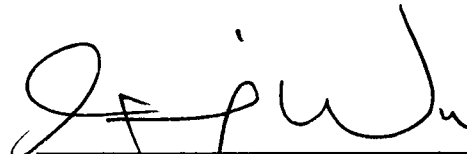


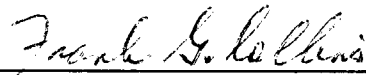
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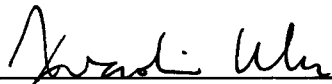


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**STREAMING EFFECT OF WALL OSCILLATION
ON BOUNDARY LAYER SEPARATION**

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

Xiaohui Wu

August 1991

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ABSTRACT

The response of boundary layer separation from smooth surface to a periodic vortical excitation imposed locally near the separation point is studied. The emphasis is on the time-averaged streaming effect of the unsteady excitation to the separating flow. The problem is formulated as a small periodic perturbation to a basic steady separating flow which is assumed given. Two types of separation model are studied, the breakaway separation model based on the numerical triple-deck solution of F.T.Smith, and the bubble separation model, which is calculated based on boundary layer equations with prescribed displacement thickness. First and second order perturbation equations are derived and simplified to ordinary differential equations near the separation point by using the triple-deck analysis. The perturbation equations are solved both analytically and numerically. It is found that the ratio of the Strouhal number St and Reynolds number Re , $\alpha = St/Re^{1/4}$, is an important parameter which determines the nature of the excited flow. For proper value of α , the streaming effect is quite strong which may delay or suppress separation. The numerical results show that there exists an optimum forcing frequency corresponding to a maximum streaming effect for each streamwise location.

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NOMENCLATURE

$A(X)$	amplitude function
$a \ll 1$	amplitude parameter
f	frequency
f_b	neutral unstable frequency of boundary layer
f_n	most unstable frequency of free shear layer
f_{sh}	most amplified frequency of "shedding" type instability
h_l	lower deck thickness
h_m	main deck thickness
h_u	upper deck thickness
l	streamwise scale of triple-deck
L	global body length
L_s	location of forcing excitation
n	circular frequency
Re	the same as Re_L
$Re_L = U_\infty L / \nu$	Reynolds number
$St = nL / U_\infty$	Strouhal number based on n
St_e	most effective excitation frequency
$St_L = fL / U_\infty$	Strouhal number based on f
U_∞	free stream velocity
u_0, V_0	steady velocity of basic flow in boundary layer coordinate
$\hat{u}, \hat{V}$	1st order amplitude of disturbance velocity
u_s, V_s	2nd order streaming velocity
(x, y)	global coordinate
$\tilde{X} = \varepsilon_0^{-3} x$	triple-deck coordinate

$X = \lambda^{-\frac{5}{4}} \tilde{X}$	normalized triple-deck coordinate, $\lambda = 0.332$
$Y = \varepsilon_0^{-4} y$	boundary layer coordinate
$z = \varepsilon_0^{-5} y$	lower-deck coordinate
α	angle of attack when refer to airfoil
$\alpha = \varepsilon_0 / \varepsilon$	an important ratio of St to Re
δ_f	thickness of oscillating layer
$\eta = \frac{1}{\sqrt{2}} \varepsilon^{-1} \varepsilon_0^{-4} y$	oscillating layer coordinate
$\varepsilon = St^{-1/2}$	frequency parameter
$\varepsilon_0 = Re^{-1/8}$	triple-deck parameter
ν	kinematic viscosity
$\omega = -\partial u / \partial y$	vorticity
$\sigma = -\nu \partial \omega / \partial y$	vorticity flux
τ	wall skin friction

CHAPTER I

INTRODUCTION

Flow separation is one of the most common phenomena in fluid dynamics. It is well known that the performance of many practical devices is greatly influenced by the separation. A typical example of the unfavorable effect of flow separation from a smooth surface is the aerodynamic stall of an airfoil at large angle of attack due to breakaway separation or a large separation bubble on the upper-surface of the airfoil. It is also known that the drag of a moving vehicle with a bluff tail is mainly determined by the size of the wake behind the body. So, it is often desirable to eliminate or suppress flow separation to gain lift and/or reduce drag. On the other hand, flow separation can also be beneficial. For instance, on the sharp-leading-edge delta wing, the separation line is fixed to the edge and two strong vortices are formed above the wing, yielding additional lift. Many examples of utilizing separation by active provocation have been reviewed extensively by Gad-el-Hak and Bushnell [1]. Moreover, it has long been known that many flying creatures can utilize the separated vortices in very smart way.

The study of flow separation and the effective ways of separation control are, therefore, very important for both theoretical and practical purposes. Many effective methods of separation control have been developed and are reviewed by Reynolds and Carr [2], as well as in [1]. An extensive review of the experimental and theoretical basis of enhancing vortex lift by unsteady excitation is given by Wu et al.[3], where it is emphasized that a thorough understanding of the relevant mechanism may shed new light on the new generation of flying vehicles with super lift. Many experiments have revealed that exciting waves may alter the process

of flow separation, promote reattachment, eliminate or suppress separation. The methods for generating exciting waves have included acoustic excitation [4-13], oscillating flaps or wires [14-18], and oscillatory surface heating [19].

The physics behind the separation control by unsteady excitation is gradually being understood. However, a complete theory is not yet available due to the complexity of the problem, the limitation of available theoretical methods (mainly perturbation method), and the difficulty and confusion in comparing experimental results with a few available theoretical analyses. It is generally believed that in an effective separation control, the instability (or vortical) waves are excited and amplified, which may induce small or large scale laminar-turbulent transition. Then the enhanced entrainment may suppress or eliminate the separation. However, this is not a unique or universal mechanism. In fact, Ahuja [6] found that acoustic excitation can be used to suppress turbulent separation. The filling of turbulent velocity profile was observed and the separation point was moved downstream; clearly, the transition mechanism is not involved. It is worth mentioning that the well-organized large scale coherent structure in the excited separating flow is another possible mechanism.

Following the general ideas addressed in [3], the present thesis concentrates on one aspect of separation control by unsteady excitation. More specifically, our focus is on the time-averaged effect of these excitations, which is known as streaming. The crucial role of streaming in separation control was first proposed by Wu [20], and the concept is applicable to both laminar and turbulent flows. In the present study, our main concern is to weaken or suppress the undesirable two-dimensional flow separation from smooth surface with the velocity streaming by applying local unsteady excitation. Our theoretical analysis and numerical computation indicate that the streaming is indeed very effective when the excitation

is applied near the separation point. The streaming can be favorable as well as unfavorable depending on the excitation modes. A favorable streaming can move the separation point downstream, or even eliminate the separation. The relation between streaming and other mechanisms relevant to the separation control is also addressed. A more general discussion of streaming can be found in [3].

