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Date December 1986

FLOW INDUCED VIBRATION
WITHIN
CENTRIFUGAL GAS COMPRESSORS

A Thesis
Presented for the
Master of Science Degree
The University of Tennessee
Knoxville, Tennessee

Thomas D. DeStefano

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ABSTRACT

The purpose of this study is to determine the effects of aerodynamically forced disturbances occurring in the impeller channels of centrifugal gas compressors. These disturbances are induced at the diffuser inlet when the tips of the impeller blades pass in front of the tips of the diffuser vanes, thereafter propagating into the impeller channels. In these channels, the disturbance waves can be amplified enough to cause severe structural damage to the walls.

Theory was developed and coded into a Fortran program that gave the steady and unsteady (disturbance) properties at all points within the impeller channel above the inducer region and in the region between the impeller and the diffuser.

The results revealed that keeping all design variables constant and changing only the ratio of diffuser vanes to impeller blades, greatly influenced the amplitude of the disturbances within the impeller channels. Consequently, with the program developed here, the optimum ratio of diffuser vanes to impeller blades can be determined without testing. This is one major finding, but it is obvious that the program will give valuable insight into many compressor design changes.

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LIST OF SYMBOLS

Symbols with subscript 0 denote steady flow variables.

Symbols with superscript, prime, denote unsteady perturbation variables.

A bar over the symbol implies a vector quantity; without, implies its magnitude.

Symbols with subscript ∞ imply variables at an upstream reference point.

ρ - density

R - ideal gas constant

T - temperature

P - pressure

r - radial distance

θ - angular distance

t - time

Z - axial distance

C_p - constant pressure specific heat

C_v - constant volume specific heat

γ - specific heat ratio

ω - angular velocity

ϕ - velocity potential

M - mach number

c - sonic speed

ℓ - axial channel thickness

n - vibration mode number

s - entropy per unit mass

1. INTRODUCTION

This paper investigates the effects of aerodynamically forced disturbances occurring within a centrifugal gas compressor. The typical centrifugal gas compressor geometry is shown in Figure 1.1. As the tips of the impeller blades pass in front of the tips of the diffuser vanes, a periodic forced acoustic disturbance is induced which propagates into the impeller channels. The possibility exists that this forced disturbance is amplified within the impeller channels if a resonance point is reached, causing high pressure fluctuations along the channel walls. Because of this, a major concern for structural damage arises. This effect may be investigated through testing with appropriate instrumentation or, as this study shows, can be predicted theoretically. This theoretical approach is more appealing than the other because, with the proper model and computer program, many types of compressor geometries and their design modifications can be investigated with comparatively little time and cost.

The field to be investigated is in a coordinate system centered at the center of the impeller and fixed to it. This is shown in Figure 1.2. The field extends radially from the impeller inlet (where it is assumed the impeller channel is radially oriented) to the tips of the diffuser vanes. Also, it extends in the angular direction from one impeller blade to its adjacent one, or from one impeller blade wake to its adjacent wake. For this field, a linear partial differential equation is developed to calculate the pertinent properties of all points within it. The field is broken into a gridwork and this equation

is solved for each grid point using finite difference techniques. A computer program was developed to do this and is presented in Appendix C.

The assumptions for the flow field of interest are:

1. irrotational flow in the inertial reference frame,
2. adiabatic flow,
3. ideal gas,
4. homentropic flow,
5. inviscid flow.

Even though these assumptions would probably not allow an accurate prediction of steady flow (i. e., mean flow) performance, because of well known viscous influences, it should be kept in mind that interest is focused on sound propagation within the impeller ducts, where the steady flow need only approximate the characteristics of the medium.

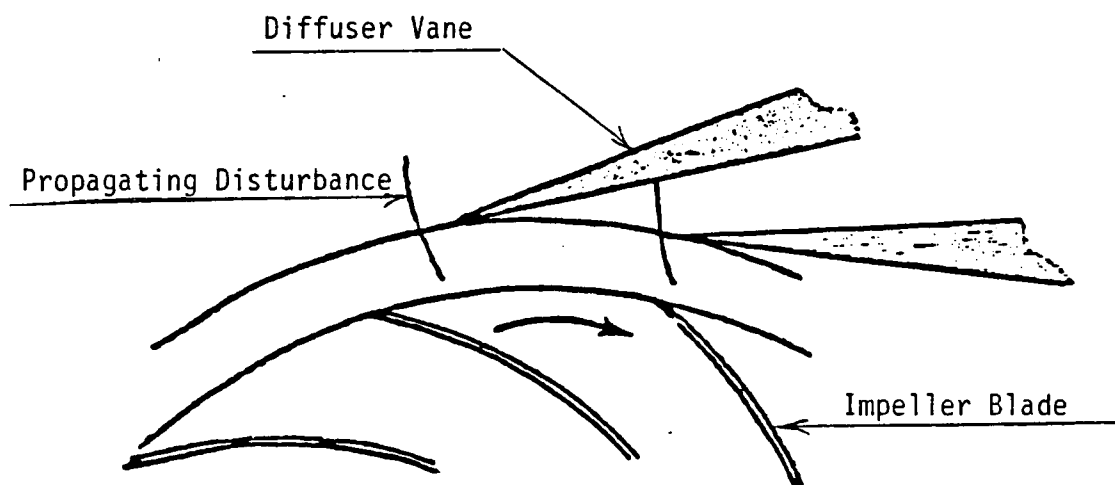


Figure 1.1 Vaned Diffuser Excitation of a Radial Impeller Exit

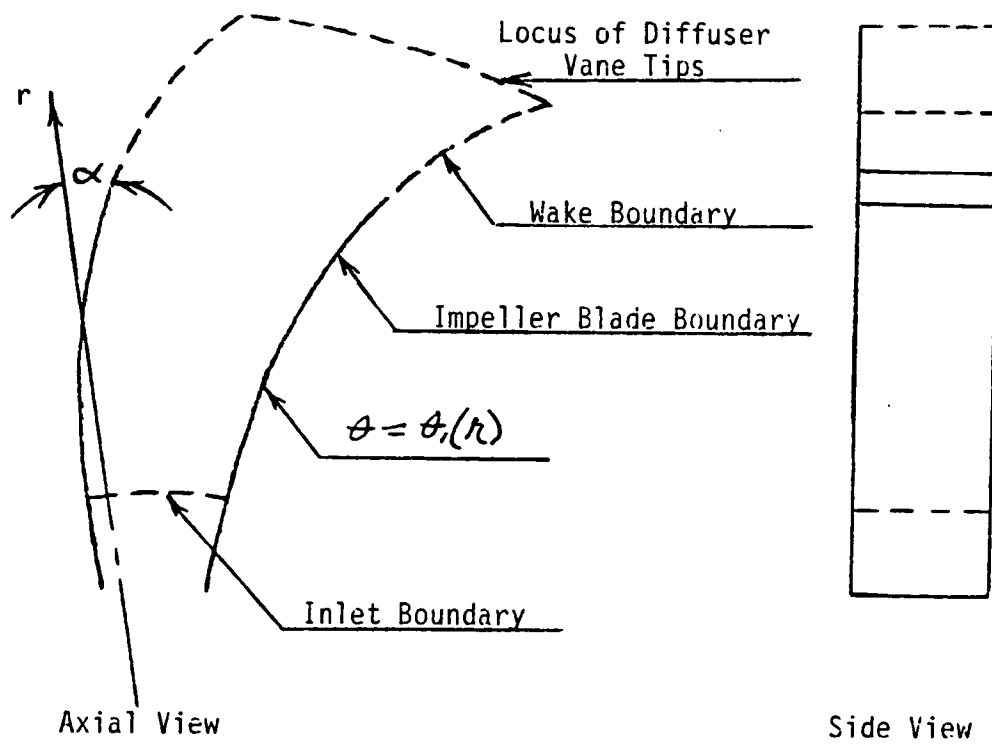


Figure 1.2 Impeller Geometry

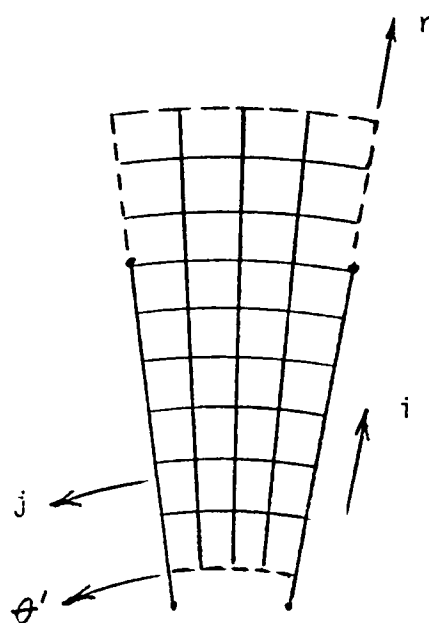


Figure 1.3 Transformed Geometry and Numerical Grid

2. GEOMETRY AND NORMALIZATION

All variables used in the solutions are non-dimensionalized by convenient normalization constants. All distances are normalized by the outer impeller radius, R . The normalization constants for the flow properties are taken to be the properties of a reference point upstream of the compressor. They are denoted by a subscript ∞ .

It was desirable to have the gridwork consisting of grids of the same shape and to have the channel walls coincide with a grid point at each radial grid step. Because most impeller blades are curved, a coordinate transformation had to be made to accomplish this. The transformation is shown in Figure 1.2 and is defined by

$$\theta' = \theta - \theta_1(r) \quad (2.1)$$

$$r' = r \quad (2.2)$$

$$Z' = Z \quad (2.3)$$

where the assumption is made that the Z dimension of the channel is constant.

3. DEVELOPMENT OF THE SOLUTION EQUATION

The governing equation of the field takes the form

$$\nabla^2 \phi - \frac{D_0^2 \phi}{Dt^2} / \hat{c}_0^2 + \left[\left(\frac{2}{\gamma - 1} \right) \frac{\nabla \hat{c}_0}{\hat{c}_0} \cdot \left(\nabla - \frac{D_0}{Dt} \hat{c}_0^{-2} \frac{D_0}{Dt} \right) \phi \right] = 0 \quad (3.1)$$

The development of this equation is described in Appendix A. Referring to Figure 1.2, equation 3.1 is written for the transformed system using equations 2.1, 2.2, and 2.3 and noting that

$$\frac{\partial}{\partial r} = \frac{\partial}{\partial r'} - \frac{\partial}{\partial \theta'} \frac{\tan \alpha}{r} \quad (3.2)$$

$$\frac{\partial^2}{\partial r^2} = \frac{\partial^2}{\partial r'^2} - \left(\frac{2 \tan \alpha}{r} \right) \frac{\partial^2}{\partial r' \partial \theta'} + \frac{\partial^2}{\partial \theta'^2} \left(\frac{\tan^2 \alpha}{r^2} \right) - \frac{\partial}{\partial \theta'} \left(\frac{d^2 \theta_1}{dr^2} \right) \quad (3.3)$$

$$\frac{\partial}{\partial \theta} = \frac{\partial}{\partial \theta'} \quad (3.4)$$

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial z'} \quad (3.5)$$

where r is normalized by R .

Incorporating equations 3.2 through 3.5 into equation 3.1 and collecting terms results in

$$\begin{aligned} & (1 - M_{r0})^2 \frac{\partial^2 \phi}{\partial r'^2} + \left[\frac{1}{r^2} + \left(\frac{d\theta_1}{dr} \right)^2 \right] \frac{\partial^2 \phi}{\partial \theta'^2} - 2 \frac{d\theta_1}{dr} \frac{\partial^2 \phi}{\partial r' \partial \theta'} + \frac{\partial^2 \phi}{\partial z'^2} \\ & + \frac{\partial \phi}{\partial r} \left[\left(\frac{1}{r} - 2iK\hat{v}_{r0}/\hat{c}_0 \right) - \frac{\partial}{\partial r} \left(\frac{\hat{v}_{r0}^2}{2} \right) + \frac{2}{(\gamma - 1)\hat{c}_0} \frac{\partial \hat{c}_0}{\partial r} \right. \\ & \left. - v_{r0}^2 \frac{\partial}{\partial r} (\hat{c}_0^{-2}) \right] + \left[- \frac{d^2 \theta_1}{dr^2} - \frac{d\theta_1}{dr} / r - \frac{2}{(\gamma - 1)\hat{c}_0} \left(\frac{\partial \hat{c}_0}{dr} \right) \frac{d\theta_1}{dr} \right. \\ & \left. + \frac{2}{(\gamma - 1)\hat{c}_0 r^2} \left(\frac{\partial \hat{c}_0}{\partial \theta} \right) \right] \frac{\partial \phi}{\partial \theta} + \left[\frac{K^2}{\hat{c}_0^2} - iK\hat{v}_{r0} \frac{\partial \hat{c}_0^{-2}}{\partial r} \right] \phi - v_{r0}^2 \frac{\partial \hat{c}_0^2}{\partial r} = 0 \end{aligned} \quad (3.6)$$

Therefore, the final governing equation has the form

$$\begin{aligned} & c_1(r) \frac{\partial^2 \phi}{\partial r'^2} + c_2(r) \frac{\partial^2 \phi}{\partial \theta'^2} + c_3(r) \frac{\partial^2 \phi}{\partial r' \partial \theta'} + \frac{\partial^2 \phi}{\partial z'^2} \\ & + c_4(r, \theta) \frac{\partial \phi}{\partial r} + c_5(r, \theta) \frac{\partial \phi}{\partial \theta} + c_6(r, \theta) \phi = 0 \end{aligned} \quad (3.7)$$

where,

$$c_6(r, \theta) = \kappa^2 \hat{c}_0^{-2} - i \kappa \hat{v}_{r0} \frac{\partial}{\partial r} (\hat{c}_0^{-2})$$

$$c_1(r) = 1 - M_{r0}^2 ; M_{r0} = \hat{v}_{r0} / \hat{c}_0$$

$$c_2(r) = \frac{1}{r^2} + \left(\frac{d\theta_1}{dr} \right)^2$$

$$c_3(r) = -2 \left(\frac{d\theta_1}{dr} \right)$$

$$c_4(r, \theta) = \frac{1}{r} - 2i \kappa \hat{v}_{r0} \hat{c}_0^{-1} - \frac{\partial}{\partial r} \left(\frac{\hat{v}_{r0}^2}{2} \right) +$$

$$\frac{2}{(\gamma - 1) \hat{c}_0} \frac{\partial \hat{c}_0}{\partial r} - \hat{v}_{r0}^2 \frac{\partial}{\partial r} (\hat{c}_0^{-2})$$

$$c_5(r, \theta) = - \frac{d^2 \theta_1}{dr^2} - \frac{1}{r} \frac{d\theta_1}{dr} - \frac{2}{(\gamma - 1) \hat{c}_0} \frac{\partial \hat{c}_0}{\partial r} \left(\frac{d\theta_1}{dr} \right) + \frac{2 \left(\frac{\partial \hat{c}_0}{\partial \theta} \right)}{(\gamma - 1) \hat{c}_0 r^2}$$

Now the assumption is made that the impeller channel width in the axial direction (Z direction) is constant just after the inducer section.

For most compressors, this is a legitimate assumption. Then the perturbation potential can be expanded into a Fourier series as follows

$$\phi(r, \theta, Z) = \sum_{n=-\infty}^{\infty} \phi_n(r, \theta) \cos K_Z^n Z$$

$$K_Z^n = \frac{n\pi}{\ell} \quad (3.8)$$

where ℓ is the width of the channel in the Z direction.

Substituting equation 3.8 into equation 3.7 results in

$$\begin{aligned} c_1(r) \frac{\partial^2 \phi_n}{\partial r^2} + c_2(r) \frac{\partial^2 \phi_n}{\partial \theta^2} + c_3(r) \frac{\partial^2 \phi_n}{\partial r \partial \theta} \\ + c_4(r, \theta) \frac{\partial \phi_n}{\partial r} + c_5(r, \theta) \frac{\partial \phi_n}{\partial \theta} + \bar{c}_6(r, \theta) \phi_n = 0 ; \end{aligned} \quad (3.9)$$

$$\bar{c}_6 = c_6 - (\kappa_Z^n)^2$$

It should be noted, the governing equation no longer has a dependence. Using equation 3.9, the ϕ_n may be solved independently.

If consideration is limited to the high-frequency cases such as are typical of the diffuser excited impeller ($K \cong 20$), then the steady flow derivatives are small compared to derivatives of the unsteady flow. The governing equation then can be simplified to

$$\begin{aligned}
 & c_1(r) \frac{\partial^2 \phi_n}{\partial r^2} + c_2(r) \frac{\partial^2 \phi_n}{\partial \theta^2} + c_3(r) \frac{\partial^2 \phi_n}{\partial r \partial \theta} \\
 & + \bar{c}_4(r, \theta) \frac{\partial \phi_n}{\partial r} + \bar{c}_5(r) \frac{\partial \phi_n}{\partial \theta} + \bar{c}_6(r, \theta) \phi_n = 0 \\
 \\
 & \bar{c}_4 = \frac{1}{r} - 2iK\hat{V}_{r0}\hat{C}_0^{-1} \\
 \\
 & \bar{c}_5 = - \left(\left(\frac{d^2 \theta_1}{dr^2} \right) + \left(\frac{d\theta_1}{dr} / r \right) \right) \\
 \\
 & \bar{c}_6 = K^2 \hat{C}_0^{-2} - (K_Z^n)^2 \tag{3.10}
 \end{aligned}$$

In this paper, equation 3.9 will be used for the solution process.

4. BOUNDARY CONDITIONS

Referring to Figure 1.2, the field of interest is broken into two regions: one inside the impeller passage bounded on the sides by the impeller blades and one outside the passage in the diffuser region bounded on the sides by the relative steady base flow streamlines. In the present investigation, the blade surfaces are not allowed to move (although this case may also be easily included) so that the blade surface boundary conditions for the perturbation potential are simply

$$\left. \frac{d\phi_n}{dn} \right|_{\theta=\theta_1} = \left. \frac{d\phi_n}{dn} \right|_{\theta=\theta_1+\theta_s} = 0 \quad (4.1)$$

In the transformed coordinates this is expressed as

$$\left(\frac{\partial \phi_n}{\partial \theta'} - \frac{r}{2} \sin 2\alpha \frac{\partial \phi_n}{\partial r} \right)_{\theta'=0, \theta_s} = 0 \quad (4.2)$$

where α is the angle between the wall surface and a radius through the point as illustrated in Figure 1.2 and θ_s is the blade spacing.

Unsteady vorticity is shed at the blade trailing edges and transported in a singular vortex sheet along the relative steady base flow stream sheets. The boundaries are therefore taken infinitesimally inside these wake sheets so as to exclude the singular region from the solution domain. It is required that the pressure and velocity normal to the stream sheet vary continuously across the sheet. Discontinuities in perturbation velocity tangent to the sheet are allowed, of course, but it is unnecessary to consider this discontinuity explicitly as part

of the calculation procedure. The requirement of blade-to-blade periodicity, with phase shift, (sometimes called the quasi-periodicity condition) can be expressed as

$$e^{i\beta} \phi(r, \theta') = \phi(r, \theta' + \theta_s) \quad (4.3)$$

where β is the time phase shift from blade to blade. This allows the continuity of normal velocity requirement along the wake surface to be expressed as

$$e^{i\beta} \left. \frac{\partial \phi}{\partial n} \right|_{\theta'=0} = \left. \frac{\partial \phi}{\partial n} \right|_{\theta'=\theta_s} \quad (4.4)$$

and continuity of pressure becomes (written in terms of the pressure coefficient)

$$e^{i\beta} p_c \Big|_{\theta'=0} = p_c \Big|_{\theta'=\theta_s} \quad (4.5)$$

In the transformed coordinates equation 4.4 is written as

$$e^{i\beta} \left(\frac{\partial \phi}{\partial \theta'} - \frac{r}{2} \sin 2\alpha \frac{\partial \phi}{\partial r} \right) \Big|_{\theta'=0} = \left(\frac{\partial \phi}{\partial \theta'} - \frac{r}{2} \sin 2\alpha \right) \Big|_{\theta'=\theta_s} \quad (4.6)$$

Using the relation for the linearized pressure coefficient,

$$p_c = -2\rho_0 \left(iK + \hat{V}_{r0} \frac{\partial}{\partial r} \right) \phi_n \quad (4.7)$$

equation 4.5 becomes

$$e^{i\beta} \left(iK + \hat{V}_{r0} \frac{\partial}{\partial r} \right) \phi_n \Big|_{\theta'=0} = \left(iK + \hat{V}_{r0} \frac{\partial}{\partial r} \right) \phi_n \Big|_{\theta'=\theta_s} \quad (4.8)$$

Collecting terms and integrating gives an expression for the jump in potential across the wake sheet (or, with appropriate phase shift, the jump across the blade spacing)

$$e^{i\beta} \phi_n \Big|_{\theta'=0} - \phi_n \Big|_{\theta'=\theta_s} = C e^{-iK} \int_{r_0}^{\psi} \frac{dr}{\hat{V}_{r0}} \quad (4.9)$$

where r_0 is any reference position along the wake and C is the constant obtained by evaluating equation 4.9 at $r = r_0$.

At the outer boundary, a potential disturbance representing the presence of the diffuser blade excitation pattern is imposed. This disturbance may then be expanded in the cosine series of equation 3.8 in the form

$$\phi^d \Big|_{r=r_e} = e^{\frac{i\beta\theta'}{\theta_s}} \sum_{n=0}^{\infty} A_n \cos K_Z^n Z \quad (4.10)$$

where the coefficients, A_n , are considered given. Again, each component may be considered individually and the total result obtained by superposition. Unfortunately, this is not sufficient to complete the boundary specification since this disturbance interacts with the impeller blade flow to scatter acoustic disturbances back through the boundary. It also causes the wake disturbances which must be included in the boundary specification. The entire boundary specification must

then consist of several parts

$$\phi_n \Big|_{r=r_e} = \phi_n^d + \phi_n^s + \phi_n^w \quad (4.11)$$

where neither the back-scattered acoustic radiation, ϕ_n^s , nor the wake disturbances, ϕ_n^w , are known at the boundary. The outer boundary condition, therefore, cannot be directly applied in the form of equation 4.11. An alternative is to find appropriate general forms for ϕ_n^s and ϕ_n^w , and then require that the numerical solution of the field equations in the near vicinity of the boundary, minus the imposed disturbance, ϕ_n^d , match the general form of $\phi_n^s + \phi_n^w$.

For this purpose, solutions of equation 3.9 are sought which are locally valid in the vicinity of the exit boundary. These may be obtained, even if somewhat crudely, after doing the following:

1. Neglect variation in steady base flow quantities in a small region about $r = r_e$, where r_e means exit radius.
2. Then consider equation 3.1 written before the simple harmonic time variation was factored out.

After steps 1, and 2 are taken, the resulting equation is

$$\nabla^2 \phi' - \frac{D_0^2 \phi'}{Dt^2} \left(\frac{1}{\hat{c}_0^2} \right) = 0 \quad (4.12)$$

Defining the transformation

$$\tilde{\theta} = \theta - v_{\theta 0} \Big|_{r_e} \left(\frac{t}{r_e} \right) \quad (4.13)$$

then

$$\left. \frac{\partial}{\partial t} \right|_{\theta} = \left. \frac{\partial}{\partial t} \right|_{\tilde{\theta}} - \frac{v_{\theta 0} \big|_{r_e}}{r_e} \frac{\partial}{\partial \tilde{\theta}} \quad (4.14)$$

so in the transformed system near $r = r_e$

$$\nabla^2 \phi' - \left(\frac{\partial}{\partial t} + v_{r0} \frac{\partial}{\partial r} \right)^2 \phi' / \hat{c}_0^2 = 0 \quad (4.15)$$

Seeking solutions of the form

$$\phi_n^m = \bar{\phi}_n^m(r) e^{i k_{\theta}^m \tilde{\theta}} e^{i \bar{K}^m t} \cos k_z^n z \quad (4.16)$$

where, to satisfy quasi-periodicity,

$$k_{\theta}^m = (\beta \pm 2m\pi) / \theta_s \quad (4.17)$$

and the Doppler frequency shift is obtained from equation 4.14

$$\bar{K}^m = K + v_{\theta 0} \big|_{r_e} \left(\frac{k_{\theta}^m}{r_e} \right) \quad (4.18)$$

yields

$$\begin{aligned} (1 - M_{r0}^2) \frac{d^2 \bar{\phi}^m}{dr^2} + \left(\frac{1}{r} - 2i \bar{K}^m M_{r0} / \hat{c}_0 \right) \frac{d \bar{\phi}^m}{dr} \\ + \left(\left(\frac{\bar{K}^m}{\hat{c}_0} \right)^2 - \left(k_z^n \right)^2 - \left(k_{\theta}^m \right)^2 / r^2 \right) \bar{\phi}^m = 0 \end{aligned} \quad (4.19)$$

where the "n" subscript of ϕ has been dropped for notational clarity and is henceforth to be understood.

Letting

$$\bar{r} = k^m r / \hat{c}_0 \quad (4.20)$$

and

$$\Gamma^2 = 1 - M_{r0}^2 \quad (4.21)$$

The equation takes the form

$$\begin{aligned} \frac{d^2 \bar{\phi}^m}{d\bar{r}^2} + \frac{1}{\Gamma^2} \left(\frac{1}{\bar{r}} - 2iM_{r0} \right) \frac{d\bar{\phi}^m}{d\bar{r}} \\ + \frac{1}{\Gamma^2} \left[1 - \left(k_Z^n \hat{c}_0 / k^m \right)^2 - \left(k_\theta^m / \bar{r} \right)^2 \right] \bar{\phi}^m = 0 \end{aligned} \quad (4.22)$$

Notice that if M_{r0} and k_Z^n equal zero, then the above equation reduces to Bessel's equation. Equation 4.22 can be transformed to standard form by the transformation

$$\bar{\phi}^m = \gamma^m \phi^m \exp \left[-\frac{1}{2} \int \frac{1}{\Gamma^2} \left(\frac{1}{\bar{r}} - 2iM_{r0} \right) d\bar{r} \right] \quad (4.23)$$

which in the present case yields

$$\bar{\phi}^m = \gamma^m \bar{r}^{-\left(\frac{1}{2\Gamma^2}\right)} \exp(iM_{r0}\bar{r}) \phi^m \quad (4.24)$$

so that

$$\frac{d^2 \phi^m}{d\bar{r}^2} - f_m(\bar{r}) \phi^m = 0 \quad (4.25)$$

where

$$f_m(\bar{r}) = \frac{-1}{\Gamma^2} \left\{ 1 - \left(K_{Z0}^n \hat{C}_0 / \bar{K}_m \right)^2 + \left(M_{r0}^2 / \Gamma^2 \right) \right. \\ \left. - \frac{(K_\theta^m)^2 - (2\Gamma^2 - 1)/4\Gamma^2}{\bar{r}^2} + \frac{iM_{r0}}{\Gamma^2 \bar{r}} \right\} \quad (4.26)$$

The Liouville-Green approximate solution of equation 4.25 (see Olver [1]) is given by

$$\gamma_m^m = C f_m^{-\frac{1}{4}}(\bar{r}) \exp \left\{ \pm \int f_m^{\frac{1}{2}}(\bar{r}) d\bar{r} \right\} \quad (4.27)$$

then

$$\bar{\phi}^m = C \bar{r}^{-\left(\frac{1}{2\Gamma^2}\right)} f_m^{-\frac{1}{4}}(\bar{r}) \exp \left\{ \left(iM_{r0} \bar{r} / \Gamma^2 \right) \pm \int f_m^{\frac{1}{2}}(\bar{r}) d\bar{r} \right\} \quad (4.28)$$

Inverting the transformation given by equation 4.13, approximating the integral in the above exponent near $r = r_e$, using equation 4.16, and transforming the result according to equation 2.1, gives finally

$$\phi^m = F_m(\bar{r}) \exp(iK_\theta^m \theta') \quad (4.29)$$

where

$$F_m(\bar{r}) = \bar{r}^{-\left(\frac{1}{2\Gamma^2}\right)} f_m^{-\frac{1}{4}} \exp \left\{ iK_\theta^m \theta_1(r) \right\} \\ \times \exp \left\{ \left(iM_{r0} / \Gamma^2 \pm f_m^{\frac{1}{2}}(r_e) \right) \bar{r} \right\} \quad (4.30)$$

and where the minus sign is taken for outward moving disturbances, the plus sign for inward.

