August 16, 1949

To the Committee on Graduate Study:

I am submitting to you a thesis written by Sung-kuei Shu entitled "Design of the Shaded-pole Motor." I recommend that it be accepted for nine quarter hours of credit in partial fulfillment of the requirements for the degree of Master of Science, with a major in Electrical Engineering.

Paul O Cromwell
Major Professor

We have read this thesis
and recommend its acceptance:

H W Seeley

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Accepted for the Committee

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Dean of the Graduate School

DESIGN OF THE SHADED-POLE MOTOR

A THESIS

Submitted to
The Committee on Graduate Study
of
The University of Tennessee
in
Partial Fulfillment of the Requirements
for the degree of
Master of Science

---

by

Sung-kuei Shu

August 1949

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I. INTRODUCTION

A shaded-pole motor is a single-phase induction motor provided with an auxiliary short-circuited winding displaced in magnetic position from the main winding. This auxiliary winding (called a "shading coil") is used for producing the starting torque.

The essential parts of a shaded-pole motor are the stator, carrying the main winding, the shading coil and the squirrel-cage rotor. Although a non-salient-pole or distributed winding can be used for the stator, it is usually built as salient pole type motor. The shading coil surrounds a portion about $\frac{1}{4}$ to $\frac{1}{2}$ of the whole pole. Usually iron wedges are used to bridge the adjacent pole tips to increase the leakage flux of the shaded poles in order to improve the motor characteristics. Sometimes these bridges are punched in one piece of the stator laminations. The squirrel-cage rotor is the same in construction as that used for polyphase induction motors.

Horsepower ratings cover a wide range from 0.00025 to 0.1. Four-pole motors are chosen for the upper range of ratings and the two-pole machine is used for medium and lower ranges.

The performance characteristics are also in a wide variety. Locked-rotor torques range from 25 to 90 per cent of full load torque. Breakdown torques (or maximum torques) are of the order of 115 to 130 per cent of full load torque. Efficiency is rarely over 25 to 30 per cent. Slip is high with 1.5 to 3.0 per cent at no load and 10 to 20 per cent at full load.

These motors are used for nearly all applications requiring an

induction motor of 1/20 hp or less such as fans, humidifiers, hair driers, etc.

Although shaded-pole motors have been widely used in practice the design of these motors is still based upon experience in many respects. P. H. Trickey has given an analysis of the electric circuits and performance calculations of shaded-pole motors (see reference 9 and 10), but only the special case of 90 electrical degrees between the effective centers of un-shaded portion of whole pole and shaded pole was considered.

The author purposes to get general solutions for magnetic and electric design characteristics for the shaded-pole motor with any angle β between the center line of the main pole axis and the shaded pole axis. These solutions are applied to a practical design and checked with laboratory results.

This paper consists of seven sections as given in the contents. Following this introduction is the field analysis. Both the stator and rotor fields are analyzed and the resultant air gap flux wave is given in a general equation. The condition for eliminating stator harmonics and the criteria for the existence of rotor harmonics are also derived. For clearity in the analysis the permeance wave is assumed in a square form. Actually this will never to be so because of the fringing and bridge effects. In the third section the four electric circuit equations (main winding, shading coil and two rotor circuits in direct and quadrature axes) are set up by means of coupled circuit theory. These equations are only solved for steady state both at standstill and running conditions. The transient solutions are quite long and complicated and nearly meaning-

less for design so they are not given here. The equivalent circuit is expressed by a diagram. The torque equation will be derived in terms of circuit constants and currents. The design procedure and performance calculations are treated with an example (1/40 hp shaded pole motor made by Robbins & Myers, Inc.) in section IV & V respectively. Most of the guiding values and empirical equations used for design are taken from Kuhlmann's book (reference 4). Some of the constants are calculated by the equations given in Trickey's paper (reference 9). Calculated results were checked experimentally.

II. FIELD ANALYSIS

1. Main pole magnetomotive force (M. M. F.):

Both the main winding and the shading coil are assumed to be concentrated around their pole surfaces, then the wave of the magnetomotive force due to either main winding current or shading coil current can be drawn in a square form as shown in Fig. 1.

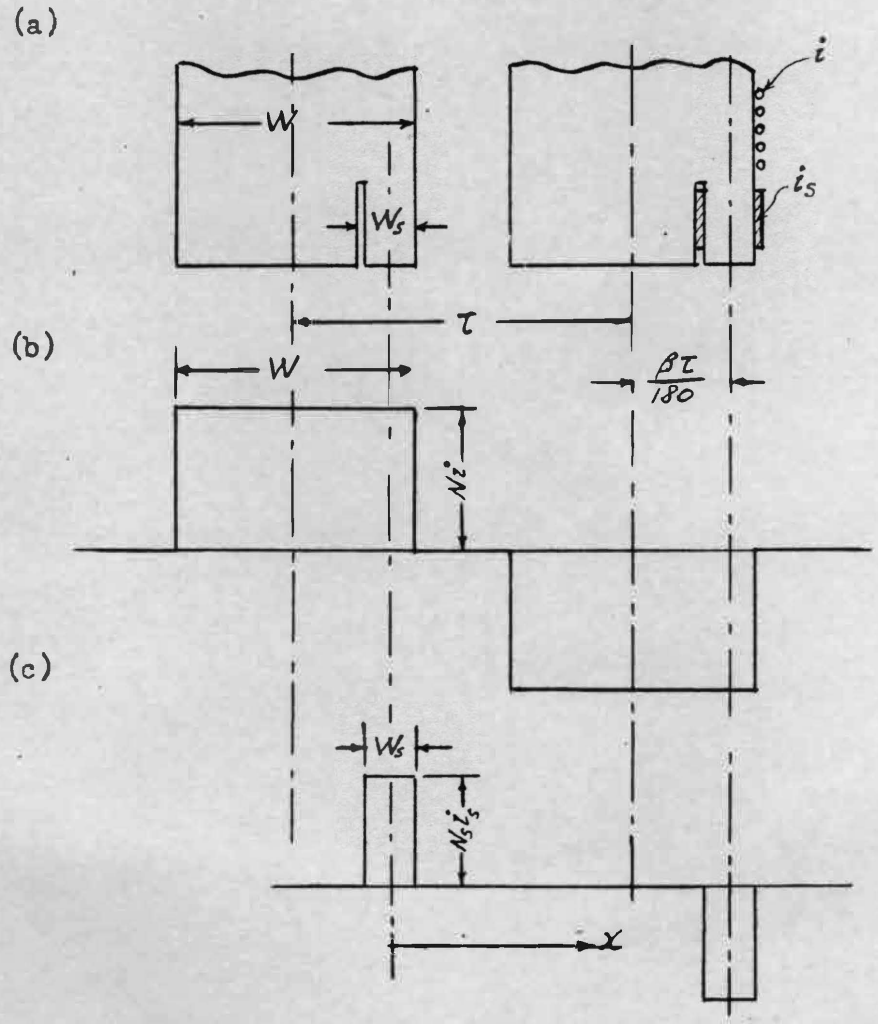


Fig. 1. M. M. F. wave of stator: (a) salient poles, (b) main pole mmf, (c) shaded pole mmf.

where W = width of main pole in inches,

W_s = width of shaded pole in inches,

τ = pole pitch in inches,

β = space angle between main pole and shaded pole in electrical degrees (with respect to the synchronous wave),

i = current (amp.) per conductor in main field winding,

i_s = current (amp.) per conductor in shading coil,

N = number of turns per pole for main field winding,

N_s = number of turns per pole for shading coil.

Express the main pole m.m.f. in cosine series (no constant and sine terms here), then the coefficients are

$$a_v = \frac{1}{\tau}\int_{-\tau}^{\tau} f(x)\cos\frac{v\pi x}{\tau}\,dx = \frac{2}{\tau}\int_{o}^{\tau} f(x)\cos\frac{v\pi x}{\tau}\,dx$$

$$= \frac{2}{\tau}\int_{o}^{\frac{W}{2}} Ni\cos\frac{v\pi x}{\tau}\,dx + \frac{2}{\tau}\int_{\tau-\frac{W}{2}}^{\tau}(-Ni)\cos\frac{v\pi x}{\tau}\,dx$$

$$= \frac{2}{v\pi}\left[Ni\sin\frac{v\pi x}{\tau}\Big|_{o}^{\frac{W}{2}} - Ni\sin\frac{v\pi x}{\tau}\Big|_{\tau-\frac{W}{2}}^{\tau}\right]$$

$$= \frac{2Ni}{v\pi}\left[\sin\frac{v\pi W}{2\tau} - \cos v\pi\sin\frac{v\pi W}{2\tau}\right]$$

Therefore $$a_v = \frac{4Ni}{v\pi}\sin\frac{v\pi W}{2\tau} \qquad \text{(where } v = \text{odd number.)}$$

Assume $i = \sqrt{2}I\sin\omega t$ (I is the effective value of main field current.), then the m.m.f. due to main field current is

$$f_m(x,t) = \frac{4\sqrt{2}NI}{\pi}\sin\omega t\sum_{\text{odd}}\frac{1}{v}\sin\frac{v\pi W}{2\tau}\cos\frac{v\pi x}{\tau}$$

$$= \frac{4\sqrt{2}NI}{2\pi}\sum_{\text{odd}}\frac{1}{v}\sin\frac{v\pi W}{2\tau}\left[\sin(\omega t + \frac{v\pi x}{\tau}) + \sin(\omega t - \frac{v\pi x}{\tau})\right]$$

Let $$F_m = \frac{4\sqrt{2}NI}{\pi} = 1.80NI.$$

Hence $$f_m(x,t) = \frac{F_m}{2} \sum_{\text{odd}} \frac{1}{v} \sin\frac{v\pi W}{2\tau} \left[\sin(\omega t + \frac{v\pi x}{\tau}) + \sin(\omega t - \frac{v\pi x}{\tau})\right], \quad (1)$$

in which $\sin\frac{v\pi W}{2\tau} = K_{pv}$ (pitch factor for vth harmonic).

2. Shaded pole M. M. F.:

Let α = time angle between i and i_s (lagging).

Assume $i_s = \sqrt{2} I_s \sin(\omega t - \beta)$ (I_s is the effective value of shading coil current.), then in a similar way as in II-1 we get the m.m.f. due to shading coil current,

$$f_s(x,t) = \frac{4\sqrt{2} N_s I_s}{2\pi} \sum_{\text{odd}} \frac{1}{v} \sin\frac{v\pi W_s}{2\tau} \left[\sin(\omega t - \alpha + \frac{v\pi x}{\tau} - v\beta) + \sin(\omega t - \alpha - \frac{v\pi x}{\tau} + v\beta)\right]$$

Let $$F_s = \frac{4\sqrt{2} N_s I_s}{\pi} = 1.80 N_s I_s.$$

$$f_s(x,t) = \frac{F_s}{2} \sum_{\text{odd}} \frac{1}{v} \sin\frac{v\pi W_s}{2\tau} \left[\sin(\omega t - \alpha + \frac{v\pi x}{\tau} - v\beta) + \sin(\omega t - \alpha - \frac{v\pi x}{\tau} + v\beta)\right] \quad (2)$$

in which $\sin\frac{v\pi W_s}{2\tau} = K_{pvs}$ (pitch factor for vth harmonic of shaded pole m.m.f.)

3. Total stator M. M. F.:

Add the magnetomotive forces for main pole and shaded pole, we get the total m.m.f. of the stator.

$$f(x,t) = f_m(x,t) + f_s(x,t).$$

In general,

$$f_1 = F_1 \sin(\omega t + x + \theta_1), \quad f_2 = F_2 \sin(\omega t + x + \theta_2),$$

$$f(x,t) = f_1 + f_2 = F \sin(\omega t + x - \lambda),$$

where $$F = \sqrt{(F_1 \cos\theta_1 + F_2 \cos\theta_2)^2 + (F_1 \sin\theta_1 + F_2 \sin\theta_2)^2}$$

$$\lambda = \tan^{-1} \frac{F_1 \sin\theta_1 + F_2 \sin\theta_2}{F_1 \cos\theta_1 + F_2 \cos\theta_2} .$$

In the same way from Eqs. (1) and (2), we get

$$f(x,t) = \sum_{\text{odd}} F_{vb} \sin(\omega t + \frac{v\pi x}{\tau} - \lambda_{vb}) + \sum_{\text{odd}} F_{vf} \sin(\omega t - \frac{v\pi x}{\tau} - \lambda_{vf}) \qquad (3)$$

where $$F_{vb} = \frac{1}{2v} \left[\left\{ F_m \sin\frac{v\pi W}{2\tau} + F_s \sin\frac{v\pi W_s}{2\tau} \cos(\alpha + v\beta) \right\}^2 + \left\{ -F_s \sin\frac{v\pi W_s}{2\tau} \sin(\alpha + v\beta) \right\}^2 \right]^{\frac{1}{2}} \qquad (3a)$$

$$F_{vf} = \frac{1}{2v} \left[\left\{ F_m \sin\frac{v\pi W}{2\tau} + F_s \sin\frac{v\pi W_s}{2\tau} \cos(\alpha - v\beta) \right\}^2 + \left\{ -F_s \sin\frac{v\pi W_s}{2\tau} \sin(\alpha - v\beta) \right\}^2 \right]^{\frac{1}{2}} \qquad (3b)$$

$$\lambda_{vb} = \tan^{-1} \frac{F_s \sin\frac{v\pi W_s}{2} \sin(\alpha + v\beta)}{F_m \sin\frac{v\pi W}{2\tau} + F_s \sin\frac{v\pi W_s}{2\tau} \cos(\alpha + v)} , \qquad (3c)$$

$$\lambda_{vf} = \tan^{-1} \frac{F_s \sin\frac{v\pi W_s}{2\tau} \sin(\alpha - v\beta)}{F_m \sin\frac{v\pi W}{2\tau} + F_s \sin\frac{v\pi W_s}{2\tau} \cos(\alpha - v\beta)} . \qquad (3d)$$

Equation (3) shows the general wave form for the stator m.m.f. of the shaded pole motor.

Since $F_{vf} \neq F_{vb}$, we get a revolving field for each harmonic as shown in Fig 2. This represents the magnitude of the revolving field forming an elliptic shape with respect to time.

If $F_{vf} > F_{vb}$, the resultant revolving field is forward; if

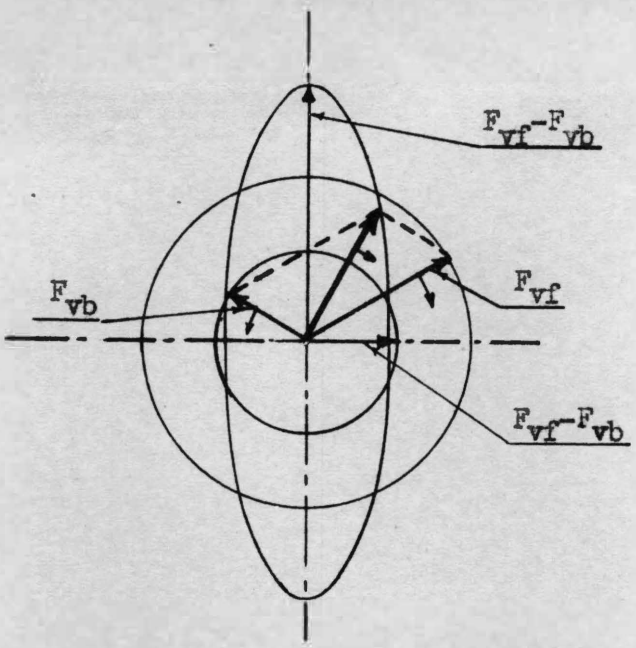


Fig. 2. Revolving field

$F_{vf} < F_{vb}$, it is backward. Usually for synchronous wave, $F_{1f} > F_{1b}$, so the resultant revolving field of the synchronous wave is forward. That means it rotates in the direction from main pole to shaded pole in the uniform air gap region.

4. Condition for eliminating harmonics of stator M.M.F.:

The backward harmonics will vanish, if $F_{vb} = 0$. From Eq. (3a)

$$\left(F_m \sin\frac{v\pi W}{2\tau}\right)^2 + \left(F_s \sin\frac{v\pi W_s}{2\tau}\right)^2 + 2\left(F_m \sin\frac{v\pi W}{2\tau}\right)\left(F_s \sin\frac{v\pi W_s}{2\tau}\right)\cos(\alpha + v\beta) = 0$$

Then
$$\cos(\alpha + v\beta) = -\frac{\left(F_m \sin\frac{v\pi W}{2\tau}\right)^2 + \left(F_s \sin\frac{v\pi W_s}{2\tau}\right)^2}{2\left(F_m \sin\frac{v\pi W}{2\tau}\right)\left(F_s \sin\frac{v\pi W_s}{2\tau}\right)} \qquad (4)$$

where v = odd number.

