-gain buffer amplifier. (top: $5\ \text{mV/div.}$, bottom: $400\ \text{mV/div.}$, $20\ \text{ns/div.}$)

Table 5.4: Preamplifier plus low-gain buffer amplifier current pulse response characteristics.

I_{IN} (μA)	V_{OUT} (V)	t_r (ns)	t_f (ns)	M_{pt}^+ (%)	M_{pt}^- (%)
120	0.6	4.8	4.6	24	8
200	1.0	4.0	4.0	28	6
300	1.5	5.2	6.0	20	5
400	2.0	7.0	5.6	14	5
500	2.5	9.6	6.2	12	3

the current pulse generator (less at higher amplitudes) has some effect on the observed risetimes, but it is clear that the preamplifier and low-gain buffer amplifier combination risetime begins to increase for output voltages greater than approximately 1.5 V. This corresponds to about 300- μA input current. For the risetimes and falltimes observed, the risetimes of the current pulse generator and the oscilloscope are almost negligible, but the risetime of the low-gain buffer amplifier has some effect. Assuming the small signal risetime of the combination is about 5 ns, correcting for the 3-ns risetime of the low-gain buffer amplifier gives 4 ns for the preamplifier risetime. This indicates an 87-MHz bandwidth for the preamplifier which is exactly the same as that predicted by the SPICE simulation. Of course the uncertainty of the small signal risetime for the amplifier combination is significant, so the actual bandwidth could be larger or smaller by as much as 10 MHz. The undershoot values, which are considerably smaller than the overshoot values, indicate that the preamplifier has less than 5 % overshoot.

The preamplifier was tested with overloading current pulses. The feedback diodes clamp the output at approximately ± 3.3 V. For pulse durations greater than 100 ns, the output begins to oscillate at a frequency greater than 100 MHz. Since the prebreakdown current pulses the preamplifier is designed to amplify have durations less than 100 ns, oscillations induced by overloading should not be a problem. The cause of this oscillation is the change in preamplifier loop transmission due to the turn-on of the feedback diodes. When the diodes begin to conduct, the effective feedback capacitance increases dramatically. This decreases the frequency of the loop transmission zero and makes the system unstable.

The response to a $-900\text{-}\mu\text{A}$, 100-ns overload pulse by the preamplifier and low-gain buffer amplifier is shown in Figure 5.22. The clamp level of the preamplifier (3.3 V) is near the maximum input signal for which the buffer amplifier can respond linearly, and even before reaching that level, the buffer amplifier step response slows considerably, and its terminated gain is slightly less than unity. The longer risetime of the amplified signal is therefore due to the buffer amplifier, and the clamped level is due to the preamplifier clamp voltage and the buffer amplifier gain. The overload response is quite good with no latching or oscillations present. The pulse is slightly lengthened (~ 10 ns), but not seriously distorted.

In an additional test, a low-level 50-MHz sinewave was coupled through a 300-k Ω resistor into the preamplifier input at the same time overloading current

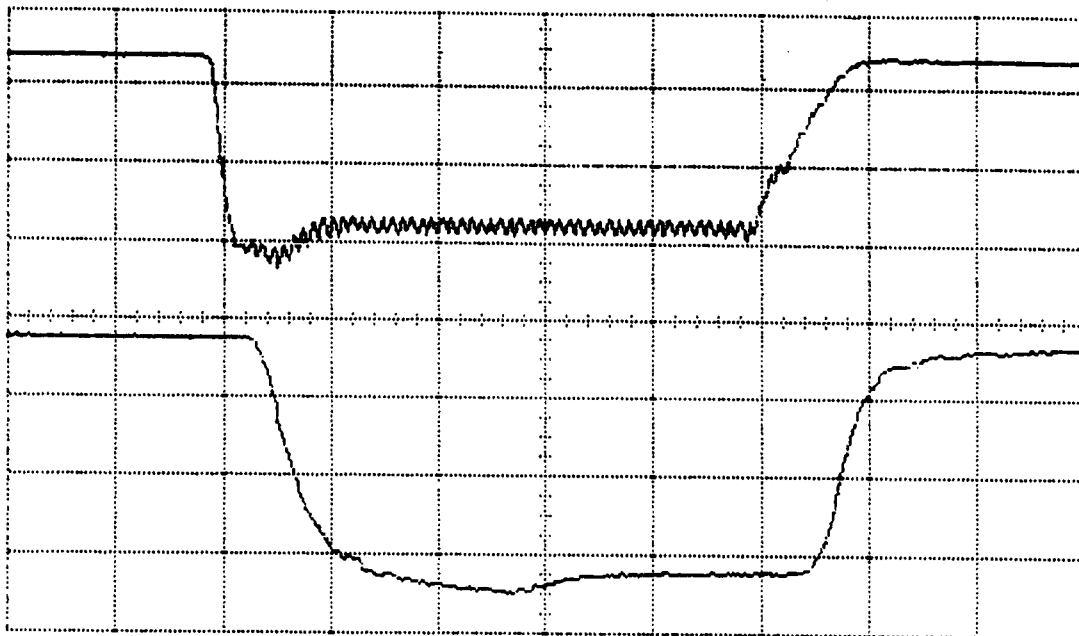


Figure 5.22: Preamplifier current pulse overload response. The top trace is the output of the current pulse generator driven directly into oscilloscope input ($50\ \Omega$, $\sim 30\ \text{pF}$) through a very short cable. The bottom trace is the signal amplified by the preamplifier and the buffer amplifier. (top: $20\ \text{mV/div.}$, bottom: $1\ \text{V/div.}$, $20\ \text{ns/div.}$)

pulses with durations ranging from 10 to 100 ns were applied. The sinewave was visible at the output of the buffer amplifier before the overloading pulse, disappeared as the preamplifier limited, and reappeared as soon as the output returned to the baseline. This indicates that the preamplifier (and low-gain buffer amplifier) return to linear operation within 20 ns after the overloading signal is removed. For longer overloading pulses that drive the preamplifier into oscillation, approximately 50 ns is required for the oscillation to die out and linear operation to be restored.

The rms output noise voltage of the preamp in a 1-Hz to 100-MHz bandwidth should be approximately $i_{ni} \times 5 \text{ k}\Omega$ or 0.125 mV, which is too small to be measured with the spectral analysis equipment described in Section 5.1.3. Of course, the preamplifier is designed to be used with additional amplification when small signals (near the noise limit) are being measured, and those amplifiers magnify the preamp noise as well. They also modify the spectrum due to their finite frequency response and noise generation as well. The additional amplification allows the *total system* noise spectrum to be measured, and that spectrum is the most important as it determines the minimum visible signal level. The measurement of the system noise spectrum is discussed in Section 5.5.

5.4 Amplifier Design and Development

Additional amplifiers are necessary to complete the measurement system. Assuming that the rms output noise voltage of the preamp is 0.125 mV, the minimum visible signal might be estimated to be five times that or 0.625 mV. The highest digitizer sensitivity setting is 10 mV/division, where 10 divisions corresponds to full scale (100 mV). The digitizer uses an 8-bit analog-to-digital converter, so full scale nominally corresponds to 256 levels, however, at higher frequencies, the accuracy of the conversion process is less and the number of effective bits drops [64]. At 20 MHz, the number of effective bits is 6, so full scale corresponds to 64 levels or one level to 1.6 mV. In order to measure current pulse height to $\pm 5\%$, the minimum pulse needs to be at least 10 levels in amplitude or approximately 16 mV. This means that an additional gain of 25 is needed to bring the calculated minimum signal up to this level.

It was decided that the most flexible arrangement would be a low-gain (~ 1) buffer amplifier following the preamplifier and two high-gain (~ 10) buffer amplifiers following in turn. All of the amplifiers could be used to measure the lowest level pulses, while letting larger pulses clip, and larger signals could be examined by reducing the number of buffer amplifiers used. For example, if only the preamplifier and the low-gain buffer are used, the lowest signal measurable to $\pm 5\%$ would be 16 mV (determined by the digitizer accuracy) or $3.2 \mu\text{A}$ referred to the input, and the largest signal would be 3 V (limited by the preamplifier

output range of ± 3 V) or $600\ \mu\text{A}$. If one $\times 10$ buffer with an output range of ± 5 V were added, then the minimum signal would be 16 mV or 320 nA referred to the output, and the maximum signal would be 5 V which would correspond to $100\ \mu\text{A}$. If two $\times 10$ buffers were used, then the minimum signal would be limited by the preamplifier noise to 125 nA, and the maximum signal would be reduced to $10\ \mu\text{A}$ due to the output range of the second $\times 10$ buffer.

Cascading amplifiers for greater gain generally results in constriction of the system bandwidth compared to the bandwidths of the individual amplifiers. For example, a preamp whose -3-dB frequency is 100 MHz, might have a -2-dB frequency of only 80 MHz. If that preamp were used with an amplifier having a -3-dB frequency of 160 MHz and a -1-dB frequency of 80 MHz, then the system -3-dB bandwidth would be only 80 MHz. This problem becomes worse as the number of amplifiers involved becomes greater. One solution to the problem is to make the bandwidths of the buffering amplifiers much greater (10 times or more) than that of the preamplifier, but that was discarded as being impractical due to the already high bandwidth of the preamplifier. It was decided to accept a small reduction in system bandwidth, and to investigate ways to minimize the reduction that would not require bandwidths considerably greater than 100 MHz for the buffer amplifiers.

The relation of the -3-dB frequency (f_{-3dB}) of an amplifier to the gain at lower frequencies depends upon the closed-loop transfer function. For a second-order system with a small damping factor, the frequency response can peak

above the mid-band value before rolling off rapidly. For larger damping factors, the response rolls off smoothly without peaking. It seems reasonable that for two systems with the same -3 -dB frequency, the gain of the system with less damping would be greater at frequencies less than f_{-3dB} . This assumption was investigated numerically for several representative cases, and the results are plotted in Figures 5.23 and 5.24. These are really plots of normalized system gain as a function of frequency normalized with respect to the system -3 -dB frequency.

From the plots, the -1 -dB frequency is about half of f_{-3dB} frequency for a first-order system or a second-order system with $\xi = 1$, but it is about 70% of f_{-3dB} for $\xi = 0.707$. A damping factor of 0.707 also has -0.5 -dB and -1.5 -dB points at 60% and 80% of f_{-3dB} respectively, no peaking in the frequency response and only little overshoot for the step response. It was decided that an acceptable system bandwidth and step response could be achieved using amplifiers with $\xi \sim 0.707$, and that the 90-MHz bandwidth of the preamp and approximately 120- to 130-MHz buffer amplifier bandwidths could give about a 70-MHz overall bandwidth for the system from preamp input to the digitizer input.

One side effect of cascading amplifiers in the manner just described is the creation of an inherent anti-aliasing filter. The system (excluding the digitizer response) gain drops rapidly above the -3 -dB frequency due to the large number of poles involved. At 100 MHz (the Nyquist frequency), the system gain is

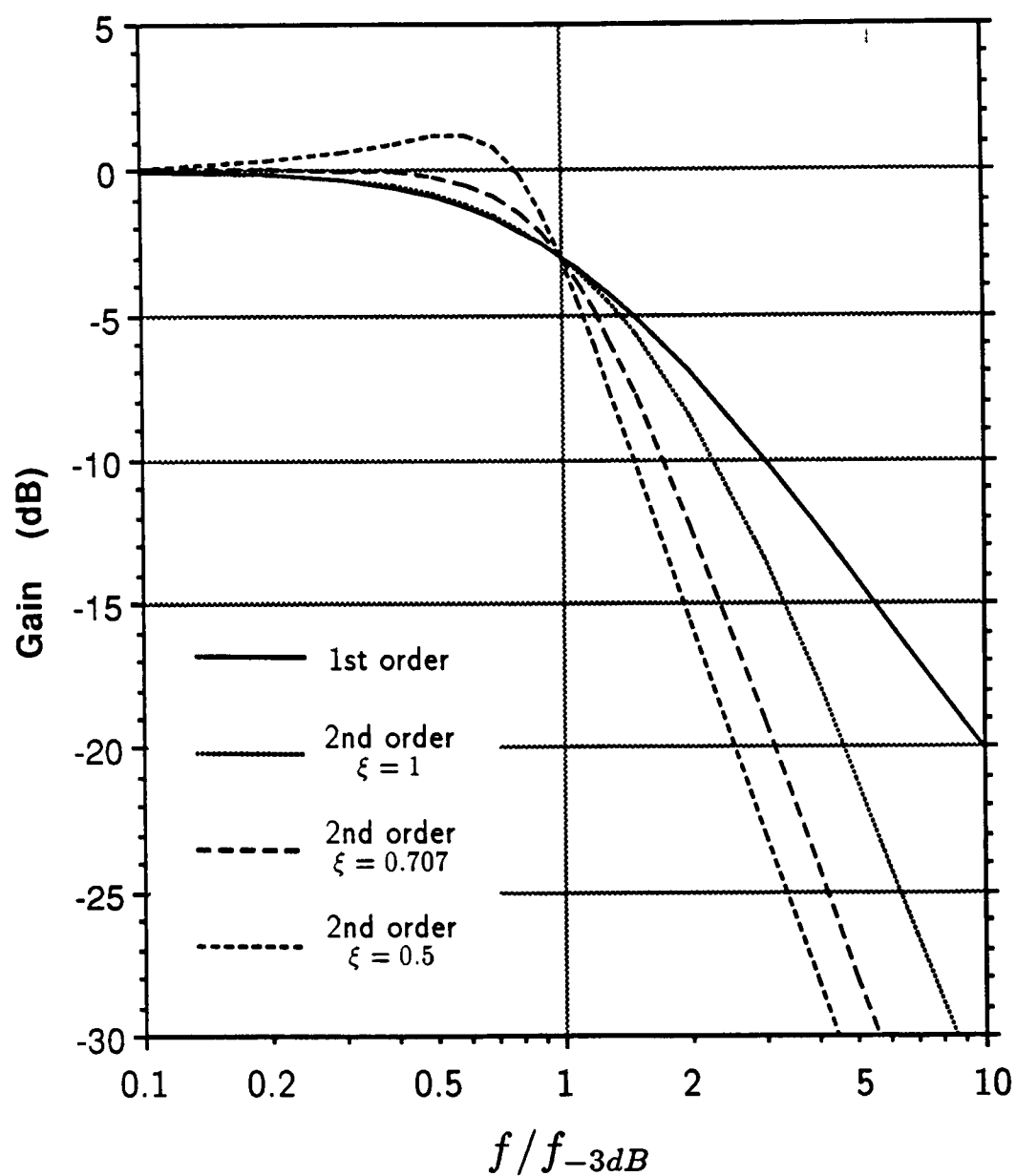


Figure 5.23: Normalized system frequency response in the vicinity of f_{-3dB} .

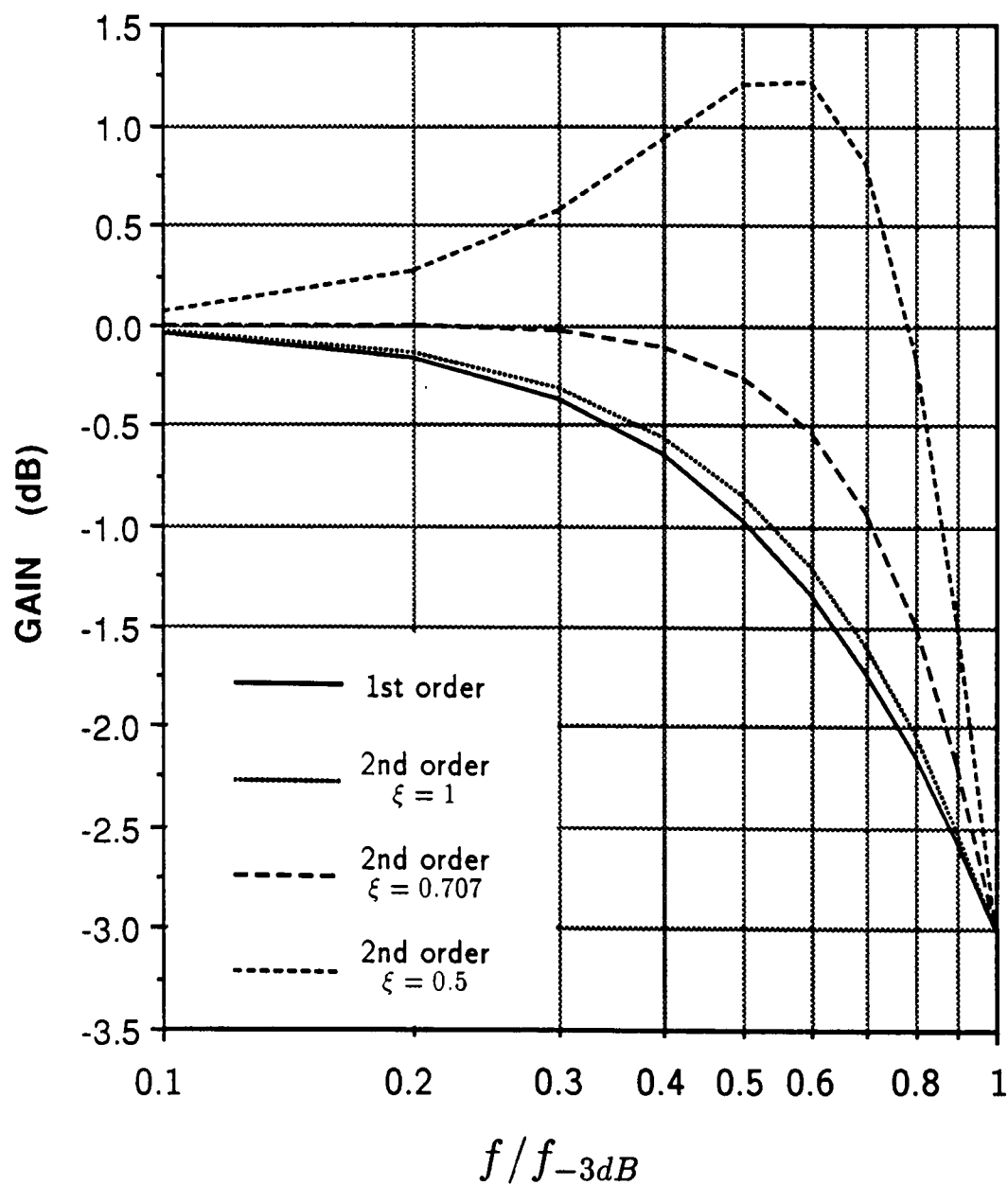


Figure 5.24: Normalized system frequency response below f_{-3dB} .

about 10 dB below its midband value, and it is 15 dB below at 120 MHz and 40 dB at 200 MHz. This should provide sufficient attenuation of high frequency signal and noise components so signal reconstruction is possible. Another side effect of the frequency response limitations of the buffering amplifiers is the attenuation of preamplifier noise at frequencies above the system bandwidth. The buffer amplifiers provide almost 10 dB of attenuation at only 120 MHz, and that represents a 10-fold reduction in output noise power spectral density at that frequency. The frequency response of the digitizer adds to the effectiveness of the anti-aliasing filter and reduces the high-frequency noise contribution further.

5.4.1 Low-Gain Buffer Amplifier

The main purpose of the low-gain buffer amplifier is to allow the preamplified signal to be coupled to the digitizer or to another amplifier through a coaxial cable terminated in $50\ \Omega$. This buffer is needed since the preamplifier is not capable of driving the cable directly. The characteristics of the preamplifier place an additional number of constraints on the low-gain buffer. It should have low input capacity in order to maintain the stability of the preamp, input resistance greater than $1\ \text{k}\Omega$ in order to not limit the maximum preamp output signal, bandwidth somewhat greater than 100 MHz to preserve the system bandwidth, and relatively low noise so as not to degrade the system noise performance. The required bandwidth was discussed in Section 5.4. It was decided that the noise power contribution of the buffer should be less than 10 to 20% of the preamp

noise as that would only increase the rms noise current by about 5 to 10%. An improvement in system noise performance of that amount could be obtained far more easily by a slight improvement in the preamp than an effort to reduce the buffer noise to a negligible amount.

A very simple configuration was chosen for the low-gain buffer amplifier. A bipolar transistor cascode loaded with a relatively low-valued resistor and followed by a complementary emitter follower output stage was used in an inverting voltage gain configuration. Both stages are very wideband, and the output stage has good current driving capabilities. It was decided that the maximum output signal should be greater than the gain times the preamp maximum output, and therefore no overload circuitry would be required. For the chosen gain of -2 , the maximum output signal would only need to be slightly greater than 6 V which should be easily obtained with the complementary output and ± 12 -V supplies. Since the buffer output impedance should be $50\ \Omega$ to match the cable, the terminated gain would be -1 .

The complete circuit of the low-gain buffer amplifier is shown in Figure 5.25, and a few comments about the design are in order. The MPSH10 and MPSH81 transistors were used for many of the same reasons they were used in the preamplifier: high f_T , low C_{cb} and sufficient breakdown voltages. The transistors were biased in the 5 to 10 mA range in order to obtain the maximum f_T . An unby-passed emitter resistor was used with Q1 to improve the open loop gain stability, and the diode bias network also associated with Q1 was added to increase the dc

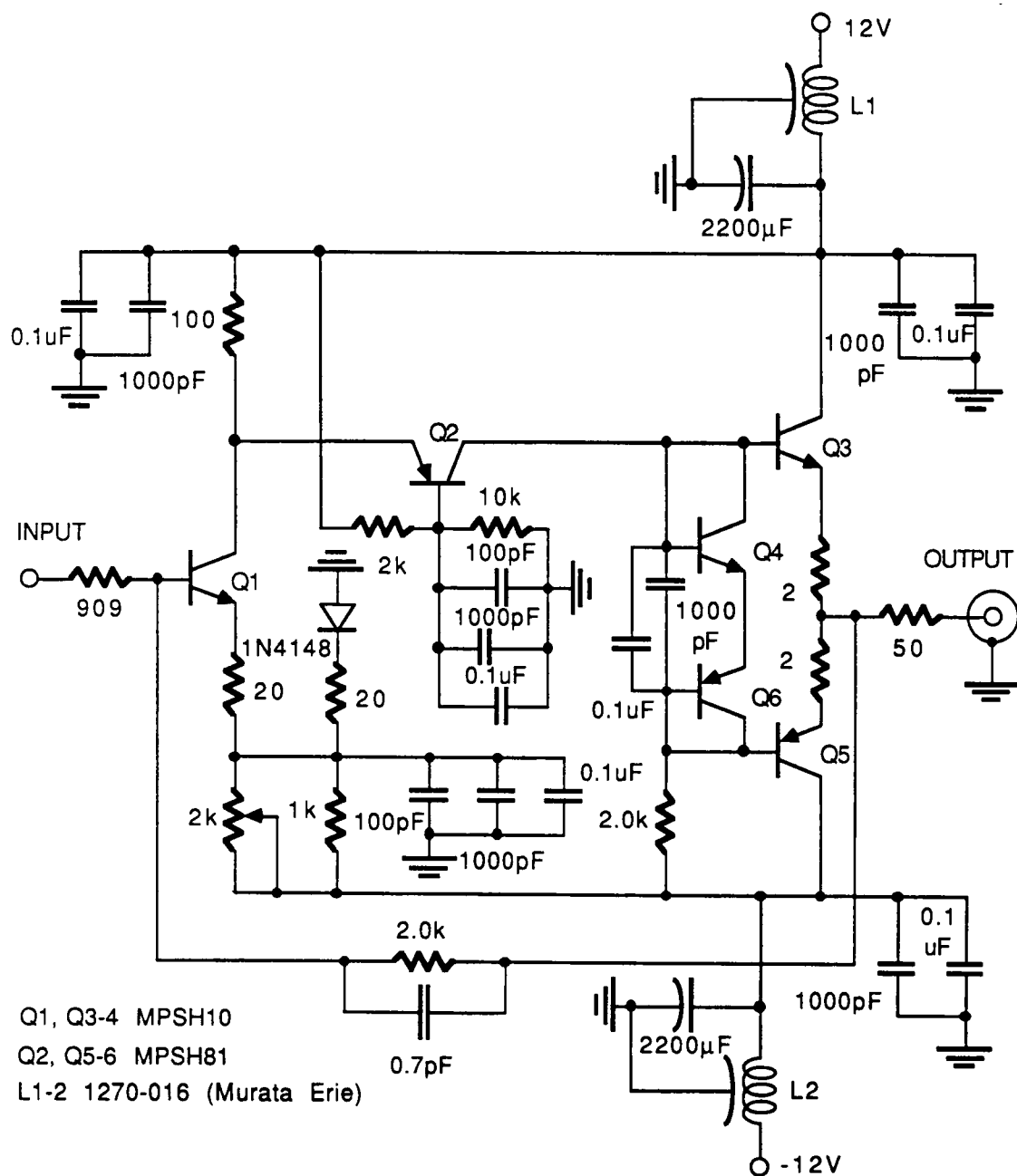


Figure 5.25: Low-gain buffer amplifier circuit.

loop transmission. Diode-connected transistors (instead of diodes) were used in the output stage biasing network for better matching of the current vs. voltage characteristics of the output transistors.

Approximate calculations concerning the buffer were performed by hand. These indicated that the amplifier had sufficient gain-bandwidth to produce a closed-loop gain of -2 and a bandwidth well above 100 MHz. They also indicated that the loop transmission unity gain crossover frequency (f_{co}) and the phase margin would be sufficiently high so the feedback network would control the closed-loop response. SPICE was used to perform more exact analyses of the open- and closed-loop responses. The open-loop analysis revealed that the midband loop transmission magnitude was 12.6 and the phase margin was 68° . The closed-loop analysis showed a frequency response with no peaking, a bandwidth of 125 MHz, and a gain of -2.04 . The -0.5 -dB and -1 -dB frequencies were 60 and 80 MHz, respectively. The small-signal step response simulation found a risetime of 3.1 ns and 3.6% overshoot. All of these results indicate that the low-gain buffer amplifier would satisfy the requirements given in Section 5.1.

The amplifier was constructed on a double-sided printed circuit board and installed in an aluminum box for shielding. The buffer amplifier was mounted directly onto the box containing the preamplifier, and the input was connected to the preamp output through a short (1.5 cm) wire except for testing purposes when a BNC connector was attached to the input along with a terminating $50\text{-}\Omega$ resistor. The ground planes of both amplifiers were connected with a short wire

paralleling the signal connection.

A number of tests were conducted in order to fully characterize the behavior of the low-gain buffer amplifier. The small-signal gain of the buffer was measured to be -1.02 ± 0.05 into a $50\ \Omega$ load. The step response was investigated using a fast pulse generator (1 ns risetime) and a 175-MHz bandwidth oscilloscope. The observed small-signal risetime was 3.4 ns which is consistent with the SPICE simulation value of 3.1 ns for the amplifier alone. Risetimes and falltimes were very similar except for the case of very large pulses where the negative transitions were somewhat faster. Approximately 10% overshoot was observed for both positive and negative pulses. An example of the amplifier response to a rectangular pulse is shown in Figure 5.26.

The risetime increases for pulses with amplitudes greater than 0.5 V and was observed to be 4.0 ns for 1 V pulses, 6 ns for 2 V pulses and about 15 ns for 3 V pulses. The output signal clips at 3.4 V and -3.2 V when loaded with $50\ \Omega$, so the assumption that the low-gain buffer maximum output level would be greater than that of the preamp is barely justified. With no load, the output levels where clipping occurred were 8.6 V and -9.2 V, so the output impedance must rise considerably above $50\ \Omega$ for large signals.

The frequency response of the low-gain buffer amplifier was measured by using an radio-frequency (RF) generator and a 100-MHz oscilloscope. The amplifier output was loaded with $50\ \Omega$, and the gain was measured with an input signal level of 250 mVp-p. Since the response of the oscilloscope begins to fall off

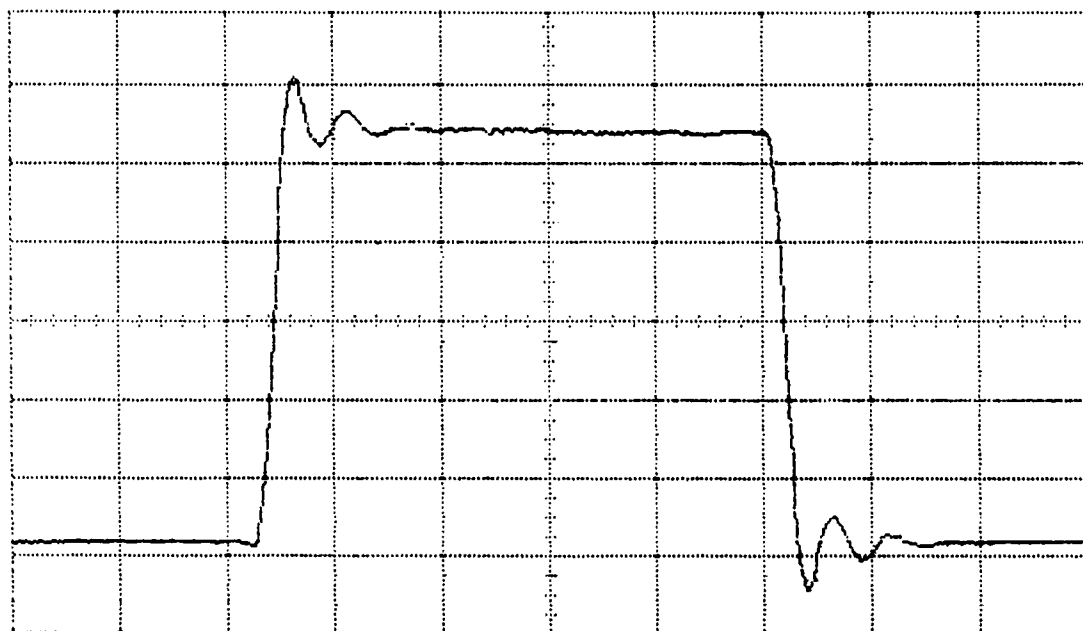


Figure 5.26: Low-gain buffer amplifier pulse response. The amplifier output was loaded with $50\ \Omega$, and the signal was captured with a 100-MHz digital oscilloscope. (200 mV/div., 20 ns/div.)

for frequencies above about 20 MHz, the scope response was measured and corrected for. The response of the amplifier is shown in Figure 5.27. The bandwidth is about 130 MHz which compares favorably with the simulated value. There is a slight amount of peaking in the response (1.3 dB) which was not predicted by the SPICE simulation. This peaking, coupled with the observed overshoot which was greater than that predicted by the simulation (10% compared with 3.6%) indicates that the phase margin is slightly poorer than predicted. This small difference could be accounted for by stray capacitance at the base of Q1 or the collector of Q2 or by inadequate modeling of the transistors.

5.4.2 High-Gain Buffer Amplifier

The main purpose of the high-gain buffer amplifier is to provide wideband gain to boost the preamplified signal to the point where it can be digitized. The gain required was estimated to be 25 in Section 5.4, and it was decided that two amplifiers with gains of approximately 10 would produce a more flexible arrangement and that such an amplifier would be easier to develop than one with a much larger gain. The amplifier should have 50 Ω input and output impedances so it can both terminate and drive coaxial cables, and, if possible, it should have a bandwidth greater than 100 MHz. It should be able to drive a signal of at least a few volts into a 50- Ω load, and recover gracefully from an overloading input signal as quickly as possible. Since prebreakdown current pulses have been observed to occur within 50 ns of one another, the amplifier overload recovery

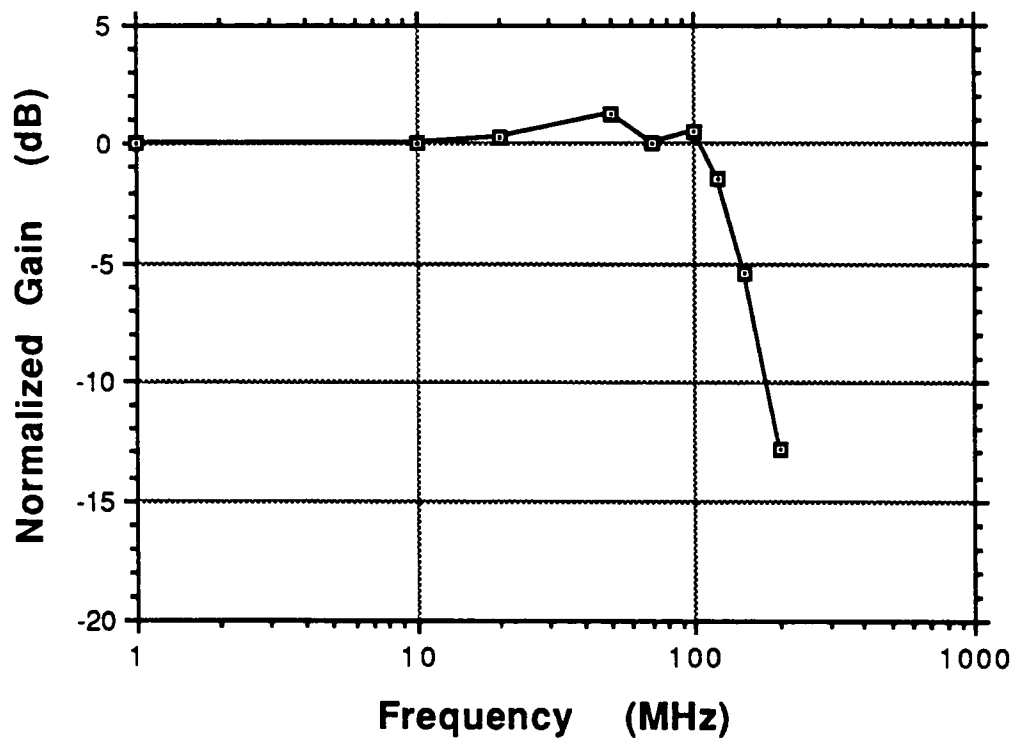


Figure 5.27: Low-gain amplifier frequency response.

should take less than that amount of time so pulses immediately following very large pulses can be observed.

Since the low-gain buffer amplifier performed very well, it was decided that an improvement of that design might produce a useful high-gain buffer. It was observed that open-loop gain and gain-bandwidth product could be increased considerably by removing the common-emitter stage unbypassed resistor. It was also noticed that the diode added to improve the dc loop transmission could be replaced by a diode-connected transistor and that the collector of that transistor could be connected to the positive power supply instead of its base with no effect on the circuit. The two transistors would then form a differential pair. Removing the emitter bypass capacitor would allow full use of the input differential to form either an inverting or non-inverting amplifier. The low-gain buffer is an inverting amplifier, but the non-inverting configuration was chosen for the high-gain buffer since it allows independent adjustment of the input impedance and the feedback network. The loss of the emitter bypass capacitor reduces the open-loop gain and the gain-bandwidth product by a factor of two for a circuit like the low-gain buffer, but that loss can be avoided when a differential input stage is used with a current mirror load to perform double-ended to single-ended conversion.

It was decided that both transistors of the differential stage should be placed in cascode arrangements for improved high-frequency performance. That decision complicates the arrangement of the mirror circuit and the conversion from a double-ended signal to a single-ended signal. The arrangement settled on has

the mirror circuit located between the non-inverting input transistor and its associated common-base stage, and is shown in Figure 5.28. That transistor, the mirror, the common-base stage and a different feedback network essentially are added to the low-gain buffer to produce the high-gain buffer circuit. An amplifier developed by Millard and Blalock [65] used a similar configuration except for the location of the current mirror which followed the common-base stage instead of preceding it.

Millard and Blalock developed some very useful approximations that are based upon the assumptions that the pole at node A in the circuit is dominant and the amplifier can be modeled as a single-pole system. Nodes A and B are separated by a small impedance, and for this approximate analysis they will be considered to be the same and referred to as node A-B. The approximations can be applied here as well despite the minor differences in the amplifier design. The gain-bandwidth product (in Hz) is

$$GBW \approx \frac{G_m}{2\pi C_T}, \quad (5.33)$$

where G_m is the transconductance from the input to node A-B, and C_T is the total capacitance to ground at node A-B. The maximum closed-loop bandwidth for a closed-loop gain A_{CL} can be estimated from the gain-bandwidth product by using

$$f_{BW} \approx \frac{GBW}{A_{CL}}. \quad (5.34)$$

This equation is not accurate for very large gains (those for which the midband

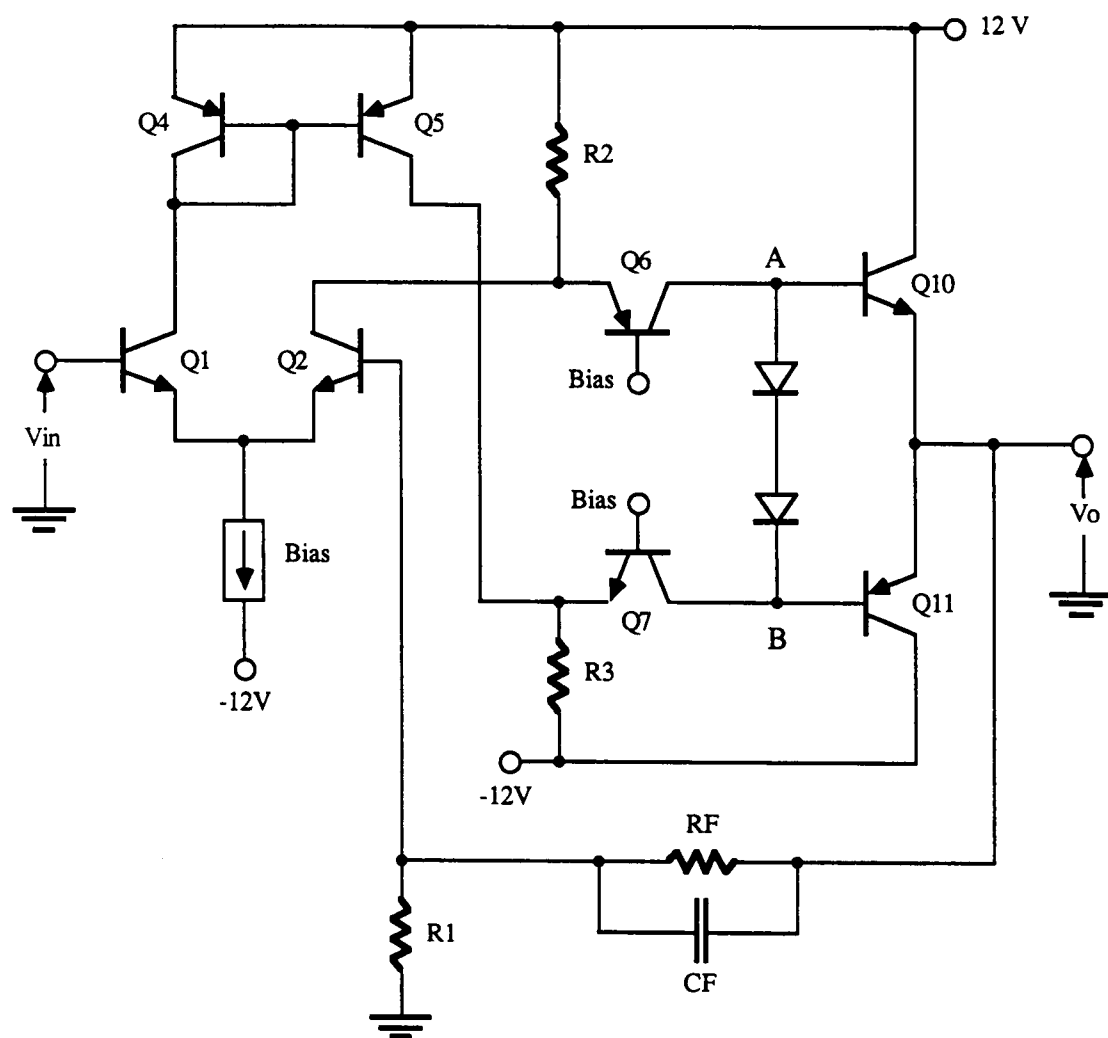


Figure 5.28: High-gain buffer amplifier basic circuit.

loop transmission is < 10) or for bandwidths approaching GBW .

The amplifier performance can be crudely estimated using these equations. C_T can be estimated as the sum of C_{cb} for Q6, Q7, Q10 and Q11, any stray capacitance and any part of C_π for Q10 and Q11 that is not bootstrapped. For the MPSH10 and MPSH81 devices, C_{cb} is about 0.8 pF, and assuming operation at 5 mA, the f_T should be approximately 1 GHz, and C_π would be 30 pF. For a 100- Ω load, most of C_π would be bootstrapped, so assuming a value of 10 pF for C_T includes an allowance of several pF for strays. If each transistor of the differential stage is also biased with $I_C = 5$ mA, then $G_m = 192$ mS and the amplifier GBW is 3.1 GHz. Assuming a closed-loop gain of 20 is needed, a closed-loop bandwidth of about 150 MHz should be possible.

A simple scheme for providing fast overload recovery for this type of amplifier was also presented by Millard and Blalock [65]. Basically it involves limiting the voltage at node A-B to some maximum and minimum values. This drastically decreases the loop transmission when an overload occurs and prevents any transistor from saturating. (Once a transistor is saturated, it can take a microsecond or more to return to operation in the active region.) Once the input signal drops below the overloading level, the amplifier rapidly switches back to linear operation. The scheme also requires biasing Q6 and Q7 with currents greater than that of Q1 or Q2.

The manner in which the amplifier responds to an overloading input can be easily understood by examining the change in the bias currents for the case of

a large positive input signal. Such a signal causes all of the differential current to switch to Q1, and Q2 cuts off. This increases the current flowing through Q6 by an amount equal to the quiescent current of Q2, and decreases the current flowing through Q7 by the same amount. This current imbalance rapidly drives the voltage at node A-B upwards until it reaches the upper clamp which sinks the current imbalance. All transistors stay in either active operation or are cut-off until the overloading input ends. The negative pulse limiting process is very similar.

It was decided that a simple, but effective method of providing clamping voltages would be to use switching diodes and Zener diode-derived reference voltages. The voltage at node A for example would be limited to the Zener voltage plus the diode forward voltage. Another clamp would be required for node B. Since the base-emitter voltage of the output transistor would be about the same as the diode forward voltage, the output would be limited to the Zener voltage. This scheme adds the junction capacitance of the two switching diodes to node A-B so low capacitance diodes should be chosen. The diodes are normally reverse biased which reduces the capacitance from the 0 V bias value, so less than 2 pF should be added when 1N4148 diodes are used.

The overload clamping circuits, biasing networks and power supply bypassing capacitors were added to the basic circuit (Figure 5.28), and following SPICE simulations, prototype construction and testing, a final design was arrived at. The complete circuit of the high-gain buffer amplifier is shown in

Figure 5.29. SPICE simulations of the loop transmission found that the midband value of T was 32, the phase margin was 59° and the crossover frequency was 109 MHz. Closed-loop small-signal frequency response simulations indicated 128-MHz bandwidth, $A_{CL} = 10.2$ at midband and no peaking. For a -1 V step output (into a $50\ \Omega$ load), SPICE predicted a 3.3 ns risetime and 5.5% overshoot. The -1 -dB and -0.5 -dB frequencies were 90 and 79 MHz, respectively. These results indicated that the high-gain buffer amplifier would satisfy the requirements given in Section 5.1.

The amplifier was constructed on a vector board with large copper strips for ground busses and installed in an aluminum box with the ground busses connected directly to the box at several points. BNC bulkhead connectors were used for both the input and output connectors, but the input connector was uninsulated while the output connector was insulated and had its ground lead connected to the ground bus near the output stage supply bypassing capacitors. RF-blocking feedthroughs were used to filter the dc supply voltages.

A number of measurements characterizing the performance of the high-gain buffer amplifier were made. The small-signal gain of the buffer was measured to be 10.1 into a $50\ \Omega$ load. The step response risetime and overshoot were measured using a fast pulse generator (1 ns risetime) and a 175-MHz bandwidth oscilloscope. The small-signal risetime was 4.8 ns (including corrections for the oscilloscope risetime), and the overshoot was less than 2%. The risetime was considerably longer than predicted by SPICE, and the overshoot observed was

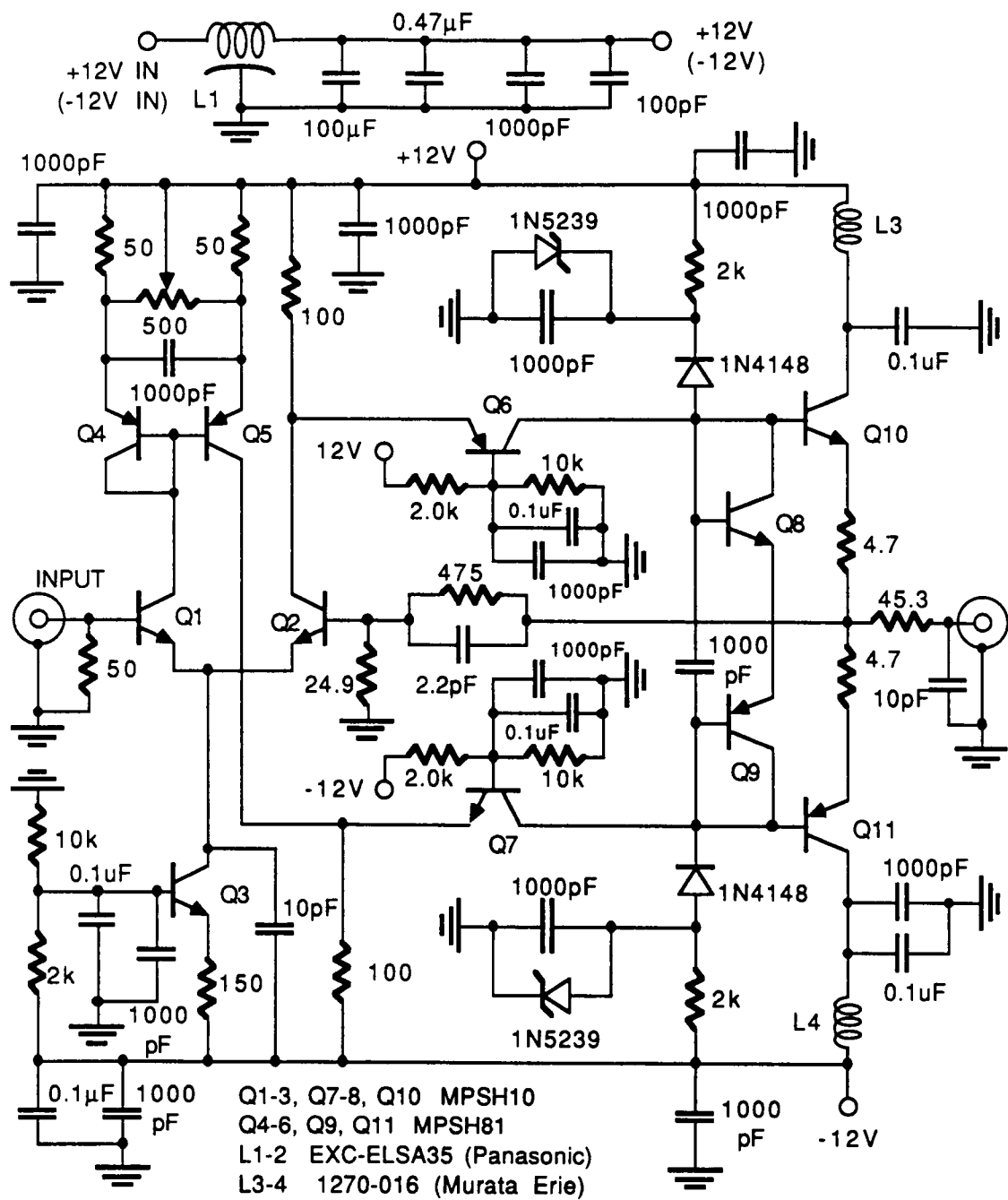


Figure 5.29: High-gain buffer amplifier circuit.

less. This indicates that the crossover frequency was lower than expected and the phase margin was greater. Risetimes and falltimes were very similar and are longer for higher amplitude pulses: 5.4 ns for 1 V and 8 ns for 2 V. An example of the amplifier response to a rectangular pulse is shown in Figure 5.30.