The appropriate form for ϕ_s is thus given by

$$\phi_s = \sum_{m=-\infty}^{\infty} C_m F_m(\bar{r}) \exp(iK_{\theta}^m \theta') \quad (4.31)$$

The solution given by equation 4.31 is continuous throughout the entire field and across the wakes. Another solution of equation 4.12, which is continuous within the bounding "wake" streamlines but is discontinuous across them, may also be obtained by setting each term of the equation to zero independently. That is, let

$$\nabla^2 \phi' = \frac{D_0^2 \phi'}{Dt^2} = 0 \quad (4.32)$$

Assume a solution of the form

$$\phi' = \bar{\phi}(r) \exp(i\lambda\theta) \exp(iKt)$$

from which is obtained the separated equation

$$\frac{d^2 \bar{\phi}}{dr^2} + \frac{1}{r} \frac{d\bar{\phi}}{dr} - \left(\frac{\lambda}{r}\right)^2 \bar{\phi} = 0 \quad (4.33)$$

Solutions of the above Cauchy-Euler equation are of the form $r^{\pm\lambda}$.

Then also the second expression of equation 4.32 is satisfied if

$$\lambda = -iKr_e / (i\hat{V}_{\theta 0} \pm \hat{V}_{r0}) \quad (4.34)$$

Letting $\lambda^{\pm}$ indicate the choice of plus or minus in equation 4.34, the

"wake" solution can now be expressed as

$$\phi^W = W_1 r^{\lambda+} e^{i\lambda^- \theta} + W_2 r^{\lambda-} e^{i\lambda^+ \theta} \quad (4.35)$$

or in terms of the transformed coordinates

$$\phi^W = W_1 r^{\lambda+} e^{i\lambda+(\theta'+\theta_1(r))} + W_2 r^{\lambda-} e^{i\lambda-(\theta'+\theta_1(r))} \quad (4.36)$$

where the constants W_1 and W_2 are to be set to satisfy the wake conditions expressed by equations 4.6 and 4.9.

This form is adequate for use in satisfying the exit excitation/radiation boundary condition, but an approximate form is a little more convenient for computation and also brings out the similarity of the above radial form to that for wakes in uniform flow expressed in rectangular cartesian coordinates [2].

This approximation is obtained by expanding the functions of r in a Taylor's series about $r = r_e$ and noting that

$$r \frac{d\theta_1}{dr} = \frac{V_{\theta 0}}{V_{r 0}} \quad (4.37)$$

The wake solution may then be expressed as

$$\phi^W = C e^{-iKr/\hat{V}_{r0}} \left(W_1 e^{i\lambda^+ \theta'} + W_2 e^{i\lambda^- \theta'} \right) \quad (4.38)$$

to first order in Δr , where C is just a constant. Letting

$$a = 1 - \exp(i[\lambda^+ \theta_s - \beta]) \quad (4.39)$$

and

$$b = 1 - \exp(i[\lambda^- \theta_s - \beta]) \quad (4.40)$$

the wake conditions may be satisfied by setting

$$w_1 = \left[\lambda^- + \frac{Kr}{2\hat{V}_{r0}} \sin 2\alpha \right] / a(\lambda^- - \lambda^+) \quad (4.41)$$

$$w_2 = (\lambda^+ + \frac{Kr}{2\hat{V}_{r0}} \sin 2\alpha) / b(\lambda^+ - \lambda^-) \quad (4.42)$$

and

$$C = \Delta\phi \quad (4.43)$$

where $\Delta\phi$ is the jump in the potential across the wake at $r = r_e$.

The entire potential disturbance in the immediate vicinity of $r = r_e$

then must be given by

$$\begin{aligned} \phi = F_0^-(\bar{r})e^{iK_{\theta}^0\theta'} + \sum_{m=-\infty}^{+\infty} \alpha_m F_m^+(\bar{r})e^{iK_{\theta}^m\theta'} \\ + e^{-iKr/\hat{V}_{r0}} \left[w_1 e^{i\lambda^+\theta'} + w_2 e^{i\lambda^-\theta'} \right] \end{aligned} \quad (4.44)$$

where the $F_m^\pm$ have been normalized by their values at $r = r_e - \Delta r$

and the plus or minus indicating the respective choice of signs in

equation 4.30. Details of the procedure for matching this solution with

the numerical solution are given in the next section.

At the lower boundary, the blades of the impeller are assumed to be radially oriented. This can always be satisfied by artificially extending the lower portion of the channel even if the actual channel is still non-radial in the region of interest. Considering the more general case here adds little to the analysis while it would greatly complicate the numerical application of the boundary condition at this end. As in the case of the outer boundary, variation in the steady base flow is neglected in a small region about $r = r_i$. This assumption is somewhat less accurate for the inlet than the exit, however, since tangential gradients may exist within the impeller channel which are not present outside of it. Nevertheless, this assumption, which is tantamount to narrow channel and high frequency assumptions, is adopted in order to render an already cumbersome boundary treatment a little more manageable.

Following the same procedure as that for ϕ^S of the exit boundary, the disturbance at the inlet boundary has the form

$$\phi = \sum_{m=0}^{\infty} \alpha_m F_m^-(r) \cos K_{\theta}^m \theta' \quad (4.45)$$

where here,

$$K_{\theta}^m = \frac{m\pi}{\theta_s} \quad (4.46)$$

so that the above cosine series satisfies the wall boundary condition (equation 4.2) in the vicinity of $r = r_i$ and F_m^- is again given by

equation 4.30 with K_{θ}^m replaced as defined by equation 4.46 and with r_e replaced by r_i .

5. DISCRETIZED EQUATIONS AND SIDE BOUNDARY CONDITIONS

If the derivatives in equation 3.9 are replaced by their centered second order difference expressions on the transformed grid network illustrated in Figure 1.3, then the discretized version of the equation becomes

$$\begin{aligned} \bar{c}_1 \phi_i^j + \bar{c}_2 \phi_i^{j+1} + \bar{c}_3 \phi_i^{j-1} + \bar{c}_4 \phi_{i+1}^j + \bar{c}_5 \phi_{i-1}^j \\ + \bar{c}_6 \left(\phi_{i+1}^{j+1} + \phi_{i-1}^{j-1} - \phi_{i-1}^{j+1} - \phi_{i+1}^{j-1} \right) = 0 \end{aligned} \quad (5.1)$$

where

$$\bar{c}_1 = -2 c_1 \delta^2 - 2c_2 + \Delta\theta^2 (c_6 - K_Z^2) \quad (5.2)$$

$$\bar{c}_2 = c_2 + (\Delta\theta) c_5/2 \quad (5.3)$$

$$\bar{c}_3 = c_2 - (\Delta\theta) c_5/2 \quad (5.4)$$

$$\bar{c}_4 = c_1 \delta^2 + \delta(\Delta\theta) c_4/4 \quad (5.5)$$

$$\bar{c}_5 = c_1 \delta^2 - \delta(\Delta\theta) c_4/2 \quad (5.6)$$

$$\bar{c}_6 = \delta c_3/4 \quad (5.7)$$

$$\delta = \Delta\theta/\Delta r \quad (5.8)$$

The discretized forms of the blade surface boundary conditions given by equation 4.1 are

$$\phi_i^2 - \phi_i^1 = \delta r_i \sin 2\alpha_i (\phi_{i+1}^2 - \phi_{i-1}^2)/4 \quad (5.9)$$

at $\theta' = 0$, and

$$\phi_i^j - \phi_i^{j-1} = \delta r_i \sin 2\alpha_i (\phi_{i+1}^{j-1} - \phi_{i-1}^{j-1})/4 \quad (5.10)$$

at $\theta' = \theta_s$. The above forms are only first order accurate. Second order accurate formulae may also be easily obtained and applied. It is much more difficult to apply second order accurate formulae in the wake boundaries, however, because of the discontinuity of ϕ and its tangent derivative. First order methods were thus used at all the side boundary points.

The wake continuity of pressure condition given by equation 4.9 takes the discretized form

$$e^{i\beta} \phi_i^1 - \phi_i^j = (e^{i\beta} \phi_{i-1}^1 - \phi_{i-1}^j) \exp \left(\frac{2iK\Delta r}{\hat{V}_{r0|i-1} + \hat{V}_{r0|i+1}} \right) \quad (5.11)$$

The continuity of normal velocity condition given by equation 4.6 becomes

$$e^{i\beta} \left[\phi_i^2 - (1 - \xi_i) \phi_i^1 \right] - \left[(1 + \xi_i) \phi_i^j - \phi_i^{j-1} \right] = 0 \quad (5.12)$$

where

$$\xi_i = iK\Delta\theta \sin 2\alpha_i / r_i \hat{V}_{r0} \quad (5.13)$$

Now letting ϕ_i represent the i^{th} vector containing the J values ϕ_i^j ; $j = 1, 2, 3, \dots, J$, then equations 5.1, 5.11, and 5.12 may be assembled in the form

$$[A]_i \phi_{i-1} + [B]_i \phi_i + [C]_i \phi_{i+1} = 0 \quad (5.14)$$

$$i = 2, 3, \dots, I-1$$

where the quantities in brackets are matrix coefficients. The first and last rows of equation 5.14, corresponding to $i = 1$ and $i = I$ respectively, are set aside for the discretized inlet and exit boundary conditions. Naturally, the excitation at the exit boundary will yield a nonhomogeneous set of equations at $i = I$ which will render the solution of the entire system unique.

6. DISCRETIZED EXCITATION AND RADIATION CONDITIONS

To provide the equations at $i = I$, the exit boundary excitation/radiation/wake expression given by equation 4.44 will now be matched to the discrete values of ϕ along the lines $i = I - 1$ and $i = I$. Adopting the summation convention except where noted, and truncating the infinite series at $m = \pm M$, equation 4.44 can be discretized at $i = I - 1$ as

$$(\delta_{j,m} - D_{j,m}) \phi_{I-1}^m = \alpha_n e^{iK_{\theta}^n j} + A e^{iK_{\theta}^0 j} \quad (6.1)$$

$$j, m = 1, 2, \dots, J$$

$$n = -M, -M+1, \dots, M$$

where A is the amplitude of the exciting disturbance, henceforth taken equal to unity, $\delta_{j,m}$ is the Kronecker delta, and $D_{j,m}$ is the discretized wake operator defined by

$$D_{j,m} = (W_1 e^{i\lambda^+ \theta j} + W_2 e^{i\lambda^- \theta j})(\delta_{1,m} - e^{i\beta} \delta_{j,m}) \quad (6.2)$$

The coefficients, α_n , of the discretized Fourier series in equation 6.1 can be obtained as

$$\alpha_n = a_{n,j} (\delta_{j,m} - D_{j,m}) \phi_{I-1}^m - \delta_{0,n} \quad (6.3)$$

$$j, m = 1, 2, \dots, J$$

$$n = -M, -M+1, \dots, M$$

where

$$a_{n,j} = \Omega_j \exp(-i K_{\theta}^n \theta_j') / (J-1) \quad (6.4)$$

No sum on j

$$\Omega_j \begin{cases} 1/2; & j = 1, J \\ 1 & ; j \neq 1, J \end{cases}$$

Then again using equation 4.44 and substituting from the above expression for α_n , the values of ϕ at I can be expressed as

$$\begin{aligned} \phi_I^i = & \left[b_{i,n} a_{n,j} (\delta_{j,m} - D_{j,m}) + \bar{D}_{j,m} \right] \phi_{I-1}^m \\ & + e^{i K_{\theta}^0 \theta_i'} \left[F_0^-(r_e) + F_0^+(r_e) \right] \end{aligned} \quad (6.5)$$

where

$$b_{i,n} = F_n^+(r_e) e^{i K_{\theta}^n \theta_i'} \quad (6.6)$$

no sum on n

and

$$\bar{D}_{j,m} = \exp\left(\frac{i K \Delta r}{\hat{V}_{r0}}\right) D_{i,m} \quad (6.7)$$

In the notation of equation 5.14, equation 6.5 may be put in the form

$$\begin{array}{c}
 \text{identity matrix} \quad \text{null} \\
 \swarrow \quad \uparrow \\
 [A]_I \phi_{I-1} + [B]_I \phi_I + [C]_I \phi_{I+1} = F_I
 \end{array} \quad (6.8)$$

Following a similar procedure for the inlet boundary, equation 4.45 leads to

$$\phi_1^m = \bar{b}_{m,n} \bar{a}_{n,j} \phi_2^j \quad (6.9)$$

where

$$\bar{b}_{m,n} = F_m^{-1}(r_i) \cos K_{\theta_m}^n$$

and

$$\bar{a}_{n,j} = A_n \Omega_j (\cos K_{\theta_j}^n) / (J-1) \quad (6.10)$$

$$\Omega_j = \begin{cases} \frac{1}{2}; & j = 1, J \\ 1; & j \neq 1, J \end{cases}$$

$$A_n = \begin{cases} 1; & n = 0 \\ 2; & n \neq 0 \end{cases}$$

In the notation of equation 5.14 this becomes

$$\begin{array}{c} \text{null} \qquad \text{identity matrix} \\ \nearrow \qquad \nearrow \\ \cancel{[A]}_1 \phi_0 + \cancel{[B]}_1 \phi_1 + [C]_1 \phi_2 = 0 \end{array} \quad (6.11)$$

The entire system may thus be put in the so-called block tridiagonal form

$$[A]_i \phi_{i-1} + [B] \phi_i + [C]_i \phi_{i+1} = F_i \quad (6.12)$$

$$i = 1, 2, \dots, I$$

The above system is solved directly by a generalization to block form of a simple scalar tridiagonal algorithm.

7. RESULTS

An example case was run, using the theory and computer program developed in this paper. The number of impeller blades was held constant at 20, while the angular velocity of the impeller, the mode number in the z direction, and the number of diffuser vanes, were varied. The impeller blades had no curvature. The results are shown in appendix E, on figures E.1 through E.32, pages 71 through 102. These figures are plots of the magnitudes and phase angles of the disturbance pressures versus radius, along the side boundaries shown on figure 1.2, page 4. The figure title symbols are explained on page 70. Hereafter, an impeller duct inlet is considered to mean the inner boundary of the region shown on figure 1.2, page 4.

Starting with the lowest angular velocity of the impeller, 10 diffuser vanes, and z -mode number of 0, figure E.18 shows that the pressure coefficient phase angles within the impeller ducts vary linearly with radius, with a positive slope. This indicates pressure disturbances travelling radially inward from the diffuser region into the impeller ducts, and thus transporting energy into them. Figure E.1 (the magnitude plot for figure E.18) shows that the disturbance amplitudes rise continuously from the exciting disturbances as they move into the ducts, indicating amplification within the impeller ducts.

It is of interest to observe the effect of increasing the impeller angular velocity while keeping the other variables constant. At the lowest angular velocity, disturbance energy propagates radially inward into the impeller ducts and then on out their inlets. Increasing the

angular velocity, shows quasi-standing wave behavior and higher pressure disturbance amplification within the ducts. The onset of quasi-standing wave behavior is seen in going from figures E.19 to E.20; the phase plots go from all positive slope to partly positive and partly negative slopes. In some plots, virtually a standing wave behavior is indicated. This is seen on figures E.6 and E.23. The pressure disturbance magnitude plot shows wave-like behavior with radius, and nodes occur where it goes to zero. The corresponding phase plot shows practically 180 degrees of phase shift at these node points, which always occurs for a standing wave pattern. With further increase in angular velocity up to the highest velocity, the propagating behavior returns, and the pressure disturbance amplitudes are amplified even further. The amplitude plots look periodic about some mean value and the phase plots have very high positive slope. This is shown on figures E.13 and E.30. These plots indicate the occurrence of more complex interactions within the ducts, but the overall effect is amplification of energy in the ducts and transmission of it out their inlets.

When the number of diffuser vanes increase, the standing wave behavior in the impeller ducts begins at lower impeller speeds. This is seen on figures E.10 and E.27 for 25 diffuser vanes and an impeller speed of .5. It seems then, the beginning of the standing wave behavior occurs at a characteristic excitation frequency, since this frequency is the number of vanes times the impeller angular velocity.

In going to higher z-mode numbers, the pressure disturbances within the ducts go from amplification to attenuation. This is seen by comparing figures E.7 and E.8. Also, this is seen by comparing figures

E.14 and E.15. For the particular compressor geometry and variables in this example case, the z-modes greater than 1 are relatively insignificant.

Figures E.16 and E.17 can be used to determine what combination of impeller speed and number of diffuser vanes (for 20 impeller blades) minimum pressure disturbances occur on the blades. For example, at a design speed of 1, the best number of diffuser vanes for 20 impeller blades is 15. These figures also indicate that with just a change of blade-to-vane number ratio the pressure disturbance magnitudes are significantly affected.

8. CONCLUSION

This paper predicts that pressure disturbances generated at the diffuser inlet region can propagate into the impeller channels with amplification, given a certain combination of diffuser vane-to-impeller blade number ratio, and impeller speed. At low speeds (dimensionless impeller angular velocity around .5) the wave energy propagating into the impeller will mostly propagate on out through the impeller inlet. At higher speeds, partial to virtually complete standing wave behavior occurs in the impeller ducts. If at the compressor design speed, this standing wave behavior constitutes the worst condition, from a structural standpoint, since more disturbance energy is entering the impeller ducts than is leaving, resulting in magnified pressure disturbances on their walls.

For compressors with geometry and variables used in the example case, that have impeller channels with axial length that do not vary radially, the lowest two z-mode eigenfunctions for pressure disturbances are the most significant.

A change in the diffuser vane-to-impeller blade number ratio can significantly change the magnitudes of the pressure disturbances within the impeller ducts, which is then an effective means to remedy intolerably high flow induced vibration within centrifugal compressors.

9. RECOMMENDATIONS

This paper can reveal the magnitudes of the pressure disturbances on the impeller blades, but will not easily show how these pressures will put stress on the blade structure. For instance, to know the net force bending a blade, the pressure disturbances on each side of it must be vectorially combined, using their magnitudes and phase angles. For those interested in these stresses, it is advisable they add extra computer algorithms that will vectorially combine the pressure disturbances across the blades, integrate the resulting differential pressures over the blade area, and then calculate blade stresses.

The axial length of the impeller channels was considered constant at any radius in the field of interest. One may want to consider a variable length with radius, as is characteristic of many centrifugal compressors. This can be accomplished by making the variable length a constant length through a coordinate transformation, and then proceeding with the rest of the analysis in these transformed coordinates.

The steady base mean flow field herein, was calculated as a function of radius only. Computer code changes should be made for flows that vary also in the other two directions.

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APPENDICES

A. DERIVATION OF THE POTENTIAL FIELD EQUATION

The development of the linear partial differential equation of the perturbation velocity potential will be presented here. This velocity potential will be a function of distance only, since the harmonic time variation will have been factored out.

For this problem the assumptions are:

1. adiabatic flow,
2. inviscid flow,
3. perfect gas,
4. homentropic flow,
5. irrotational flow with respect to the inertial reference frame.

Figure 1.2 is a schematic of the compressor geometry denoting the field we are to develop the equation for, and the coordinate systems of interest. It is desired to develop the equation for a rotating coordinate system fixed to the impeller and centered at the impeller centerline.

The equation of continuity for the inertial coordinate system is

$$\frac{D\rho}{Dt} + \rho(\nabla \cdot \bar{V}_a) = 0 \quad A.1$$
$$\frac{\partial \rho}{\partial t} + \bar{V}_a \cdot \nabla \rho + \rho(\nabla \cdot \bar{V}_a)$$

where

ρ = density

$\bar{V}_a$ = velocity vector in the inertial coordinate system

$\bar{V}$ = velocity vector relative to the rotating coordinate system

$$\nabla \equiv \frac{\partial}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial}{\partial \theta} \hat{\theta} + \frac{\partial}{\partial Z} \hat{Z}$$

$\hat{r}, \hat{\theta}, \hat{Z}$ are unit vectors for inertial cylindrical coordinate system

r, θ, t are for inertial system

r', θ', t' are for rotating system

$\bar{\omega}$ is the angular velocity vector of the impeller

Therefore,

$$\bar{V}_a = \bar{V} + \bar{\omega} \times \bar{r} \quad \text{A.2}$$

At this point, note the applicable coordinate system transformation to be

$$r = r'$$

$$\theta = \theta' + \omega' t' \quad \text{A.3}$$

$$t = t'$$

then

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} \frac{\partial t}{\partial t'} + \frac{\partial}{\partial \theta} \frac{\partial \theta}{\partial t'} + \frac{\partial}{\partial r} \frac{\partial r}{\partial t'} + \frac{\partial}{\partial Z} \frac{\partial Z}{\partial t'} \quad \text{A.4}$$

which reduces to

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + \omega \frac{\partial}{\partial \theta} \quad \text{A.5}$$

rearranging

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t'} - \omega \frac{\partial}{\partial \theta} \quad \text{A.6}$$

Then, equation A.1 becomes

$$\frac{\partial \rho}{\partial t} + (\bar{V} + \bar{\omega} \times \bar{r}) \cdot \nabla \rho + \rho [\bar{\nabla} \cdot (\bar{V} + \bar{\omega} \times \bar{r})] \quad \text{A.7}$$

Noting that

$$\bar{\omega} \times \bar{r} = \omega r \hat{\theta} \quad \text{A.8}$$

and

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho}{\partial t'} - \omega \frac{\partial \rho}{\partial \theta} \quad \text{A.9}$$

we have

$$\begin{aligned} \frac{\partial \rho}{\partial t'} - \omega \frac{\partial \rho}{\partial \theta} + \omega r \hat{\theta} \cdot \nabla \rho + \bar{V} \cdot \nabla \rho + \rho \bar{\nabla} \cdot \bar{V} \\ + \rho \bar{\nabla} \cdot \omega r \hat{\theta} = 0 \end{aligned} \quad \text{A.10}$$

The last term on the left-hand side of equation A.10 vanishes and

$$\omega r \hat{\theta} \cdot \nabla \rho = \omega \frac{\partial \rho}{\partial \theta} \quad \text{A.11}$$

then, equation A.10 becomes

$$\frac{\partial \rho}{\partial t'} + \bar{V} \cdot \nabla \rho + \rho \nabla \cdot \bar{V} = 0 \quad \text{A.12}$$

and since $t' = t$

$$\frac{\partial \rho}{\partial t} + \bar{V} \cdot \nabla \rho + \rho \nabla \cdot \bar{V} = 0 \quad \text{A.13}$$

At this point, it has been shown that the continuity equation is valid when written with velocity relative to a rotating coordinate system with fixed center.

Using equation A.13, we allow each property to be broken into steady and unsteady parts. The unsteady part is considered to be a small perturbation from the steady part.

$$\begin{aligned} \rho &= \rho_0(r, \theta, Z) + \rho'(r, \theta, Z, t) \\ \bar{V} &= \bar{V}_0(r, \theta, Z) + \bar{V}'(r, \theta, Z, t) \\ |\bar{V}'| &\ll |\bar{V}_0| \quad \rho' \ll \rho_0 \end{aligned} \quad \text{A.14}$$

Substituting these into equation A.13, and expanding and neglecting terms of second order, results in

$$\frac{\partial \rho'}{\partial t} + \nabla \cdot (\rho_0 \bar{V}_0) + \nabla \cdot (\rho' \bar{V}_0) + \nabla \cdot (\rho_0 \bar{V}') = 0 \quad \text{A.15}$$

The second term on the right-hand side of equation A.15 is the steady flow expression of the continuity equation which is zero. Then

$$\frac{\partial \rho'}{\partial t} + \bar{V}_0 \cdot \nabla \rho' + \rho' (\nabla \cdot \bar{V}_0) + \nabla \cdot (\rho_0 \bar{V}') = 0 \quad \text{A.16}$$

Also, from the steady flow continuity equation

$$\nabla \cdot \bar{V}_0 = \frac{-1}{\rho_0} (\bar{V}_0 \cdot \nabla \rho_0) \quad \text{A.17}$$

Substituting equation A.17 into equation A.16 results in

$$\begin{aligned} \frac{\partial \rho'}{\partial t} + \bar{V}_0 \cdot \nabla \rho' - \frac{\rho'}{\rho_0} (\bar{V}_0 \cdot \nabla \rho_0) + \nabla \cdot (\rho_0 \bar{V}') \\ = 0 \end{aligned} \quad \text{A.18}$$

this can be rewritten as

$$\rho_0 \left(\frac{D}{Dt} \left(\frac{\rho'}{\rho_0} \right) + \left(\bar{V}_0 \cdot \nabla \frac{\rho'}{\rho_0} \right) \right) + \nabla \cdot (\rho_0 \bar{V}') = 0 \quad \text{A.19}$$

Since these equations are being developed for a field being forced with a simple harmonic time variation and because it is assumed the transients have died out, all unsteady properties have the same simple harmonic time variation as the forcing function.

Therefore, let

$$\begin{aligned} \rho' &= \rho'^* e^{i\omega t} \\ p' &= p'^* e^{i\omega t} \\ \bar{V}' &= \bar{V}'^* e^{i\omega t} \end{aligned} \quad \text{A.20}$$

Substituting equations A.20 into equation A.19 results in

$$\rho_0 \left(i\omega \left(\frac{\rho'^*}{\rho_0} \right) + \bar{V}_0 \cdot \nabla \left(\frac{\rho'^*}{\rho_0} \right) \right) + \nabla \cdot (\rho_0 \bar{V}'^*) = 0 \quad \text{A.21}$$

Now define the operator D_0/Dt to be

$$\frac{D_0(\quad)}{Dt} = i\omega(\quad) + \bar{V}_0 \cdot \nabla(\quad) \quad A.22$$

Then, equation A.21 can be written as

$$\rho_0 \left(\frac{D_0}{Dt} \left(\frac{\rho'^*}{\rho_0} \right) \right) + \nabla \cdot (\rho_0 \bar{V}'^*) = 0 \quad A.23$$

From a thermodynamic equation for the entropy of a perfect gas

$$ds = C_v \frac{dP}{P} - C_p \frac{d\rho}{\rho} \quad A.24$$

where:

s is entropy per unit mass

P is pressure

C_v is constant volume specific heat

C_p is constant pressure specific heat

Considering each property of equation A.24 to be the sum of a steady and unsteady part as was done before, and neglecting second-order terms, equation A.24 becomes

$$s' = C_v P'/P_0 - C_p \rho'/\rho_0 \quad A.25$$

$$\rho'/\rho_0 = P'/\gamma P_0 - s'/C_p$$

γ is the specific heat ratio. Assuming simple harmonic time variation, equation A.25 becomes

$$\frac{\rho'^{*}}{\rho_0} = \frac{p'^{*}}{\gamma p_0} - \frac{s'^{*}}{c_p} \quad \text{A.26}$$

Since the flow is isentropic

$$\frac{D_0}{Dt} (s'^{*}) = 0 \quad \text{A.27}$$

therefore

$$\frac{D_0}{Dt} \left(\frac{\rho'^{*}}{\rho_0} \right) = \frac{D_0}{Dt} \left(\frac{p'^{*}}{\gamma p_0} \right) \quad \text{A.28}$$

From the expression for the speed of a small disturbance in a perfect gas

$$\gamma p_0 = \rho_0 c_0^2 \quad \text{A.29}$$

where c_0 is local speed of sound of the steady flow field.

Substituting equation A.29 into equation A.28

$$\frac{D_0}{Dt} \left(\frac{\rho'^{*}}{\rho_0} \right) = \frac{D_0}{Dt} \left(\frac{p'^{*}}{\rho_0 c_0^2} \right) \quad \text{A.30}$$

In Appendix B, equation B.18 was developed from the momentum equation which is applicable here. It is

$$\frac{p'^{*}}{\rho_0} = - \frac{D_0}{Dt} (\phi'^{*}) \quad \text{A.31}$$

where ϕ'^{*} is the magnitude of the perturbation velocity potential.

Using the typical notation found here, it was developed by breaking the velocity potential in the rotating coordinate system into steady and unsteady parts, and then assuming harmonic time variation, i.e.

$$\begin{aligned}\phi &= \phi_0(r, \theta, Z) + \phi'(r, \theta, Z, t) \\ \phi' &= \phi'^* e^{i\omega t}\end{aligned}\tag{A.32}$$

Substituting equation A.31 into equation A.30

$$\frac{D_0}{Dt} \left(\frac{\rho'^*}{\rho_0} \right) = \frac{D_0}{Dt} \left(\frac{1}{C_0^2} \frac{D_0}{Dt} \phi'^* \right)\tag{A.33}$$

Now, substitute equation A.33 into equation A.23 getting

$$-\rho_0 \left[\frac{D_0}{Dt} \left(\frac{1}{C_0^2} \frac{D_0}{Dt} \phi'^* \right) \right] + \nabla \cdot (\rho_0 \nabla \phi'^*) = 0\tag{A.34}$$

Equation A.34 is now cast into non-dimensional form using the following normalizations:

$$\begin{aligned}\hat{C}_0 &= \frac{C_0}{C_\infty} & \hat{V}_{\theta 0} &= V_{\theta 0}/C_\infty \\ \hat{\rho}_0 &= \frac{\rho_0}{\rho_\infty} & \hat{\bar{V}}' &= \nabla \phi \\ K &= \frac{\omega R}{C_\infty} & \hat{\ell} &= \ell/R \\ \phi &= \phi'^*/R C_\infty & r &= r/R \\ \hat{\bar{V}}_0 &= \bar{V}_0/C_\infty \\ \hat{V}_{r0} &= V_{r0}/C_\infty\end{aligned}$$

All values with subscript ∞ are for a reference state of the working gas upstream of the compressor

R is the impeller outer radius

V_{r0} is the radial component of the steady flow velocity in the rotating coordinate system.