Similarly the condition for eliminating the forward harmonics is

$$\cos(\alpha - v\beta) = - \frac{(F_m \sin\frac{v\pi W}{2\tau})^2 + (F_s \sin\frac{v\pi W_s}{2\tau})^2}{2(F_m \sin\frac{v\pi W}{2\tau})(F_s \sin\frac{v\pi W_s}{2\tau})} \qquad (5)$$

In the design we wish to eliminate all backward harmonics and make the synchronous forward wave as large as possible.

5. Harmonics of rotor M. M. F.:

In general the rotor of the shaded pole motor is always of the squirrel cage type, so the following analysis is based upon squirrel-cage rotor design only.

The rotor current induced by the vth stator space harmonic is

$$i_{2v} = \sqrt{2}\, I_{2v} \sin(S_v wt) \qquad (6)$$

where $S_v = 1 \mp v(1 - s)$. (see 6-b)

S_v means the slip of the rotor with respect to the vth stator space harmonic. s is the slip referred to the synchronous wave; - sign used for forward v and + sign for backward v. Consider the uth of these harmonics due to the vth stator main pole field (produced by main winding current). Its mmf is

$$f_{um}(x_2, t) = F_{um} \sin S_v wt \cos\frac{u\pi x_2}{\tau}$$

$$= \frac{F_{um}}{2} \left\{ \sin(S_v wt + \frac{u\pi x_2}{\tau}) + \sin(S_v wt - \frac{u\pi x_2}{\tau}) \right. \qquad (7)$$

where $F_{um} = \frac{4\sqrt{2} N_2 I_{vm}}{u\pi}$,

N_2 = number of coils (turns) per phase per pole pair

= $\frac{1}{2}$ for squirrel-cage rotor,

I_{vm} = R.M.S. rotor current induced by vth stator main pole field.

The uth mmf of these harmonics due to vth shaded pole field (produced by the shading coil current) is

$$f_{us}(x_2,t) = F_{us} \sin(S_v \omega t - v\alpha) \cos(\frac{u\pi x_2}{\tau} - u\beta)$$

$$= \frac{F_{us}}{2} \left\{ \sin(S_v \omega t + \frac{u\pi x_2}{\tau} - v\alpha - u\beta) \right.$$

$$\left. + \sin(S_v \omega t - \frac{u\pi x_2}{\tau} - v\alpha - u\beta) \right\} \qquad (8)$$

where $F_{us} = \frac{4\sqrt{2}\ N_2 I_{vs}}{u}$

I_{vs} = R.M.S. rotor current induced by vth stator shaded pole field.

Then the uth mmf of these harmonics (for zero phase) due to total vth stator field (both main pole and shaded pole) is the sum of Eqs. (7) and (8).

$$f_{uo}(x_2,t) = f_{um}(x_2,t) + f_{us}(x_2,t)$$

$$= F_{ub} \sin(S_v \omega t + \frac{u\pi x_2}{\tau} - \lambda_{ub})$$

$$+ F_{ub} \sin(S_v \omega t - \frac{u\pi x_2}{\tau} - \lambda_{ub}) \qquad (9)$$

where $F_{ub} = \frac{1}{2} \left[\left\{ F_{um} + F_{us} \cos(v\alpha + u\beta) \right\}^2 + \left\{ -F_{us} \sin(v\alpha + u\beta) \right\}^2 \right]^{\frac{1}{2}}$ (9a)

$$F_{uf} = \frac{1}{2} \left[\left\{ F_{um} + F_{us} \cos(v\alpha - u\beta) \right\}^2 + \left\{ -F_{us} \sin(v\alpha - u\beta) \right\}^2 \right]^{\frac{1}{2}} \qquad (9b)$$

$$\lambda_{ub} = \tan^{-1} \frac{F_{us} \sin(v\alpha - u\beta)}{F_{um} + F_{us} \cos(v\alpha - u\beta)} \qquad (9c)$$

$$\lambda_{uf} = \tan^{-1} \frac{F_{us} \sin(v\alpha - u\beta)}{F_{um} + F_{us} \cos(v\alpha - u\beta)}, \qquad (9d)$$

v = odd number.

For the squirrel-cage rotor there are $2Q_2/P$ phases (Q_2 is the total number of rotor slots, P is the number of poles of stator.), the time angle between two adjacent phases is $2\pi v/m_2$ electrical degrees, and the phase angle is $2\pi u/m_2$. Then the mmf of uth harmonic for c phase is

$$f_{uc}(x_2,t) = F_{ub}\sin\left[S_v\omega t + \frac{u\pi x_2}{\tau} - (u+v)\frac{2\pi c}{m_2} - \lambda_{ub}\right] + F_{uf}\sin\left[S_v\omega t - \frac{u\pi x_2}{\tau} + (u-v)\frac{2\pi c}{m_2} - \lambda_{uf}\right] \tag{10}$$

where $m_2 = 2Q_2/P$ = number of phases for rotor.

Then the total uth mmf of the rotor due to total vth stator field is the sum of these magnetomotive forces for m_2 phases:

$$\begin{aligned} f_u(x_2,t) &= f_{u0}(x_2,t) + f_{u1}(x_2,t) + \cdots + f_{uc}(x_2,t) + \cdots + f_{u(m_2-1)}(x,t) \\ &= F_{ub}\left\{\sum_{c=0}^{m_2-1}\cos(u+v)\frac{2\pi c}{m_2}\sin(S_v\omega t + \frac{u\pi x_2}{\tau} - \lambda_{ub}) - \sum_{c=0}^{m_2-1}\sin(u+v)\frac{2\pi c}{m_2}\cos(S_v\omega t + \frac{u\pi x_2}{\tau} - \lambda_{ub})\right\} \\ &\quad + F_{uf}\left\{\sum_{c=0}^{m_2-1}\cos(u-v)\frac{2\pi c}{m_2}\sin(S_v\omega t - \frac{u\pi x_2}{\tau} - \lambda_{uf}) + \sum_{c=0}^{m_2-1}\sin(u-v)\frac{2\pi c}{m_2}\cos(S_v\omega t - \frac{u\pi x_2}{\tau} - \lambda_{uf})\right\} \end{aligned} \tag{11}$$

Let $$a_b = \sum_{c=0}^{m_2-1}\cos(u+v)\frac{2\pi c}{m_2}, \qquad b_b = -\sum_{c=0}^{m_2-1}\sin(u+v)\frac{2\pi c}{m_2},$$

$$a_f = \sum_{c=0}^{m_2-1}\cos(u-v)\frac{2\pi c}{m_2}, \qquad b_f = \sum_{c=0}^{m_2-1}\sin(u-v)\frac{2\pi c}{m_2},$$

then $$f_u(x_2,t) = F_{ub}(a_b^2 + b_b^2)^{\frac{1}{2}}\sin(S_v\omega t + \frac{u\pi x_2}{\tau} - \lambda_{ub} + \phi_{ub}) + F_{uf}(a_f^2 + b_f^2)^{\frac{1}{2}}\sin(S_v\omega t - \frac{u\pi x_2}{\tau} - \lambda_{uf} + \phi_{uf}) \tag{12}$$

where $\phi_{ub} = \tan^{-1}\frac{b_b}{a_b}$, $\qquad \phi_{uf} = \tan^{-1}\frac{b_f}{a_f}$.

Since in a symmetrical polyphase winding ($m_2 > 2$), we only have the uth harmonic rotating in one direction or the other, therefore either $(a_b^2 + b_b^2)^{\frac{1}{2}}$ or $(a_f^2 + b_f^2)^{\frac{1}{2}}$ should be equal to zero.

Assume $a_b = b_b = 0$, then

$$(u + v)\frac{2\pi}{m_2} = K(2\pi/m_2), \qquad K = u + v = \pm 1, \pm 2, \pm 3, \ldots$$

$\neq$ multiple of m_2.

a_f and b_f will also both equal to zero, unless

$$(u - v)\frac{2\pi}{m_2} = (K - 2v)\frac{2\pi}{m_2} = 2\pi K_2$$

$$K_2 = (u - v)/m_2 = (K - 2v)/m_2 = 0, \pm 1, \pm 2, \pm 3, \ldots$$

$$a_f = \sum_{c=0}^{m_2-1} \cos(2\pi K_2) = m_2,$$

$$b_f = \sum_{c=0}^{m_2-1} \sin(2\pi K_2) = 0,$$

Therefore $\qquad \sqrt{(a_f^2 + b_f^2)} = m_2$

Assume $a_f = b_f = 0$, then

$$(u - v)\frac{2\pi}{m_2} = K(2\pi/m_2), \qquad K = u - v = \pm 1, \pm 2, \pm 3, \ldots$$

$\neq$ multiple of m_2.

a_b and b_b will also both equal to zero, unless

$$(u + v)\frac{2\pi}{m_2} = (K + 2v)\frac{2\pi}{m_2} = 2\pi K_2,$$

$$K_2 = (u + v)/m_2 = (K + 2v)/m_2 = 0, \pm 1, \pm 2, \pm 3, \ldots$$

$$a_b = \sum_{c=0}^{m_2-1} \cos(2\pi K_2) = m_2$$

$$b_b = \sum_{c=0}^{m_2-1} \sin(2\pi K_2) = 0$$

Therefore $\sqrt{(a_b^2 - b_b^2)} = m_2$

The criteria for the existence of rotor harmonic u:

Case 1: For backward waves of u,

$$(u - v)/m_2 = (K - 2v)/m_2 = K_2 = \pm \text{ integers including zero.} \tag{13}$$

$K = u + v = \pm$ integers except zero and multiple of m_2.

Case 2: For forward waves of u,

$$(u + v)/m_2 = (K + 2v)/m_2 = K_2 = \pm \text{ integers including zero.} \tag{14}$$

$K = u - v = \pm$ integers except zero and multiple of m_2.

In both cases $m_2 = 2Q_2/p$ for squirrel cage rotor.

6. Stator flux wave:

The general formula for flux density is

$$B = \phi/(\text{area}) = 0.4\pi\,(AT)/l \text{ gausses,}$$

where AT = ampere turns,

l = length of magnetic circuit in cms.

Change l to inches and B to lines per sq. in., then

$$B = \frac{0.4\,(AT)}{1 \times 2.54}(2.54)^2 = \frac{3.2(AT)}{l} \text{ lines per sq. in.} \tag{15}$$

(a) Flux due to the stator M. M. F.:

Eq. (3) shows the stator mmf for a salient-pole machine. It is

the combination of two discontinuous square waves and must go to zero beyond the poles. If we assume the air gap under the poles is uniform, then the resultant flux wave must be the same in shape as that of the mmf.

From Eq. (15) the amplitudes of flux waves for vth forward and backward harmonics are:

$$B_{vf} = \frac{3.2F_{vf}}{\delta K_c K_s}$$

and $$B_{vb} = \frac{3.2F_{vb}}{\delta K_c K_s} \qquad (16)$$

where δ = air gap in inches,

K_c= Carter's coefficient for the slots,

K_s= ratio of ampere turns per pole to ampere turns per air gap,

F_{vf} and F_{vb} are given by Eqs. (3a) and (3b).

Then from Eqs. (3) and (16), the flux distribution wave due to stator mmf is

$$b_s(x_1,t) = \sum_{odd} B_{vb} \sin(wt + v\pi x_1/\tau - \lambda_{vb}) + \sum_{odd} B_{vf} \sin(wt - v\pi x_1/\tau - \lambda_{vf}) \qquad (17)$$

Equation (17) is only true for the resultant flux wave but not for individual harmonics. For individual harmonics it must be modified by permeance wave. (see 7-b.)

The velocity of the traveling waves is

$$\frac{dx_1}{dt} = \pm \frac{w\tau}{v\pi} = \pm \frac{2f_1\tau}{v} \text{ inches/sec.} \qquad (18)$$

where + sign is used for forward waves and - sign for backward waves. f_1 is the frequency of applied voltage.

(b) Stator flux wave referred to rotor:

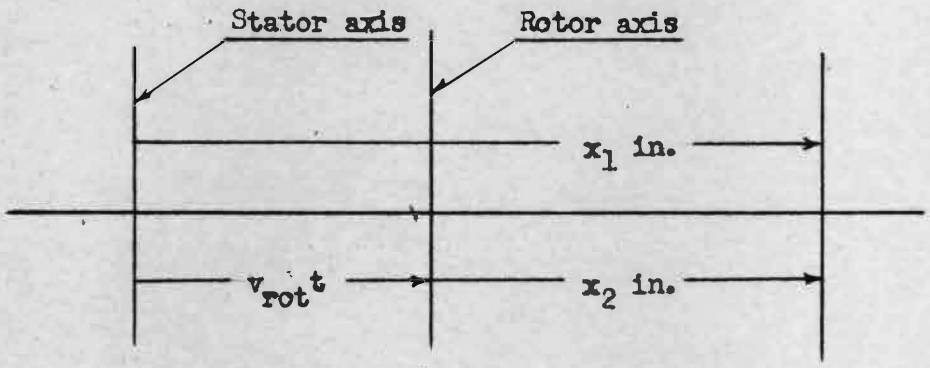


Fig. 3. Relation of stator & rotor axes.

$$x_1 = v_{rot}t + x_2 = (1 - s)\frac{\tau wt}{\pi} + x_2.$$

Inserting this into Eq. (17), we have

$$b_s(x_2,t) = \sum_{\text{odd}} B_{vb} \sin\left[\{1 + v(1-s)\}wt + v\pi x_2/\tau - \lambda_{vb}\right]$$

$$+ \sum_{\text{odd}} B_{vf} \sin\left[\{1 - v(1-s)\}wt - v\pi x_2/\tau - \lambda_{vf}\right],$$

or

$$b_s(x_2,t) = \sum_{\text{odd}} B_{vb} \sin(S_{vb}wt + v\pi x_2/\tau - \lambda_{vb})$$

$$+ \sum_{\text{odd}} B_{vf} \sin(S_{vf}wt - v\pi x_2/\tau - \lambda_{vf}), \qquad (19)$$

where $S_{vb} = 1 + v(1 - s)$ = slip of rotor with respect to the vth stator backward harmonic. (19a)

$S_{vf} = 1 - v(1 - s)$ = slip of rotor with respect to the vth stator forward harmonic. (19b)

s is the slip with respect to synchronous wave and its value is from zero to one. Therefore S_{vb} is always positive but S_{vf} may be positive or negative. When S_{vf} is negative this means the wave

of B_{vf} is also going backward with respect to the rotor.

The velocities of stator flux wave with respect to rotor are:

$$\frac{dx_2}{dt} = \frac{S_{vf} w \tau}{v \pi} = \frac{2 S_{vf} f_1 \tau}{v} \text{ inches/sec.},$$
$$\frac{dx_2}{dt} = -\frac{S_{vb} w \tau}{v \pi} = -\frac{2 S_{vb} f_1 \tau}{v} \text{ inches/sec.} \qquad (20)$$

The first one is used for forward waves of v and the second is used for backward waves of v.

7. Rotor flux wave:

(a) Rotor flux for smooth air gap machine:

If the machine were non-salient pole type (i.e., with smooth air gap), then the amplitude of the rotor flux distribution wave for uth forward or backward harmonic would be:

$$B_{uf} = \frac{3.2 F_{uf} m_2}{\delta K_c K_s},$$
$$B_{ub} = \frac{3.2 F_{ub} m_2}{\delta K_c K_s}, \qquad (21)$$

where F_{uf} and F_{ub} are given by Eqs. (9a) and (9b), δ, K_c, K_s are same as before. Then the rotor flux distribution wave of uth harmonic would be (from Eq. 12)

$$b'_{uf}(x_2,t) = B_{uf} \sin(S_v w t - u\pi x_2/\tau - \lambda_{uf} - \phi_{uf})$$
$$b'_{ub}(x,t) = B_{ub} \sin(S_v w t + u\pi x_2/\tau - \lambda_{ub} + \phi_{ub}) \qquad (22)$$

where $S_v = 1 \pm v(1 - s)$

(b) Rotor flux wave for shaded-pole motor:

Actually all shaded-pole motors are salient-pole type, so Eq. (22) must be modified by the wave of permeance.

Assume the permeance of air path between rotor and stator outside the pole shoe space is 1/5 times that under poles (uniform air gap), then the wave of permeance may be shown as Fig. 4.

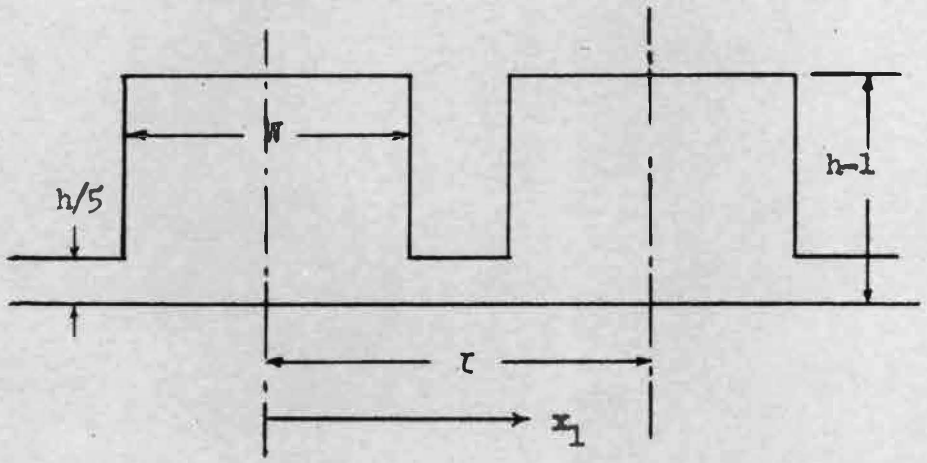


Fig. 4. Permeance Wave.