In Chapter 2, a critical review of the current understanding of the relevant physical phenomena is presented, the concept of streaming and the triple-deck structure near separation are introduced, and the problem is simplified to be suitable for theoretical approach. In Chapter 3, the analytical formulation of the problem is derived by using the perturbation method and some theoretical results are given. Chapter 4 presents our numerical study of the problem, including discussion of the numerical schemes and results for both breakaway and bubble separation models. Concluding remarks and suggestion of further work are given in the last Chapter.

CHAPTER II

PHYSICAL BACKGROUND

First, here we present the physical background necessary for understanding the streaming effect of forced excitation on flow separation. We shall review some typical experimental results and the current understanding of the physics relevant to separation control by forced excitations. Although many experiments have been reviewed in [1], for our purpose it is still necessary to analyze the experiments more systematically. The scaling of the natural unstable frequencies in terms of the Reynolds number is first derived in §2.1, based on which the relevant experiments are reviewed in §2.2. It will be shown that the relation between various instabilities of a separating flow and the effective control is not at all simple, and that instability is not the whole story. Then, in §2.3 we introduce the concept of streaming and discuss its relation to separation control. The excitation methods are considered in §2.4, in which we also simplify the formulation of the problem for this study. Finally, we introduce the triple-deck structure of steady separation in §2.5.

2.1 Instabilities and Scaling of the Natural Frequencies

One of the central problems in separation controls by unsteady excitation is to find the optimum forcing frequencies that produce the most effective control. This requires a knowledge of the correct scaling rules for the excitation frequencies with respect to Reynolds number. Since it is generally observed that the excitations become most effective when the forcing frequencies are locked onto the natural unstable frequencies of the separating flow, the scaling rules of the natural unstable frequencies are analyzed first, which can be used as a guideline of reviewing

experimental results.

Throughout this study we assume the flow is incompressible, the scaling of natural frequencies then means relating the reduced frequencies (Strouhal number) St to the Reynolds number Re . Note that defining Re and St based on different characteristic length scales will lead to different scaling. Two types of characteristic length are often used, the global length, such as the body scale or the width of the wake, and the local length, such as the initial momentum thickness of the separating shear layer. In the following we derive the scaling of frequency based on the body scale, which is easier to use in practice.

Three types of instability are often considered in literature to relate to an effective control: the instability of boundary layer (the Rayleigh and Tollmien-Schlichting type), the instability of free shear layer (the Kelvin-Helmholtz type) and the “shedding” type instability of the entire separation bubble [13]. Many experimentalists referred to only one of these when they explained their results. However, it will be evident later that it is possible to excite different instabilities at the same frequency range, and thus the problem may become much more complicated.

First, we consider the scaling for boundary layer instability. Let U_∞ , L , ν be the free stream velocity, a global characteristic body length, and the kinematic viscosity, respectively. From the asymptotic study of the boundary layer stability with non-parallel flow effect by Smith [21], and Bodonyi and Smith [22], for the lower and upper branches of the marginal stability curve we have,

$$Re_x \propto \varepsilon^8 \quad \text{and} \quad Re_x \propto \varepsilon^{10},$$

where $Re_x = U_\infty x / \nu$ is the Reynolds number based on the distance x of separation point from the point where boundary layer starts to grow, and $\varepsilon = F^{1/6}$ with

$F = \nu n / U_\infty^2$, where n is the neutral unstable circular frequency. Then, if we define

$$Re_L = \frac{U_\infty L}{\nu} \quad \text{and} \quad St_L = \frac{f_b L}{U_\infty},$$

where $f_b = n/2\pi$, we find $F = 2\pi St_L / Re_L$, hence it can be easily shown that for the lower branch,

$$St_L \propto \frac{1}{2\pi} Re_L^{\frac{1}{2}} \left(\frac{x}{L}\right)^{-\frac{3}{2}}, \quad (2.1a)$$

and for the upper branch

$$St_L \propto \frac{1}{2\pi} Re_L^{\frac{2}{3}} \left(\frac{x}{L}\right)^{-\frac{3}{2}}. \quad (2.1b)$$

It should be noted that Eqs. (2.1a, b) give the lower and upper bounds of the natural unstable frequencies of boundary layer for a fixed Reynolds number rather than the scaling of the most unstable frequency. The factor 2π is retained in the formulas in order to emphasize the difference between the circular frequency n and frequency f_b because it can be large enough to lead to different qualitative assessment (see §2.2). Eqs. (2.1a, b) imply some complexity in the scaling due to the presence of x/L , which is determined by, say, the geometry and the incidence of an airfoil.

Second, for laminar and turbulent free shear layers, we have the following relation for the most amplifying frequency f_n [23]:

$$\frac{f_n \theta}{U_a} = c_1 \quad (\text{constant}), \quad (2.2)$$

where U_a is the averaged velocity of those on the two sides and θ is the initial momentum thickness of the shear layer. For a free shear layer leaving trailing edge, the constant c_1 was found experimentally and numerically to be 0.032 for a laminar shear layer or 0.044–0.048 for a turbulent shear layer (see [23]). Note that the constant might be different from the above ones for a free shear layer

leaving a smooth surface, but Eq. (2.2) is still valid. Then, for a laminar shear layer, consider the relation between θ and Re_L

$$\theta \propto \left(\frac{\nu x}{U_a}\right)^{\frac{1}{2}} \propto Re_L^{-\frac{1}{2}} \left(\frac{x}{L} / \frac{U_a}{U_\infty}\right)^{\frac{1}{2}} L,$$

we have, for the Kelvin-Helmholtz instability,

$$St_L \propto c_1 Re_L^{\frac{1}{2}} \left(\frac{U_a}{U_\infty}\right)^{\frac{3}{2}} \left(\frac{x}{L}\right)^{-\frac{1}{2}}, \quad (2.3)$$

and c_1 is kept in (2.3) since it is very small.

Finally, the scaling for “shedding” type instability was proposed by Sigurdson and Roshko [13] as

$$\frac{f_{sh} h}{U_s} = c_2 \text{ (constant)}, \quad (2.4)$$

where f_{sh} is the most amplified “shedding” frequency and U_s is the velocity just outside the shear layer at separation, or

$$St_L = c_2 \frac{L}{h} \frac{U_s}{U_\infty}, \quad (2.5)$$

where h is the height of the separation bubble or the half width of wake behind bluff bodies. The constant c_2 in (2.4) and (2.5) is 0.08 approximately. For flow over airfoils, we may estimate that [24, 25]

$$St_L \sin \alpha = 0.15 \sim 0.19, \quad (2.6)$$

where α is the angle-of-attack.