C_0 is the local speed of sound in the rotating coordinate system.

From equation A.22 for the D_0/Dt operator

$$\frac{D_0}{Dt} (\phi'^*) = i\omega\phi'^* + \bar{V}_0 \cdot \nabla\phi'^* \quad A.35$$

This can be rewritten in non-dimensional form as

$$\frac{D_0}{Dt} (\phi) = iK\phi + \hat{V}_{r0} \frac{\partial\phi}{\partial r} + \frac{\hat{V}_{\theta 0}}{r} \frac{\partial\phi}{\partial\theta} \quad A.36$$

Using these normalization definitions and equation A.36, equation A.34 becomes

$$\frac{1}{\hat{\rho}_0} \nabla \cdot (\hat{\rho}_0 \nabla\phi) - \frac{D_0}{Dt} \left(\frac{D_0\phi}{Dt} / \hat{C}_0^2 \right) = 0 \quad A.37$$

expanding

$$\nabla^2\phi + \frac{\nabla\hat{\rho}_0}{\hat{\rho}_0} \cdot \nabla\phi - \left(\frac{1}{\hat{C}_0^2} \right) \frac{D_0}{Dt} \left(\frac{D_0\phi}{Dt} \right) - \frac{D_0}{Dt} \left(\frac{1}{\hat{C}_0^2} \right) \frac{D_0\phi}{Dt} \quad A.38$$

= 0

rearranging

$$\nabla^2 - \frac{D_0^2\phi}{Dt^2} \hat{C}_0^{-2} + \left[\frac{\nabla\hat{\rho}_0}{\hat{\rho}_0} \cdot \left(\nabla - \frac{D_0}{Dt} \hat{C}_0^{-2} \frac{D_0}{Dt} \right) \phi \right] = 0 \quad A.39$$

and because the flow is homentropic

$$\frac{\nabla \hat{\rho}_0}{\hat{\rho}_0} = \left(\frac{2}{\gamma - 1} \right) \frac{\nabla \hat{c}_0}{\hat{c}_0} \quad \text{A.40}$$

γ is the specific heat ratio. Substituting equation A.40 into equation A.39 results in

$$\nabla^2 \phi - \frac{D_0^2 \phi}{Dt^2} / \hat{c}_0^2 + \left(\frac{2 \nabla \hat{c}_0}{(\gamma - 1) \hat{c}_0} \cdot \left(\nabla - \frac{D_0}{Dt} \hat{c}_0^{-2} \frac{D_0}{Dt} \right) \phi \right) \quad \text{A.41}$$

This is the equation that was sought. It is used to find the solution of all perturbation properties in the field.

B. DERIVATION OF THE PRESSURE COEFFICIENT

For the solution of the field of interest, it is desirable to express the unsteady pressure perturbations in terms of a dimensionless value called the pressure coefficient, denoted by P_c , where

$$P_c = \frac{p'^*}{\frac{1}{2} \rho_\infty C_\infty^2} \quad \text{B.1}$$

p'^* is the unsteady perturbation pressure. It is a complex number with a magnitude and phase angle. ρ_∞ and C_∞ are the density and sonic speed respectively of the upstream reference point. Here, an equation will be developed to give P_c in terms of ϕ , where ϕ is the dimensionless perturbation velocity potential, which is a complex number with a phase angle and magnitude.

For the field that solutions are desired for, which is in a rotating cylindrical coordinate system, as described in Figure 1.2 page 4 the momentum equation is

$$\frac{\partial \bar{V}}{\partial t} + \bar{V} \cdot \nabla \bar{V} + 2\bar{\omega} \times \bar{V} - \omega^2 r \hat{r} = \frac{-\nabla P}{\rho} \quad \text{B.2}$$

where the assumptions are:

1. inviscid flow,
2. adiabatic flow,
3. perfect gas,
4. irrotational flow in the inertial reference frame,
5. and homentropic flow.

$\bar{V}$ is the velocity vector relative to the rotating coordinate

system fixed to the impeller and centered at the impeller center.

$\bar{\omega}$ is the angular velocity vector of the impeller.

$\hat{r}, \hat{\theta}, \hat{z}$ are for the rotating coordinate system.

∇ is for the rotating coordinate system.

Equation B.2 is rewritten

$$\frac{\partial \bar{V}}{\partial t} + \nabla \left(\frac{V^2}{2} \right) + (\nabla \times \bar{V} \times \bar{V}) - 2\bar{\omega} \times \bar{V} - \omega^2 r \hat{r} = \frac{-\nabla P}{\rho} \quad \text{B.3}$$

The $\nabla \times \bar{V}$ in the last equation is the vorticity in the relative coordinate system, which must be $-2\bar{\omega}$ in order for the vorticity in the inertial system to be zero. Therefore

$$\frac{\partial \bar{V}}{\partial t} + \nabla \left(\frac{V^2}{2} \right) + [-2\bar{\omega} \times \bar{V} + 2\bar{\omega} \times \bar{V}] - \omega^2 r \hat{r} = -\nabla P / \rho \quad \text{B.4}$$

The third term in brackets of equation B.4 vanishes identically, and the equation becomes

$$\frac{\partial \bar{V}}{\partial t} + \nabla \left(\frac{V^2}{2} \right) - \omega^2 r \hat{r} = \frac{-\nabla P}{\rho} \quad \text{B.5}$$

The term $\omega^2 r \hat{r}$ can be written $\nabla \left(\frac{\omega^2 r^2}{2} \right)$. Also, because the flow is homentropic, it is then barotropic, which allows $\frac{\nabla P}{\rho}$ to be written $\nabla \int dP/\rho$.

Therefore, equation B.5 can be written as follows:

$$\frac{\partial \bar{V}}{\partial t} + \nabla \left(\frac{V^2}{2} \right) - \nabla \left(\frac{\omega^2 r^2}{2} \right) + \nabla \int dP/\rho = 0 \quad \text{B.6}$$

Since the flow is irrotational, a velocity potential exists, which is defined as $\frac{\partial \bar{V}}{\partial t} = \frac{\partial}{\partial t} \nabla \phi$. Now dotting equation B.6 with a unit vector in the steady flow streamline direction, $\hat{\ell}$, and incorporating ϕ results in

$$\frac{d}{d\ell} \left(\frac{\partial \phi}{\partial t} \right) + \frac{d}{d\ell} \left(\frac{V^2}{2} \right) + \frac{d}{d\ell} \left(\frac{\omega^2 r^2}{2} \right) + \frac{d}{d\ell} \int \frac{dP}{\rho} = 0 \quad B.7$$

integrating equation B.7

$$\frac{\partial \phi}{\partial t} + \frac{V^2}{2} + \frac{\omega^2 r^2}{2} + \int_{P_\infty}^P dP/\rho = f(t) \quad B.8$$

where $f(t)$ is some function of time, and P_∞ is the pressure at the same reference point that ρ_∞ and C_∞ are calculated at. Evaluating equation B.8 at the reference point and noting that it will be assumed $\omega = 0$ at the reference point gives

$$f(t) = V_\infty^2/2 \quad B.9$$

where V_∞ is the velocity at the reference point. It follows that equation B.8 becomes

$$\int_{P_\infty}^P dP/\rho = \left\{ \frac{\partial \phi}{\partial t} + \frac{1}{2} \left(V^2 + \omega^2 r^2 - V_\infty^2 \right) \right\} (-1) \quad B.10$$

Now let P' be an unsteady pressure perturbation from the local steady value P_0 and let V'_ℓ and V'_n be unsteady velocity perturbations along and normal to the steady flow streamline. Equation

B.10 can then be expressed as

$$\int_{P_{\infty}}^{P_0} dP/\rho + \int_{P_0}^{P_0+P'} dP/\rho = \quad \text{B.11}$$

$$- \left[\frac{\partial}{\partial t} (\phi_0 + \phi') + \frac{1}{2} \left((V_0 + V'_\ell)^2 + V_n'^2 + \omega^2 r^2 - V_{\infty}^2 \right) \right]$$

where

$$\phi = \phi_0(r, \theta, Z) + \phi'(r, \theta, Z, t)$$

$$V^2 = (V_0 + V'_\ell)^2 + V_n'^2$$

$$|V'_\ell| \ll |V_0| ; \quad |V_n'| \ll |V_0|$$

Noting that if there is steady flow only, equation B.11 becomes

$$\int_{P_{\infty}}^{P_0} dP/\rho = -\left(\frac{1}{2}\right) \left(V_0^2 + \omega^2 r^2 - V_{\infty}^2 \right) \quad \text{B.12}$$

With equation B.12 and neglecting second order terms, equation B.11 can be written

$$\int_{P_0}^{P_0+P'} dP/\rho = -\left(\frac{\partial \phi'}{\partial t} + V_0 V'_\ell \right) \quad \text{B.13}$$

Since the flow is barotropic, let $\frac{1}{\rho} = F(P)$ be expanded into a Taylor's series about $P = P_0$;

$$F(P) = F(P_0) + F'(P_0)(P - P_0) \dots [0] (P - P_0)^2 \quad \text{B.14}$$

then

$$\int_{P_0}^{P_0+P'} dP/\rho = \int_{P_0}^{P_0+P'} dP [F(P_0) + F'(P_0)(P - P_0) + \dots [0] (P - P_0)^2] \quad \text{B.15}$$

$$= F(P_0)P' + F'(P_0)(P')^2/2 + \dots [0] (P')^3$$

so then to first order in perturbation quantities, equation B.13 becomes

$$\frac{P'}{\rho_0} = - \left(\frac{\partial \phi'}{\partial t} + V_0 V'_\ell \right) \quad \text{B.16}$$

Now assume the unsteady values have simple harmonic time variation where

$$P' = P'^* e^{i\omega t} ; \quad \phi' = \phi'^* e^{i\omega t}$$

$$V'_\ell = V'_\ell^* e^{i\omega t} \quad \text{B.17}$$

Then equation B.16 becomes

$$\frac{P'^*}{\rho_0} = - \left(i\omega \phi'^* + V_0 \frac{\partial \phi'^*}{\partial \ell} \right) \quad \text{B.18}$$

Equation B. 18 is made dimensionless by introducing the normalizing parameters as follows

$$P_c = P'^*/\frac{1}{2} \rho_\infty C_\infty^2 ; \quad \hat{\rho}_0 = \rho_0/C_\infty$$

$$\hat{V}_0 = V_0/C_\infty ; \quad \phi = \phi'^*/R C_\infty$$

$$K = \omega R/C_\infty ; \quad \hat{\ell} = \ell/R \quad \text{B.19}$$

where

All values with subscript ∞ are the upstream reference point.

R is the impeller outer radius.

C_∞ is the reference point sonic speed.

ℓ is distance in the streamline direction.

Then equation B.18 becomes

$$P_c = -2\hat{\rho}_0 [iK\phi + \hat{V}_0 \frac{\partial \phi}{\partial \hat{\ell}}] \quad \text{B.20}$$

Once ϕ is found for any point in the field equation B.20 will give the pressure coefficient there.

C. COMPUTER PROGRAM DESCRIPTION

The computer program is written for execution on a Control Data Corporation CYBER 176 computer. One subroutine is accessed from the International Mathematical and Statistical Library (IMSL). The subroutine, LEQTIC, is a complex linear equation solver, and it or its equivalent are available on most major computer systems.

Input to the program is accomplished by use of a "namelist". The user simply creates a named disk file containing the input data in the format illustrated in program IND whose sample input listing and description of input variables are included at the end of the program listing which is presented in Appendix D.

The current program contains no provision for calculating two-dimensional steady base flows or for calculating certain base flow gradients which would be needed by the unsteady flow analysis in that case. A one-dimensional steady base flow calculation is imbedded in the current analysis for convenience of the user. Addition of a two-dimensional steady base flow to the present code can be easily accomplished.

Program input is adequately described in the listing of program IND with the following modifications:

1. $(2) \times (\text{MODE}) + 1$ equals the number of modes in the acoustic radiation expansion.
2. Z THICK equals dimensionless axial channel thickness (axial channel thickness/impeller radius).

The program output consists of a reflection of the input data, a detailed printout of the one-dimensional steady base flow field along

with certain parameters pertinent to the subsequent unsteady flow analysis, and finally, a printout of the pressure coefficient and velocity potential along selected constant "theta" grid lines. A supplemental printout of the unsteady velocity field can be triggered by an input variable as explained in Appendix D., page 69.

D. COMPUTER CODE LISTING

PROGRAM RDFLOW

```

PROGRAM RADFLO(OUTPUT,TAPE5,TAPE8,TAPE10=OUTPUT)
COMMON/V1/VR(120),VTH(120),V(120),P(120),RO(120),
1 C(120),R1(120),R2(120),TH1(120),TH2(120),
2 DTHDR(120),RM(120),ALPHA(120),RI,R,DELR,SUP
COMMON/S1/N1,GAM,RMPRI,N,NN,AOMEG,NNN,
1 K,KZ,MODE,NMODE,KWAKE1,KWAKE2,BETA,DELT,KRX,
2 JMAX,ZTHICK,NZMODE,NBLADE,NVANE
COMPLEX KWAKE1,KWAKE2,KRX
NAMELIST/DATIN/R1,TH1,GAM,RMPRI,N1,N,
1 NN,JMAX,AOMEG,MODE,NBLADE,NVANE,ZTHICK,NZMODE,
2 SUP
READ(5,DATIN)
CALL STEADY
CALL COEF
CALL EIGEN
CALL SOLVE
CALL OUT
STOP
END

```

PROGRAM STEADY

```

SUBROUTINE STEADY
DIMENSION ADEG(120)
C ADEG IS ALPHA IN DEGREES
COMMON/V1/VR(120),VTH(120),V(120),P(120),RO(120),
1 C(120),R1(120),R2(120),TH1(120),TH2(120),
2 DTHDR(120),RM(120),ALPHA(120),RI,R,DELR,SUP
COMMON/S1/N1,GAM,RMPRI,N,NN,AOMEG,NNN,
1 K,KZ,MODE,NMODE,KWAKE1,KWAKE2,BETA,DELT,KRX,
2 JMAX,ZTHICK,NZMODE,NBLADE,NVANE
COMPLEX KWAKE1,KWAKE2,KRX
WRITE(10,500)
500 FORMAT(25X,*INPUT VALUES ARE AS FOLLOWS*//24X,*R1,15X,*TH1*)
DO 501 I=1,N1
WRITE(10,502) R1(I),TH1(I)
502 FORMAT(24X,F7.3,8X,F7.3)
501 CONTINUE
WRITE(10,503) GAM,RMPRI,N1,N,NN,AOMEG,
1 MODE,NBLADE,NVANE,ZTHICK,NZMODE,SUP
503 FORMAT(3X,*GAM=*,F8.3/3X,*RMPRI=*,F8.3/3X,*N1=*,I8/
1 3X,*N=*,I8/3X,*NN=*,I8/3X,*AOMEG=*,F8.3/3X,*MODE=*,I8/
2 3X,*NBLADE=*,I6/3X,*NVANE=*,I6/3X,*ZTHICK=*,F8.3/
3 3X,*NZMODE=*,I6/3X,*SUP=*,F5.3)
DELR=(1.0-R1(1)/R1(N1))/(FLOAT(N)-1.0)
CALL INTERP

```

```

CALL DERIV
WRITE(10,102) DTHDR(1),DTHDR(3)
102  FORMAT(2F7.4)
      GAM1=GAM/(GAM-1.0)
      GAM2=1.0/GAM
      GAM3=(GAM-1.0)/(2.0*GAM)
      GAM4=(GAM-1.0)/2.0
      DO 2 I=1,N
      A=(GAM4)*((AOMEG*R2(1))**2.0)+1.0
      B=1.0+(GAM4)*(RMPRI**2.0)
      P(I)=(A/B)**GAM1
      V(I)=RMPRI*(P(I)**GAM3)
      E=SQRT(1.0+((R2(I)*DTHDR(I))**2.0))
      VR(I)=V(I)/E
      VTH(I)=(V(I)*R2(I)*DTHDR(I))/E
      RO(I)=P(I)**GAM2
2     C(I)=P(I)**GAM3
      GAM5=2.0/(GAM-1.0)
      GAM6=(2.0*GAM)/(GAM1.0)
      C1=(RO(N)*VR(N))**2.0
      C2=(AOMEG+VTH(N))**2.0
      EE=AOMEG**2.0
      EE0=VTH(N)/AOMEG
      TAU=(GAM-1.0)*EE*(1.0+EE0)+1.0
      KK=N+1
      RM(KK)=RMPRI
      NNN=N+NN
      DO 30 I=KK,NNN
      ICOUNT=0
      RM(I)=RM(I-1)
      R2(I)=R2(I-1)+DELR
32     AA1=RM(I)**2.0
      ICOUNT=ICOUNT+1
      AA2=1.0+GAM*AA1
      C(I)=SQRT(TAU/AA2)
      RO(I)=C(I)**GAM5
      EE1=1.0/(R2(I)**2.0)
      EE2=1.0/(C(I)**2.0)
      EE3=1.0/(RO(I)**2.0)
      EE4=EE1*EE2*(EE3*C1+C2)
      RMNEW=SQRT(EE4)
      IF(ICOUNT.GT.100) GO TO 33
      IF(ABS(RMNEW-RM(I)).GT..0005) GO TO 31
      GO TO 33
31     RM(I)=RMNEW
      GO TO 32
33     P(I)=C(I)**GAM6
      VR(I)=SQRT(C1)/(RO(I)*R2(I))
      EE5=AOMEG*R2(I)
      VTH(I)=(SQRT(C2)/R2(I))-EE5
      V(I)=SQRT(VR(I)**2.+VTH(I)**2.)

```

```

      DTHDR(I)=VTH(I)/(VR(I)*R2(I))
      TH2(I)=TH2(I-1)+((DTHDR(I)+DTHDR(I-1))/2.0)*DELR
30    CONTINUE
      DO 50 I=KK,NNN
C CALC. OF REL MACH NO.
      F1=VTH(I)**2.0
      F2=VR(I)**2.0
50    RM(I)=SQRT(F1+F2)/C(I)
      WRITE(10,700)
700   FORMAT(20X,*THE STEADY FLOW SOLUTION*)
      WRITE(10,3)
3     FORMAT(1H0,3X,*RADIUS*,6X,*R-VEL.*,2X,*TAN. VEL.*.2X,*DENSITY*,
1     2X,*PRESSURE*,2X,*SONIC VEL*,2X,*MACH NO.*)
      DO 81 I=1,N
01    RM(I)=RMPRI
      DO 4 I=1,NNN
      WRITE(10,5) R2(I),VR(I),VTH(I),RO(I),P(I),C(I),RM(I)
5     FORMAT(1X,7F10.4)
4     CONTINUE
      WRITE(10,82)
82    FORMAT(4X,*RADIUS*,3X,*TOT ANGLE*,
1     3X,*DTHDR*,4X,*RT ANGLE*
      DO 83 I=1,NNN
      ALPHA(I)=ATAN(R2(I)*DTHDR(I))
      ADEG(I)=ALPHA(I)*57.4
      TH2(I)=TH2(I)*57.4
      WRITE(10,84) R2(I),TH2(I),DTHDR(I),ADEG(I)
84    FORMAT(1X,4F10.4)
      TH2(I) = TH2(I)/57.4
83    CONTINUE
      WRITE(10,103)
103   FORMAT(1X,*END STEADY*)
      RETURN
      END

```

PROGRAM INTERP

```

      SUBROUTINE INTERP
      COMMON/V1/VR(120),VTH(120),V(120),P(120),RO(120),
1 C(120),R1(120),R2(120),TH1(120),TH2(120),DTHDR(120),
2 RM(120),ALPHA(120),RI,R,DELR,SUP
      COMMON/S1/N1,GAM,RMPRI,N,NN,AOMEG,NNN,
1 K,KZ,MODE,NMODE,KWAKE1,KWAKE2,BETA,DELT,KRX,
2 JMAX,ZTHICK,N2MODE,NBLADE,NVANE
      COMPLEX KWAKE1,KWAKE2,KRX
      TH2(N)=TH1(N1)
      TH2(1)-TH1(1)
      R2(1)=R1(1)/R1(N1)
      R2(N)=R1(1)/R1(1)
      I=1
      K=N-2

```

```

DO 40 J=1,K
R2(J+1)=R2(J)+DELR
IF(R2(J+1).GT.R1(I+1)) I=I+1
AA=TH1(I+1)-TH1(I)
BB=R2(J+1)-R1(I)
CC=R1(I+1)-R1(I)
40 TH2(J+1)=((AA*BB)/CC)+TH1(I)
RETURN
END

```

PROGRAM DERIV

```

SUBROUTINE DERIV
COMMON/V1/VR(120),VTH(120),V(120),P(120),RO(120),
1 C(120),R1(120),R2(120),TH1(120),TH2(120),DTHDR(120),
2 RM(120),ALPHA(120),RI,R,DELR,SUP
COMMON/S1/N1,GAM,RMPRI,N,NN,AOMEG,NNN,
1 K,KZ,MODE,NMODE,KWAKE1,KWAKE2,BETA,DELT,KRX,
2 JMAX,ZTHICK,NZMODE,NBLADE,NVANE
COMPLEX KWAKE1,KWAKE2,KRX
DTHDR(1)=(TH2(2)-TH2(1))/DELR
L=N-1
DO 41 I=2,L
41 DTHDR(I)=(TH2(I+1)-TH2(I-1))/(2.0*DELR)
DTHDR(N)=(TH2(N)-TH2(N-1))/DELR
RETURN
END

```

PROGRAM COEF

```

SUBROUTINE COEF
LEVEL 2,COF1,COF2,COF3,COF4,COF5,COF6
COMMON/COF/COF1(120,60),COF2(120,60),COF3(120,60),
1 COF4(120,60),COF5(120,60),COF6(120,60)
COMMON/S1/N1,GAM,RMPRI,N,NN,AOMEG,NNN,
1 K,KZ,MODE,NMODE,KWAKE1,KWAKE2,BETA,DELT,KRX,
2 JMAX,ZTHICK,NZMODE,NBLADE,NVANE
COMMON/S2/A1,A2,A3,A4,A5,A6,C3,CI,SPACE
COMMON/V1/VR(120),VTH(120),V(120),P(120),RO(120),
1 C(120),R1(120),R2(120),TH1(120),TH2(120),DTHDR(120),
2 RM(120),ALPHA(120),RI,R,DELR,SUP
REAL K,KZ
COMPLEX COF1,COF4,COF5,A4,A6,CI,KWAKE1,KWAKE2,KRX,C3
102 FORMAT(1X,*START COEF*)
PI=3.1416
DELT=6.283/(NBLADE*(JMAX-1))
IMAX=NNN
IEDGE=N
GAM=1.4
CI=CMPLX(0.,1.)
GRATIO=DELT/DELR

```

```

      BETA=2.0*PI*(FLOAT(NVANE)/NBLADE
C  CALCULATE THE COEFFICIENTS OF THE DIFFERENTIAL EQN.
      IL=IMAX-1
      K=AOMEG*NVANE
      KZ=NZMODE*PI/ZTHICK
      DO 1 I=2,IL
        A1=1.0-(RM(I)*COS(ALPHA(I)))**2.0
        A2=1.0/R2(I)**2.0+DTHDR(I)**2.0
        A3=-2.0*DTHDR(I)
        DVRDR=.5*(VR(I+1)-VR(I-1))/DELR
        DCDR=.5*(C(I+1)-C(I-1))/DELR
        A4=1.0/R2(I)-2.0*CI*K*VR(I)/C(I)**2.0-VR(I)*DVRDR
1      +2.0/(GAM-1.0)*DCDR/C(I)+2.*VR(I)**2.0/C(I)**3.0*DCDR
        A5=-.5*(DTHDR(I+1)-DTHDR(I-1))/DELR-DTHDR(I)/R2(I)
1      -2.0/(GAM-1.0)*DCDR/C(I)*DTHDR(I)
        A6=K**2.0/C(I)**2.0+2.0*CI*K*VR(I)/C(I)**3.0*DCDR
1      -KZ**2.0
C  NOW COMPUTE THE COEFFICIENTS OF THE FINITE DIFFERENCE EQN.
        COF1(I,1)=-2.0*A1*GRATIO**2.0-2.0*A2*DELT**2.0*A6
        COF2(I,1)=A2+.5*DELT*A5
        COF3(I,1)=A2-.5*DELT*A5
        COF4(I,1)=A1*GRATIO**2.0+.5*GRATIO*DELT*A4
        COF5(I,1)=A1*GRATIO**2.0-.5*GRATIO*DELT*A4
        COF6(I,1)=.25*GRATIO*A3
C  FOR THE SIMPLIFIED STEADY FIELD CALCULATION THERE IS NO
C  VARIATION OF STEADY FIELD QUANTITIES WITH THETA, THEREFORE
      DO 2 J=2,JMAX
        COF1(I,J)=COF1(I,1)
        COF2(I,J)=COF2(I,1)
        COF3(I,J)=COF3(I,1)
        COF4(I,J)=COF4(I,1)
        COF5(I,J)=COF5(I,1)
2      COF6(I,J)=COF6(I,1)
1      CONTINUE
      RETURN
      END

```

PROGRAM EIGEN

```

      SUBROUTINE EIGEN
      COMMON/S1/N1,GAM,RMPRI,N,NN,AOMEG,NNN,
1      K,KZ,MODE,NMODE,KWAKE1,KWAKE2,BETA,DELT,KRX,
2      JMAX,ZTHICK,NZMODE,NBLADE,NVANE
      COMMON/V1/VR(120),VTH(120),V(120),P(120),RO(120),
1      C(120),R1(120),R2(120),TH1(120),TH2(120),DTHDR(120),
2      RM(120),ALPHA(120),RI,R,DELR,SUP
      COMMON/S2/A1,A2,A3,A4,A5,A6,C3,CI,SPACE
      COMMON/V2/KRI(15),KRE(15),KTE(15),KR,KT
      REAL K,KZ,KT,KTE,KR,MR,KBAR
      COMPLEX C3,A4,A6,KRI,KPE,CI,KWAKE1,KWAKE2,ROOT,
2      KRK,CROOT,F1,F2,F,CF

```

```

F(RBAR) = -1./B2*(C1+MR**2/B2-(KT**2-C2)/RBAR**2)
PI=3.1416
SPACE=6.2832/(FLOAT(NBLADE))
NMODE=2*MODE+1
C CALCULATE INLET EIGENVALUES
B2 = 1.-RM(2)**2
P=.5/B2
RBAR1 = R2(1)*K/C(1)
RBAR2 = R2(2)*K/C(1)
SIGN = RBAR1/ABS(RBAR1)
C2=(2.*B2-1.)/(4.*B2)
C1=1.-(KZ*C(1)/K)**2
MR = RM(2)
DO 1 I=1,NMODE
KT = (I-1)*NBLADE/2
F1 = F(RBAR1)
F2 = F(RBAR2)
C3 = CI*MR/B2 + SIGN*CSQRT(F2)
CF = CSQRT(F2/F1)
KRI(I)=(RBAR2/RBAR1)**P*CSQRT(CF)*CEXP(C3*(RBAR1-RBAR2))
1 CONTINUE
NL=NNN-1
B2=1.-RM(NL)**2
P = .5/B2
C2 = (2.*B2-1.)/(4.*B2)
MR = RM(NL)
DO 2 I=1,NMODE
KT = (I-(MODE+1))*NBLADE+NVANE
KTE(I) = KT
KBAR = K +VTH(NL)*KT/R2(NL)
RBAR1 = R2(NL)**KBAR/C(NL)
RBAR2 = R2(NNN)+KBAR/C(NL)
SIGN = RBAR1/ABS(RBAR1)
C1 = 1.-(KZ*C(NL)/KBAR)**2
F1 = F(RBAR1)
F2 = F(RBAR2)
C3 = CI*MR/B2 - SIGN*CSQRT(F1)
A4 = CI*MR/B2 + SIGN*CSQRT(F1)
I1=I-1
CF=CSQRT(F1/F2)
IF(I1.EQ.MODE) KRX=(RBAR1/RBAR2)**P*CSQRT(CF)
1 *CEXP(A4*(RBAR2-RBAR1)+CI*KT*(TH2(NNN)-TH2(NL)))
KRE(I)=(RBAR1/RBAR2)**P*CSQRT(CF)
1 *CEXP(C3*(RBAR2-RBAR1)+CI*KT*(TH2(NNN)-TH2(NL)))
2 CONTINUE
KWAKE1 = K*R2(NL)/(VR(NL)+CI*VTH(NL))*(-CI)
KWAKE2 = -K*R2(NL)/(VR(NL)-CI*VTH(NL))*(-CI)
RETURN
END