1N5239 Zener diodes were used to provide overload limiting voltages of 9.1 V and -9.1 V. The maximum output voltage developed across a $50\ \Omega$ load should be approximately half of those values (4.5 V and -4.5 V) due to the $50\ \Omega$ output impedance of the amplifier. The actual limits were measured to be 4.0 V and -4.0 V. The response due to overloading rectangular pulses was investigated. For an overloading pulse with duration greater than about 40 ns, the output is an ordinary-looking pulse with a risetime and a falltime of approximately 20 ns, which are not drastically greater than for a non-overloaded output pulse of 2-V amplitude. The pulse is stretched by an amount equal to the falltime. There does not appear to be any significant delay added by the overload clamping circuit. For pulses with shorter durations, the amount of stretching is greater as a percentage of the input pulse duration, but is still due only to the falltime lengthening. Examples of the response to -1 V and 1 V input pulses are shown in Figure 5.31.

The frequency response of the high-gain buffer amplifier was also measured using an radio-frequency (RF) generator and a 100-MHz oscilloscope. The amplifier output was loaded with $50\ \Omega$, and the gain was measured for an output signal level of 250 mVp-p. The results were corrected for the response of the

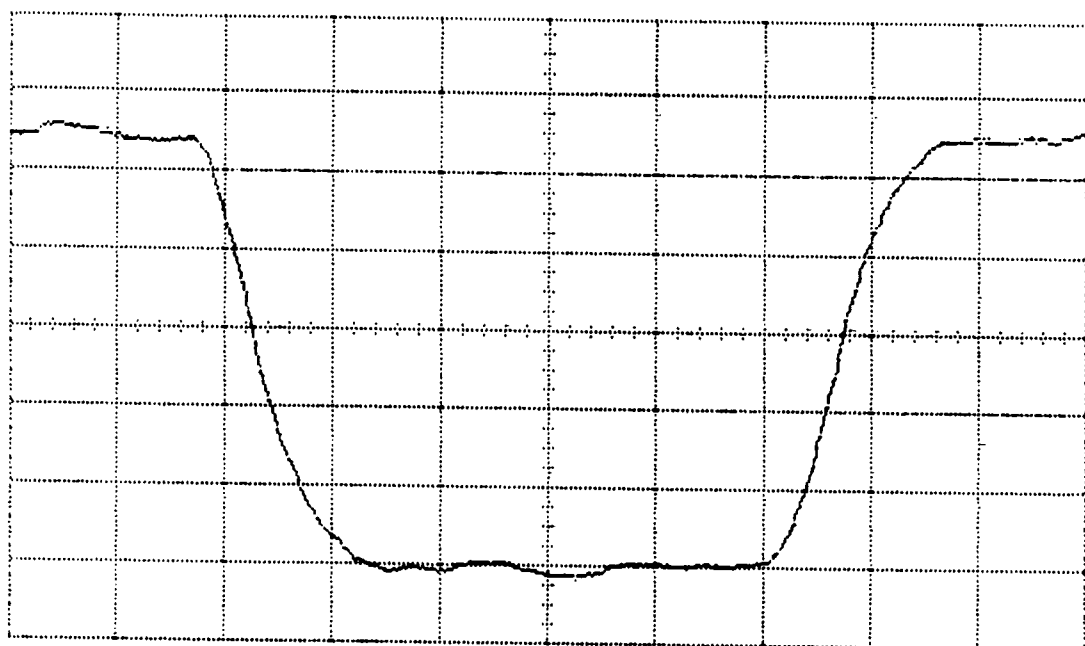


Figure 5.30: High-gain buffer amplifier pulse response. The amplifier output was loaded with $50\ \Omega$, and the signal was captured with a 100-MHz digital oscilloscope. (100 mV/div., 5 ns/div.)

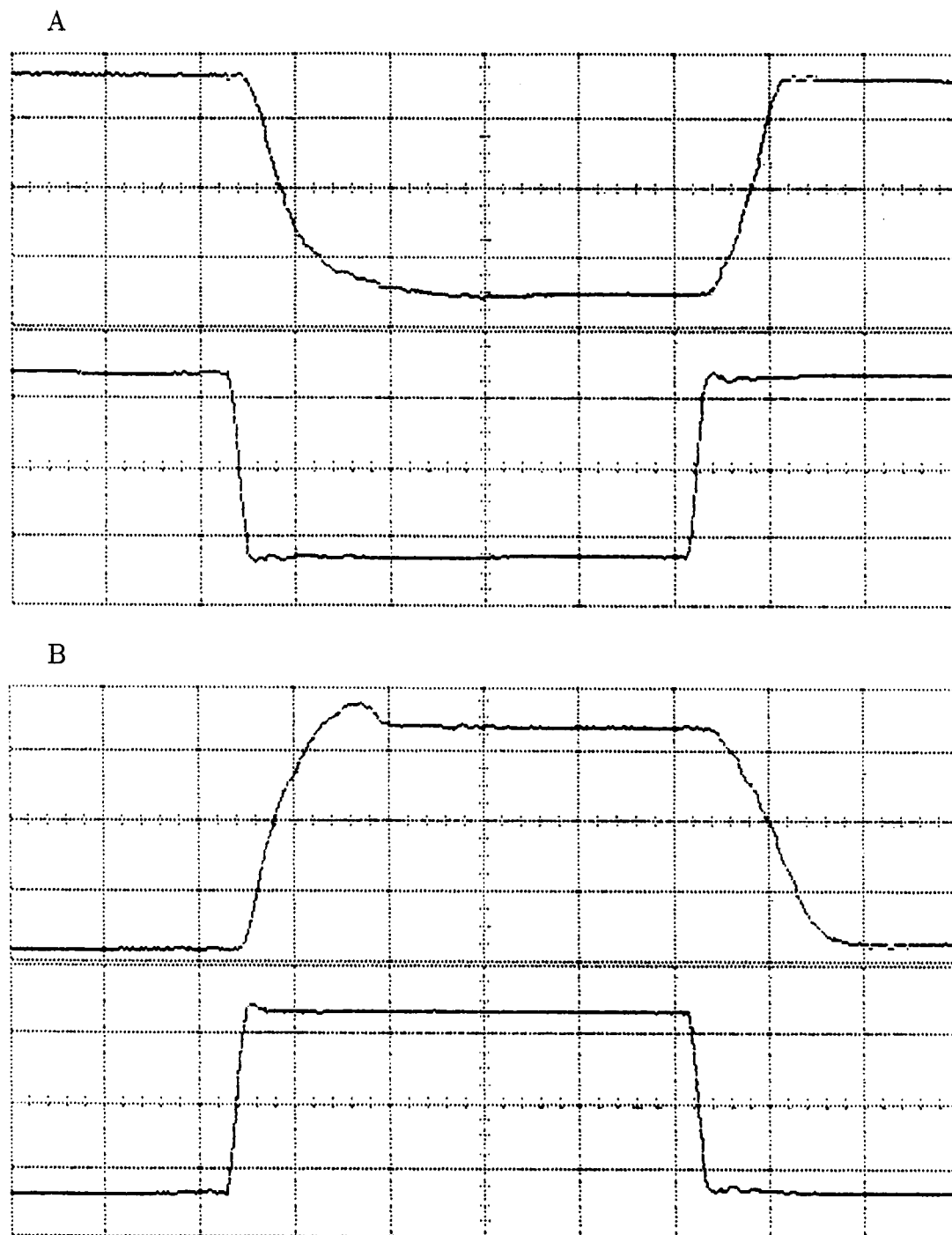


Figure 5.31: High-gain buffer amplifier response to overload pulses. (A) -1 V input pulse. (top: 1.25 V/div., bottom: 400 mV/div, 20 ns/div.) (B) 1 V input pulse. (top: 1.25 V/div., bottom: 400 mV/div, 20 ns/div.) For both cases, the amplifier output was loaded with $50\ \Omega$, and the signal was captured with a 100 -MHz digital oscilloscope.

oscilloscope, and the response of the amplifier is shown in Figure 5.32. The bandwidth is about 90 MHz which is somewhat less than the simulated value (128 MHz), but consistent with the observed risetime. There was no peaking in the frequency response, as predicted by the SPICE simulation. Although the bandwidth was not as large as hoped for, it was deemed adequate. Additional development or tuning might increase it to near the predicted value.

5.5 System Testing

The amplifiers described in Sections 5.3 and 5.4 were combined into the system shown in Figure 5.33. A commercial wideband amplifier (ORTEC 535) was added to the system as an additional high-gain buffer. This is necessary for examining the system noise performance and for viewing very small current pulses. It has gain of 10 V/V, approximately 120-MHz bandwidth, fast overload recovery and can drive a $50\ \Omega$ load to $\pm 5\text{ V}$ [66]. These specifications are essentially the same as the requirements for the high-gain buffer amplifier described at the beginning of Section 5.4.

A number of tests were performed using variations on the basic system. Step response tests were conducted in which the current pulse generator was used as the source and the response of the amplifiers were recorded using a digital oscilloscope. The system frequency response was found by combining the measured responses of the individual amplifiers. Finally, the system noise performance was

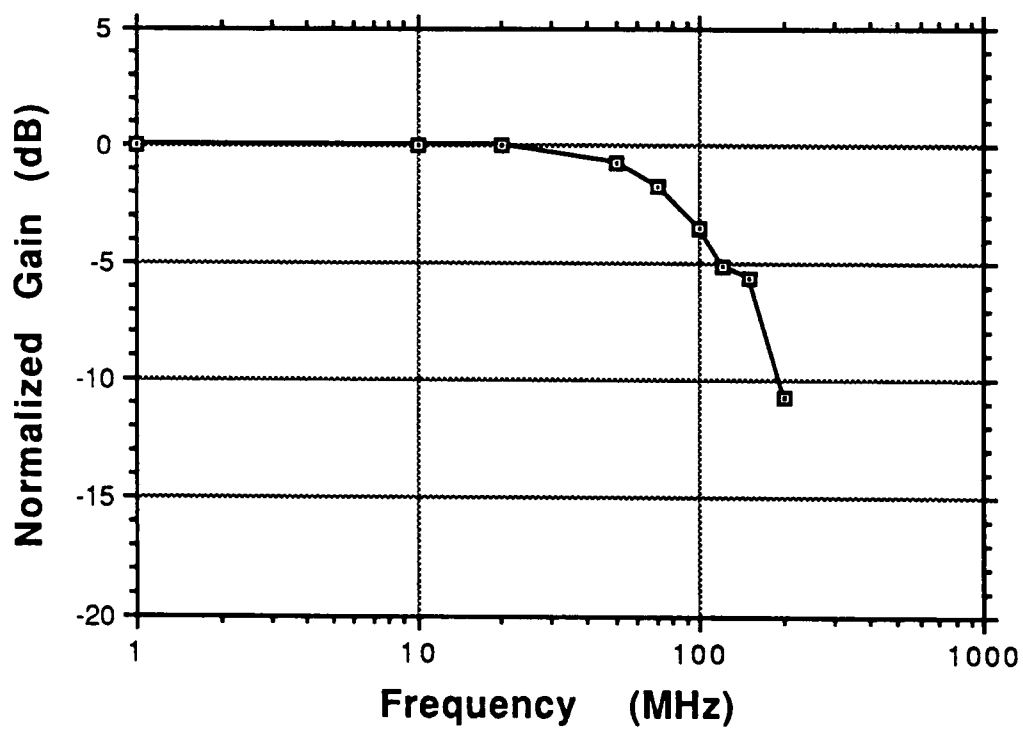


Figure 5.32: High-gain buffer amplifier frequency response.

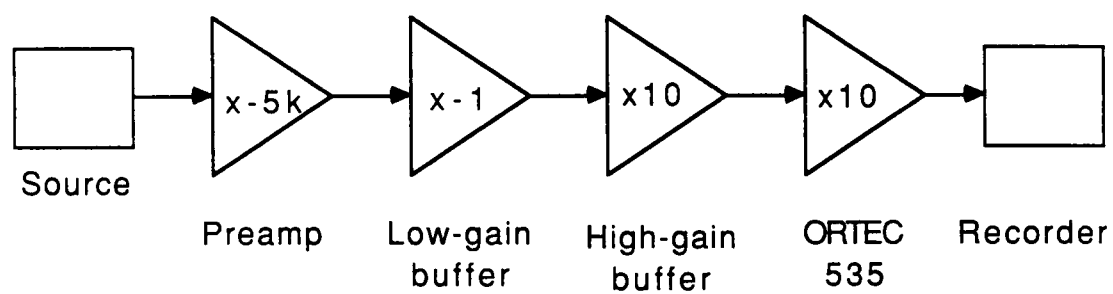


Figure 5.33: Amplification system test arrangement.

measured by finding the total output noise power after digitization and referring that back to the preamplifier input as an rms input noise current. The details and results of these tests follow.

5.5.1 System Step Response

The step response of the preamplifier, low-gain buffer and high-gain buffer was measured using the current pulse generator as the source and a 100-MHz digital storage oscilloscope as the recorder. The generator was adjusted to produce a $-200\ \mu\text{A}$, 20-ns duration pulse, and since that level would overload the high-gain buffer, a $-20\ \text{dB}$, wideband attenuator was connected between the buffer amplifiers. ($-200\ \mu\text{A}$ was chosen since the generator risetime is longer for lower levels.) The response is shown in Figure 5.34. The pulse shape is very much like that of Figure 5.21 (response of the preamplifier and low-gain buffer only) save only a slightly longer risetime. The risetime (corrected for the oscilloscope characteristics) is 5.5 ns, and the overshoot (measured on the positive transition) is about 5%.

The system was also tested with overloading current pulses. Figure 5.35(A) shows the response to the same $-200\ \mu\text{A}$ pulse described above, but with the attenuator removed. For that case, the preamplifier and low-gain buffer operated linearly, but the high-gain buffer was overloaded. The output pulse resembles Figure 5.31(A) (high-gain buffer overload output), but the shorter duration pulse caused some ringing after the pulse. Figure 5.35(B) also shows the system

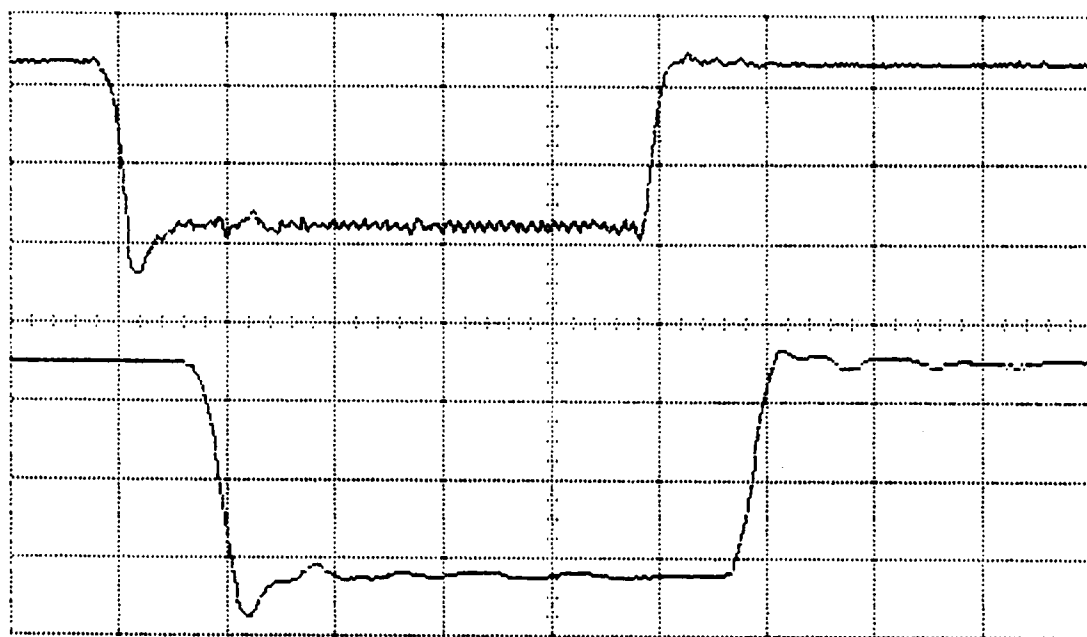


Figure 5.34: System current pulse response. The top trace is the output of the current pulse generator driven directly into oscilloscope input ($50\ \Omega$, $\sim 30\ \text{pF}$) through a very short cable. The bottom trace is the signal amplified by the preamplifier, low-gain buffer and high-gain buffer. (top: $5\ \text{mV/div.}$, bottom: $400\ \text{mV/div.}$, $20\ \text{ns/div.}$)

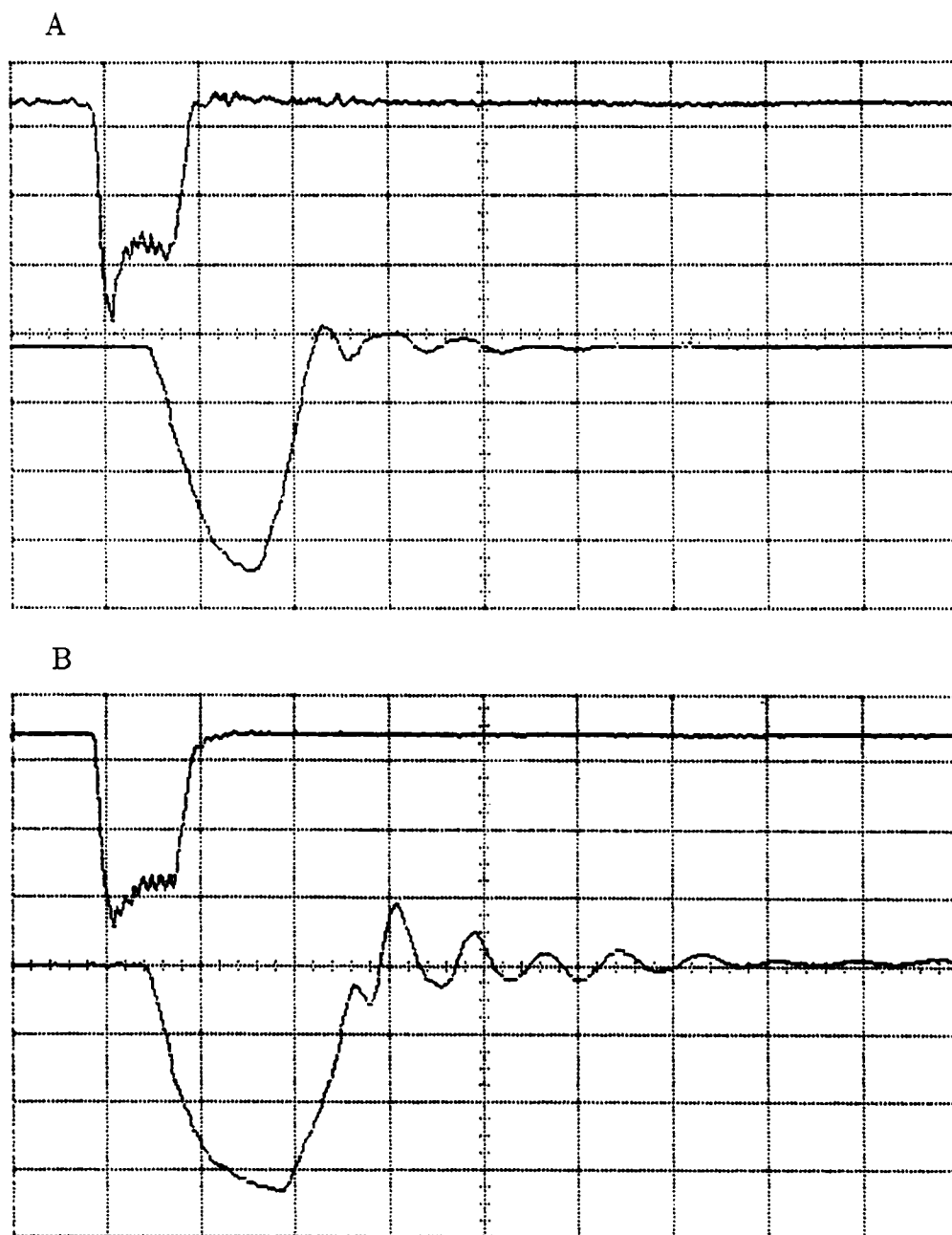


Figure 5.35: System current pulse overload responses. (A) The top trace is the output of the current pulse generator driven directly into oscilloscope input ($50\ \Omega$, $\sim 30\ \text{pF}$) through a very short cable. The bottom trace is the signal amplified by the preamplifier, low-gain buffer and high-gain buffer. (top: $5\ \text{mV/div.}$, bottom: $1.25\ \text{V/div.}$, $20\ \text{ns/div.}$) (B) The top trace is the output of the current pulse generator driven directly into oscilloscope input ($50\ \Omega$, $\sim 30\ \text{pF}$) through a very short cable. The bottom trace is the signal amplified by the preamplifier, low-gain buffer and high-gain buffer. (top: $20\ \text{mV/div.}$, bottom: $1.00\ \text{V/div.}$, $20\ \text{ns/div.}$)

response to a $-900\ \mu\text{A}$, 20-ns duration pulse. For that pulse, all of the amplifiers involved were overloaded, and the same behavior was observed, except that the ringing was more pronounced.

5.5.2 System Frequency Response

The frequency responses for all of the amplifiers, including the digitizer amplification, are given in Table 5.5. The responses were normalized with respect to the midband gain of each amplifier. All of the responses were measured using an RF signal generator and an oscilloscope, except that of the preamplifier. Due to the difficulty in producing a sinusoidal current with known amplitude and introducing it into the preamplifier, and since the simulated risetime and overshoot matched the experimental values, it was decided that the simulated (SPICE) frequency response could be used.

The frequency response for the amplification system or any part of it can be found by multiplying together the responses of the amplifiers involved. The responses of several different combinations are shown in Figures 5.36 and 5.37. For a system consisting of the preamplifier, the low-gain buffer and the high-gain buffer, the bandwidth is 65 MHz. When the ORTEC 535 amplifier is added, the bandwidth increases to 70 MHz, due to the peaking of the ORTEC 535 frequency response. When the digitizer is also added, the system bandwidth drops back to 65 MHz. These bandwidths are consistent with the 70 MHz determined in Section 5.4 as being a reasonable goal for the system bandwidth at the input of

Table 5.5: Normalized frequency response of system amplifiers.

f (MHz)	Normalized Gains				
	current preamp	low-gain buffer	high-gain buffer	ORTEC 535 amp	TEK 7612D digitizer
1	1.00	1.00	1.00	1.00	1.00
10	1.00	1.00	1.00	1.00	0.98
20	0.99	1.03	1.00	1.00	0.96
50	0.90	1.16	0.91	1.00	0.95
70	0.80	1.01	0.81	1.11	0.92
100	0.63	1.07	0.67	1.16	0.85
120	0.50	0.85	0.55	0.67	0.80
150	0.36	0.54	0.52	0.34	0.55
200	0.24	0.23	0.29	0.15	0.40

the digitizer. The system bandwidth could be improved somewhat by increasing the bandwidth of the high-gain buffer.

5.6 System Noise Performance

The output noise power spectral density for the amplification system (Figure 5.33) was found using the noise measurement technique used in Section 5.1.3. Connectors with a total capacitance of 4 pF were attached to the preamp for the purpose of simulating the test cell capacitance. The measured noise power spectral density was referred back to the output of the preamplifier by dividing by the midband power gains of the low-gain buffer, high-gain buffer and ORTEC 535 amplifier, and that spectrum is shown in Figure 5.38 along with

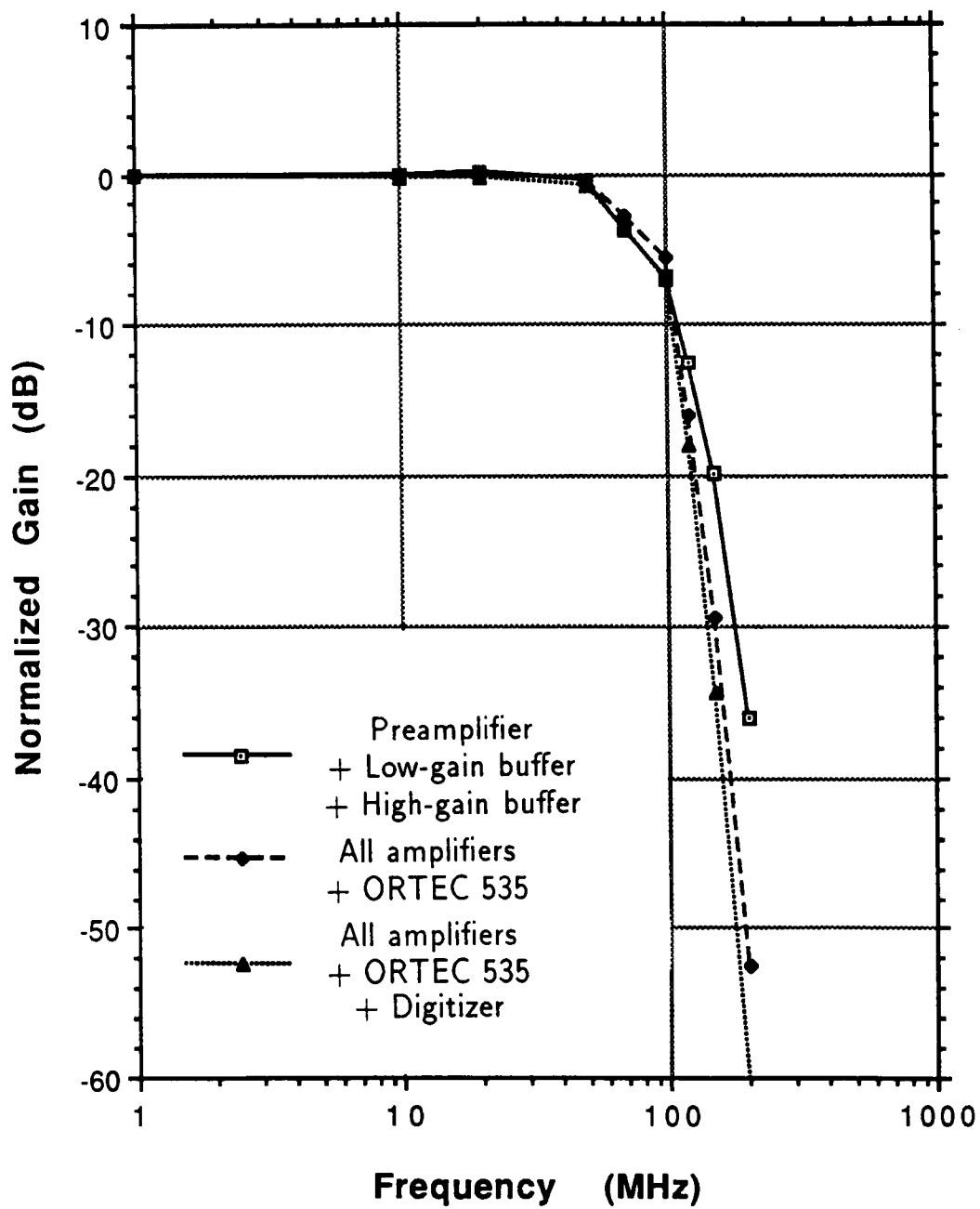


Figure 5.36: Amplification system frequency responses.

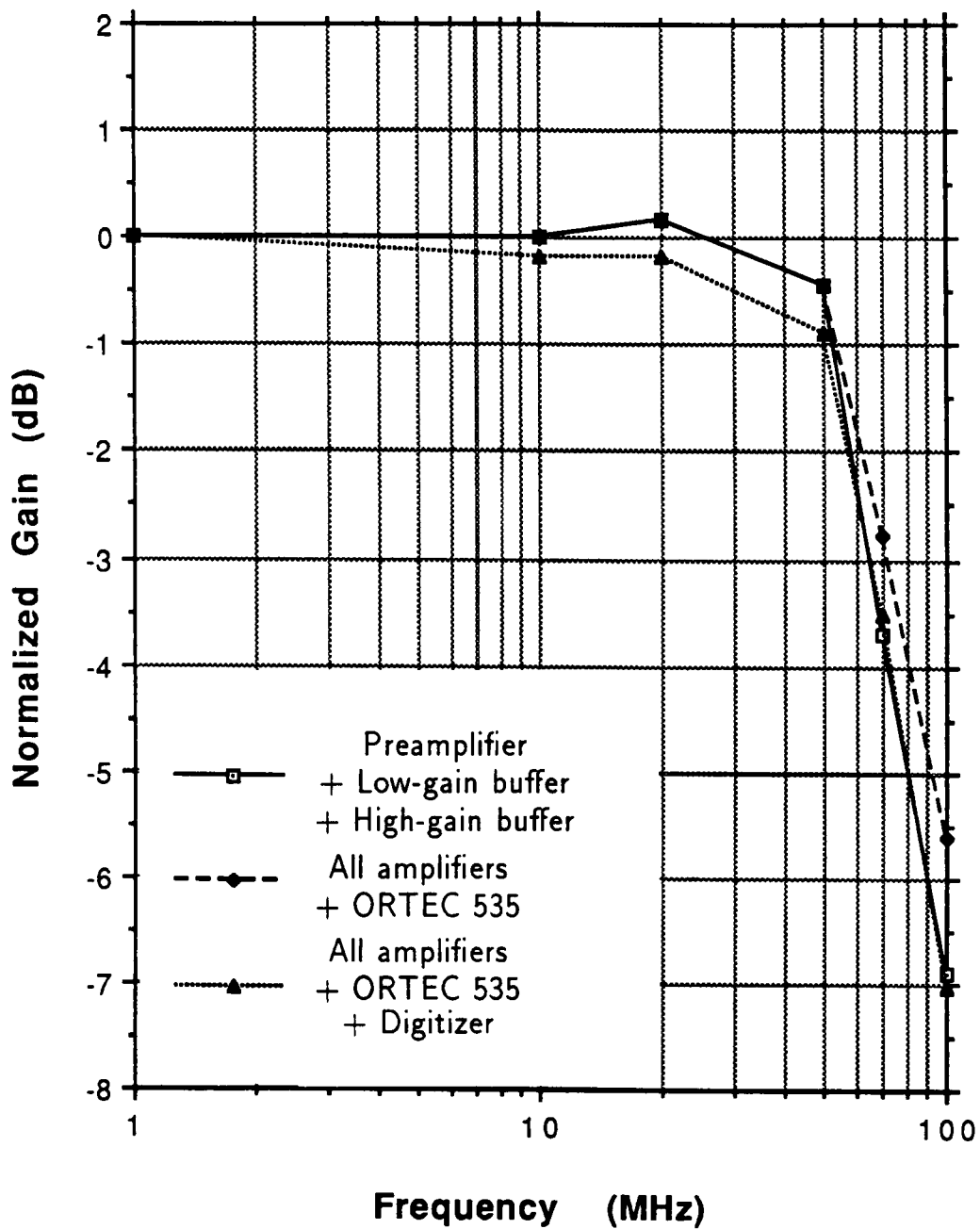


Figure 5.37: Amplification system frequency responses below 100 MHz.

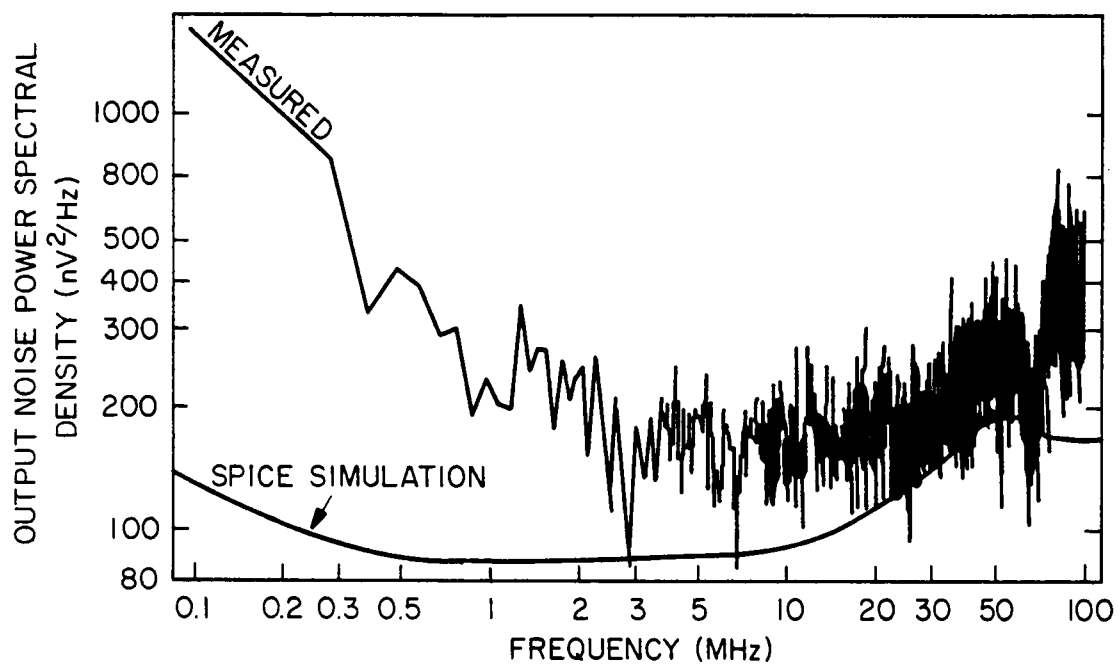


Figure 5.38: Measured output noise power spectral density for transimpedance preamplifier.

the preamplifier output noise PSD predicted by SPICE. The measured noise PSD is approximately twice that predicted by the simulation for frequencies above a few MHz and differs even more at lower frequencies.

There are a number of reasons why the measured PSD is greater than that predicted. For the lower frequencies, the discrepancy is due to the modeling of the $1/f$ -type noise as having an f^{-1} dependency when it is actually $f^{-1.37}$. The simulation also did not include the noise due to the additional amplifiers. The predicted preamplifier e_{no} was 0.125 mV rms, which equals the thermal noise produced by a 9.4 k Ω resistor in a 100-MHz noise bandwidth, so the additional amplifiers do not have to be extremely low noise amplifiers, but they do make a small contribution to the random noise. More importantly, there was a barely visible parasitic oscillation (due to the high-gain buffer) with a frequency near 100 MHz which appears as the large peak just below 100 MHz. This peak may also include the noise power of frequencies just above 100 MHz which has been aliased to that location.

The preamplifier generates a significant amount of noise in the range 100 MHz to 200 MHz, but the noise power above approximately 120 MHz is strongly attenuated by the frequency response of the additional amplifiers. As shown in Figure 5.39, the response of the additional amplifiers has little effect on the noise spectrum below 100 MHz, but provides 12 dB attenuation at only 120 MHz for the complete system. This reduces the noise power by more than a factor of 10 at 120 MHz, and the reduction is greater for higher frequencies, so approximating

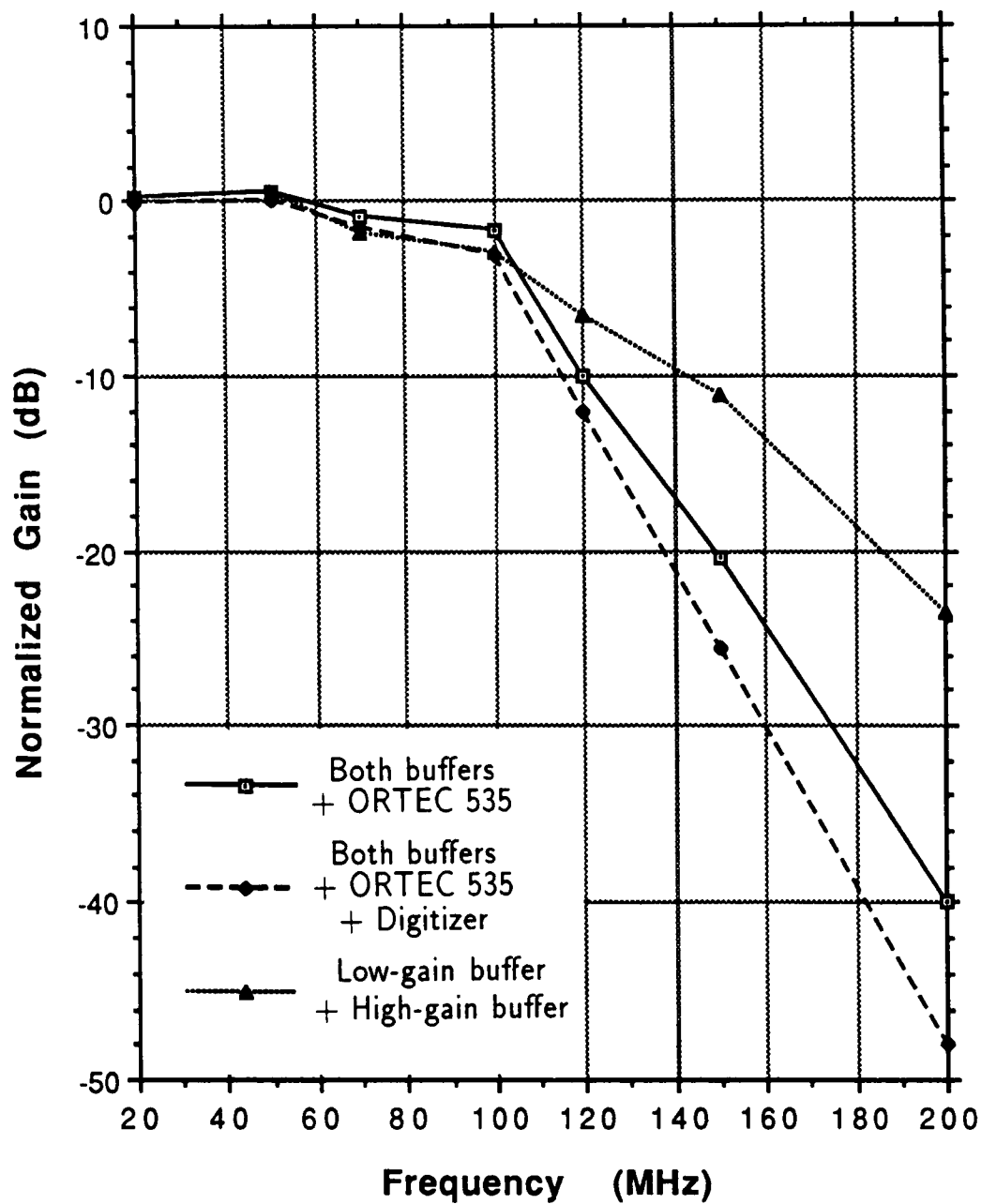


Figure 5.39: Normalized frequency response of amplifier combinations following the preamplifier.

the preamplifier noise bandwidth as 100 MHz was not unreasonable. It should be noted that the SPICE simulated preamplifier PSD shown in Figure 5.38 differs from that shown in Figure 5.20 since the PSD shown in Figure 5.38 has been corrected for the frequency response of the rest of the amplification system.

The total noise power shown in Figure 5.38 was integrated, and the square root of the total was divided by the preamplifier transimpedance gain to give $i_{ni} = 47$ nA rms. Since the system used to measure this noise is essentially the same as that to be used for prebreakdown current measurements, the measured noise should be achievable in practice. 47 nA rms is almost twice that estimated for the preamp (25.1 nA rms), but considering the approximations involved in the estimate and the presence of the parasitic oscillation, the agreement is reasonable. The measured value of i_{ni} means that current pulses with amplitudes as low as 235 nA will be detectable for a signal-to-noise of five.

CHAPTER 6

SIGNAL RECONSTRUCTION

6.1 Continuous Signal Recovery Through Interpolation

A transient digitizer is a convenient way to capture and save a waveform. The digitizer samples the waveform periodically, converts the amplitude to a digital number and stores that value (a sample) for each time. Thus a continuous analog signal is converted into a discrete time signal which is essentially a long string of numbers or samples. A plot of the values as a function of time ($t = nT$, where n is the sample number and T is the sampling interval), produces a reasonable picture of the waveform. This digital representation of a waveform can be stored, filtered, averaged with similar waveforms or even converted back to analog form.

For signals like prebreakdown current pulses that are very fast and are composed of only a few samples, a simple plot of the samples versus time provides only a rough picture of the original continuous signal. By considering the effect of the digitization process on a high frequency signal, the effects on the high frequency components of a pulse can be easily visualized. For example, a digitizer with a minimum sampling interval of 5 ns would sample a 67-MHz signal (which has a period of 15 ns) three times per period. Three points per cycle does not

make a very good plot of a sinewave. One might conclude that something on the order of ten points per cycle would be needed to get a good representation of the waveform. Fortunately, the sampling theorem states that signals with frequencies less than the Nyquist frequency ($f_n = 1/2T$) can be reconstructed [67], so for a 200-megasample/s digitizer, the maximum signal frequency is 100 MHz. There is an important restriction, however: the signal must be bandlimited and contain no frequencies greater than f_n .

Since the digitizer can only capture a finite number of samples (m), the duration of the sampled portion of the waveform is mT , where T is the sampling interval. A waveform identical on the range 0 to mT to the one digitized but periodic in mT would produce exactly the same results when sampled. Fourier's theorem states that a periodic signal can be represented as a weighted sum of sinewaves (Fourier series), and since the signal is bandlimited, only a finite number of sines are needed to completely represent the periodic signal. This summation would also represent the signal that was digitized over the range 0 to mT , *for any value of t* , and not just the sample points. Actually a discrete Fourier transform (DFT) would be used to compute the Fourier coefficients of a sampled data waveform, but the idea is the same. The signal can be transformed to a finite spectrum and the spectrum back to a signal. Both forms represent the same information.

The basic idea behind signal reconstruction is that the continuous signal can be represented as a sum of sinewaves and that the sampled points contain enough

information to find that series. Since the value of the continuous signal at any time can be found from the series, it can also be found from the sampled values. The procedure of finding the values of the signal at times between the sampled points is called *interpolation*. Frequency domain interpolation procedures find the DFT of the sampled data, manipulate it and then use an inverse DFT to recover the values of the signal at intermediate times. Time domain interpolation procedures such as Whittaker's reconstruction can be found in many texts on digital signal processing [67,68,69,70] and operate directly on the sampled data to find intermediate values. If enough intermediate values are found, then for practical purposes, the continuous signal has been reconstructed.

The time domain methods are simpler computationally than the frequency domain methods as they do not require calculation of a DFT or inverse DFT. A very straightforward approach to time-domain interpolation described by Jackson [69] was selected as it appeared to be a method that could be implemented fairly easily and not require an excessive amount of computation. This interpolation process increases the sampling rate by an integer factor L which amounts to reconstructing $L - 1$ points between every pair of data points. The actual process consists of two steps. The first is the insertion of $L - 1$ zeroes between each pair of samples. The second is the lowpass filtering of the augmented data. The filtered output is the reconstructed data.

6.2 Theoretical Basis for the Interpolation Process

The procedure to interpolate one point halfway between each pair of data points consists of adding a zero between each pair and digitally lowpass filtering the resulting string of values. Examination of the resulting frequency spectra justifies this seemingly strange procedure. The basic idea is to increase the sampling rate and therefore the number of points per cycle without changing the frequency spectrum of the signal. This process can be repeated to give additional points between the samples. More than one point between samples can be calculated with one use of the same procedure if more than one zero is added between samples, but several repetitions using only one zero give more accurate results.