It needs to be pointed out that the above scaling can be used to obtain the qualitative behavior and rough estimation of the natural frequencies rather than the quantitative results, because some quantities in the scaling (e.g. x , θ , U_s and h) will be changing when the flow is being excited. The interesting point is that, the

scaling for both boundary layer and Kelvin-Helmholtz instabilities are explicitly dependent on the Reynolds number and the dependency is stronger for the latter one, while the scaling for the “shedding” type instability shows that, such a dependency coming from L/h and U_s/U_∞ is much weaker. For separating flow over an airfoil, the effect of incidence angle α is shown clearly in (2.5) by $L/h \propto 1/\sin \alpha$. By contrast, this kind of dependency on α is rather implicit in (2.1a, b) and (2.3). One should be careful to compare the magnitudes of the reduced frequencies of different instability types because the small constants appearing in (2.2) and (2.4). In general, the natural frequencies of a free shear layer and boundary layer are higher than that of shedding (if the wake is not too narrow) but it is hard to distinguish which is higher between the two former types.

2.2 Review of the Experimental Results

Unsteady excitation methods have been used in many separation control experiments. Usually these methods were mainly for suppressing or eliminating separation rather than provoking it. Different means are used to generate the exciting waves, including acoustic excitation [4-13], periodic forcing of the velocity field via an oscillating flap or wire [14-18], and also oscillatory surface heating [19]. The basic model flows tested in experiments include separating flow over airfoils, backward step separation, leading edge separation from flat plates, etc. It should be stressed that different model flows might involve quite different mechanisms for effective control; in particular, the control of the separation from a smooth surface might be very different from that of the separation from a fixed separation point (2-D flow) or line.

As mentioned in §2.1, many authors explained their experiments by instabilities of separating flows, and in most cases only one instability mechanism was

referred to because of the apparent mismatch of the imposed forcing frequency with other types of instabilities. However, as it will be clear soon, in these experiments the direct matching between reduced forcing frequencies and natural unstable ones derived in §2.1 are often poor and from the available experimental data it is rather difficult to identify which instability is really excited.

Reisenthel et al. [17] and Nagib et al. [15] studied the control of backward step separation by the oscillation of a spoiler-like flap inside the region of separation. It was reported that the effective frequency scales with the height of the separation bubble, which is in agreement with the scaling of the instability of “shedding” type proposed by Sigurdson and Roshko [13]. Huang et al. [10] were able to reduce the leading edge separation “drastically” by introducing acoustic excitation from a narrow gap in the vicinity of the separation point (which is called internal excitation) with the frequency near the fundamental vortex shedding frequency or its subharmonics. Nishioka et al. [12] argued that for a flat plate at high incidence, the separation is more likely controlled by the “shedding” type instability because the effective frequency is lower than the natural frequency of the free shear layer. The frequency variation trend with angle-of-attack in their experiments is in qualitative agreement with Eq. (2.6). However, the direct matching with the scaling rule (Eq. (2.5)) is not obvious. In fact, Sigurdson and Roshko [13] found that the effective reduced frequency is four times of f_{sh} predicted by Eq. (2.4), though the frequencies are still lower than the initial Kelvin-Helmholtz frequencies of the separating free shear layer.

It is worth mentioning that, the “shedding” type instability of the entire bubble may closely relate to the global instability of the wake flow behind bluff bodies closely, and this can be seen by the considering the wake flow behind a circular cylinder. When Reynolds number is low, the wake of the cylinder is

of closed bubble type and the excitation induces oscillation of the closed bubble but with no vortex shedding. As the Reynolds number increases, the periodic vortex shedding behind the cylinder is induced by small disturbance due to the global instability of the closed bubble type wake flow. The same reasoning may be applied to explain the backward step separation bubble control experiments since the flow structure are similar to the bubble type wake behind a cylinder, except that the separation points are generally fixed and the bubble interacts with its image vortex. Then the most amplified shedding frequency is just the most unstable frequency of the global instability of the entire bubble.

When the vortex shedding is established behind a bluff body, the unsteady excitation can alter the wake flow structure. It has been shown very clearly in [26] that the wake structure changes with the excitation frequency. The excitation at natural shedding frequency makes the wake to reduce and become more well organized [26, 27]. This is different from the “shedding” type instability mentioned above. The key mechanism for such a case is the wake flow resonance [27, 28].

The Kelvin-Helmholtz (K-H) instability is considered as another possible effective mechanism in separation controls. The excellent review by Ho and Huerre [23] provides a complete account of the current knowledge of the perturbed free shear layer. The “collective interaction” process proposed by Ho and Nosseir [29] is especially useful for explaining the separation control from the viewpoint of free shear layer instability. They pointed out that at a low forcing frequency (sub-harmonic of f_n in Eq. (2.2)), many small vortices roll into large-scale structures and a train of large vortices is formed. The experiment by Bhattacharjee et al [8] showed that, when forcing at a reduced frequency (based on the height of the backward step and free stream velocity) in the range of 0.2~0.4, the spreading rate of the separated mean flow was increased and the reattachment length was

decreased by 10%. It was pointed out that forcing at a subharmonic of the vortex passage frequency rather than the vortex passage frequency itself can enhance the spreading rate of the mixing layer. The frequency range is several times higher than f_{sh} .

The control of separation from smooth surface (in which case the movement of separation point should be considered) are mainly tested on airfoils. Zaman et al. [9] conducted a careful study on a low- Re (4×10^4) airfoil using external acoustic excitation and found that for the angle of attack $\alpha < 8^\circ$ (pre-stall region), the effective forcing frequency St_e (or effective St_L) was in the range of $5 \sim 1$ or lower; while for $\alpha \geq 18^\circ$ (post-stall region), the St_e range is about $4 \sim 25$, and the amplitude of excitation had to be large. They also observed that the transverse velocity fluctuations near the airfoil induced by wind tunnel resonance was crucially important for effective control in the post-stall region. The frequency ranges were lower than the natural frequency of a free shear layer as predicted by (2.3). But it was argued that, for small α , the St_e -range, $5 \sim 1$, was well below $Re_L^{\frac{1}{4}} (\simeq 14)$, and the separation was from smooth surface, so that the receptivity mechanism proposed by Goldstein [30] could be used to explain the generation of amplified instability waves in the free shear layer, which in turn influenced the separating flow. However, we note that for high angle-of-attack ($\alpha \geq 18^\circ$), Zaman's argument might fail because a large portion of the St_e -range, $4 \sim 25$ is not much smaller than $Re_L^{\frac{1}{4}}$, although in this case it is still appropriate to classify the leading edge separation as separation from a smooth surface because the separation point on the airfoil can be moved to downstream (as far as 80% chord in [18]), unlike the separation from a flat plate with sharp leading edge or from a backward step with a fixed separation point. But the effects of subharmonic frequencies mentioned above make the free shear layer instability remain as a

possible mechanism for the high α case without the receptivity of Goldstein [30] kind. On the other hand, another receptivity mechanism proposed by Goldstein [31], which make it possible for low frequency disturbances to be able to excite a much higher Tollmien-Schlichting (T-S) frequency due to the growth of boundary layer around leading edge, might also be important in explaining this experiment (by using (2.1) the forcing frequencies are found to be indeed lower than the T-S frequencies). The “shedding” type instability seems not be important in this case, since St_e does not scale with α as in (2.6), but St_e does have a trend to vary as α , as indicated by the last terms of (2.1) and (2.3). This is a typical example of the fact that different instabilities may be involved in one experiment while it is difficult to know which is more important.