```

```

PROGRAM SOLVE
  SUBROUTINE SOLVE
    LEVEL 2,COF1,COF2,COF3,COF4,COF5,COF6
    COMMON/COF/COF1(120,60),COF2(120,60),COF3(120,60),
1 COF4(120,60),COF5(120,60),COF6(120,60)
    COMMON/S1/N1,GAM,RMPRI,N,NN,AOMEG,NNN,
1 K,KZ,MODE,NMODE,KWAKE1,KWAKE2,BETA,DELT,KRX,
2 JMAX,ZTHICK,NZMODE,NBLADE,NVANE
    COMMON/S2/A1,A2,A3,A4,A5,A6,C3,CI,SPACE
    COMMON/V1/VR(120),VTH(120),V(120),P(120),RO(120),
1 C(120),R1(120),R2(120),TH1(120),TH2(120),DTHDR(120),
2 RM(120),ALPHA(120),RI,R,DELR,SUP
    COMMON/V2/KRI(15),KRE(15),KTE(15),KR,KT
    COMMON/V3/PHI(120,60)
    DIMENSION AA(60),BD(60),CC(60),DD(60),EE(60),FF(60),
1 AAA(60,60),BBB(60,60),CCC(60,60),DDD(60,60),EEE(60,60)
    COMPLEX AA,BB,CC,DD,EE,FF,
1 AAA,BBB,CCC,DDD,EEE,
2 PHI,A,B,W1,W2,KWAKE1,KWAKE2,CI,KRI,KRE,EIB,EMIB,COF1,
3 COF4,COF5,C3,KRX,A4,A6,C1
    REAL K,KZ,KT,KTE,KR
  C BEGIN CALCULATION OF EXIT RADIATION EQUATIONS
  C CALCULATE THE WAKE CONSTANTS,W1 AND W2
    GRATIO=DELT/DELR
    IMAX=NNN
    IEDGE=N
    PI=3.1416
    DO 1 I=1,60
      AA(I)=0.0
      BB(I)=0.0
      CC(I)=0.0
      DD(I)=0.0
      EE(I)=0.0
      FF(I)=0.0
      DO 1 J=1,60
        AAA(I,J)=0.0
        BBB(I,J)=0.0
        CCC(I,J)=0.0
        DDD(I,J)=0.0
        EEE(I,J)=0.0
1      CONTINUE
      CALL IMSL
      A=1.-CEXP(CI*(KWAKE1*SPACE-BETA))
      B=1.-CEXP(CI*(KWAKE2*SPACE-BETA))
      C1=-CI*K*R2(IMAX-1)/VR(IMAX-1)*COS(ALPHA(IMAX-1))
1      *SIN(ALPHA(IMAX-1))
      W1=(CI*KWAKE2-C1)/(CI*A*(KWAKE2-KWAKE1))
      W2=(CI*KWAKE1-C1)/(CI*B*(KWAKE1-KWAKE2))
  C CALCULATE THE WAKE MATRIX
    EIB=CEXP(CI*BETA)
    EMIB=CEXP(-CI*BETA)

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      A=CEXP(-K/VR(IMAX-1)*CI*DELR)
      DO 2 J=1,JMAX
        DDD(J,1)=W1*CEXP(CI*KWAKE1*(J-1)*DELT)
1      +W2*CEXP(CI*KWAKE2*(J-1)*DELT)
        DDD(J,JMAX)=-EMIB*DDD(J,1)
        EEE(J,1)=A*DDD(J,1)
        EEE(J,JMAX)=A*DDD(J,JMAX)
2      CONTINUE
C CALCULATE THE MATRIX A(N,J)
      DO 3 J=1,JMAX
        FAC=1./(JMAX-1)
        IF(J.EQ.1.OR.J.EQ.JMAX)FAC=.5*FAC
        DO 3 N=1,NMODE
          AAA(N,J)=FAC*CEXP(-CI*KTE(N)*(J-1)*DELT)
3      CONTINUE
C CALCULATE THE MATRIX B(I,N)
      DO 4 N=1,NMODE
        A = KRE(N)
        DO 4 I=1,JMAX
          BBB(I,N)=A*CEXP(CI*KTE(N)*(I-1)*DELT)
4      CONTINUE
C CALCULATE C(J,M)=I(J,M)=D(J,M)
      DO 5 M=1,JMAX
        DO 5 J=1,JMAX
          I=0
          IF (J.EQ.M) I=1
5      CCC(J,M)=I-DDD(J,M)
C PERFORM THE MATRIX MULTIPLICATION B(I,N)*A(N,J)*C(J,M)
C FIRST A*C AND STORE RESULT BACK IN C
      CALL CMATX (NMODE,JMAX,JMAX,AAA,CCC,CCC)
C NOW B*C AND STORE RESULT IN B
      CALL CMATX(JMAX,NMODE,NMODE,JMAX,BBB,CCC,BBB)
C NOW ADD DBAR(I,M)
      CALL CMATAD(JMAX,JMAX,BBB,EEE,EEE)
C CALCULATE THE DIFFUSER GENERATED DISTURBANCE, I.E., THE
C NON-HOMOGENEOUS PART OF THE SYSTEM
      DO 6 I=1,JMAX
        TH=(I-1)*DELT
        AA(I)=CEXP(CI*KTE(MODE+1)*TH)*(KRX-KRE(MODE+1))
6      CONTINUE
C THE EXIT BOUNDARY RADIATION AND EXCITATION
C EQUATIONS ARE NOW COMPLETE. THE FORM IS
C PHI(IMAX,I)=BB(I,M) PHI(IMAX,M)+AA(I)
C
C STORE THE MATRICES ON DISK FOR LATER USE
      WRITE(8) AA,EEE
C STARTING JUST INSIDE THE EXIT BOUNDARY, SWEEP THROUGH
C THE FIELD TOWARD THE INLET SETTING UP THE FIELD
C EQUATIONS AND BOUNDARY CONDITIONS
C
C FIRST SET THE FIELD EQUATIONS

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      NL=IMAX-2
      DO 100 L=1,NL
        I=IMAX-L
C REINITIALIZE VECTOR
      DO 101 LL=1,60
        DO 101 M=1,60
          AAA(M,LL)=0.0
          BBB(M,LL)=0.0
          CCC(M,LL)=0.0
          DDD(M,LL)=0.0
101    CONTINUE
        JL=JMAX-1
        DO 102 J=2,JL
          AAA(J,J-1)=COF6(I,J)
          AAA(J,J)=COF5(I,J)
          AAA(J,J+1)=-COF6(I,J)
          BBB(J,J-1)=COF3(I,J)
          BBB(J,J)=COF1(I,J)
          BBB(J,J+1)=COF2(I,J)
          CCC(J,J-1)=-COF6(I,J)
          CCC(J,J)=COF4(I,J)
          CCC(J,J+1)=COF6(I,J)
102    CONTINUE
C THE FIRST AND LAST ROWS OF THE ABOVE EQUATIONS ARE
C DETERMINED BY THE SIDE BOUNDARY CONDITIONS WHICH ARE IN
C TURN, DIFFERENT INSIDE THE CHANNEL AND OUTSIDE THE
C CHANNEL. THEREFORE BRANCH BASED ON THIS.
      IF(I.GT.IEDGE) GO TO 103
C INSIDE THE CHANNEL---APPLY BLADE SURFACE
C BOUNDARY CONDITION---NORMAL VELOCITY = 0.0
      FAC=.5*GRATIO*R2(I)*SIN(ALPHA(I))*COS(ALPHA(I))
C SET BOUNDARY EQUATION AT THETA = 0
      BBB(1,1)=-1.0
      BBB(1,2)=1.0
      AAA(1,2)=FAC
      CCC(1,2)=-FAC
C SET BOUNDARY EQUATION AT THETA = SPACE
      BBB(JMAX,JMAX)=1.0
      BBB(JMAX,JMAX-1)=-1.0
      AAA(JMAX,JMAX-1)=FAC
      CCC(JMAX,JMAX-1)=-FAC
      GO TO 104
103    CONTINUE
C OUTSIDE THE CHANNEL IN THE DIFFUSER REGION. APPLY
C QUASI-PERIODICITY, CONTINUITY OF PRESSURE AND NORMAL
C VELOCITY CONDITIONS
C
C FIRST APPLY QUASI-PERIODICITY AND CONTINUITY OF
C PRESSURE CONDITION
      A=CEXP(-CI*K*2.*DELR/(VR(I)+VR(I-1)))
      BBB(1,1)=EIB

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        BBB(1,JMAX)=-1.0
        AAA(1,1)=-EIB*A
        AAA(1,JMAX)=A
C NOW APPLY QUASI-PERIODICITY ALONG WITH CONTINUITY OF
C NORMAL VELOCITY
        A=R2(I)*COS(ALPHA(I))*SIN(ALPHA(I))*CI*K*DELT/VR(I)
        BBB(JMAX,1)=EIB*(A-1.0)
        BBB(JMAX,2)=EIB
        BBB(JMAX,JMAX)=-(A+1.0)
        BBB(JMAX,JMAX-1)=1.0
104    CONTINUE
C END OF FIELD AND SIDE BOUNDARY EQUATION SET UP
C
C NOW CALCULATE BACK-SUBSTITUTION MATRIX EEE AND
C VECTOR AA FROM BLOCK TRIDIAGONAL SOLVER
C RECURSION FORMULA
        CALL CMATX (JMAX,JMAX,JMAX,JMAX,CCC,EEE,DDD)
        IF(I.LT.IEDGE.OR.I.GT.IEDGE+1) GO TO 400
400    CONTINUE
        CALL CMATAD(JMAX,JMAX,BBB,DDD,DDD)
        IF(I.LT.IEDGE.OR.I.GT.IEDGE+1) GO TO 401
401    CONTINUE
C INVERT THE DDD MATRIX USING CDC SUPPLIED LIBRARY
C SUBROUTINE
C
C FIRST SET UP IDENTITY MATRIX---USE BBB SINCE IT
C IS NO LONGER NEEDED
        DO 105 KK=1,60
        DO 105 J=1,60
105    BBB(J,KK)=0.0
        DO 106 J=1,60
106    BBB(J,J)=1.0
        CALL LEQ1C(DDD,JMAX,60,BBB,JMAX,60,0,BB,IER)
C BBB NOW CONTAINS THE INVERSE OF DDD
C
C CHANGE SIGN OF BBB ELEMENTS
        DO 107 KK=1,JMAX
        DO 107 J=1,JMAX
107    BBB(J,KK)=-BBB(J,KK)
C COMPLETE THE RECURSION CALCULATION
        CALL CMATX(JMAX,JMAX,JMAX,JMAX,BBB,AAA,EEE)
        CALL CMATX(JMAX,JMAX,JMAX,JMAX,BBB,CCC,CCC)
        CALL MATVEC(JMAX,JMAX,CCC,AA,AA)
C NOW STORE RESULTS ON DISK
        WRITE(8) AA,EEE
100    CONTINUE
C END OF FIELD CALCULATION
C
C CALCULATE THE INLET RADIATION MATRIX
C FIRST CALCULATE A(N,J)
        DO 110 J=1,JMAX
        FAC=1./(JMAX-1)

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```

      IF(J.EQ.1.OR.J.EQ.JMAX) FAC=.5*FAC
      FAC2=1.
      DO 110 N=1,NMODE
      AAA(N,J)=FAC2*FAC*COS((N-1)*PI*(J-1)*DELT/SPACE)
      FAC2=2.
110    CONTINUE
C
C NOW CALCUALTE B(M,N)
      DO 111 N=1,NMODE
      DO 111 M=1,JMAX
      BBB(M,N)=COS((N-1)*PI*(M-1)*DELT/SPACE)*KRI(N)
111    CONTINUE
C FORM THE RADIATION MATRIX BY B(M,N)*A(N,J)
      CALL CMATX(JMAX,NMODE,NMODE,JMAX,BBB,AAA,CCC)
C CCC IS NOW THE INLET RADIATION MATRIX
C
C NOW CAN COMPUTE THE INLET VALUES OF THE PERTUBATION
C POTENTIAL
      CALL CMATX (JMAX,JMAX,JMAX,JMAX,CCC,EEE,AAA)
      DO 118 J=1,JMAX
      DO 118 I=1,JMAX
      FAC=0.0
      IF(I.EQ.J)FAC=1.0
      BBB(I,J)=FAC-AAA(I,J)
118    CONTINUE
      CALL MATVEC(JMAX,JMAX,CCC,AA,DD)
C INVERT BBB
C FIRST MUST SET UP IDENTITY MATRIX
      DO 121 KK=1,60
      DO 121 J=1,60
121    DDD(J,KK)=0.0
      DO 122 J=1,60
122    DDD(J,J)=1.0
      CALL LEQTIC(BBB,JMAX,60,DDD,JMAX,60,0,BB,IER)
C COMPUTE INLET VALUES OF PHI
      CALL MATVEC(JMAX,JMAX,DDD,DD,CC)
      DO 125 J=1,JMAX
125    PHI(1,J)=CC(J)
C CAN SOLVE THE REMAINDER OF THE PHI FIELD BY BACK-
C SUBSTITUTION
C FIRST MUST RETRIEVE MATRIX EEE AND VECTOR AA FROM DISK
C WHERE THEY HAVE BEEN STORED FOR EACH I, SEQUENTIALLY.
      DO 130 I=2,IMAX
C BACK UP READER TO TOP OF LAST AA+EE
      BACKSPACE 8
      READ(8)AA,EEE
      CALL MATVEC(JMAX,JMAX,EEE,CC,BB)
      DO 131 J=1,JMAX
      CC(J)=BB(J)+AA(J)
      PHI(I,J)=CC(J)
131    CONTINUE
      BACKSPACE 8

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```

130    CONTINUE
      RETURN
      END

```

```

PROGRAM  MATVEC
      SUBROUTINE MATVEC (NR,NC,A,VEC1,VEC2)
C NC(NUMBER OF COLUMNS IN MATRIX A)MUST EQUAL LENGTH OF VEC1.
      DIMENSION A(60,60),VEC1(60),VEC2(60),VEC3(60)
      COMPLEX A,VEC1,VEC2,VEC3
      DO 1 I=1,NR
      VEC3(I)=0.0
      DO 1 J=1,NC
1      VEC3(I)=A(I,J)*VEC1(J)+VEC3(I)
      DO 2 I=1,NR
2      VEC1(I)=VEC3(I)
      RETURN
      SUBROUTINE CMATX(NR1,NC1,NR2,NC2,A,B,C)
      DIMENSION A(60,60),B(60,60),C(60,60),D(60,60)
      COMPLEX A,B,C,D
      DO 1 I=1,NR1
      DO 2 J=1,NC2
      D(I,J)=0.0
      DO 3 K=1,NC1
3      D(I,J)=A(I,K)*B(K,J)+D(I,J)
2      CONTINUE
1      CONTINUE
C STORE THE RESULT IN MATRIX C.
      DO 4 I=1,NR1
      DO 4 J=1,NC2
4      C(I,J)=D(I,J)
      RETURN
      END
      SUBROUTINE CMATAD(NR,NC,A,B,C)
      DIMENSION A(60,60),B(60,60),C(60,60)
      COMPLEX A,B,C
      DO 1 I=1,NR
      DO 1 J=1,NC
      C(I,J)=A(I,J)+B(I,J)
      RETURN
      END

```

```

PROGRAM  OUT
      SUBROUTINE OUT
      DIMENSION DPHI(120),SM1(120),SP1(120),SM2(120),SP2(120)
      COMMON/V3/PHI(120,60)
      COMMON/V1/VR(120),VTH(120),V(120),P(120),RO(120),
1      C(120),R1(120),R2(120),TH1(120),TH2(120),
2      DTHDR(120),RM(120),ALPHA(120),RI,R,DELR,SUP
      COMMON/S1/N1,GAM,RMPRI,NN,NN,AOMEG,NNN,
1      K,KZ,MODE,NMODE,KWAKE1,KWAKE2,BETA,DELT,KRX,
2      JMAX,ZTHICK,NZMODE,NBLADE,NVANE

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```

COMPLEX PHI,Z,DPHI,PC,DPHI2,VTAN,VNORM
COMPLEX KWAKE1,KWAKE2,KRX,CI
REAL K
CI=CMPLX(0.,1.0)
CPNORM=CABS(2.*RO(NNN-1)*(CI*K+VR(NNN-1)*(KRX-1.)/DELRR))
DO 200 J=1,JMAX
DO 200 I=1,NNN
200 PHI(I,J)=PHI(I,J)/CPNORM
WRITE(10,300)
300 FORMAT(1X,*START OUT*)
WRITE(10,600)
600 FORMAT(18X,*THE UNSTEADY PERTURBATION SOLUTION*)
Z=CMPLX(0.,1.)
JSKIP=JMAX/2
DO 1 J=1,JMAX,JSKIP
THETA=FLOAT(J-1)*DELT
WRITE(10,3) J,THETA
3 FORMAT(35X,*J=*,13,*THETA=*,F6.2)
WRITE(10,4)
4 FORMAT(1H0,23X,*PRESSURE COEF.*,23X,*POTENTIAL*/
1 3X,*I*,2X,*RADIUS*,2X,*MAG.*,3X,*PHASE*,4X,*REAL*,
2 4X,*IMAG.*,3X,*MAG.*,3X,*PHASE*,4X,*REAL*,4X,*IMAG.*)
C PC IS THE COMPLEX PRESS. COEF.
C PCR IS REAL PART OF PRESS. COEF.
C PCI IS IMAG. PART OF PRESS. COEF.
C PHI IS THE COMPLEX POTENTIAL
C PHIR IS THE REAL PART OF PHI
C PHII IS THE IMAG. PART OF PHI
DO 2 I=1,NNN
C DPHI IS THE PARTIAL OF PHI W/R TO R
IF(I.EQ.1) GO TO 6
IF(I.EQ.NNN) GO TO 7
GO TO 9
6 DPHI(I)=(PHI(I+1,J)-PHI(I,J))/DELRR
GO TO 8
7 DPHI(I)=(PHI(I,J)-PHI(I-1,J))/DELRR
GO TO 8
9 DPHI(I)=(PHI(I+1,J)-PHI(I-1,J))/(2.0*DELRR)
8 PC=(-2.0*RO(I))*((Z*K*PHI(I,J))+(VR(I)*(DPHI(I))))
PCR=REAL(PC)
PCI=AIMAG(PC)
PHIR=REAL(PHI(I,J))
PHII=AIMAG(PHI(I,J))
C PCM IS THE MAG. OF THE PRESS. COEF.
C PHIM IS THE MAG. OF THE POTENTIAL
C PH1 IS THE PHASE OF THE PRESS. COEF.
C PH2 IS THE PHASE OF THE POTENTIAL
PCM=CABS(PC)
PHIM=CABS(PHI(I,J))
PH1=777.
PH2=777.
IF(PCR.EQ.0.0) GO TO 101

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      PH1=(ATAN2(PCI,PCR))*57.296
101  IF (PHIR.EQ.0.0) GO TO 102
      PH2=(ATAN2(PHII,PHIR))*57.296
102  CONTINUE
      IF(J.EQ.1)SM1(I)=PCM
      IF(J.EQ.1)SP1(I)=PH1
      IF(J.EQ.JMAX)SM2(I)=PCM
      IF(J.EQ.JMAX)SP2(I)=PH1
      WRITE(10,5) I,R2(I),PCM,PH1,PCR,PCI,PHIM,PH2,PHIR,PHII
5    FORMAT(1X,13,1X,F5.3,1X,F7.2,1X,F7.2,1X,F7.2,
1    1X,F7.2,1X,F7.4,1X,F7.2,1X,F7.4,1X,F7.4)
2    CONTINUE
      IF(SUP.EQ.0.0) GO TO 1
      WRITE(10,10)
10   FORMAT(1H0 23X,*TANGENT VELOCITY*,23X,
1     *NORMAL VELOCITY *,//3X,*I*,2X,*RADIUS*,
2     2X,*MAG.*,3X,*PHASE*,3X,*REAL*,4X,*IMAG.*,
3     3X,*MAG*,4X,*PHASE*,3X,*REAL*,4X,*IMAG.*)
C DPHI2 IS THE PARTIAL OF PHI IN THE THETA DIR.
C (IN TRANSFORMED SYSTEM)
      DO 100 I=1,NNN
      IF(J.EQ.1) GO TO 11
      IF(J.EQ.JMAX) GO TO 12
      GO TO 13
11   DPHI2=(PHI(I,J+1)-PHI(I,J))/DELT
      GO TO 14
12   DPHI2=(PHI(I,J)-PHI(I,J-1))/DELT
      GO TO 14
13   DPHI2=(PHI(I,J+1)-PHI(I,J-1))/(2.*DELT)
C VTAN IS THE PERT. VEL. TAN. TO A STREAMLINE
C VNORM IS THE PERT. VEL. NORMAL TO A STREAMLINE
14   VTAN=(VR(I)/V(I))*DPHI(I)
      XX=1.0/(R2(I)*COS(ALPHA(I)))
      VNORM=-1.0*(DPHI(I)*SIN(ALPHA(I)))+(DPHI2*XX)
C VTANM IS THE MAG. OF VTAN
C VNORMM IS THE MAG. OF VNORM
C PH3 IS THE PHASE OF VTAN
C PH4 IS THE PHASE OF VNORM
C VTANR IS THE REAL PART OF VTAN
C VNORMR IS THE REAL PART OF VNORM
C VTANI IS THE IMAG. PART OF VTAN
C VNORMI IS THE IMAG. PART OF VNORM
      VTANM=CABS(VTAN)
      VNORMM=CABS(VNORM)
      VTANR=REAL(VTAN)
      VNORMR=REAL(VNORM)
      VTANI=AIMAG(VTAN)
      VNORMI=AIMAG(VNORM)
      PH3=777.
      PH4=777.
      IF(VTANR.EQ.0.0) GO TO 103
      PH3=(ATAN2(VTANI,VTANR))*57.296

```

```

103   IF(VNORMR.EQ.0.0) GO TO 104
      PH4=(ATAN2(VNORMI,VNORMR))*57.296
104   CONTINUE
      WRITE(10,15)I,R2(I),VTANM,PH3,VTANR,
1    1 VTANI,VNORMM,PH4,VNORMR,VNORMI
15   1 FORMAT(1X,I3,1X,F5.3,1X,F7.2,1X,F7.2,
1    1 1X,F7.2,1X,F7.2,1X,F7.2,1X,F7.2,1X,
2    2 F7.2,1X,F7.2)
100   CONTINUE
1    CONTINUE
      WRITE(10,1000)
1000  FORMAT (1H0,20X,*SURFACE PRESSURE DIST *//
1    1 13X,*PRESSURE SURFACE*,10X,*SUCTION SURFACE*//
2    2 3X,*1*,2X,*RADIUS*,2X,*MAG.*,3X,*PHASE*,
3    3 15,*MAG.*,3X,*PHASE*)
      NL=NNN-1
      DO 1100 I=1,NL
1010  WRITE(10,1010) I,R2(I),SM1(I),SP1(I),SM2(I),SP2(I)
1100  FORMAT(1X,I3,1X,F5.3,1X,F7.2,1X,F7.0,15X,F7.2,1X,F7.0)
      CONTINUE
      RETURN
      END

```

PROGRAM IND

```

C GAM IS THE SPECIFIC HEAT RATIO
C RMPRIM IS THE RELATIVE MACH NO IN THE IMPELLER.
C N1 IS THE NUMBER OF STATIONS ALONG THE BLADE THAT
C ARE BEING SUPPLIED WITH RADIAL AND ANGULAR
C COORDINATES. R1 BEING THE RADIAL COORDINATE AND
C THI BEING THE ANGULAR COORDINATE.
C
C N IS THE NUMBER OF RADIAL GRID POINTS WITHIN THE
C IMPELLER BLADE REGION. (INCLUDING THE MAX. R POINT)
C
C NN IS THE NUMBER OF RADIAL GRID POINTS WITHIN
C THE DIFFUSER REGION.
C
C JMAX IS THE NUMBER OF ANGULAR GRID POINTS BETWEEN
C BLADES.
C
C AOMEG IS THE NON-DIMENSIONAL ANGULAR VELOCITY WHICH
C EQUALS (ANG. VEL. OF IMP.)*(IMP. RADIUS)/(SPEED OF
C SOUND AT UP STREAM INFINITY).
C
C MODE IS THE NUMBER OF MODES OF ACOUSTIC RADIATION
C EXPANSION.
C
C NBLADE IS THE NUMBER OF IMPELLER BLADES.
C
C NVANE IS THE NUMBER OF DIFFUSER VANES.
C

```

C ZTHICK IS THE THICKNESS OF THE IMPELLER CHANNEL
C IN THE AXIAL DIRECTION.
C
C NZMODE IS THE NUMBER OF HALF WAVE-LENGTHS IN
C THE AXIAL DIRECTION.
C
C SUP IS FOR SUPPLEMENTARY DATA. LET SUP=
C 0.0 IF NOT WANTED, OR 1.0 IF WANTED.
 DATIN R1(1)=3.0,4.0,6.0,
 THI(1)=1.0,1.0,1.0,
 GAM=1.4,
 RMPRI=.6,
 N1=3,
 N=60,
 NN=15,
 JMAX=31,
 AOMEG=2.0,
 MODE=1,
 NBLADE=20,
 NVANE=25,
 ZTHICK=.10,
 NZMODE=0,
 SUP=0.0

E. RESULTS OF SAMPLE CASES

The computer program presented in Appendix D. was run for a number of hypothetical centrifugal gas compressors. The results are presented here in figures E.1 through E.32. In all the results the impeller blades had no curvature. The figures are all for the same geometry of compressor, and the same upstream properties. To see their effects, the following were varied:

1. number of impeller blades,
2. number of diffuser vanes,
3. impeller blade angular velocity,
4. mode number.

In all of figures E.1 through E.32, the meaning of the symbols are:

1. NB equals number of impeller blades.
2. NV equals number of diffuser vanes.
3. WB equals impeller blade dimensionless angular velocity,
 $\frac{\omega R}{C_\infty}$ Where ω equals the angular velocity of the
impeller, R equals the impeller outer radius, C_∞ equals
the speed of sound at the upstream reference point.
4. NZ equals the mode number of acoustical expansion in the z
direction of the impeller channel (duct mode number).