The discrete time Fourier transform $X(j\Omega)$ or frequency spectrum of a discrete time signal $x(n)$ is given by

$$X(j\Omega) = \sum_{n=-\infty}^{\infty} x(n) \exp^{-j\Omega n}, \quad (6.1)$$

where Ω is the normalized digital frequency in radians. For a signal $x(n)$ band-limited to frequencies less than some frequency f_1 , which is in turn less than f_n , the frequency spectrum would resemble Figure 6.1(A). The frequency axis is labeled both in terms of f (the continuous analog frequency in Hz) and Ω . These frequencies are related by $\Omega = 2\pi T f$. Thus the sampling frequency, f_s , which is the inverse of T , corresponds to 2π and the Nyquist frequency corresponds to π . The spectrum is 2π periodic in Ω , but all the information needed to reconstruct

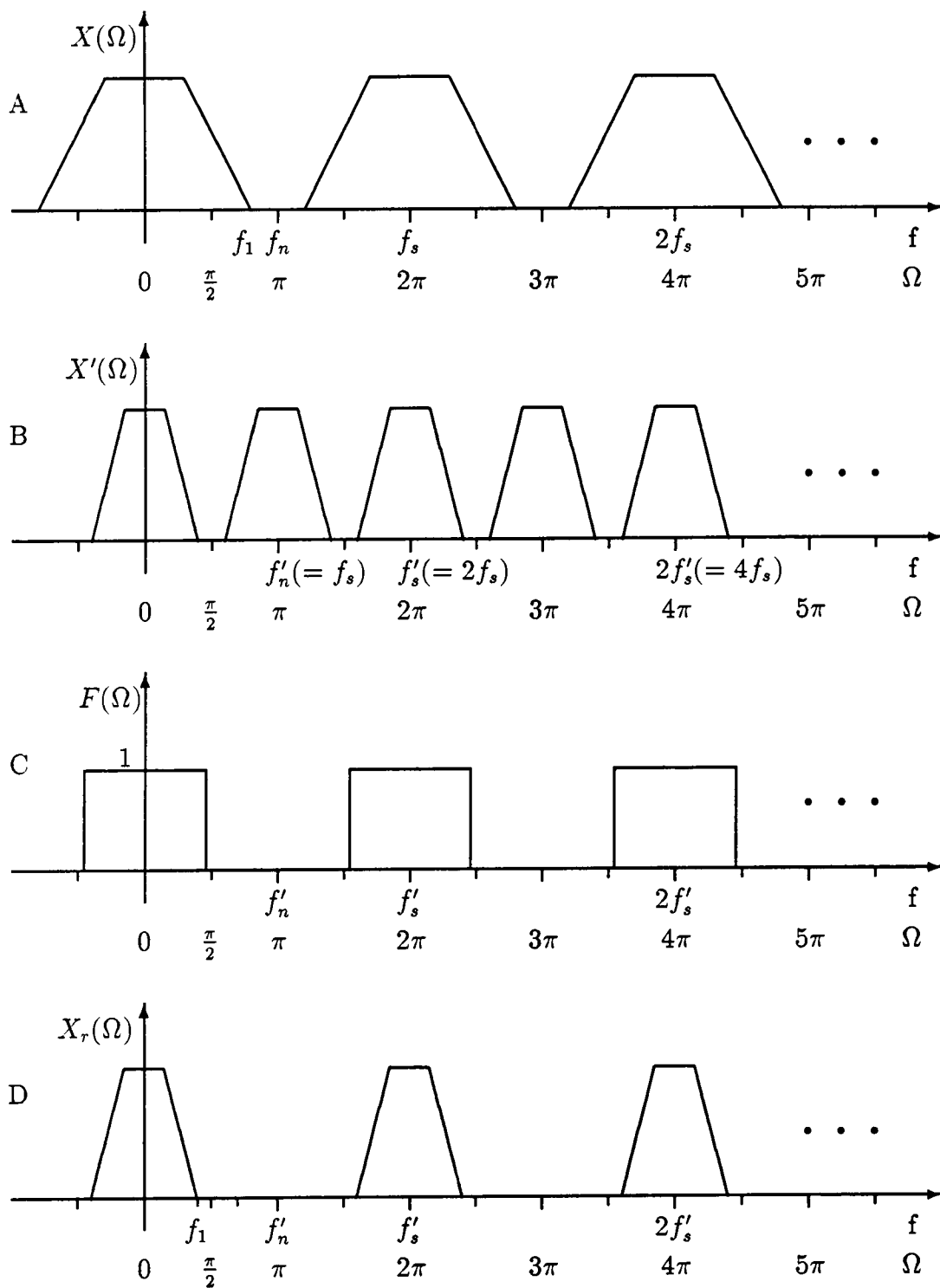


Figure 6.1: Power spectra for the interpolation process. (A) Power spectrum of the original signal. (B) Spectrum after adding zeroes. (C) Digital lowpass filter characteristics. (D) Reconstructed signal power spectrum.

the signal is contained between 0 and π .

Since the time between the original samples $x(n)$ was T , the addition of a zero sample between each original sample effectively doubles the sampling rate, and the time between samples is now $T/2$. If the original sampling frequency is denoted by f_s and the new frequencies and spectrum are denoted by the addition of a prime, then $f'_n = f_s$ and $f'_s = 2f_s$. The addition of zeroes to the waveform distorts the spectrum, but in a controlled sort of way. The original spectrum is compressed by a factor of two in terms of the digital frequency Ω , but for analog frequencies less than $f_n = f'_n/2$, the spectrum $X'(\Omega)$ is the same (Figure 6.1(B)). This spectral compression is proven mathematically in Appendix F.

To produce a spectrum that is the same as the original up to f_1 and zero from f_1 up to f'_n , requires a lowpass filter as shown in Figure 6.1(C). Such a filter must pass frequencies up to and greater than f_1 but stop frequencies above $2f_n - f_1$ (f_n now corresponds to $\pi/2$) up to f'_n . The signal created by adding zeroes is passed through the filter and the resulting spectrum is the product of the distorted spectrum and the filter frequency response. As shown in Figure 6.1(D), it is the same as the original except that the effective sampling rate has been doubled. All the information about the signal is contained in the range of frequencies from 0 to f_n . No new information has been added to the signal.

6.3 An Example of Reconstruction Through Interpolation

When the interpolated signal is represented as a function of time, an improvement has been made since there are now twice as many more closely spaced samples of the original continuous signal. Half are the original samples and half are reconstructed, and together they provide a more useful representation of the continuous signal. As an example, the signal shown in Figure 6.2 was sampled every $2\mu\text{s}$ and then reconstructed. This signal is made up of two frequencies, 100 kHz and 167 kHz which are only somewhat less than the Nyquist frequency of 250 kHz. If the continuous signal is reconstructed by connecting the sampled points with straight lines as in Figure 6.3, a poor representation of the original signal is the result, but if the interpolation procedure is used, the better picture shown in Figure 6.4 results. It is important to note that this procedure does not create or gain any information not present in the sampled data, but only makes a better representation of it.

In theory, the interpolation procedure could be carried out a great many times until the signal was essentially continuous. In practice only several times are necessary to give a very satisfactory picture of the original signal. This type of processing is used with some digital storage oscilloscopes so the scope can accurately display transient waveforms containing frequencies nearly up to the Nyquist frequency [71]. When this technique is not used, 10 samples per cycle of a sinusoidal signal are required to give reasonable accuracy with linear

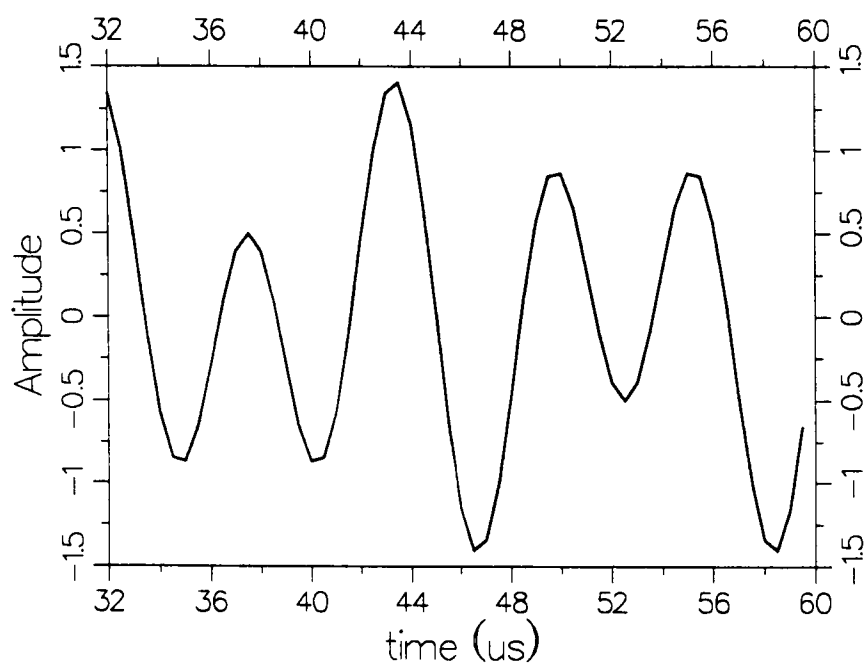


Figure 6.2: Original signal.

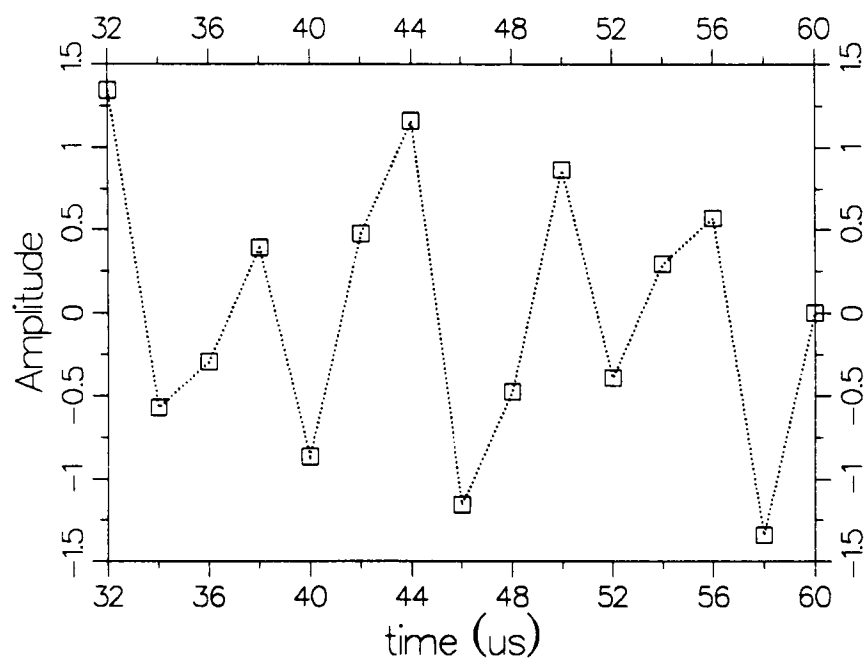


Figure 6.3: Sampled signal.

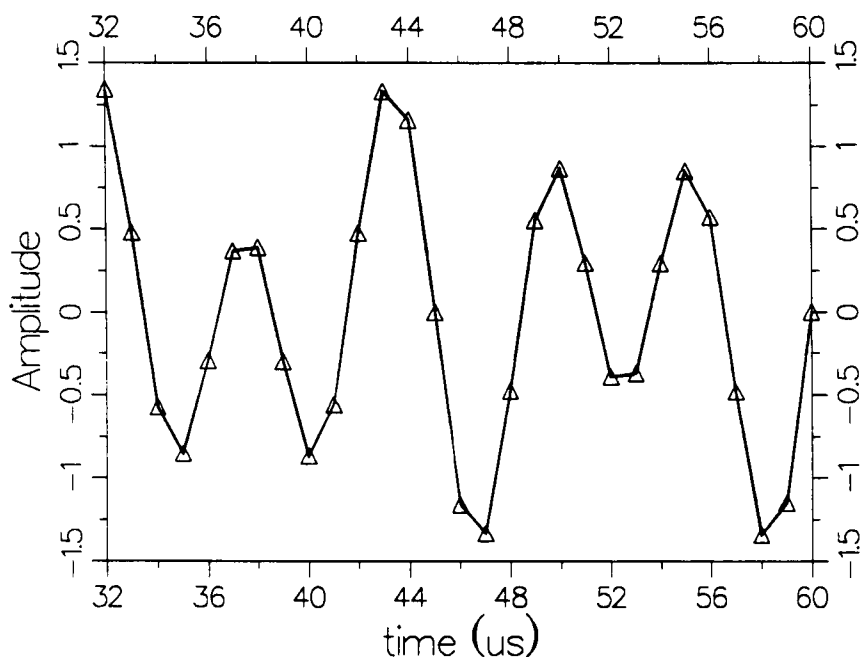


Figure 6.4: Reconstructed signal.

interpolation (connect the dots), and thus the signal is limited to one-fifth of the Nyquist frequency. Accurate reconstruction through interpolation minimizes the speed needed from the analog-to-digital converter to achieve a particular equivalent analog bandwidth and obtains the highest possible bandwidth from a digitizer.

6.4 Development of Digital Lowpass Filters for Interpolation

The lowpass filter that was shown in Figure 6.1 had an ideal lowpass characteristic, but as described in the text, the requirements for the filter are not that strict. Actually the filter must pass frequencies up to f_1 and stop frequencies

above $2f_n - f_1$. If the highest frequency components of the signal approach the Nyquist frequency, as is the case for this application, then $f_1 \approx f_n$ and a very narrow transition band is required. The filter should be flat in the passband to prevent distortion of the signal and have reasonably large attenuation in the stopband to remove the reconstruction artifacts in the spectrum.

There are a number of different ways to implement a digital lowpass filter, but Jackson [69] reported that it was usually advantageous to use a finite impulse response (FIR) filter in direct form due mainly to the savings in the number of multiplications required for the interpolation process. One very direct way to produce an FIR filter is to take the inverse discrete-time Fourier transform of the desired frequency response. This process yields the impulse response sequence or coefficients of the filter [72]. For example, if an ideal lowpass filter with a cutoff frequency of Ω_c is desired, the the frequency response is

$$H(\Omega) = \begin{cases} 1 & |\Omega| \leq \Omega_c \\ 0 & |\Omega| > \Omega_c \end{cases} \quad (6.2)$$

on the interval from $-\pi$ to π and is 2π periodic in Ω . The inverse discrete-time Fourier transform of $H(\Omega)$ is given by

$$h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(\Omega) e^{j\Omega n} d\Omega, \quad (6.3)$$

and substituting for $H(\Omega)$ gives

$$h(n) = \frac{1}{2\pi} \int_{-\Omega_c}^{\Omega_c} e^{j\Omega n} d\Omega. \quad (6.4)$$

Integrating this and simplifying gives

$$h(n) = \begin{cases} \frac{\sin(n\Omega_c)}{n\pi} & n \neq 0 \\ \Omega_c/\pi & n = 0 \end{cases}, \quad (6.5)$$

or

$$h(n) = \frac{\Omega_c}{\pi} \operatorname{sinc}\left(\frac{n\Omega_c}{\pi}\right), \quad (6.6)$$

where

$$\operatorname{sinc}(x) = \frac{\sin \pi x}{\pi x}. \quad (6.7)$$

A plot of $h(n)$ for the case of $\Omega_c = \pi/2$ is shown in Figure 6.5. $h(n)$ has two important features that are worth noting. Since the sinc function is an even function, $h(n) = h(-n)$. For the case $\Omega_c = \pi/2$, $h(n) = 0$ when n is even, except for $n = 0$.

Obviously $h(n)$ cannot be implemented in its entirety since it is an infinite sequence. One possible solution is to simply truncate $h(n)$ and only use a finite number of its terms for the filter. This is not an unreasonable approach as the strength of the coefficients decrease with $|n|$. Mathematically, the truncation can be described as the multiplication of the $h(n)$ with a windowing function or window $w(n)$ which is given by

$$w(n) = \begin{cases} w(n) = 1 & |n| \leq m \\ w(n) = 0 & |n| > m \end{cases}, \quad (6.8)$$

where m is an integer. The multiplication yields a windowed filter $h_w(n)$ which

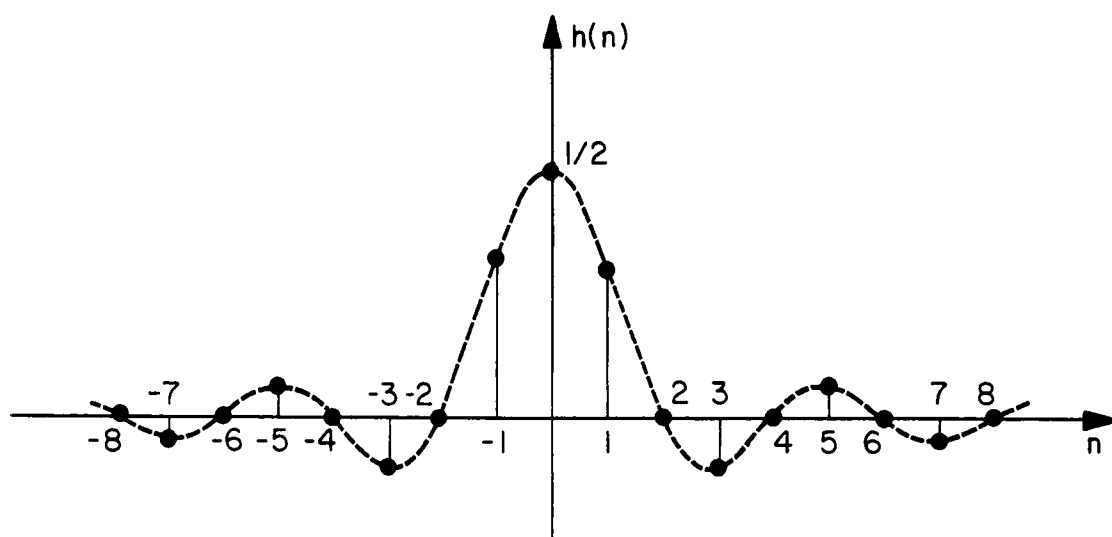


Figure 6.5: Ideal lowpass filter coefficients for $\Omega_c = \pi/2$.

is given by

$$h_w(n) = h(n)w(n) = \begin{cases} h_w(n) = h(n) & |n| \leq m \\ h_w(n) = 0 & |n| > m \end{cases} \quad (6.9)$$

This limits the filter to $2m + 1$ terms ranging from $h(-m)$ to $h(m)$.

Unfortunately there is a problem with this simple approach. The time domain multiplication of the ideal filter $h(n)$ with the rectangular window $w(n)$ corresponds to the frequency domain convolution of their Fourier transforms $H(j\Omega)$ and $W(j\Omega)$ [73]. $H(j\Omega)$ was given in Eq. 6.2, and $W(j\Omega)$ can be found by taking the Fourier transform of $w(n)$:

$$W(j\Omega) = \sum_{n=-\infty}^{\infty} w(n)e^{-j\Omega n} = \sum_{n=-m}^m e^{-j\Omega n}. \quad (6.10)$$

This series can be expressed as a fraction:

$$W(j\Omega) = \frac{e^{j\Omega m} - e^{-j\Omega(m+1)}}{1 - e^{-j\Omega}}, \quad (6.11)$$

which can then be reorganized as

$$W(j\Omega) = \frac{e^{j\Omega(m+1/2)} - e^{-j\Omega(m+1/2)}}{e^{j\Omega/2} - e^{-j\Omega/2}}. \quad (6.12)$$

The differences of complex exponentials are recognizable as sine terms, and $W(j\Omega)$ reduces to

$$W(j\Omega) = \frac{\sin[(m + \frac{1}{2})\Omega]}{\sin(\Omega/2)}, \quad (6.13)$$

which resembles the sinc function for Ω near zero, but is 2π periodic.

The frequency response of the windowed filter, $H_w(\Omega)$, is given by the convolution of $H(j\Omega)$ and $W(j\Omega)$ or

$$H_w(j\Omega) = H(j\Omega) * W(j\Omega) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(j\Theta)W[j(\Omega - \Theta)] d\Theta. \quad (6.14)$$

If $W(j\Omega)$ were an impulse train, then $H_w(j\Omega)$ would be the same as $H(j\Omega)$, but since it is not, the ideal lowpass characteristic is degraded. This effect is shown in Figure 6.6 for the case of a lowpass with $\Omega_c = \pi/2$ and a window with $m = 4$ (9 terms). The resulting filter has considerable ripple in the passband and relatively wide transition region. Re-plotting the filter frequency response with a logarithmic magnitude scale (Figure 6.7) reveals that the minimum stopband attenuation is only about 20 dB.

The width of the transition region can be decreased by increasing the number of terms as shown in Figure 6.8, but this does not increase the stopband attenuation or reduce the passband ripple. The ideal lowpass FIR filter no longer has ideal characteristics when only a finite number of terms are used.

A similar problem arises in spectral analysis when digitized data is processed with a Fast Fourier transform (FFT). The use of a finite number of data points is equivalent to multiplying an infinite amount of data by a rectangular window. This multiplication in the time domain is equivalent to convolution in the frequency domain. Usually the signal is thought of as having impulse-like characteristics, so the most dramatic effect of the rectangular window is to broaden each spectral peak. Another effect is the shift of energy from the signal to frequencies that are very different from that of the signal. Both of these effects are termed spectral leakage and are illustrated in Figure 6.9 for the very simple case of a dc signal (an impulse at the origin in frequency space) and the same window shown in Figure 6.6. Spectral leakage can prevent the detection of low strength

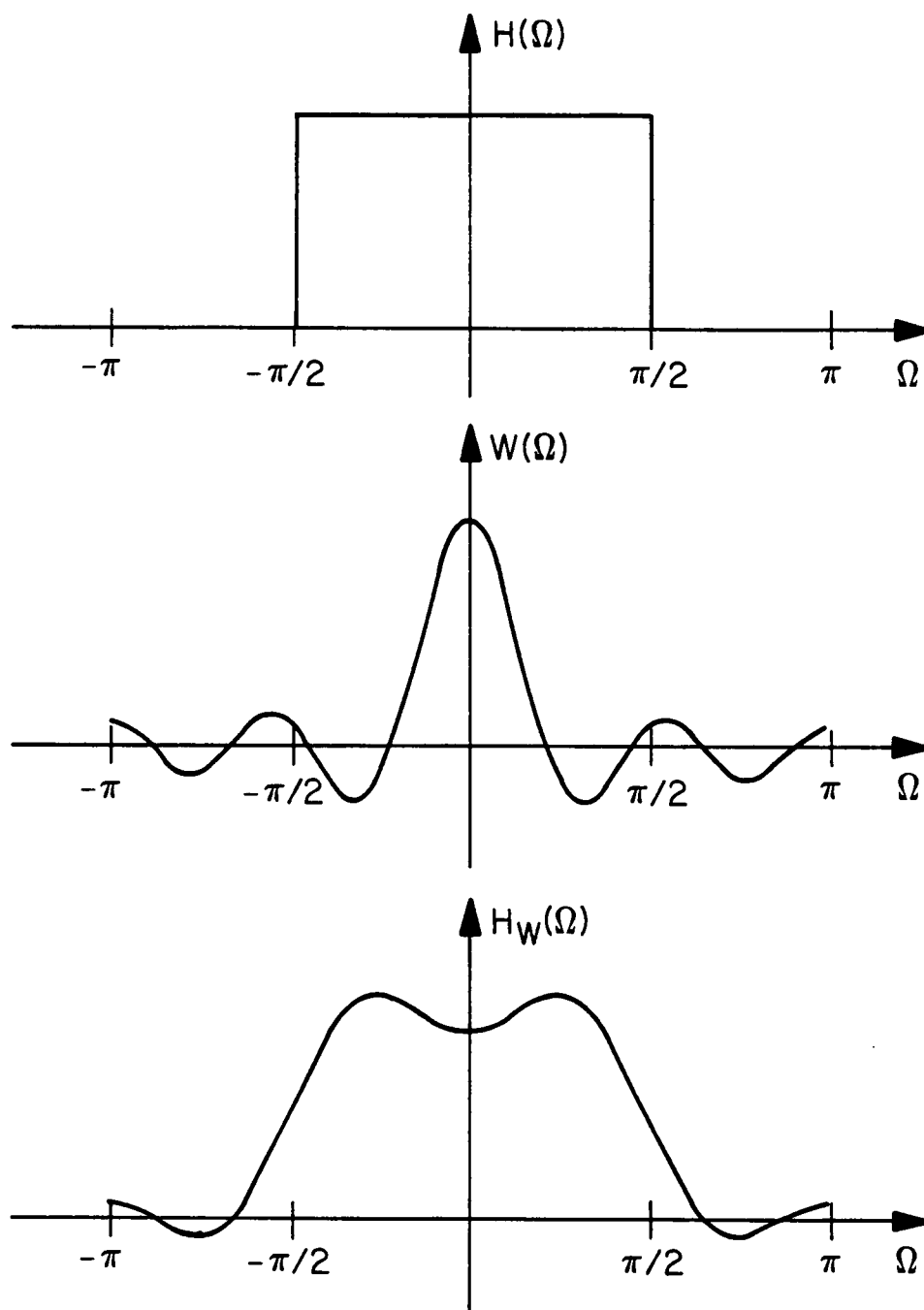


Figure 6.6: Convolution of rectangular window with ideal lowpass filter.

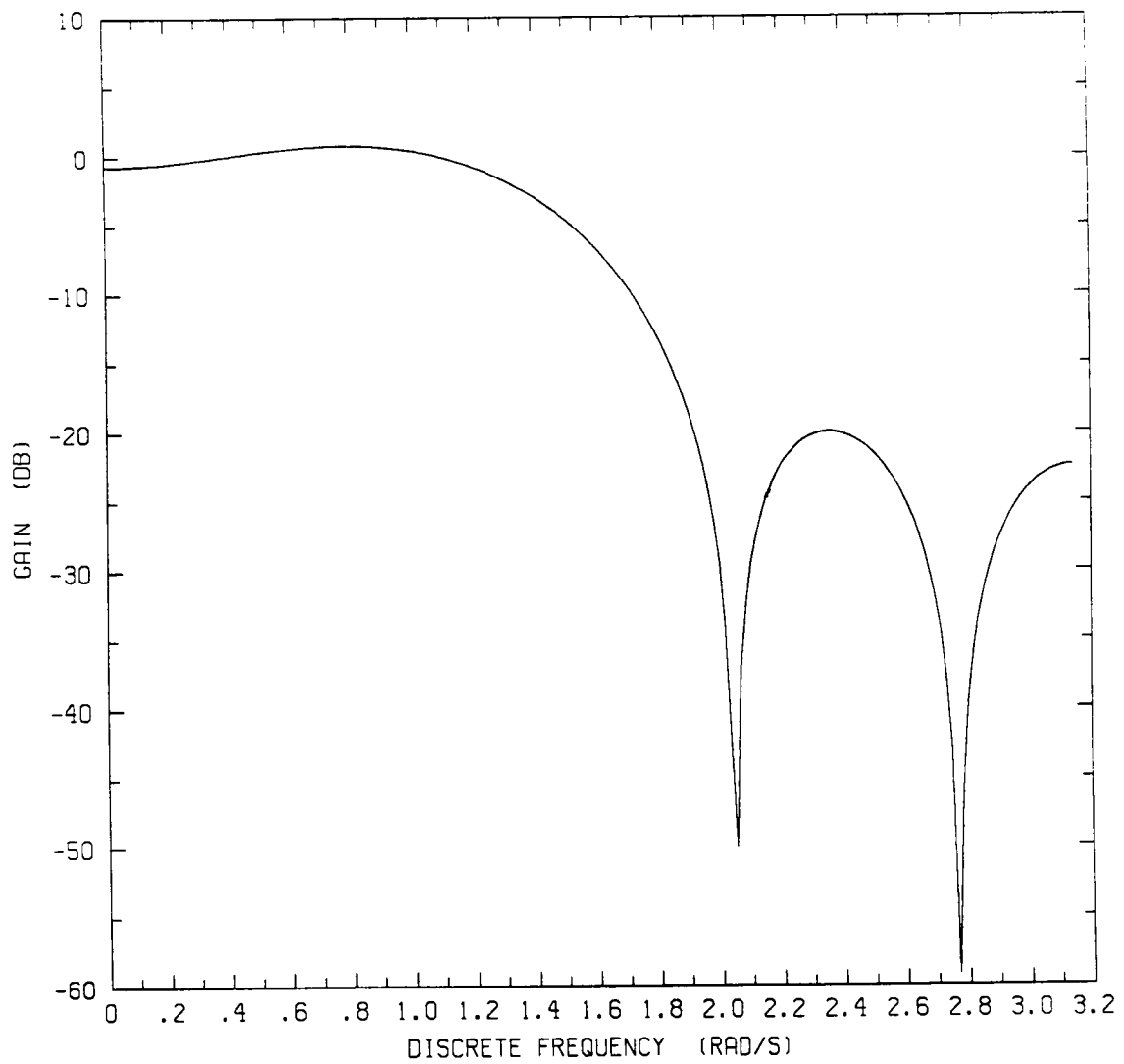


Figure 6.7: Frequency response of rectangular window-based, 9-term FIR filter.

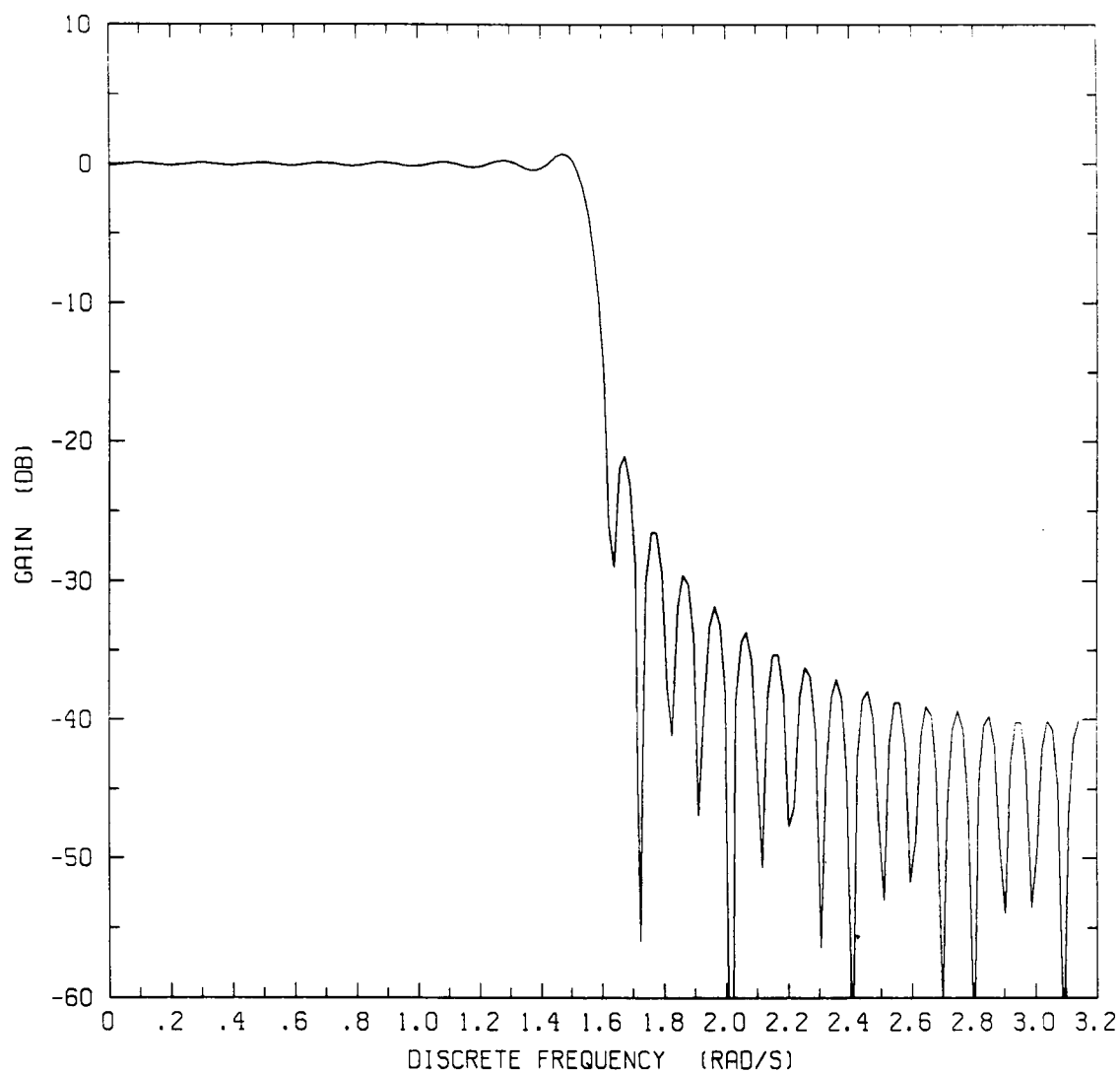


Figure 6.8: Frequency response of rectangular window-based, 65-term FIR filter.

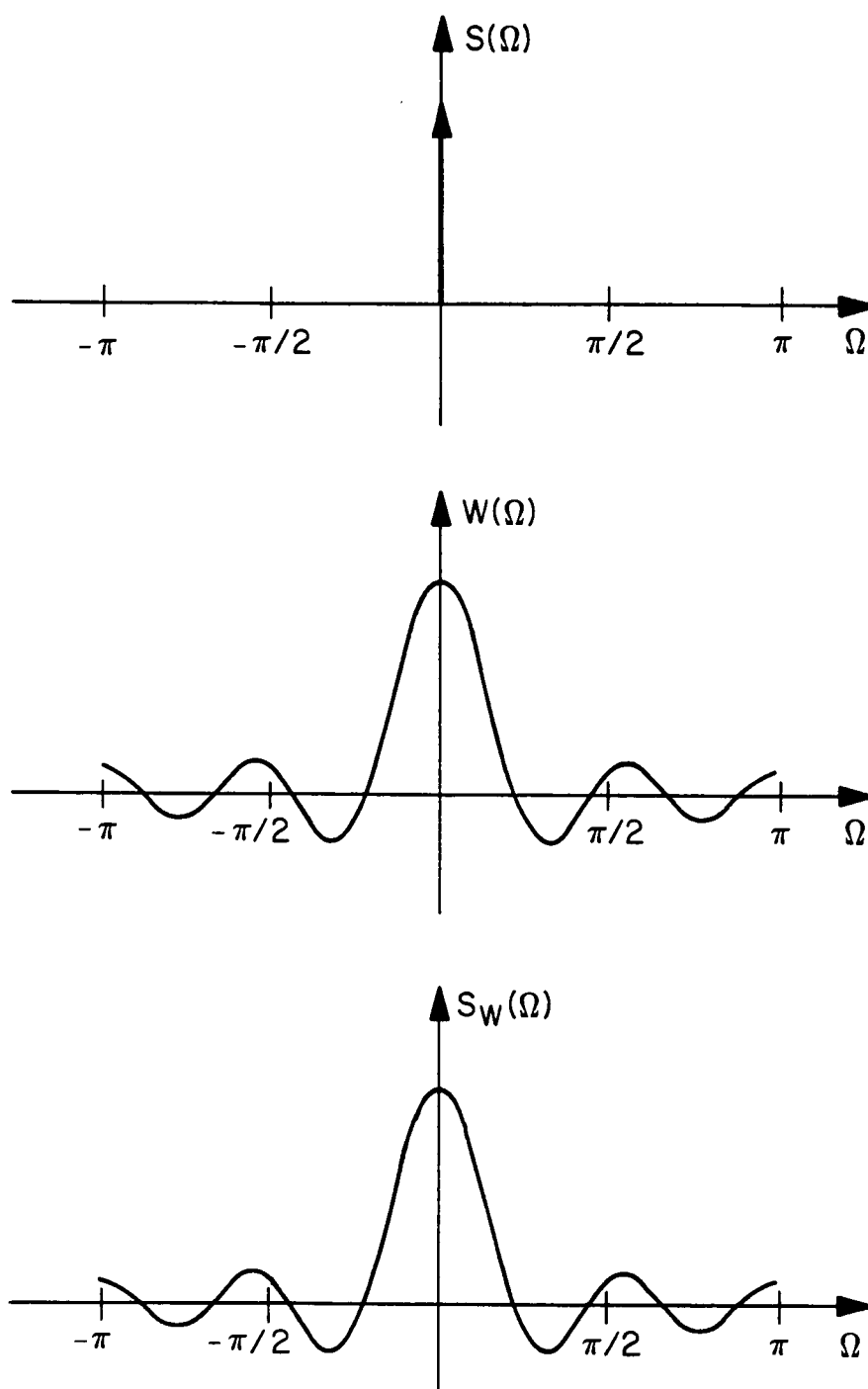


Figure 6.9: Spectral line broadening due to data windowing. The signal spectrum $S(j\Omega)$ is convolved with the Fourier transform of the window $W(j\Omega)$ to give the spectrum of the windowed signal, $S_W(j\Omega)$.

signals in the presence of strong signals or the resolution of spectral lines with nearly the same frequency. The solution to this problem is the use of a window function with better frequency domain characteristics than the rectangular window.

Figure 6.10 shows the magnitude of the Fourier transform of the same 9-term window used earlier and can be used to illustrate the window characteristics needed for lowered spectral leakage. The width of the “main lobe” determines the degree of spectral line broadening, while the amplitude of the “side lobes” relative to that of the main lobe determines the minimum strength for a signal to be easily detected above the spectral leakage of a strong signal. An ideal window would have zero main lobe width and side lobes of zero amplitude, so a “good” window is one with a very narrow main lobe and side lobes with very low amplitudes. The width of the main lobe can be reduced by increasing the number of terms used in the window: the width is proportional to $1/N$, where N is the number of terms, but the maximum side lobe level cannot be reduced by increasing N as it depends only upon the windowing function.

Upon re-examination of Figures 6.6 and 6.9, it is clear that the features of a window that are desirable for creating a lowpass filter are the same ones that are desired for spectral analysis. A window with a narrow main lobe produces a filter with a narrow transition region. A window with low amplitude side lobes produces a filter with low passband ripple and high stop band minimum attenuation. Thus any windows developed for spectral analysis may also be useful

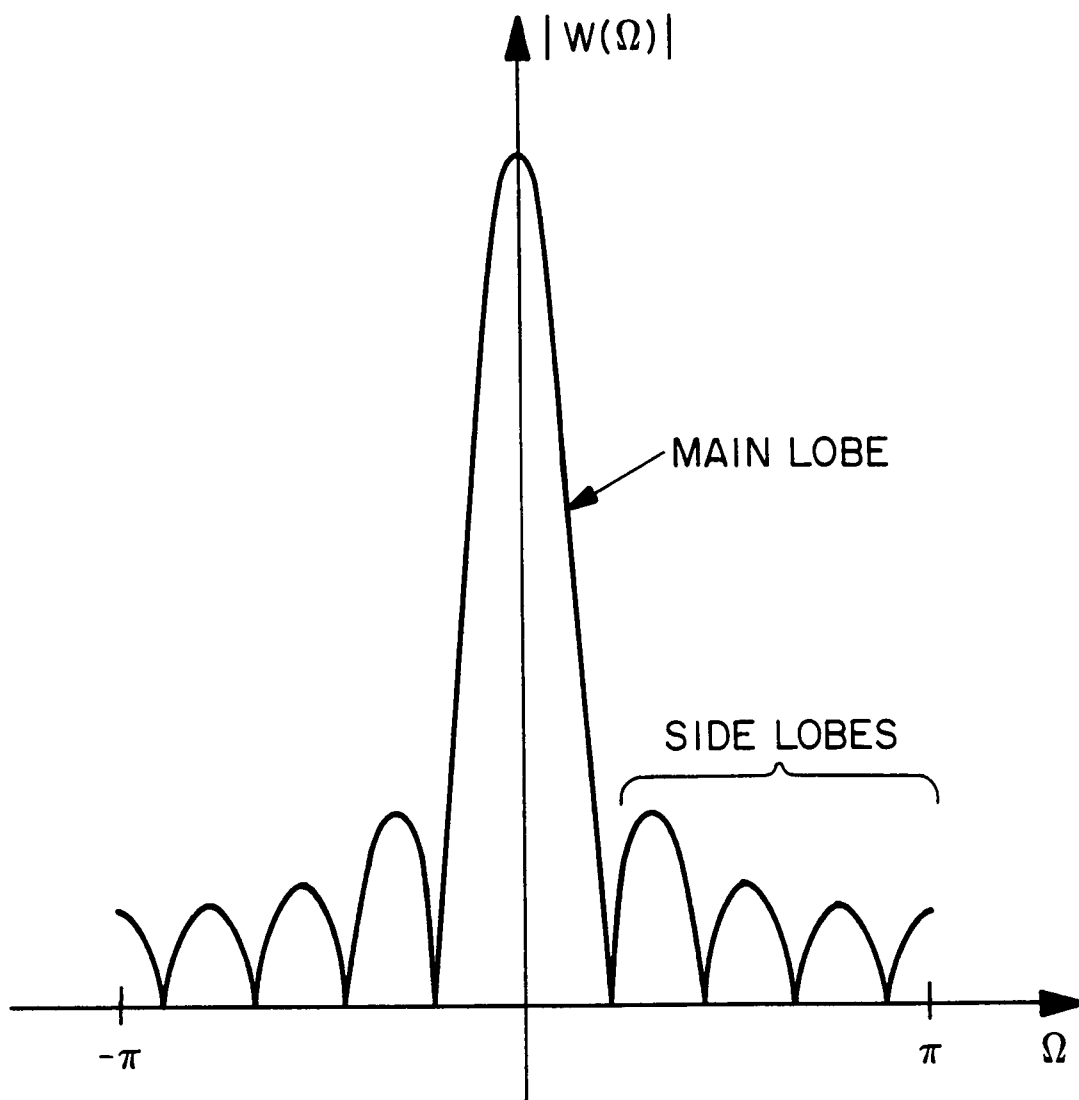


Figure 6.10: Magnitude of rectangular window spectrum.

for developing lowpass filters. Fortunately, a great many windows have been developed for spectral analysis, and their characteristics have been studied in depth. Harris [73] gave a very good review of the use of windows for spectral analysis. He presented and compared many window functions and their characteristics, and found the performance of the Kaiser-Bessel window to be as good as or superior to that of any other window studied.

The Kaiser-Bessel window is defined by

$$w(n) = \frac{I_0 \left[\pi \alpha \sqrt{1 - \left(\frac{n}{m} \right)^2} \right]}{I_0[\pi \alpha]} \quad (6.15)$$

for

$$0 \leq |n| \leq m. \quad (6.16)$$

$I_0(x)$ is the zero-order modified Bessel function of the first kind and is given by

$$I_0(x) = \sum_{k=0}^{\infty} \left\{ \frac{(x/2)^k}{k!} \right\}^2. \quad (6.17)$$

The parameter α allows some adjustment of the window. A lower value of α (like $\alpha = 2$) gives a relatively narrow main lobe, though not as narrow as a rectangular window, while a higher value of α (like $\alpha = 3$) gives a wider main lobe and lower side lobes.

A number of $\pi/2$ cut-off FIR filters were developed using the Kaiser-Bessel window. Equations 6.15 and 6.6 were used to generate the window coefficients and ideal lowpass filter coefficients, respectively. These coefficients were multiplied together according to Equation 6.9 to give the filter coefficients. An

example of the filter resulting from this process is given in Table 6.1 for the case of $m = 16$ which produces a 33-term filter. From this table, it is easy to see that the window has little effect on the filter coefficients for n near zero, but the window gradually tapers the filter coefficients to zero as $|n|$ approaches m . The frequency response of this filter is shown in Figure 6.11, and that of a similar, but 65-term filter is shown in Figure 6.12.

Filters were generated for the cases of $\alpha = 2$ and $\alpha = 3$ for m values of 4, 8, 16 and 32, and compared to rectangular filters of the same length. To estimate the suitability of these filters for signal reconstruction via interpolation, the requirements placed on the filter need to be considered again. For use in signal reconstruction, a filter needs to pass frequencies up to Ω_1 and stop frequencies above $\pi - \Omega_1$, where Ω_1 is the maximum signal frequency and π is twice the Nyquist frequency of the reconstructed signal. These are the same requirements outlined in at the first of this section, only restated in terms of the normalized digital frequency, and apply to the case where one point is to be interpolated between each pair of samples. Unfortunately, the exact spectrum of the signal (the prebreakdown currents) is not known and the source of the data (the amplification system and digitizer) does not have a brick-wall cut-off characteristic, so Ω_1 is not known. In fact, since the system has response and noise above 100 MHz, which corresponds to $\pi/2$ for the reconstructed signal, Ω_1 would appear to be greater than $\pi/2$, and the filter requirements would be impossible to satisfy (since the signal exceeded the Nyquist frequency). From

Table 6.1: Lowpass filter and window coefficients. $h(n)$ is an ideal lowpass filter, $w(n)$ is a Kaiser-Bessel window with $\alpha = 2$ and $h_w(n)$ is a lowpass filter created by combining $h(n)$ and $w(n)$. All the sequences have 33 terms.

n	$h(n)$	$w(n)$	$h_w(n)$	n	$h(n)$	$w(n)$	$h_w(n)$
-16	0.00000	0.01148	0.00000	1	0.50000	1.00000	0.50000
-15	-0.02122	0.02989	-0.00063	2	0.31831	0.98880	0.31475
-14	0.00000	0.05797	0.00000	3	0.00000	0.95585	0.00000
-13	0.02449	0.09720	0.00238	4	-0.10610	0.90299	-0.09581
-12	0.00000	0.14851	0.00000	5	0.00000	0.83312	0.00000
-11	-0.02894	0.21209	-0.00614	6	0.06366	0.74999	0.04775
-10	0.00000	0.28725	0.00000	7	0.00000	0.65791	0.00000
-9	0.03537	0.37236	0.01317	8	-0.04547	0.56139	-0.02553
-8	0.00000	0.46486	0.00000	9	0.00000	0.46486	0.00000
-7	-0.04547	0.56139	-0.02553	10	0.03537	0.37236	0.01317
-6	0.00000	0.65791	0.00000	11	0.00000	0.28725	0.00000
-5	0.06366	0.74999	0.04775	12	-0.02894	0.21209	-0.00614
-4	0.00000	0.83312	0.00000	13	0.00000	0.14851	0.00000
-3	-0.10610	0.90299	-0.09581	14	0.02449	0.09720	0.00238
-2	0.00000	0.95585	0.00000	15	0.00000	0.05797	0.00000
-1	0.31831	0.98880	0.31475	16	-0.02122	0.02989	-0.00063
0	0.50000	1.00000	0.50000				

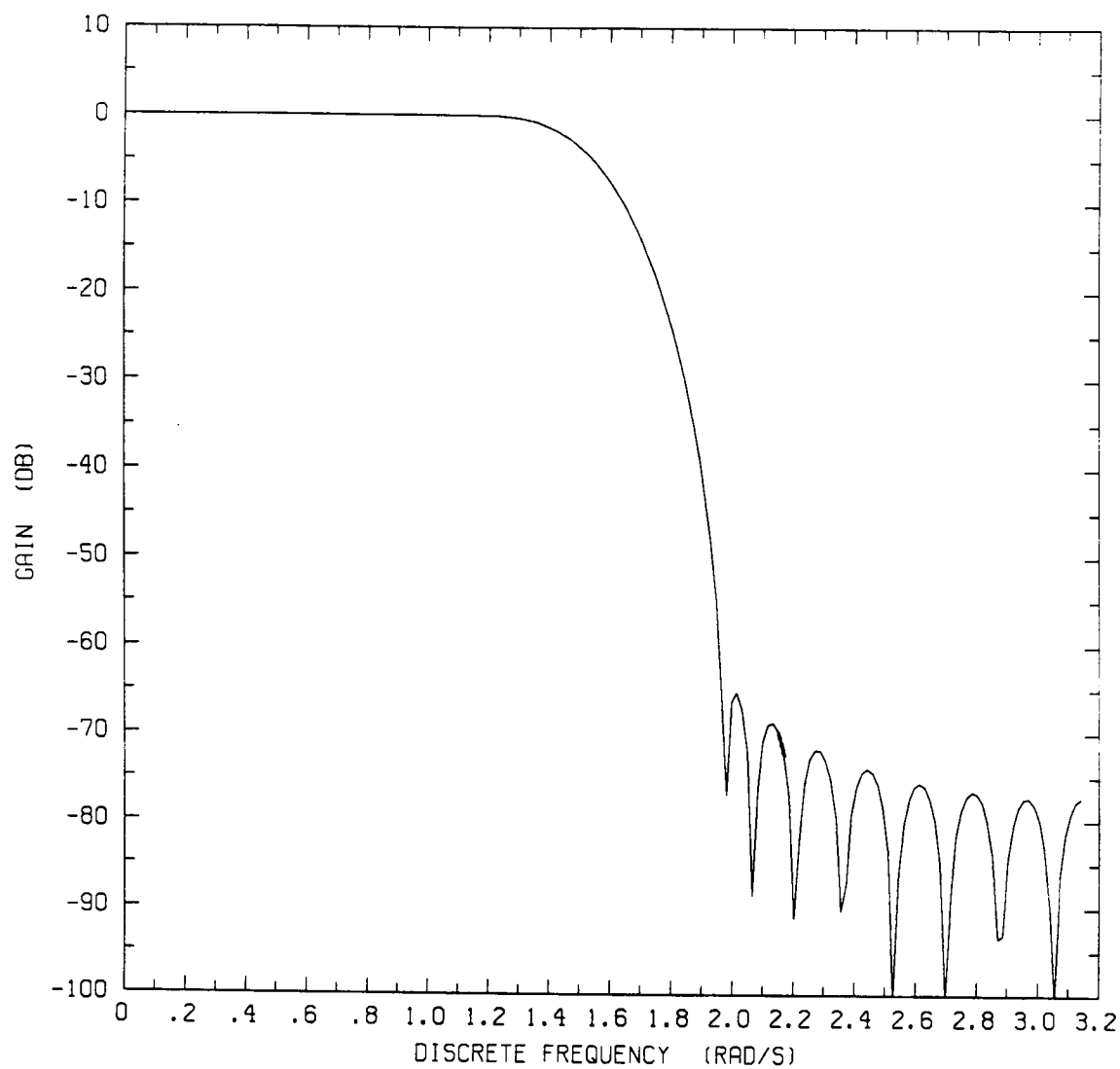


Figure 6.11: Frequency response of Kaiser-Bessel $\alpha = 2$ window-based, 33-term FIR filter.