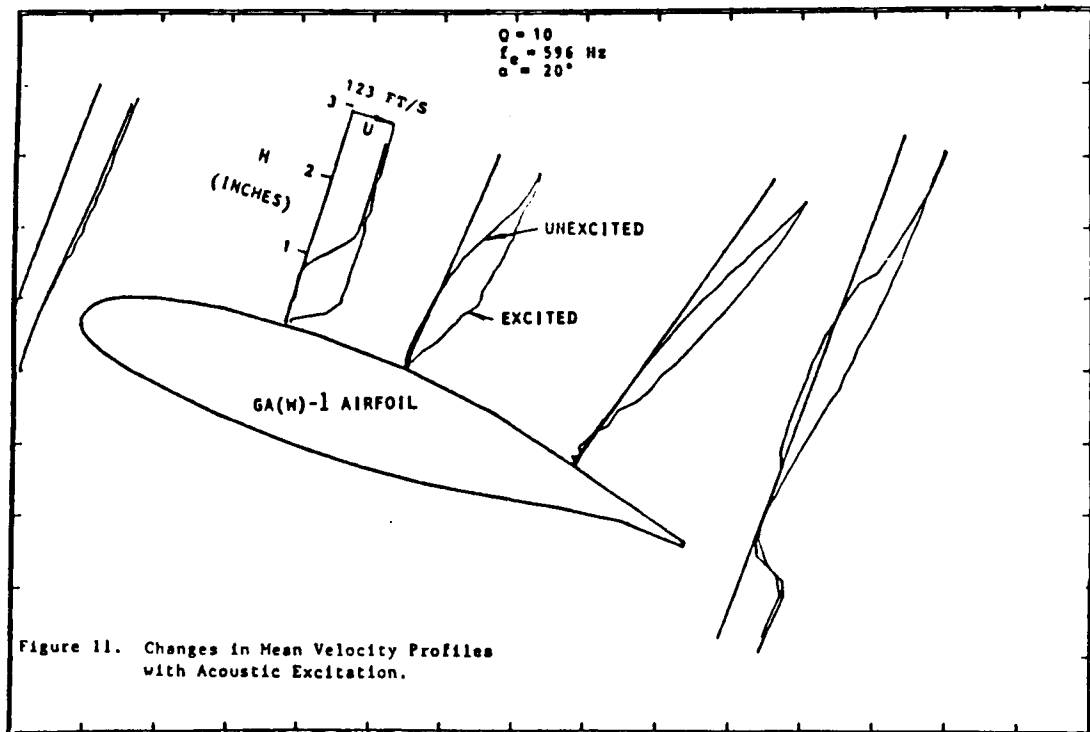
Hsiao et al. [11] studied the effect of the internal excitation on the leading edge separation of an airfoil at the post-stall regime as well as the separation over a circular cylinder. Their Re_L varied from 2.0×10^4 to 3.0×10^5 . The St_e -range was about 0.5~4.0, much narrower than $4 \sim 25$ as in Zaman's case. The interesting point is that, for both airfoil and cylinder, the most effective reduced frequency is always equal to 2, almost independent of Re_L and α . The success of the control was explained by shear layer instability, which is however not quite consistent with Eq. (2.4). The drop in lift coefficient for pre-stall α ($\simeq 15^\circ$) was observed by Hsiao et al. and Zaman et al. It was also observed that, at low α , the excitation effect is very weak, similar to the results obtained by Ahuja [6].

The experimental study of the role of T-S wave in the separation control has recently been done by a group of Russian researchers (see [32] and the references cited there). Their work indicates that T-S waves are indeed excited by either acoustic excitation or oscillating ribbon, and these vortical waves determine largely the flow structure in the region of separation. The generation of perturbations at

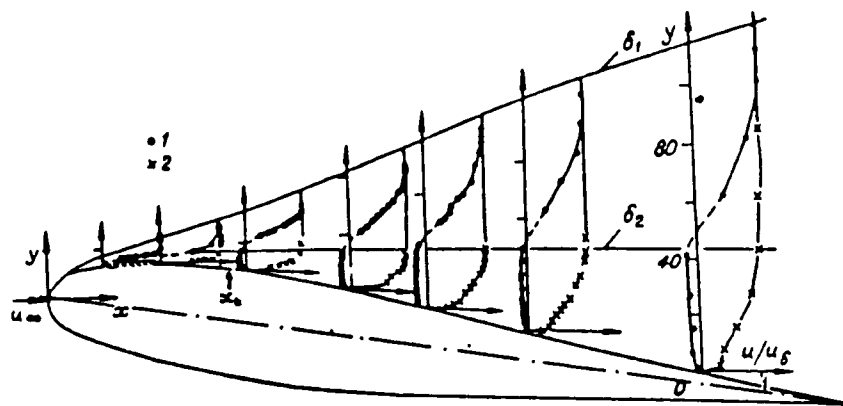
the start of the region of adverse pressure gradient was found to be decisive, while the perturbation in the accelerating flow was not important because of their strong damping. Transition to turbulence was observed, and its effect was shown mainly in the turbulent reattachment of separation bubble . On the other hand, it was emphasized that the filling of the velocity profile did not result from the flow turbulization but took place in a laminar layer. Similar to this observation, Ahuja [7] has proved that even turbulent separating flow was controllable by acoustic excitation. Therefore, the transition might not be the unique mechanism in the filling of velocity profiles (Fig. 1).

The above review clearly shows that the matching of experimental results with the scaling rates of §2.1 is by no means as simple as one expected. As pointed out by Smith [33], the separation links with various instabilities intimately, "partly because of the inflectional profiles and associated critical layers produced and partly because of strong nonparallelism and nonlinear interactions", various instabilities do have important effects on separation controls. The effective frequencies are usually lower than that of T-S wave and the K-H instability , but are higher than the "shedding" frequency. The reason behind this is still not clear, but the study of various receptivity mechanisms and the role of subharmonic of vortex forming frequencies seems promising. In fact, Smith [34] stressed that one type of instability can induce other types and the combined instability behavior of separating flow should not be ignored.