Assume

$$P(x_1) = a_o + \sum_1^\infty a_n \cos n\pi x_1/\tau + \sum_1^\infty b_n \sin n\pi x_1/\tau$$

where
$$a_o = \frac{1}{2\tau}\int_o^{2\tau} P(x_1)\,dx_1 = \frac{1}{2\tau}\left[\int_o^{\frac{W}{2}} h\,dx_1 + \int_{\frac{W}{2}}^{\tau-\frac{W}{2}} \frac{h}{5}\,dx_1 + \int_{\tau-\frac{W}{2}}^{\tau+\frac{W}{2}} h\,dx_1 + \int_{\tau+\frac{W}{2}}^{2\tau-\frac{W}{2}} \frac{h}{5}\,dx_1 + \int_{2\tau-\frac{W}{2}}^{2\tau} h\,dx_1\right]$$
$$= \frac{h}{5}\left(\frac{4W}{\tau} + 1\right),$$

$$a_n = \frac{1}{\tau}\int_o^{2\tau} P(x_1)\cos\frac{n\pi x_1}{\tau}\,dx_1 = \frac{1}{\tau}\left[\int_o^{\frac{W}{2}} h\cos\frac{n\pi x_1}{\tau}\,dx_1 + \int_{\frac{W}{2}}^{\tau-\frac{W}{2}} \frac{h}{5}\cos\frac{n\pi x_1}{\tau}\,dx_1 + \int_{\tau-\frac{W}{2}}^{\tau+\frac{W}{2}} h\cos\frac{n\pi x_1}{\tau}\,dx_1 + \int_{\tau+\frac{W}{2}}^{2\tau-\frac{W}{2}} \frac{h}{5}\cos\frac{n\pi x_1}{\tau}\,dx_1 + \int_{2\tau-\frac{W}{2}}^{2\tau} h\cos\frac{n\pi x_1}{\tau}\,dx_1\right]$$

$$= \frac{8h}{5n\pi}\left(\sin\frac{n\pi W}{2\tau} + \cos n\pi \sin\frac{n\pi W}{2\tau}\right)$$

$$= \frac{16h}{5n\pi}\sin\frac{n\pi W}{2\tau}, \quad \text{when } n = \text{even number}$$

$$= 0, \quad \text{when } n = \text{odd number.}$$

$$b_n = \frac{1}{\tau}\int_0^{2\tau} P(x_1)\sin\frac{n\pi x_1}{\tau}\,dx_1 = 0 \text{ for } n = \text{any integer.}$$

Therefore

$$P(x_1) = a_o + \sum_{n=\text{even}}^{\infty} a_n \cos\frac{n\pi x_1}{\tau}$$

$$= \frac{h}{5}\left(\frac{4W}{\tau} + 1\right) + \sum_{n=\text{even}}^{\infty}\left(\frac{16h}{5n\pi}\sin\frac{n\pi W}{2\tau}\right)\cos\frac{n\pi x_1}{\tau} \qquad (23)$$

where $h = 1$.

Shift $P(x_1)$ referred to rotor,

$$x_1 = (1 - s)\frac{\tau\omega t}{\pi} + x_2,$$

then
$$P(x_2, t) = a_o + \sum_{n=\text{even}}^{\infty} a_n \cos\left\{(1 - s)n\omega t + \frac{n\pi x_2}{\tau}\right\} \qquad (24)$$

The actual rotor flux distribution wave of uth harmonic due to vth stator field is the product of Eqs. (22) and (24),

$$b_{uf}(x_2, t) = b'_{uf}(x_2, t)P(x_2, t)$$

$$= a_o B_{uf}\sin(S_v\omega t - \frac{u\pi x_2}{\tau} - \lambda_{uf} + \phi_{uf})$$

$$+ B_{uf}\sum_{n=\text{even}}^{\infty} a_n \sin(S_v\omega t - \frac{u\pi x_2}{\tau} - \lambda_{uf} + \phi_{uf}) \text{ times}$$

$$\cos\left\{(1 - s)n\omega t + \frac{n\pi x_2}{\tau}\right\}$$

or

$$b_{uf}(x_2,t) = a_o B_{uf} \sin(S_v wt - \frac{u\pi x_2}{\tau} - \lambda_{uf} - \phi_{uf})$$
$$+ \frac{B_{uf}}{2} \sum_{n\text{-even}}^{\infty} a_n \Big[\sin\{(S_v + (1-s)n)wt - (u-n)\frac{\pi x_2}{\tau} - \lambda_{uf} + \phi_{uf}$$
$$+ \sin\{(S_v - (1-s)n)wt - (u+n)\frac{\pi x_2}{\tau} - \lambda_{uf} + \phi_{uf}\}\Big] \tag{24}$$

Similarly,

$$b_{ub}(x_2,t) = a_o B_{ub} \sin(S_v wt + \frac{u\pi x_2}{\tau} - \lambda_{ub} + \phi_{ub})$$
$$+ \frac{B_{ub}}{2} \sum_{n\text{-even}}^{\infty} a_n \Big[\sin\{(S_v + (1-s)n)wt + (u+n)\frac{\pi x_2}{\tau} - \lambda_{ub} + \phi_{ub}\}$$
$$+ \sin\{(S_v - (1-s)n)wt + (u-n)\frac{\pi x_2}{\tau} - \lambda_{ub} + \phi_{ub}$$
(25)

Notice that either $b_{uf}(x_2,t)$ or $b_{ub}(x_2,t)$ contains one normal traveling wave and two sets of infinite series traveling waves with different speeds.

The velocities of $b_{uf}(x_2,t)$ with respect to rotor are (for three different kinds of waves):

$$\frac{dx_2}{dt} = \frac{2S_v f_1 \tau}{u},$$
$$\frac{dx_2}{dt} = \frac{2(S_v+(1-s)n)f_1\tau}{u-n}, \tag{26}$$
and
$$\frac{dx_2}{dt} = \frac{2(S_v-(1-s)n)f_1\tau}{u+n},$$

where $f_1 = w/2\pi$.

The velocities of $b_{ub}(x_2,t)$ with respect to rotor are:

$$\frac{dx_2}{dt} = -\frac{2S_v f_1 \tau}{u}$$

$$\frac{dx_2}{dt} = -\frac{2(S_v+(1-s)n)f_1\tau}{u+n}, \qquad \frac{dx_2}{dt} = -\frac{2(S_v-(1-s)n)f_1\tau}{u-n}. \tag{27}$$

(c) Rotor Flux Waves Referred to Stator:

$$x_2 = x_1 - v_{rot}t = x_1 - (1-s)\frac{\tau wt}{\pi}.$$

Then from Eqs. (24) and (25), we get

$$b_{uf}(x_1,t) = a_o B_{uf} \sin\left\{(S_v + u(1-s))wt - \frac{u\pi x_1}{\tau} - \lambda_{uf} + \phi_{uf}\right\} + \frac{B_{uf}}{2}\sum_{n\text{-even}}^{\infty} a_n\left[\sin\left\{(S_v + u(1-s))wt - (u-n)\frac{\pi x_1}{\tau} - \lambda_{uf} + \phi_{uf}\right\} + \sin\left\{(S_v + u(1-s))wt - (u+n)\frac{\pi x_1}{\tau} - \lambda_{uf} + \phi_{uf}\right\}\right] \tag{28}$$

and

$$b_{ub}(x_1,t) = a_o B_{ub} \sin\left\{(S_v - u(1-s))wt - \frac{u\pi x_1}{\tau} - \lambda_{ub} + \phi_{ub}\right\} + \frac{B_{ub}}{2}\sum_{n\text{-even}}^{\infty} a_n\left[\sin\left\{(S_v - u(1-s))wt + (u+n)\frac{\pi x_1}{\tau} - \lambda_{ub} + \phi_{ub}\right\} + \sin\left\{(S_v - u(1-s))wt + (u-n)\frac{\pi x_1}{\tau} - \lambda_{ub} + \phi_{ub}\right\}\right] \tag{29}$$

The velocities of rotor waves with respect to stator are:

For $b_{uf}(x_1,t)$,

$$\frac{dx_1}{dt} = \frac{2(S_v+u(1-s))f_1\tau}{u}, \qquad \frac{dx_1}{dt} = \frac{2(S_v+u(1-s))f_1\tau}{u-n}, \tag{30}$$

and

$$\frac{dx_1}{dt} = \frac{2(S_v+u(1-s))f_1\tau}{u+n}.$$

For $b_{ub}(x_1,t)$,

$$\frac{dx_1}{dt} = -\frac{2(S_v-u(1-s))f_1\tau}{u},$$

$$\frac{dx_1}{dt} = -\frac{2(S_v-u(1-s))f_1\tau}{u+n}, \qquad (31)$$

and

$$\frac{dx_1}{dt} = -\frac{2(S_v-u(1-s))f_1\tau}{u-n}.$$

(d) Total rotor flux:

The total rotor flux wave referred to stator is the summation of $b_{uf}(x_1,t)_v$ (Eq. 28) and $b_{ub}(x_1,t)_v$ (Eq. 29) with both u and v from one to infinity:

$$b_r(x_1,t) = \sum_{v\text{-odd}} \sum_{u} \left\{ b_{uf}(x_1,t)_v + b_{ub}(x_1,t)_v \right\} \qquad (32)$$

Notice that $b_{uf}(x_1,t)_v$ and $b_{ub}(x_1,t)_v$ do not exist for the same u (see II-5) and the criteria for the existence of u is given by Eqs. (13) and (14).

The net air gap flux is the vector sum of total stator flux (Eq. 17) and total rotor flux (Eq. 32),

$$b_g(x_1,t) = b_s(x_1,t) + b_r(x_1,t). \qquad (33)$$

III. CIRCUIT ANALYSIS

1. Circuit Equations:

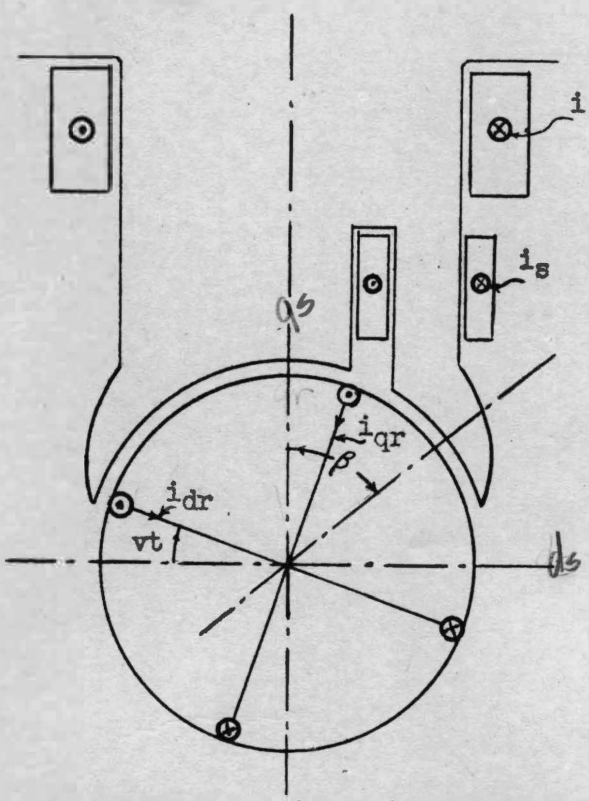


Fig. 5. Diagram of Pole and Rotor in Shaded Pole Motor.

Let β = angle in electrical degrees between main pole axis and shaded pole axis,

i = stator current in main winding,

i_s = stator current in shading coil,

i_{dr} = rotor current in the winding referred to the direct axis,

i_{qr} = rotor current in the winding referred to the quadrature axis,

m = mutual inductance between main winding and shading coil,

M = mutual inductance between main winding and rotor winding in direct axis,

M_s = mutual inductance between shading coil and rotor winding

in the direct axis of shaded pole,

$r + lp$ = main winding impedance,

$r_s + l_s p$ = shading coil impedance,

$R + p(l_o + l_2)$ = rotor impedance in direct axis,

$R + p(l_o - l_2)$ = rotor impedance in quadrature axis.

The circuit equations can be written as follows:

For main coil circuit:

$$e = (r+lp)i + mpi_s + Mp \cos vt\, i_{dr} + Mp \cos(vt+90^\circ)\, i_{qr}$$

For shading coil circuit:

$$e_s = 0 = (r_s+l_s p)i_s + mpi + M_s p \cos(vt-\beta)\, i_{dr} + M_s p \cos(vt+90^\circ-\beta) i_{qr}$$

For rotor circuit referred to direct axis:

$$e_{dr} = 0 = (R+p(l_o+l_2 \cos 2vt))i_{dr} + 0 + Mp \cos vt\, i + M_s p \cos(vt-\beta) i_s$$

For rotor circuit referred to quadrature axis:

$$e_{qr} = 0 = \left\{ R+p(l_o+l_2 \cos 2(vt+90^\circ)) \right. \; i_{qr} + 0$$
$$+ Mp \cos(vt+90^\circ) i + M_s p \cos(vt+90^\circ-\beta)$$

Put the above four expressions in box notation and we get the following equations:

	i	i_s	i_{dr}	i_{qr}	
e	$r+lp$	mp	$Mp\cos vt$	$-Mp\sin vt$	(34)
$e_s = 0$	mp	r_s+l_sp	$M_sp\cos(vt-\beta)$	$-M_sp\sin(vt-\beta)$	(35)
$e_{dr} = 0$	$Mp\cos vt$	$M_sp\cos(vt-\beta)$	$R+p(l_o+l_2\cos 2vt)$	0	(36)
$e_{qr} = 0$	$-Mp\sin vt$	$-M_sp\sin(vt-\beta)$	0	$R+p(l_o-l_2\cos 2vt)$	(37)

The transient solution of these equations may be obtained by operational methods, but it is quite long and complicated and nearly useless for design. Therefore only the steady state solutions are given in the following paragraphs.

2. Steady State Equations Referred to Stationary Axes:

For steady state, replace all p by +jw.

Transfer all rotor voltages and currents in the stationary axes (direct and quadrature axis) by keeping the angle vt equal to zero and introducing the speed voltages in the rotor circuits. Then the above equations are reduced into the following form :

	i	i_s	i_d	i_q
e	$r+jwl$	jwm	jwM	0
$e_s=0$	jwm	r_s+jwl_s	$jwM_s \cos\beta$	$jwM_s \sin\beta$
$e_d=0$=transform v. +speed v.	jwM + 0	$jwM_s \cos\beta$ $+(1-s)wM_s \sin\beta$	$R+jw(l_o+l_2)$ + 0	0 $+(1-s)w(l_o-l_2)$
$e_q=0$=transform v. +speed v.	0 $-(1-s)wM$	$jw\ M_s \sin\beta$ $-(1-s)wM_s \cos\beta$	0 $-(1-s)w(l_o+l_2)$	$R+jw(l_o-l_2)$ +0

where s = slip as decimal,

i_d = rotor current in direct axis,

i_q = rotor current in quadrature axis.

Let $X = wl$ = self reactance of main winding,

$X_s = wl_s$ = self reactance of shading coil,

$X_d = w(l_o+l_2)$ = self reactance of rotor winding in direct axis,

$X_q = w(l_o-l_2)$ = self reactance of rotor winding in quadrature axis,

$X_m = wm$ = mutual reactance between main winding and shading coil,

$X_{md} = wM$ = mutual reactance between main winding and rotor winding in direct axis,

$X_{sd} = wM_s \cos\beta$ = mutual reactance between shading coil and

rotor winding in direct axis,

$X_{sq} = w\mathcal{M}_s \sin\beta$ = mutual reactance between shading coil and rotor winding in quadrature axis,

$S = 1 - s = v/w$ = speed as decimal of synchronous speed.