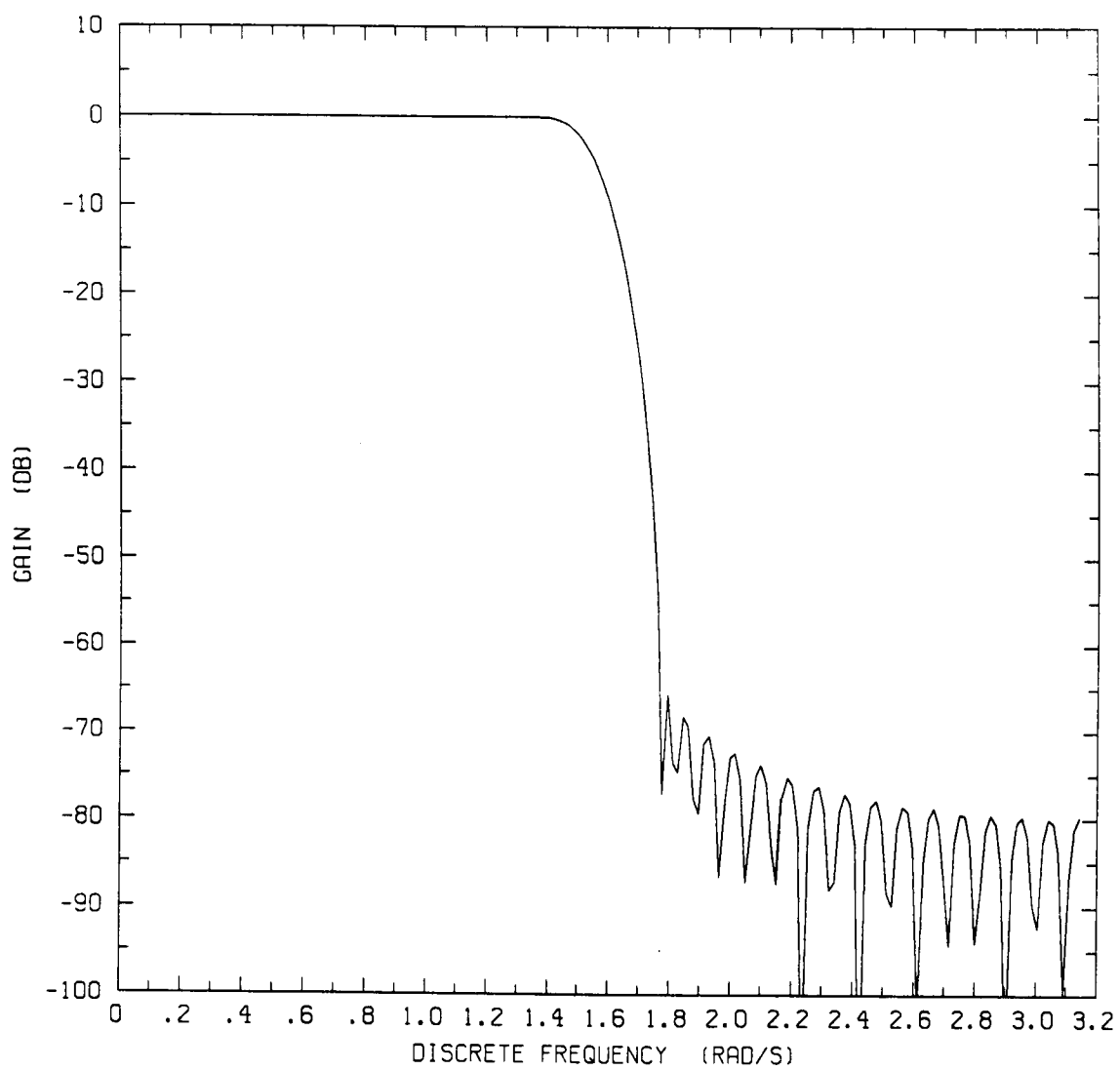


Figure 6.12: Frequency response of Kaiser-Bessel $\alpha = 2$ window-based, 65-term FIR filter.

a practical standpoint, however, since the system bandwidth is well less than $\pi/2$ and the expected signals are broadband, most of the amplified signal should lie below $\pi/2$, and it should be possible to design a filter to allow accurate reconstruction of the signal components up to and probably somewhat above the system bandwidth.

The measured system bandwidth was approximately 70 MHz, which corresponds to 70% of $\pi/2$ when signal reconstruction is begun by zero padding, so it seems reasonable that the upper frequency limit of the filter passband should be somewhat greater, 80% of $\pi/2$, for example. The lower frequency limit of the filter stopband should be an equivalent amount greater than $\pi/2$, which would be 120% of $\pi/2$ for the same example. These somewhat arbitrary limits were used as a basis to compare filter performance, and the results are given in Table 6.2. The passband ripple is the difference between the maximum and minimum filter gain for frequencies between 0 and $0.8\pi/2$, and it was decided that it should be less than 0.5 dB in order to not degrade the system frequency response. The minimum stopband attenuation is the minimum attenuation for frequencies above $1.2\pi/2$ up to π , and it was estimated that values less than 20 dB and probably even 40 dB would not provide sufficient attenuation of reconstruction artifacts.

It is obvious from the data that the rectangular window-based filters have excessive passband ripple and probably have insufficient stopband attenuation. The shorter Kaiser-Bessel window-based filters (9 and 17 term) also have ex-

Table 6.2: Filter characteristics. "K-B" indicates a Kaiser-Bessel window.

Window Type	Number of Terms	Passband Ripple (dB)	Minimum Stopband Attenuation (dB)
rectangular	9	2.1	17.5
rectangular	17	1.2	20.7
rectangular	33	0.8	26
rectangular	65	0.5	30
K-B, $\alpha = 2$	9	3.2	10.2
K-B, $\alpha = 2$	17	1.5	16.0
K-B, $\alpha = 2$	33	0.1	36.5
K-B, $\alpha = 2$	65	0.00	68
K-B, $\alpha = 3$	9	3.3	9.3
K-B, $\alpha = 3$	17	2.0	13.9
K-B, $\alpha = 3$	33	0.4	26.4
K-B, $\alpha = 3$	65	0.00	95

cessive passband ripple and insufficient stopband attenuation. (The passband response for these filters does not actually ripple, but decreases monotonically from unity.) For any of the longer Kaiser-Bessel window-based filters, the passband ripple is negligible, and the stopband attenuation is greater than that possible with a rectangular window. It was decided that the question of filter suitability should be determined by processing a test signal and examining the accuracy of the reconstructed signal.

6.5 Use of FIR Filters in Interpolation

To fully understand and appreciate the simplicity of this interpolation process, it is helpful to visualize the process of digital filtering with an FIR filter. The filtering process is described mathematically by

$$y(n) = \sum_{k=-m}^m h(k)x(n-k), \quad (6.18)$$

where $h(n)$ is a filter sequence, $x(n)$ is a signal and $y(n)$ is the filtered output sequence. Causality is not an issue here since the data will be filtered after capture of the entire sequence. Since $h(n) = h(-n)$ for the filters developed by windowing the ideal lowpass filter, $y(n)$ can be rewritten as

$$y(n) = \sum_{k=-m}^m h(-k)x(n-k). \quad (6.19)$$

This summation can be diagrammed as shown in Figure 6.13. The filter can be thought of as an operator ‘sliding’ on top of the signal to produce an output.

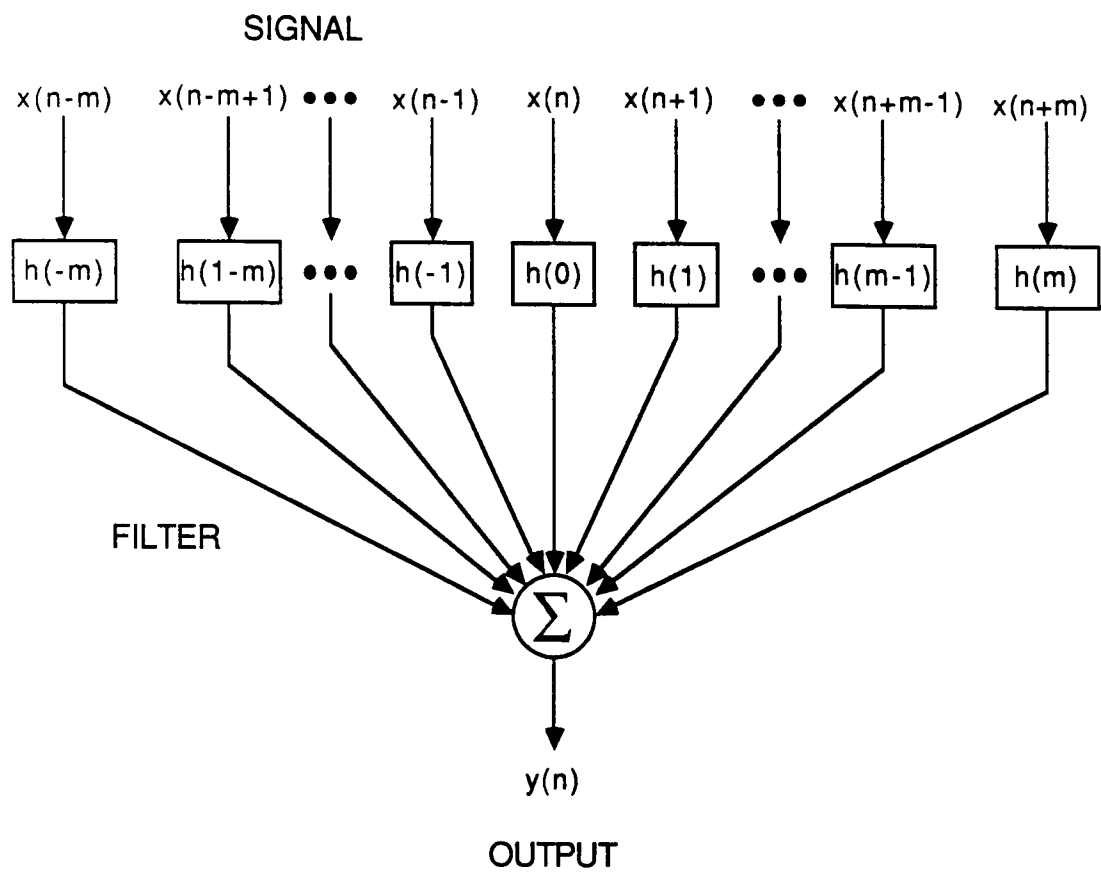


Figure 6.13: Block diagram of symmetric FIR filter.

Note that $x(n)$, $h(0)$ and $y(n)$ are aligned. The figure shows that the filtering process requires a minimum of m samples preceding and m samples following $x(n)$ or, in other words, at least enough samples to 'fill' the filter. Another way of looking at the process is as the taking of a weighted average with the filter coefficients being equal to the weights times the number of points being averaged.

When the data to be filtered is padded with zeroes for interpolation, the filtering process can be divided into two cases. The first case occurs when the index n corresponds to a point that is an original sample, and the second case occurs when n corresponds to an added zero or, in other words, to a point that is to be reconstructed. Points included in the first case do not actually need to be re-computed, except possibly as a check on the interpolation procedure, but it is instructive to examine that case.

Figure 6.14 is much like Figure 6.13, except that only the center of the filter is shown and the data is represented as either a zero or as an "x" (non-zero data point). All the non-zero data points are multiplied by even-indexed filter coefficients, which are zero, except for $h(0)$, and all the other data points are zero, thus

$$y(n) = h(0)x(n). \quad (6.20)$$

This relation determines the scale factor needed for the filter so the gain $y(n)/x(n)$, (for $x(n) \neq 0$) of the interpolation process is unity. This scaling can be accomplished by dividing all $h(n)$ by $h(0)$. Scaling is necessary so the inter-

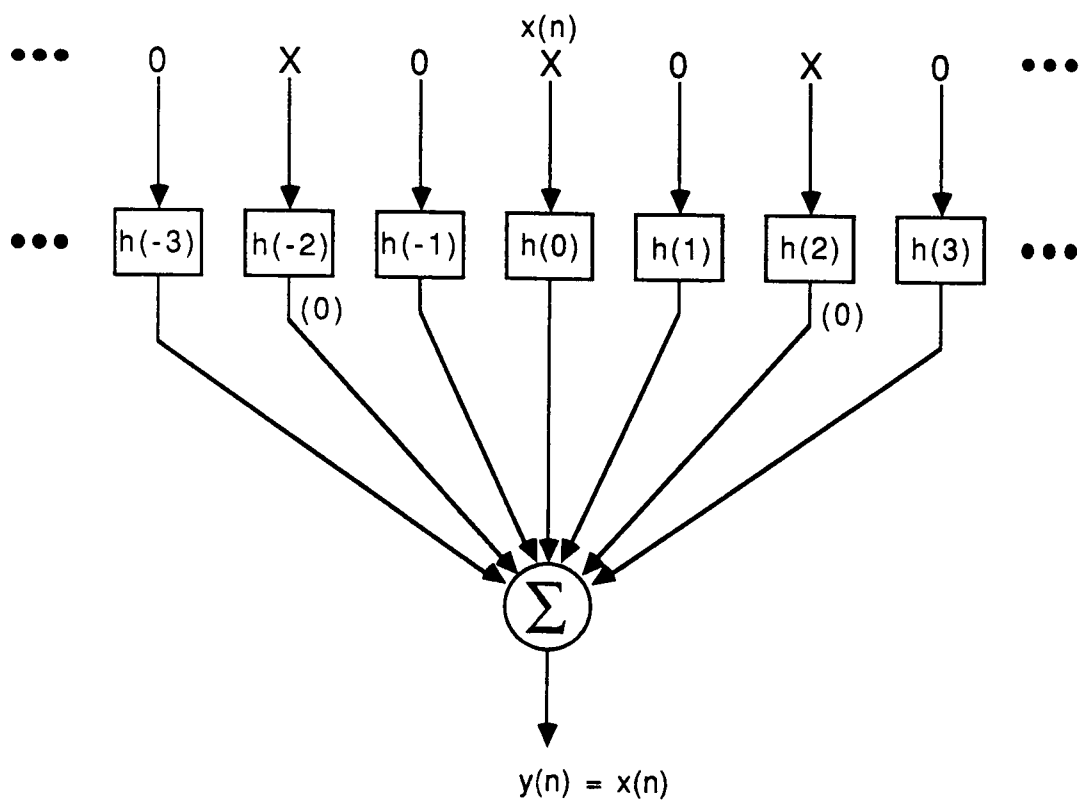


Figure 6.14: FIR filtering of zero-padded data to reconstruct an original point.

polated points and the original points will mesh properly without re-computing the original data points (to a new scale).

The second case of filtering for interpolation is shown in Figure 6.15. In this case, all the non-zero filter coefficients line up with non-zero data (except for $h(0)$), so approximately m data points contribute to $y(n)$. The greatest weight is given to $x(n - 1)$ and $x(n + 1)$, so the process is not terribly different from linear interpolation in which half the total weight would go to $x(n - 1)$ and the other half would go to $x(n + 1)$. Computation of an interpolated $x(n)$ value requires just m multiplications and additions when m is even and $m + 1$ when it odd.

6.6 A Practical Signal Reconstruction Procedure

A three stage reconstruction procedure was developed and tested. The first two stages are virtually identical, and each uses the zero-padding, FIR-filtering interpolation process to double the sampling rate (and the number of samples). The third stage produces an approximation of a continuous signal by linearly interpolating the data. Since the maximum frequency signal that can be reconstructed has more than two points per cycle and that number is quadrupled (to more than eight) by the first two stages, reasonable accuracy can be obtained with linear interpolation, and additional stages of the more complex interpolation process are not really necessary.

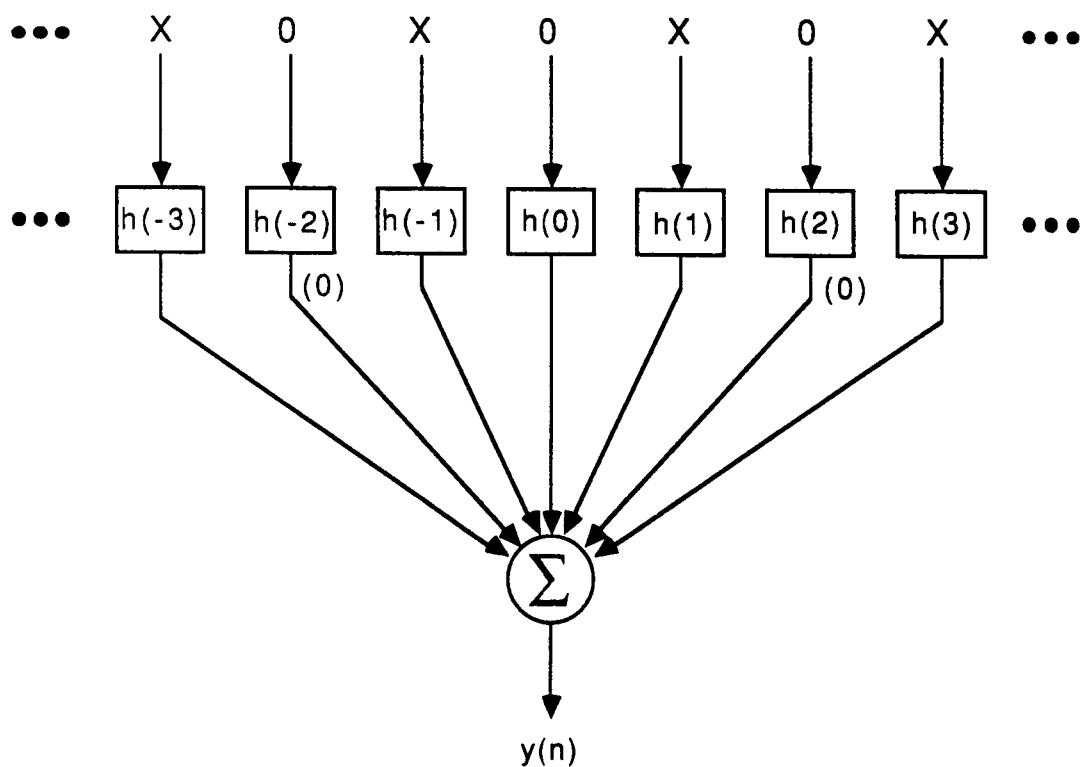


Figure 6.15: FIR filtering of zero-padded data to interpolate a point not originally sampled.

The signal reconstruction procedure begins with software plotting the original waveform in its entirety. The user chooses the portion that is to be reconstructed, and it is padded with zeroes. The entire waveform (2048 samples) is not automatically interpolated since the time required for the computations and plotting is quite long (more than 10 minutes) on the small computer used for data acquisition and processing. Actually the software uses a slightly longer portion of the waveform than that chosen since the filtering process shortens it by approximately the length of the filter. (About m points before and after a point $x(n)$ are required to fill the filter so it will perform correctly.) The zero-padded sequence is passed through the lowpass filter, and the output sequence is zero-padded again and sent through the filter once more. This output sequence is then plotted with the points connected with straight lines.

The procedure was tested by reconstructing a test signal and comparing the result to that obtained with a digital storage oscilloscope with an analog bandwidth comparable to that of the digitizer plug-in amplifiers (about 100 MHz). For repetitive signals, the digital oscilloscope has a sampling rate high enough so linear interpolation provides an accurate view of the signal. The test signal used was the current pulse generator output amplified by the preamplifier and both buffer amplifiers, and was shown as Figure 5.34 in Chapter 5. The same type of pulse was amplified and digitized, and is shown in Figure 6.16 with the sampled points connected with straight lines (linear interpolation). Obviously some details that are present in Figure 5.34 are lacking in Figure 6.16.

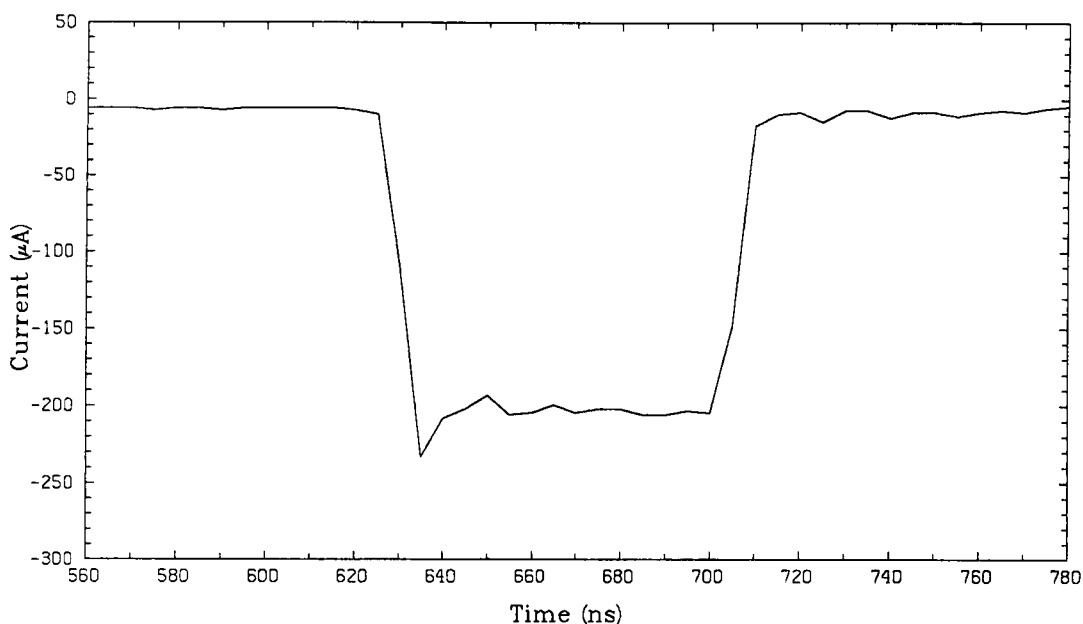


Figure 6.16: Test signal reconstructed with linear interpolation only.

The reconstruction procedure was carried out using the 33-term Kaiser-Bessel window-based filter given in Table 6.1, and the resulting signal resembled Figure 5.34 considerably better than Figure 6.16 does. Marginally better results were obtained with the 65-term Kaiser-Bessel $\alpha = 2$ window-based filter, and that signal is shown in Figure 6.17. The overshoot and ringing visible in the reconstructed signal resembles that visible in Figure 5.34 as well as could be expected considering that the digitizer and digital oscilloscope used do not have exactly the same frequency response. The signal reconstruction used here has allowed a single-shot signal to be displayed with details visible that would normally require a much higher digitizing rate.

To illustrate a case where the lowpass filter has insufficient stopband atten-

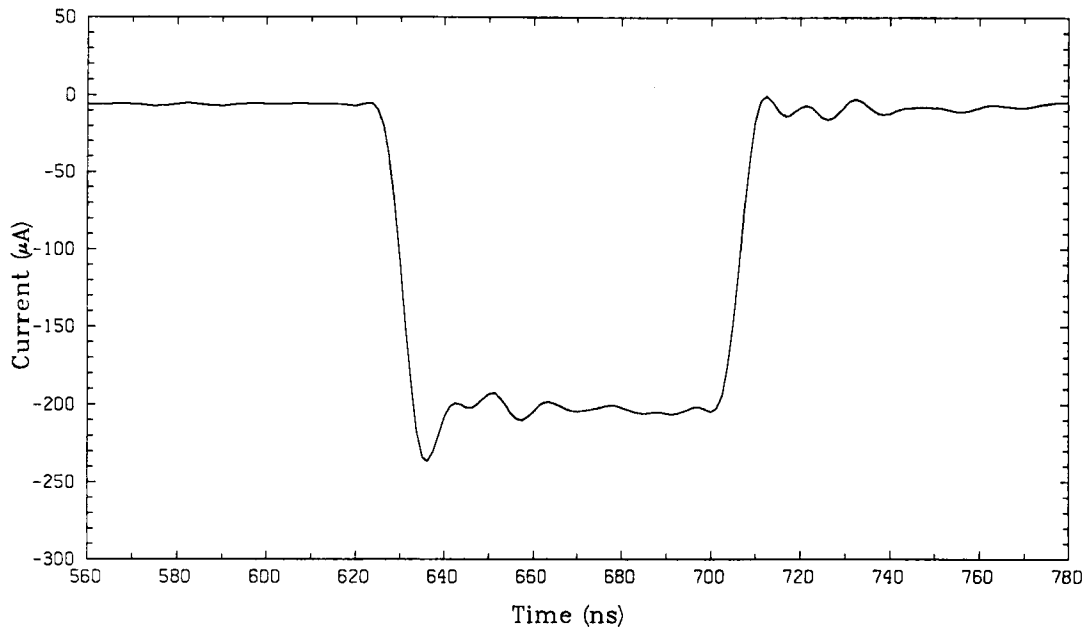


Figure 6.17: Test signal reconstructed with complete interpolation procedure.

uation, the signal was recreated using the 17-term rectangular window-based filter. The reconstructed signal is shown in Figure 6.18, and it is obvious that the filter did not completely remove high frequency artifacts created by the addition of the zeroes to the data. Figure 6.1 showed that low frequency parts of the signal spectrum (dc, for example) are duplicated near π when zeroes are added between the data points. The pulse is somewhat like a shift of the dc level, so artifacts are created near π , and they are attenuated by the lowpass filter, but not strongly enough to be negligible.

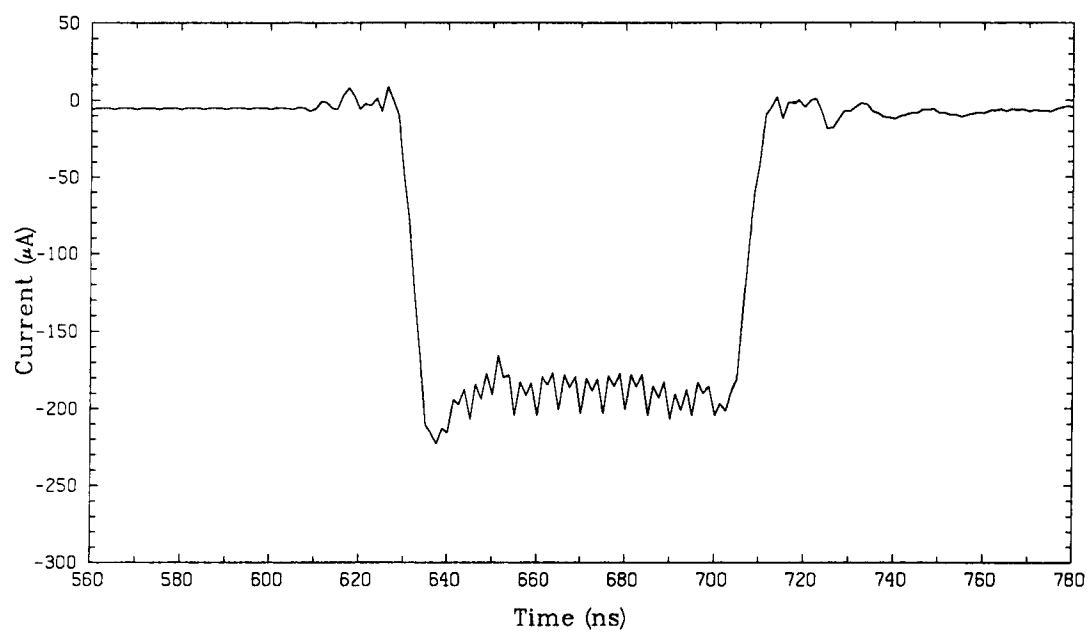


Figure 6.18: Test signal reconstructed using filter with insufficient attenuation.

CHAPTER 7

PREBREAKDOWN CURRENT MEASUREMENTS

The measurement system was used to make a number of measurements of prebreakdown currents in hexane in order to verify and to demonstrate its capabilities. Hexane was chosen for the tests since it had been studied extensively, and the data described in Chapters 1 and 2 was available for comparison. This chapter describes the results of those measurements and several tests performed to characterize the complete measurement system.

7.1 Measurement System Testing

The measurement system configuration described in Chapter 2 was assembled, and the test cell (Chapter 3) was filled with research grade hexane. The preamplifier and buffer amplifiers described in Chapter 5 were connected, but the ORTEC 535 amplifier was not used since it was found the additional amplification was not needed even for the smallest visible current pulses. The calculations of Section 5.5 indicated that a gain of 25 was needed from the buffer amplifiers, but that estimate was based on the theoretical rms noise current of the preamplifier and not the actual rms noise current of the system (which was almost twice as large). Therefore the required gain was only half as much, and that of

the high-gain buffer was sufficient. The measurement system that was actually used is shown in Figure 7.1, and it is basically the same system that was given in Figure 2.4.

To protect the preamplifier from damage in the event of a complete breakdown of the test cell, a protection circuit was inserted between the test cell and the preamplifier input. This circuit (Figure 7.2) is basically the same as that described by Blalock [25], and has only a slight effect on the performance of the measurement system. At frequencies below 100 MHz, the small capacitors involved (C_S , the preamp input capacitance and the protection circuit capacitances) have impedances at least several times that of $50\ \Omega$, so a reasonable approximation for the effect of the protection circuit on the preamplifier behavior would be obtained by simply neglecting the $50\text{-}\Omega$ resistors and adding the protection circuit capacitance to the source (test cell) capacitance. The gas discharge tube capacitance is 0.7 pF , and the capacitance of each 1N4148 is approximately 1.2 pF , so the total capacitance added was approximately 1.9 pF .

The rms input noise current for the system was measured by capturing several waveforms with the high-voltage supply turned off, computing the rms value of the (noise) signal and referring that value back to the preamplifier input. The measured value was 60 nA rms , which is slightly greater than the 47 nA rms noise current measured using spectral analysis. The test cell capacitance (2.8 pF without preamplifier plus 1.9 pF due to the protection circuit plus 1 pF assumed due to the connection for a total of 5.7 pF) was somewhat greater than that of

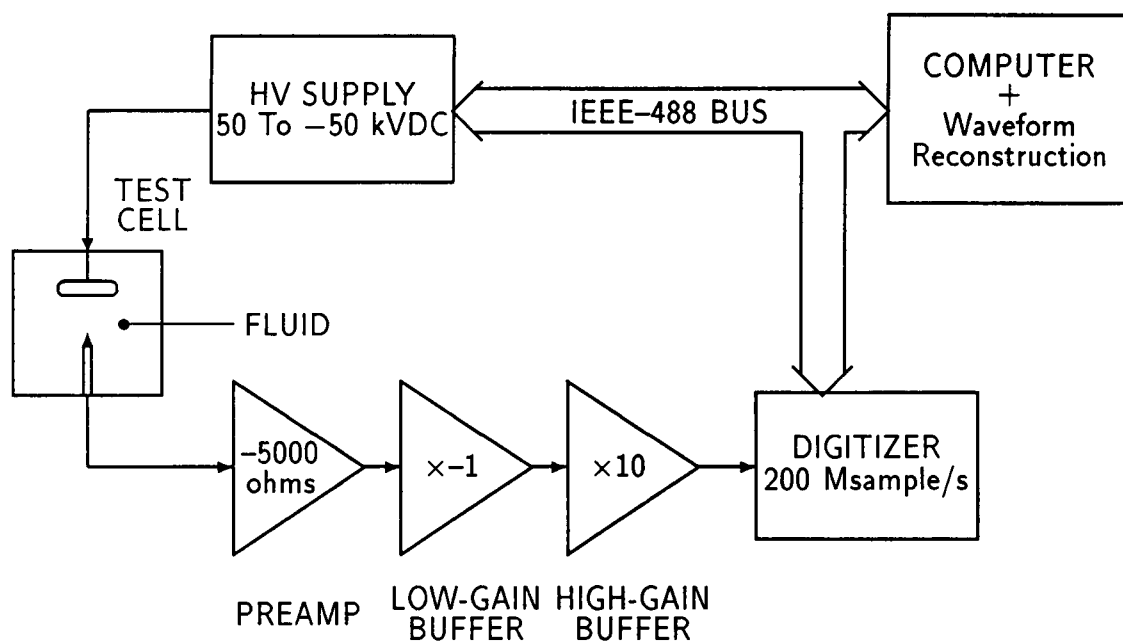


Figure 7.1: Prebreakdown current measurement system.

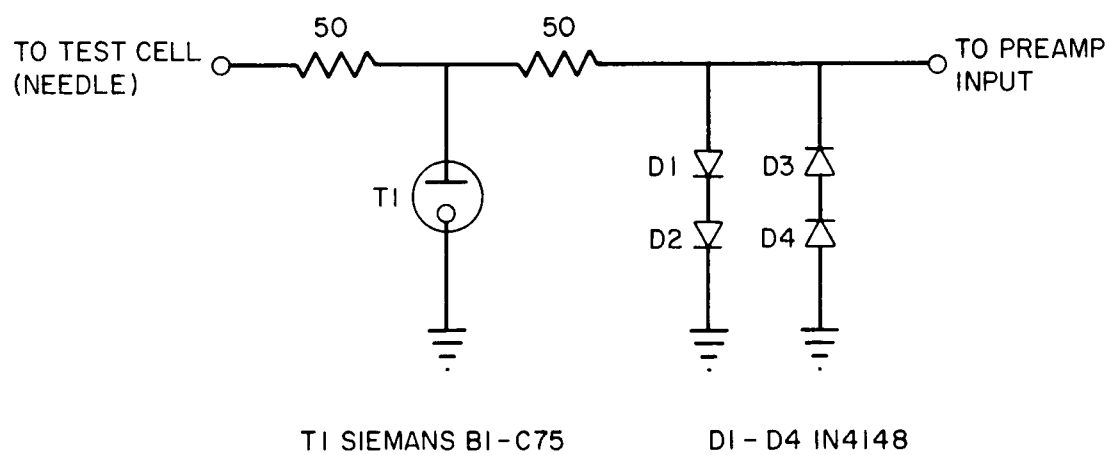


Figure 7.2: Preamplifier protection circuit.

the source capacitance (4 pF) used to simulate the test cell for spectral analysis, so a larger i_{ni} would be expected. It is also possible that connection of the amplifier ground to the test cell and high-voltage supply ground could have increased the level of the amplifier parasitic oscillations and therefore produced some added noise.

Since the prebreakdown current pulses were expected to be impulse-like, the impulse response of the system was measured. A (repetitive) quasi-impulse was introduced into the system (including the test cell but excluding the digitizer and signal reconstruction procedure), and the response was recorded with a 100-MHz sampling oscilloscope. The quasi-impulse was produced by capacitively coupling the output of a fast-risetime rectangular voltage pulse generator to the needle electrode. The current flowing through this capacitance (and into the preamplifier) was the derivative of the voltage signal – a positive and a negative impulse. The coupling capacitor was actually the capacitance between a short (5 cm) length of wire approximately 5 cm distant from the needle, and the RC time constant due to this very small capacitance and the 50- Ω output impedance of the generator was small enough so the current signal was essentially an impulse for a system with a 65 MHz bandwidth.

The measured system impulse response is shown in Figure 7.3. The full width at half (the) maximum pulse height (FWHM) is 6.3 ns, and the pulses are followed by ringing with a maximum amplitude of about 12% of the pulse amplitude. Some of the observed ringing was probably due to the impulse source

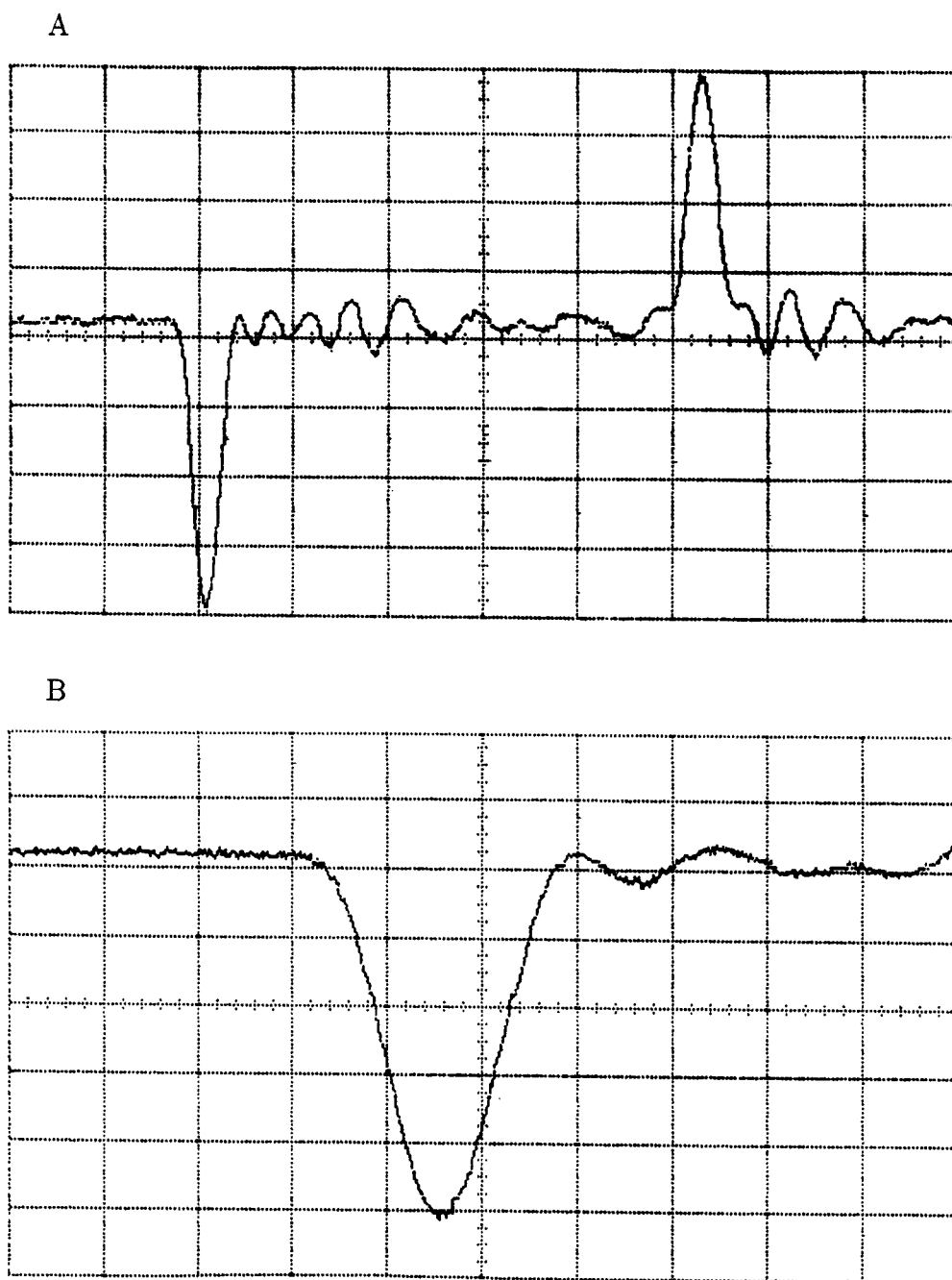


Figure 7.3: Partial system impulse response. (A) Positive and negative impulse responses. (25 mV/div, 20 ns/div.) (B) Expanded view of the response to a negative impulse. (20 mV/div, 5 ns/div.)

as the ringing could be changed somewhat by changing location of the connection of the source ground with the test cell ground.

The same signal source was used to test the complete measurement system (cell, amplifiers, digitizer and signal reconstruction procedure). The response of the system is shown in Figures 7.4 and 7.5. The FWHM of the negative impulse response is 6.8 ns which is approximately the same that was found using the digital oscilloscope. The reconstructed pulses (Figure 7.5) exhibit ringing following the pulses much like that observed with the sampling oscilloscope (Figure 7.3), but also exhibit 'ringing' preceding the pulse which is not present in Figure 7.3 or the unreconstructed waveform (Figure 7.4). This is apparently an artifact of the reconstruction process and is probably due to high frequency signal components being aliased to lower frequencies due to the imperfect anti-aliasing filter. The distortion arises not only from the creation of low frequency components, but also from the lack of the high frequency components.

7.2 Prebreakdown Current Measurements

The system was used to measure prebreakdown currents in hexane for several different voltages of both polarities. For all of the results described here the fluid was maintained at atmospheric pressure and room temperature. The purpose of these measurements was to demonstrate the capabilities of the measurement system. The emphasis of the experiments was therefore placed on capturing

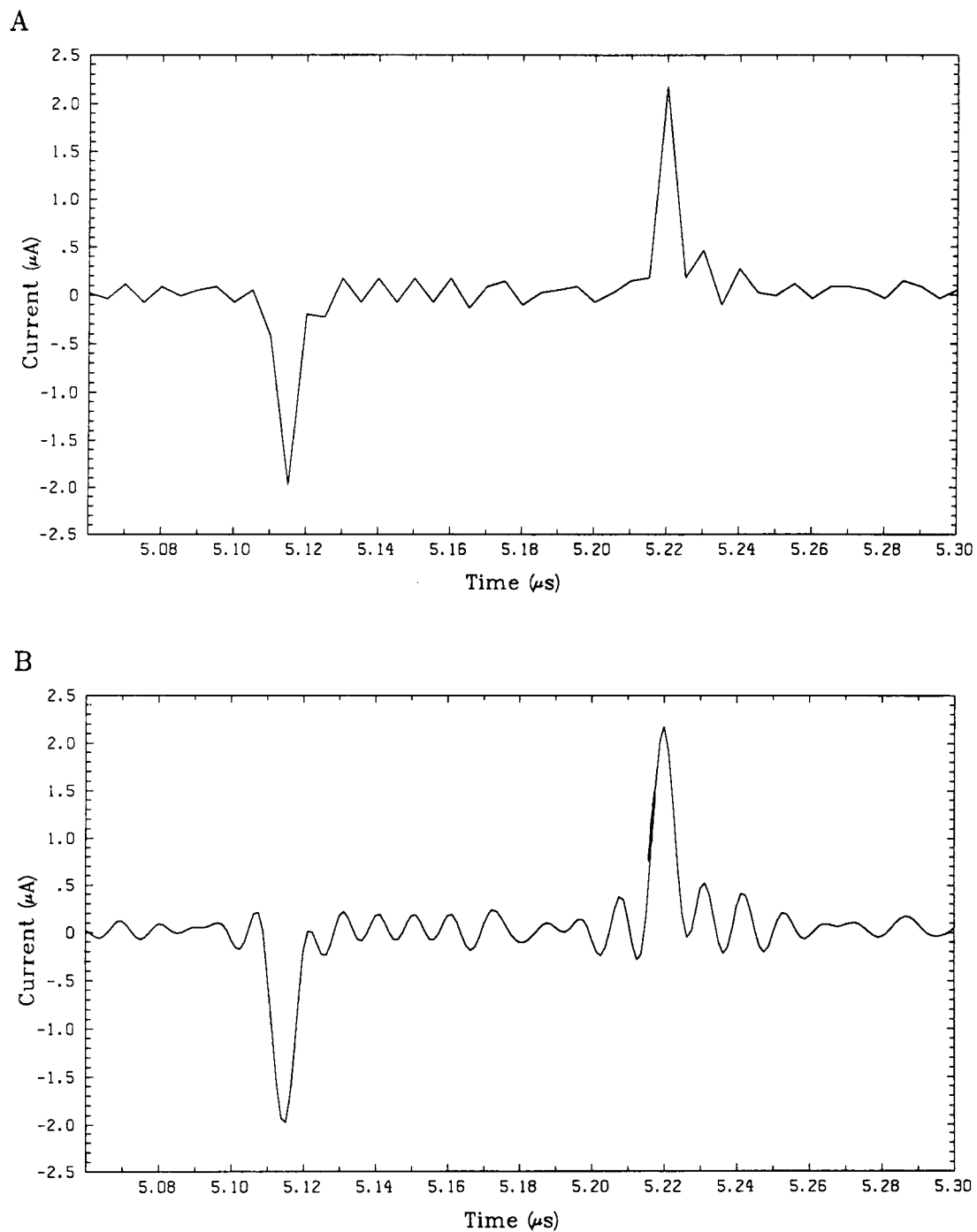


Figure 7.4: System impulse responses. (A) Response without signal reconstruction. (B) Response with signal reconstruction.