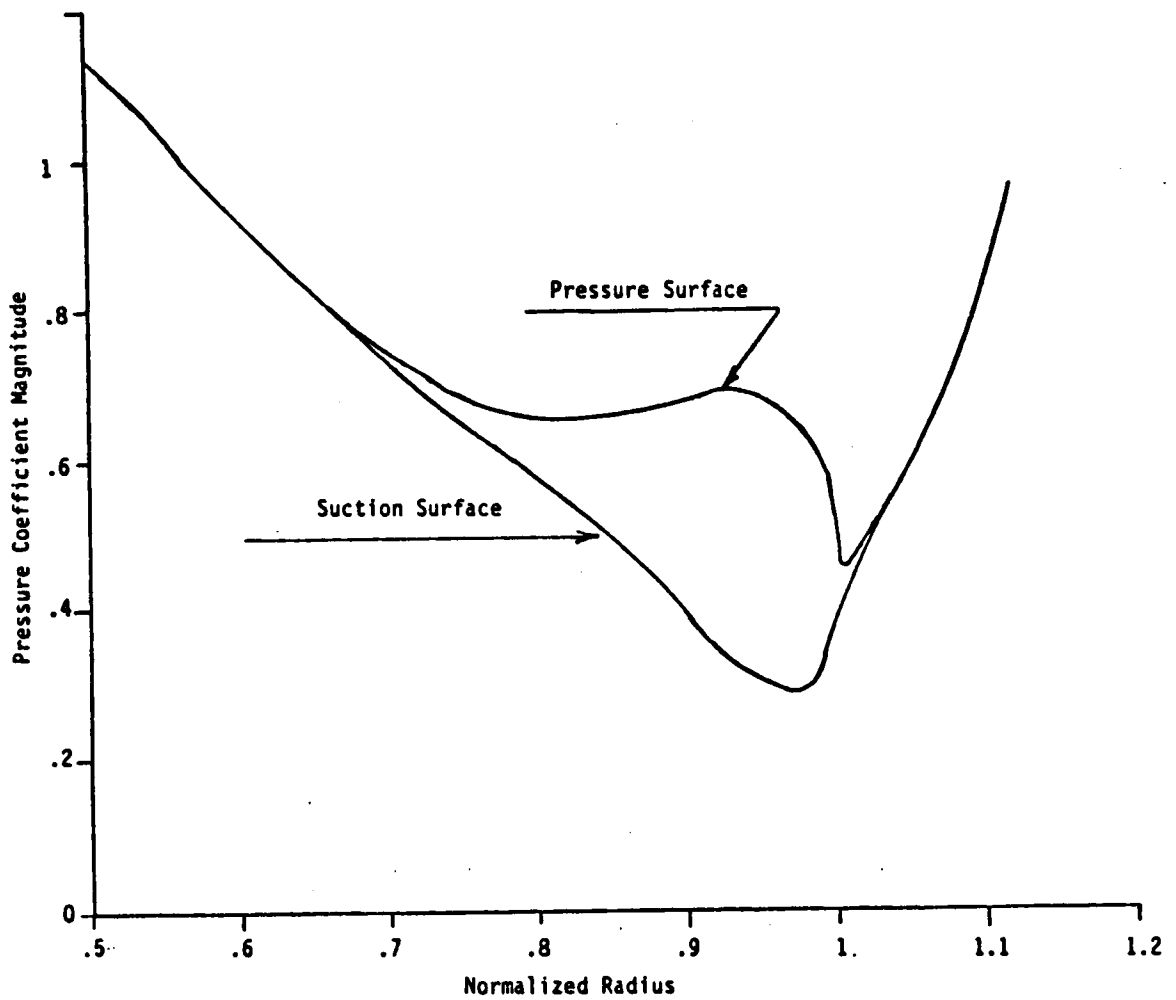


Figure E.1 Pressure Coefficient Magnitude versus Normalized Radius;
NB = 20, NV = 10, WB = .5, NZ = 0

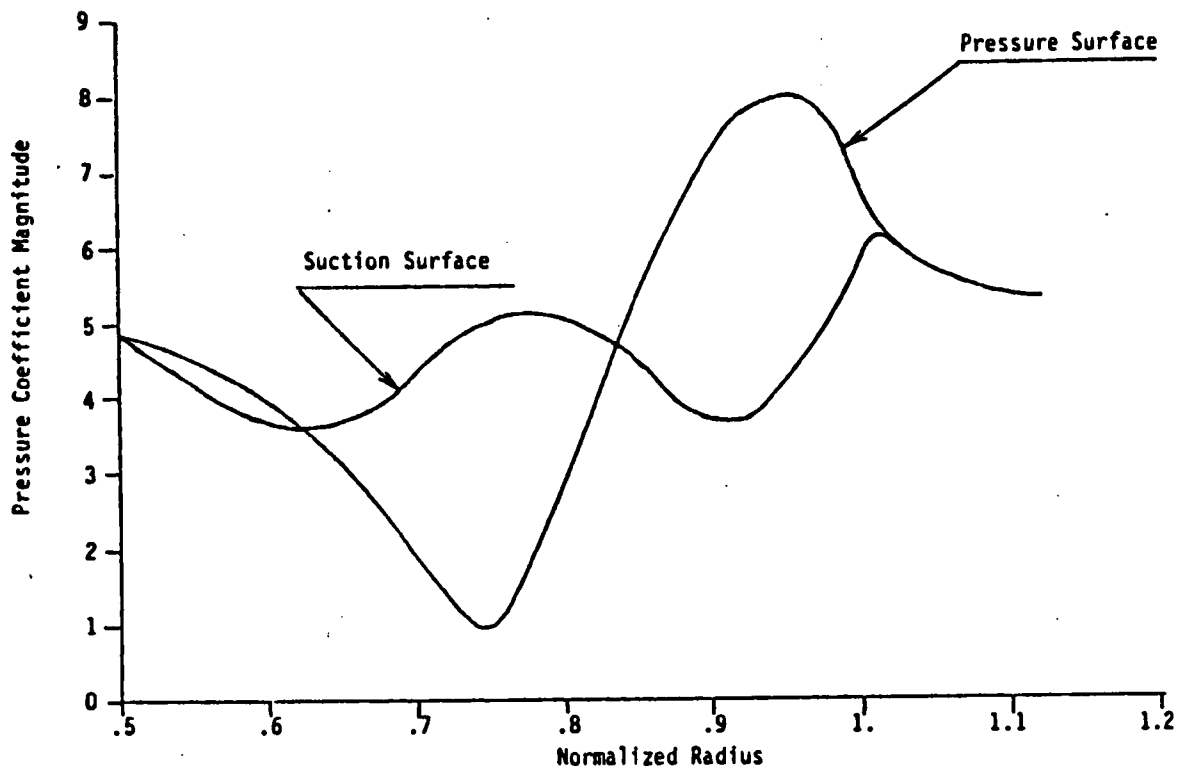


Figure E.2 Pressure Coefficient Magnitude versus Normalized Radius;
NB = 20, NV = 10, WB = 1, NZ = 0

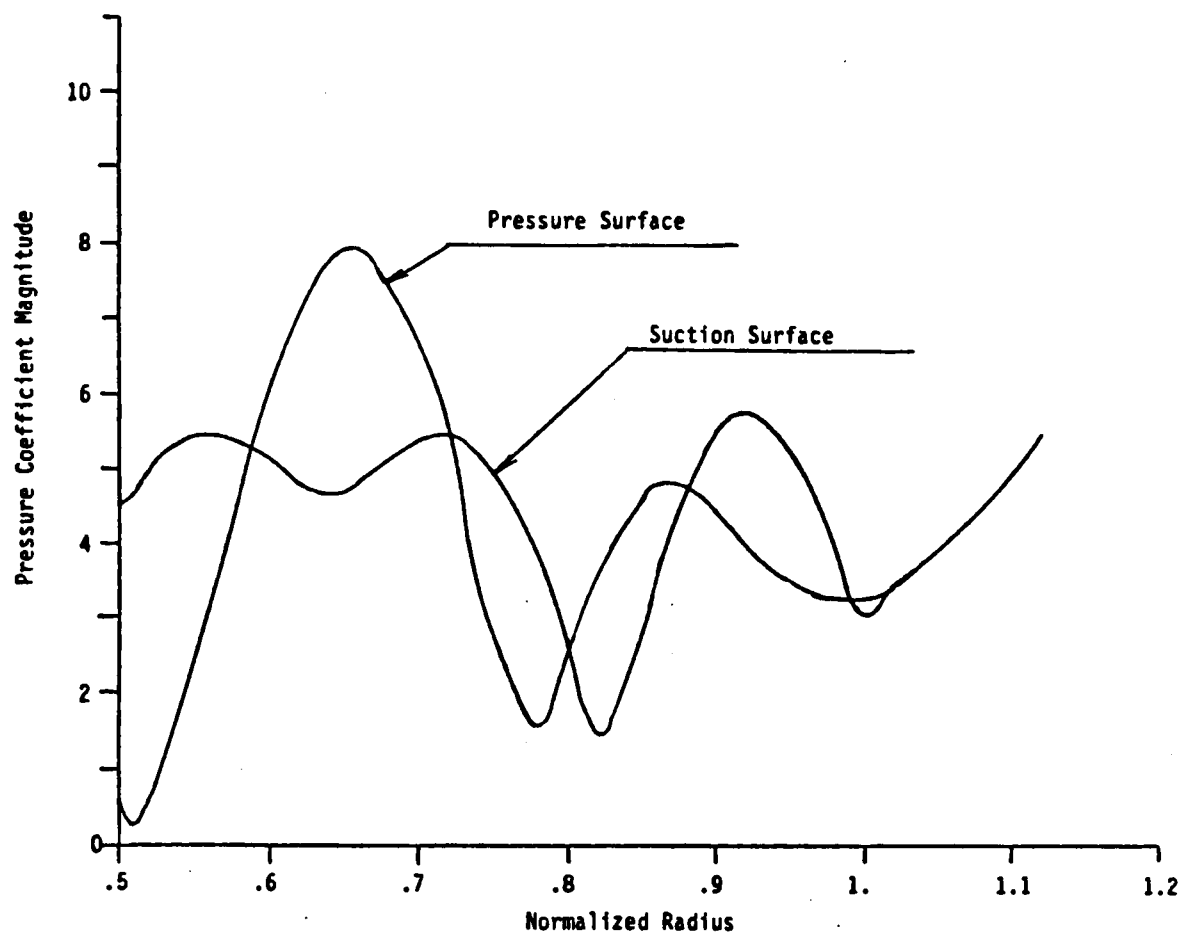


Figure E.3 Pressure Coefficient Magnitude versus Normalized Radius;
NB = 20, NV = 10, WB = 1.5, NZ = 0

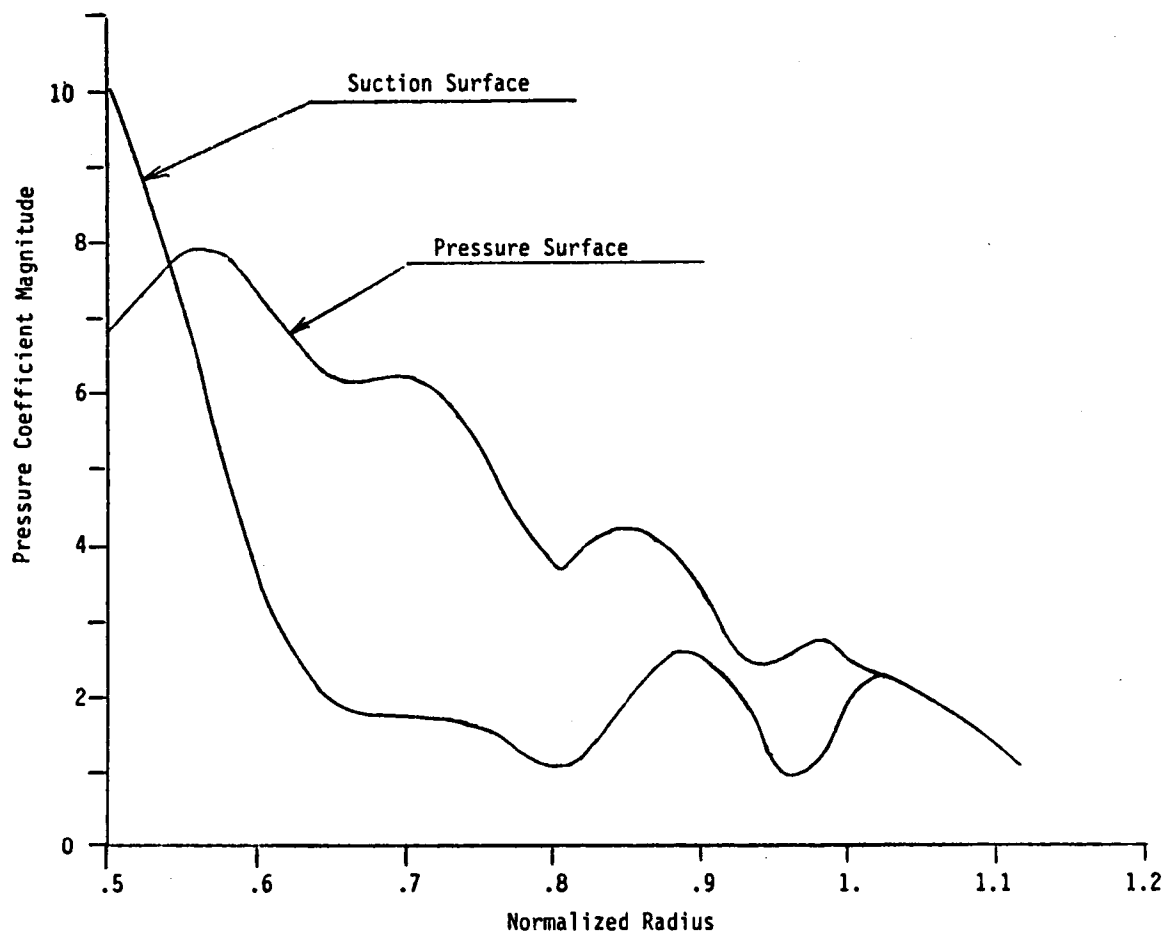


Figure E.4 Pressure Coefficient Magnitude versus Normalized Radius;

NB = 20, NV = 10, WB = 2, NZ = 0

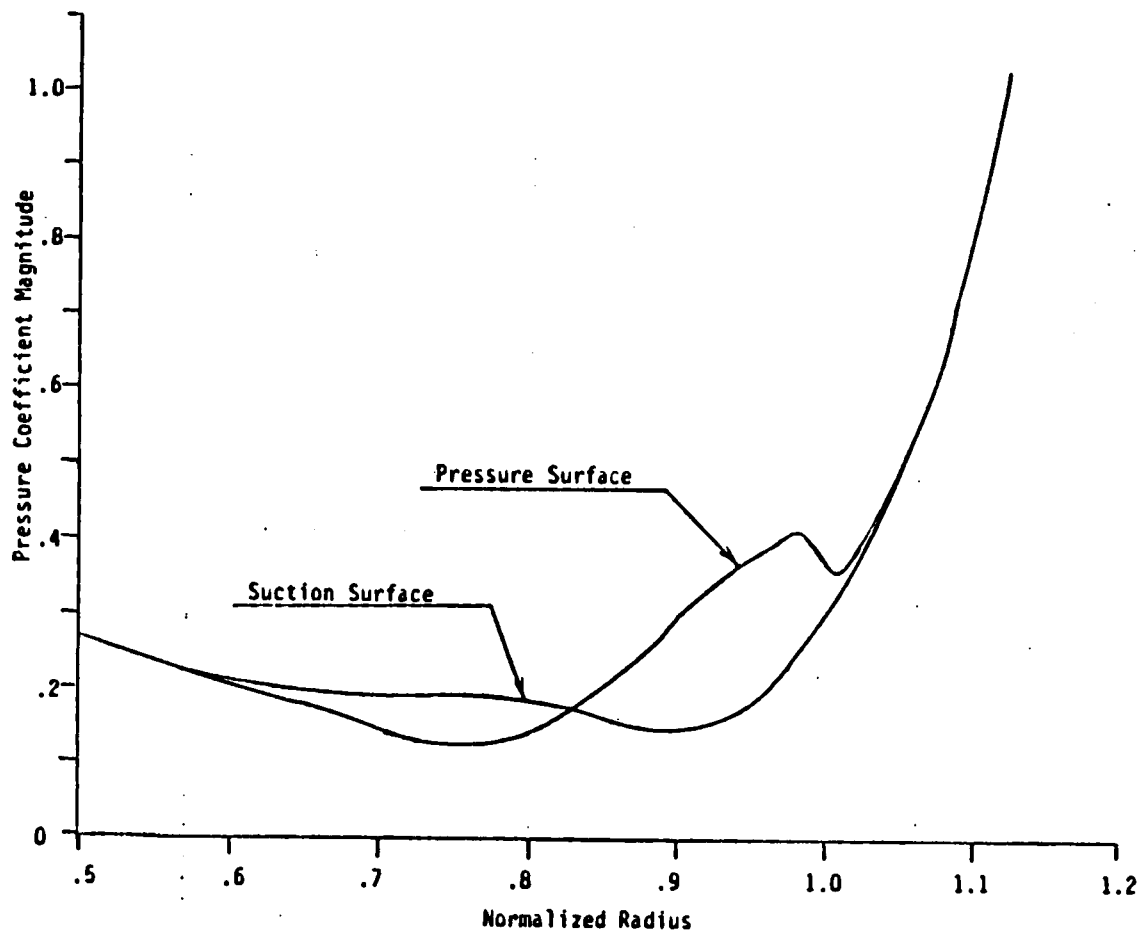


Figure E.5 Pressure Coefficient Magnitude versus Normalized Radius;
NB = 20, NV = 15, WB = .5, NZ = 0

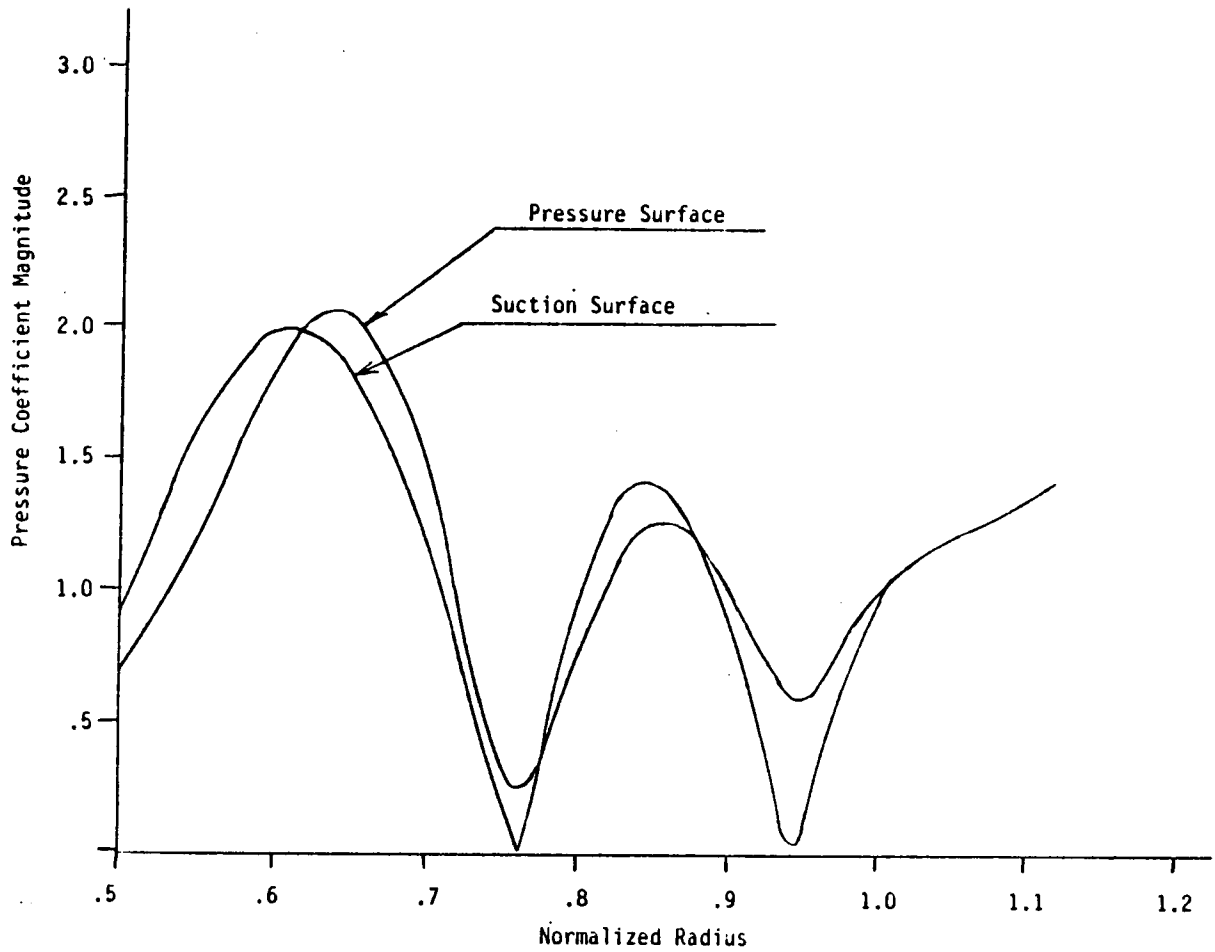


Figure E.6 Pressure Coefficient Magnitude versus Normalized Radius;
NB = 20, NV = 15, WB = 1, NZ = 0

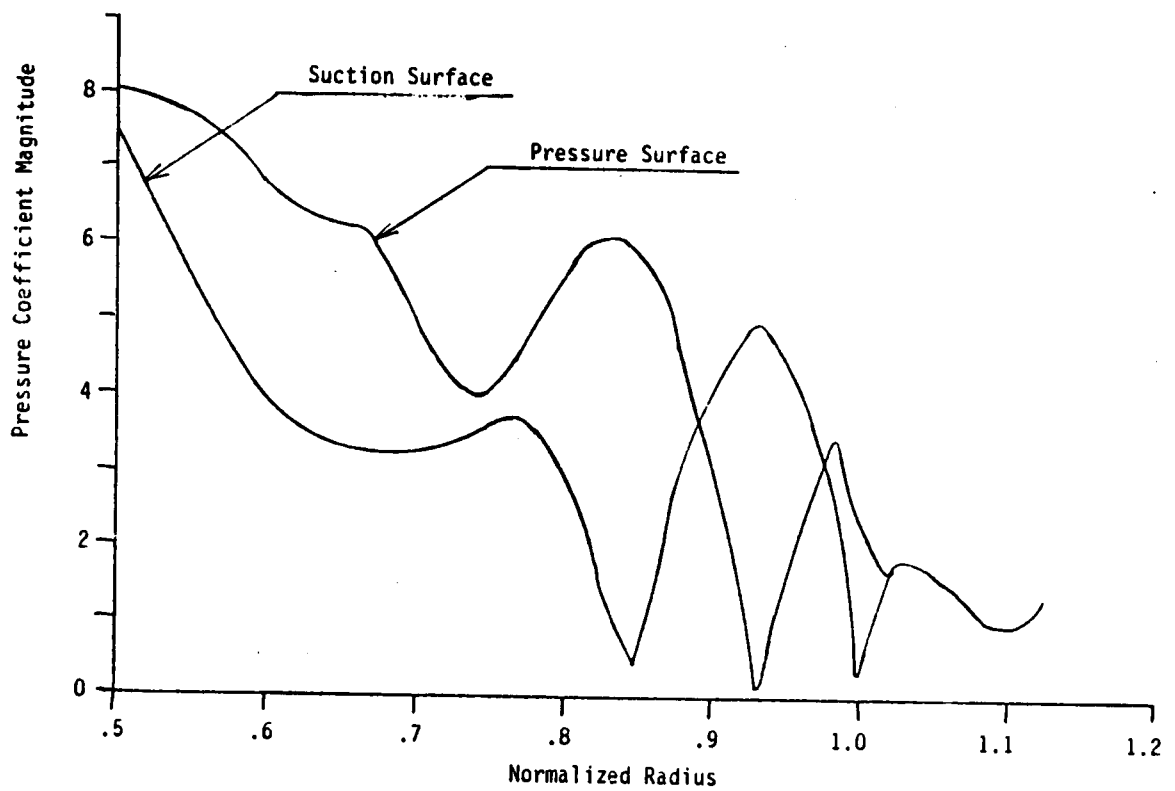


Figure E.7 Pressure Coefficient Magnitude versus Normalized Radius;

NB = 20, NV = 15, WB = 1.5, NZ = 0

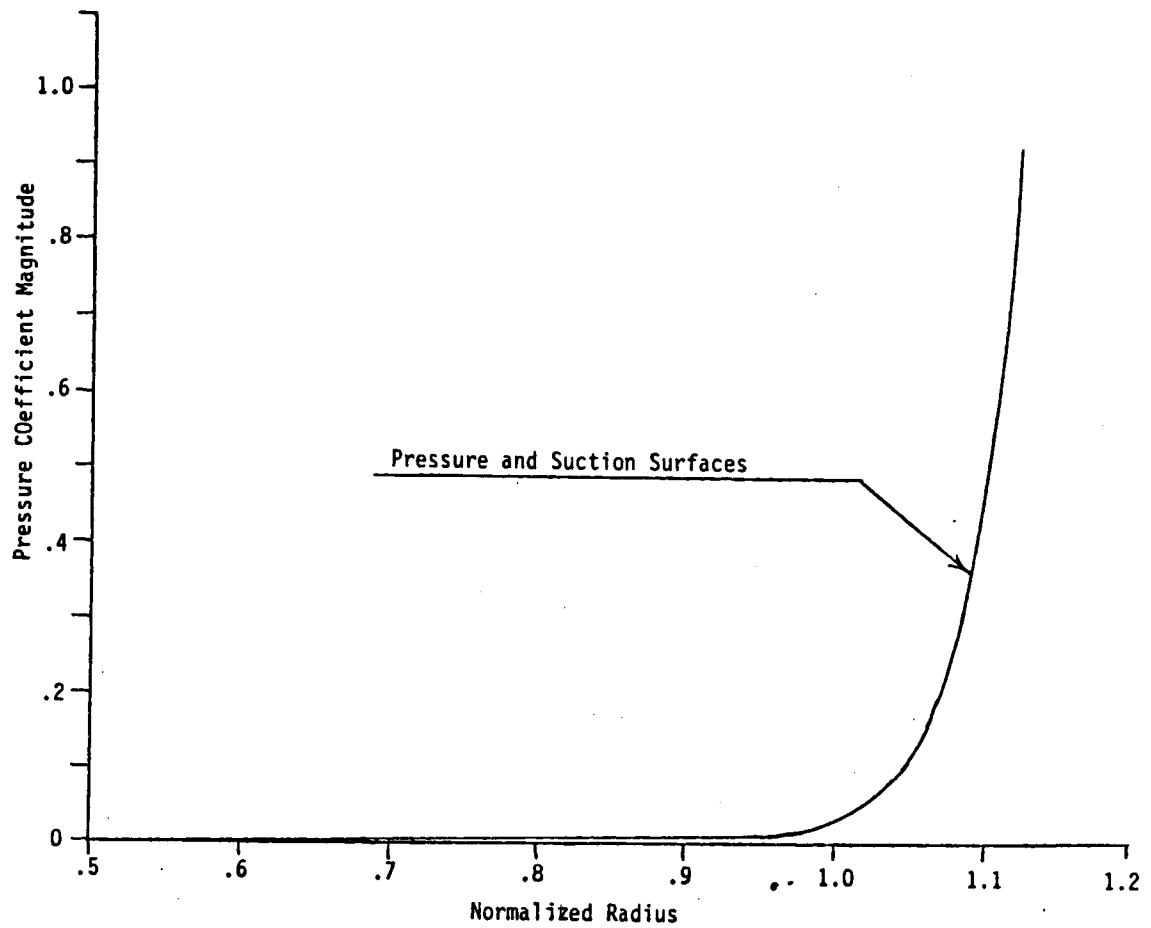


Figure E.8 Pressure Coefficient Magnitude versus Normalized Radius;
NB = 20, NV = 15, WB = 1.5, NZ = 1

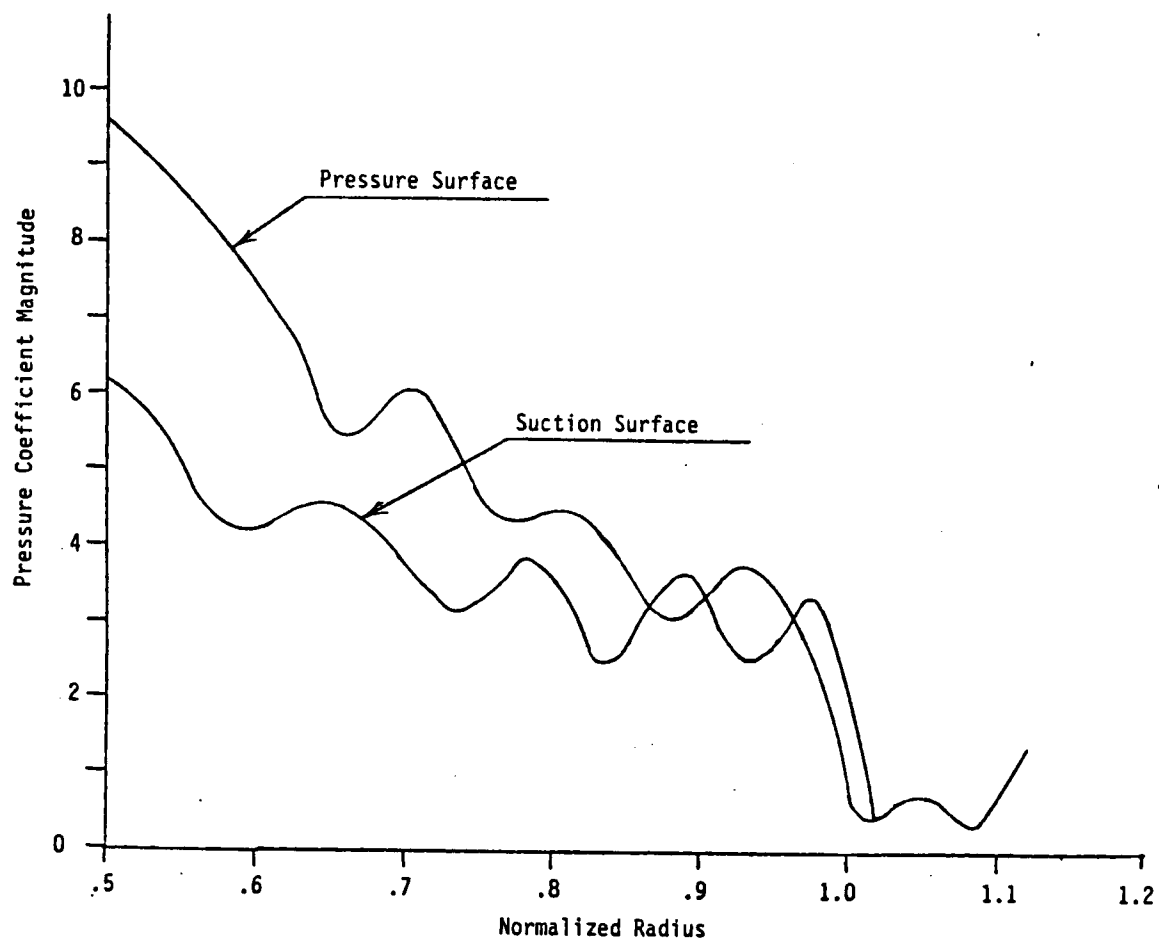


Figure E.9 Pressure Coefficient Magnitude versus Normalized Radius;
NB = 20, NV = 15, WB = 2, NZ = 0