Then we get the following equations:

	i	i_s	i_d	i_q	
e	$r+jX$	jX_m	jX_{md}	0	(38)
$e_s=0$	jX_m	r_s+jX_s	jX_{sd}	jX_{sq}	(39)
$e_d=0$	jX_{md}	$SX_{sq}+jX_{sd}$	$R+jX_d$	SX_q	(40)
$e_q=0$	$-SX_{md}$	$-SX_{sd}+jX_{sq}$	$-SX_d$	$R+jX_q$	(41)

3. Solutions of Steady State equations (Eqs. 38 to 41) for all currents:

$$\Delta = \begin{vmatrix} r+jX & jX_m & jX_{md} & 0 \\ jX_m & r_s+jX_s & jX_{sd} & jX_{sq} \\ jX_{md} & SX_{sq}+jX_{sd} & R+jX_d & SX_q \\ -SX_{md} & -SX_{sd}+jX_{sq} & -SX_d & R+jX_q \end{vmatrix}$$

$$\Delta = U_1 + jV_1 + SU_2 + jSV_2 = U - jV \qquad (42)$$

$$i = \frac{e}{\Delta}(A_1 + jB_1 + SA_2 + jSB_2) = \frac{e}{\Delta}(A + jB), \qquad (43)$$

$$i_s = \frac{e}{\Delta}(C_1 + jD_1 + SC_2 + jSD_2) = \frac{e}{\Delta}(C + jD), \qquad (44)$$

$$i_d = \frac{e}{\Delta}(E_1 + jF_1 + SE_2 + jSF_2) = \frac{e}{\Delta}(E + jF), \qquad (45)$$

$$i_q = \frac{e}{\Delta}(G_1 + jH_1 + SG_2 + jSH_2) = \frac{e}{\Delta}(G + jH), \quad [1] \qquad (46)$$

where $A_1 = -r_s(X_dX_q - R^2) - RX_s(X_d + X_q) + R(X_{sd}^2 + X_{sq}^2)$,

$B_1 = r_sR(X_d + X_q) - X_s(X_dX_q - R^2) + (X_qX_{sd}^2 + X_dX_{sq}^2)$,

$A_2 = S\, r_sX_dX_q$,

$B_2 = S(X_sX_dX_q - X_qX_{sd}^2 - X_dX_{sq}^2)$,

$A = A_1 + SA_2$,

$B = B_1 + SB_2$,

$C_1 = R(X_m(X_d + X_q) - X_{md}X_{sd})$,

$D_1 = X_m(X_dX_q - R^2) - X_qX_{md}X_{sd}$,

$C_2 = 0$,

$D_2 = -SX_q(X_mX_d - X_{md}X_{sd}) - RX_{md}X_{sq}$,

$C = C_1$,

$D = D_1 + SD_2$,

$E_1 = R(X_sX_{md} - X_mX_{sd}) + r_sX_qX_{md}$,

$F_1 = X_q(X_sX_{md} - X_mX_{sd}) - X_{md}(r_sR + X_{sq}^2)$,

$E_2 = -Sr_sX_qX_{md}$,

$F_2 = RX_mX_{sq} - SX_q(-X_mX_{sd} + X_sX_{md}) + SX_{md}X_{sq}^2$,

---

[1] The evaluation of Δ and the determination of currents are given in Appendix I.

$$E = E_1 + SE_2,$$

$$F = F_1 + SF_2,$$

$$G_1 = -RX_mX_{sq},$$

$$H_1 = -X_{sq}(X_mX_d - X_{md}X_{sd}),$$

$$G_2 = r_sRX_{md},$$

$$H_2 = R(-X_mX_{sd} + X_sX_{md}) + SX_{sq}(X_mX_d - X_{md}X_{sd}),$$

$$G = G_1 + SG_2,$$

$$H = H_1 + SH_2,$$

$$U_1 = rA_1 - XB_1 - X_mD_1 - X_{md}F_1,$$

$$V_1 = XA_1 + rB_1 + X_mC_1 + X_{md}E_1,$$

$$U_2 = rA_2 - XB_2 - X_mD_2 - X_{md}F_2,$$

$$V_2 = XA_2 + rB_2 + X_{md}E_2,$$

$$U = U_1 + SU_2,$$

$$V = V_1 + SV_2.$$

4. Standstill Solution of Steady State Equation:

At standstill, the speed S = 0, then

$$\Delta = U_1 + jV_1 \tag{47}$$

$$i = \frac{e}{\Delta}(A_1 + jB_1) \tag{48}$$

$$i_s = \frac{e}{\Delta}(C_1 + jD_1), \tag{49}$$

$$i_d = \frac{e}{\Delta}(E_1 + jF_1), \tag{50}$$

$$i_q = \frac{e}{\Delta}(G_1 + jH_1). \tag{51}$$

5. Equivalent Circuit:

Let $a_1 = i_q/i$, $a_2 = i_q/i_s$,

$a_3 = i_d/i_s$, $a_4 = i_d/i_q$.

Then from Eqs. (5) to (8), we can draw the equivalent circuit diagram as shown in Fig. 6.

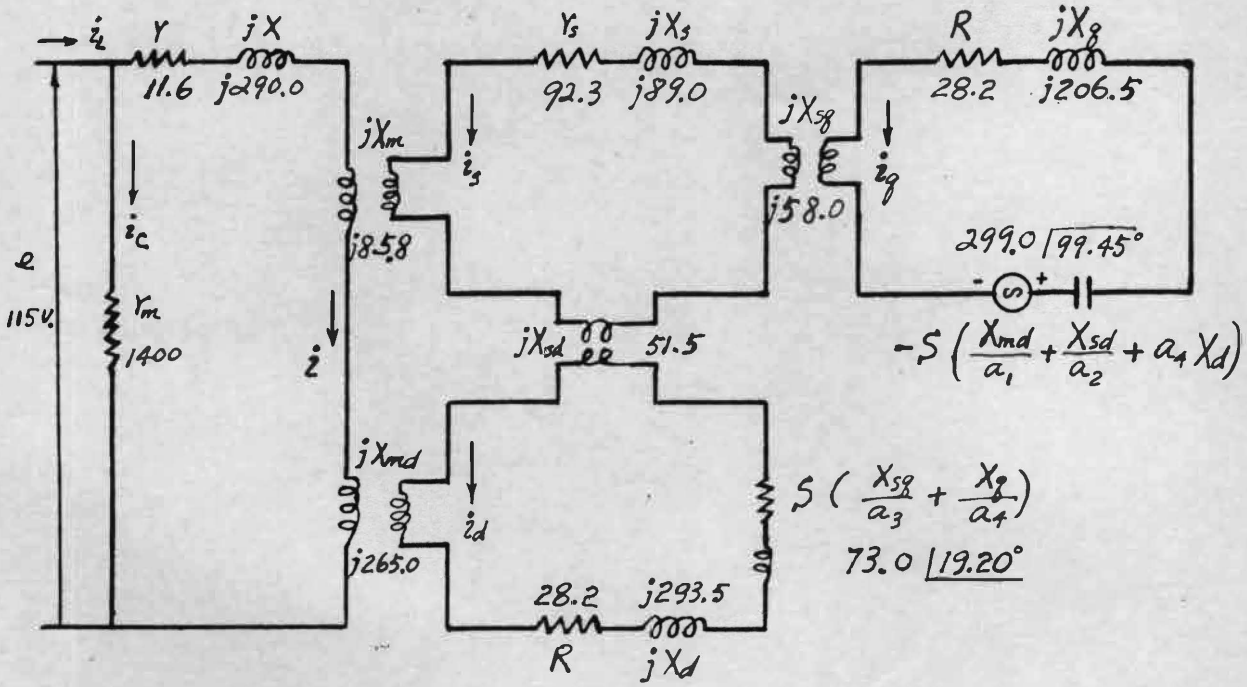


Fig. 6. Equivalent Circuit.

The numerical values are given for the special machine at 1550 rpm. (see V-7.)

6. Torque Equation:

The general equation of voltage for a winding rotating in a varying magnetic field is:

$$e_a = R_a i_a + \frac{d\phi}{dt} + \nu\psi,$$

where e_a = applied voltage (volts),

i_a = applied current (amperes),

R_a = resistance of the winding (ohms),

ϕ = flux-linkages (10^8 line-linkage),

ψ = flux density (10^{-8} line-linkages per radian),

ν = angular velocity (radians per second).

The electrical power flow equation is:

$$e_a i_a = R_a i_a^2 + (\frac{d\phi}{dt}) i_a + \nu\psi\, i_a$$

where $e_a i_a$ = power input (watts),

$R_a i_a^2$ = power lost in the form of heat,

$(\frac{d\phi}{dt})\, i_a$ = the rate of increase of the stored magnetic energy,

$\nu\psi i_a$ = mechanical power output (watts).

Since H.P. output $= \frac{2\pi(T/16)(rpm)}{33,000} = \frac{\nu\psi i_a}{746} = \frac{2\pi(rpm)}{60} \times \frac{\psi i_a}{746}$

then the torque

$$T = \frac{33,000 \times 16}{60 \times 746} \psi i_a = 11.8 \psi i_a \text{ Oz-ft.} \qquad (52)$$

Apply Eq. (52) to the shaded-pole motor of P poles, we get the total torque as:

$$T = \frac{11.8P}{\pi}\left\{i_d(M_s \sin\beta\, i_s + (l_o - l_2)i_q)\right.$$

$$\left. - i_q(Mi + M_s \cos\beta\, i_s + (l_o - l_2)i_d)\right\} \text{ oz-ft.}$$

$$= 0.01P\left\{i_d(X_{sq}i_s + X_q i_q) - i_q(X_{md}i + X_{sd}i_s + X_d i_d)\right\} \text{ oz-ft.,} \quad (53)$$

since for 60-cycle,

$$\frac{11.8}{w\pi} = \frac{11.8}{2\pi 60} = 0.01.$$

The above equation shows the instantaneous torque in terms of instantaneous currents.

The average torque developed can be expressed in terms of effective currents as the real part of

$$T_d = 0.01P\left\{I_d^*(X_{sq}I_s + X_q I_q) - I_q^*(X_{md}I + X_{sd}I_s + X_d I_d)\right\} \text{ oz-ft.} \quad (54)$$

where the effective currents, I, I_s, I_d and I_q, can be calculated by the same equations (Eqs. 43 to 46) used for instantaneous currents with E (effective voltage) instead of e. I_d^* and I_q^* are the conjugates of I_d and I_q respectively.

IV. EXAMPLE OF DESIGN CALCULATIONS

The procedure in the design of a shaded-pole motor is different from that for other type induction motors. The stator is something like that used in direct current machines but with shaded pole structure attached to the main pole. The rotor is always of the squirrel-cage type which is the same as for other induction motors.

The procedure for the design can be illustrated clearly by an example:

Rating: 1/40 hp, 1550 rpm, 115 volts, 60 cycles.

Specification:

Full load efficiency- $\geq 22.5\%$

Full load power factor $\geq 60\%$

Locked rotor torque (at rated voltage) $\geq 35\%$ of full load torque

Locked rotor current ≤ 2.0 amp.

Maximum torque (or breakdown torque) $\geq 118\%$ of full load torque

Temperature rise of the winding (for continuous load operation) $\leq 40^{\circ}C$

1. Stator Design:

(a) Main dimensions:

Number of poles,

$$P = 120\ f/n \tag{55}$$

For $f = 60$ cycles and $n = 1800$ rpm,

$$P = \frac{120 \times 60}{1800} = 4.$$

The output equation for fractional-horsepower motor is

$$\frac{D^2 L n}{watt} = C \qquad (56)$$

where D = air gap diameter of the stator in inches,

L = length of the laminated core in inches.

The output constant C can be found from Fig. 7 with known watts/n.

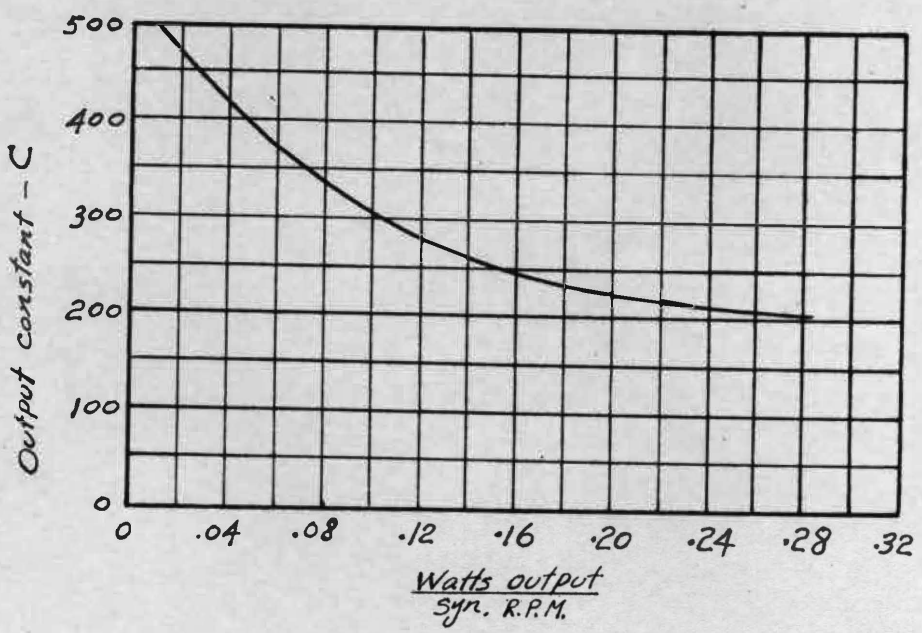


Fig. 7. Output Constant for Single-phase Motors.

Now, $$\frac{\text{watts output}}{\text{syn. rpm.}} = \frac{0.025 \times 746}{180} = 0.01037.$$

From Fig. 7, C = 510.

Then $$D^2L = (\text{watts}) \times C/n = 0.01037 \times 510 = 5.29.$$

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In general, L/D = 0.5 to 10. Take L/D = 1.0 for this machine, then $D = L = (5.29)^{1/3} = 1.74$ inches, use 1.75 inches.

(b) Magnetic circuit in stator:

Pole pitch

$$\tau = \pi D/P = 3.14 \times 1.75/4 = 1.375 \text{ inches.}$$

Assume $\frac{\text{pole arc}}{\text{pole pitch}} = \psi = 0.78$

then pole arc = 0.78x1.375 = 1.07 inches.

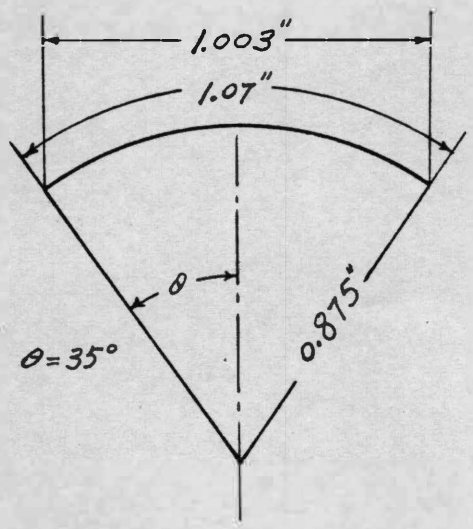


Fig. 8. Pole Arc.

From Fig. 8, pole width is

$$W_p = 2 \times 0.875 \sin\theta = 1.75 \sin\left(\frac{1.07}{1.75}\frac{180}{}\right) = 1.003 \text{ inches.}$$

Assume that the pole flux density is 36,700 lines per square inch and the lamination factor is 0.93, then the useful flux per pole,

$$\phi = 1.75 \times 1.003 \times 36{,}700 \times 0.93 = 60{,}000 \text{ lines.}$$

Let the radial length of pole, $l_p = 0.625"$. (It should be checked by the space for main winding and shading coil.)