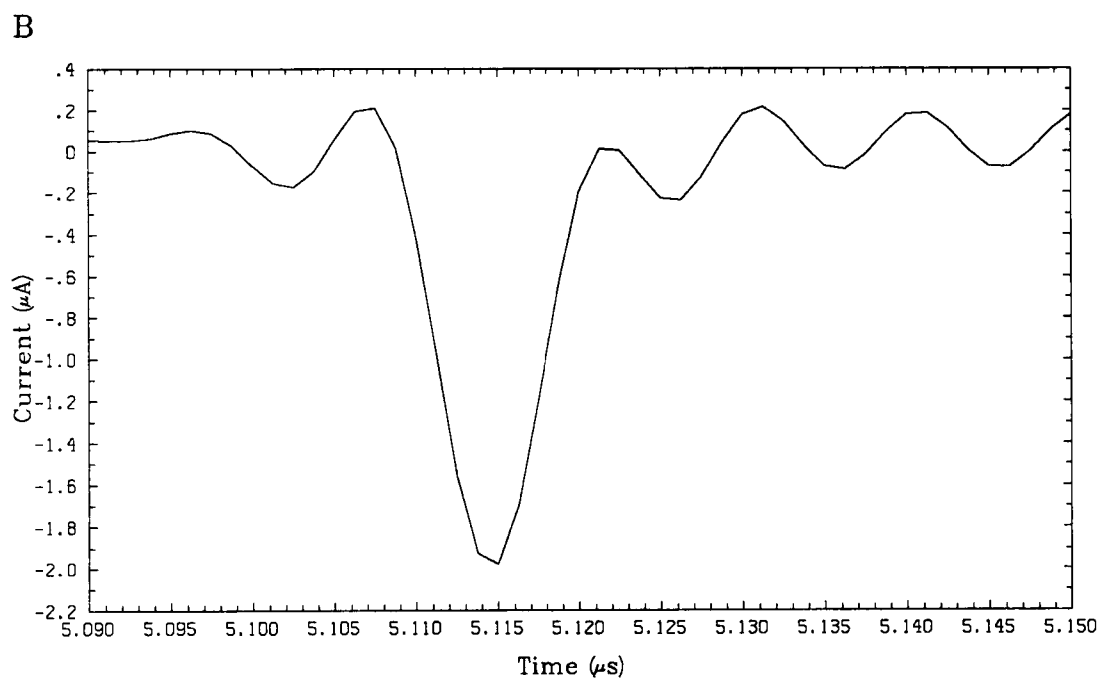
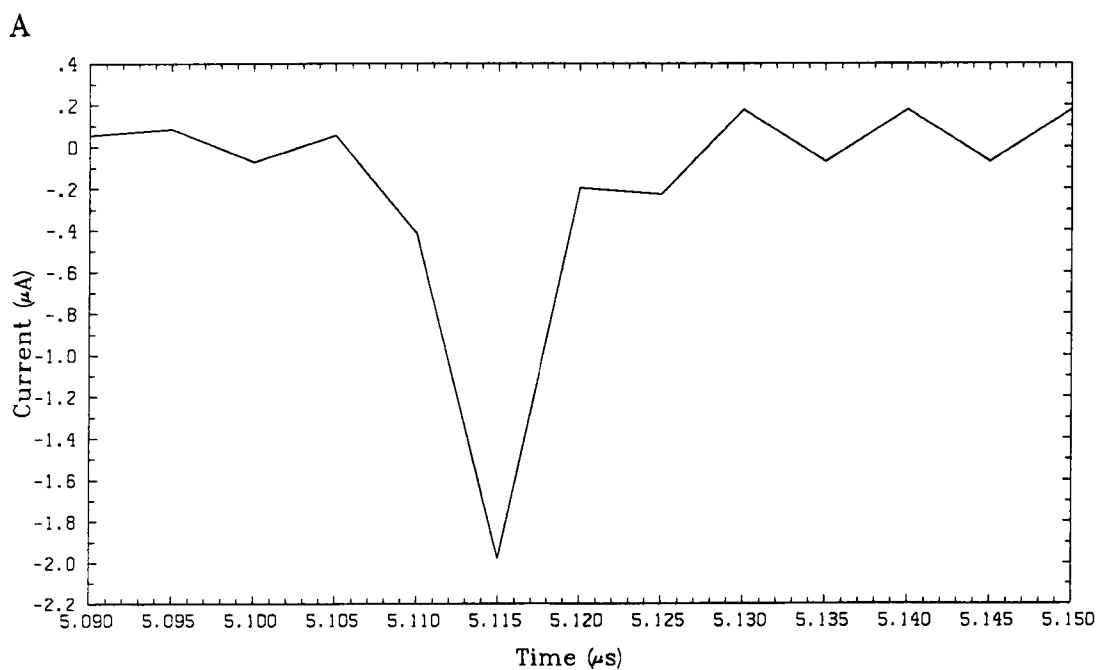


Figure 7.5: System negative impulse response. (A) Response without signal reconstruction. (B) Response with signal reconstruction.

extremely small pulses in order to illustrate the low noise level of the system and on capturing characteristic bursts of current pulses like those shown in Figures 1.3 and 2.1 in order to demonstrate the improved pulse separation possible due to the greater system bandwidth.

Figure 7.6 shows a single pulse captured by the measurement system. The pulse has an amplitude of 820 nA and a FWHM of 7.9 ns. The shape of the pulse appears to be approximately the same as the system impulse response (Figure 7.5(B)), but the ringing is not evident because of the noise level. The peak-to-peak noise level is about 200 nA, so pulses with amplitudes comparable to this (820 nA) are easily visible, and a pulse of 235 nA (the minimum detectable pulse as defined and calculated earlier) would be more than twice as large as a typical noise peak. However, since the noise is random, occasional peaks could be considerably larger than average. For greater certainty in identifying current pulses, it might be more prudent to define the minimum detectable pulse as being 10 times the rms noise level rather than 5 times.

Figure 7.7 shows a current pulse produced with negative high voltage stress (positive needle). This pulse has an amplitude of about 2 μ A and a FWHM of 7.2 ns. The shape of the pulse also appears to be like the system impulse response, but it is interesting to note that it is just slightly wider than the impulse response. Since the pulse is considerably larger than that shown in Figure 7.6, the ringing (which is a fraction of the peak amplitude) is greater than the noise.

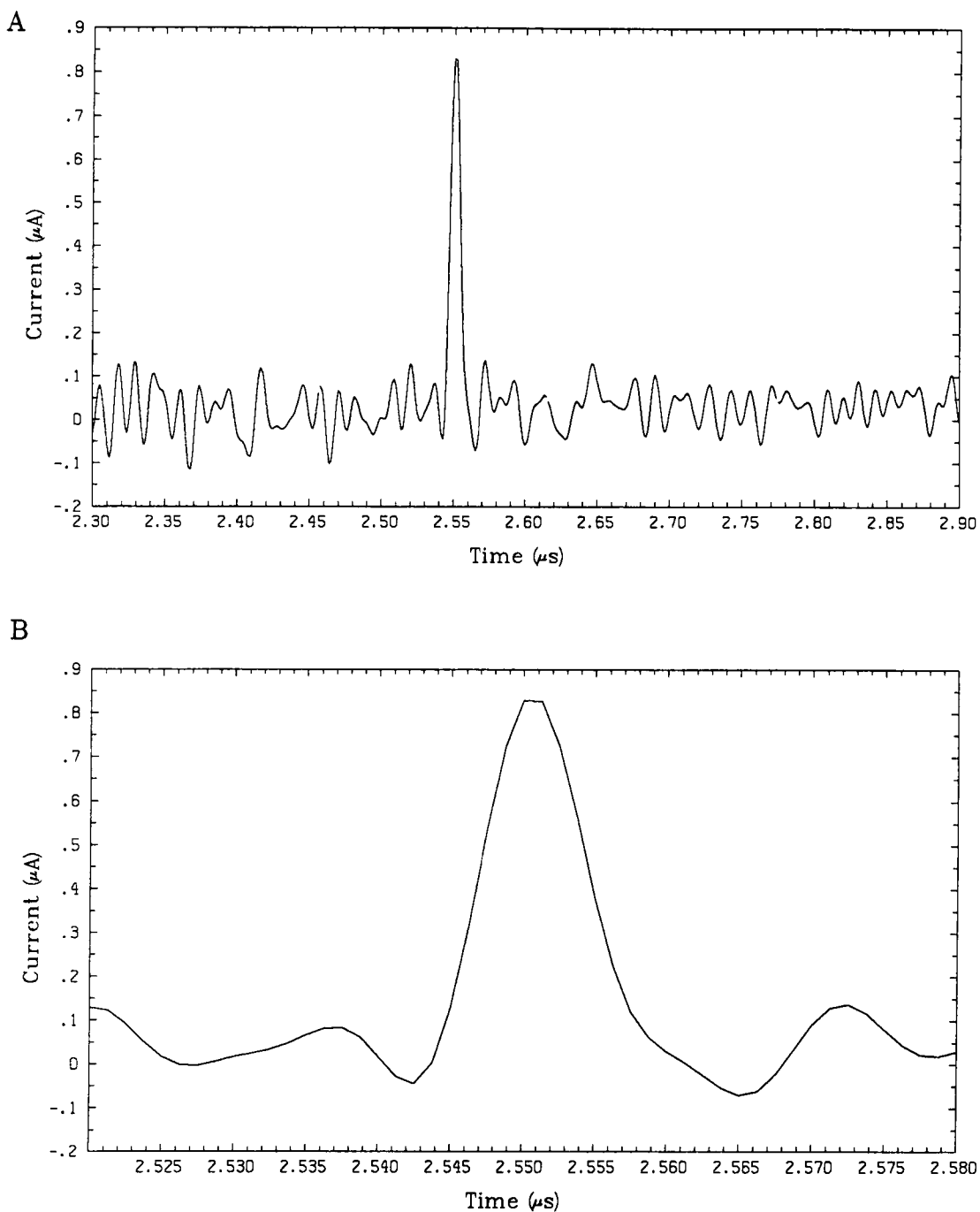


Figure 7.6: Prebreakdown current pulse in hexane for the negative needle case. 8.2 kV was applied to the high-voltage electrode. (A) This view shows the pulse plus the background noise. (B) This view shows the pulse with an expanded time scale.

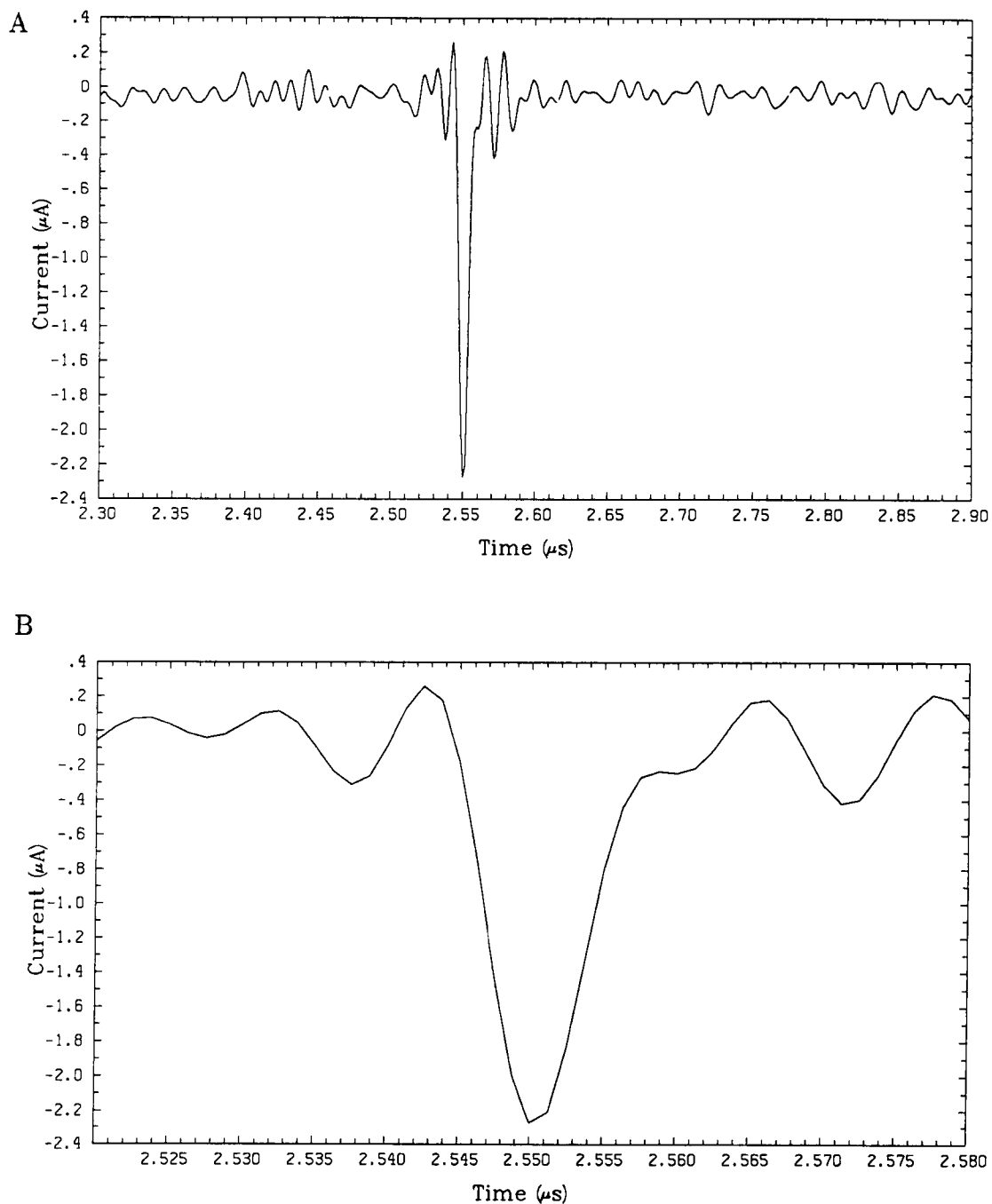


Figure 7.7: Prebreakdown current pulse in hexane for the positive needle case. -10.0 kV was applied to the high-voltage electrode. (A) This view shows the pulse plus the background noise. (B) This view shows the pulse with an expanded time scale.

A burst of current pulses is shown in Figure 7.8(A). The burst exhibits the usual increase in pulse amplitude with time. The pulses range in amplitude from a few μA to more than $20 \mu\text{A}$, and both pre- and post-pulse ringing is evident for the larger pulses. All the pulses appear to have the same general shape – the same as the system impulse response. The first two pulses overlap slightly and have peaks only 15 ns apart. Had these pulses been detected with the system described in Chapter 2, the overlap would probably be so great that they would not be distinguishable as two pulses.

Another burst of current pulses is shown in Figure 7.8(B). This burst also exhibits the characteristics of increasing pulse amplitude and spacing with time. Here too, the pulse shapes appear to be the system impulse response, except for the first pulse which is somewhat wider. Closer inspection leaves one wondering if it is not two overlapping pulse with peaks separated by less than 10 ns.

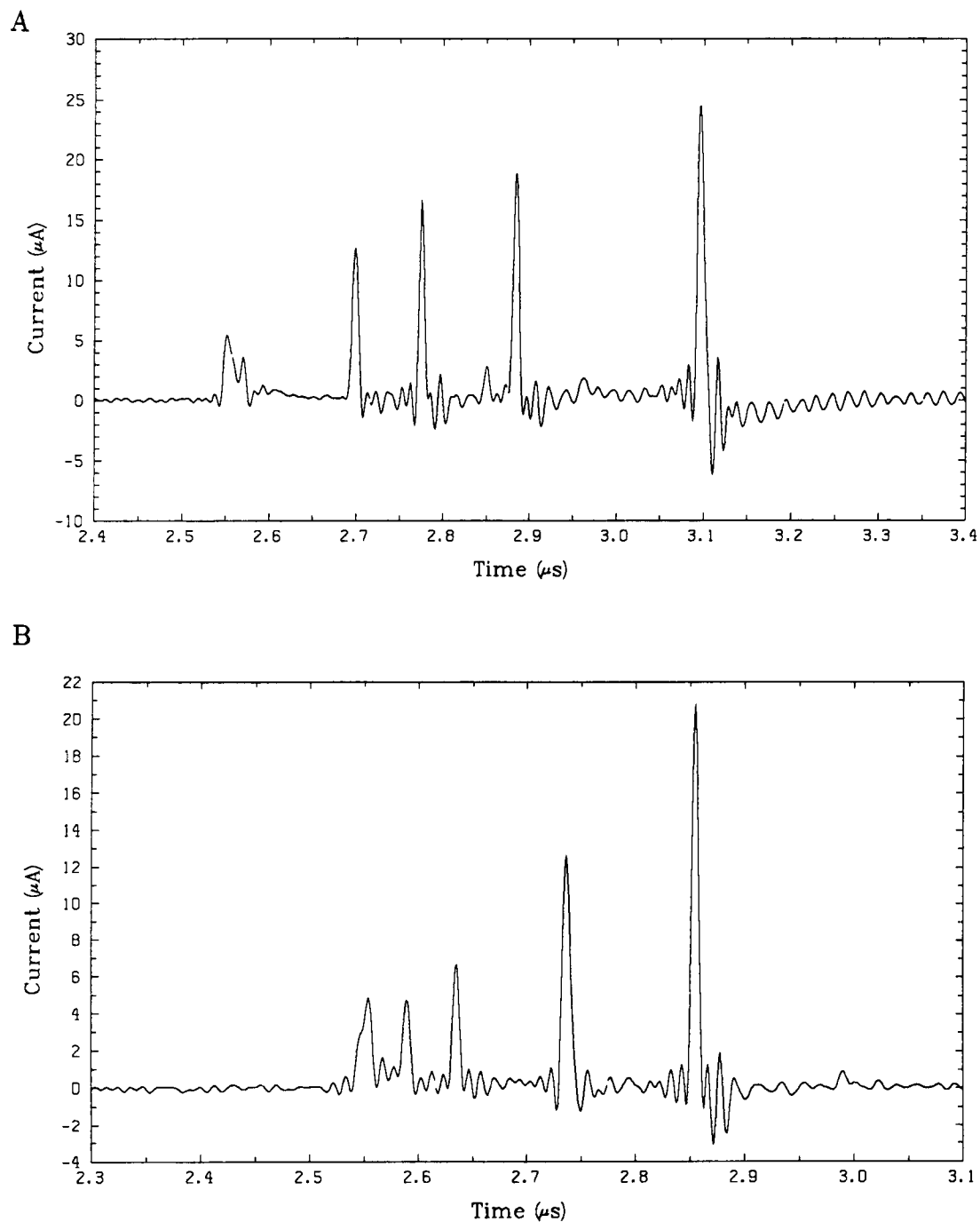


Figure 7.8: Prebreakdown current pulse bursts in hexane. 10.0 kV was applied to the high-voltage electrode. (A) Burst with $0.55\text{-}\mu\text{s}$ duration. (B) Burst with $0.3\text{-}\mu\text{s}$ duration.

CHAPTER 8

CONCLUSIONS AND FUTURE WORK

This dissertation has described the development of a system optimized to measure prebreakdown currents in liquid dielectrics. Tools needed for that development were also described. The measurement system was tested extensively with both artificially generated signals and actual prebreakdown currents and found to provide both lower noise and wider bandwidth than previously used systems.

A number of tools were developed to aid in the development of the measurement system. CAD tools useful for the design and analysis of low-noise, wideband transimpedance and voltage-sensitive amplifiers were developed. The transimpedance amplifier tool was used extensively in the preamplifier design process to estimate the stability, bandwidth and noise performance, and these results were confirmed by algebraic and SPICE analyses. Instruments were developed and used to measure C_{IN} , g_m and $S_e(\omega)$ for low-noise FETs used as preamplifier input devices. The measurement of these parameters for a number of different transistors was necessary so a device minimizing the preamplifier noise could be selected. Finally, a current pulse generator was developed in order to provide a source mimicking the characteristics of the prebreakdown

currents and the test cell – low risetime, high output resistance and low shunt capacitance.

The optimized system configuration is not very different from those previously used, but several of the elements were redesigned or were completely new designs. A low capacitance test cell with an electrodes arranged for improved prebreakdown current collection efficiency was developed and used. A transimpedance preamplifier and two buffer amplifiers were developed to provide very low noise amplification of the currents. Interpolation techniques were adapted for use in reconstruction of an effectively continuous signal from the digitized current waveforms.

The main development effort was devoted to the preamplifier. A study of the noise performance theoretically possible for different preamplifier configurations revealed that the transimpedance preamplifier was superior for this application and that GaAs MESFETs appeared to be the best input devices from a noise standpoint, primarily because of their low input capacitance to transconductance ratio. In fact, the quantity C_{IN}^2/g_m proved to be a figure of merit for FETs used as input devices of transimpedance amplifiers designed to have low rms input noise current. A transimpedance preamplifier based on these findings was designed and built.

A signal reconstruction procedure was developed and shown to provide advantages over reconstruction with simple linear interpolation. This procedure is based on zero-padding the discrete signal to increase the sampling rate and

then lowpass filtering to remove high frequency artifacts. This leaves a signal with the same frequency content as the original, but sampled at a higher rate. It was found that two applications of this procedure followed by linear interpolation gave very good results for low risetime rectangular signals and allowed the determination of the pulse width for very narrow pulses. One drawback to this method is that it introduces some ringing preceding impulse-like signals (like prebreakdown current pulses).

It is worth noting that the frequency response of an analog measurement system (amplifiers plus an analog or digital sampling oscilloscope) extends well above its -3 dB bandwidth, albeit at reduced gain, but that of a digital system (amplifiers, anti-aliasing filter, digitizer and signal reconstruction) has an absolute cutoff at one half of the sampling frequency. This means that the responses of the two systems to the same signal are different even if the frequency responses are the same up to the -3 dB point because the high-frequency components are completely missing for an ideal digital system, and attenuated and aliased to lower frequencies for a practical digital system. The degree of difference depends not only on how close the -3 dB bandwidth is to the Nyquist frequency, but also on the signal. An in-band sinewave can be captured, reconstructed and displayed using the digital system without noticeable differences from the analog system. The response due to a rectangular pulse may not be very different because the frequency content of the pulse is much greater at low frequencies, while the response to an impulse is somewhat different since its spectrum is

white.

The measurement system (including the digitizer) that was developed has a 65 MHz bandwidth, a gain of 50000 V/A and a 60 nA rms input noise current when the test cell contains hexane. This system allows the measurement of pre-breakdown currents with amplitudes ranging from 300 nA to more than 500 μ A. The observed prebreakdown current pulses appear to be essentially the impulse response of the system and have FWHM of approximately 7.5 ns. This narrow pulse width allows the time resolution of pulses not previously possible.

8.1 Recommendations for Future Work

After extensive use of the transimpedance amplifier CAD tool, it is clear that at least two modifications would allow more accurate modeling. The $1/f$ -type noise model could be improved by using a variable power instead of being limited to only $1/f$ and $1/f^2$ terms. This would allow more accurate modeling of the GaAs MESFET $1/f^{1.37}$ noise, for example. Modeling of the system frequency response, output noise power spectral densities and rms input noise current could be improved by adding features to simulate the frequency response of additional gain stages or filters. As noted in Section 5.5, the buffer amplifiers and digitizer frequency responses have a considerable effect on the amount of noise power that is contributed from frequencies above the system bandwidth. Another area worth investigating would be the modeling of the gain block. It was assumed

that the gain block had zero output impedance and infinite differential input impedance (for the voltage-sensitive amplifier), and more realistic values might yield different gain and noise characteristics at high frequencies.

Alternatives to the preamplifier overload protection circuit (diode feedback elements) should be studied. Limiting the number of diodes to a practical value also limited the size of the feedback resistor and therefore the preamplifier noise performance. Use of a limiting scheme not involving the feedback network, might allow the use of a larger feedback resistor and obtain better noise performance. This would also eliminate the possibility of the preamplifier being driven into oscillations by a long-duration overloading pulse. Elimination of the long string of diodes would allow better control of the feedback capacitance for response tuning purposes.

Several improvements to the buffer amplifiers seem worth investigating. The low-gain buffer could be improved so that its noise contribution to the system is negligible and not just small. The system bandwidth could be increased by improving the high-gain buffer to have a slightly underdamped response and a higher bandwidth. Increasing the output drive capabilities of both buffers would increase the dynamic range of the system, and perhaps more importantly, not decrease the bandwidth for large signals. The use of darlington-connected transistors in the output stages might accomplish that.

Since the use of this measurement system has established that the current pulses are impulse-like, it would be useful to rethink the system design in terms

of impulse response instead of the commonly used step response. The settling time should be adjusted to be short so that ringing after a large pulse would not obscure following smaller pulses. Since an impulse consists mainly of high-frequency components, it would be very useful to conduct further study of the test cell, interconnection and preamp input impedances, including the grounding network, and determine their effects on the system response. As the interpolation process proved useful in reconstructing the prebreakdown current pulse waveforms, it would be useful to develop better tests of the accuracy of the reconstruction for signals of that type.

The bandwidth of the measurement system was ultimately limited by the sampling rate of the digitizer, but faster digitizers are becoming available. The techniques developed here could be applied to the design of a considerably wider-band system (200 MHz or more) with the hope of capturing the actual shape of the prebreakdown current pulses. This knowledge might reveal the mechanisms of prebreakdown charge flow, especially its initiation. An alternate approach to learning more about prebreakdown currents would be to develop a charge-sensitive system (also very fast) that would be capable of measuring the charge of each pulse. This charge information is important [30,31,32] in describing the development of the low density region associated with prebreakdown currents.

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APPENDICES

APPENDIX A

DERIVATIONS OF TRANSIMPEDANCE AMPLIFIER LOOP TRANSMISSION, SIGNAL AND NOISE GAINS

A.1 Transimpedance Amplifier Ideal Closed-Loop Gain

The model that was developed in Chapter 4 for the transimpedance amplifier is shown in Figure A.1. The ideal closed-loop gain can be found by noting that since the inverting input is a virtual ground, the total signal current flows through the feedback impedance. The output voltage is therefore the voltage developed across the impedance, and the ideal closed-loop or transimpedance gain is

$$A_{CL}^{ideal}(s) = \frac{V_o}{I_s} = -R_F \parallel \frac{1}{C_F s} = \frac{-R_F}{1 + R_F C_F s}. \quad (1.1)$$

By letting $\omega_F = 1/(R_F C_F)$ the ideal closed-loop gain becomes

$$A_{CL}^{ideal}(s) = \frac{-R_F}{1 + s/\omega_F}. \quad (1.2)$$

A.2 Transimpedance Amplifier Loop Transmission

The loop transmission is found by breaking the feedback loop at the output of the gain block (a zero impedance point) and then calculating the gain around the

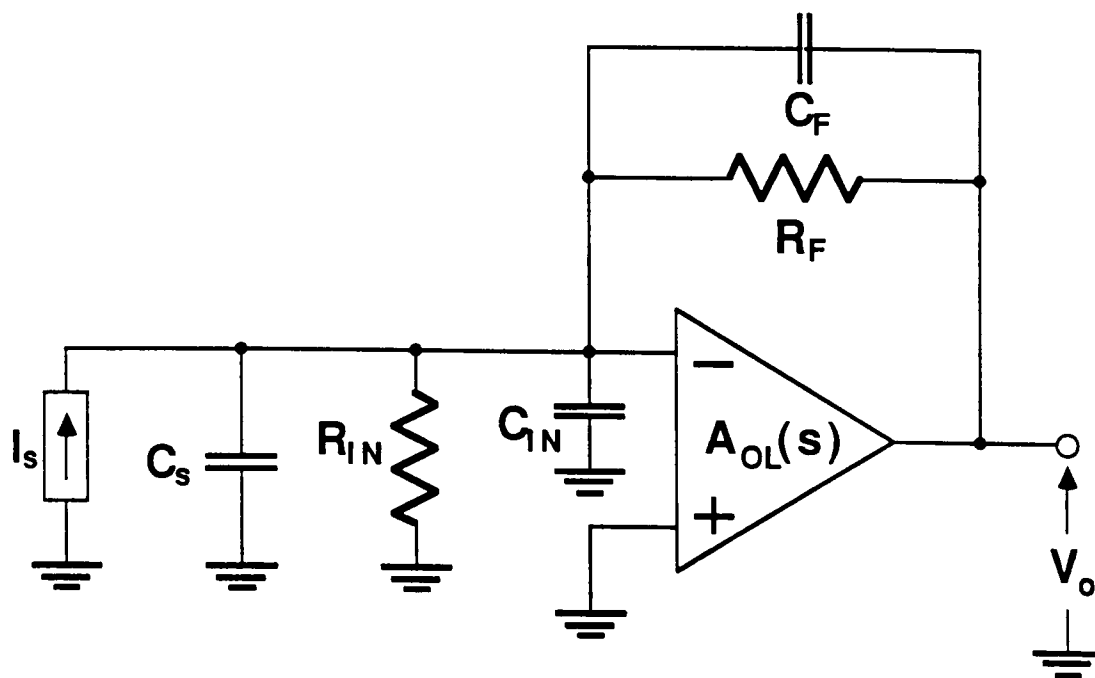


Figure A.1: Transimpedance amplifier circuit model.

loop. Figure A.2 shows the feedback loop broken and a test generator inserted. The loop transmission $T(s)$ equals the returned voltage (V_{RET}) divided by V_{TEST} . From the figure it is clear that the loop transmission is the product of $-A_{OL}(s)$ and an attenuation due to Z_F and Z_I . Thus

$$T(s) = -A_{OL}(s) \left(\frac{Z_I}{Z_I + Z_F} \right), \quad (1.3)$$

and since $Z_F = R_F \parallel 1/(sC_F)$ and $Z_I = R_{IN} \parallel 1/(sC_S) \parallel 1/(sC_{IN})$, the loop transmission is

$$T(s) = \frac{-A_{OL}(s) \left(\frac{R_{IN}}{1 + sR_{IN}(C_S + C_{IN})} \right)}{\left(\frac{R_{IN}}{1 + sR_{IN}(C_S + C_{IN})} \right) + \left(\frac{R_F}{1 + sR_F C_F} \right)} \quad (1.4)$$

in terms of the amplifier parameters. Rearranging this expression gives

$$T(s) = -A_{OL}(s) \left(\frac{R_{IN}}{R_{IN} + R_F} \right) \left(\frac{1 + sR_F C_F}{1 + s \left(\frac{R_{IN} R_F}{R_{IN} + R_F} \right) (C_F + C_S + C_{IN})} \right), \quad (1.5)$$

which can be simplified to

$$T(s) = -A_{OL}(s) \left(\frac{R_{IN}}{R_{IN} + R_F} \right) \left(\frac{1 + sR_F C_F}{1 + sR_{IF} C_{IF}} \right) \quad (1.6)$$

by letting

$$R_{IF} = R_{IN} \parallel R_F = \frac{R_{IN} R_F}{R_{IN} + R_F}, \quad (1.7)$$

and

$$C_{IF} = C_F + C_S + C_{IN}. \quad (1.8)$$

Letting $\omega_F = 1/(R_F C_F)$ (as before) and $\omega_{IF} = 1/(R_{IF} C_{IF})$ results in

$$T(s) = -A_{OL}(s) \left(\frac{R_{IN}}{R_{IN} + R_F} \right) \left(\frac{1 + s/\omega_F}{1 + s/\omega_{IF}} \right). \quad (1.9)$$

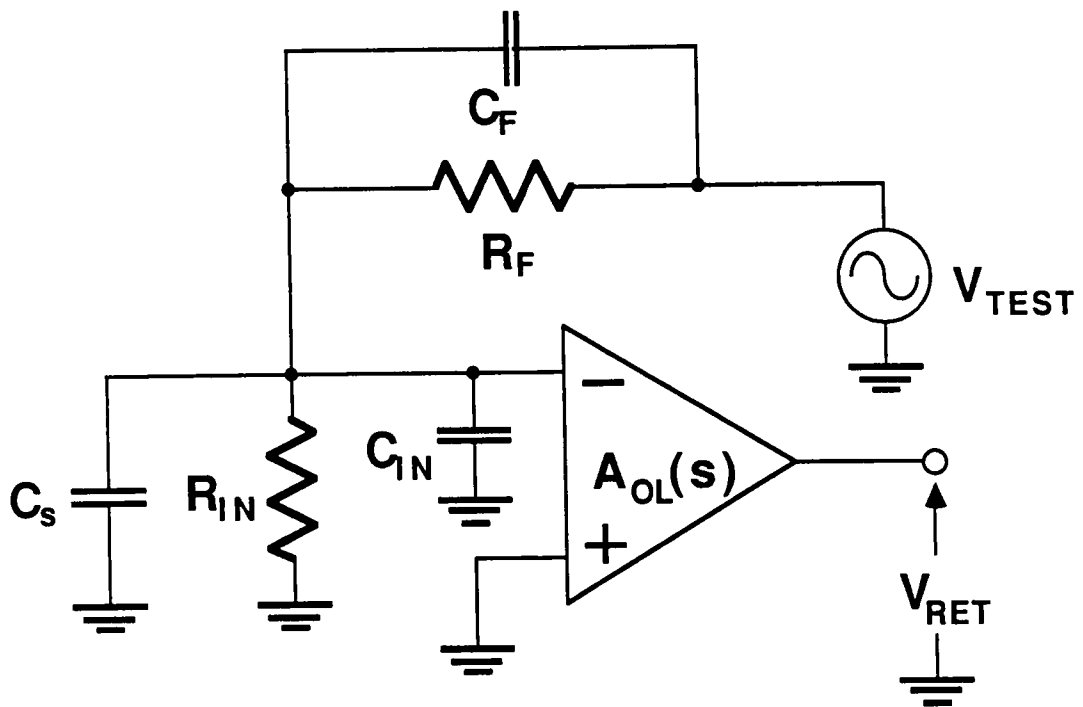


Figure A.2: Transimpedance amplifier loop transmission circuit.

A.3 Transimpedance Amplifier Noise Gains

The model for the transimpedance amplifier (Figure A.1) was extended to include sources that model the noise behavior. This model is shown in Figure A.3. The noise due to the gain block is represented by a voltage source and two current sources, and the thermal noise of the feedback resistor is represented by another voltage source. The voltage or current gain for each noise source is found in this section. These gains are needed to calculate noise power gains and eventually an equivalent input noise current for the amplifier.

A.3.1 Noise Gain for Equivalent Input Noise Voltage Source

The equivalent input noise voltage source for the gain block has power spectral density $S_e(f)$ and a circuit containing this source (labeled e_n) is shown as Figure A.4. From this figure it is clear that the ideal closed-loop gain for this source is

$$A_{CLe}^{ideal}(s) = \frac{R_{IN} \parallel \frac{1}{sC_I} + R_F \parallel \frac{1}{sC_F}}{R_{IN} \parallel \frac{1}{sC_I}}, \quad (1.10)$$

where $C_I = C_{IN} + C_F$. This expression reduces to

$$A_{CLe}^{ideal}(s) = \frac{R_F \left(\frac{R_F + R_{IN}}{R_F R_{IN}} + s(C_F + C_I) \right)}{1 + sR_F C_F}. \quad (1.11)$$

By letting $C_{IF} = C_I + C_F$ and $R_{IF} = R_{IN} \parallel R_F$ (as before) the ideal closed-loop gain becomes

$$A_{CLe}^{ideal}(s) = \left(\frac{R_F + R_{IN}}{R_{IN}} \right) \left(\frac{1 + sR_{IF}C_{IF}}{1 + sR_F C_F} \right). \quad (1.12)$$

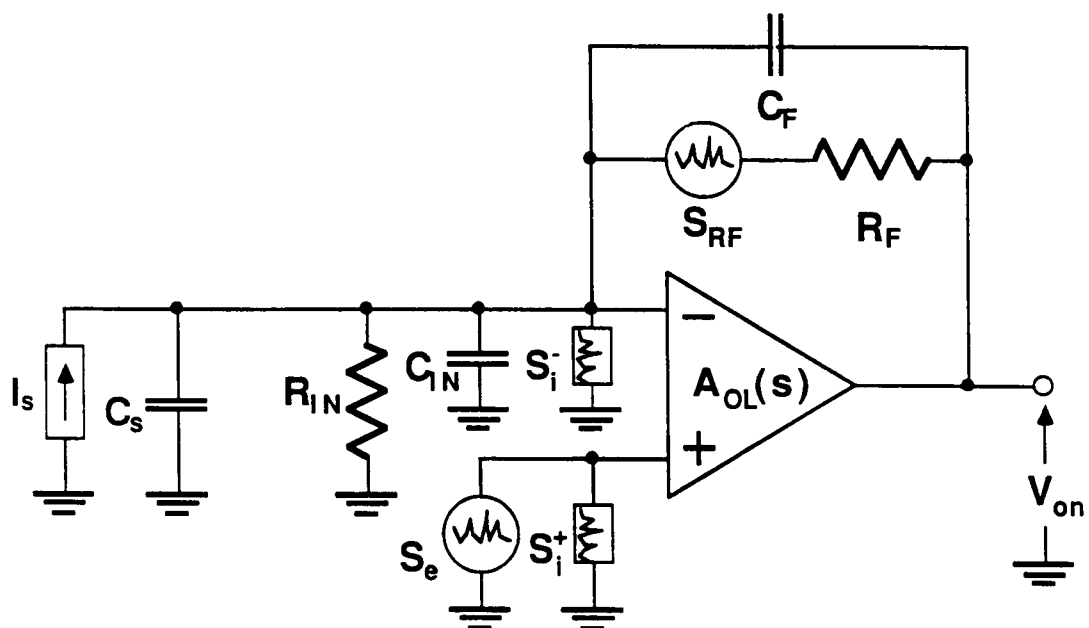


Figure A.3: Transimpedance amplifier noise model.

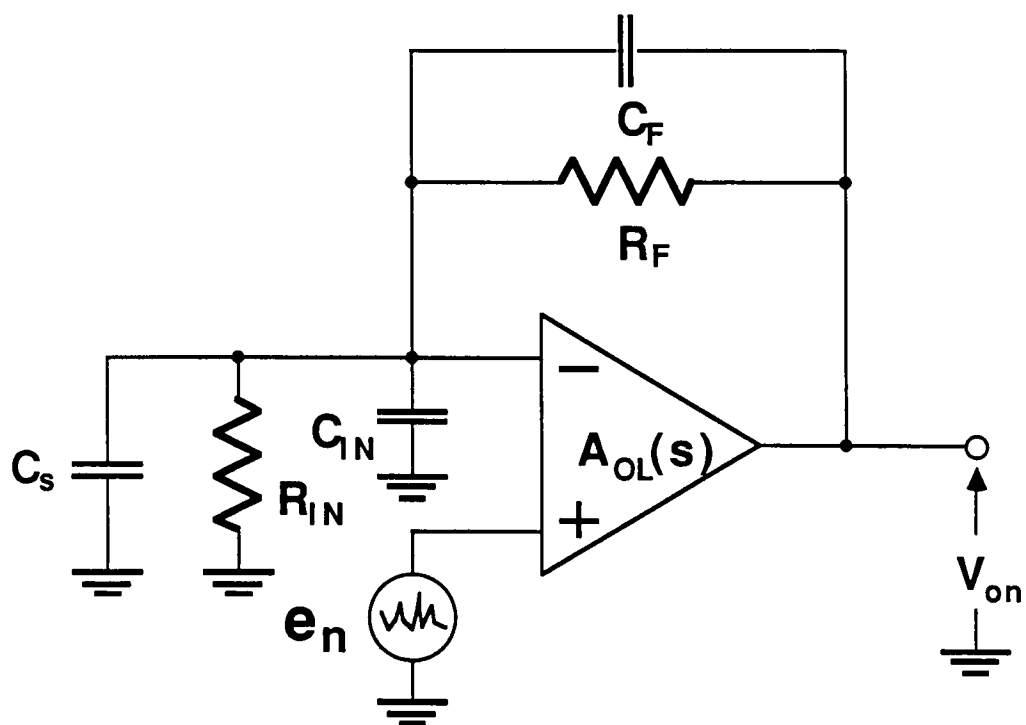


Figure A.4: Transimpedance amplifier with equivalent input noise voltage source.

Again letting $\omega_F = 1/(R_F C_F)$ and $\omega_{IF} = 1/(R_{IF} C_{IF})$, gives

$$A_{CLe}^{ideal}(s) = \left(1 + \frac{R_F}{R_{IN}}\right) \left(\frac{1 + s/\omega_{IF}}{1 + s/\omega_F}\right). \quad (1.13)$$

A.3.2 Noise Gain for Equivalent Input Noise Current Source

The equivalent input noise current source for the gain block inverting input has power spectral density $S_i^-(f)$ and a circuit containing this source (labeled i_n^-) is shown as Figure A.5. By comparing this figure to Figure A.1, it clear that the magnitude of the ideal closed-loop gain for this source is the same as that for the signal source. (The noise source does not have an assumed polarity as only the noise power produced is important.) Thus

$$A_{CLi}^{ideal}(s) = \frac{R_F}{1 + R_F C_F s}. \quad (1.14)$$

By substituting $\omega_F = 1/(R_F C_F)$ the ideal closed-loop gain becomes

$$A_{CLi}^{ideal}(s) = \frac{R_F}{1 + s/\omega_F}. \quad (1.15)$$

A.3.3 Noise Gain for Feedback Resistor Thermal Noise Source

The feedback resistor thermal noise source has power spectral density $S_{RF}(f)$ and a circuit containing this source (labeled e_{RF}) is shown as Figure A.6. Since the inverting input of the gain block is a virtual ground, the output voltage is simply the voltage developed across C_F . The ideal closed-loop gain is then

$$A_{CLRF}^{ideal}(s) = \frac{\frac{1}{sC_F}}{R_F + \frac{1}{sC_F}} = \frac{1}{1 + sR_F C_F}. \quad (1.16)$$

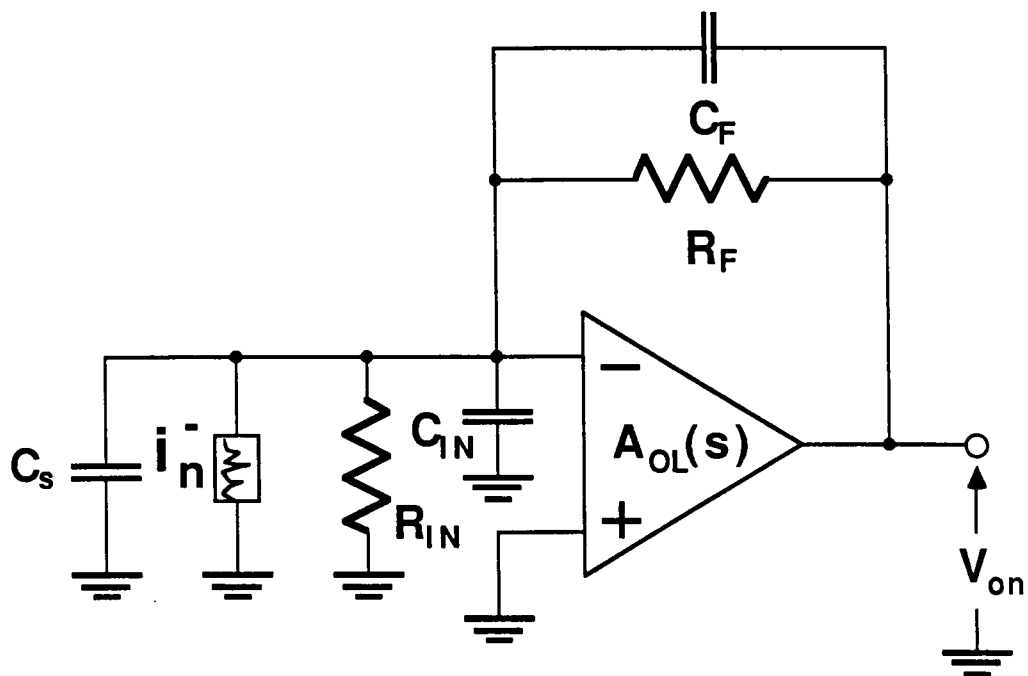


Figure A.5: Transimpedance amplifier with equivalent input noise current source.

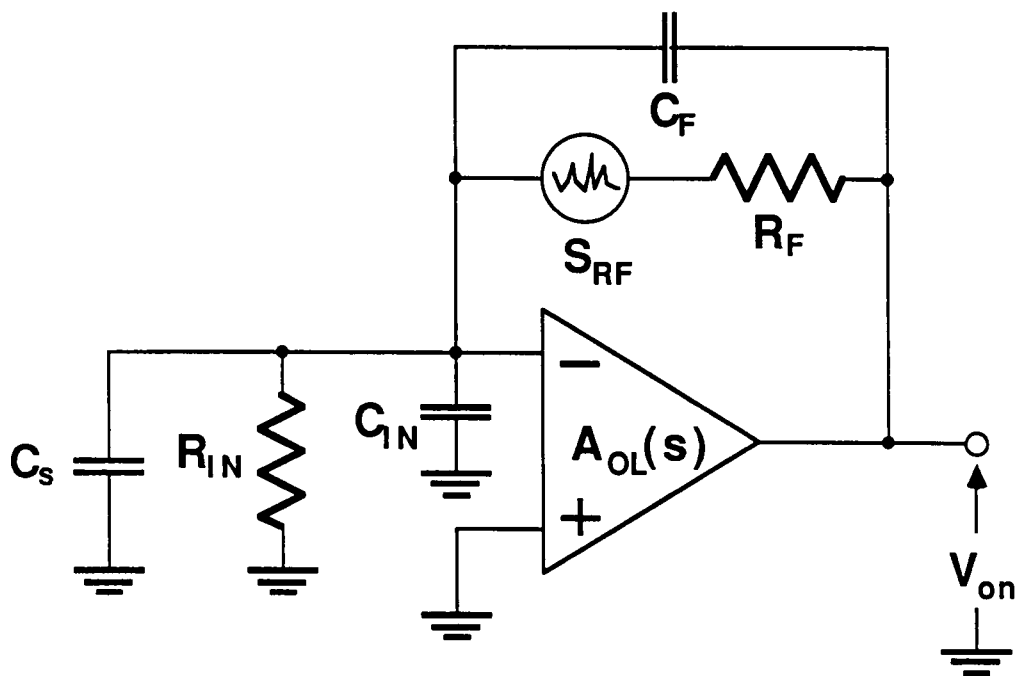


Figure A.6: Transimpedance amplifier with feedback resistor thermal noise source.