In comparing to the point of view of "adding momentum" near the wall [1], the above phenomena may be better explained from the vorticity point of view, for which an excellent example was due to Lighthill [35]. The vortical waves and vortex interactions play important roles in the process of separation control by unsteady excitation. Thus, three problems can be singled out [36]: (1) The



(a)



(b)

Figure 1. Velocity profile filling due to excitation without turbulization. (a) Filling of turbulent velocity profile (Ahuja, 1983). (b) Filling of laminar velocity profile before transition point x_t , $\bullet$ no excitation, $\times$ excited.

receptivity problem, (2) hydrodynamic instability of separating flows, and (3) the effect of amplifying vortical disturbances. The focus of the present study is on the third problem, the effect of the amplifying vortical disturbances. We shall look at the problem from the point of view of steady streaming to be discussed in the next section. The relation of streaming and instabilities will also be covered.

2.3 Steady Streaming

In many experiments (e.g. [7],[12] and [18]) the shifting of separation points and the filling of velocity profiles due to excitations are observed. This can certainly be attributed in part to the transition to turbulence induced by instabilities. Then the separation control is achieved because of the enhanced turbulent entrainment. However, this is not the whole story; the excitation might not cause a premature transition [5], while the filling of velocity profiles may not be associated with transition [36], and the filling can be achieved for turbulent flow [7]. In these cases, the formation of coherent structure or large vortices may be a possible mechanism to suppress the turbulent separation.

Another mechanism that has never been explored before is the steady streaming effect, which is due to the viscosity and nonlinearity of the flow. Basically, the streaming is the non-vanishing, time averaged, steady effect generated in a periodically oscillating flow. Therefore, it is natural to consider its effect in separation controls by periodic forcing excitations. It is worth mentioning that this concept is valid for both laminar and turbulent flow, and is not limited to velocity but also valid for other physical quantities, such as vorticity and temperature (although the word "streaming" came from the velocity consideration and widely used in acoustics).

The streaming phenomenon was first studied theoretically for the oscillating flow with vanishing mean (with respect to time) by Rayleigh [37], who derived the classical streaming law (Eq. (3.20) below). Stuart [38] introduced the concept of inner and outer layers, as well as the streaming Reynolds member $Re_s = U_\infty^2 / \nu$. The problem was also studied theoretically and numerically by Riley [39] and coworkers [40]. For flow with non-vanishing mean, the problem is more difficult and was studied by Lin [41] and others. Numerical calculation of the streaming effect (including temperature streaming) for an attached boundary layer was presented by Telionis and Romaniuk [42], and a rather complete account on the theoretical and numerical results can be found in Telionis's book [43].

In the framework of perturbation theory, streaming is a higher order steady effect of the periodic perturbation with small amplitude, which can be handled by carrying out perturbation expansion of the Navier-Stokes equation to the second order. Because most of the theoretical works on streaming have been limited to perturbation techniques which can only gives small effects, there might be some doubt regarding its role in separation control. However, as Lin [41] has proved, for large amplitude excitation, streaming is still a valid concept, and this can be demonstrated by an excellent experiment due to Taneda [44] (Fig. 2).

In this experiment, the velocity induced by the rotatory oscillation of a cylinder was even larger than the free stream velocity. The wake behind the cylinder was completely eliminated and the resulted flow looks almost like a potential flow. No instability or resonance appeared in the experiment, hence the result can be best explained by the steady streaming effect, although it is certainly not explainable by perturbation methods (X.H.Wu 1991, unpublished). It should also be noted that even with a low excitation level, the excited waves may be amplified by instabilities or other resonance mechanisms, so that a strong streaming can still

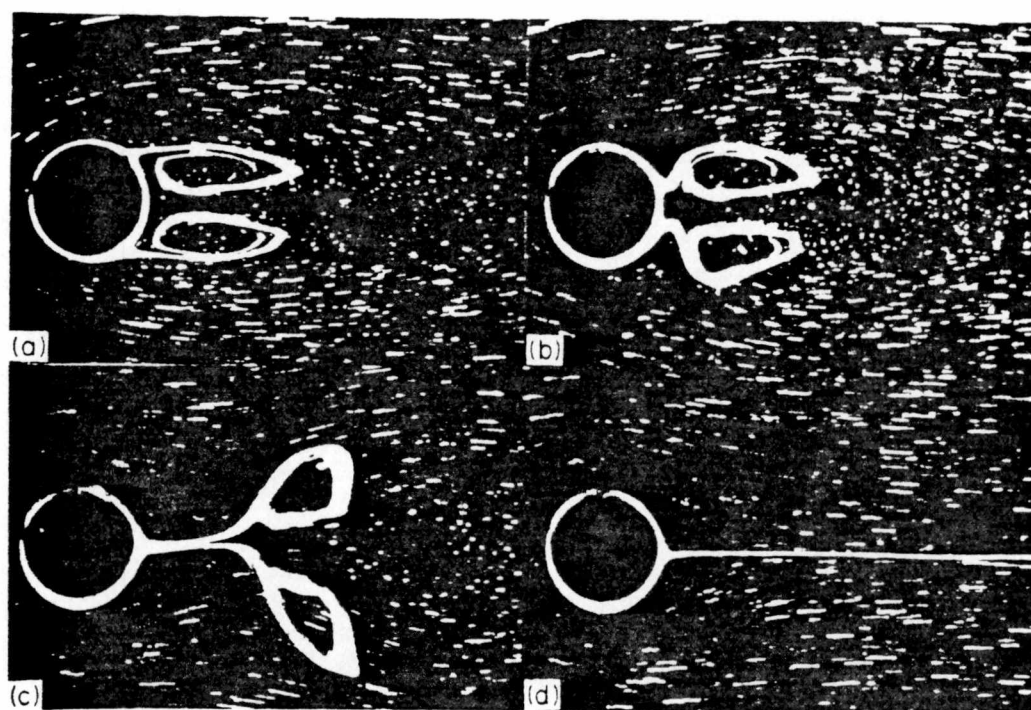


Figure 2. Streamlines and streaklines around a circular cylinder performing a rotatory oscillation about its axis in a uniform flow (Taneda, 1977). $d = 1\text{cm}$, $U_\infty = 0.33\text{cm/s}$, $Re_L = 35$, $f = 2\text{Hz}$, $St_L = 6$, $\theta = 45\text{deg}$, $x = U_\infty t$. (a) $x/d = 0$ (b) 1.5 (c) 3.4 (d) 12.

be achieved. For example, Kurosaka [45] used the streaming effect to explain the Ranque-Hilsh vortex tube effect, which involves an amplification of streaming by resonance [54]. In the experiment of Nishioka et al. [12] the time-averaged velocity profile near separation was appreciably changed by the forcing excitation, which can be attributed to the streaming effect (Fig. 3).