Assume the leakage factor of the pole, $\lambda = 1.2$, then the flux passing through the stator yoke,

$$\frac{\phi\lambda}{2} = \frac{60,000 \times 1.2}{2} - 36,000 \text{ lines.}$$

Assume flux density in yoke, $B_y = 88,500$ lines/sq.in., then the depth of yoke,

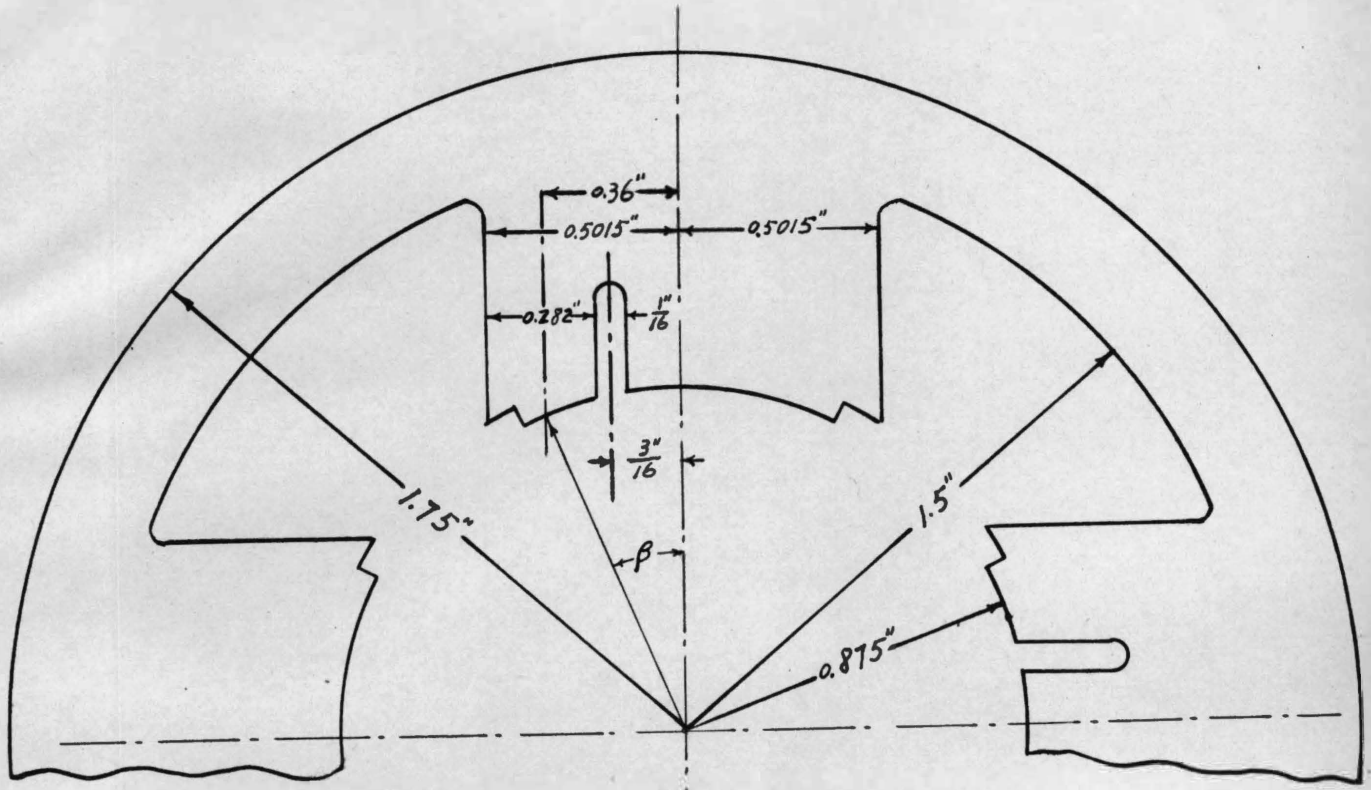
$$d_y = \frac{\phi\lambda/2}{0.93 L B_y} = \frac{36,000}{0.93 \times 1.75 \times 88,500} = 0.25".$$

The outside diameter of stator lamination,

$$D_o = D + 2l_p + 2d_y = 1.75 + 2 \times 0.625 + 2 \times 0.25$$

$$= 3.50".$$

The shape of stator lamination is shown in Fig. 9.



β = 24.2° mech. degrees
= 48.4° elec. degrees ,

Shaded pole arc = 0.313",
Pole arc = 1.07".

Fig. 9. Field Punching (Twice Size).

(c) Main winding:

The voltage equation,

$$E = 4fPN\phi \times 10^{-8} \text{ volts.} \qquad (57)$$

Therefore, the number of turns per pole,

$$N = \frac{E \times 10^{8}}{4fP\phi} = \frac{115 \times 10^{8}}{4 \times 60 \times 4 \times 60{,}000} = 200 \text{ turns/pole.}$$

For the given efficiency and power factor to be met, the full load current is

$$I = \frac{hp \times 746}{E \times \text{eff.} \times pf} = \frac{0.025 \times 746}{115 \times 0.225 \times 0.60} = 1.2 \text{ amp.}$$

In general the current density is from 2500 to 3000 amp. per square inch. Let the current density of the main winding, A_m = 3000 amp./sq.in., then the section area of wire,

$$S_m = I/A_m = 1.2/3000 = 0.0004 \text{ sq.in,}$$

Choose #23 enamel copper wire:

Bare diameter = 0.0226 inch,

Insulated diameter = 0.0242 inch,

Area of each wire, S_m = 0.000401 sq. in.

(d) Shading coil:

A slot of 1/16 inch wide is cut on the pole for shading coil as shown in Fig. 9.

Use a copper ring for each shading coil,

Cross section area of the copper ring

$$= 0.125 \times 3/64 - 0.00587 \text{ sq.in.}$$

2. Rotor Design:

(a) Air gap length and rotor diameter:

The empirical equation[2] for the air gap length of fractional horsepower motors is:

$$\delta = 0.005 + 0.00035D + 0.001L + 0.003V/1000, \qquad (58)$$

where the peripheral velocity, $V = \pi D n/12 = 3.14 \times 1.75 \times 1550/12$ = 710 ft./min. Therefore

$$\delta = 0.005 + 0.00035 \times 1.75 + 0.001 \times 1.75 + 0.003 \times 710/1000$$
$$= 0.00949 \text{ in., use } 0.008 \text{ in.}$$

The outside diameter of the rotor,

$$D_r = D - 2\delta = 1.75 - 2 \times 0.008 = 1.734 \text{ in.}$$

(b) Cage winding:

To avoid pulsations and oscillations of the air gap flux in a salient-pole machine the number of slots per pole should be equal to a whole number plus $\frac{1}{2}$. For this machine we choose $4\frac{1}{2}$ slots per pole, then the total number of slots,

$$Q_2 = 4 \times 4\tfrac{1}{2} = 18.$$

Assume the total ampere-turns of rotor to be 10 per cent less

---

[2] Kuhlmann, J. H, Design of Electrical Apparatus (New York: John Wiley & Sons, Inc., 1937), P. 352.

than that of the main winding of stator, the ratio of the total section area of bars in rotor to the total section area of the main winding is:

$$S_{cb}/S_{cm} = 0.9A_m/A_b, \qquad (59)$$

where A_m and A_b are the current densities in the main winding and in the rotor bars respectively.

If the rotor bars are made of the same material as that of the stator winding, the current density of rotor may be higher than that of stator because the mean length of rotor winding is shorter and the ventilation is better. Now cast aluminum cage winding is used for the rotor, the current density must be half that of copper, since aluminum has only 50% conductivity.

From 1(c), we have

$$A_m = 3{,}000 \text{ amp./sq.in.}$$

and $$S_{cm} = 0.000401x4x2x200 = 0.642 \text{ sq.in.}$$

Assume the current density of bars, $A_b = 4500/2 = 2250$ amp./sq.in., then

$$S_{cb} = 0.90x0.642x3000/2250 = 0.770 \text{ sq.in.}$$

Section area per bar,

$$S_b = 0.770/18 = 0.0427 \text{ sq.in.}$$

Choose the slot size as shown in Fig. 10, the area for each slot,

$$S_b = \tfrac{1}{2}x3.14(\overline{0.172^2} - \overline{0.115^2})/4 + 0.165(0.172 + 0.115)/2$$
$$= 0.0405 \text{ sq.in.}$$

then $S_{cb}=18 \times 0.0405 = 0.73$ sq.in.

The rotor skew is 0.375 in. per bar.

The section area of each end ring[3] is

$$S_{er} = \frac{0.32\ S_{cb}}{P} \frac{A_b}{A_{er}} \qquad (60)$$

Assume the current density of the end ring, $A_{er}=1.1A_b$, then

$$S_{er} = \frac{0.32 \times 0.73}{4} \times \frac{1}{1.1} = 0.053 \text{ sq.in.}$$

The size of end ring is shown in Fig. 10.

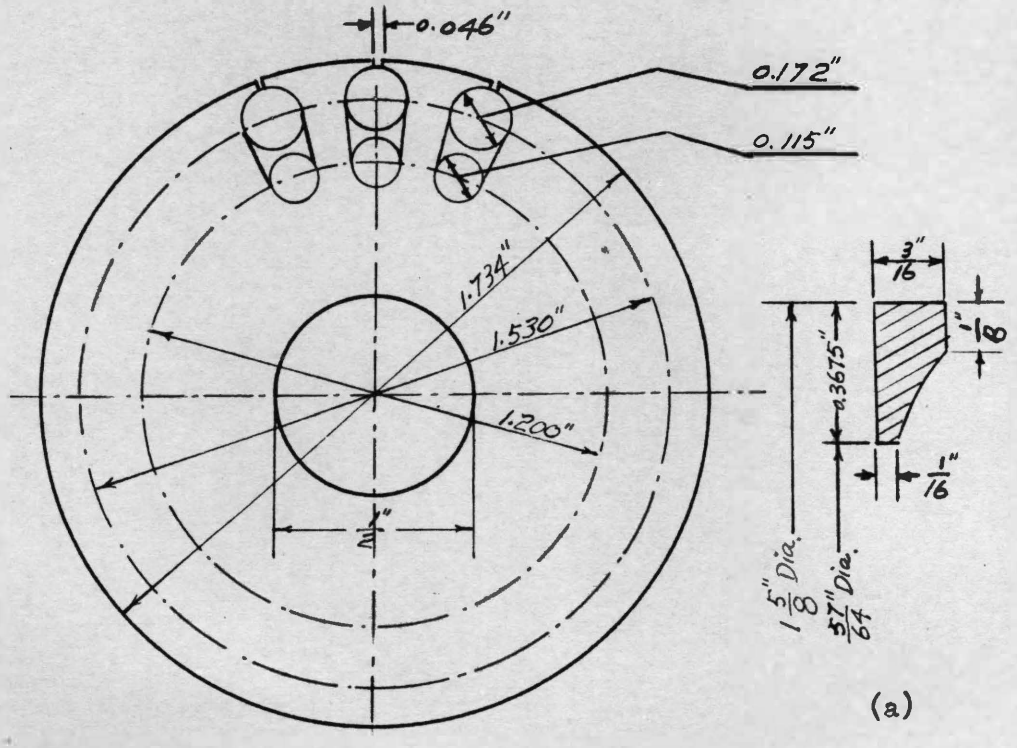


Fig. 10. Rotor Punching. (a) Cross Section of End Ring.

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[3] Ibid., p. 303.

(c) Magnetic circuit:

Minimum tooth width,

$$W_t = 3.14 \times 1.2/18 - 0.115 = 0.0945 \text{ in.}$$

Maximum tooth density,

$$B_t = \frac{\phi P}{0.93 L W_t Q_2} = \frac{60{,}000 \times 4}{0.93 \times 1.75 \times 0.0945 \times 18} = 86{,}800 \text{ lines/sq.in.}$$

Diameter of shaft = 0.500 in.

Depth of rotor yoke,

$$d_{yr} = \tfrac{1}{2}(1.200 - 0.115 - 0.500) - 0.2925 \text{ in.}$$

Flux density in rotor yoke,

$$B_{yr} = \frac{\phi/2}{0.93 L d_{yr}} = \frac{60{,}000/2}{0.93 \times 1.75 \times 0.2925} = 63{,}000 \text{ lines/sq.in.}$$

V OPERATING CHARACTERISTICS

(Calculated from Design Data)

Before calculating any characteristic curve we should find out all the motor constants (referred to main winding) appeared in Eqs. (38) to (41). Then we can solve those equations for all currents and calculate operating characteristics.

1. Resistances:

(a) Resistance of main winding:

$$r = \frac{PNL_m\rho}{S_m \times 10^6} \text{ ohms,} \qquad (61)$$

where $\rho = 0.690$ at $25^\circ C$

$= 0.796$ at $65^\circ C$,

$N = 200$ turns,

$S_m = 0.000401$ sq.in. (see IV-1-c)

$P = 4$ (see IV-1-a)

L_m = mean turn length

$= 2(L + W_p + 2\text{x}\text{thickness of coil})$

$= 2(1.75 + 1.003 + 2\text{x}0.45) = 7.306$ inches.

Therefore $$r = \frac{4\text{x}200\text{x}7.306\text{x}0.692}{0.000401\text{x}10^6} = 10.1 \text{ ohms at } 25^\circ C$$

$= 10.1\text{x}1.15 = 11.6$ ohms at $65^\circ C$.

(b) Resistance of shading coil:

$$r_s = \frac{PN_sL_s\rho}{S_s\text{x}10^6}(N/N_s)^2 \qquad (62)$$

where N_s = 1 turn, (see IV-1-d)

S_s = 0.00587 sq.in., (see IV-1-d)

L_s = mean turn length

= 2(L + W_s + 2xthickness of coil)

= 2(1.75 + 0.282 + 2x3/64) = 4.2516 inches.

Therefore $r_s = \frac{4x1x4.2516x0.692}{0.00587x10^6} x(200/1)^2 = 80.3$ ohms at 25°C

= 80.3x1.15 = 92.3 ohms at 65°C

(c) Rotor resistance:

The rotor resistance[4] is

$$R = \frac{l_b}{S_b Q_2} + \frac{0.637 D_{er}}{P^2 S_{er}} \frac{(2PN)^2 m(2\rho)}{10^6} \text{ ohms,} \qquad (63)$$

where Q_2 = 18

S_b = 0.0405 sq.in. (see IV-2-b)

S_{er} = 0.0530 sq.in. (see IV-2-b)

l_b = length of each bar

= $(\overline{1.75^2} + \overline{0.375^2})^{\frac{1}{2}}$ = 1.79 inches,

D_{er} = diameter of end ring = 1.33 inches,

m = 2 for single phase motor,

Therefore

$$R = \frac{1.79}{0.0405x18} + \frac{0.637x1.33}{4^2x0.0530} \frac{(2x4x200)^2 x2x1.384}{10^6}$$

= 24.5 ohms at 25°C

= 24.5x1.15 = 28.2 ohms at 65°C

---

[4] Ibid., p. 314.

2. Reactance:

(a) Self reactance of main winding (o.c. reactance):

$$X = wl = E/I_m = \text{open-circuit reactance} \qquad (64)$$

where E = impressed voltage,

I_m = magnetizing current = $AT/\sqrt{2}N$.

The magnetic ampere turns, AT, contain the following parts:

Air gap ampere turns[5]

$$AT_g = B_g \delta K_c \times 0.313 \qquad (65)$$

where $B_g = \phi/(\text{gap area}) = 60{,}000/1.07 \times 1.75$

$= 32{,}000$ lines/sq.in.,

$\delta = 0.008$ inch,

$K_c = \dfrac{t_{lr}}{W_{tr_1} + \delta y}$ = Carter's coefficient (or air gap coefficient) for rotor slots.

$t_{lr} = \pi D_r/N_b = 3.14 \times 1.734/18 = 0.3025$ inch

= tooth pitch,

$W_{tr} = 0.3025 - 0.046 = 0.2565$ inch

= tooth width,

$y = 2.5$ from Fig. 11 corresponding to (slot opening)/(air gap length) = $0.046/0.008 = 5.75$.

Then $K_c = 0.3025/(0.2565 + 0.008 \times 2.5) = 1.09$.

Therefore $AT_g = 32{,}000 \times 0.008 \times 1.09 \times 0.313 = 87.50$ amp.turns per pole.

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[5] Ibid., p. 63.

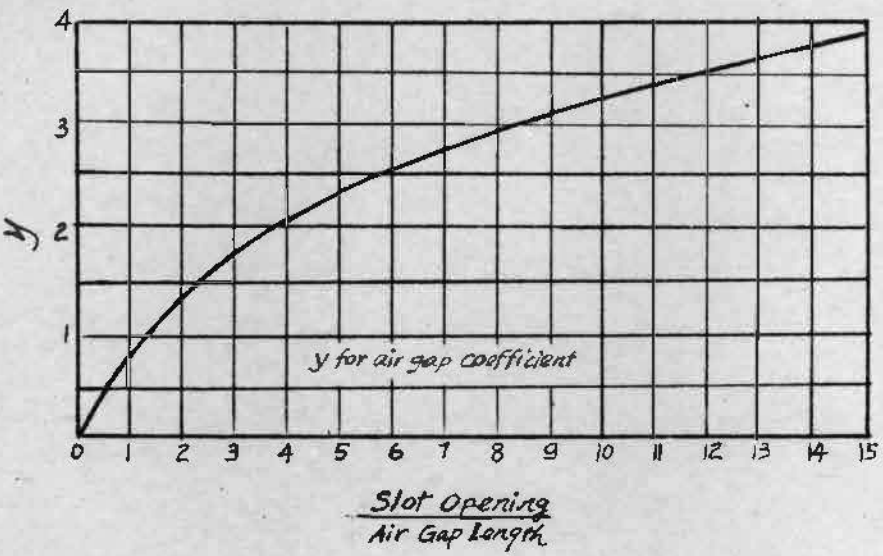


Fig. 11. Air Gap Coefficient.

Rotor tooth ampere turns:

B_t = 86,300 lines/sq.in. (see IV-2-c)

From Fig. 12, we get at_t = 13 amp. turns per inch.

The length of tooth,

$$l_t = \tfrac{1}{2}(0.172 + 0.115) + 0.165 = 0.3085 \text{ inch.}$$

Therefore $AT_t = 13 \times 0.3085 = 4.01$ amp. turns per pole.