By substituting $\omega_F = 1/(R_FC_F)$ the ideal closed-loop gain becomes

$$A_{CLRF}^{ideal}(s) = \frac{1}{1 + s/\omega_F}. \quad (1.17)$$

APPENDIX B

DERIVATIONS OF VOLTAGE-SENSITIVE AMPLIFIER LOOP TRANSMISSION, SIGNAL AND NOISE GAINS

B.1 Voltage-Sensitive Amplifier Ideal Closed-Loop Gain

The model that was developed in Chapter 4 for the voltage-sensitive amplifier is shown in Figure B.1. The ideal closed-loop gain can be found by noting that it is the product of the impedance loading the current source (across which a voltage is developed) and the voltage gain of the feedback amplifier. The impedance seen by the source is the parallel combination of R_D , $1/(sC_S)$, and Z_{IN}^+ , and denoting that impedance by Z_D gives

$$Z_D = R_D \parallel R_{IN}^+ \parallel \frac{1}{s(C_{IN}^+ + C_S)}. \quad (2.18)$$

Letting $R_I = R_D \parallel R_{IN}^+$ and $C_I = C_{IN}^+ + C_S$ allows Z_D to be written as

$$Z_D = \frac{R_I}{1 + sR_IC_I} = \frac{R_I}{1 + s/\omega_I}, \quad (2.19)$$

where $\omega_I = 1/(R_IC_I)$.

The amplifier is an ordinary non-inverting type, and the voltage gain (from

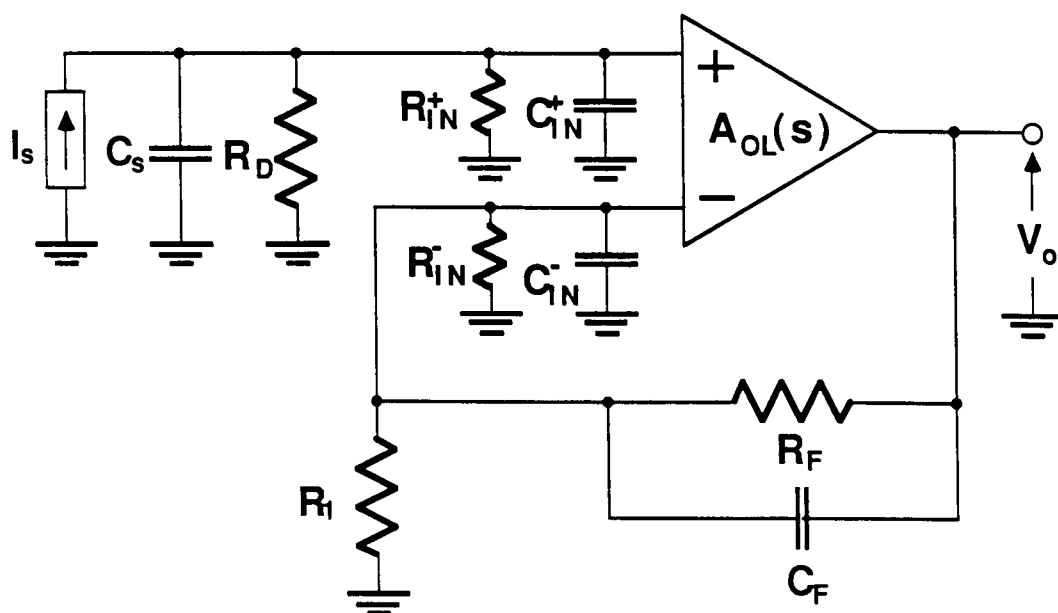


Figure B.1: Voltage-sensitive amplifier circuit model.

the non-inverting input to the output) is

$$A_v(s) = 1 + \frac{Z_F}{Z_X}, \quad (2.20)$$

where

$$Z_F = R_F \parallel \frac{1}{C_F s} = \frac{R_F}{1 + s R_F C_F}, \quad (2.21)$$

$$Z_X = R_1 \parallel R_{IN}^- \parallel \frac{1}{C_{IN}^- s} = \frac{R_X}{1 + s R_X C_{IN}^-}, \quad (2.22)$$

and R_X is the parallel combination of R_{IN}^- and R_1 . Substituting for Z_F and Z_X gives

$$A_v(s) = 1 + \frac{\frac{R_F}{1 + s R_F C_F}}{\frac{R_X}{1 + s R_X C_{IN}^-}}. \quad (2.23)$$

Rearranging this expression gives

$$A_v(s) = \left(1 + \frac{R_F}{R_X}\right) \left(\frac{1 + s \left(\frac{R_X R_F}{R_X + R_F} \right) (C_F + C_{IN}^-)}{1 + s R_F C_F} \right), \quad (2.24)$$

which can be reduced to

$$A_v(s) = \left(1 + \frac{R_F}{R_X}\right) \left(\frac{1 + s R_{XF} C_{XF}}{1 + s R_F C_F} \right), \quad (2.25)$$

by letting

$$R_{XF} = R_X \parallel R_F = R_1 \parallel R_{IN}^- \parallel R_F, \quad (2.26)$$

and

$$C_{XF} = C_F + C_{IN}^-. \quad (2.27)$$

Letting $\omega_F = 1/(R_F C_F)$ (as before) and $\omega_{XF} = 1/(R_{XF} C_{XF})$ results in

$$A_v(s) = \left(1 + \frac{R_F}{R_X}\right) \left(\frac{1 + s/\omega_{XF}}{1 + s/\omega_F} \right), \quad (2.28)$$

or

$$A_v(s) = \left(1 + \frac{R_F}{R_{IN}^- \parallel R_1}\right) \left(\frac{1 + s/\omega_{XF}}{1 + s/\omega_F}\right), \quad (2.29)$$

where the role of R_1 in determining the low frequency gain is more obvious.

The current-to-voltage gain is the product of the detection impedance and the voltage gain, therefore

$$A_{CL}^{ideal}(s) = \frac{V_o}{I_s} = Z_D A_v(s). \quad (2.30)$$

Substituting for Z_D and $A_v(s)$ gives

$$A_{CL}^{ideal}(s) = \left(\frac{R_I}{1 + s/\omega_I}\right) \left(1 + \frac{R_F}{R_X}\right) \left(\frac{1 + s/\omega_{XF}}{1 + s/\omega_F}\right). \quad (2.31)$$

B.2 Voltage-Sensitive Amplifier Loop Transmission

The loop transmission is found by breaking the feedback loop at the output of the gain block (a zero impedance point) and then calculating the gain around the loop. Figure B.2 shows the feedback loop broken and a test generator inserted. The loop transmission $T(s)$ equals the returned voltage V_{RET} divided by V_{TEST} . From the figure it is clear that the loop transmission is the product of $-A_{OL}(s)$ and an attenuation due to Z_F and Z_X . Thus

$$T(s) = -A_{OL}(s) \left(\frac{Z_X}{Z_X + Z_F}\right), \quad (2.32)$$

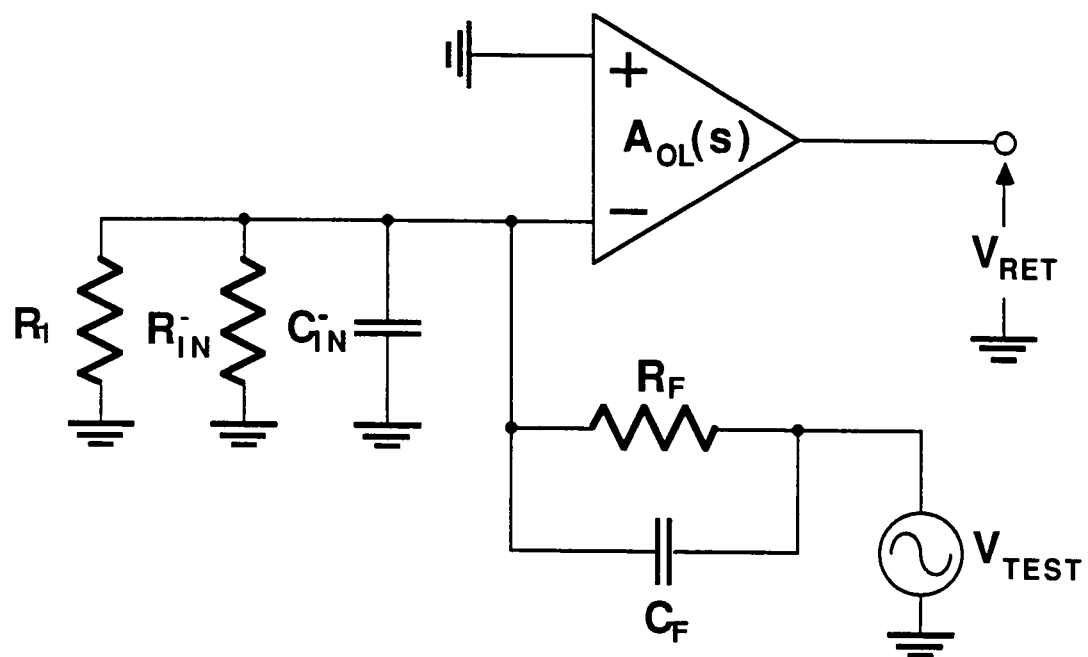


Figure B.2: Voltage-sensitive amplifier loop transmission circuit.

and since $Z_F = R_F \parallel 1/(sC_F)$ and $Z_X = R_{IN}^- \parallel R_1 \parallel 1/(sC_{IN}^-)$, the loop transmission is

$$T(s) = \frac{-A_{OL}(s) \left(\frac{R_{IN}^- \parallel R_1}{1 + s(R_{IN}^- \parallel R_1)C_{IN}^-} \right)}{\left(\frac{R_{IN}^- \parallel R_1}{1 + s(R_{IN}^- \parallel R_1)C_{IN}^-} \right) + \left(\frac{R_F}{1 + sR_FC_F} \right)} \quad (2.33)$$

in terms of the amplifier parameters. Rearranging and simplifying this expression gives

$$T(s) = -A_{OL}(s) \left(\frac{R_X}{R_X + R_F} \right) \left(\frac{1 + sR_FC_F}{1 + s \left(\frac{R_X R_F}{R_X + R_F} \right) (C_F + C_{IN}^-)} \right), \quad (2.34)$$

which can be further simplified to

$$T(s) = -A_{OL}(s) \left(\frac{R_X}{R_X + R_F} \right) \left(\frac{1 + sR_FC_F}{1 + sR_{XF}C_{XF}} \right) \quad (2.35)$$

by again using $R_{XF} = R_X \parallel R_F$, and $C_{XF} = C_F + C_{IN}^-$. Substituting $\omega_F = 1/(R_FC_F)$ and $\omega_{XF} = 1/(R_{XF}C_{XF})$ results in

$$T(s) = -A_{OL}(s) \left(\frac{R_X}{R_X + R_F} \right) \left(\frac{1 + s/\omega_F}{1 + s/\omega_{XF}} \right). \quad (2.36)$$

B.3 Voltage-Sensitive Amplifier Noise Gains

The model for the voltage-sensitive amplifier (Figure B.1) was extended to include sources modeling the noise behavior. This model is shown in Figure B.3. The noise due to the gain block is represented by a voltage source and two current sources, and the thermal noise of the detection, feedback and gain-setting resistors are represented additional voltage sources. The ideal voltage-to-voltage or current-to-voltage gain for each of these noise sources is found in this

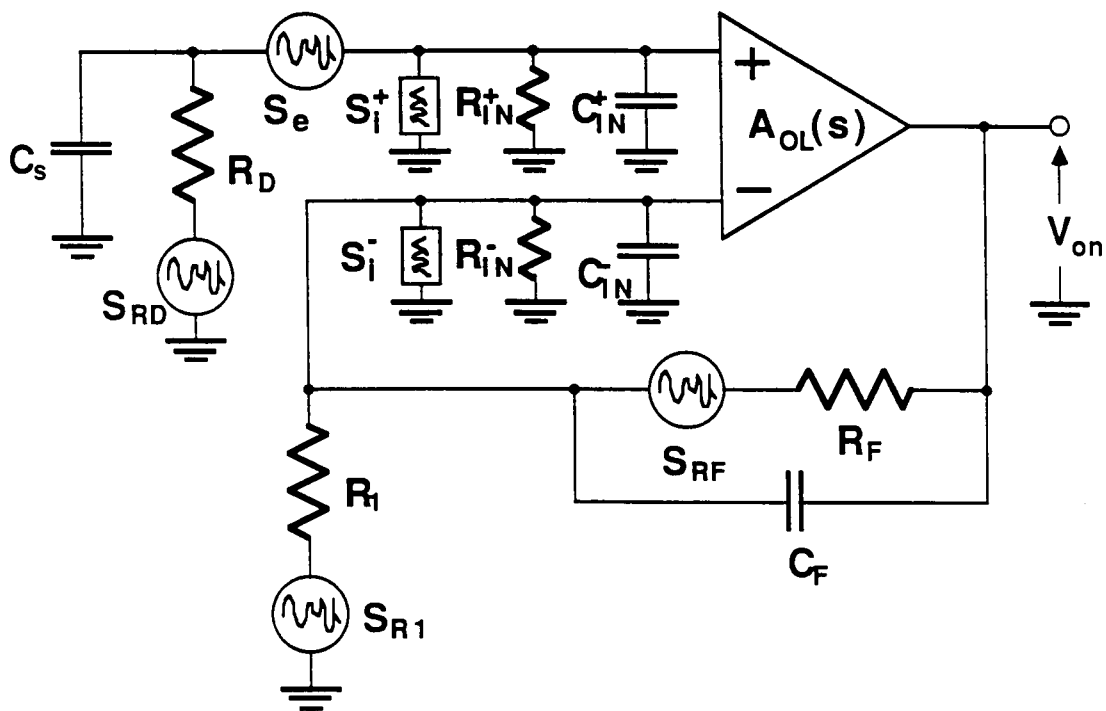


Figure B.3: Voltage-sensitive amplifier noise model.

section. These gains are needed to calculate noise power gains and eventually an equivalent input noise current for the amplifier.

B.3.1 Gain for Detection Resistor Noise Source

The thermal noise of the detection resistor R_D can be represented by a source with power spectral density $S_{RD}(f) = 4kTR_D$. A diagram of the amplifier circuit showing only this noise source (labeled e_{RD}) is shown as Figure B.4. The ideal closed-loop gain (in V/V) for this source is the product of two terms: an attenuation due to the input circuit and the voltage gain of the feedback amplifier. The gain from the source to the non-inverting terminal is

$$A_{gRD}(s) = \frac{\frac{1}{sC_S} \parallel \frac{1}{sC_{IN}^+} \parallel \frac{1}{R_{IN}^+}}{R_D + \frac{1}{sC_S} \parallel \frac{1}{sC_{IN}^+} \parallel \frac{1}{R_{IN}^+}}, \quad (2.37)$$

which can be simplified to

$$A_{gRD}(s) = \frac{\frac{R_{IN}^+}{R_D + R_{IN}^+}}{1 + sR_IC_I}, \quad (2.38)$$

by letting $C_I = C_{IN}^+ + C_S$ and $R_I = R_D \parallel R_{IN}^+$, as before. Using $\omega_I = 1/(R_IC_I)$ finally gives

$$A_{gRD}(s) = \left(\frac{R_{IN}^+}{R_D + R_{IN}^+} \right) \left(\frac{1}{1 + s/\omega_I} \right). \quad (2.39)$$

The voltage gain of the feedback amplifier (found in Section B.1) is

$$A_v(s) = \left(1 + \frac{R_F}{R_{IN}^- \parallel R_1} \right) \left(\frac{1 + s/\omega_{XF}}{1 + s/\omega_F} \right), \quad (2.40)$$

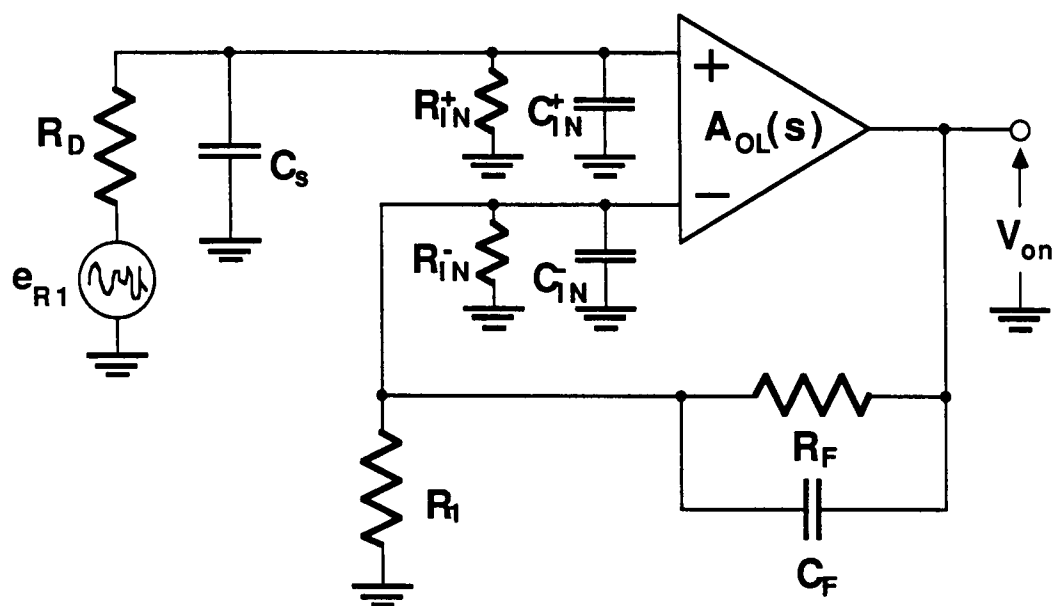


Figure B.4: Voltage-sensitive amplifier with detection resistor noise source.

and therefore the ideal closed-loop gain for the e_{RD} noise source is

$$A_{CLRD}^{ideal}(s) = A_{gRD}(s)A_v(s), \quad (2.41)$$

or

$$A_{CLRD}^{ideal}(s) = \left(\frac{R_{IN}^+}{R_D + R_{IN}^+} \right) \left(\frac{1}{1 + s/\omega_I} \right) \left(1 + \frac{R_F}{R_{IN}^- \parallel R_1} \right) \left(\frac{1 + s/\omega_{XF}}{1 + s/\omega_F} \right). \quad (2.42)$$

B.3.2 Gain for Equivalent Input Noise Voltage Source

The equivalent input noise voltage source for the gain block has power spectral density $S_e(f)$, and a circuit diagram of the amplifier showing this source (labeled e_n) is shown as Figure B.5. From this figure, it is clear that like the gain for the e_{RD} source, the gain is the product of an attenuation due to the input circuit and the voltage gain of the feedback amplifier. The gain from the source to the non-inverting terminal is

$$A_{ge}(s) = \frac{Z_{IN}^+}{Z_S + Z_{IN}^+}, \quad (2.43)$$

where $Z_S = R_D \parallel 1/(sC_S)$. Substituting for Z_S and Z_{IN}^+ gives

$$A_{ge}(s) = \frac{R_{IN}^+ \parallel \frac{1}{sC_{IN}^+}}{R_D + \parallel \frac{1}{sC_S} + R_{IN}^+ \parallel \frac{1}{sC_{IN}^+}}, \quad (2.44)$$

which can be simplified to

$$A_{ge}(s) = \left(\frac{R_{IN}^+}{R_D + R_{IN}^+} \right) \left(\frac{1 + sR_D C_S}{1 + sR_I C_I} \right), \quad (2.45)$$

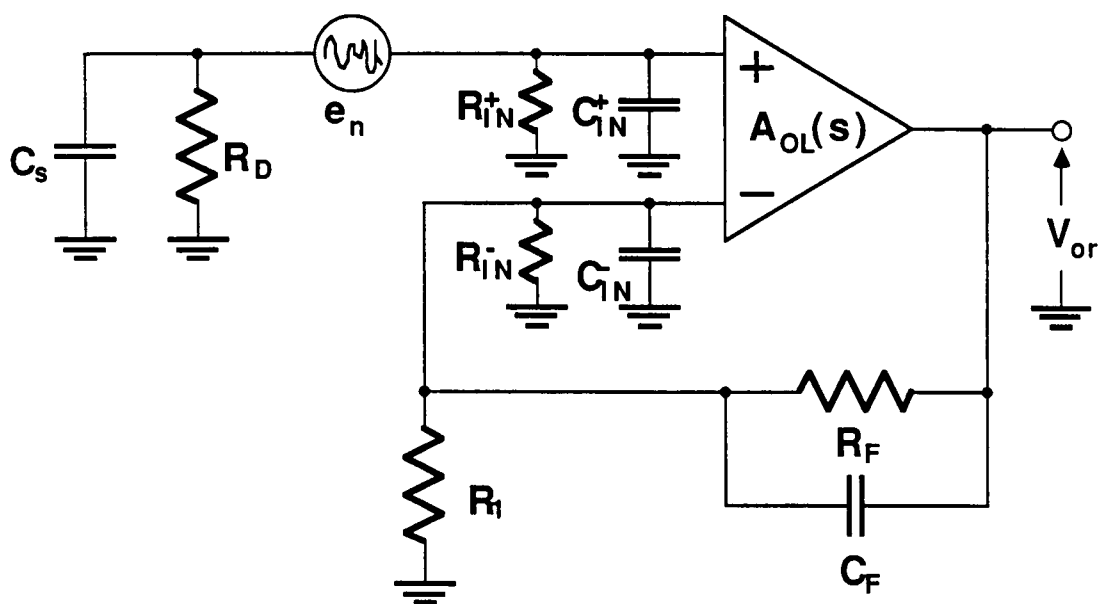


Figure B.5: Voltage-sensitive amplifier with equivalent input noise voltage source.

by again letting $C_I = C_{IN}^+ + C_S$ and $R_I = R_D \parallel R_{IN}^+$. Letting $\omega_D = 1/(R_D C_S)$ and $\omega_I = 1/(R_I C_I)$ gives

$$A_{ge}(s) = \left(\frac{R_{IN}^+}{R_D + R_{IN}^+} \right) \left(\frac{1 + s/\omega_D}{1 + s/\omega_I} \right). \quad (2.46)$$

The ideal closed-loop gain (V/V) for the e_n noise source is

$$A_{CLe}^{ideal}(s) = A_{ge}(s)A_v(s), \quad (2.47)$$

where $A_v(s)$ is given by Eq. 2.29, and substituting for $A_{ge}(s)$ and $A_v(s)$ gives

$$A_{CLe}^{ideal}(s) = \left(\frac{R_{IN}^+}{R_D + R_{IN}^+} \right) \left(\frac{1 + s/\omega_D}{1 + s/\omega_I} \right) \left(1 + \frac{R_F}{R_{IN}^- \parallel R_1} \right) \left(\frac{1 + s/\omega_{XF}}{1 + s/\omega_F} \right). \quad (2.48)$$

B.3.3 Gain for Non-inverting Terminal Equivalent Input Noise Current Source

The equivalent input noise current source for the gain block non-inverting input has power spectral density $S_i^+(f)$, and a circuit diagram of the amplifier showing this source (labeled i_n^+) is given as Figure B.6. From this figure, it can be seen that the current-to-voltage gain for this source is the same as for the current signal source, thus

$$A_{CLi+}^{ideal}(s) = \left(\frac{R_I}{1 + s/\omega_I} \right) \left(1 + \frac{R_F}{R_X} \right) \left(\frac{1 + s/\omega_{XF}}{1 + s/\omega_F} \right). \quad (2.49)$$

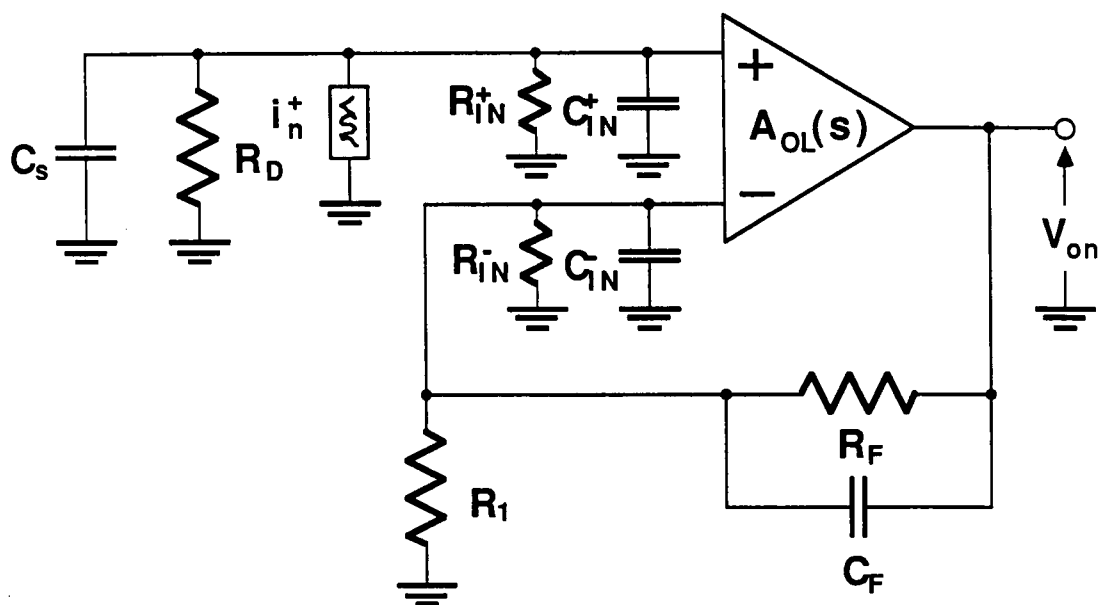


Figure B.6: Voltage-sensitive amplifier with non-inverting terminal equivalent input noise current source.

B.3.4 Gain for Inverting Terminal Equivalent Input Noise Current Source

The equivalent input noise current source for the gain block inverting input has power spectral density $S_i^-(f)$, and a circuit diagram of the amplifier including this source (labeled i_n^-) is shown in Figure B.7. The circuit is altered to show only the components needed for calculating the gain for this source. Since the inverting input is a virtual ground in the configuration shown here, the noise current of the i_n^- source flows through the feedback impedance, developing a voltage at the output of the amplifier. Thus the ideal closed-loop gain (V/A) for this source is

$$A_{CLi-}^{ideal}(s) = Z_F = \frac{R_F}{1 + s/\omega_F}, \quad (2.50)$$

where $\omega_F = 1/(R_FC_F)$.

B.3.5 Gain for Gain-Setting Resistor Noise Source

The thermal noise of the gain-setting resistor (R_1) can be represented by a source with power spectral density $S_{R1}(f) = 4kTR_1$. A circuit diagram of the amplifier including this source (labeled e_{R1}) is shown in Figure B.8. The circuit is simplified to show only the components needed for calculating the gain for this source. Since the inputs are virtual grounds for this configuration, Z_{IN}^- does not effect the ideal gain, and the amplifier functions as an inverting voltage amplifier

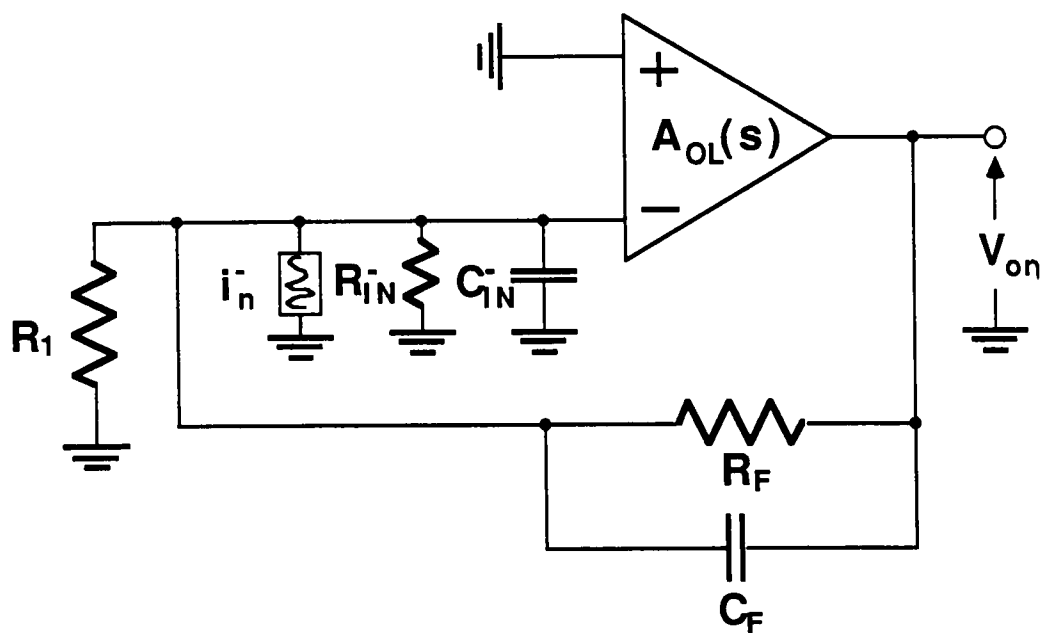


Figure B.7: Voltage-sensitive amplifier with inverting terminal equivalent input noise current source.

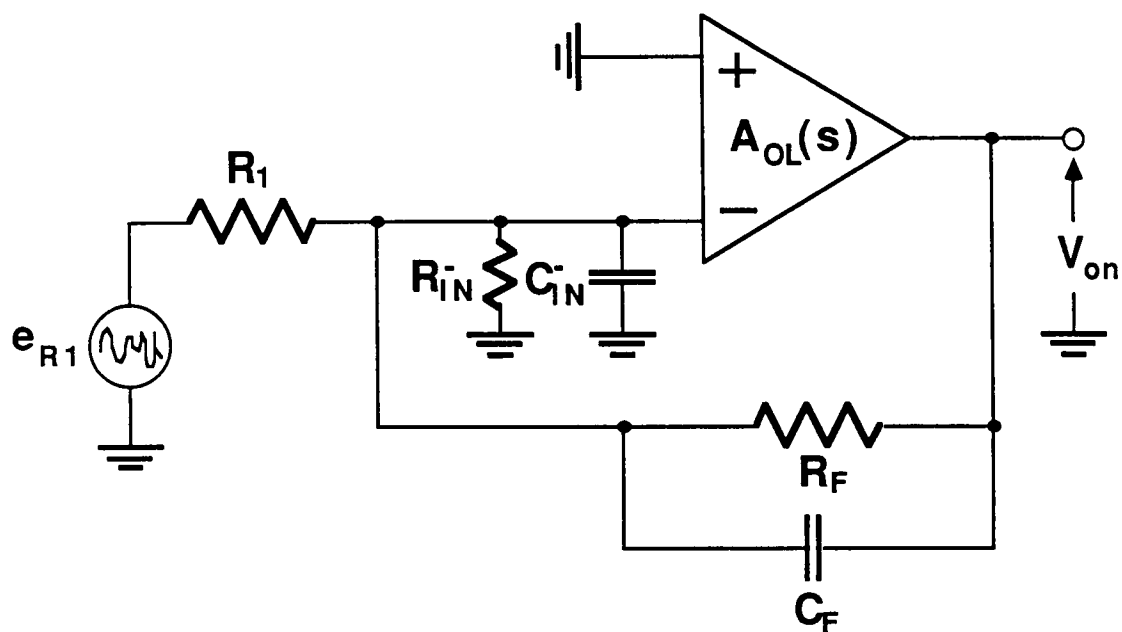


Figure B.8: Voltage-sensitive amplifier with gain-setting resistor thermal noise voltage source.

with ideal closed-loop gain (V/V)

$$A_{CLR1}^{ideal}(s) = \frac{Z_F}{R_1} = \frac{1}{R_1} \left(\frac{R_F}{1 + sR_FC_F} \right). \quad (2.51)$$

This can be simplified to

$$A_{CLR1}^{ideal}(s) = \frac{R_F}{R_1} \left(\frac{1}{1 + s/\omega_F} \right), \quad (2.52)$$

where $\omega_F = 1/(R_FC_F)$.

B.3.6 Gain for Feedback Resistor Noise Source

The thermal noise of the feedback resistor (R_F) can be represented by a source with power spectral density $S_{RF}(f) = 4kTR_F$. A simplified circuit diagram of the amplifier useful for calculating the gain of this source (labeled e_{RF}) is shown in Figure B.9. Since the inputs are virtual grounds for this configuration, R_1 and Z_{IN}^- do not effect the ideal gain, and the output voltage is simply the voltage developed across C_F by e_{RF} . Thus the ideal closed-loop gain (V/V) is

$$A_{CLRF}^{ideal}(s) = \frac{1/(sC_F)}{R_F + 1/(sC_F)} = \frac{1}{1 + s/\omega_F}, \quad (2.53)$$

where $\omega_F = 1/(R_FC_F)$.

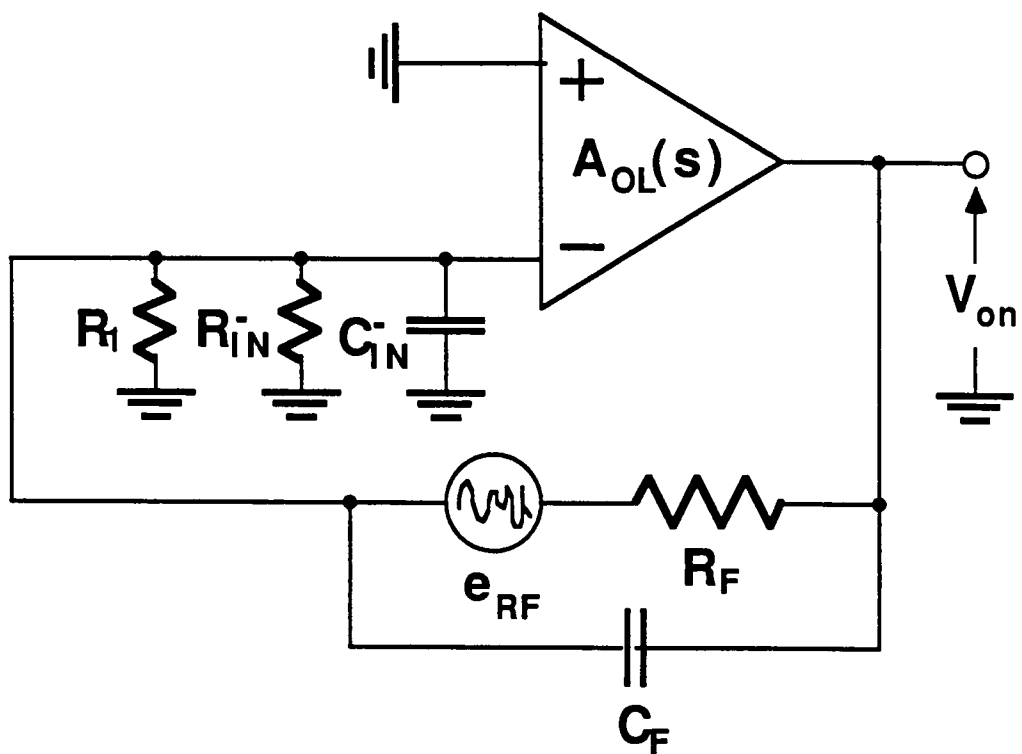


Figure B.9: Voltage-sensitive amplifier with feedback resistor thermal noise voltage source.

APPENDIX C

TRAMP ORGANIZATION, OPERATION AND LISTINGS

TRAMP is a FORTRAN program which performs frequency and noise analysis of transimpedance amplifiers described using the general model of Chapter 4. This appendix describes the organization and operation of TRAMP and includes a complete listing of the code.

C.1 Organization

TRAMP has a modular organization. It is subdivided into a main program, seven frequency response subroutines and four noise computation subroutines. The main program, named TRAMP, contains the user interface (input and output routines) and logic which decides which subroutines need to be called to perform the calculations chosen by the user. The frequency response routines perform calculations such as finding $A_{CL}(f)$ or the phase margin and are listed in Table C.1. The more basic routines such as LOOP, for example, are used by higher level routines like PHASEM. Similarly, the noise computation routines perform calculations such as finding $S_{noe}(f)$ and are given in Table C.2.

Table C.1: Frequency response subroutines used by TRAMP.

Name	Purpose
RECPOL	convert a complex number from rectangular form to polar form
IDEAL	calculate the ideal closed-loop gain for a single value of s and the amplifier parameters
LOOP	calculate the loop transmission for a single value of s and the amplifier parameters
ACTUAL	calculate the actual closed-loop gain $A_{CL}(s)$ given the values of $T(s)$ and $A_{CL}^{ideal}(s)$
ACTUAL2	calculate the actual closed-loop gain $A_{CL}(s)$ by first calling LOOP and IDEAL to get $T(s)$ and $A_{CL}^{ideal}(s)$
PHASEM	find the crossover frequency and phase margin by using LOOP iteratively
BANDWI	find the closed-loop -3dB frequency (the bandwidth) by using ACTUAL2 iteratively

C.2 Operation

Most of the frequency response and noise computation subroutines are straightforward implementations of the equations developed for the transimpedance amplifier general model. RECPOL, LOOP, ACTUAL, ACTUAL2, ENOISE, INOISE and RNOISE fall into that category and will not be discussed further as their operation is understandable from the model equations and the comments that are a part of each listing. The operation of TRAMP, PHASEM, BANDWI and INTQUAD is not so obvious and is the topic of the remainder of this section.

The main program (TRAMP) presents the user with two layers of menus.

Table C.2: Noise computation subroutines used by TRAMP.

Name	Purpose
ENOISE	calculate input noise power spectral density S_e , ideal closed-loop gain, loop transmission and actual closed-loop gain for this source, and output noise power spectral density S_{noe} for a single frequency
INOISE	calculate input noise power spectral density S_i , ideal closed-loop gain, loop transmission and actual closed-loop gain for this source, and output noise power spectral density S_{noi} for a single frequency
RNOISE	calculate input noise power spectral density S_{RF} , ideal closed-loop gain, loop transmission and actual closed-loop gain for this source, and output noise power spectral density S_{noRF} for a single frequency
INTQUAD	calculate total noise power at amplifier output for source S_e , S_i or S_{RF} by using ENOISE, INOISE or RNOISE to generate values of S_{noe} , S_{noi} or S_{noRF} and integrating the noise power spectral density over the desired bandwidth

The highest level menu, called 'MAIN', lets the user choose to proceed to the amplifier parameter menu, the frequency analysis menu, the noise analysis menu or exit. From the amplifier parameter menu, the user can display, set or modify the amplifier parameters. From the frequency analysis menu, the user can choose to have the bandwidth or phase margin calculated or to generate a table of frequencies and the corresponding loop transmissions and gains. From the noise analysis menu, the user can select calculation of the noise power spectral density at one or more frequencies or the total noise power in a bandwidth.

The operation of TRAMP is relatively simple as TRAMP contains mainly input and output statements, logic that implements the menu structure and subroutine calls. TRAMP also contains two common blocks that allow the amplifier parameters to be shared with the subroutines and a set of default parameters which are useful to demonstrate the program's operation and to ease setup. As a typical example of the operation, consider the case where a list of frequencies and gains are desired. The user goes through the main menu to reach the frequency analysis menu and chooses the list option. The program then gets the frequency range and frequency increment from the user and calculates the frequencies for which the gains will be calculated. For each frequency, LOOP is called to get $T(f)$, then IDEAL is called to get $A_{CL}^{ideal}(f)$ and finally ACTUAL is called to get $A_{CL}(f)$. When all the calculations are completed, the amplifier parameters and the frequency response table are printed out, and the program return to the frequency analysis menu.

PHASEM finds the phase margin by using the bisection method to solve $|T(f)| = 1$ for $f = f_{co}$ and consequently solving for the phase of $T(f_{co})$ and then the phase margin. The routine requires two frequency guesses, f_{lo} and f_{hi} , to begin. These should bracket f_{co} ($f_{hi} > f_{co} > f_{lo}$) so that $|T(f_{hi})| < 1$ and $|T(f_{lo})| > 1$. $T(f)$ values are found by calling LOOP, and $|T(f)|$ values are then computed by RECPOL. If $|T(f_{hi})| \not< 1$ (which implies that $f_{hi} \not> f_{co}$), then f_{hi} is doubled and $|T(f_{hi})|$ checked repeatedly until $|T(f_{hi})| < 1$. If $|T(f_{lo})| \not> 1$, then f_{lo} is halved and $|T(f_{lo})|$ checked repeatedly until $|T(f_{lo})| > 1$. Once f_{lo} and f_{hi} bracket f_{co} , the bisection method is started. A new frequency, f_{new} is formed by averaging f_{lo} and f_{hi} , and $|T(f_{new})|$ is found. If $|T(f_{new})| < 0.99$, then f_{hi} is replaced by f_{new} , and if $|T(f_{new})| > 1.01$, then f_{lo} is replaced by f_{new} . In either case, the procedure then repeats with the formation of a new f_{new} . If $|T(f_{new})| = 1 \pm 0.01$, then f_{new} is f_{co} within a few percent, and the routine proceeds to calculate the phase margin from the phase of $T(f_{new})$.

BANDWI finds the upper -3dB frequency through much the same process as PHASEM finds the phase margin, except in this case ACTUAL2 and RECPOL are used to get $|A_{CL}(f)|$, and $|A_{CL}(f)| = |A_{CL}(0)|/\sqrt{2}$ is solved for $f = f_{BW}$. Two frequency guesses, f_{lo} and f_{hi} , bracketing f_{BW} are required to begin. This condition is tested by checking to see if $|A_{CL}(f_{hi})| < |A_{CL}(0)|/\sqrt{2}$ and $|A_{CL}(f_{lo})| > |A_{CL}(0)|/\sqrt{2}$. If these conditions are satisfied, the bisection procedure begins, but if not, f_{lo} is decreased or f_{hi} is increased until they are. The bisection procedure proceeds until $|A_{CL}(f)|$ is within 0.2% of $|A_{CL}(0)|/\sqrt{2}$,

and this frequency is recorded as f_{BW} .

INTQUAD integrates the output noise power spectral density due to a single (selectable) noise source over a range of frequencies to give the total noise power. The integration process closely follows the procedure given by [48, pages 155–158] for an adaptive quadrature method based on Simpson's composite rule. The procedure begins by calculating an initial estimate of the integral using Simpson's rule. Next, the overall tolerance is set to 0.1% of the initial estimate of the integral, which should lead to an error bound of $\sim 0.01\%$ for the final approximation, if the initial estimate is of the same order of magnitude as the actual integral. For this application (integrating noise power spectral densities), it was felt that accuracy expressed as a fraction of the integral was more appropriate than an absolute tolerance which might be unnecessarily strict for a noisy amplifier yet result in poor relative accuracy for a low-noise amplifier. Setting the tolerance to 0.1% of the initial estimate was empirically determined to give accuracy much better than 1% for the type of relatively smooth functions expected as noise power spectral densities even though the initial estimate might differ greatly from the actual integral.

The adaptive quadrature procedure continues by dividing the integration interval into two equal parts and then using Simpson's rule to estimate the integral of each subinterval. If the sum of these two estimates agrees with initial estimate to within the desired tolerance, then the sum is the final approximation of the integral. If not, each subinterval is subdivided into two parts, and Simpson's

rule is applied to each. Once again, the sum of the approximations for these subintervals is compared to the approximation for the higher level subinterval. If they differ by less than the tolerance (which is divided by two, since the interval was halved), then the sum is the final approximation of the integral on the subinterval. This process continues until the interval is divided into subintervals small enough that each approximation is within its tolerance. It is important to note that the subintervals may be various sizes. The approximations for the lowest level subintervals are summed to give an approximation of the integral for the entire interval.

Besides the adaptive quadrature procedure, INTQUAD contains logic to select the noise power spectral density that is to be integrated. The adaptive quadrature procedure evaluates the desired noise power spectral density by calling a function subprogram which in turn calls ENOISE, INOISE or RNOISE depending upon the value of a control variable passed from the main program.