Moreover, in the studies of controlling the reattachment of airfoil and backstep flows, Francis et al. [46] and Koga et al. [13] found that the moving velocity U_c of the reattachment point was related to the flow parameters by

$$\frac{U_c}{U_\infty} \simeq 0.5 \sqrt{\frac{fL}{U_\infty}} \quad \text{and} \quad \frac{U_c}{U_\infty} \simeq \frac{fh}{U_\infty} \quad (2.7a, b)$$

respectively, where h is the height of the backstep. These two results are apparently inconsistent. But Vakili (private communication, 1985) noticed that the two curves would be well correlated if h is of the order of upstream boundary thickness, i.e., $h \sim \sqrt{\nu L / U_\infty}$, since then, by (2.7b),

$$\frac{U_c}{U_\infty} \sim \frac{f}{U_\infty} \sqrt{\frac{\nu L}{U_\infty}} = \sqrt{\frac{f\nu}{U_\infty^2}} \sqrt{\frac{fL}{U_\infty}},$$

which is in agreement with (1.7a) except for a dimensionless factor. Wu et al. [3] notice that this factor is, however, precisely the dimensionless streaming layer thickness, or

$$\sqrt{\frac{f\nu}{U_\infty^2}} = Re_s^{-\frac{1}{2}},$$

where Re_s is the streaming Reynolds number. Hence the streaming effect does play an important role (though implicitly in this example) in the shear flow control by forcing excitation.

Accidentally we note that the streaming Reynolds number Re_s appears in the linear stability theory as $F = 1/Re_s$ (see §2.1), which may imply that the

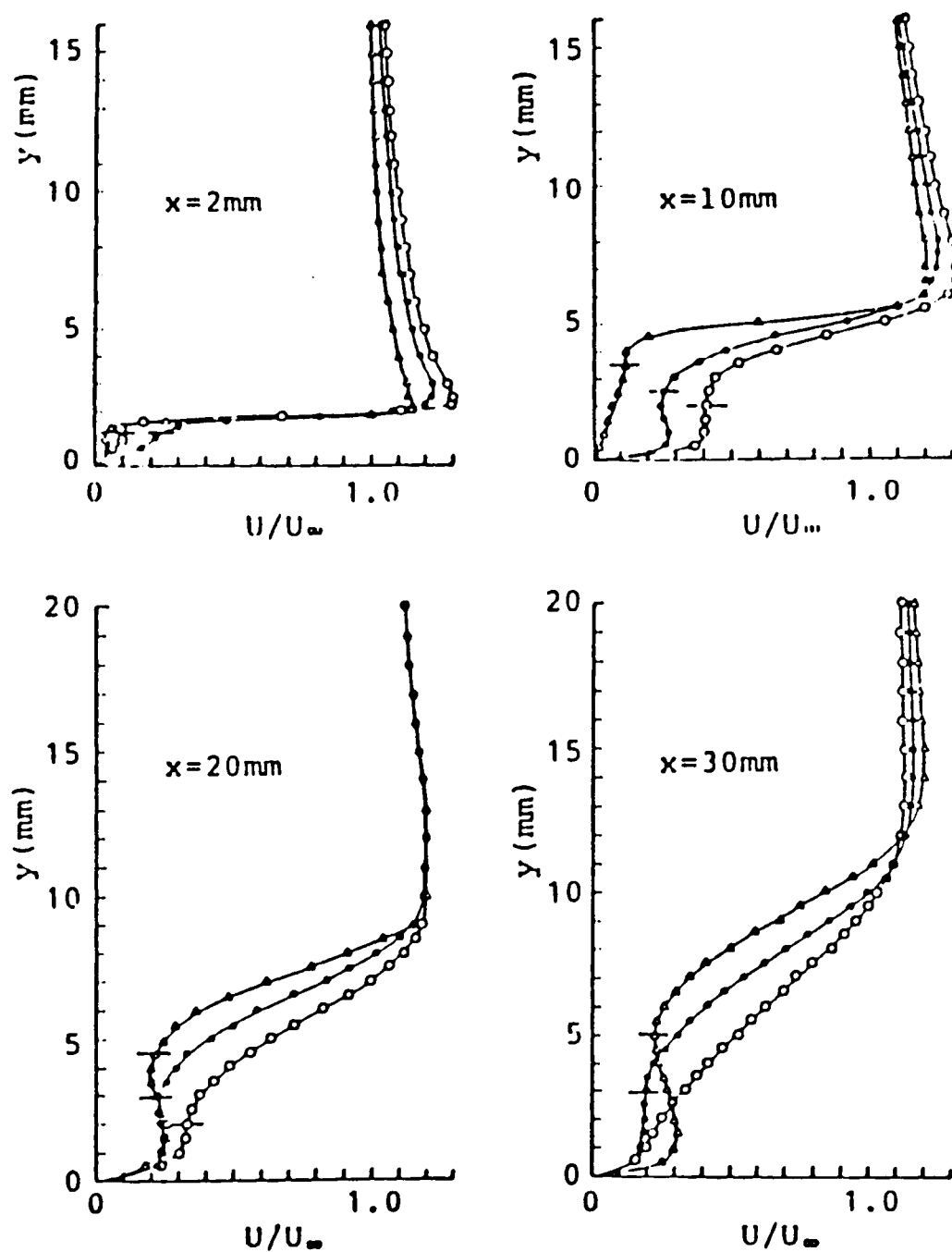


Figure 3. Mean velocity profile distribution with and without excitation on a flat plate with angle of attack (Nishioka, 1990). $Re_L = 4 \times 10^4$, $\alpha = 12\text{deg}$; Δ no excitation, $\circ$ excitation at 200Hz, $\bullet$ excitation at 600Hz.

streaming is inherently linked with instability. In terms of Re_s , we could rewrite Eqs. (2.1a, b) as

$$Re_s \propto Re_L^{\frac{3}{4}} \left(\frac{x}{L}\right)^{\frac{3}{4}} \quad \text{and} \quad Re_s \propto Re_L^{\frac{3}{5}} \left(\frac{x}{L}\right)^{\frac{3}{5}}$$

for the lower and upper branches of the marginal stability curves. The interaction of streaming and instability will be further demonstrated in Chapter 3. Finally, we note that besides the amplification mechanisms mentioned above (e.g. instability waves, resonance), the strong upwelling velocity near the separation point is a very important amplifier of the streaming effect in separation control. We shall come to this fact in Chapter 3.