Rotor yoke ampere turns:

B_{yr} = 63,000 lines/sq.in. (see IV-2-c)

From Fig. 12, we get $at_{yr} = 4$ amp. turns per inch.

$$l_{yr} = \frac{(D_r - 2l_t - d_{yr})\pi}{2P} = \frac{(1.734 - 2 \times 0.3085 - 0.2925)\pi}{2 \times 4}$$

$$= 0.334 \text{ inch.}$$

Therefore $AT_{yr} = 4 \times 0.334 = 1.34$ amp. turns per pole.

Pole ampere turns:

$B_p = 36{,}700$ lines/sq.in. (see IV-1-b)

From Fig. 12, we get $at_p = 2$ amp. turns/inch,

$l_p = 1.500 - 0.875 = 0.625$ inch.

Therefore $AT_p = 2 \times 0.625 = 1.25$ amp. turns per pole.

Stator yoke ampere turns:

$B_y = 88{,}500$ lines/sq.in. (see IV-1-b)

From Fig. 12, $at_y = 14$ amp. turns per inch,

$$l_y = \frac{(D_{yi} + d_y)}{2P} = \frac{3.14(3.00 + 0.25)}{2 \times 4} = 1.275 \text{ inches.}$$

Therefore $AT_y = 14 \times 1.275 = 17.90$ amp. turns per pole.

Total ampere turns per pole,

$$AT = AT_g + AT_t + AT_{yr} + AT_p + AT_y$$

$$= 87.50 + 4.01 + 1.34 + 1.25 + 17.90$$

$$= 112.0 \text{ ampere turns per pole.}$$

$$I_m = 112 / 2 \times 200 = 0.396 \text{ amp. (effective)}$$

Then, by Eq. (64) we get

$$X = 115/0396 = 290.0 \text{ ohms.}$$

(b) Mutual reactance between main winding and rotor winding in direct axis (magnetizing reactance):

$$X_{md} = wM = X - (X_1 + 2X_b + 2X_{sk}) \qquad (66)$$

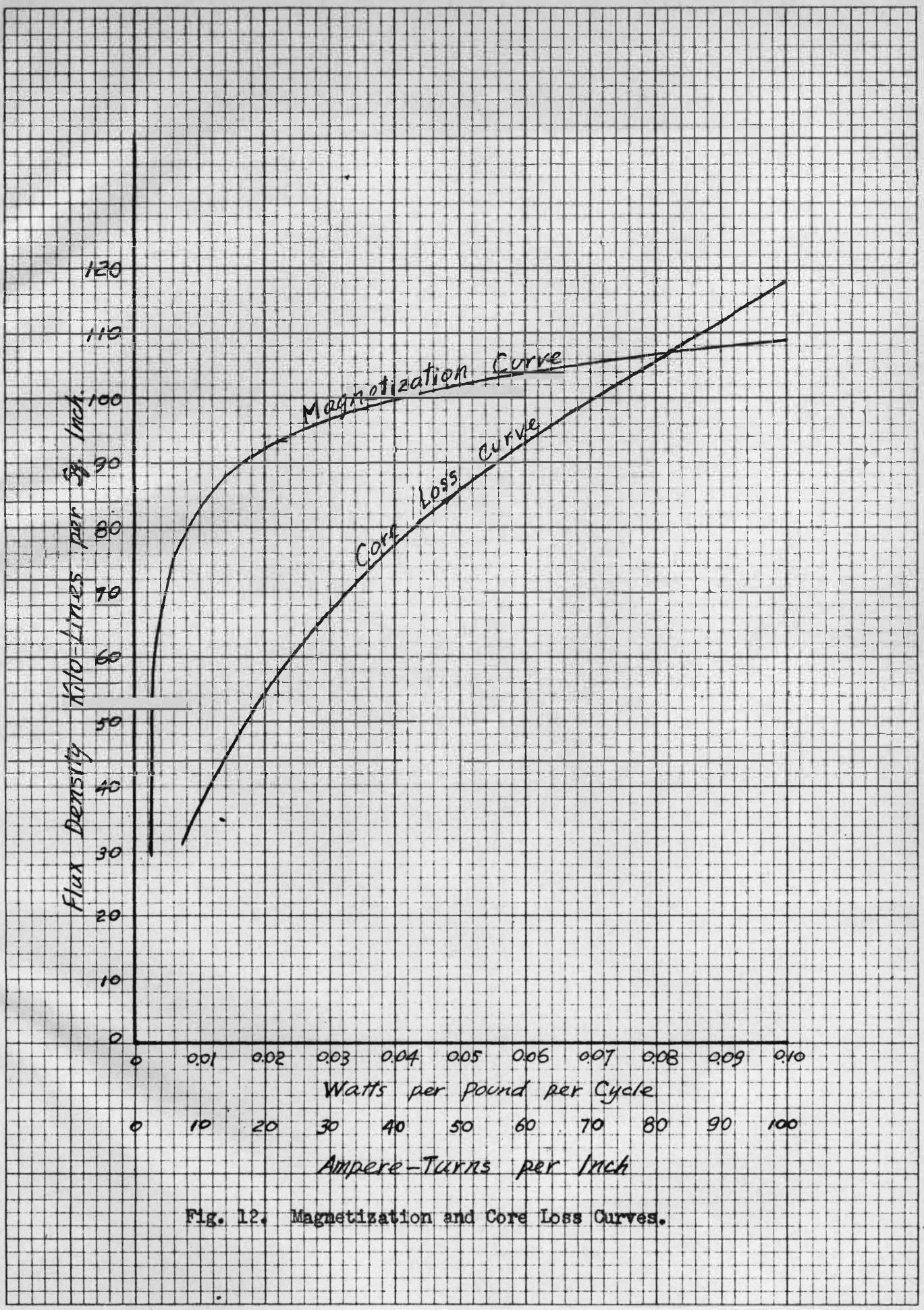


Fig. 12. Magnetization and Core Loss Curves.

where X_1 = primary leakage reactance,

X_b = bridge leakage reactance,

X_{sk} = skew leakage reactance.

These leakage reactances can be calculated by Trickey's formulae[6] as follows:

$$X_1 = 2\pi f(2PN)^2 10^{-8}(3.19LK_s/2P + (\text{span})/P). \qquad (67)$$

where the span of primary coil is equal to the width of pole plus the thickness of coil,

span = 1.003 + 0.450 = 1.453 inches.

Referring to Fig. 13, for d = 0, c = 0, a_1/a_2 = 0.375/1.188 = 0.316, b/a_2 = 0.56/1.188 = 0.473, the slot constant, K_s, is then

$K_s = F = 0.70.$

Therefore $X_1 = 2x3.14x60(2x4x200)^2 10^{-8}(3.19x1.75x0.70/2x4 + 1.453/4)$

$= 8.22$ ohms.

$$X_b = 2\pi f(2PN)^2 10^{-8}(3.19L/4P \times l_g/d_1) \qquad (68)$$

where d_1 = distance between adjacent tips of poles

$= \tau$ - (pole arc) = 1.375 - 1.070 = 0.305 inch,

$l_g = \delta K_c = 0.008x1.09 = 0.00872$ inch.

Therefore $X_b = 2x3.14x60(2x4x200)^2 10^{-8}(3.19x1.75x0.00872/4x4x0.305)$

---

[6] Trickey, P. H, "An Analysis of the Shaded Pole Motor," A.I.E.E. Transactions, 55:1014, September 1936.

$= 0.0965$ ohm, say 0.10 ohm.

$$X_{sk} = X_{md}(1 - C_{sk})/C_{sk}, \tag{69}$$

where $$C_{sk} = \frac{\sin\theta_{sk}/2}{\theta_{sk}/2} = \frac{\sin(0.865/2)}{0.865/2} = 0.970,$$

since θ_{sk} = rotor bar skew in radians

$$\theta_{sk} = \frac{\pi P}{N_b} \times \frac{0.375 N_b}{D_r} = 4 \times 0.375/1.734 = 0.865 \text{ radian.}$$

Therefore $X_{sk} = X_{md}(1 - 0.970)/0.970 = 0.031 X_{md}$.

Substituting these three leakage reactances and X into Eq. (66), we obtain

$$X_{md} = 290.0 - (8.22 + 2\times0.10 + 2\times0.031 X_{md}).$$

Therefore $X_{md} = 265.0$ ohms.

(c) Mutual reactance between main winding and shading coil:

$$X_m = X_b + X_{sk} + wM_s, \tag{70}$$

where $X_b = 0.10$ ohm,

$X_{sk} = 0.031 X_{md} = 0.031 \times 265.0 = 8.22$ ohms,

$wM_s = X_{md}$(shaded pole arc)/(pole arc)

$= 265 \times 0.313/1.070 = 77.50$ ohms.

Therefore $X_m = 0.10 + 8.22 + 77.50 = 85.82$ ohms, say 85.8 ohms.

(d) Self-reactance of shading coil:

$$X_s = X_{ls} + X_m, \tag{71}$$

where the shading coil leakage reactance was given by Trickey[7] as following equations:

$$X_{ls} = 2\pi f(2PN)^2 10^{-8}(3.19LK_s/4P + (\text{span})/P) , \qquad (72)$$

in which the span of shading coil = 0.282 + 3/64 = 0.329. Then

$$X_{ls} = 2x3.14x60(2x4x200)^2 10^{-8}(3.19x1.75x0.7/4x4 + 0.329/4)$$

$$= 3.15 \text{ ohms.}$$

Therefore X_s = 3.15 + 85.80 = 88.95 ohms, say 89.0 ohms.

(e) Rotor self-reactance in the direct axis:

$$X_d = w(l_o + l_2) = X_{lr} + X_{md}. \qquad (73)$$

The rotor leakage reactance can be divided into three components (neglecting leakage reactance of the end connection for squirrel cage rotor):

The slot leakage reactance[8] is

$$X_{slot} = 2\pi f(2PN)^2 10^{-8}\ 6.38LK_s/N_b, \qquad (74)$$

in which the slot constant, K_s, can be found from Fig. 13 (on the next page). For a_1/a_2 = 0.172/0.115 = 1.50 and b/a_2 = 0.165/0.115 = 1.435, we get

$$F = 0.40$$

and $$K_s = 0.40 + 0.016/0.046 + 0.172/(0.046+0.172)$$

$$= 1.57.$$

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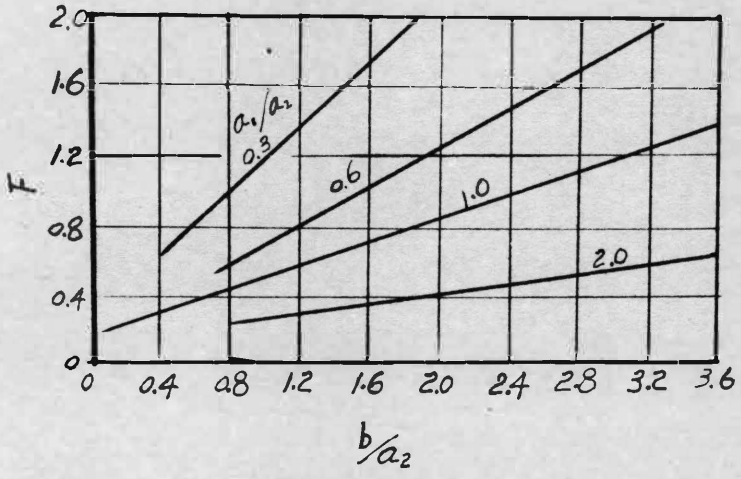
[7] Ibid., p. 1014.

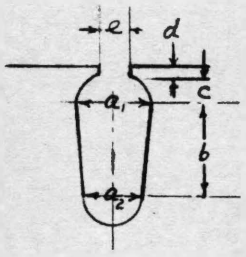
[8] Ibid., p. 1014.

Substituting K_s into Eq. (74), we obtain

$$X_{slot} = 2x3.14x60(2x4x200)^2 10^{-8} x6.38x1.75x1.57/18$$

$$= 9.40 \text{ ohms.}$$





Slot

Fig. 13. Curves for Obtaining the Slot Leakage Constant,

$$K_s = F + d/e + 2c/(e+a_1).$$

The skew leakage reactance is

$$X_{sk} = 8.22 \text{ ohms.} \quad \text{(se V-2-c)}$$

The zig zag leakage reactance[9] is

$$X_z = \frac{5}{6} X_{md}(P/Q_2)^2 \qquad (75)$$

$$= \frac{5}{6}x265(4/18)^2 = 10.90 \text{ ohms.}$$

Then

$$X_{lr} = X_{slot} + X_{sk} + X_z$$

$$= 9.40 + 8.22 + 10.90 = 28.5 \text{ ohms, say.}$$

---

[9]Kuhlmann, op. cit., p. 221.

From V-2-b we have $X_{md} = 265.0$ ohms. By Eq. (73) we obtain

$$X_d = 28.5 + 265.0 = 293.5 \text{ ohms.}$$

(f) Rotor self-reactance in the quadrature axis:

$$X_q = X_{er} + X_{mq}. \tag{76}$$

The magnetizing reactance in quadrature axis is

$$X_{mq} = X_{md}A_q/A_d, \tag{77}$$

where A_q is the ratio of the maximum value of the fundamental to the maximum value of the air gap flux wave when a salient pole machine is excited only by a sine-wave mmf in the quadrature axis and A_d is the ratio of the maximum value of the fundamental to the maximum value of the air gap flux wave when a salient pole machine is excited only by a sine-wave mmf in the direct axis.

Assume the permeance of air gap in the region between two adjacent pole tips is 1/5 of that under the pole, then the formula[10] for A_q is

$$A_q = 1/5\ (1 + 4\psi) - 4 \sin(\psi\pi)/5\pi, \tag{78}$$

where ψ = (pole arc)/(pole pitch) = 0.78 (see IV-1-b)

Hence $A_q = (1 + 4x0.78)/5 - 4 \sin(0.78\pi)/5\pi = 0.662.$

The value of A_d can be expressed[11] as

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[10] Langsdorf, S. H, Theory of A. C. Machinery (New York: McGraw-Hill Book Company, 1940), p. 378.

[11] See Appendix II.

$$A_d = (1 + 4\psi)/5 + 4 \sin\psi\pi/5 \qquad (79)$$

$$= 1/5 + 4x0.78/5 + 4 \sin(0.78\pi)/5\pi = 0.986.$$

Then $X_{md} = 265x0.662/0.986 = 178.0$ ohms.

Therefore $X_q = 28.5 + 178.0 = 206.5$ ohms.

3. List of Constants:

All the constants calculated above are listed as follows:

$r = 10.1$ ohms at $25^\circ C$

$= 11.6$ ohms at $65^\circ C$

$r_s = 80.3$ ohms at $25^\circ C$

$= 92.3$ ohms at $65^\circ C$

$R = 24.5$ ohms at $25^\circ C$

$= 28.5$ ohms at $65^\circ C$

$X = 290$ ohms

$X_s = 89.0$ ohms

$X_m = 85.8$ ohms

$X_d = 293.5$ ohms

$X_q = 206.5$ ohms

$X_{md} = 265.0$ ohms

$X_{sd} = wM_s \cos\beta = 77.5 \cos 48.4^\circ = 51.5$ ohms

$X_{sq} = wM_s \sin\beta = 77.5 \sin 48.4^\circ = 58.0$ ohms

4. Core Loss:

The weight of pole laminations,

$$G_p = PLl_p W_p x0.93x0.278 \qquad (80)$$

$$G_p = 4x1.75x0.625x1.003x0.93x0.278 = 1.138 \text{ lbs.}$$

From Fig. 12 (p. 47), the core loss per pound per cycle is 0.01 watt for B_p = 36,700 lines per sq.in. Then the core loss in poles,

$$W_{cp} = G_p \, fx0.01 = 1.138x60x0.01 = 0.683 \text{ watt.}$$

The iron weight of stator yoke (referred to Fig. 9),

$$G_y = 0.25\pi(3.0 + 0.25)1.75x0.278 = 1.24 \text{ lbs.}$$

From Fig. 12, for B_y = 88,500 lines per sq.in., the core loss per pound per cycle is 0.054 watt. Then the core loss in stator yoke,

$$W_{cy} = 1.24x60x0.054 = 4.02 \text{ watts.}$$

The core loss of rotor and additional losses (as surface losses in the teeth due to variation in the air gap density, losses due to slot filing, losses due to non-uniform flux distribution and losses in the end plates and end-brackets) can be estimated as 0.8 to 1.2 times that of the stator. For this motor assume the total core loss is 2 times the stator core losses,

$$W_c = 2(W_{cp} + W_{cy}) = 2(0.683 + 4.02) = 9.41 \text{ watts.}$$

The component current for core loss is

$$I_c = W_c/E \qquad (81)$$

$$= 9.41/115 = 0.082 \text{ ampere.}$$

5. Friction and Windage Loss:

For the small motor, the friction and windage loss is generally assumed to be 8% of the rated output at rated speed and varies as the square of the speed.

W_{fw} = 0.08x18.65 = 1.49 watts at 1550 rpm,

since rated output = 1/40 hp = 18.65 watts.