C.3 TRAMP Listing

```

C      PROGRAM TRAMP
C      written 2/28/89 by Alan Wintenberg (after NEWAMP5)
C      updated 2/28/89
C
C      It must be linked with ACTUAL, ACTUAL2, BANDWI, IDEAL, LOOP,
C      PHASEM, RECPOL, ENOISE, INOISE, RNOISE and INTQUAD
C
C      amplifier parameters are located in common block amp1
C      amplifier parameters are located in common block amp2
C
      COMPLEX*8 s,t,b1,b2,ai,act
      character*6 lbl(13),nlbl(6)

```

```

dimension f(200),tm(200),tp(200),aim(200),aip(200),actm(200)
dimension actp(200),sno(4,200)
real*4 par(13),npar(6)
real*4 ke1,ke2,ke3,ki1,ki2,ki3
C
C   Make individual parameter names equivalent to array par
equivalence (par(1),a0)
equivalence (par(2),rf)
equivalence (par(3),rin)
equivalence (par(4),cf)
equivalence (par(5),cs)
equivalence (par(6),cin)
equivalence (par(7),wz1)
equivalence (par(8),wz2)
equivalence (par(9),wp1)
equivalence (par(10),wp2)
equivalence (par(11),wp3)
equivalence (par(12),wp4)
equivalence (par(13),tau)
C
C   Make noise parameter names equivalent to array npar
equivalence (npar(1),ke1)
equivalence (npar(2),ke2)
equivalence (npar(3),ke3)
equivalence (npar(4),ki1)
equivalence (npar(5),ki2)
equivalence (npar(6),ki3)
C
C   set up amplifier parameter common block
common /amp1/ a0,rf,rin,cf,cs,cin,wz1,wz2,wp1,wp2,wp3,wp4,tau
C
C   a0  =  amplifier dc gain
C   rf  =  feedback resistor
C   rin =  input resistance
C   cf  =  feedback capacitance
C   cs  =  source capacitance
C   cin =  input capacitance (probably in parallel with cs)
C   wz1 =  amplifier zero 1
C   wz2 =  amplifier zero 2
C   wp1 =  amplifier pole 1
C   wp2 =  amplifier pole 2
C   wp3 =  amplifier pole 3
C   wp4 =  amplifier pole 4
C   tau =  delay term -- not implemented
C
C   set up amplifier noise parameter common block
common /amp2/ ke1,ke2,ke3,ki1,ki2,ki3

```

```

C
C   ke1 = equivalent noise voltage source linear coefficient
C   ke2 = equivalent noise voltage source 1/f coefficient
C   ke3 = equivalent noise voltage source 1/f*f coefficient
C   ki1 = equivalent noise current source linear coefficient
C   ki2 = equivalent noise current source 1/f coefficient
C   ki3 = equivalent noise current source 1/f*f coefficient
C
C   Assign labels to amplifier parameters
C
      data lbl /'a0','rf','rin','cf','cs','cin','wz1','wz2','wp1',
c'wp2','wp3','wp4','tau'/
      data nlbl /'ke1','ke2','ke3','ki1','ki2','ki3'/
C
C   assign default parameters
30 rin=1.0e9
   cin=1.0e-12
   cs=3.0e-12
   rf=1.0e4
   cf=159.0e-12
   a0=1.0e4
   wz1=1.0e14
   wz2=1.0e14
   wp1=628.0
   wp2=1.0e7
   wp3=1.0e14
   wp4=1.0e14
   tau=0.0
   ke1=1.0e-16
   ke2=0.0
   ke3=0.0
   ki1=1.0e-22
   ki2=0.0
   ki3=0.0
   iu=5
C
C   **** Main Menu ****
C
50 type *,' '
   type *,'   MAIN MENU'
   type *,' '
   type *,'Enter : '
   type *,' A       Amplifier parameter menu'
   type *,' F       for Frequency analysis'
   type *,' N       for Noise analysis'
   type *,' Q       to Quit'
   accept 55,ans

```



```

55 format(a1)
   if(ans .eq. 1ha) go to 80
   if(ans .eq. 1hf) go to 120
   if(ans .eq. 1hn) go to 3000
   if(ans .eq. 1hq) go to 6000
   go to 50
C
C   **** Set Amplifier Parameter Menu ****
C
80 type *,' '
   type *,'   AMPLIFIER PARAMETERS'
   type *,'Enter : '
   type *,' C       to Change all parameters'
   type *,' D       to Display current parameters'
   type *,' N       to display current Noise parameters'
   type *,' R       to Return to main menu'
   type *,' S       to Set to default values'
   accept 55,ans
   if(ans .eq. 1hc) go to 90
   if(ans .eq. 1hd) go to 100
   if(ans .eq. 1hn) go to 110
   if(ans .eq. 1hr) go to 50
   if(ans .eq. 1hs) go to 30
   go to 80
90 type *,'Enter Rin and Cs'
   accept *,rin,cs
   type *,'Enter Rf and Cf'
   accept *,rf,cf
   type *,'Enter A0 and Cin'
   accept *,a0,cin
   type *,'Enter wz1 and wz2'
   accept *,wz1,wz2
   type *,'Enter wp1, wp2, wp3, and wp4'
   accept *,wp1,wp2,wp3,wp4
   type *,'Enter tau'
   accept *,tau
   go to 80
C   Display current parameters
100 type 102,'#','NAME','VALUE','#','NAME','VALUE'
102 format(/8x,a1,5x,a4,6x,a5,11x,a1,5x,a4,6x,a5/)
   do 106 i=1,6
   j=i+6
   type 104,i,lbl(i),par(i),j,lbl(j),par(j)
104 format(7x,i2,5x,a6,3x,pe10.3,6x,i2,5x,a6,3x,pe10.3)
106 continue
   j=13
   type 107,j,lbl(j),par(j)

```

```

107 format(7x,i2,5x,a6,3x,pe10.3/)
108 type *,'Change # ? (0 for no change)'
    accept *,i
    if(i .le. 0) go to 80
    if(i .gt. 13) go to 80
    type *,'Current ',lbl(i),' value is',par(i),'      New value ?'
    accept *,par(i)
    type *,'More ? (y or n)'
    accept 55,ans
    if(ans .eq. 1hy) go to 108
    go to 80
C   Display current noise parameters
110 type 112,'#','NAME','VALUE'
112 format(/8x,a1,5x,a4,6x,a5/)
    do 116 i=1,6
    type 114,i,nlbl(i),npar(i)
114 format(7x,i2,5x,a6,3x,pe10.3)
116 continue
118 type *,'Change # ? (0 for no change)'
    accept *,i
    if(i .le. 0) go to 80
    if(i .gt. 13) go to 80
    type *,'Current ',nlbl(i),' value is',npar(i),'      New value ?'
    accept *,npar(i)
    type *,'More ? (y or n)'
    accept 55,ans
    if(ans .eq. 1hy) go to 118
    go to 80
C
C   **** Frequency Analysis Menu ****
C
120 type *,'    FREQUENCY ANALYSIS'
    type *,' '
    type *,'Enter : '
    type *,' B      for Bandwidth'
    type *,' L      to set printout Location'
    type *,' P      for Phase margin'
    type *,' R      to Return to Main menu'
    type *,' V      for a list of Values'
    accept 55,ans
    if(ans .eq. 1hb) go to 2000
    if(ans .eq. 1hl) go to 600
    if(ans .eq. 1hp) go to 1000
    if(ans .eq. 1hr) go to 50
    if(ans .eq. 1hv) go to 125
    go to 120
125 type *,'enter start f, stop f '

```

```

        accept *,f1,f2
        type *,'linear or log spacing (0 or 1) ? '
        accept *,ispc
        if (ispc .ne. 1) go to 150
        type *,'enter number of points per decade'
        accept *,npd
        spc=10.0**(1.0/npd)
C      get number of points
        np=nint(log10(f2/f1)/log10(spc))+1
        f(1)=f1
        do 130 i=2,np
        f(i)=f(i-1)*spc
130    continue
        go to 200
150    type *,'Enter number of points'
        accept *,np
        spc=(f2-f1)/float(np-1)
        do 170 i=1,np
        f(i)=f1+float(i-1)*spc
170    continue
200    type *,'calculating ...'
        do 300 i=1,np
        s=(0,6.28)*f(i)
        call loop(s,t)
        call ideal(s,ai)
        call actual(ai,t,act)
C      call actual2(s,act)
C
        call recpol(t,tm(i),tp(i))
        call recpol(ai,aim(i),aip(i))
        call recpol(act,actm(i),actp(i))
C
300    continue
C
C      Print out results
        write (iu,320)
320    format(/5x,'Frequency Response for Transimpedance Amplifier')
        write (iu,330)rin,cs
330    format(/ 5x,'Rin = ',pe10.3,5x,'Cs = ',pe10.3)
        write (iu,332)rf,cf
332    format(/ 5x,'Rf = ',pe10.3,5x,'Cf = ',pe10.3)
        write (iu,333)cin
333    format(/ 5x,'Cin = ',pe10.3)
        write (iu,334)a0,tau
334    format(/ 5x,'A0 = ',pe10.3,5x,'tau = ',pe10.3)
        write (iu,335)wz1,wz2
335    format(/ 5x,'wz1 = ',pe10.3,5x,'wz2 = ',pe10.3)

```

```

        write (iu,336)wp1,wp2
336 format(/ 5x,'wp1 = ',pe10.3,5x,'wp2 = ',pe10.3)
        write (iu,338)wp3,wp4
338 format(/ 5x,'wp3 = ',pe10.3,5x,'wp4 = ',pe10.3)
        write (iu,390)
390 format(//7x,'F',8x,'T',9x,'<T',10x,'Ai',8x,'<Ai',9x,'Aact',
c6x,'<Aact'//)
        do 500 i=1,np
            write (iu,400)f(i),tm(i),tp(i),aim(i),aip(i),actm(i),actp(i)
400 format (2x,pe10.3,2x,3(pe10.4,0pf8.2,3x))
500 continue
        go to 120
600 iu=5
        type *, 'Enter 1 to put results in file, 0 for screen'
        accept *,ans
        if (ans .eq. 1) iu=30
        go to 120
1000 type *, 'will now find the phase margin'
        type *, 'enter flo,fhi'
        accept *,flo,fhi
        call phasem(flo,fhi,pm,fco,0,ict)
        type *,ict,' iterations were required'
        type *, 'Phase margin = ',pm,' and f cross over = ',fco
        go to 120
2000 type *, 'will now find the bandwidth'
        type *, 'enter flo,fhi'
        accept *,flo,fhi
        call bandwi(flo,fhi,fbw,0,ict)
        type *,ict,' iterations were required'
        type *, 'Bandwidth = ',fbw
        go to 120
C
C      **** Noise Menu ****
C
3000 type *, 'Noise calculations'
        type *, ' '
        type *, 'Enter : '
        type *, ' L      to set printout Location'
        type *, ' R      to Return to main menu'
        type *, ' S      for Spot noise analysis'
        type *, ' T      for Total noise analysis'
        type *, ' V      for list of spot Values'
        accept 55,ans
        if(ans .eq. 1hl) go to 3020
        if(ans .eq. 1hr) go to 50
        if(ans .eq. 1hs) go to 3050
        if(ans .eq. 1ht) go to 4000

```

```

        if(ans .eq. 1hv) go to 3500
        go to 3000
3020 type *, 'Enter 1 to put results in file, 0 for screen'
        accept *, ans
        if (ans .eq. 1) iu=30
        go to 3000
3050 type *, 'enter f for noise calculation'
        accept *, fn
        call enoise(fn, se, aei, te, aecl, seo)
        call inoise(fn, si, aii, ti, aicl, sio)
        call rnoise(fn, sr, ari, tr, arcl, sro)
C      Print out results
        write(iu, 3100) fn
3100 format(/5x, 'Noise analysis for f = ', pe10.3, ' Hz')
        write(iu, 3110)
3110 format(8x, 'Source', 5x, 'Si', 10x, 'Aci', 10x, 'T', 12x, 'A', 11x, 'So')
        write(iu, 3120) se, aei, te, aecl, seo
3120 format(10x, 'E', 5(3x, pe10.4))
        write(iu, 3130) si, aii, ti, aicl, sio
3130 format(10x, 'I', 5(3x, pe10.4))
        write(iu, 3140) sr, ari, tr, arcl, sro
3140 format(10x, 'R', 5(3x, pe10.4))
        go to 3000
C
C      Calculate list of spot noise values
C
3500 type *, 'enter start f, stop f '
        accept *, f1, f2
        type *, 'linear or log spacing (0 or 1) ? '
        accept *, ispc
        if (ispc .ne. 1) go to 3550
        type *, 'enter number of points per decade'
        accept *, npd
        spc=10.0**(1.0/npd)
C      get number of points
        np=nint(log10(f2/f1)/log10(spc))+1
        f(1)=f1
        do 3530 i=2, np
        f(i)=f(i-1)*spc
3530 continue
        go to 3600
3550 type *, 'Enter number of points'
        accept *, np
        spc=(f2-f1)/float(np-1)
        do 3570 i=1, np
        f(i)=f1+float(i-1)*spc
3570 continue

```

```

C
3600 type *, 'calculating output noise...'
      do 3700 i=1,np
      call enoise(f(i),se,aei,te,aecl,sno(1,i))
      call inoise(f(i),si,aii,ti,aicl,sno(2,i))
      call rnoise(f(i),sr,ari,tr,arcl,sno(3,i))
      sno(4,i)=sno(1,i)+sno(2,i)+sno(3,i)
3700 continue
C
C   Print out results
C
      write(iu,3710)
3710 format(/5x,'Output Noise Spectra (V*V / Hz)')
      write(iu,3720)
3720 format(8x,'Frequency',5x,'Seo',10x,'Sio',10x,'Sro',10x,'So')
      do 3800 i=1,np
      write(iu,3730)f(i),sno(1,i),sno(2,i),sno(3,i),sno(4,i)
3730 format(5x,5(3x,pe10.4))
3800 continue
      go to 3000
C
C   Calculate Total Noise
C
4000 type *, 'enter start, stop freqs'
      accept *,f1,f2
      call intquad(1,f1,f2,tote,0,num1)
      rmse=1.0e6*sqrt(tote)/rf
      type *, 'enoise total = ',tote,' V**2, Ini = ',rmse,' uArms'
      call intquad(2,f1,f2,toti,0,num1)
      rmsi=1.0e6*sqrt(toti)/rf
      type *, 'inoise total = ',toti,' V**2, Ini = ',rmsi,' uArms'
      call intquad(3,f1,f2,totr,0,num1)
      rmsr=1.0e6*sqrt(totr)/rf
      type *, 'rnoise total = ',totr,' V**2, Ini = ',rmsr,' uArms'
C   get total noise due to all sources
      totrms=tote+toti+totr
C   get total equivalent input noise current
      rmstot=1.0e6*sqrt(totrms)/rf
      write(iu,4005)f1,f2
4005 format(/5x,'Output noise power between ',pe10.3,' and ',
      cpe10.3,' Hz')
      write(iu,4010)tote,rmse
4010 format(/5x,'enoise total = ',pe12.4,' V**2, Ini = ',
      cOpf12.6,' urms')
      write(iu,4020)toti,rmsi
4020 format(5x,'inoise total = ',pe12.4,' V**2, Ini = ',
      cOpf12.6,' urms')

```

```

        write(iu,4030)totr,rmsr
4030 format(5x,'noise total = ',pe12.4,' V**2, Ini = ',
        c0pf12.6,' urms'/)
        write(iu,4040)totrms,rmstot
4040 format(5x,' grand total = ',pe12.4,' V**2, Ini = ',
        c0pf12.6,' urms'/)
        go to 3000
6000 stop
        end

```

C.4 RECPOL Listing

```

        subroutine RECPOL(r,am,p)
C
C      SUBROUTINE TO CONVERT COMPLEX NUMBER INTO POLAR FORM
C      written 10/29/87 by Alan Wintenberg
C      updated 12/07/87
C
C      This subroutine converts a complex*8 number into two real
C      values am and p representing the magnitude and the phase of
C      the complex number respectively. The phase ranges from +90
C      degrees to -270 degrees.
C
C      r   = complex value      input
C      am  = magnitude          output
C      p   = phase in degrees   output
C
        complex*8 r
        x=r
        y=(0,-1.0)*r
        if(x .eq. 0.0) go to 110
        am=sqrt(x**2+y**2)
        p=atan(y/x)*57.2958
        if (x .ge. 0.0) goto 100
        p=p-180.0
100 return
110 am=abs(y)
    x=90.0
    p=sign(x,y)
    return
    end

```

C.5 IDEAL Listing

```
Subroutine IDEAL(s,ai)
C
C SUBROUTINE TO EVALUATE IDEAL CLOSED-LOOP GAIN FOR SIMPLE
C TRANSIMPEDANCE AMPLIFIER
C
C A. L. WINTENBERG      10/28/87
C   UPDATED            12/07/87
C
C s      = complex frequency
C ai     = ideal closed-loop gain
C
C COMPLEX*8 s,ai
C
C common /amp1/ a0,rf,rin,cf,cs,cin,wz1,wz2,wp1,wp2,wp3,wp4,tau
C
C a0     = amplifier dc gain
C rf     = feedback resistor
C rin    = input resistance
C cf     = feedback capacitance
C cs     = source capacitance
C cin    = input capacitance (probably in parallel with cs)
C wz1    = amplifier zero 1
C wz2    = amplifier zero 2
C wp1    = amplifier pole 1
C wp2    = amplifier pole 2
C wp3    = amplifier pole 3
C wp4    = amplifier pole 4
C tau    = delay term
C
C ai=rf/(1.0+s*rf*cf)
C return
C end
```

C.6 LOOP Listing

```
Subroutine LOOP (s,t)
C
C SUBROUTINE TO EVALUATE LOOP TRANSMISSION FOR SIMPLE
C TRANSIMPEDANCE AMPLIFIER
C
C A. L. WINTENBERG      10/27/87
```



```

C      UPDATED          12/7/87
C
C      s      = complex frequency
C      t      = loop transmission
C
C      complex*8 b1,b2,t,s
C
C      common /amp1/ a0,rf,rin,cf,cs,cin,wz1,wz2,wp1,wp2,wp3,wp4,tau
C
C      a0      = amplifier dc gain
C      rf      = feedback resistor
C      rin     = input resistance
C      cf      = feedback capacitance
C      cs      = source capacitance
C      cin     = input capacitance (probably in parallel with cs)
C      wz1     = amplifier zero 1
C      wz2     = amplifier zero 2
C      wp1     = amplifier pole 1
C      wp2     = amplifier pole 2
C      wp3     = amplifier pole 3
C      wp4     = amplifier pole 4
C      tau     = delay term
C
C      b1=rin/(1+s*(rin*(cs+cin)))
C      b2=rf/(1+s*(rf*cf))
C      t=a0*(b1/(b1+b2))
C      t=t*(1+s/wz1)*(1+s/wz2)
C      t=t/((1+s/wp1)/((1+s/wp2)/((1+s/wp3)/((1+s/wp4)
C      return
C      end

```

C.7 ACTUAL Listing

```

C      Subroutine ACTUAL(ai,t,act)
C
C      SUBROUTINE TO EVALUATE ACTUAL CLOSED-LOOP GAIN
C
C      A. L. WINTENBERG      10/28/87
C      UPDATED              10/28/87
C
C
C      ai      = ideal closed-loop gain
C      t      = loop transmission
C      act     = actual closed-loop gain
C
C

```

```

C
COMPLEX*8 t,ai,act
act=ai*(t/(1.0+t))
return
end

```

C.8 ACTUAL2 Listing

```

Subroutine ACTUAL2(s,act)
C
C SUBROUTINE TO EVALUATE ACTUAL CLOSED-LOOP GAIN ( #2 )
C
C A. L. WINTENBERG      12/07/87
C   UPDATED            12/07/87
C
C
C s      = complex frequency
C act    = actual closed-loop gain
C
C
COMPLEX*8 s,t,ai,act
call loop(s,t)
call ideal(s,ai)
act=ai*(t/(1.0+t))
return
end

```

C.9 PHASEM Listing

```

Subroutine PHASEM(flo,fhi,pm,fco,iprt,num)
C
C SUBROUTINE TO FIND PHASE MARGIN FOR SIMPLE
C   TRANSIMPEDANCE AMPLIFIER
C
C A. L. WINTENBERG      11/01/87
C   UPDATED            12/07/87
C
C
C flo = initial guess lower frequency
C fhi = initial guess upper frequency
C pm  = phase margin
C fco = crossover frequency
C iprt = print control for intermediate results

```

```

C          1 for yes, 0 for no
C  num  =  number of bisections used
C
C  COMPLEX*8 s,tlo,thi,slo,shi,snew,tnew
C
C  common /amp1/ a0,rf,rin,cf,cs,cin,wz1,wz2,wp1,wp2,wp3,wp4,tau
C
C  a0  =  amplifier dc gain
C  rf  =  feedback resistor
C  rin =  input resistance
C  cf  =  feedback capacitance
C  cs  =  source capacitance
C  cin =  input capacitance (probably in parallel with cs)
C  wz1 =  amplifier zero 1
C  wz1 =  amplifier zero 2
C  wp1 =  amplifier pole 1
C  wp2 =  amplifier pole 2
C  wp3 =  amplifier pole 3
C  wp4 =  amplifier pole 4
C  tau =  delay term
C
C  slo=(0.0,1)*(flo*6.28318)
C  shi=(0.0,1)*(fhi*6.28318)
C  i=1
50 call LOOP(slo,tlo)
   call LOOP(shi,thi)
   call recpol(tlo,tlom,tlop)
   call recpol(thi,thim,thip)
   if(iprt .ne. 1) go to 70
   type *, 'bisection # ', i
   type *, 'slo = ', slo, ' t = ', tlom, tlop
   type *, 'shi = ', shi, ' t = ', thim, thip
70  i=i+1
   if(tlom .le. 1) go to 100
   if(thim .lt. 1) go to 200
   shi=2.0*shi
   go to 50
100 slo=slo/2.0
   go to 50
200 snew=(slo+shi)/2.0
   call LOOP(snew,tnew)
   call RECPOL(tnew,tnewm,tnewp)
   if(tnewm .gt. 1.01) goto 300
   if(tnewm .ge. 0.99) goto 400
   shi=snew
   go to 50
300 slo=snew

```

```

        go to 50
400 pm=180.0+tnewp
    call recpol(snew,wco,wpc)
    fco=wco/6.28318
    num=i-1
    if(iprt .ne. 1) go to 500
    type *, 's = ',snew
    type *, 'crossover f = ',fco
    type *, 'phase margin = ',pm
500 return
    end

```

C.10 BANDWI Listing

```

Subroutine BANDWI(flo,fhi,fbw,irpt,num)
C
C SUBROUTINE TO FIND CLOSED-LOOP BANDWIDTH FOR SIMPLE
C TRANSIMPEDANCE AMPLIFIER
C
C A. L. WINTENBERG    12/07/87
C UPDATED            01/18/88
C
C flo = initial guess lower frequency
C fhi = initial guess upper frequency
C fbw = closed loop bandwidth in Hz
C irpt = print control for intermediate results
C        1 for yes, 0 for no
C num = number of bisections used
C
C COMPLEX*8 s,adc,alo,ahi,slo,shi,snew,anew
C
C common /amp1/ a0,rf,rin,cf,cs,cin,wz1,wz2,wp1,wp2,wp3,wp4,tau
C
C a0 = amplifier dc gain
C rf = feedback resistor
C rin = input resistance
C cf = feedback capacitance
C cs = source capacitance
C cin = input capacitance (probably in parallel with cs)
C wz1 = amplifier zero 1
C wz2 = amplifier zero 2
C wp1 = amplifier pole 1
C wp2 = amplifier pole 2
C wp3 = amplifier pole 3
C wp4 = amplifier pole 4

```

```

C      tau = delay term
C
      slo=(0.0,1)*(flo*6.28318)
      shi=(0.0,1)*(fhi*6.28318)
      call ACTUAL2((0.0,0.0),adc)
      call recpol(adc,adcm,adcp)
      if(iprt .ne. 1) go to 40
      type *, 'dc gain is ',adcm
40  i=1
50  call ACTUAL2(slo,alo)
      call ACTUAL2(shi,ahi)
      call recpol(alo,alom,alop)
      call recpol(ahi,ahim,ahip)
      if(iprt .ne. 1) go to 70
      type *, 'bisection # ',i
      type *, 'slo = ',slo,' gain = ',alom,alop
      type *, 'shi = ',shi,' gain = ',ahim,ahip
70  i=i+1
      if(alom .le. 0.707*adcm) go to 100
      if(ahim .lt. 0.707*adcm) go to 200
      shi=2.0*shi
      go to 50
100 slo=slo/2.0
      go to 50
200 snew=(slo+shi)/2.0
      call ACTUAL2(snew,anew)
      call RECPOL(anew,anewm,anewp)
C      value is ok when within 0.2% of -3dB
      if(anewm .gt. 0.7085*adcm) goto 300
      if(anewm .ge. 0.7057*adcm) goto 400
      shi=snew
      go to 50
300 slo=snew
      go to 50
400 call recpol(snew,fbw,fbwp)
      fbw=fbw/6.28318
      num=i-1
      if(iprt .ne. 1) go to 500
      type *, 'closed-loop bandwidth = ',fbw
500 return
      end

```

C.11 ENOISE Listing

Subroutine ENOISE(f,se,aclim,tm,actm,seo)

```

C
C SUBROUTINE TO EVALUATE OUTPUT NOISE SPECTRUM DUE TO
C EQUIVALENT INPUT NOISE VOLTAGE SOURCE S_e OF SIMPLE
C TRANSIMPEDANCE AMPLIFIER
C
C A. L. WINTENBERG      12/09/87
C   UPDATED            12/09/87
C
C
C f      = frequency (in Hz)
C se     = input noise power spectrum (V**2/Hz) due to en
C aclim  = ideal closed-loop gain for this source
C tm     = loop transmission for this source
C actm   = actual closed-loop gain for this source (V/V)
C seo    = output noise power spectrum (at f)
C
C real*4 ke1,ke2,ke3,ki1,ki2,ki3
C COMPLEX*8 s,t,b,acli,act
C
C common /amp1/ a0,rf,rin,cf,cs,cin,wz1,wz2,wp1,wp2,wp3,wp4,tau
C
C a0     = amplifier dc gain
C rf     = feedback resistor
C rin    = input resistance
C cf     = feedback capacitance
C cs     = source capacitance
C cin    = input capacitance (probably in parallel with cs)
C wz1    = amplifier zero 1
C wz2    = amplifier zero 2
C wp1    = amplifier pole 1
C wp2    = amplifier pole 2
C wp3    = amplifier pole 3
C wp4    = amplifier pole 4
C tau    = delay term
C
C common /amp2/ ke1,ke2,ke3,ki1,ki2,ki3
C ke1    = equivalent noise voltage source linear coefficient
C ke2    = equivalent noise voltage source 1/f coefficient
C ke3    = equivalent noise voltage source 1/f*f coefficient
C ki1    = equivalent noise current source linear coefficient
C ki2    = equivalent noise current source 1/f coefficient
C ki3    = equivalent noise current source 1/f*f coefficient
C
C se=ke1+ke2/f+ke3/f**2
C s=(0.0,1.0)*(6.283185*f)
C call loop(s,t)
C call recpol(t,tm,tp)

```

```

C      calculate ideal closed-loop gain for this source
      b=(1.0+s*rin*(cs+cin))/(1.0+s*rf*cf)
      acli=(1.0,0)+rf/rin*b
      call recpol(acli,aclim,aclip)
C      calculate actual closed-loop gain
      act=acli*(t/(1.0+t))
      call recpol(act,actm,actp)
      seo=actm**2*se
      return
      end

```

C.12 INOISE Listing

```

      Subroutine INOISE(f,si,aclim,tm,actm,sio)
C
C      SUBROUTINE TO EVALUATE OUTPUT NOISE SPECTRUM DUE TO
C      EQUIVALENT INPUT NOISE CURRENT SOURCE S_i OF SIMPLE
C      TRANSIMPEDANCE AMPLIFIER
C
C      A. L. WINTENBERG      12/16/87
C      UPDATED              12/16/87
C
C      f      = frequency (in Hz)
C      si     = input noise power spectrum (A**2/Hz) due to in
C      aclim  = ideal closed-loop gain for this source (V/A)
C      tm     = loop transmission for this source
C      actm   = actual closed-loop gain (mag) for this souce (V/A)
C      sio    = output noise voltage spectrum (at f) due to S_i
C
      real*4 ke1,ke2,ke3,ki1,ki2,ki3
      COMPLEX*8 s,t,b,acli,act
C
      common /amp1/ a0,rf,rin,cf,cs,cin,wz1,wz2,wp1,wp2,wp3,wp4,tau
C
C      a0     = amplifier dc gain
C      rf     = feedback resistor
C      rin    = input resistance
C      cf     = feedback capacitance
C      cs     = source capacitance
C      cin    = input capacitance (probably in parallel with cs)
C      wz1    = amplifier zero 1
C      wz2    = amplifier zero 2
C      wp1    = amplifier pole 1
C      wp2    = amplifier pole 2

```

```

C      wp3 = amplifier pole 3
C      wp4 = amplifier pole 4
C      tau = delay term
C
C      common /amp2/ ke1,ke2,ke3,ki1,ki2,ki3
C      ke1 = equivalent noise voltage source linear coefficient
C      ke2 = equivalent noise voltage source 1/f coefficient
C      ke3 = equivalent noise voltage source 1/f*f coefficient
C      ki1 = equivalent noise current source linear coefficient
C      ki2 = equivalent noise current source 1/f coefficient
C      ki3 = equivalent noise current source 1/f*f coefficient
C
C      si=ki1+ki2/f+ki3/f**2
C      s=(0.0,1.0)*(6.283185*f)
C      call loop(s,t)
C      call recpol(t,tm,tp)
C      calculate ideal closed-loop gain for this source
C      acli=rf/((1.0,0.0)+s*(rf*cf))
C      call recpol(acli,aclim,aclip)
C      calculate actual closed-loop gain
C      act=acli*(t/(1.0+t))
C      call recpol(act,actm,actp)
C      sio=actm**2*si
C      return
C      end

```

C.13 RNOISE Listing

```

C      Subroutine RNOISE(f,sr,aclim,tm,actm,sro)
C
C      SUBROUTINE TO EVALUATE OUTPUT NOISE SPECTRUM S_R DUE TO
C      FEEDBACK RESISTOR RF OF SIMPLE TRANSIMPEDANCE AMPLIFIER
C
C      A. L. WINTENBERG      12/16/87
C      UPDATED              2/28/89
C
C      f      = frequency (in Hz)
C      sr     = noise power spectrum (V**2/Hz) due to 4kTR
C      aclim  = ideal closed-loop gain for this source (V/V)
C      tm     = loop transmission for this source
C      actm   = actual closed-loop gain (mag) for this souce (V/V)
C      sro    = output noise voltage spectrum (at f) due to RF
C
C      real*4 ke1,ke2,ke3,ki1,ki2,ki3

```



```

COMPLEX*8 s,t,acli,act
C
common /amp1/ a0,rf,rin,cf,cs,cin,wz1,wz2,wp1,wp2,wp3,wp4,tau
C
C   a0  =  amplifier dc gain
C   rf  =  feedback resistor
C   rin =  input resistance
C   cf  =  feedback capacitance
C   cs  =  source capacitance
C   cin =  input capacitance (probably in parallel with cs)
C   wz1 =  amplifier zero 1
C   wz2 =  amplifier zero 2
C   wp1 =  amplifier pole 1
C   wp2 =  amplifier pole 2
C   wp3 =  amplifier pole 3
C   wp4 =  amplifier pole 4
C   tau =  delay term
C
common /amp2/ ke1,ke2,ke3,ki1,ki2,ki3
C   ke1 =  equivalent noise voltage source linear coefficient
C   ke2 =  equivalent noise voltage source 1/f coefficient
C   ke3 =  equivalent noise voltage source 1/f*f coefficient
C   ki1 =  equivalent noise current source linear coefficient
C   ki2 =  equivalent noise current source 1/f coefficient
C   ki3 =  equivalent noise current source 1/f*f coefficient
C
C
C   assume T=20 degrees celsius   S_R = 4kTR
C   sr=1.66e-20*rf
C   s=(0.0,1.0)*(6.283185*f)
C   call loop(s,t)
C   call recpol(t,tm,tp)
C   calculate ideal closed-loop gain for this source
C   acli=1.0/((1.0,0.0)+s*(rf*cf))
C   call recpol(acli,aclim,aclip)
C   calculate actual closed-loop gain
C   act=acli*(t/(1.0+t))
C   call recpol(act,actm,actp)
C   sro=actm**2*sr
C   return
C   end

```

C.14 INTQUAD Listing

```

Subroutine INTQUAD(ictrl,aa,b,app,iprt,maxlev)

```

```

C
C Subroutine to approximate integral using adaptive quadrature
C Alan Wintenberg 12/17/87
C
C ictrl = control code
C 1 = calculate noise for e source
C 2 = calculate noise for i source
C 3 = calculate noise for r source
C aa = left endpoint
C b = right endpoint
C app = value of integral
C iprt = print control 1 for print results 0 not
C maxlev = maximum level of bisection used
C
C
C dimension a(100),h(100),fa(100),fb(100)
C dimension fc(100),s(100),l(100),tol(100)
C
C n=30
C
C App=0.0
C i=1
C a(i)=aa
C h(i)=(b-aa)/2.0
C fa(i)=fff(ictrl,a)
C fc(i)=fff(ictrl,aa+h(i))
C fb(i)=fff(ictrl,b)
C make first approximation of integral
C s(i)=h(i)*(fa(i)+4*fc(i)+fb(i))/3.0
C set tolerance to 1/1000 of this initial estimate
C tol(i)=s(i)/1000.0
C l(i)=1
C maxlev=l(i)
C
C
C
C 100 if(i .le. 0) go to 9999
C fd=fff(ictrl,a(i)+h(i)/2.0)
C fe=fff(ictrl,a(i)+1.5*h(i))
C s1=h(i)*(fa(i)+4*fd+fc(i))/6.0
C s2=h(i)*(fc(i)+4*fe+fb(i))/6.0
C icount=icount+2
C Save data at this level
C v1=a(i)
C v2=fa(i)
C v3=fc(i)
C v4=fb(i)
C v5=h(i)

```

```

        v6=tol(i)
        v7=s(i)
        v8=l(i)
C      Delete the level
        i=i-1
C      See if desired accuracy was acheived
        if (abs(s1+s2-v7) .lt. v6) go to 400
        if (v8 .gt. n) go to 300
C      for neither test passed, add one level
C      right half subinterval first
        i=i+1
        a(i)=v1+v5
        fa(i)=v3
        fc(i)=fe
        fb(i)=v4
        h(i)=v5/2.0
        tol(i)=v6/2.0
        s(i)=s2
        l(i)=v8+1
        if(l(i) .gt. maxlev) maxlev=l(i)
C
C
C      left half subinterval now
        i=i+1
        a(i)=v1
        fa(i)=v2
        fc(i)=fd
        fb(i)=v3
        h(i)=h(i-1)
        tol(i)=tol(i-1)
        s(i)=s1
        l(i)=l(i-1)
C
        go to 100
300 type *, 'Maximum level exceeded'
    return
400 app=app+(s1+s2)
    go to 100
9999 if(iprt .ne. 1) go to 10000
    type *, 'integral is = ',app
    type *, 'maxium level attained = ',maxlev
10000 return
    end
C
    function fff(ictrl,x)
        y=0.0
        if(ictrl .eq. 1) call enoise(x,s1,s2,s3,s4,y)

```

```
if(ictrl .eq. 2) call inoise(x,s1,s2,s3,s4,y)
if(ictrl .eq. 3) call rnoise(x,s1,s2,s3,s4,y)
fff=y
return
end
```

APPENDIX D

VAMP ORGANIZATION, OPERATION AND LISTINGS

VAMP is a FORTRAN program which performs frequency and noise analysis of voltage-sensitive amplifiers described using the general model of Chapter 4. This appendix describes the organization and operation of VAMP and includes a listing of the code.

D.1 Organization

The organization of VAMP is very similar to and in fact is patterned after TRAMP. VAMP is subdivided into a main program, seven frequency response subroutines and seven noise computation subroutines. The main program, named VAMP, contains the user interface (input and output routines) and logic that decides and calls the subroutines needed to perform the calculations chosen by the user. The frequency response routines are adapted from the TRAMP routines that perform the same function for the transimpedance amplifier and have similar names. ACTUAL and RECPOL are exactly the same routines, and their listings are not repeated here. The frequency response rou-

tines are described in Table D.1. Similarly, the noise computation routines are adapted from the corresponding TRAMP routines, with the exceptions of RD-NOIS, R1NOIS and INOISP which simulate the noise of sources not present in the transimpedance amplifier model. The noise routines are listed in Table D.2.

Table D.1: Frequency response subroutines used by VAMP.

Name	Purpose
RECPOL	convert a complex number from rectangular form to polar form
IDEALV	calculate the ideal closed-loop gain for a single value of s and the amplifier parameters
LOOPV	calculate the loop transmission for a single value of s and the amplifier parameters
ACTUAL	calculate the actual closed-loop gain $A_{CL}(s)$ given the values of $T(s)$ and $A_{CL}^{ideal}(s)$
ACTUAL3	calculate the actual closed-loop gain $A_{CL}(s)$ by first calling LOOPV and IDEALV to get $T(s)$ and $A_{CL}^{ideal}(s)$
PHASMV	find the crossover frequency and phase margin by using LOOPV iteratively
BANDWV	find the closed-loop -3dB frequency (the bandwidth) by using ACTUAL3 iteratively

D.2 Operation

Like the routines used with TRAMP, most of the frequency response and noise computation subroutines are straightforward implementations of the equations developed for the voltage-sensitive amplifier general model. LOOPV, ACTUAL3, ENOISV, INOISV, R1NOIS, RDNOIS and RFNOIS fall into that cat-

Table D.2: Noise computation subroutines used by VAMP.

Name	Purpose
ENOISV	calculate input noise power spectral density S_e , ideal closed-loop gain, loop transmission and actual closed-loop gain for this source, and output noise power spectral density S_{noe} for a single frequency
INOISN	calculate input noise power spectral density S_i^- , ideal closed-loop gain, loop transmission and actual closed-loop gain for this source, and output noise power spectral density S_{noi-} for a single frequency
INOISP	calculate input noise power spectral density S_i^+ , ideal closed-loop gain, loop transmission and actual closed-loop gain for this source, and output noise power spectral density S_{noi+} for a single frequency
R1NOIS	calculate input noise power spectral density S_{R1} , ideal closed-loop gain, loop transmission and actual closed-loop gain for this source, and output noise power spectral density S_{noR1} for a single frequency
RDNOIS	calculate input noise power spectral density S_{RD} , ideal closed-loop gain, loop transmission and actual closed-loop gain for this source, and output noise power spectral density S_{noRD} for a single frequency
RFNOIS	calculate input noise power spectral density S_{RF} , ideal closed-loop gain, loop transmission and actual closed-loop gain for this source, and output noise power spectral density S_{noRF} for a single frequency
INTNOV2	calculate total noise power at amplifier output for source S_e , S_i^+ , S_i^- , S_{R1} , S_{RD} or S_{RF} by using ENOISV, INOISP, INOISN, R1NOIS, RDNOIS or RFNOIS to generate values of S_{noe} , S_{noi+} , S_{noi-} , S_{noR1} , S_{noRD} or S_{noRF} and integrating the noise power spectral density over the desired bandwidth

egory and will not be discussed further as their operations are described sufficiently by the model equations and the comments that are a part of each listing.

The operations of VAMP, PHASMV, BANDWV and INTNOV2 are essentially the same as those of TRAMP, PHASEM, BANDWI and INTQUAD that were described in Appendix C. VAMP has exactly the same menu structure as TRAMP and the same logic is used in all calculations. VAMP differs only in the subroutines that are called to perform a particular calculation and in the parameters gotten from the user (those needed to describe the voltage-sensitive amplifier). The only differences between PHASMV and BANDWV and their counterparts PHASEM and BANDWI are that PHASMV calls LOOPV instead of LOOP and BANDWV calls ACTUAL3 instead of ACTUAL2. INTNOV2 differs from INTQUAD only in the noise power spectral density generating subroutines that are called.

D.3 VAMP Listing

```
C  PROGRAM VAMP
C  written 01/03/89 by Alan Wintenberg (after TRAMP)
C  updated 01/03/89
C
C  VAMP must be linked with RECPOL, IDEALV, LOOPV, ACTUAL,
C  ACTUAL3, PHASMV, BANDWV, ENOISV, INOISN, INOISP, R1NOIS,
C  RDNOIS, RFNOIS and INTNOV2
C
C
C  amplifier parameters are located in common block amp1
C  amplifier parameters are located in common block amp2
C
C  COMPLEX*8 s,t,b1,b2,ai,act
C  character*6 lbl(17),nlbl(6)
C  dimension f(200),tm(200),tp(200),aim(200),aip(200),actm(200)
```



```

dimension actp(200),sno(4,200)
real*4 par(17),npar(6)
real*4 ke1,ke2,ke3,ki1,ki2,ki3
C
C   Make individual parameter names equivalent to array par
C   array par stores amplifier parameters
equivalence (par(1),a0)
equivalence (par(2),rinp)
equivalence (par(3),cinp)
equivalence (par(4),rinm)
equivalence (par(5),cinm)
equivalence (par(6),wz1)
equivalence (par(7),wz2)
equivalence (par(8),wp1)
equivalence (par(9),wp2)
equivalence (par(10),wp3)
equivalence (par(11),wp4)
equivalence (par(12),tau)
equivalence (par(13),rd)
equivalence (par(14),cs)
equivalence (par(15),r1)
equivalence (par(16),rf)
equivalence (par(17),cf)
C
C   Make noise parameter names equivalent to array npar
C   array npar stores noise parameters
equivalence (npar(1),ke1)
equivalence (npar(2),ke2)
equivalence (npar(3),ke3)
equivalence (npar(4),ki1)
equivalence (npar(5),ki2)
equivalence (npar(6),ki3)
C
C   set up amplifier parameter common block
common /amp1/ a0,rinp,cinp,rinm,cinm,wz1,wz2,wp1,wp2,wp3,wp4,tau,
crd,cs,r1,rf,cf
C
C   a0   = amplifier dc gain
C   rinp = input resistance (non-inverting terminal)
C   cinp = input capacitance (non-inverting terminal)
C   rinm = input resistance (inverting terminal)
C   cinm = input capacitance (inverting terminal)
C   wz1  = amplifier zero 1
C   wz2  = amplifier zero 2
C   wp1  = amplifier pole 1
C   wp2  = amplifier pole 2
C   wp3  = amplifier pole 3

```

```

C    wp4 = amplifier pole 4
C    tau = delay term -- not implemented
C    rd = detection resistance
C    cs = source capacitance
C    r1 = gain-setting resistor
C    rf = feedback resistor
C    cf = feedback capacitance
C
C
C    set up amplifier noise parameter common block
C    common /amp2/ ke1,ke2,ke3,ki1,ki2,ki3
C
C    ke1 = equivalent noise voltage source linear coefficient
C    ke2 = equivalent noise voltage source 1/f coefficient
C    ke3 = equivalent noise voltage source 1/f*f coefficient
C    ki1 = equivalent noise current source linear coefficient
C    ki2 = equivalent noise current source 1/f coefficient
C    ki3 = equivalent noise current source 1/f*f coefficient
C
C    Assign labels to amplifier parameters
C
C    data lbl /'a0','rinp','cinp','rinm','cinm','wz1','wz2','wp1',
c'wp2','wp3','wp4','tau','rd','cs','r1','rf','cf'/
C    data nlbl /'ke1','ke2','ke3','ki1','ki2','ki3'/
C
C    assign default parameters
30 a0=1.0e4
    rinp=1.0e9
    cinp=1.0e-12
    rinm=1.0e9
    cinm=1.0e-12
    wz1=1.0e14
    wz2=1.0e14
    wp1=628.0
    wp2=1.0e7
    wp3=1.0e14
    wp4=1.0e14
    tau=0.0
    rd=50.0
    cs=3.0e-12
    r1=100.0
    rf=9900.0
    cf=15.90e-12
    ke1=1.0e-16
    ke2=0.0
    ke3=0.0
    ki1=1.0e-22

```

```

ki2=0.0
ki3=0.0
iu=5
C
C
C
**** Main Menu ****
50 type *,' '
   type *,'   MAIN MENU'
   type *,' '
   type *,'Enter : '
   type *,' A       Amplifier parameter menu'
   type *,' F       for Frequency analysis'
   type *,' N       for Noise analysis'
   type *,' Q       to Quit'
   accept 55,ans
55 format(a1)
   if(ans .eq. 1ha) go to 80
   if(ans .eq. 1hf) go to 120
   if(ans .eq. 1hn) go to 3000
   if(ans .eq. 1hq) go to 6000
   go to 50
C
C
C
**** Set Amplifier Parameter Menu ****
80 type *,' '
   type *,'   AMPLIFIER PARAMETERS'
   type *,'Enter : '
   type *,' C       to Change all parameters'
   type *,' D       to Display current parameters'
   type *,' N       to display current Noise parameters'
   type *,' R       to Return to main menu'
   type *,' S       to Set to default values'
   accept 55,ans
   if(ans .eq. 1hc) go to 90
   if(ans .eq. 1hd) go to 100
   if(ans .eq. 1hn) go to 110
   if(ans .eq. 1hr) go to 50
   if(ans .eq. 1hs) go to 30
   go to 80
90 type *,'Enter Rinp and Cinp'
   accept *,rinp,cinp
   type *,'Enter Rinm and Cinm'
   accept *,rinm,cinm
   type *,'Enter A0 and tau'
   accept *,a0,tau
   type *,'Enter wz1 and wz2'
   accept *,wz1,wz2

```

```

type *, 'Enter wp1, wp2, wp3, and wp4'
accept *, wp1, wp2, wp3, wp4
type *, 'Enter R1, Rf and Cf'
accept *, r1, rf, cf
type *, 'Enter Rd and Cs'
accept *, rd, cs
go to 80
C Display current parameters
100 type 102, '#', 'NAME', 'VALUE', '#', 'NAME', 'VALUE'
102 format(/8x, a1, 5x, a4, 6x, a5, 11x, a1, 5x, a4, 6x, a5/)
do 106 i=1, 8
j=i+9
type 104, i, lbl(i), par(i), j, lbl(j), par(j)
104 format(7x, i2, 5x, a6, 3x, pe10.3, 6x, i2, 5x, a6, 3x, pe10.3)
106 continue
j=9
type 107, j, lbl(j), par(j)
107 format(7x, i2, 5x, a6, 3x, pe10.3/)
108 type *, 'Change # ? (0 for no change)'
accept *, i
if(i .le. 0) go to 80
if(i .gt. 17) go to 80
type *, 'Current ', lbl(i), ' value is', par(i), '      New value ?'
accept *, par(i)
type *, 'More ? (y or n)'
accept 55, ans
if(ans .eq. 1hy) go to 108
go to 80
C Display current noise parameters
110 type 112, '#', 'NAME', 'VALUE'
112 format(/8x, a1, 5x, a4, 6x, a5/)
do 116 i=1, 6
type 114, i, nlbl(i), npar(i)
114 format(7x, i2, 5x, a6, 3x, pe10.3)
116 continue
118 type *, 'Change # ? (0 for no change)'
accept *, i
if(i .le. 0) go to 80
if(i .gt. 13) go to 80
type *, 'Current ', nlbl(i), ' value is', npar(i), '      New value ?'
accept *, npar(i)
type *, 'More ? (y or n)'
accept 55, ans
if(ans .eq. 1hy) go to 118
go to 80
C
C