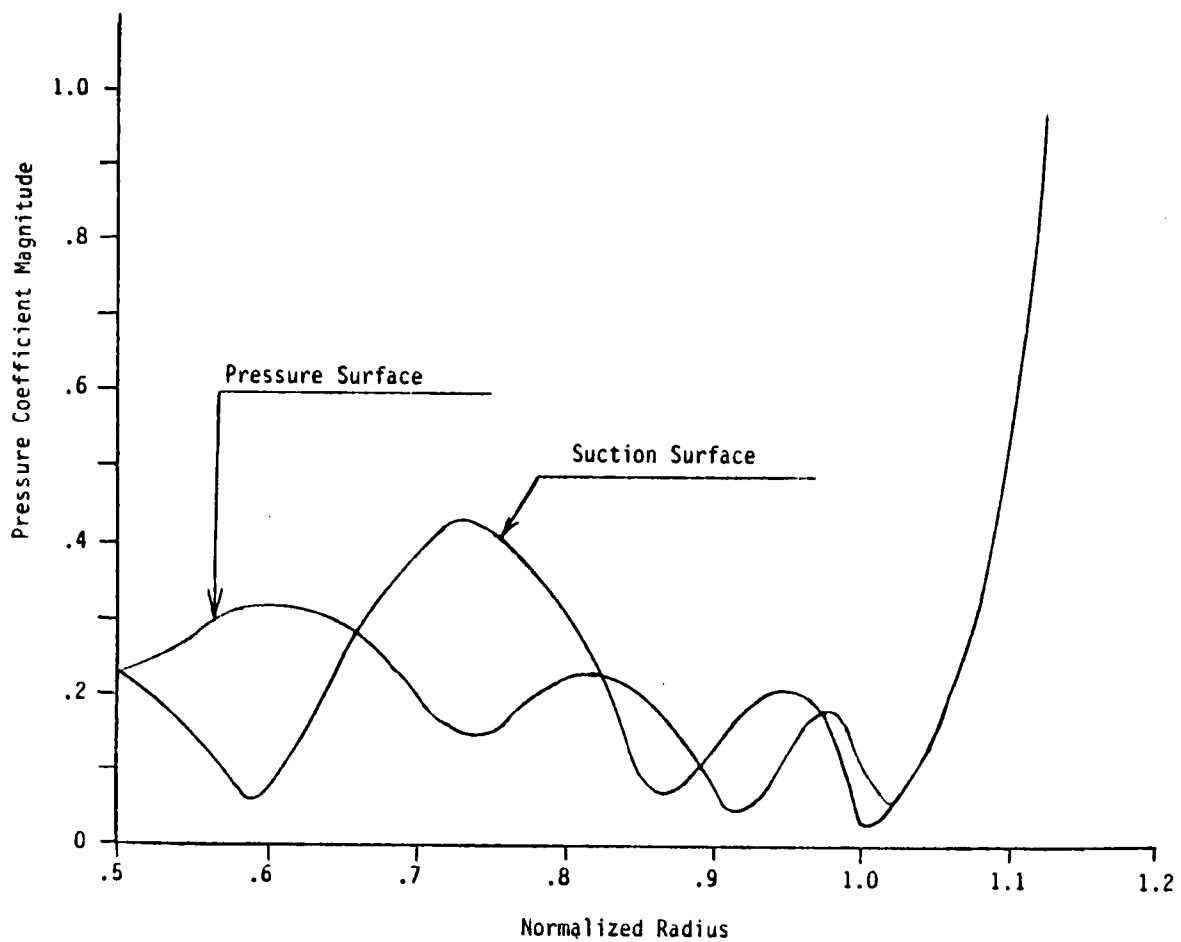


Figure E.10 Pressure Coefficient Magnitude versus Normalized Radius;

NB = 20, NV = 25, WB = .5, NZ = 0

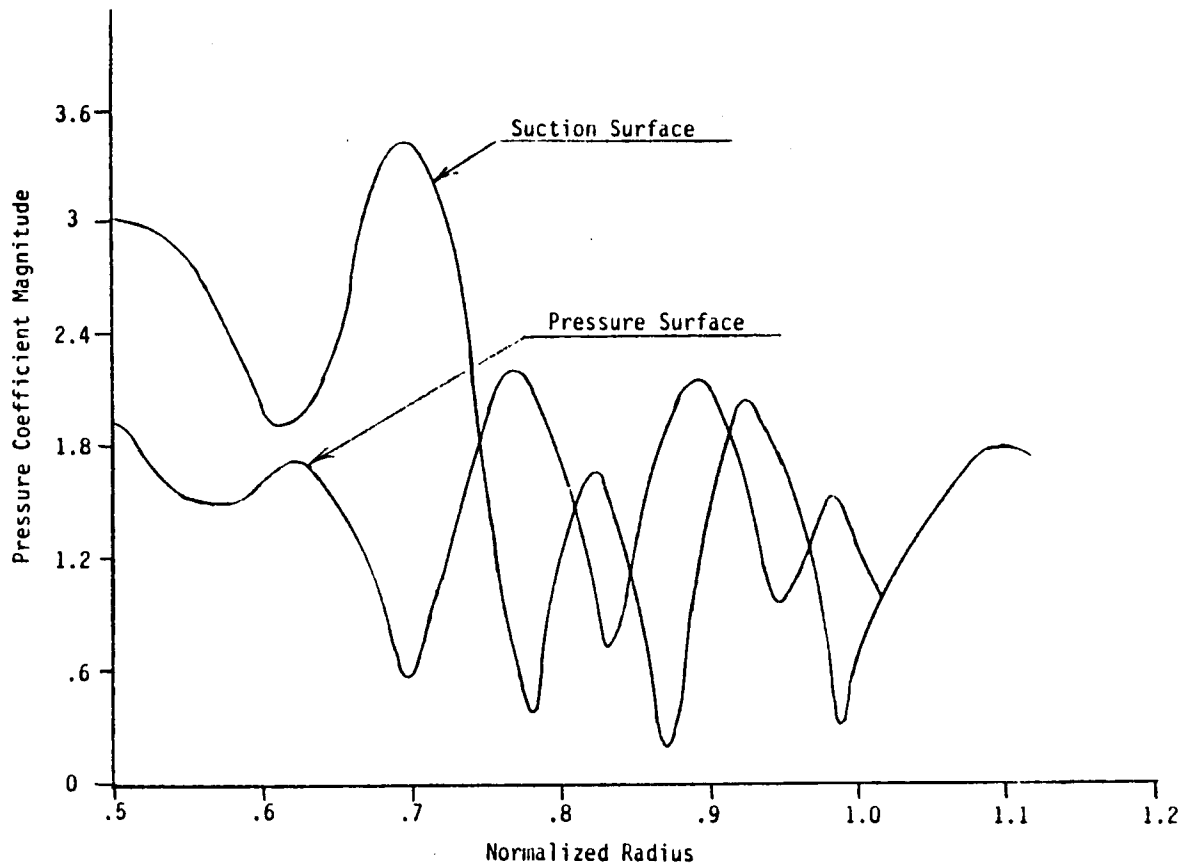


Figure E.11 Pressure Coefficient Magnitude versus Normalized Radius;
NB = 20, NV = 25, WB = 1, NZ = 0

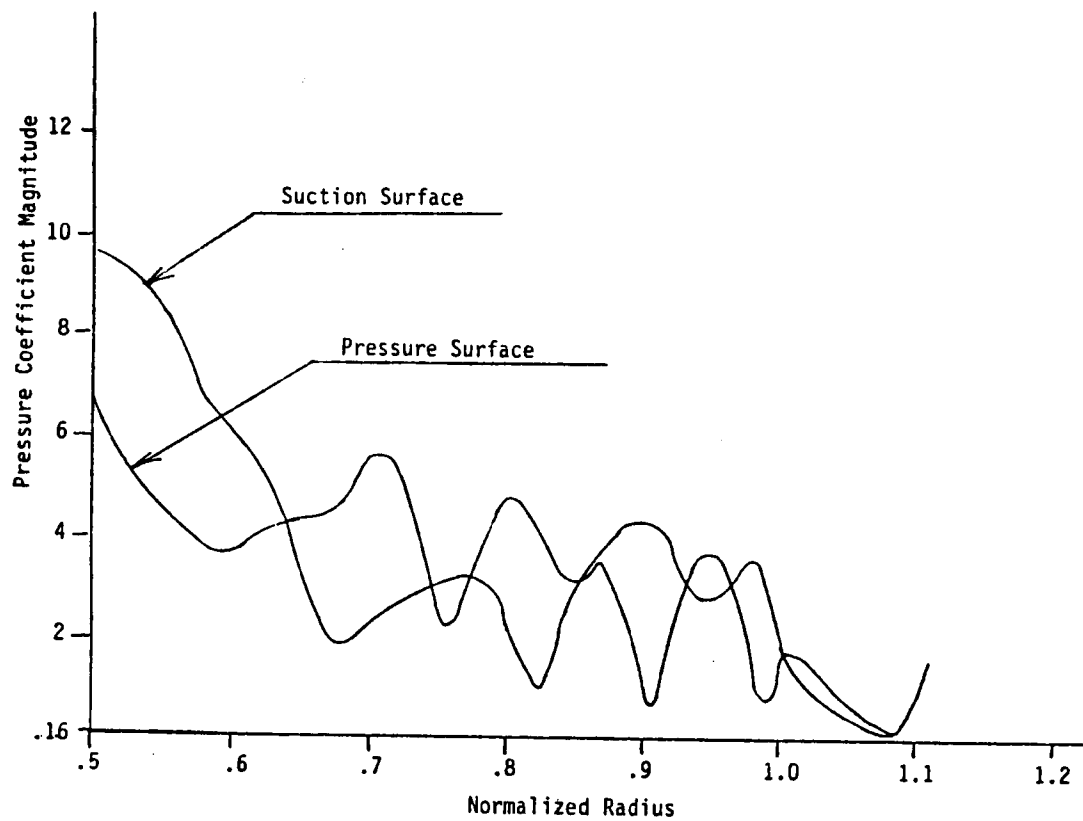


Figure E.12 Pressure Coefficient Magnitude versus Normalized Radius;
 NB = 20, NV = 25, WB = 1.5, NZ = 0

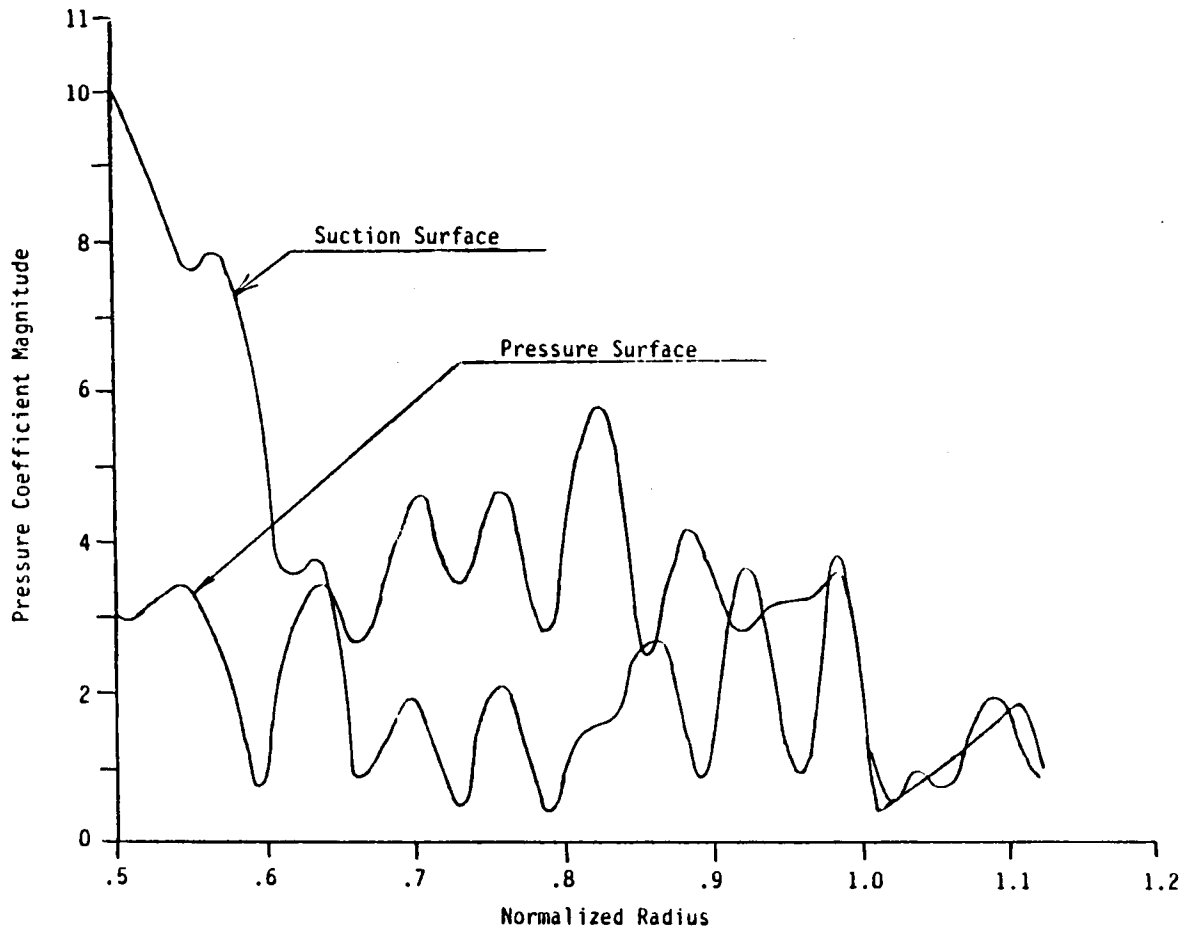


Figure E.13 Pressure Coefficient Magnitude versus Normalized Radius;

NB = 20, NV = 25, WB = 2, NZ = 0

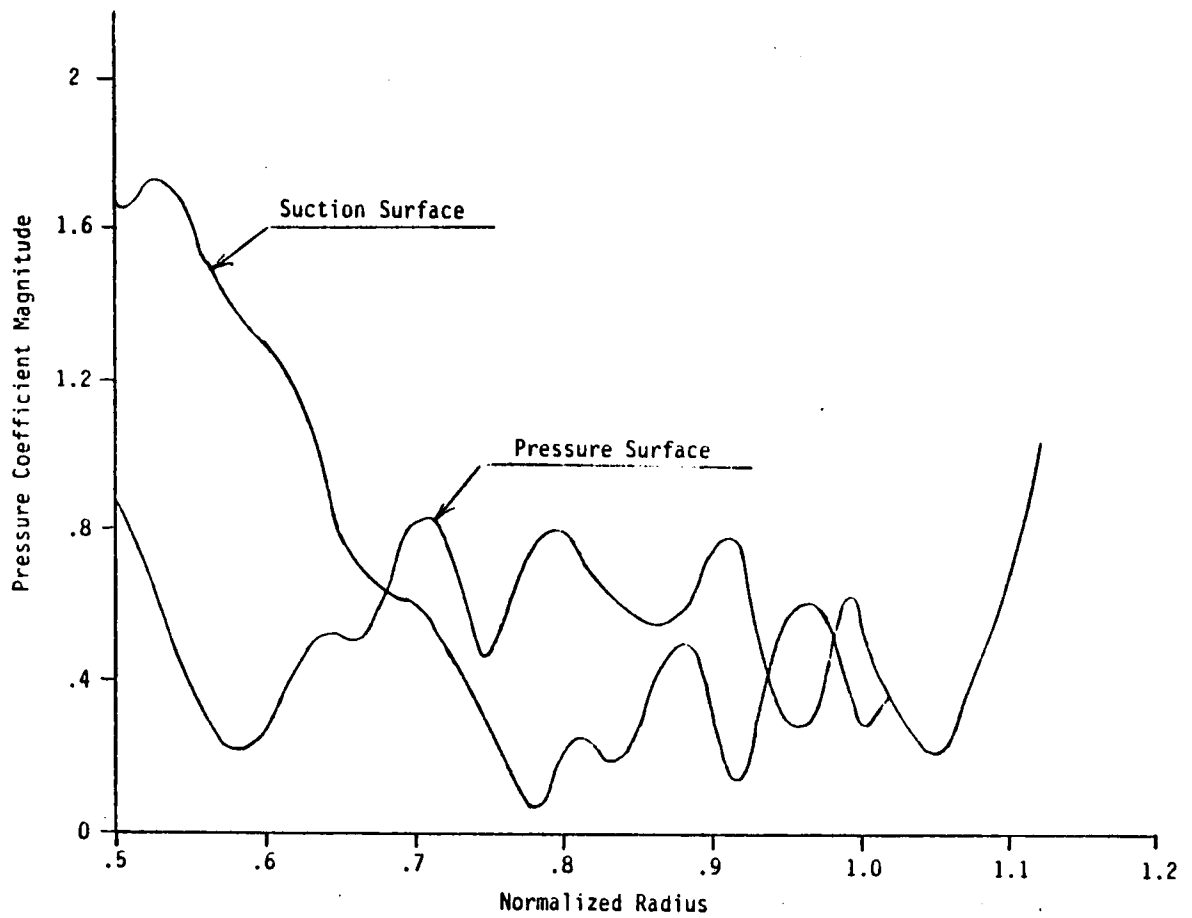


Figure E.14 Pressure Coefficient Magnitude versus Normalized Radius;
NB = 20, NV = 25, WB = 2, NZ = 1

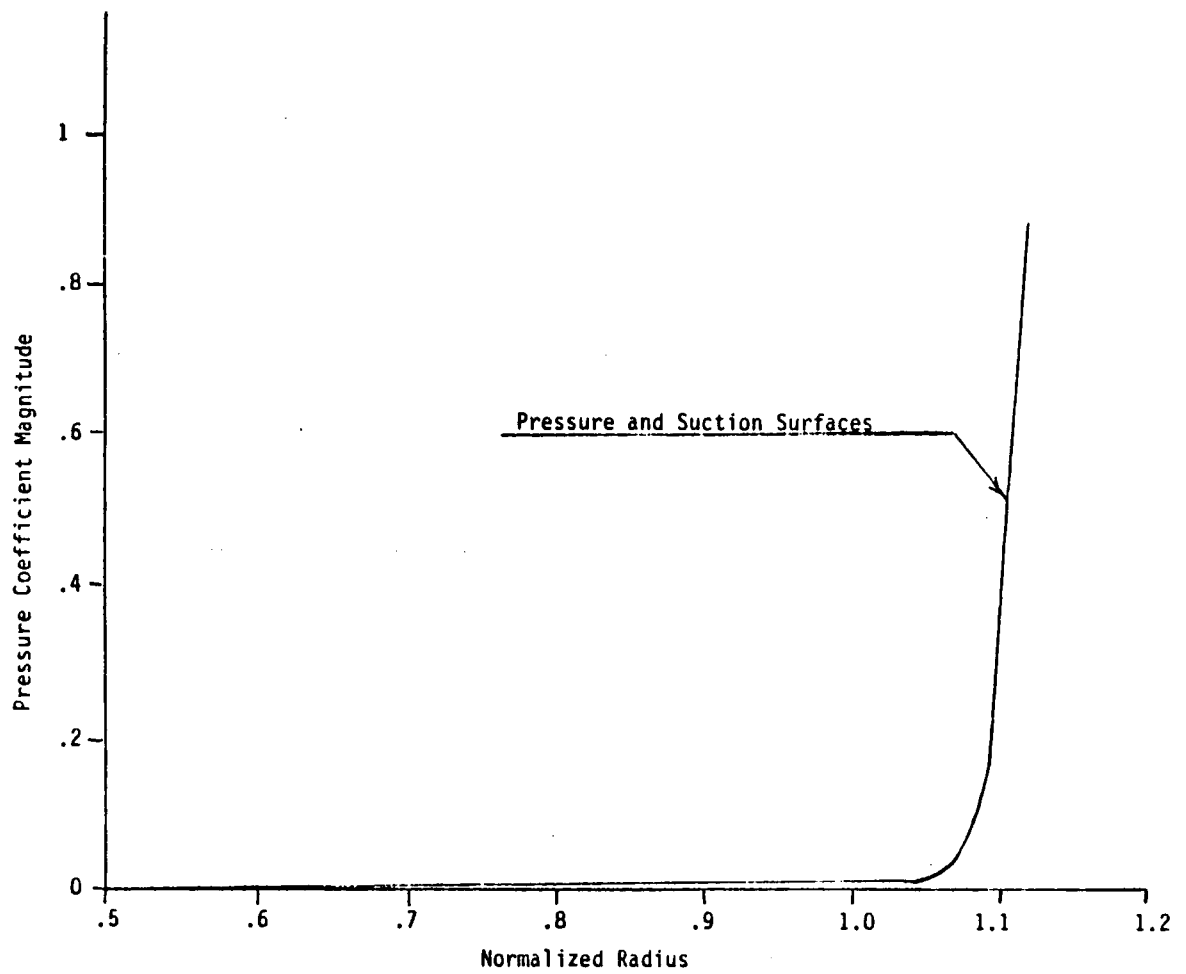


Figure E.15 Pressure Coefficient Magnitude versus Normalized Radius;

NB = 20, NV = 25, WB = 2, NZ = 2

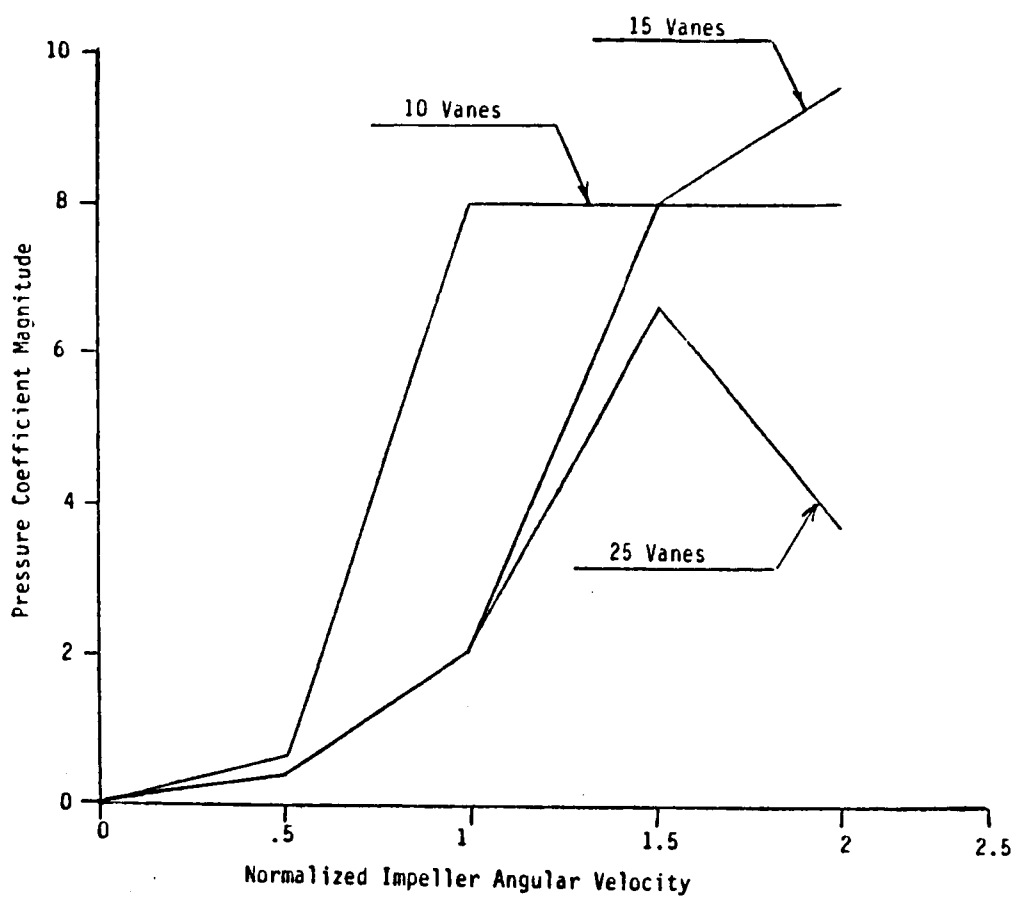


Figure E.16 Maximum Pressure Coefficient Magnitudes within the Impeller Region on the Pressure Surface, versus Normalized Impeller Speed, for Different Numbers of Diffuser Vanes

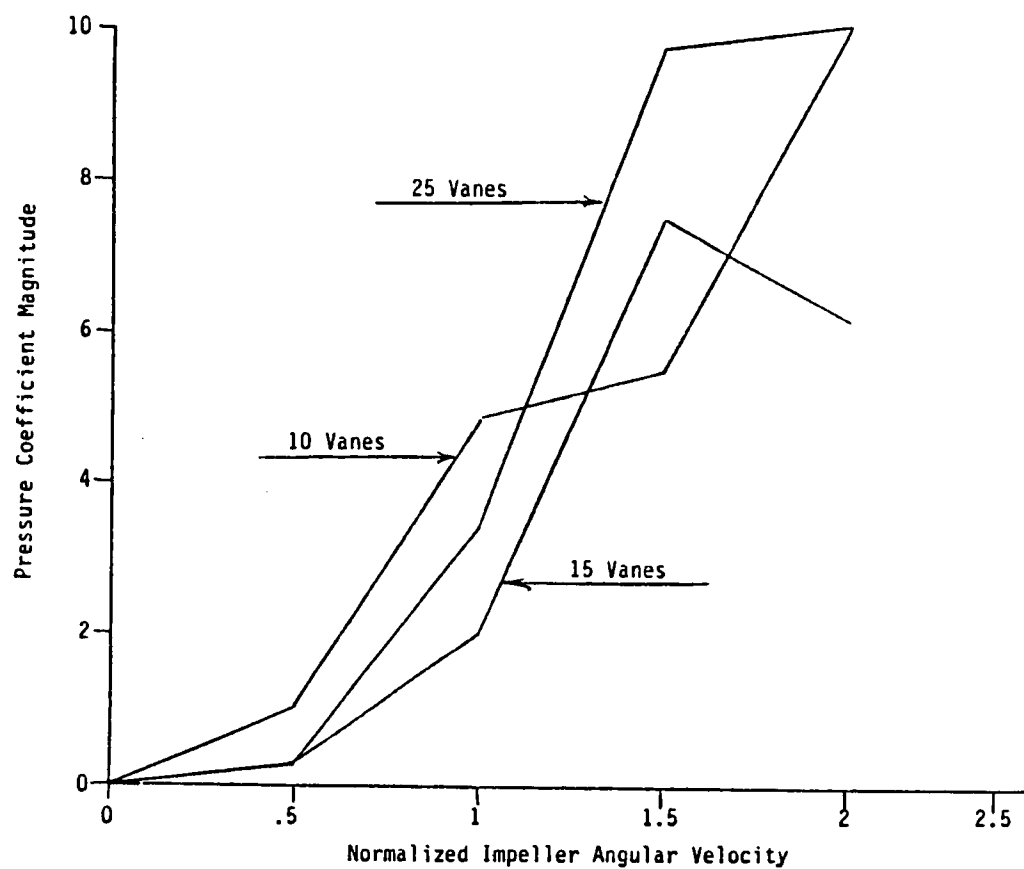


Figure E.17 Maximum Pressure Coefficient Magnitudes within the Impeller Region on the Suction Surface, versus Normalized Impeller Speed, for Different Numbers of Diffuser Vanes