2.4 Forcing Excitation

Various forcing excitation methods have been used to generate the forcing waves and to control the separation. As reviewed in Ref. [47], the forcing wave can be either a longitudinal wave (including sound wave) or a transverse (vortical) wave, and the exciting source can be either away from the separation point or close to it (Fig. 4). Nishioka and Morkovin [48] have pointed out some inherent drawbacks in using distant sources of sound waves. By contrast, the internal sound wave excitation was found to save much more energy compare to the external one [5, 11]. For vortical waves, they emphasized and will be shown below that what is really relevant is the amplitude variation (rather than the amplitude itself) of vortical waves.

A sound wave can excite the desired effect only after it is transferred to a vortical wave by the wall adherence condition, which can be seen from the familiar formula [35]

$$\left. \frac{\partial p}{\partial x} \right|_{y=0} = \sigma(x) \quad \text{and} \quad \left. \frac{\partial p}{\partial y} \right|_{y=0} = \frac{\partial \tau(x)}{\partial x},$$

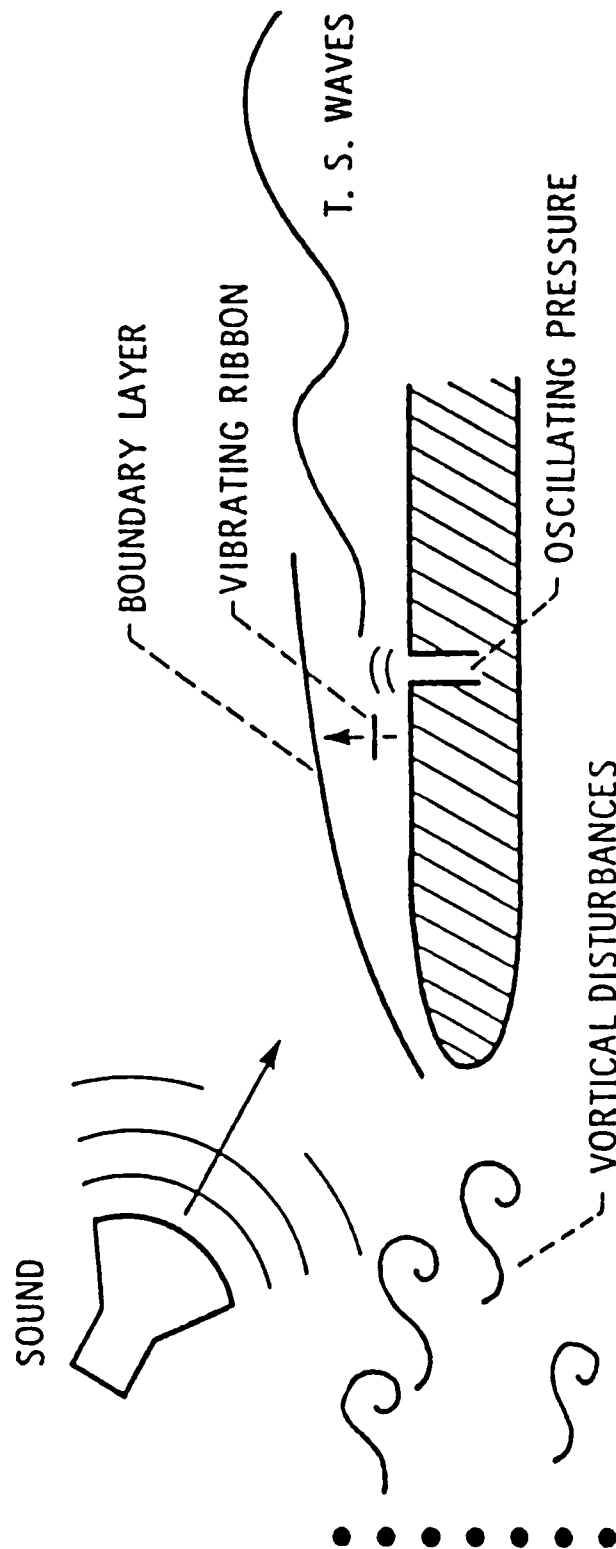


Figure 4. Various method of unsteady excitation (Goldstein, 1989).

where τ is the skin friction and $\sigma = -\partial\omega/\partial y$ (here ω is the vorticity) is the vorticity flux which governs the curvature of velocity profile near the wall.

Indeed, a comparison of earlier experiments on backstep flow control such as [14], [16] and [18] shows that applying a forcing vortical wave directly and locally is more effective. For separating flow over an airfoil, Zaman [8] emphasized that the transverse velocity fluctuation (which is induced by the wind tunnel resonance in this particular case) rather than the pressure fluctuation near the surface of the airfoil is very important. Accordingly, Bar-Sever [18] used the oscillating wire method to generate the velocity fluctuation or the vortical wave directly near the leading edge, and he reported that the separation point can move from the leading edge to as far as 80% chord.

Therefore, in the present analysis we assume that a local tangential oscillation is directly produced by the vibration of a small piece of the wall, of which a possible mechanism is sketched in Fig. 5. This model was chosen due to its simplicity, but still, it might not be the best method. Other methods, like the oscillating flap used in backstep flow control, or the oscillating wire method of Bar-Sever, could be more effective because they could generate very strong vortical disturbances in comparison to the method we used here, but the structure of the excited flow is much more complicated and hard to analysis. In Chapter 3, we will further simplify the forcing model to the level amenable to theoretical study.

2.5 Steady Separation and Triple-Deck Structure

As mentioned in the introduction, the triple-deck structure of steady separation (from a smooth surface) is one of the most important discoveries in the understanding of boundary layer separation. The well-known Goldstein singularity of the boundary layer equation at the separation point can be removed by