```

```

C      **** Frequency Analysis Menu ****
C
120 type *, '    FREQUENCY ANALYSIS'
    type *, ' '
    type *, 'Enter : '
    type *, ' B      for Bandwidth'
    type *, ' L      to set printout Location'
    type *, ' P      for Phase margin'
    type *, ' R      to Return to Main menu'
    type *, ' V      for a list of Values'
    accept 55,ans
    if(ans .eq. 1hb) go to 2000
    if(ans .eq. 1hl) go to 600
    if(ans .eq. 1hp) go to 1000
    if(ans .eq. 1hr) go to 50
    if(ans .eq. 1hv) go to 125
    go to 120
125 type *, 'enter start f, stop f '
    accept *,f1,f2
    type *, 'linear or log spacing (0 or 1) ? '
    accept *,ispc
    if (ispc .ne. 1) go to 150
    type *, 'enter number of points per decade'
    accept *,npd
    spc=10.0**(1.0/npd)
C    get number of points
    np=nint(log10(f2/f1)/log10(spc))+1
    f(1)=f1
    do 130 i=2,np
    f(i)=f(i-1)*spc
130 continue
    go to 200
150 type *, 'Enter number of points'
    accept *,np
    spc=(f2-f1)/float(np-1)
    do 170 i=1,np
    f(i)=f1+float(i-1)*spc
170 continue
200 type *, 'calculating ...'
    do 300 i=1,np
    s=(0,6.28)*f(i)
    call loopv(s,t)
    call idealv(s,ai)
    call actual(ai,t,act)
C
    call recpol(t,tm(i),tp(i))
    call recpol(ai,aim(i),aip(i))

```

```

        call recpol(act,actm(i),actp(i))
C
300 continue
C
C    Print out results
    write (iu,320)
320 format(//5x,'Frequency Response for Voltage-Sensitive Amplifier')
    write (iu,330)rinp,cinp
330 format(/ 5x,'Rinp = ',pe10.3,5x,'Cinp = ',pe10.3)
    write (iu,331)rinm,cinm
331 format(/ 5x,'Rinm = ',pe10.3,5x,'Cinm = ',pe10.3)
    write (iu,334)a0,tau
334 format(/ 5x,'A0 = ',pe10.3,5x,'tau = ',pe10.3)
    write (iu,335)wz1,wz2
335 format(/ 5x,'wz1 = ',pe10.3,5x,'wz2 = ',pe10.3)
    write (iu,336)wp1,wp2
336 format(/ 5x,'wp1 = ',pe10.3,5x,'wp2 = ',pe10.3)
    write (iu,338)wp3,wp4
338 format(/ 5x,'wp3 = ',pe10.3,5x,'wp4 = ',pe10.3)
    write (iu,340)rd,cs
340 format(/ 5x,'Rd = ',pe10.3,5x,'Cs = ',pe10.3)
    write (iu,342)rf,cf
342 format(/ 5x,'Rf = ',pe10.3,5x,'Cf = ',pe10.3)
    write (iu,344)r1
344 format(/ 5x,'R1 = ',pe10.3)
    write (iu,390)
390 format(//7x,'F',8x,'T',9x,'<T',10x,'Ai',8x,'<Ai',9x,'Aact',
c6x,'<Aact'//)
    do 500 i=1,np
        write (iu,400)f(i),tm(i),tp(i),aim(i),aip(i),actm(i),actp(i)
400 format (2x,pe10.3,2x,3(pe10.4,0pf8.2,3x))
500 continue
    go to 120
600 iu=5
    type *,'Enter 1 to put results in file, 0 for screen'
    accept *,ans
    if (ans .eq. 1) iu=30
    go to 120
1000 type *,'will now find the phase margin'
    type *,'enter flo,fhi'
    accept *,flo,fhi
    call phasem(flo,fhi,pm,fco,0,ict)
    type *,ict,' iterations were required'
    type *,'Phase margin = ',pm,' and f cross over = ',fco
    go to 120
2000 type *,'will now find the bandwidth'
    type *,'enter flo,fhi'

```

```

        accept *,flo,fhi
        call bandwv(flo,fhi,fbw,0,ict)
        type *,ict,' iterations were required'
        type *,'Bandwidth = ',fbw
        go to 120
C
C      **** Noise Menu ****
C
3000 type *,'Noise calculations'
      type *,' '
      type *,'Enter : '
      type *,' L      to set printout Location'
      type *,' R      to Return to main menu'
      type *,' S      for Spot noise analysis'
      type *,' T      for Total noise analysis'
      type *,' V      for list of spot Values'
      accept 55,ans
      if(ans .eq. 1hl) go to 3020
      if(ans .eq. 1hr) go to 50
      if(ans .eq. 1hs) go to 3050
      if(ans .eq. 1ht) go to 4000
      if(ans .eq. 1hv) go to 3500
      go to 3000
3020 type *,'Enter 1 to put results in file, 0 for screen'
      accept *,ans
      if (ans .eq. 1) iu=30
      go to 3000
3050 type *,'enter f for noise calculation'
      accept *,fn
      call enoise(fn,se,aei,te,aecl,seo)
      call inoise(fn,si,aii,ti,aicl,sio)
      call rnoise(fn,sr,ari,tr,arcl,sro)
C      Print out results
      write(iu,3100)fn
3100 format(//5x,'Noise analysis for f = ',pe10.3,' Hz'//)
      write(iu,3110)
3110 format(8x,'Source',5x,'Si',10x,'Aci',10x,'T',12x,'A',11x,'So'//)
      write(iu,3120)se,aei,te,aecl,seo
3120 format(10x,'E',5(3x,pe10.4))
      write(iu,3130)si,aii,ti,aicl,sio
3130 format(10x,'I',5(3x,pe10.4))
      write(iu,3140)sr,ari,tr,arcl,sro
3140 format(10x,'R',5(3x,pe10.4))
      go to 3000
C
C      Calculate list of spot noise values
C

```

```

3500 type *, 'enter start f, stop f '
      accept *, f1, f2
      type *, 'linear or log spacing (0 or 1) ? '
      accept *, ispc
      if (ispc .ne. 1) go to 3550
      type *, 'enter number of points per decade'
      accept *, npd
      spc=10.0**(1.0/npd)
C      get number of points
      np=nint(log10(f2/f1)/log10(spc))+1
      f(1)=f1
      do 3530 i=2,np
        f(i)=f(i-1)*spc
3530 continue
      go to 3600
3550 type *, 'Enter number of points'
      accept *, np
      spc=(f2-f1)/float(np-1)
      do 3570 i=1,np
        f(i)=f1+float(i-1)*spc
3570 continue
C
3600 type *, 'calculating output noise...'
      do 3700 i=1,np
        call enoise(f(i),se,aei,te,aecl,sno(1,i))
        call inoise(f(i),si,aii,ti,aicl,sno(2,i))
        call rnoise(f(i),sr,ari,tr,arcl,sno(3,i))
        sno(4,i)=sno(1,i)+sno(2,i)+sno(3,i)
3700 continue
C
C      Print out results
C
      write(iu,3710)
3710 format('//5x, 'Output Noise Spectra (V*V / Hz)')
      write(iu,3720)
3720 format(8x, 'Frequency', 5x, 'Seo', 10x, 'Sio', 10x, 'Sro', 10x, 'So')
      do 3800 i=1,np
        write(iu,3730)f(i),sno(1,i),sno(2,i),sno(3,i),sno(4,i)
3730 format(5x,5(3x,pe10.4))
3800 continue
      go to 3000
C
C      Calculate Total Noise
C
4000 type *, 'enter start, stop freqs'
      accept *, f1, f2
      call intquad(1,f1,f2,tote,0,num1)

```



```

rmse=1.0e6*sqrt(tote)/rf
type *, 'enoise total = ',tote,' V**2, Ini = ',rmse,' uArms'
call intquad(2,f1,f2,toti,0,num1)
rmsi=1.0e6*sqrt(toti)/rf
type *, 'inoise total = ',toti,' V**2, Ini = ',rmsi,' uArms'
call intquad(3,f1,f2,totr,0,num1)
rmsr=1.0e6*sqrt(totr)/rf
type *, 'rnoise total = ',totr,' V**2, Ini = ',rmsr,' uArms'
C   get total noise due to all sources
    totrms=tote+toti+totr
C   get total equivalent input noise current
    rmstot=1.0e6*sqrt(totrms)/rf
    write(iu,4005)f1,f2
4005 format(/5x,'Output noise power between ',pe10.3,' and ',
    cpe10.3,' Hz')
    write(iu,4010)tote,rmse
4010 format(/5x,'enoise total = ',pe12.4,' V**2, Ini = ',
    c0pf12.6,' urms')
    write(iu,4020)toti,rmsi
4020 format(5x,'inoise total = ',pe12.4,' V**2, Ini = ',
    c0pf12.6,' urms')
    write(iu,4030)totr,rmsr
4030 format(5x,'rnoise total = ',pe12.4,' V**2, Ini = ',
    c0pf12.6,' urms')
    write(iu,4040)totrms,rmstot
4040 format(5x,' grand total = ',pe12.4,' V**2, Ini = ',
    c0pf12.6,' urms')
    go to 3000
6000 stop
    end

```

D.4 IDEALV Listing

```

Subroutine IDEALV(s,ai)
C
C   SUBROUTINE TO EVALUATE IDEAL CLOSED-LOOP GAIN FOR SIMPLE
C   VOLTAGE-SENSITIVE AMPLIFIER
C
C   A. L. WINTENBERG      01/02/89
C   UPDATED              01/02/89
C
C
C   s   = complex frequency
C   ai  = ideal closed-loop gain
C

```

```

COMPLEX*8 s,ai,av,zd,zf,zx
C
common /amp1/ a0,rinp,cinp,rinm,cinm,wz1,wz2,wp1,wp2,wp3,wp4,tau,
crd,cs,r1,rf,cf
C
C    a0  =  amplifier dc gain
C    rinp = input resistance (non-inverting terminal)
C    cinp = input capacitance (non-inverting terminal)
C    rinm = input resistance (inverting terminal)
C    cinm = input capacitance (inverting terminal)
C    wz1  =  amplifier zero 1
C    wz2  =  amplifier zero 2
C    wp1  =  amplifier pole 1
C    wp2  =  amplifier pole 2
C    wp3  =  amplifier pole 3
C    wp4  =  amplifier pole 4
C    tau  =  delay term
C    rd   =  detection resistance
C    cs   =  source capacitance
C    r1   =  gain-setting resistor
C    rf   =  feedback resistor
C    cf   =  feedback capacitance
C
C    calculate actual detection impedance
    r=(rd*rinp)/(rd+rinp)
    zd=(1.0,0.0)/(1.0/r + (cinp+cs)*s)
C    calculate voltage gain of amplifier
    rx=(rinm*r1)/(rinm+r1)
    zf=rf/(1.0+s*rf*cf)
    zx=rx/(1.0+s*rx*cinm)
    av=1.0+zf/zx
C    get ideal closed-loop (current) gain
    ai=zd*(1.0+zf/zx)
    return
end

```

D.5 LOOPV Listing

```

Subroutine LOOPV(s,t)
C
C    SUBROUTINE TO EVALUATE LOOP TRANSMISSION FOR SIMPLE
C    VOLTAGE-SENSITIVE AMPLIFIER
C
C    A. L. WINTENBERG    01/02/89
C    UPDATED            01/02/89

```

```

C
C
C      s      = complex frequency
C      t      = loop transmission
C
C      complex*8 b1,b2,t,s
C
C      common /amp1/ a0,rinp,cinp,rinm,cinm,wz1,wz2,wp1,wp2,wp3,wp4,tau,
crd,cs,r1,rf,cf
C
C      a0      = amplifier dc gain
C      rinp    = input resistance (non-inverting terminal)
C      cinp    = input capacitance (non-inverting terminal)
C      rinm    = input resistance (inverting terminal)
C      cinm    = input capacitance (inverting terminal)
C      wz1     = amplifier zero 1
C      wz1     = amplifier zero 2
C      wp1     = amplifier pole 1
C      wp2     = amplifier pole 2
C      wp3     = amplifier pole 3
C      wp4     = amplifier pole 4
C      tau     = delay term
C      rd      = detection resistance
C      cs      = source capacitance
C      r1      = gain-setting resistor
C      rf      = feedback resistor
C      cf      = feedback capacitance
C
C
C      rx=(rinm*r1)/(rinm+r1)
C      b1=rx/(1+s*(rx*cinm))
C      b2=rf/(1+s*(rf*cf))
C      t=a0*(b1/(b1+b2))
C      t=t*(1+s/wz1)*(1+s/wz2)
C      t=t/(1+s/wp1)/(1+s/wp2)/(1+s/wp3)/(1+s/wp4)
C      return
C      end

```

D.6 ACTUAL3 Listing

```

Subroutine ACTUAL3(s,act)
C
C      SUBROUTINE TO EVALUATE ACTUAL CLOSED-LOOP GAIN FOR SIMPLE
C      VOLTAGE-SENSITIVE AMPLIFIER
C

```

```

C      A. L. WINTENBERG      01/03/89
C      UPDATED              01/03/89
C
C      s      = complex frequency
C      ai     = idela closed-loop gain
C      t      = loop transmission
C      act    = actual closed-loop gain
C
C      COMPLEX*8 s,ai,t,act
C
C      get ideal closed-loop gain
C      call idealv(s,ai)
C      get loop transmission
C      call loopv(s,t)
C      combine Ai(s) and T(s) to get actual gain
C      call actual(ai,t,act)
C
C      return
C      end

```

D.7 PHASMV Listing

```

      Subroutine PHASMV(flo,fhi,pm,fco,iprt,num)
C
C      SUBROUTINE TO FIND PHASE MARGIN FOR SIMPLE
C      VOLTAGE-SENSITIVE AMPLIFIER
C
C      A. L. WINTENBERG      01/03/89 (after PHASEM)
C      UPDATED              01/03/89
C
C      flo = initial guess lower frequency
C      fhi = initial guess upper frequency
C      pm  = phase margin
C      fco = crossover frequency
C      iprt = print control for intermediate results
C             1 for yes, 0 for no
C      num = number of bisections used
C
C      COMPLEX*8 s,tlo,thi,slo,shi,snew,tnew
C
C
C      slo=(0.0,1)*(flo*6.28318)
C      shi=(0.0,1)*(fhi*6.28318)
C      i=1

```

```

50 call LOOPV(slo,tlo)
   call LOOPV(shi,thi)
   call recpol(tlo,tlom,tlop)
   call recpol(thi,thim,thip)
   if(iprt .ne. 1) go to 70
   type *, 'bisection # ', i
   type *, 'slo = ', slo, ' t = ', tlo, tlop
   type *, 'shi = ', shi, ' t = ', thim, thip
70 i=i+1
   if(tlom .le. 1) go to 100
   if(thim .lt. 1) go to 200
   shi=2.0*shi
   go to 50
100 slo=slo/2.0
   go to 50
200 snew=(slo+shi)/2.0
   call LOOPV(snew,tnew)
   call RECPOL(tnew,tnewm,tnewp)
   if(tnewm .gt. 1.01) goto 300
   if(tnewm .ge. 0.99) goto 400
   shi=snew
   go to 50
300 slo=snew
   go to 50
400 pm=180.0+tnewp
   call recpol(snew,wco,wpc)
   fco=wco/6.28318
   num=i-1
   if(iprt .ne. 1) go to 500
   type *, 's = ', snew
   type *, 'crossover f = ', fco
   type *, 'phase margin = ', pm
500 return
   end

```

D.8 BANDWV Listing

```

Subroutine BANDWV(flo,fhi,fbw,irpt,num)
C
C SUBROUTINE TO FIND CLOSED-LOOP BANDWIDTH FOR SIMPLE
C VOLTAGE-SENSITIVE AMPLIFIER
C
C A. L. WINTENBERG 01/03/89 (after BANDWI)
C UPDATED 01/03/89
C

```

```

C   flo = initial guess lower frequency
C   fhi = initial guess upper frequency
C   fbw = closed loop bandwidth in Hz
C   irpt = print control for intermediate results
C           1 for yes, 0 for no
C   num = number of bisections used
C
C   COMPLEX*8 s,adc,alo,ahi,slo,shi,snew,anew
C
C
C   slo=(0.0,1)*(flo*6.28318)
C   shi=(0.0,1)*(fhi*6.28318)
C   call ACTUAL3((0.0,0.0),adc)
C   call recpol(adc,adcm,adcp)
C   if(iprt .ne. 1) go to 40
C   type *, 'dc gain is ',adcm
40  i=1
50  call ACTUAL3(slo,alo)
C   call ACTUAL3(shi,ahi)
C   call recpol(alo,alom,alop)
C   call recpol(ahi,ahim,ahip)
C   if(iprt .ne. 1) go to 70
C   type *, 'bisection # ',i
C   type *, 'slo = ',slo,' gain = ',alom,alop
C   type *, 'shi = ',shi,' gain = ',ahim,ahip
70  i=i+1
C   if(alom .le. 0.707*adcm) go to 100
C   if(ahim .lt. 0.707*adcm) go to 200
C   shi=2.0*shi
C   go to 50
100 slo=slo/2.0
C   go to 50
200 snew=(slo+shi)/2.0
C   call ACTUAL3(snew,anew)
C   call RECPOL(anew,anewm,anewp)
C   value is ok when within 0.2% of -3dB
C   if(anewm .gt. 0.7085*adcm) goto 300
C   if(anewm .ge. 0.7057*adcm) goto 400
C   shi=snew
C   go to 50
300 slo=snew
C   go to 50
400 call recpol(snew,fbw,fbwp)
C   fbw=fbw/6.28318
C   num=i-1
C   if(iprt .ne. 1) go to 500
C   type *, 'closed-loop bandwidth = ',fbw

```

```

500 return
end

```

D.9 ENOISV Listing

```

Subroutine ENOISV(f,se,aclim,tm,actm,seo)
C
C SUBROUTINE TO EVALUATE OUTPUT NOISE SPECTRUM DUE TO
C EQUIVALENT INPUT NOISE VOLTAGE SOURCE Se OF SIMPLE
C VOLTAGE-SENSITIVE AMPLIFIER
C
C A. L. WINTENBERG      01/03/89  (after ENOISE)
C   UPDATED            01/03/89
C
C f      = frequency (in Hz)
C se     = input noise power spectrum (V**2/Hz) due to en
C aclim  = ideal closed-loop gain for this source
C tm     = loop transmission for this source
C actm   = actual closed-loop gain for this souce (V/V)
C seo    = output noise power spectrum (at f)
C
C real*4 ke1,ke2,ke3,ki1,ki2,ki3
C COMPLEX*8 s,t,za,zd,acli,act
C
C common /amp1/ a0,rinp,cinp,rinm,cinm,wz1,wz2,wp1,wp2,wp3,wp4,tau,
C crd,cs,r1,rf,cf
C
C a0    = amplifier dc gain
C rinp  = input resistance (non-inverting terminal)
C cinp  = input capacitance (non-inverting terminal)
C rinm  = input resistance (inverting terminal)
C cinm  = input capacitance (inverting terminal)
C wz1   = amplifier zero 1
C wz2   = amplifier zero 2
C wp1   = amplifier pole 1
C wp2   = amplifier pole 2
C wp3   = amplifier pole 3
C wp4   = amplifier pole 4
C tau   = delay term
C rd    = detection resistance
C cs    = source capacitance
C r1    = gain-setting resistor
C rf    = feedback resistor

```

```

C      cf  =  feedback capacitance
C
C      common /amp2/ ke1,ke2,ke3,ki1,ki2,ki3,ki4,ki5,ki6
C      ke1 = equivalent noise voltage source linear coefficient
C      ke2 = equivalent noise voltage source 1/f coefficient
C      ke3 = equivalent noise voltage source 1/f*f coefficient
C      ki1 = equivalent noise current source (+) linear coefficient
C      ki2 = equivalent noise current source (+) 1/f coefficient
C      ki3 = equivalent noise current source (+) 1/f*f coefficient
C      ki4 = equivalent noise current source (-) linear coefficient
C      ki5 = equivalent noise current source (-) 1/f coefficient
C      ki6 = equivalent noise current source (-) 1/f*f coefficient
C
C      se=ke1+ke2/f+ke3/f**2
C      s=(0.0,1.0)*(6.283185*f)
C      call loopv(s,t)
C      call recpol(t,tm,tp)
C      calculate ideal closed-loop gain for this source
C      rx=(rinm*r1)/(rinm+r1)
C      za=rinp/(1.0+s*(rinp*cinp))
C      zd=rd/(1.0+s*(rd*cs))
C      acli=(1.0+rf*(1.0/rx+cinm*s))/(1.0+s*(rf*cf))*(za/(za+zd))
C      call recpol(acli,aclim,aclip)
C      calculate actual closed-loop gain
C      act=acli*(t/(1.0+t))
C      call recpol(act,actm,actp)
C      seo=actm**2*se
C      return
C      end

```

D.10 INOISN Listing

```

      Subroutine INOISN(f,si,aclim,tm,actm,sio)
C
C      SUBROUTINE TO EVALUATE OUTPUT NOISE SPECTRUM DUE TO
C      EQUIVALENT INPUT NOISE CURRENT SOURCE S_in OF SIMPLE
C      VOLTAGE-SENSITIVE AMPLIFIER (inverting input)
C
C      A. L. WINTENBERG      01/24/89  (after INOISE)
C      UPDATED              01/24/89
C
C
C      f      =  frequency (in Hz)
C      si     =  input noise power spectrum (A**2/Hz) due to in
C      aclim  =  ideal closed-loop gain for this source  (V/A)

```



```

C      tm  =  loop transmission for this source
C      actm =  actual closed-loop gain (mag) for this souce (V/A)
C      sio  =  output noise voltage spectrum (at f) due to S_i
C
C      real*4 ke1,ke2,ke3,ki1,ki2,ki3,ki4,ki5,ki6
C      COMPLEX*8 s,aol,za,zb,zf,acli,act
C
C      common /amp1/ a0,rinp,cinp,rinm,cinm,wz1,wz2,wp1,wp2,wp3,wp4,tau,
C      crd,cs,r1,rf,cf
C
C      a0  =  amplifier dc gain
C      rinp =  input resistance (non-inverting terminal)
C      cinp =  input capacitance (non-inverting terminal)
C      rinm =  input resistance (inverting terminal)
C      cinm =  input capacitance (inverting terminal)
C      wz1  =  amplifier zero 1
C      wz1  =  amplifier zero 2
C      wp1  =  amplifier pole 1
C      wp2  =  amplifier pole 2
C      wp3  =  amplifier pole 3
C      wp4  =  amplifier pole 4
C      tau  =  delay term
C      rd   =  detection resistance
C      cs   =  source capacitance
C      r1   =  gain-setting resistor
C      rf   =  feedback resistor
C      cf   =  feedback capacitance
C
C      common /amp2/ ke1,ke2,ke3,ki1,ki2,ki3,ki4,ki5,ki6
C      ke1  =  equivalent noise voltage source linear coefficient
C      ke2  =  equivalent noise voltage source 1/f coefficient
C      ke3  =  equivalent noise voltage source 1/f*f coefficient
C      ki1  =  equivalent noise current source (+) linear coefficient
C      ki2  =  equivalent noise current source (+) 1/f coefficient
C      ki3  =  equivalent noise current source (+) 1/f*f coefficient
C      ki4  =  equivalent noise current source (-) linear coefficient
C      ki5  =  equivalent noise current source (-) 1/f coefficient
C      ki6  =  equivalent noise current source (-) 1/f*f coefficient
C
C      si=ki4+ki5/f+ki6/f**2
C      s=(0.0,1.0)*(6.283185*f)
C      call loopv(s,t)
C      call recpol(t,tm,tp)
C      calculate ideal closed-loop gain for this source
C      acli=rf/((1.0,0.0)+s*(rf*cf))
C      call recpol(acli,aclim,aclip)
C      calculate actual closed-loop gain

```

```

act=acli*(t/(1.0+t))
call recpol(act,actm,actp)
sio=actm**2*si
return
end

```

D.11 INOISP Listing

```

Subroutine INOISP(f,si,aclim,tm,actm,sio)
C
C SUBROUTINE TO EVALUATE OUTPUT NOISE SPECTRUM DUE TO
C EQUIVALENT INPUT NOISE CURRENT SOURCE S_ip OF SIMPLE
C VOLTAGE-SENSITIVE AMPLIFIER (non-inverting input)
C
C A. L. WINTENBERG      01/24/89 (after INOISE)
C   UPDATED            01/24/89
C
C
C f      = frequency (in Hz)
C si     = input noise power spectrum (A**2/Hz) due to in
C aclim  = ideal closed-loop gain for this source (V/A)
C tm     = loop transmission for this source
C actm   = actual closed-loop gain (mag) for this souce (V/A)
C sio    = output noise voltage spectrum (at f) due to S_ip
C
C real*4 ke1,ke2,ke3,ki1,ki2,ki3,ki4,ki5,ki6
C COMPLEX*8 s,aol,za,zb,zf,acli,act
C
C common /amp1/ a0,rinp,cinp,rinm,cinm,wz1,wz2,wp1,wp2,wp3,wp4,tau,
C crd,cs,r1,rf,cf
C
C a0    = amplifier dc gain
C rinp  = input resistance (non-inverting terminal)
C cinp  = input capacitance (non-inverting terminal)
C rinm  = input resistance (inverting terminal)
C cinm  = input capacitance (inverting terminal)
C wz1   = amplifier zero 1
C wz2   = amplifier zero 2
C wp1   = amplifier pole 1
C wp2   = amplifier pole 2
C wp3   = amplifier pole 3
C wp4   = amplifier pole 4
C tau   = delay term
C rd    = detection resistance
C cs    = source capacitance

```

```

C      r1  =  gain-setting resistor
C      rf  =  feedback resistor
C      cf  =  feedback capacitance
C
C      common /amp2/ ke1,ke2,ke3,ki1,ki2,ki3,ki4,ki5,ki6
C      ke1 =  equivalent noise voltage source linear coefficient
C      ke2 =  equivalent noise voltage source 1/f coefficient
C      ke3 =  equivalent noise voltage source 1/f*f coefficient
C      ki1 =  equivalent noise current source (+) linear coefficient
C      ki2 =  equivalent noise current source (+) 1/f coefficient
C      ki3 =  equivalent noise current source (+) 1/f*f coefficient
C      ki4 =  equivalent noise current source (-) linear coefficient
C      ki5 =  equivalent noise current source (-) 1/f coefficient
C      ki6 =  equivalent noise current source (-) 1/f*f coefficient
C
C      si=ki1+ki2/f+ki3/f**2
C      s=(0.0,1.0)*(6.283185*f)
C      call loopv(s,t)
C      call recpol(t,tm,tp)
C      calculate ideal closed-loop gain for this source
C      the gain is the same as for the signal source, so use that routine
C      call idealv(s,acli)
C      call recpol(acli,aclim,aclip)
C      calculate actual closed-loop gain
C      act=acli*(t/(1.0+t))
C      call recpol(act,actm,actp)
C      sio=actm**2*si
C      return
C      end

```

D.12 R1NOIS Listing

```

      Subroutine R1NOIS(f,sr,aclim,tm,actm,sro)
C
C      SUBROUTINE TO EVALUATE OUTPUT NOISE SPECTRUM S_R1 DUE TO
C      GAIN-SETTING RESISTOR R1 OF SIMPLE VOLTAGE-SENSITIVE AMPLIFIER
C
C      A. L. WINTENBERG      01/05/89  (after RNOISE)
C      UPDATED              01/05/89
C
C      f      =  frequency (in Hz)
C      sr     =  noise power spectrum (V**2/Hz) due to 4kTR
C      aclim  =  ideal closed-loop gain for this source  (V/V)
C      tm     =  loop transmission for this source
C      actm   =  actual closed-loop gain (mag) for this souce  (V/V)

```

```

C      sro = output noise voltage spectrum (at f) due to R1
C
C      real*4 ke1,ke2,ke3,ki1,ki2,ki3
C      COMPLEX*8 s,t,acli,act
C
C      common /amp1/ a0,rinp,cinp,rinm,cinm,wz1,wz2,wp1,wp2,wp3,wp4,tau,
C      crd,cs,r1,rf,cf
C
C      a0 = amplifier dc gain
C      rinp = input resistance (non-inverting terminal)
C      cinp = input capacitance (non-inverting terminal)
C      rinm = input resistance (inverting terminal)
C      cinm = input capacitance (inverting terminal)
C      wz1 = amplifier zero 1
C      wz2 = amplifier zero 2
C      wp1 = amplifier pole 1
C      wp2 = amplifier pole 2
C      wp3 = amplifier pole 3
C      wp4 = amplifier pole 4
C      tau = delay term
C      rd = detection resistance
C      cs = source capacitance
C      r1 = gain-setting resistor
C      rf = feedback resistor
C      cf = feedback capacitance
C
C      common /amp2/ ke1,ke2,ke3,ki1,ki2,ki3,ki4,ki5,ki6
C      ke1 = equivalent noise voltage source linear coefficient
C      ke2 = equivalent noise voltage source 1/f coefficient
C      ke3 = equivalent noise voltage source 1/f*f coefficient
C      ki1 = equivalent noise current source (+) linear coefficient
C      ki2 = equivalent noise current source (+) 1/f coefficient
C      ki3 = equivalent noise current source (+) 1/f*f coefficient
C      ki4 = equivalent noise current source (-) linear coefficient
C      ki5 = equivalent noise current source (-) 1/f coefficient
C      ki6 = equivalent noise current source (-) 1/f*f coefficient
C
C
C      assume T=20 degrees celsius  S_R = 4kTR
C      sr=1.66e-20*r1
C      s=(0.0,1.0)*(6.283185*f)
C      call loopv(s,t)
C      call recpol(t,tm,tp)
C      calculate ideal closed-loop gain for this source
C      acli=(RF/R1)/((1.0,0.0)+s*(rf*cf))
C      call recpol(acli,aclim,aclip)
C      calculate actual closed-loop gain

```

```

act=acli*(t/(1.0+t))
call recpol(act,actm,actp)
sro=actm**2*sr
return
end

```

D.13 RDNOIS Listing

```

C
C      Subroutine RDNOIS(f,sr,aclim,tm,actm,sro)
C
C      SUBROUTINE TO EVALUATE OUTPUT NOISE SPECTRUM S_RD DUE TO
C      DETECTION RESISTOR RD OF SIMPLE VOLTAGE-SENSITIVE AMPLIFIER
C
C      A. L. WINTENBERG      01/05/89  (after RNOISE)
C      UPDATED              01/05/89
C
C      f      = frequency (in Hz)
C      sr      = noise power spectrum (V**2/Hz) due to 4kTR
C      aclim = ideal closed-loop gain for this source (V/V)
C      tm      = loop transmission for this source
C      actm = actual closed-loop gain (mag) for this souce (V/V)
C      sro = output noise voltage spectrum (at f) due to RD
C
C      real*4 ke1,ke2,ke3,ki1,ki2,ki3
C      COMPLEX*8 s,t,acli,act,za
C
C      common /amp1/ a0,rinp,cinp,rinm,cinm,wz1,wz2,wp1,wp2,wp3,wp4,tau,
C      crd,cs,r1,rf,cf
C
C      a0 = amplifier dc gain
C      rinp = input resistance (non-inverting terminal)
C      cinp = input capacitance (non-inverting terminal)
C      rinm = input resistance (inverting terminal)
C      cinm = input capacitance (inverting terminal)
C      wz1 = amplifier zero 1
C      wz1 = amplifier zero 2
C      wp1 = amplifier pole 1
C      wp2 = amplifier pole 2
C      wp3 = amplifier pole 3
C      wp4 = amplifier pole 4
C      tau = delay term
C      rd = detection resistance
C      cs = source capacitance
C      r1 = gain-setting resistor
C      rf = feedback resistor

```

```

C      cf      = feedback capacitance
C
C      common /amp2/ ke1,ke2,ke3,ki1,ki2,ki3,ki4,ki5,ki6
C      ke1 = equivalent noise voltage source linear coefficient
C      ke2 = equivalent noise voltage source 1/f coefficient
C      ke3 = equivalent noise voltage source 1/f*f coefficient
C      ki1 = equivalent noise current source (+) linear coefficient
C      ki2 = equivalent noise current source (+) 1/f coefficient
C      ki3 = equivalent noise current source (+) 1/f*f coefficient
C      ki4 = equivalent noise current source (-) linear coefficient
C      ki5 = equivalent noise current source (-) 1/f coefficient
C      ki6 = equivalent noise current source (-) 1/f*f coefficient
C
C
C      assume T=20 degrees celsius  S_R = 4kTR
C      sr=1.66e-20*rd
C      s=(0.0,1.0)*(6.283185*f)
C      call loopv(s,t)
C      call recpol(t,tm,tp)
C      calculate ideal closed-loop gain for this source
C      za=rinp/(1.0+s*(rinp*(cinp+cs)))
C      acli=(1.0+(RF/R1)/((1.0,0.0)+s*(rf*cf)))*(za/(rd+za))
C      call recpol(acli,aclim,aclip)
C      calculate actual closed-loop gain
C      act=acli*(t/(1.0+t))
C      call recpol(act,actm,actp)
C      sro=actm**2*sr
C      return
C      end

```

D.14 RFNOIS Listing

```

      Subroutine RFNOIS(f,sr,aclim,tm,actm,sro)
C
C      SUBROUTINE TO EVALUATE OUTPUT NOISE SPECTRUM S_RF DUE TO
C      FEEDBACK RESISTOR RF OF SIMPLE VOLTAGE-SENSITIVE AMPLIFIER
C
C      A. L. WINTENBERG      01/03/89  (after RNOISE)
C      UPDATED              01/03/89
C
C      f      = frequency (in Hz)
C      sr      = noise power spectrum (V**2/Hz) due to 4kTR
C      aclim = ideal closed-loop gain for this source  (V/V)
C      tm      = loop transmission for this source
C      actm = actual closed-loop gain (mag) for this souce  (V/V)

```

```

C      sro = output noise voltage spectrum (at f) due to RF
C
C      real*4 ke1,ke2,ke3,ki1,ki2,ki3
C      COMPLEX*8 s,t,acli,act
C
C      common /amp1/ a0,rinp,cinp,rinm,cinm,wz1,wz2,wp1,wp2,wp3,wp4,tau,
C      crd,cs,r1,rf,cf
C
C      a0 = amplifier dc gain
C      rinp = input resistance (non-inverting terminal)
C      cinp = input capacitance (non-inverting terminal)
C      rinm = input resistance (inverting terminal)
C      cinm = input capacitance (inverting terminal)
C      wz1 = amplifier zero 1
C      wz2 = amplifier zero 2
C      wp1 = amplifier pole 1
C      wp2 = amplifier pole 2
C      wp3 = amplifier pole 3
C      wp4 = amplifier pole 4
C      tau = delay term
C      rd = detection resistance
C      cs = source capacitance
C      r1 = gain-setting resistor
C      rf = feedback resistor
C      cf = feedback capacitance
C
C      common /amp2/ ke1,ke2,ke3,ki1,ki2,ki3,ki4,ki5,ki6
C      ke1 = equivalent noise voltage source linear coefficient
C      ke2 = equivalent noise voltage source 1/f coefficient
C      ke3 = equivalent noise voltage source 1/f*f coefficient
C      ki1 = equivalent noise current source (+) linear coefficient
C      ki2 = equivalent noise current source (+) 1/f coefficient
C      ki3 = equivalent noise current source (+) 1/f*f coefficient
C      ki4 = equivalent noise current source (-) linear coefficient
C      ki5 = equivalent noise current source (-) 1/f coefficient
C      ki6 = equivalent noise current source (-) 1/f*f coefficient
C
C
C      assume T=20 degrees celsius  S_R = 4kTR
C      sr=1.66e-20*rf
C      s=(0.0,1.0)*(6.283185*f)
C      call loopv(s,t)
C      call recpol(t,tm,tp)
C      calculate ideal closed-loop gain for this source
C      acli=1.0/((1.0,0.0)+s*(rf*cf))
C      call recpol(acli,aclim,aclip)
C      calculate actual closed-loop gain

```

```

act=acli*(t/(1.0+t))
call recpol(act,actm,actp)
sro=actm**2*sr
return
end

```

D.15 INTNOV2 Listing

```

Subroutine INTNOV2(ictrl,aa,b,app,iprt,maxlev)
C
C Subroutine to approximate integral using adaptive quadrature
C Alan Wintenberg 01/24/89 (after INTQUAD, basically the same)
C
C ictrl = control code
C 1 = calculate noise for e source
C 2 = calculate noise for ip source
C 3 = calculate noise for in source
C 4 = calculate noise for rd source
C 5 = calculate noise for rf source
C 6 = calculate noise for r1 source
C aa = left endpoint
C b = right endpoint
C app = value of integral
C iprt = print control 1 for print results 0 not
C maxlev = maximum level of bisection used
C
C
C dimension a(100),h(100),fa(100),fb(100)
C dimension fc(100),s(100),l(100),tol(100)
C
C n=30
C
C App=0.0
C i=1
C a(i)=aa
C h(i)=(b-aa)/2.0
C fa(i)=fff(ictrl,a)
C fc(i)=fff(ictrl,aa+h(i))
C fb(i)=fff(ictrl,b)
C make first approximation of integral
C s(i)=h(i)*(fa(i)+4*fc(i)+fb(i))/3.0
C set tolerance to 1/1000 of this initial estimate
C tol(i)=s(i)/1000.0
C l(i)=1
C maxlev=l(i)

```



```

C
C
100 if(i .le. 0) go to 9999
    fd=fff(ictrl,a(i)+h(i)/2.0)
    fe=fff(ictrl,a(i)+1.5*h(i))
    s1=h(i)*(fa(i)+4*fd+fc(i))/6.0
    s2=h(i)*(fc(i)+4*fe+fb(i))/6.0
    icount=icount+2
C    Save data at this level
    v1=a(i)
    v2=fa(i)
    v3=fc(i)
    v4=fb(i)
    v5=h(i)
    v6=tol(i)
    v7=s(i)
    v8=l(i)
C    Delete the level
    i=i-1
C    See if desired accuracy was acheived
    if (abs(s1+s2-v7) .lt. v6) go to 400
    if (v8 .gt. n) go to 300
C    for neither test passed, add one level
C    right half subinterval first
    i=i+1
    a(i)=v1+v5
    fa(i)=v3
    fc(i)=fe
    fb(i)=v4
    h(i)=v5/2.0
    tol(i)=v6/2.0
    s(i)=s2
    l(i)=v8+1
    if(l(i) .gt. maxlev) maxlev=l(i)
C
C
C    left half subinterval now
    i=i+1
    a(i)=v1
    fa(i)=v2
    fc(i)=fd
    fb(i)=v3
    h(i)=h(i-1)
    tol(i)=tol(i-1)
    s(i)=s1
    l(i)=l(i-1)
C

```

```

        go to 100
300 type *, 'Maximum level exceeded'
    return
400 app=app+(s1+s2)
    go to 100
9999 if(iprt .ne. 1) go to 10000
    type *, 'integral is = ', app
    type *, 'maxium level attained = ', maxlev
10000 return
    end
C
function fff(ictrl,x)
y=0.0
if(ictrl .eq. 1) call enoisv(x,s1,s2,s3,s4,y)
if(ictrl .eq. 2) call inoisp(x,s1,s2,s3,s4,y)
if(ictrl .eq. 3) call inoism(x,s1,s2,s3,s4,y)
if(ictrl .eq. 4) call rdnois(x,s1,s2,s3,s4,y)
if(ictrl .eq. 5) call rfnois(x,s1,s2,s3,s4,y)
if(ictrl .eq. 6) call rinois(x,s1,s2,s3,s4,y)
fff=y
return
end

```

APPENDIX E

PREAMPLIFIER SPICE LISTING

This appendix contains the SPICE listing for the transimpedance preamplifier. The dual gate GaAs MESFET is modeled using a subcircuit consisting of two cascoded FETs. The preamplifier actually has four of the devices connected in parallel, so to simplify the SPICE model, an area factor of 4.0 is specified for the FETs making up the subcircuit.

```
CURRENT PREAMPLIFIER --- CA1.6 DESIGN
*   CURRENT AMPLIFIER CA1.6
*   CREATED 5/9/89
*   MODIFIED 26JUNE89 TO USE IMPROVED GAASFET MODEL
*   MODIFIED 4SEPTEMBER89 FOR SINGLE FB RESISTOR
*   WITH IDSS = 85MA AND VP = -3.55V
*   CLOSED-LOOP ANALYSIS
*
*   DUAL GATE GAASFET SUBCIRCUIT -- ACTUALLY 4 IN PARALLEL
*
.SUBCKT MRF966 1 2 4 5
*   NODES: GATE1, SOURCE1, GATE2 AND DRAIN2
J1 3 1 2 JFET1 4.0
J2 5 4 3 JFET1 4.0
*
*   MODEL FOR INDIVIDUAL FET
*
.MODEL JFET1 NJF VTO=-3.55 BETA=0.00675 LAMBDA=.02 CGS=1.6PF CGD=1.6PF
+KF=.44P
.ENDS MRF966
*
*
*   MAIN CIRCUIT
*
*   SUPPLY VOLTAGES
*
```

```

VCC 11 0 12
VDD 10 0 -12
*
* SIGNAL SOURCE
*
VS 0 99 DC 0
IS 99 1 AC 1 PULSE(0 100UA 5NS 1NS 1NS 40NS)
CS 1 0 6PF
*
* PREAMPLIFIER
*
X1 1 2 2 3 MRF966
RD 11 3 100
RS 2 0 54.9
CJ1 2 0 0.1UF
Q1 5 4 3 MPSH81
R1 4 11 15K
R2 4 0 10K
CQ1 4 0 .10UF
RC1 5 10 1.1K
CSTRAY1 5 0 0.5PF
Q2 11 5 6 MPSH10
RE2 6 10 1K
RLOAD 6 0 1K
*
* FEEDBACK NETWORK
*
RF 6 1 5.11K
D1 6 20 D1N4148
D2 20 21 D1N4148
D3 21 22 D1N4148
D4 22 23 D1N4148
D5 23 24 D1N4148
D6 24 1 D1N4148
D7 1 25 D1N4148
D8 25 26 D1N4148
D9 26 27 D1N4148
D10 27 28 D1N4148
D11 28 29 D1N4148
D12 29 6 D1N4148
*
* MODEL FOR DIODE
*
.MODEL D1N4148 D IS=1E-8 RS=5 N=2 TT=4NS CJO=1PF BV=100
*
* MODELS FOR TRANSISTORS
*

```

```

.MODEL MPSH81 PNP BF=74 BR=2.5 IS=1.6E-15 RB=14 RC=35 RE=1.2 VA=60 VB=15
+IK=100M C2=900 NE=1.8 IKR=15M C4=331 NC=3.2 TF=1.2E-10 TR=5NS CJE=2.1PF
+PE=0.9 ME=0.33 CJC=3.3PF PC=0.7 MC=0.7
*
.MODEL MPSH10 NPN BF=83 BR=2.5 IS=4.4E-16 RB=60 RC=4 RE=1 VA=70 VB=30
+IK=160M C2=340 NE=1.7 IKR=30M C4=70 NC=2.2 TF=1.55E-10 TR=10NS
CJE=1.8PF
+PE=0.9 ME=0.33 CJC=1.25PF PC=0.8 MC=0.26
*
* ANALYSIS
*
.WIDTH IN=80 OUT=80
.NODESET V(1)=0
.NOISE V(6) IS 10
.AC DEC 10 10K 1000MEG
.TRAN 1NS 65NS
.PRINT AC VM(5) VP(5) VM(6) VP(6)
.PRINT NOISE ONOISE INOISE
.PRINT TRAN I(VS) V(6)
.END

```

APPENDIX F

INTERPOLATION PROOF

When interpolation is used to double the effective sampling rate, a zero is added between each pair of data points. The only effect this addition has on the frequency spectrum is a factor of two compression in terms of Ω (the digital frequency). A low pass filter is then used to remove the spurious parts of the spectrum and recover the original.

The nature of the distortion in the frequency domain is not at all obvious even though the time domain waveform is easily pictured with the zeroes added. A proof that the only effect is the frequency compression follows.

If $x(n)$ is a discrete time signal, then its DFT, $X(j\Omega)$, is given by

$$X(j\Omega) = \sum_{n=-\infty}^{\infty} x(n) \exp^{-j\Omega n}, \quad (6.54)$$

where Ω is the normalized digital frequency in radians. When $x(n)$ is modified by the addition of a zero between each pair of samples, the new signal is denoted by $x'(n)$ and is given by $x'(n) = x(n/2)$ when n is even and by $x'(n) = 0$ when n is odd. The DFT of this modified signal is

$$X'(j\Omega) = \sum_{n=-\infty}^{\infty} x'(n) \exp^{-j\Omega n}. \quad (6.55)$$

This can be separated into two parts with one summation for even index

values and one for the negative ones:

$$X'(j\Omega) = \sum_{\substack{n=-\infty \\ (n \text{ even})}}^{\infty} x'(n) \exp^{-j\Omega n} + \sum_{\substack{n=-\infty \\ (n \text{ odd})}}^{\infty} x'(n) \exp^{-j\Omega n}. \quad (6.56)$$

Since $x'(n) = x(n/2)$ when n is even and $x'(n) = 0$ when n is odd, Eq. 6.56 reduces to

$$X'(j\Omega) = \sum_{\substack{n=-\infty \\ (n \text{ even})}}^{\infty} x(n/2) \exp^{-j\Omega n}. \quad (6.57)$$

Letting $m = n/2$ and $\Theta = 2\Omega$ and substituting into Eq. 6.57 gives

$$X'(j\Theta/2) = \sum_{m=-\infty}^{\infty} x(m) \exp^{-j\Theta m}. \quad (6.58)$$

Letting $n = m$ and $\Omega = \Theta$ and substituting into Eq. 6.58 gives

$$X'(j\Omega/2) = \sum_{n=-\infty}^{\infty} x(n) \exp^{-j\Omega n}. \quad (6.59)$$

Since the right hand expression is the same as Eq. 6.54,

$$X'(j\Omega/2) = X(j\Omega). \quad (6.60)$$

Thus $X'(j\Omega)$ varies twice as rapidly as does $X(j\Omega)$.

VITA

Alan L. Wintenberg was born in Oak Ridge, Tennessee, on June 30, 1958. He grew up there and was graduated from Oak Ridge High School in June 1976. That fall he entered the University of Tennessee, Knoxville and received the Bachelor of Science degree in Engineering Physics in June 1981 and the Master of Science degree with an major in Electrical Engineering in December 1984. While an undergraduate, he was also a coop student with the Physics Division of Oak Ridge National Laboratory, and as a graduate student he worked part time and performed his thesis research with the Fusion Energy Division.

In January 1985, he began work as an instructor in the Electrical Engineering Department of the University of Tennessee, Knoxville, and enrolled in the doctoral program. While in this program, he also worked as a research and teaching assistant and as manager of the Dielectrics Research Laboratory. The Doctor of Philosophy degree was awarded in December 1989.

The author is a member of Phi Kappa Phi and Tau Beta Pi Honor Societies, the IEEE, and the IEEE Dielectrics and Electrical Insulation Society.