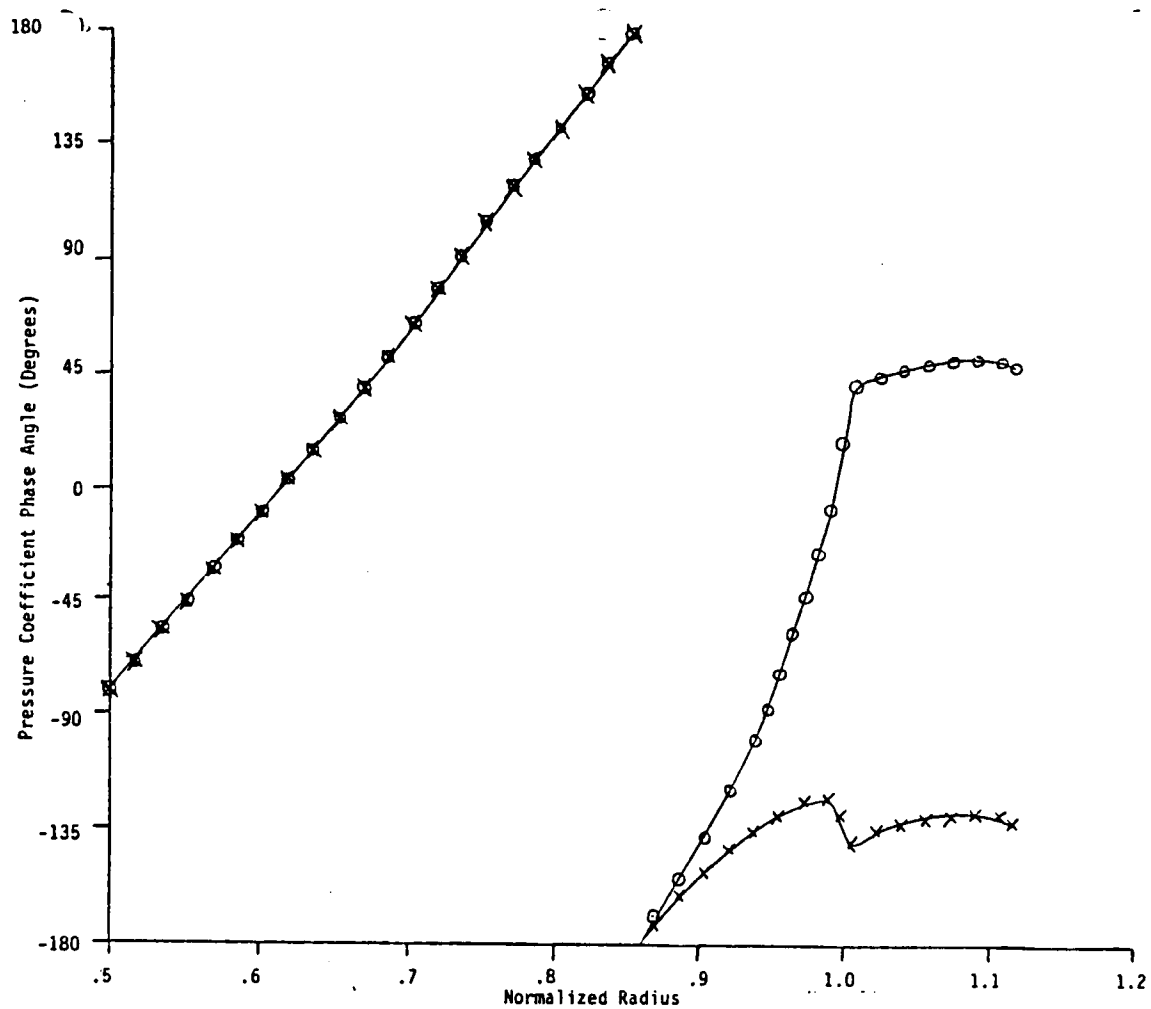


Figure E.18 Pressure Coefficient Phase Angle Versus Radius;

NB=20, NV=10, WB=.5, NZ=0

x Pressure Surface

o Suction Surface

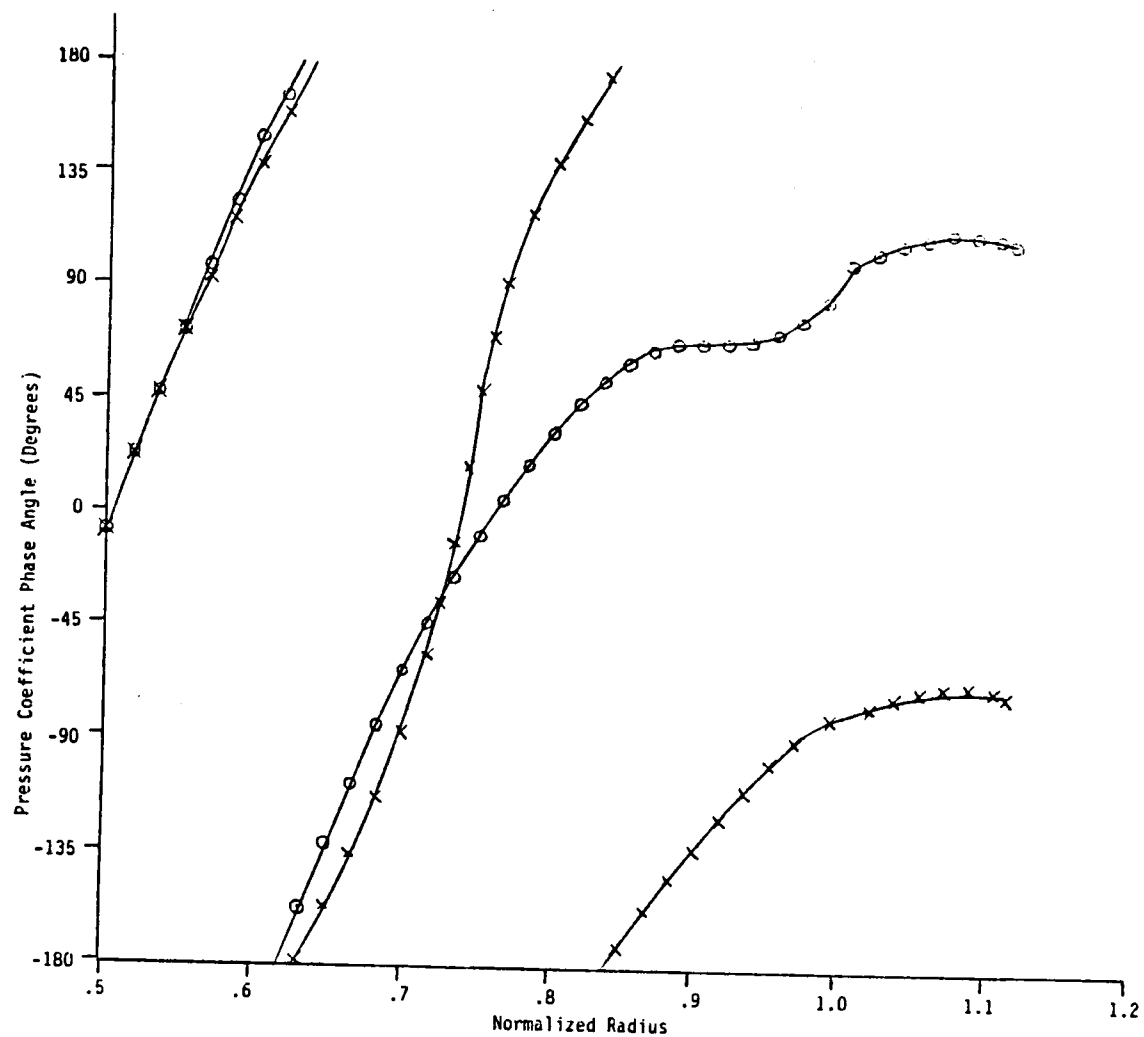


Figure E.19 Pressure Coefficient Phase Angle Versus Radius;

NB=20, NV=10, WB=1, NZ=0

x Pressure Surface

o Suction Surface

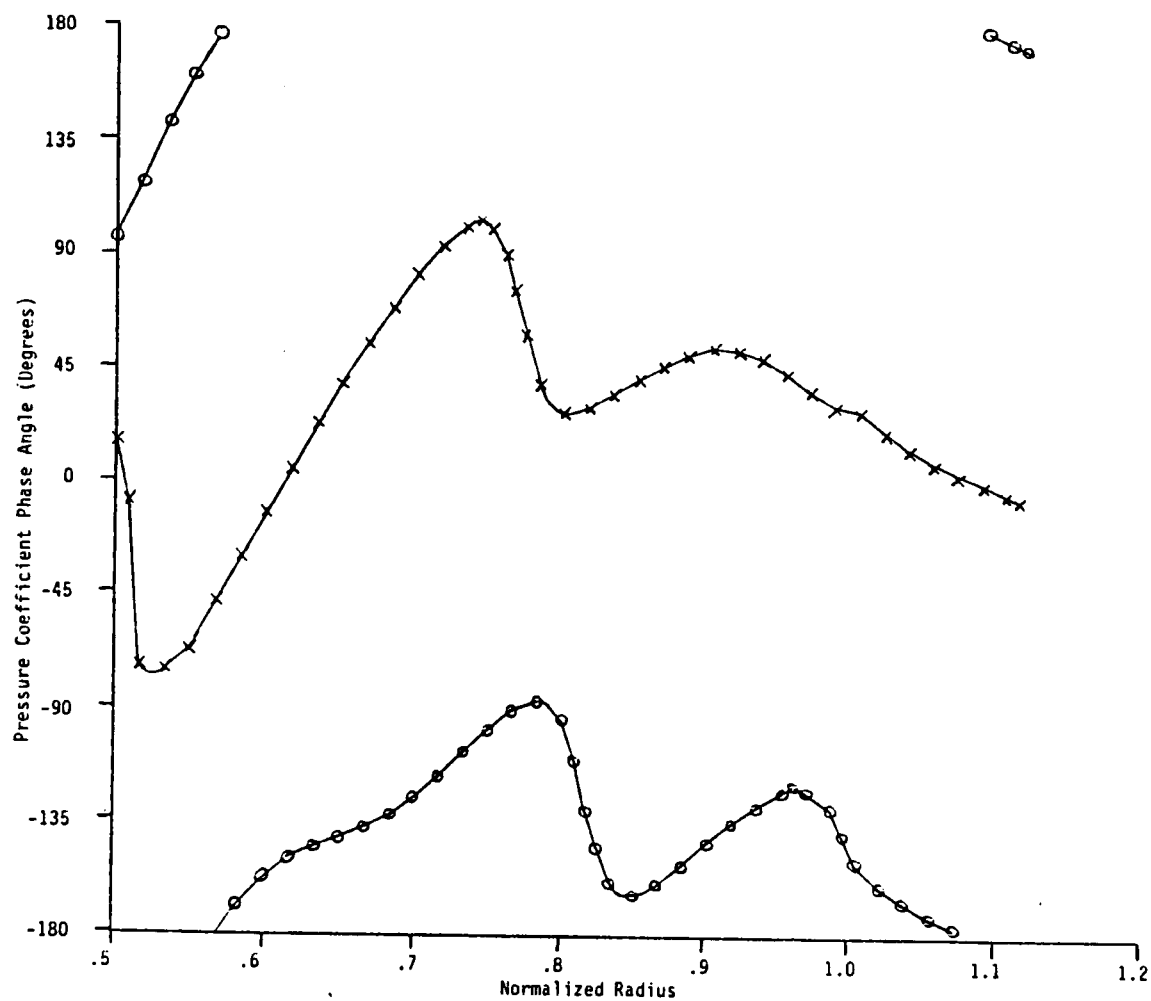


Figure E.20 Pressure Coefficient Phase Angle Versus Radius;

NB=20, NV=10, WB=1.5, NZ=0

x Pressure Surface

o Suction Surface

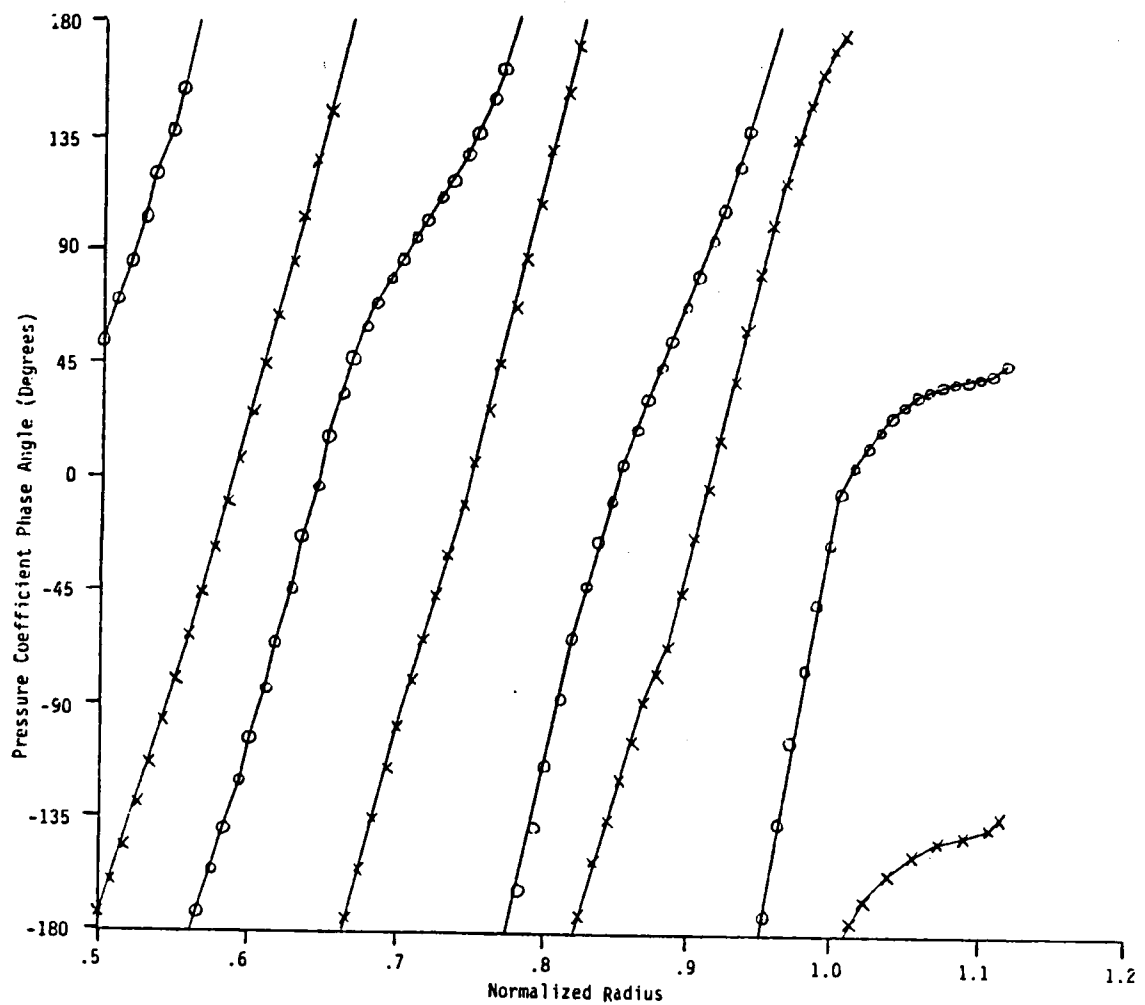


Figure E.21 Pressure Coefficient Phase Angle Versus Radius;

NB=20, NV=10, WB=2, NZ=0

x Pressure Surface

o Suction Surface

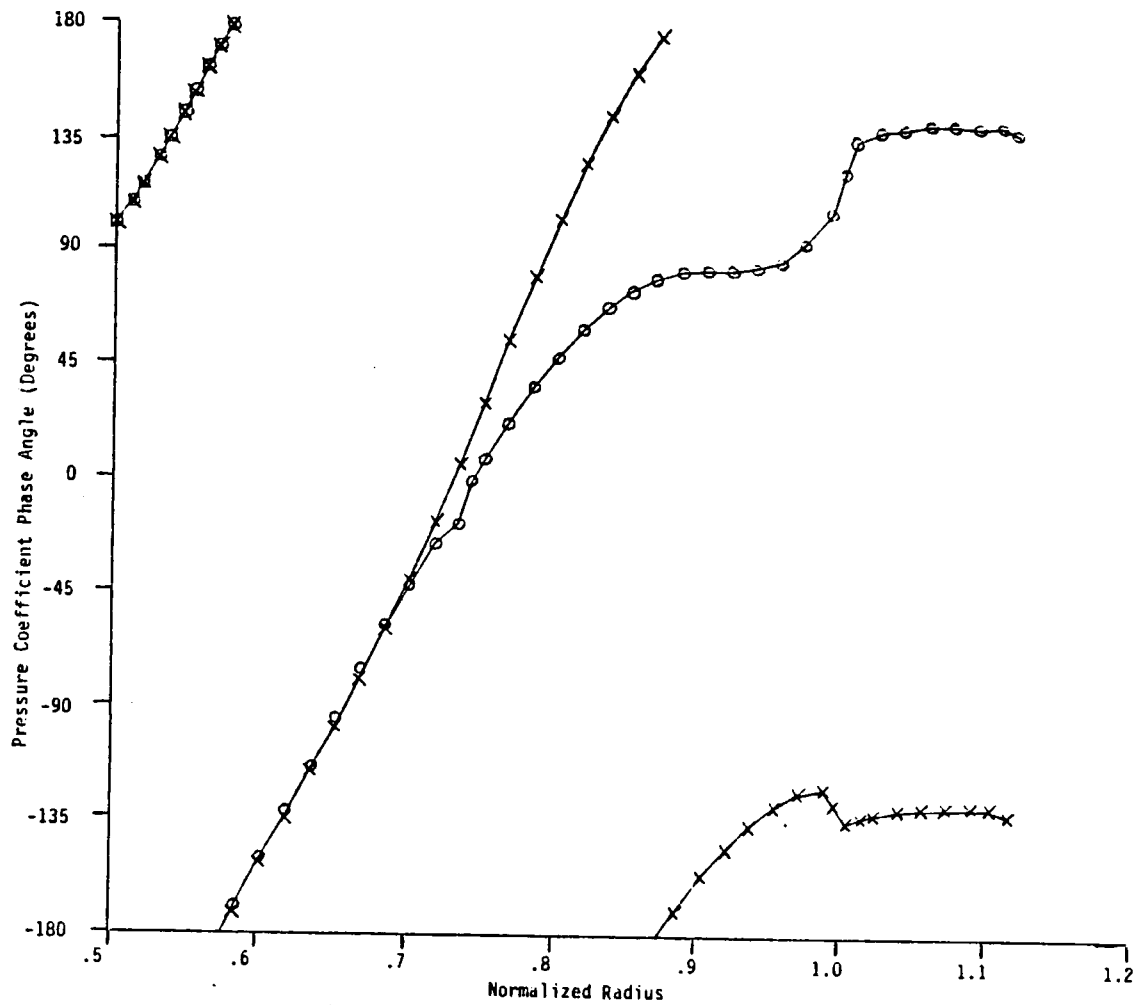


Figure E.22 Pressure Coefficient Phase Angle Versus Radius;

NB=20, NV=15, WB=.5, NZ=0

x Pressure Surface

o Suction Surface

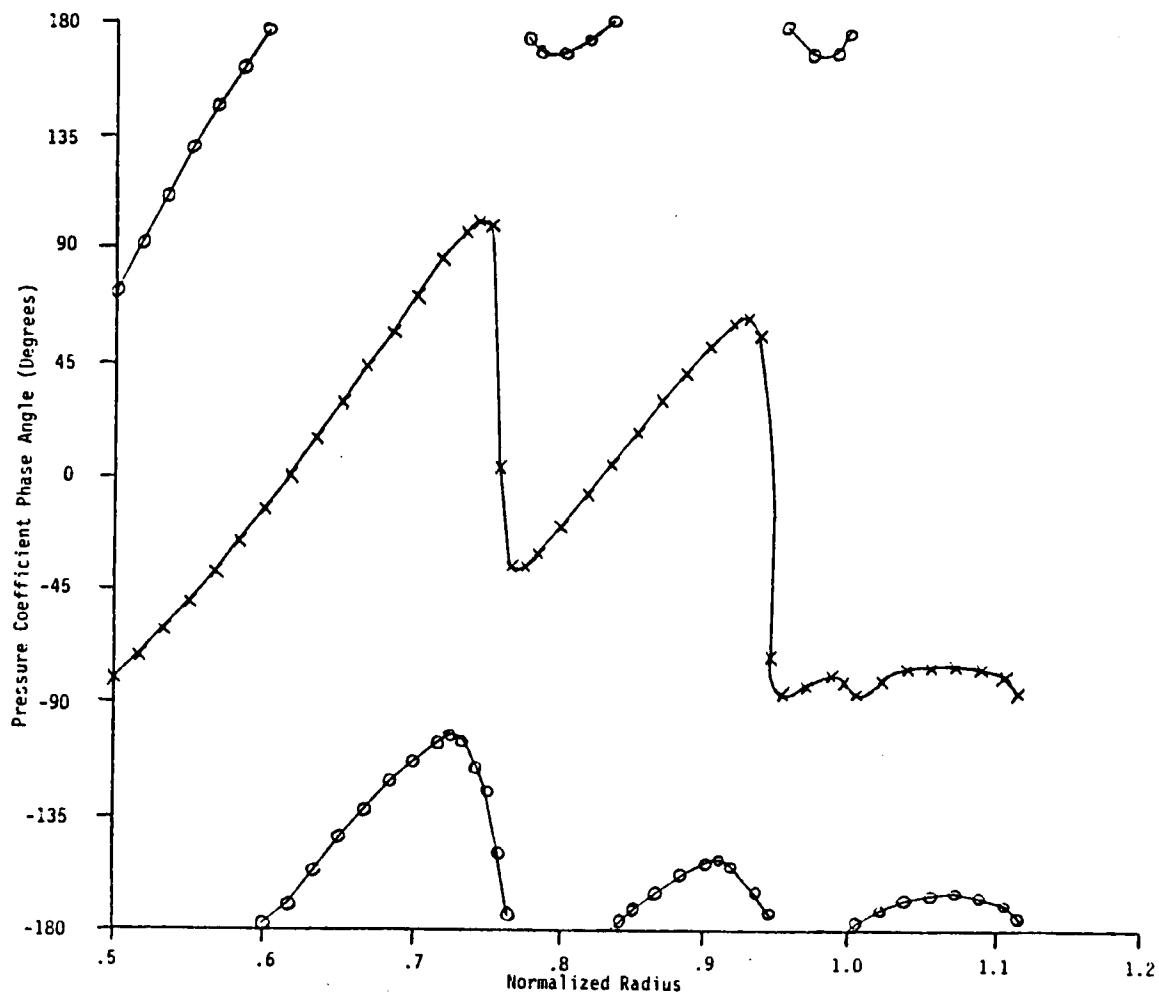


Figure E.23 Pressure Coefficient Phase Angle Versus Radius;

NB=20, NV=15, WB=1, NZ=0

x Pressure Surface

o Suction Surface

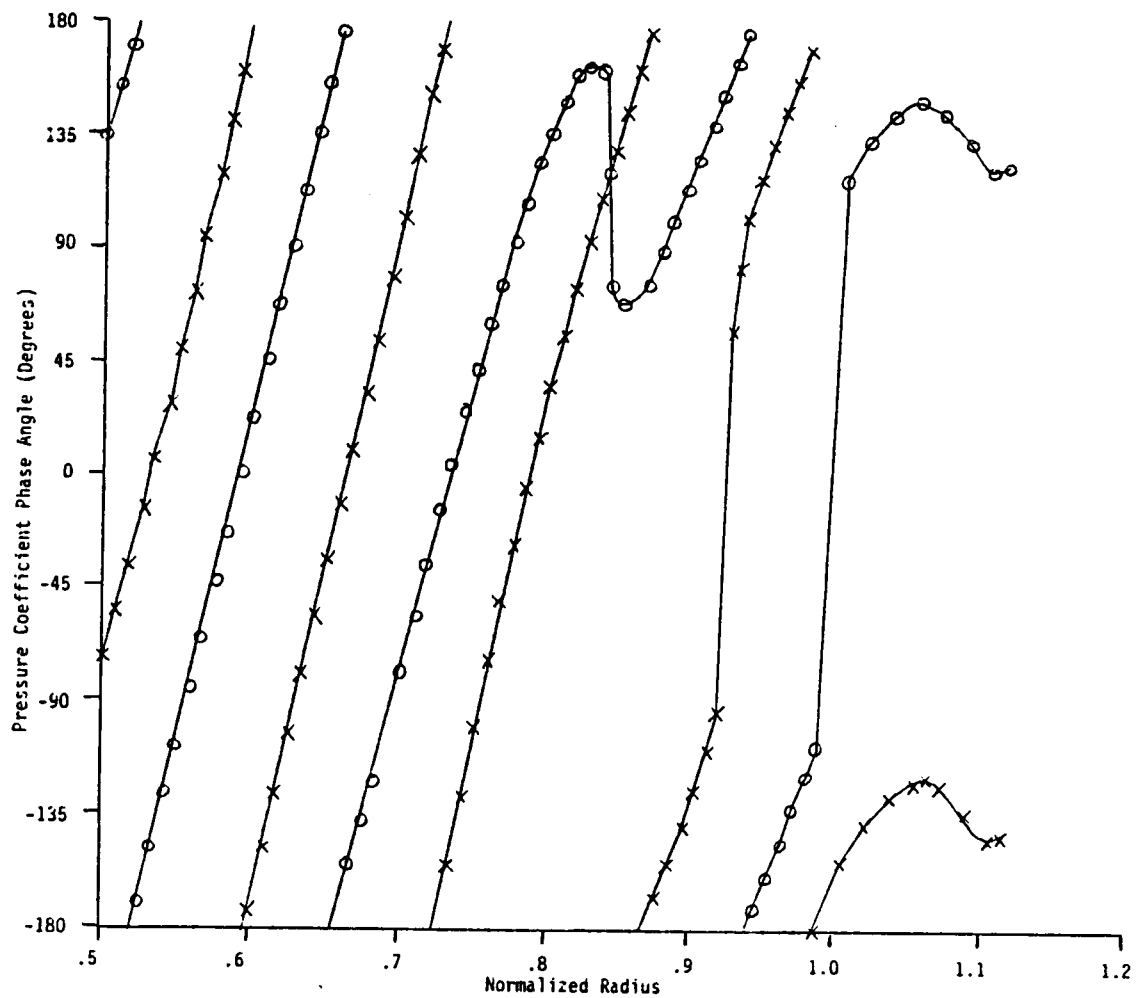


Figure E.24 Pressure Coefficient Phase Angle Versus Radius;

NB=20, NV=15, WB=1.5, NZ=0

x Pressure Surface

o Suction Surface

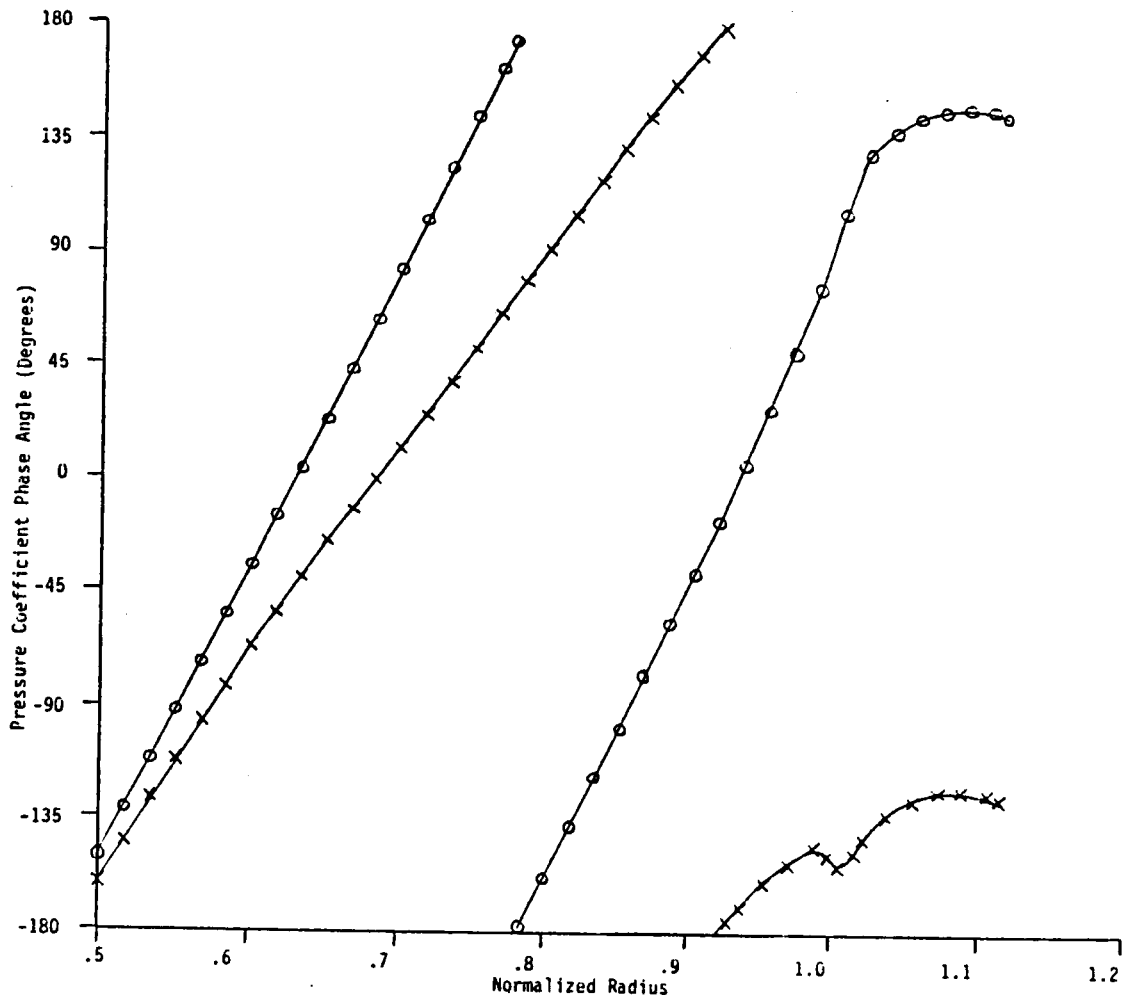


Figure E.25 Pressure Coefficient Phase Angle Versus Radius;