6. Performance Calculations:

First of all, calculate the following factors (from A to V) in terms of S. Then compute the results in the calculation sheet.

$$A_1 = -r_s(X_d X_q - R^2) - RX_s(X_d + X_q) + R(X_{sd}^2 + X_{sq}^2)$$

$$= -92.3(293.5x206.5 - \overline{28.2}^2) - 28.2x89.0(293.5 + 206.5) + 28.2(\overline{51.5}^2 + \overline{58.0}^2)$$

$$= -6.605x10^6,$$

$$B_1 = r_s R(X_d + X_q) - X_s(X_d X_q - R^2) + (X_q X_{sd}^2 + X_d X_{sq}^2)$$

$$= 92.3x28.2(293.5 + 206.5) - 89.0(293.5x206.5 - 28.2^2) + (206.5x\overline{51.5}^2 + 293.5x\overline{58.0}^2)$$

$$= -2.495x10^6,$$

$$A_2 = r_s X_d X_q S = 92.3x293.5x206.5S$$

$$= 5.600x10^6 S,$$

$$B_2 = (X_s X_d X_q - X_q X_{sd}^2 - X_d X_{sq}^2)S$$

$$= (89.0x293.5x206.5 - 206.5x\overline{51.5}^2 - 293.5x\overline{58.0}^2)S$$

$$= 3.865x10^6 S,$$

$$C_1 = R(X_m(X_d + X_q) - X_{md} X_{sd})$$

$$= 28.2(85.8x(293.5+206.5) - 265.0x51.5)$$

$$= 0.825x10^6,$$

$$D_1 = X_m(X_dX_q - R^2) - X_qX_{md}X_{sd}$$
$$= 85.8(293.5\text{x}206.5 - \overline{28.2^2}) - 206.5\text{x}265.0\text{x}51.5$$
$$= 2.310\text{x}10^6$$

$$C_2 = 0$$

$$D_2 = -RX_{md}X_{sq} - (X_mX_d - X_{md}X_{sd})X_qS$$
$$= -28.2\text{x}265.0\text{x}58.0 - (85.8\text{x}293.5 - 265.0\text{x}51.5)\text{x}206.5S$$
$$= -(0.434 + 2.381S)\text{x}10^6,$$

$$E_1 = R(X_sX_{md} - X_mX_{sd}) + r_sX_qX_{md}$$
$$= 28.2(89.0\text{x}265.0 - 85.8\text{x}51.5) + 92.3\text{x}206.5\text{x}265.0$$
$$= 5.590\text{x}10^6,$$

$$F_1 = X_q(X_sX_{md} - X_mX_{sd}) - X_{md}(r_sR + X_{sq}^2)$$
$$= 206.5(89.0\text{x}265.0 - 85.8\text{x}51.5) - 265.0(92.3\text{x}28.2 + \overline{58.0^2})$$
$$= 2.380\text{x}10^6,$$

$$E_2 = -r_sX_qX_{md}S = -92.3\text{x}206.5\text{x}265.0S$$
$$= -5.060\text{x}10^6S,$$

$$F_2 = RX_mX_{sq} - (X_q(X_sX_{md} - X_mX_{sd}) - X_{md}X_{sq}^2)S$$
$$= 28.2\text{x}85.8\text{x}58.0 - (206.5(89.0\text{x}265.0 - 85.8\text{x}51.5)$$
$$- 265.0\text{x}\overline{58.0^2})S$$
$$= (0.140 - 3.068S)\text{x}10^6,$$

$$G_1 = -RX_mX_{sq} = -28.2\text{x}85.8\text{x}58.0$$
$$= -0.141\text{x}10^6,$$

$$H_1 = -X_{sq}(X_m X_d - X_{md}X_{sd})$$

$$= -58.0(85.8x293.5 - 265.0x51.5)$$

$$= -0.670x10^6,$$

$$G_2 = r_s R X_{md} = 92.3x28.2x265.0$$

$$= 0.690x10^6,$$

$$H_2 = R(X_s X_{md} - X_m X_{sd}) + (X_m X_d - X_{md}X_{sd})X_{sq}S$$

$$= 28.2(89.0x265.0 - 85.8x51.5) + (85.8x293.5 - 265.0x51.5)$$

$$x58.0S$$

$$=(0.540 + 0.670S)x10^6,$$

$$U_1 = rA_1 - XB_1 - X_m D_1 - X_{md}F_1$$

$$= (-11.6x6.605 + 290.0x2.495 - 85.8x2.310 - 265.0x2.380)10^6$$

$$= -181.5x10^6,$$

$$V_1 = XA_1 + rB_1 + X_m O_1 + X_{md}E_1$$

$$= (-290.0x6.605 - 11.6x2.495 + 85.8x0.825 + 265.0x5.590)10^6$$

$$= -397.2x10^6,$$

$$U_2 = rA_2 - XB_2 - X_m D_2 - X_{md}F_2$$

$$= (11.6x5.600S - 290.0x3.865S - 85.8(0.434+2.381S)$$

$$- 265.0(0.140-3.068S))x10^6$$

$$= 0.10 - 37.5Sx10^6$$

$$V_2 = XA_2 + rB_2 + X_{md}E_2$$

$$=(290x5.600S + 11.6x3.865S -265.0x5.060S)x10^6$$

$$= 327.8Sx10^6.$$

TABLE I

CALCULATION SHEET

Item	Quantity	S (% of syn. speed)			
		0	0.86 (1550 rpm)	0.90 (1620 rpm)	0.967 (1740 rpm)
1	r (hot)	11.6	11.6	11.6	11.6
2	r_s (hot)	92.3	92.3	92.3	92.3
3	R (hot)	28.2	28.2	28.2	28.2
4	X	290.0	290.0	290.0	290.0
5	X_s	89.0	89.0	89.0	89.0
6	X_m	85.8	85.8	85.8	85.8
7	X_d	293.5	293.5	293.5	293.5
8	X_q	206.5	206.5	206.5	206.5
9	X_{md}	265.0	265.0	265.0	265.0
10	X_{sd}	51.5	51.5	51.5	51.5
11	X_{sq}	58.0	58.0	58.0	58.0
12	$A_1 \times 10^{-6}$	-6.605	-6.605	-6.605	-6.605
13	$B_1 \times 10^{-6}$	-2.495	-2.495	-2.495	-2.495
14	$A_2 \times 10^{-6} = 5.600S$	0	4.810	5.040	5.410
15	$B_2 \times 10^{-6} = 3.865S$	0	3.320	3.480	3.740
16	$A \times 10^{-6} = (A_1 + SA_2)10^{-6}$	-6.605	-2.465	-2.075	-1.375
17	$B \times 10^{-6} = (B_1 + SB_2)10^{-6}$	-2.495	0.360	0.635	1.123
18	$(A + jB)10^{-6}$	$7.07/\underline{200.7}^\circ$	$2.498/\underline{171.7}^\circ$	$2.170/\underline{163.0}^\circ$	$1.775/\underline{140.7}^\circ$

TABLE I, CONTINUED

Item	Quantity	S (% of syn. speed)			
		0	0.86 (1550 rpm)	0.90 (1620 rpm)	0.967 (1740 rpm)
19	$C_1 \times 10^{-6}$	0.825	0.825	0.825	0.825
20	$D_1 \times 10^{-6}$	2.310	2.310	2.310	2.310
21	$C_2 \times 10^{-6}$	0	0	0	0
22	$D_2 \times 10^{-6} = -(0.434+2.381S)$	-0.434	-2.484	-2.576	-2.736
23	$C \times 10^{-6} = (C_1+SC_2)10^{-6}$	0.825	0.825	0.825	0.825
24	$D \times 10^{-6} = (D_1+SD_2)10^{-6}$	2.310	0.170	-0.010	-0.335
25	$(C + jD)10^{-6}$	$2.45/\underline{70.35}^\circ$	$0.842/\underline{11.64}^\circ$	$0.825/\overline{0.7}^\circ$	$0.890/\overline{22.1}^\circ$
26	$E_1 \times 10^{-6}$	5.590	5.590	5.590	5.590
27	$F_1 \times 10^{-6}$	2.380	2.380	2.380	2.380
28	$E_2 \times 10^{-6} = -5.060S$	0	-4.350	-4.600	-4.900
29	$F_2 \times 10^{-6} = 0.140-3.068S$	0.140	-2.500	-2.620	-2.825
30	$E \times 10^{-6} = (E_1+SE_2)10^{-6}$	5.590	1.850	1.490	0.865
31	$F \times 10^{-6} = (F_1+SF_2)10^{-6}$	2.380	0.230	0.020	-0.350
32	$(E + jF)10^{-6}$	$6.07/\underline{23.08}^\circ$	$1.86/\underline{7.1}^\circ$	$\underline{1.49/0.77}^\circ$	$0.938/\overline{22.6}^\circ$
33	$G_1 \times 10^{-6}$	-0.141	-0.141	-0.141	-0.141
34	$H_1 \times 10^{-6}$	-0.670	-0.670	-0.670	-0.670
35	$G_2 \times 10^{-6}$	0.690	0.690	0.690	0.690
36	$H_2 \times 10^{-6} = 0.54+0.67S$	0.540	1.116	1.143	1.188

TABLE I, CONTINUED

Item	Quantity	S (% of syn. speed) 0	0.86 (1550 rpm)	0.90 (1620 rpm)	0.967 (1740 rpm)
37	$G \times 10^{-6} = (G_1+SG_2)10^{-6}$	-0.141	0.453	0.479	0.525
38	$H \times 10^{-6} = (H_1+SH_2)10^{-6}$	-0.670	0.290	0.360	0.479
39	$(G + jH)10^{-6}$	$0.684\angle 258.1^\circ$	$0.539\angle 32.6^\circ$	$0.600\angle 36.9^\circ$	$0.710\angle 42.4^\circ$
40	$u_1 \times 10^{-6}$	-181.5	-181.5	-181.5	-181.5
41	$v_1 \times 10^{-6}$	-397.2	-397.2	-397.2	-397.2
42	$u_2 \times 10^{-6} = 0.10-37.5S$	0.10	-32.1	-33.6	-36.2
43	$v_2 \times 10^{-6} = 327.8S$	0	282.0	295.0	317.0
44	$u \times 10^{-6} = (u_1+Su_2)10^{-6}$	-181.5	-209.1	-211.7	-216.5
45	$v \times 10^{-6} = (v_1+Sv_2)10^{-6}$	-397.2	-155.0	-131.5	-91.0
46	$(u + jv)10^{-6} = \Delta \times 10^{-6}$	$437.2\angle 245.43^\circ$	$260.0\angle 216.55^\circ$	$249.5\angle 211.85^\circ$	$235.0\angle 202.8^\circ$
47	E	115	115	115	115
48	$I = (47)(18)/(46)$	$1.87\angle 44.73^\circ$	$1.105\angle 44.85^\circ$	$1.00\angle 48.85^\circ$	$0.867\angle 62.1^\circ$
49	$I_s = (47)(25)/(46)$	$0.644\angle 175.08^\circ$	$0.372\angle 204.91^\circ$	$0.380\angle 212.55^\circ$	$0.435\angle 224.9^\circ$
50	$I_d = (47)(32)/(46)$	$1.595\angle 222.35^\circ$	$0.822\angle 209.45^\circ$	$0.686\angle 211.08^\circ$	$0.459\angle 225.4^\circ$
51	$I_q = (47)(39)/(46)$	$0.180\angle 12.67^\circ$	$0.238\angle 183.95^\circ$	$0.2762\angle 174.95^\circ$	$0.347\angle 160.4^\circ$

TABLE I, CONTINUED

Item	Quantity	S (% of syn. speed) 0	0.86 (1550 rpm)	0.90 (1620 rpm)	0.967 (1740 rpm)
52	$X_{aq} I_s I_d^*$	$59.5/47.27^\circ$	$17.72/4.54^\circ$	$15.1/1.47^\circ$	$11.58/0.5^\circ$
53	$X_q I_q I_d^*$	$59.4/235.02^\circ$	$40.4/25.5^\circ$	$39.2/36.13^\circ$	$32.9/65.0^\circ$
54	$-X_{md} II_q^*$	$-89.2/57.4^\circ$	$-69.8/139.1^\circ$	$-73.3/126.1^\circ$	$-79.5/98.3^\circ$
55	$-X_{sd} I_s I_q^*$	$-5.96/187.75^\circ$	$-4.56/209^\circ$	$-5.42/37.6^\circ$	$-7.77/64.5^\circ$
56	$-X_d I_d I_q^*$	$-84.2/235.02^\circ$	$-57.4/25.5^\circ$	$-55.7/36.13^\circ$	$-46.7/65.0^\circ$
57	Re (52) + ... + (56)	12.4	51.0	40.6	14.0
58	$T_d = 0.04 \times (57)$	0.496	2.040	1.625	0.560
59	$P_d = 16ST_d$ (w.)	0	28.10	23.40	8.66
60	W_{fw}	0	1.49	1.63	1.89
61	Output (w.) = (59) - (61)	0	27.61	21.77	6.77
62	Core loss	9.41	9.41	9.41	9.41
63	I_c	0.082	0.082	0.082	0.082
64	$I_L = (63) + (48)$	$1.93/40.0^\circ$	$1.165/42.0^\circ$	$1.040/45.5^\circ$	$0.910/57.6^\circ$
65	rI_L^2	43.20	15.75	12.55	9.60
66	$r_s I_s^2$	38.30	12.78	13.32	17.48
67	$R I_d^2$	71.70	19.08	13.28	5.94
68	$R I_q^2$	0.91	1.60	2.16	3.40
69	Total loss = (60) + (62) + (65) up to (68)	164.52	60.11	52.35	47.72

TABLE I, CONTINUED

Item	Quantity	S (% of syn. speed)			
		0	0.86 (1550 rpm)	0.90 (1620 rpm)	0.967 (1740 rpm)
70	Input = (69) + (61)	164.52	87.72	74.12	54.49
71	Eff. = (61)/(70)	0	0.315	0.293	0.123
72	P.F. = (70)/EI_L	0.740	0.655	0.620	0.520
73	H.P. output = (61)/746	0	0.0370	0.0291	0.0091

In the above calculation sheet, the currents, I, I_s, I_d and I_q are calculated by Eqs. (43) to (46). The line current I_L is the vector sum of I and I_c (core loss current).

The average torque developed, T_d, is obtained by Eq. (54).

The power developed is

$$P_d = \frac{2\ (T_d/16)\times 1800S}{33{,}000} \times 746 \text{ watts} = 16ST_d \text{ watts}. \qquad (82)$$

The power output is equal to the power developed minus friction and windage loss and the power input is equal to the power output plus total losses. The efficiency and power factor are calculated in the usual way.

The calculated performance curves are drawn in Fig. 14 with broken lines.