NB=20, NV=15, WB=1.5, NZ=1

x Pressure Surface

o Suction Surface

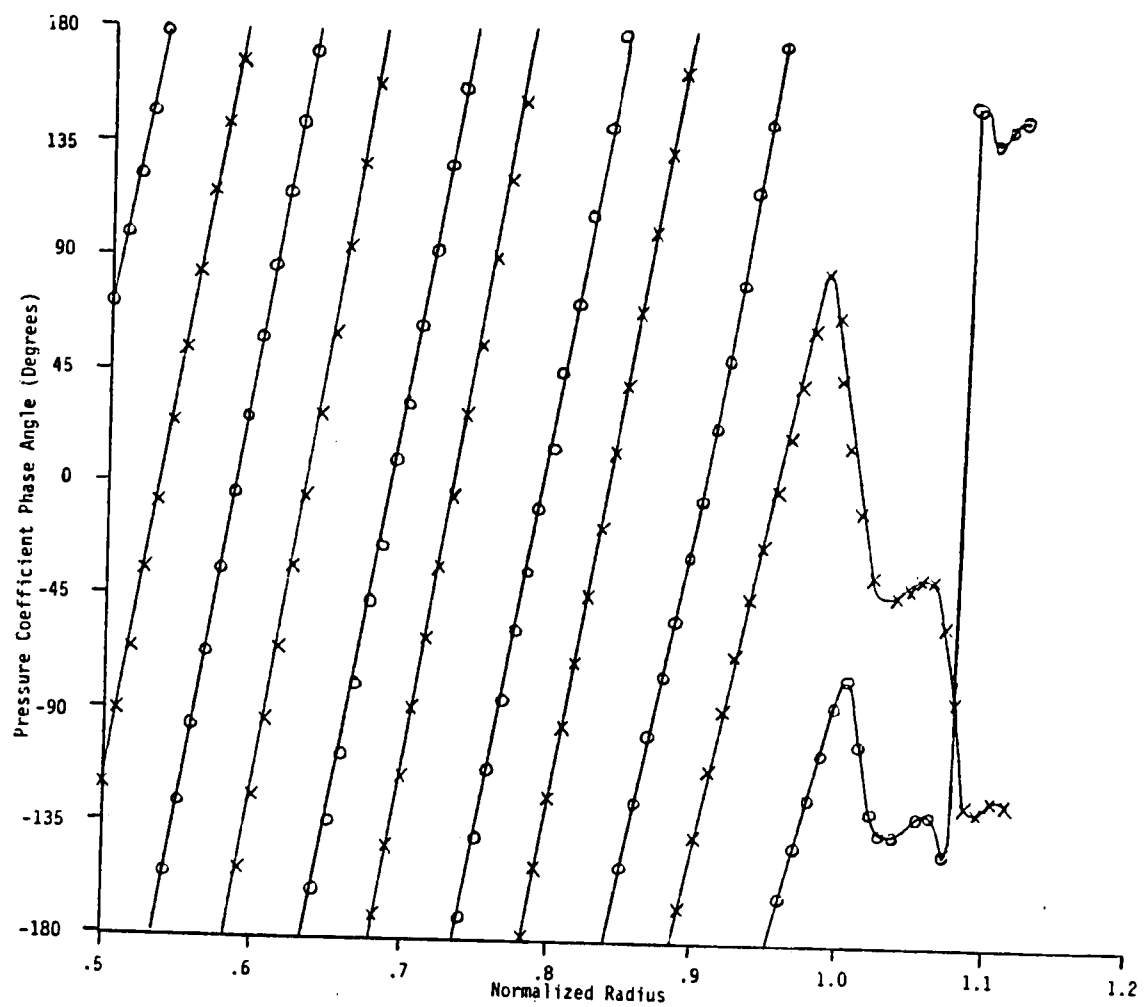


Figure E.26 Pressure Coefficient Phase Angle Versus Radius;

NB=20, NV=15, WB=2, NZ=0

x Pressure Surface

o Suction Surface

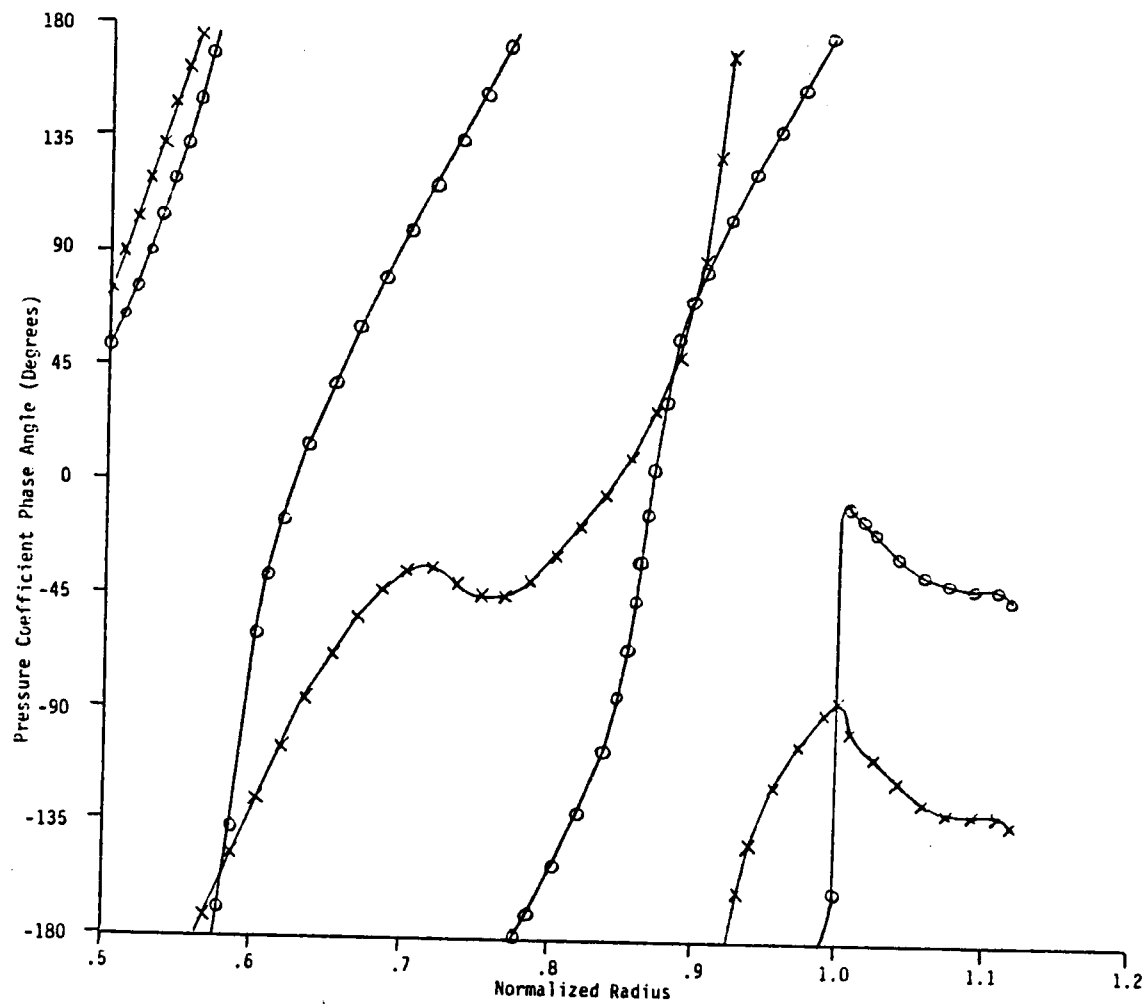


Figure E.27 Pressure Coefficient Phase Angle Versus Radius;

NB=20, NV=25, WB=.5, NZ=0

x Pressure Surface

o Suction Surface

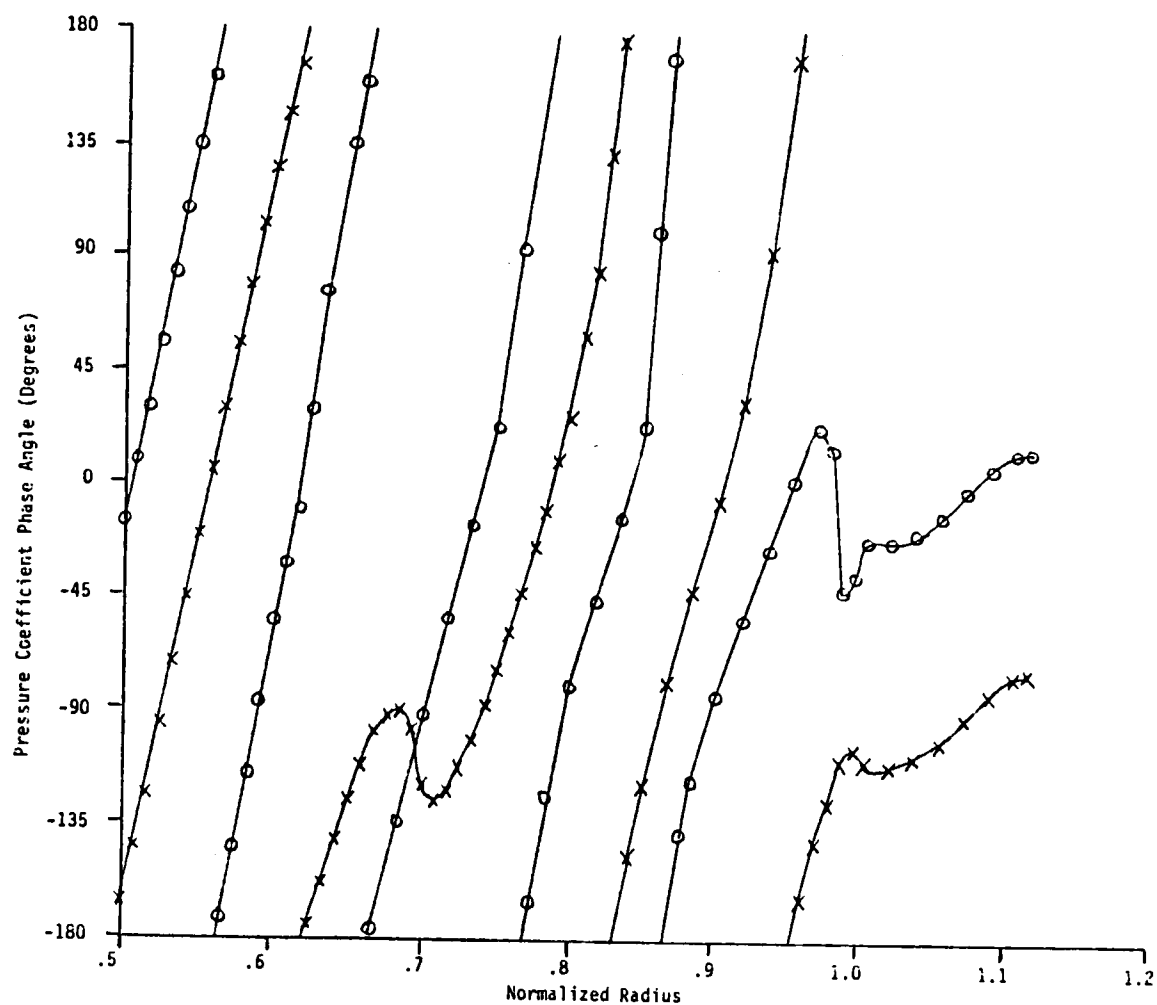


Figure E.28 Pressure Coefficient Phase Angle Versus Radius;

NB=20, NV=25, WB=1, NZ=0

x Pressure Surface

o Suction Surface

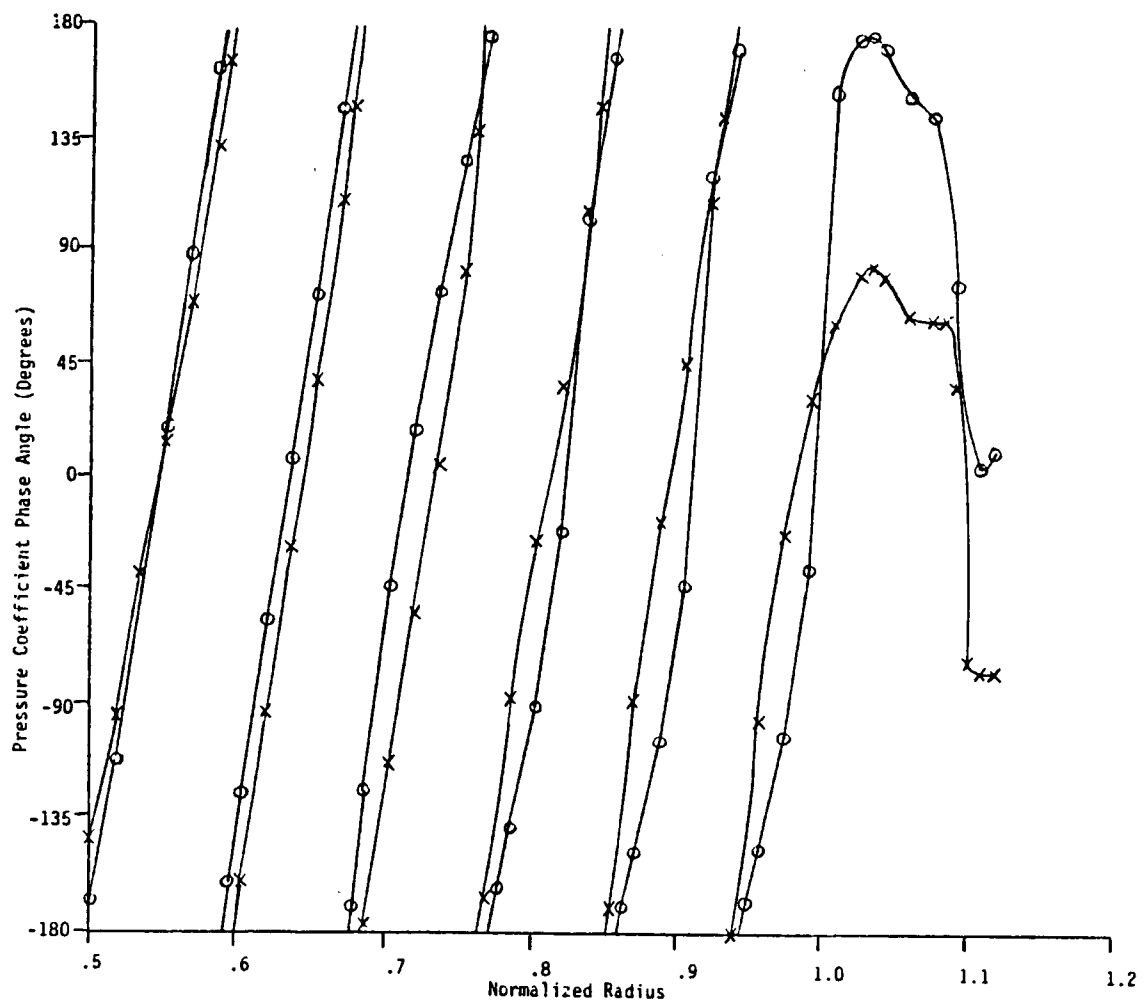


Figure E.29 Pressure Coefficient Phase Angle Versus Radius;

NB=20, NV=25, WB=1.5, NZ=0

x Pressure Surface

o Suction Surface

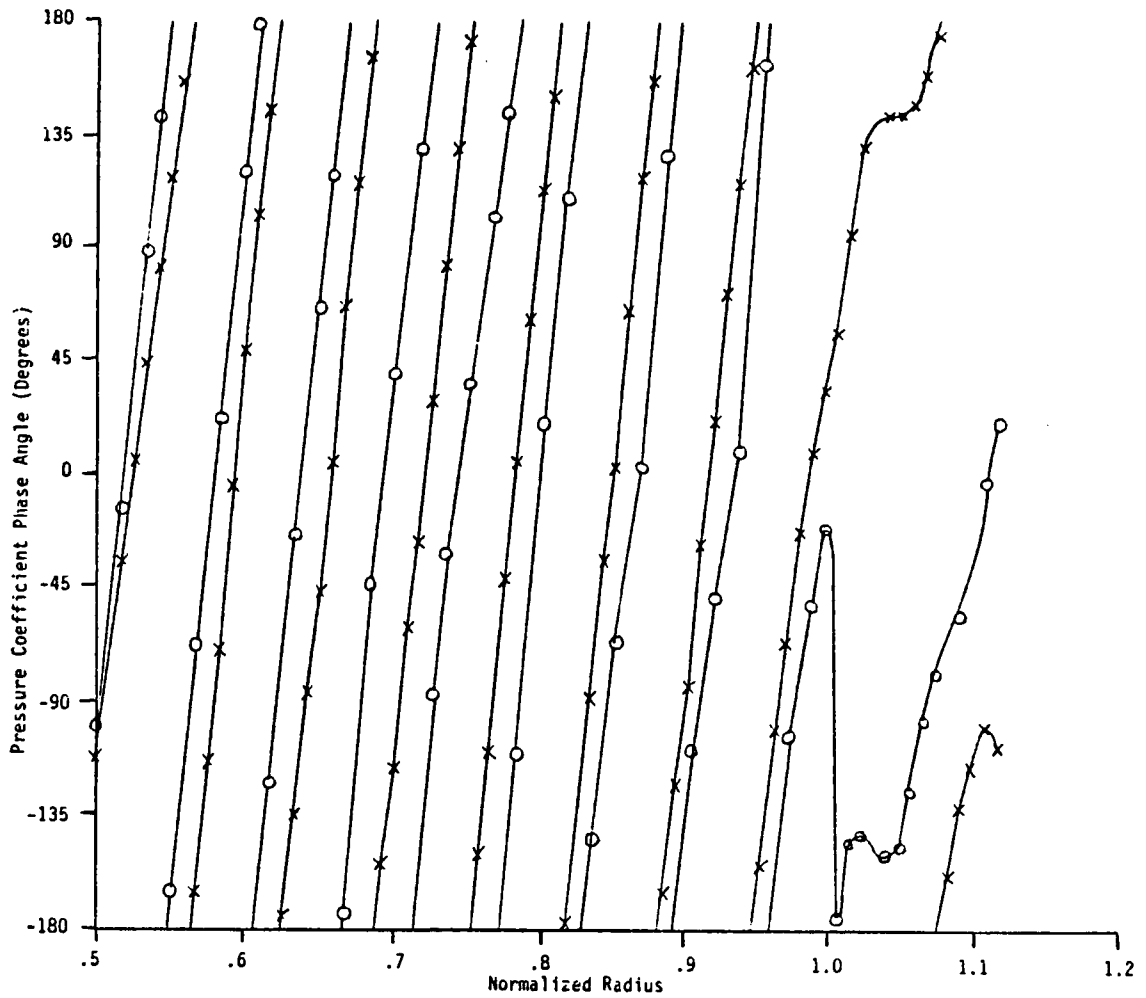


Figure E.30 Pressure Coefficient Phase Angle Versus Radius;

NB=20, NV=25, WB=2, NZ=0

x Pressure Surface

o Suction Surface

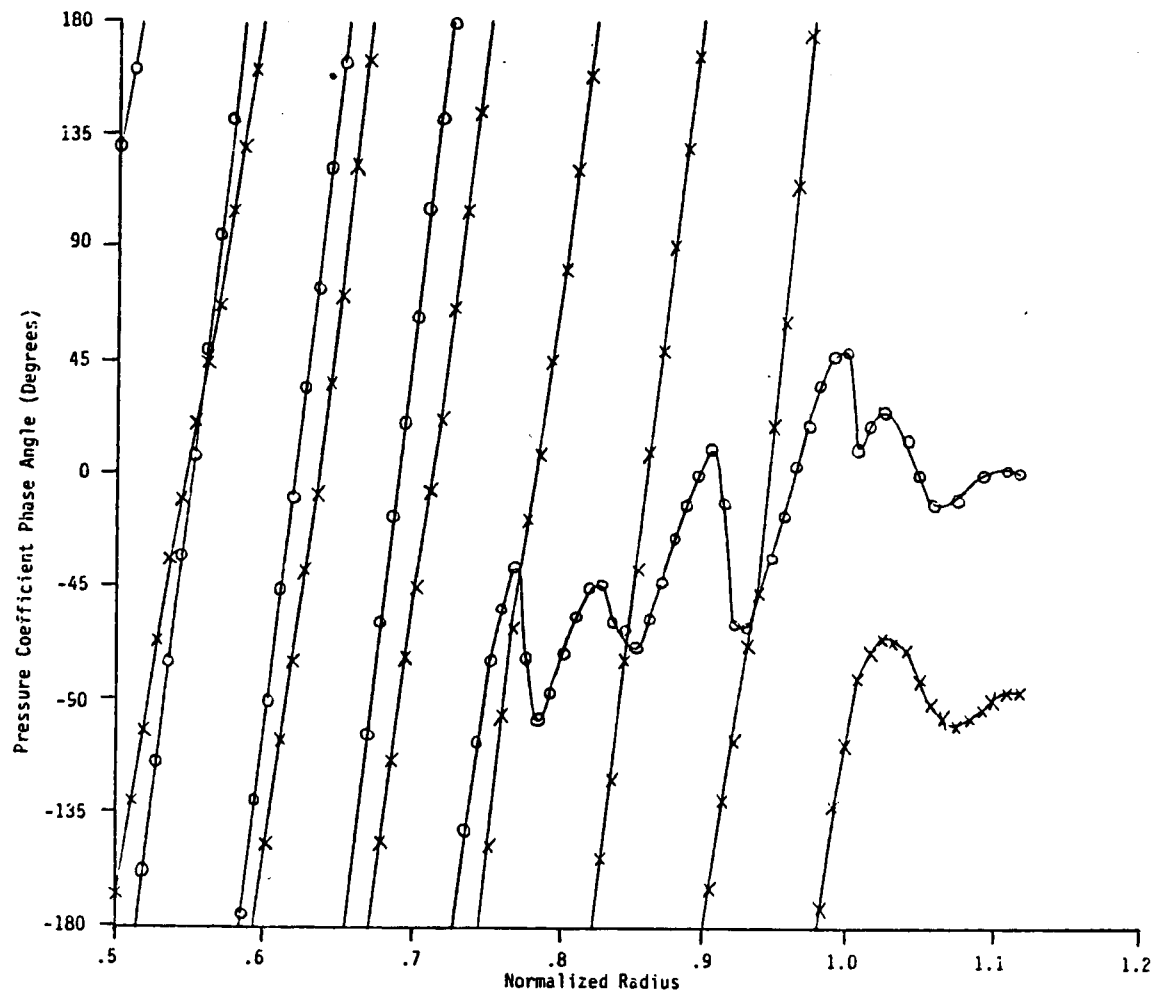


Figure E.31 Pressure Coefficient Phase Angle Versus Radius;

NB=20, NV=25, WB=2, NZ=1

x Pressure Surface

o Suction Surface

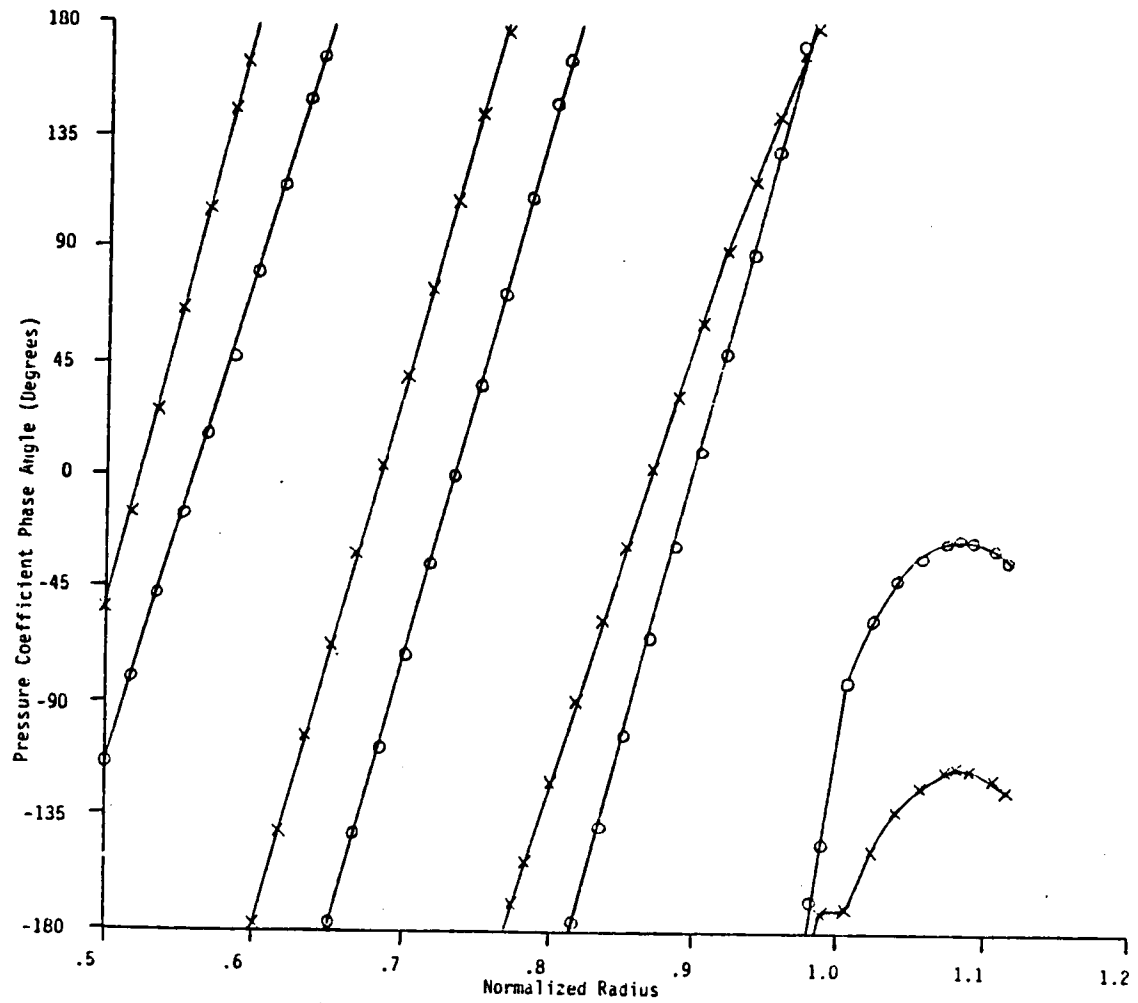


Figure E.32 Pressure Coefficient Phase Angle Versus Radius;

NB=20, NV=25, WB=2, NZ=2

x Pressure Surface

o Suction Surface

VITA

Thomas D. DeStefano was born in Altoona, Pennsylvania in 1949. He graduated from Altoona High School in June, 1967. He entered Pennsylvania State University in January, 1968 and in January of 1972, he received a Bachelor of Science Degree in Mechanical Engineering. Also upon graduation from this university he was commissioned a Second Lieutenant in the United States Army as a result of R.O.T.C. training.

In September, 1972, he began work at the Ford Motor Company, in Dearborn, Michigan, as an Engine Test Engineer. Spending a year there, he decided to seek more challenging work and accepted employment in September, 1973, with Southern Research Institute in Birmingham, Alabama, as a Thermal Research Engineer. He remained there approximately a year, gaining valuable analytical experience. From then on until March, 1981, he worked for various aerospace companies as a Stress and Structures Engineer, until which time he entered the University of Tennessee Space Institute to work toward a Master of Science Degree in Mechanical Engineering. He finished all the required course work for this degree in June, 1982. From this time to September, 1986, he worked as a Contract Engineer in aircraft structural analysis. He completed his Master's Thesis in September in 1986, and received a Master of Science Degree in Mechanical Engineering from the aforementioned university in March, 1987.