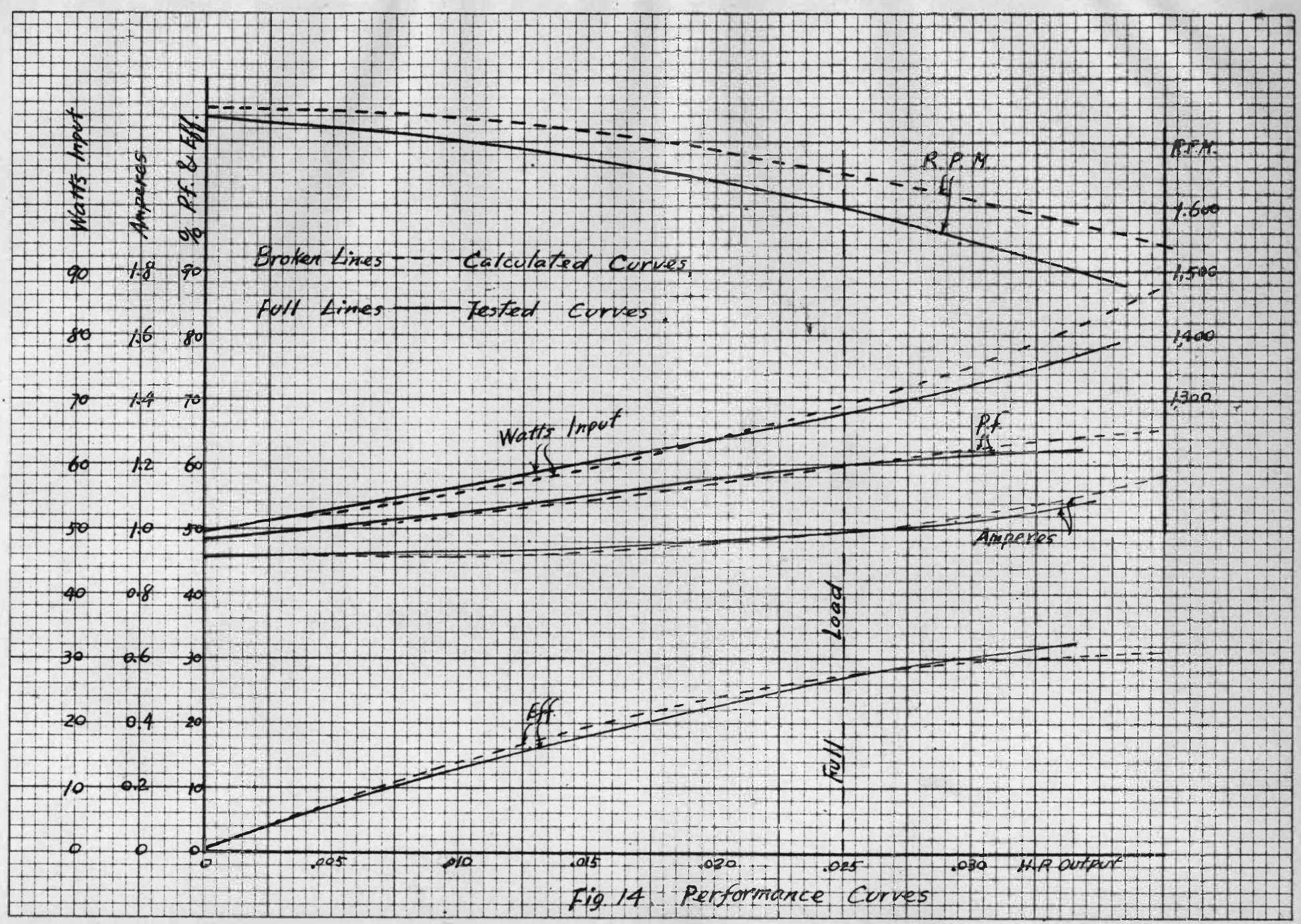


Fig. 14 Performance Curves

7. Constants for Equivalent Circuit:

The ratios of transformation for the four currents at 1550 rpm are:

$$a_1 = I_q/I = \frac{0.238/183.95^\circ}{1.105/44.85^\circ} = 0.215/139.10^\circ,$$

$$a_2 = I_q/I_s = \frac{0.238/183.95^\circ}{0.372/204.91^\circ} = 0.640/20.96^\circ$$

$$a_3 = I_d/I_s = \frac{0.822/209.45^\circ}{0.372/204.91^\circ} = 2.210/4.54^\circ$$

$$a_4 = I_d/I_q = \frac{0.822/209.45^\circ}{0.238/183.95^\circ} = 3.450/25.50^\circ$$

The equivalent impedance,

$$-S(X_{md}/a_1 + X_{sd}/a_2 + a_4 X_d)$$

$$= -0.86(265.0/0.215/139.10^\circ + 51.5/0.640/20.96^\circ + 3.450/25.5^\circ \times 293.5)$$

$$= -49.0 - j294.2 = 299/99.45^\circ.$$

The equivalent impedance,

$$S(X_{sq}/a_3 + X_q/a_4) = 0.86(58.0/2.210/4.54^\circ + 206.5/3.450/25.50^\circ)$$

$$= 69.0 + j24.0 = 73.0/19.20^\circ.$$

The equivalent core loss resistance is:

$$r_m = (\text{core loss})/I_c^2 = 9.41/(0.082)^2 = 1400 \text{ ohms.}$$

The other constants have been given in Table I.

The equivalent circuit for this special machine at 1550 rpm is shown in Fig. 6 by numerical values.

IV. LABORATORY TEST RESULTS

1. Equipment:

This test was carried on in the Electrical Engineering Laboratory of the University of Tennessee. The following equipment was used: ammeter, voltmeter, wattmeter, stroboscope, prony brake and scale.

2. Circuit Diagram:

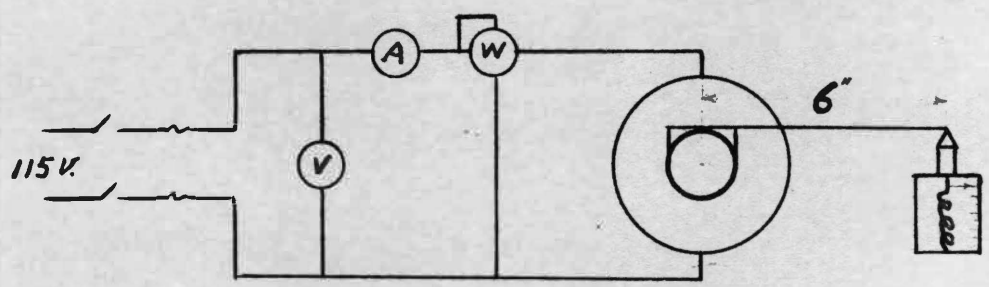


Fig. 15. Circuit Diagram for Test.

3. Procedure:

(a) Instruments connected as shown in Fig. 15 The motor was operated for about twenty minutes in order to warm the machine to its operating temperature.

(b) Increase the load from zero to breakdown point and take readings of voltmeter, ammeter, wattmeter, stroboscope (for rpm) and scale reading for each load.

(c) With the rotor locked the same readings as (b) were obtained except the speed (equal to zero).

4. Data and results:

TABLE II

TEST DATA AND RESULTS

V	I	W	R.P.M.	oz	T	P.F.	H.P.	Eff.
115	0.90	49.6	1730	0.00	0.00	0.478	0.00	0.00
115	0.92	54.6	1715	0.70	0.35	0.516	0.00715	0.0988
115	0.96	60.0	1655	1.30	0.65	0.543	0.0128	0.1590
115	1.020	72.0	1545	3.20	1.60	0.615	0.0294	0.3040
115	1.080	77.0	1500	3.80	1.90	0.620	0.0339	0.3280
115	1.55	114.0	0	0.90	0.45	0.640	0	0

where $T = \frac{1}{2} \times (oz)$ oz.ft.

$P.F. = W/VI,$

$$H.P. = \frac{2\pi T \times (R.P.M.)}{16 \times 33{,}000} = 0.0000119T \times (R.P.M.),$$

$$Eff. = \frac{(H.P.) \times 746}{W}.$$

These laboratory results are shown in Fig. 14 with full lines.

VII. CONCLUSION

For preliminary design it is not necessary to apply the field analysis as given in Section II. In precise design we can improve the field form by means of analysis. As pointed out in II-3 the harmonics of mmf may be eliminated by the conditions of Eqs. (10) and (11). Although these harmonics can not be eliminated in any given case, we can make them much smaller in design by analysis of the field.

The circuit analysis (Section III) must be used in preliminary design for calculating the electrical characteristics. For fractional horsepower motor design, the output should be calculated from torque developed which is obtained through this circuit analysis. Since the output is very small, we can not get it accurately by estimating the constant losses (W_c and W_{fw}) as the method used for larger motor design.

Figure 14 shows that all the calculated performance curves are close to the laboratory results.

The calculated speed regulation is rather better than the actual value. This error may be due to the torque equation (Eq. 54). Actually the rotor windings are not concentrated in direct and quadrature axes and the torque equation must be modified by a distribution factor (less than unity). For this special machine a factor about 0.7 is suitable for multiplying the torque equation which can bring the calculated speed regulation close to the test results.

The other difference is that the locked current and input calculated are larger than the laboratory results (see Calculation Sheet and Test Data.). This error may be due to the temperature of the test

machine which was below the assumed operating temperature (65° C) in design.

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BIBLIOGRAPHY

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APPENDIX I

APPENDIX I

Evaluation of Δ and Determination of Currents.

$$\Delta = \begin{vmatrix} r+jX & jX_m & jX_{md} & 0 \\ jX_m & r_s+jX_s & jX_{sd} & jX_{sq} \\ jX_{md} & SX_{sq}+jX_{sd} & R+jX_d & SX_q \\ -SX_{md} & -SX_{sd}+jX_{sq} & -SX_d & R+jX_q \end{vmatrix}$$

$$= (r+jX)\left\{\begin{array}{l} (r_s+jX_s\,[R^2-X_dX_q+jR(X_d+X_q)+S^2X_dX_q] \\ -jX_{sd}[SRX_{sq}-X_qX_{sd}+jRX_{sd}+jSX_qX_{sq}+S^3X_qX_{sd}-jSX_qX_{sq}] \\ +jX_{sq}[-S^2X_dX_{sq}-jSX_dX_{sd}+SRX_{sd}+X_dX_{sq}-jRX_{sq}+jSX_dX_{sd}] \end{array}\right\}$$

$$-jX_m\left\{\begin{array}{l} jX_m\,[R^2-X_dX_q+jR(X_d+X_q)+S^2X_dX_q] \\ -jX_{sd}[jRX_{md}-X_qX_{md}+S^2X_qX_{md}\,] \\ +jX_{sq}[-jSX_dX_{md}+SRX_{md}+jSX_dX_{md}] \end{array}\right\}$$

$$+jX_{md}\left\{\begin{array}{l} jX_m[SRX_{sq}-X_qX_{sd}+jRX_{sd}+jSX_qX_{sq}+S^2X_qX_{sd}-jSX_qX_{sq}] \\ -(r_s+jX_s)[jRX_{md}-X_qX_{md}+S^2X_qX_{md}] \\ +jX_{sq}[-jSX_{md}X_{sd}-X_{md}X_{sq}+S^2X_{md}X_{sq}+jSX_{md}X_{sd}] \end{array}\right\}$$

$$\Delta = (r+jX)\left\{\begin{array}{l} [-r_s(X_dX_q-R^2)-RX_s(X_d+X_q)+R(X_{sd}^2+X_{sq}^2) \\ +j[r_sR(X_d+X_q)-X_s(X_dX_q-R^2)+X_qX_{sd}^2+X_dX_{sq}^2 \\ +S[r_sX_dX_qS] \\ +jS[S(X_sX_dX_q-X_qX_{sd}^2-X_dX_{sq}^2)] \end{array}\right\}$$

$$+jX_m\left\{\begin{array}{l} R\left[X_m(X_dX_q)-X_{md}X_{sd}\right]+j\left[X_m(X_dX_q-R^2)-X_qX_{md}X_{sd}\right] \\ +S(0)+jS\left[-RX_{md}X_{sq}-(X_mX_d-X_{md}X_{sd})X_qS\right] \end{array}\right\}$$

$$+jX_{md}\left\{\begin{array}{l} \left[R(X_sX_{md}-X_mX_{sd})+r_sX_qX_{md}\right] \\ +j\left[X_q(X_sX_{md}-X_mX_{sd})-X_{md}(r_sR+X_{sq}^2)\right] \\ +S\left[-Sr_sX_qX_{md}\right] \\ +jS\left[RX_mX_{sq}-(X_q(X_sX_{md}-X_mX_{sd})-X_{md}X_{sq}^2)S\right] \end{array}\right\}.$$

Let $A_1=-r_s(X_dX_q-R^2)-RX_s(X_d+X_q)+R(X_{sd}^2+X_{sq}^2)$,

$B_1=r_sR(X_d+X_q)-X_s(X_dX_q-R^2)+(X_qX_{sd}^2+X_dX_{sq}^2)$,

$A_2=r_sX_dX_qS$,

$B_2=(X_sX_dX_q-X_qX_{sd}^2-X_dX_{sq}^2)S$,

$A=A_1+A_2S$,

$B=B_1+B_2S$;

$C_1=R\left[X_m(X_d+X_q)-X_{md}X_{sd}\right]$,

$D_1=X_m(X_dX_q-R^2)-X_qX_{md}X_{sd}$,

$C_2=0$,

$D_2=-RX_{md}X_{sq}-(X_mX_d-X_{md}X_{sd})X_qS$,

$C=C_1$,

$D=D_1+D_2S$;

$$E_1 = R(X_s X_{md} - X_m X_{sd}) + r_s X_g X_{md} ,$$

$$F_1 = X_g (X_s X_{md} - X_m X_{sd}) - X_{md} (r_s R + X_{sg}^2) ,$$

$$E_2 = - r_s X_g X_{md} S ,$$

$$F_2 = R X_m X_{sg} - [X_g (X_s X_{md} - X_m X_{sd}) - X_{md} X_{sg}^2] S ,$$

$$E = E_1 + E_2 ,$$

$$F = F_1 + F_2 S ;$$

$$G_1 = - R X_m X_{sg} ,$$

$$H_1 = - X_{sg} (X_m X_d - X_{md} X_{sd})$$

$$G_2 = r_s R X_{md}$$

$$H_2 = R (X_s X_{md} - X_m X_{sd}) + [X_{sg} (X_{md} - X_{md} X_{sd})] S ,$$

$$G = G_1 + G_2 S$$

$$H = H_1 + S H_2 .$$

Then $$\Delta = (r + jX)[A_1 + jB_1 + S A_2 + jS B_2] + jX_m [C_1 + jD_1 + S(0) + jS D_2]$$

$$+ j X_{md} [E_1 + jF_1 + S E_2 + jS F_2]$$

$$= [rA_1 - XB_1 - X_m D_1 - X_{md} F_1] + j[XA_1 + rB_1 + X_m C_1 + X_{md} E_1]$$

$$+ S[rA_2 - XB_2 - X_m D_2 - X_{md} F_2] + jS[XA_2 + rB_2 + X_{md} E_2]$$

$$\Delta = U_1 + jV_1 + SU_2 + jSV_2 = U + jV$$

$$i = \frac{e}{\Delta}[A_1 + jB_1 + SA_2 + jSB_2] = \frac{e}{\Delta}(A + jB)$$

$$i_s = \frac{e}{\Delta}[C_1 + jD_1 + 0 + jSD_2] = \frac{e}{\Delta}(C + jD)$$

$$i_d = \frac{e}{\Delta}[E_1 + jF_1 + SE_2 + jSF_2] = \frac{e}{\Delta}(E + jF)$$

$$i_q = -\frac{e}{\Delta}\left\{\begin{array}{l} jX_m[-S^2X_dX_{sq} - jSX_dX_{sd} + SRX_{sd} + X_dX_{sq} - jRX_{sq} + jSX_dX_{sd}] \\ -(r_s + jX_s)[-jSX_dX_{md} + SRX_{md} + jSX_dX_{md}] \\ +jX_{sd}[-jSX_{md}X_{sd} - X_{md}X_{sq} + S^2X_{md}X_{sq} + jSX_{md}X_{sd}] \end{array}\right\}$$

$$= \frac{e}{\Delta}\left\{\begin{array}{l} -RX_mX_{sq} + j[-X_{sq}(X_mX_d - X_{md}X_{sd})] + S(r_sRX_{md}) \\ +jS[R(X_sX_{md} - X_mX_{sd}) + X_{sq}(X_mX_d - X_{md}X_{sd})S] \end{array}\right\}$$

$$= \frac{e}{\Delta}(G_1 + jH_1 + SG_2 + jSH_2) = \frac{e}{\Delta}(G + jH)$$

APPENDIX II

APPENDIX II

Derivation of A_d (Eq. 79.)

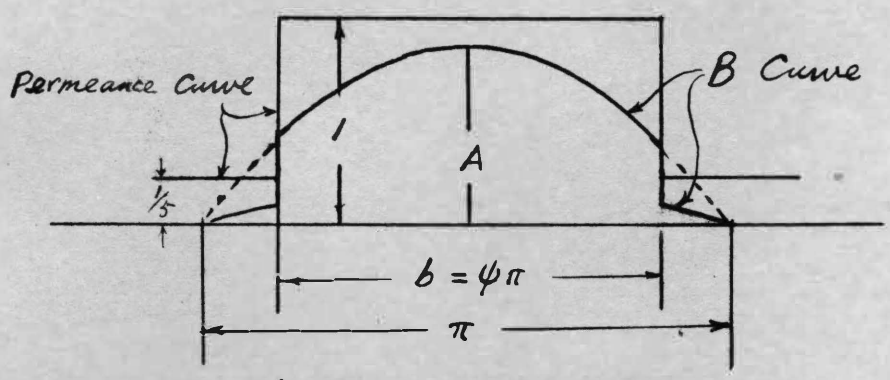


Fig. 16. Curves for Obtaining A_d.

$$A_d = \frac{\frac{2}{\pi}\int_0^{\pi} B \sin\alpha \, d\alpha}{A}$$

Assume $mmf = A\sin\alpha$, Then $B = PA\sin\alpha$.

$$\therefore \quad A_d = \frac{2}{\pi A}\left\{\int_0^{\frac{\pi-b}{2}} \frac{1}{5} A\sin^2\alpha \, d\alpha + \int_{\frac{\pi-b}{2}}^{\frac{\pi+b}{2}} A\sin^2\alpha \, d\alpha + \int_{\frac{\pi+b}{2}}^{\pi} \frac{1}{5} A\sin^2\alpha \, d\alpha\right\}$$

$$= \frac{2}{4\pi}\left\{\frac{1}{5}\left[\pi - b - \sin(\pi-b) + \left[\pi + b - \sin(\pi+b) - \pi + b + \sin(\pi-b)\right] + \frac{1}{5}\left[2\pi - \pi - b + \sin(\pi-b)\right]\right\},$$

$$\text{or} \quad A = \frac{1}{5}(1+4\psi) + \frac{4}{5\pi}\sin\psi\pi$$