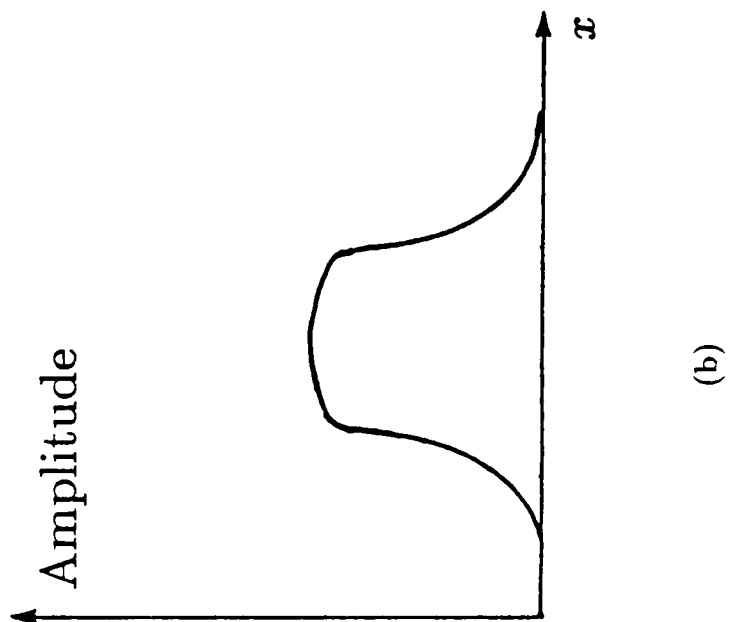
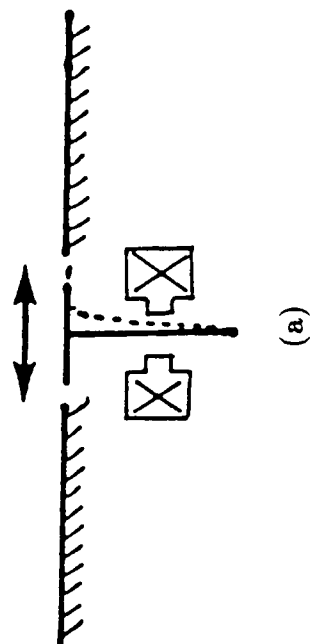


Figure 5. A local source of vortical waves. (a) The mechanism to realized the local source
(b) Sketch of amplitude distribution in streamwise direction.

more careful matched asymptotic analysis where the classical boundary layer is replaced by a three-layer structure. An excellent comprehensive review on this topic is given by Smith [49].

Principally, we may expect that higher order singularities would still occur because the approximation of the Navier-Stokes equation at any level is obtained by neglecting certain physical aspect of the fluid motion and this ignorance could turn out to be the reason for a singularity to occur in that approximation. Nevertheless, the triple-deck structure is good enough to reflect the strong viscous-inviscid interaction near the separation point. We shall use the triple-deck theory as the basis of this study.

According to the triple-deck theory, the flow displays a three layer structure near the separation point (Fig. 6). They are the lower deck, main deck and upper deck. The lower deck accounts for the viscous effect, where the governing equation has the same form as the classical boundary layer equation, the main deck transmits the flow deflection generated by the lower deck to the outer edge of the boundary layer, and the upper deck is governed by an elliptic differential equation which provides the upstream and downstream influence. The transmission of the deflection of the streamline due to separation is reflected in the pressure-displacement relation, which is the key essence of the triple-deck theory. In this way, the strong effect of the separation on the local pressure gradient is taken into account and the parabolic boundary layer equation is ellipticalized so that the strong interaction between the viscous layer and the outer inviscid layer is treated correctly and the singularity is removed.

The scales of the three decks are determined by the small parameter $\varepsilon_0 = Re^{-1/8}$. For the heights h_l , h_m and h_u of the lower, main and upper deck respec-

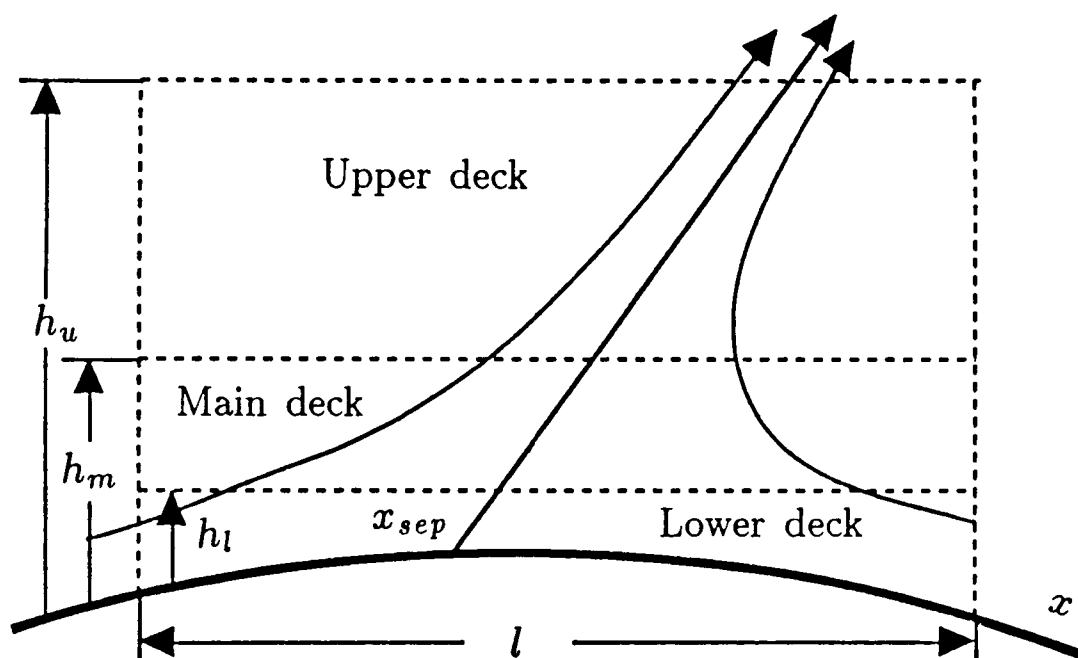


Figure 6. The triple-deck structure. $h_l = \varepsilon_0^5 L$, $h_m = \varepsilon_0^4 L$, $h_u = l = \varepsilon_0^3 L$.

tively, we have,

$$h_l = \varepsilon_0^5 L, \quad h_m = \varepsilon_0^4 L, \quad h_u = \varepsilon_0^3 L,$$

where L is the characteristic length. The main deck has the same scale as the boundary layer thickness. Unlike the boundary layer, however, the streamwise scale of the triple-deck is confined in a length scale $l = \varepsilon_0^3 L$ around the separation point.

Here we would like to point out that the triple-deck structure described above is constructed only for steady laminar boundary layer separation. For unsteady separation, the unsteady nonlinear form of the triple-deck should be used. This unsteady triple-deck structure controls both separation and the Tollmien-Schlichting instability [21]. Thus, the problem is unsteady, highly nonlinear and non-parallel, which is beyond the scope of this preliminary study. Fortunately, even though our problem relates to unsteady separation, it is highly localized and our primary concern is the time averaged steady streaming effect. Hence, we may still use the steady triple-deck analysis to describe the basic steady separating flow and obtain the correct order of magnitude for relevant physical quantities and then add the periodic perturbations to the flow. As a consequence, the instability problem is excluded.

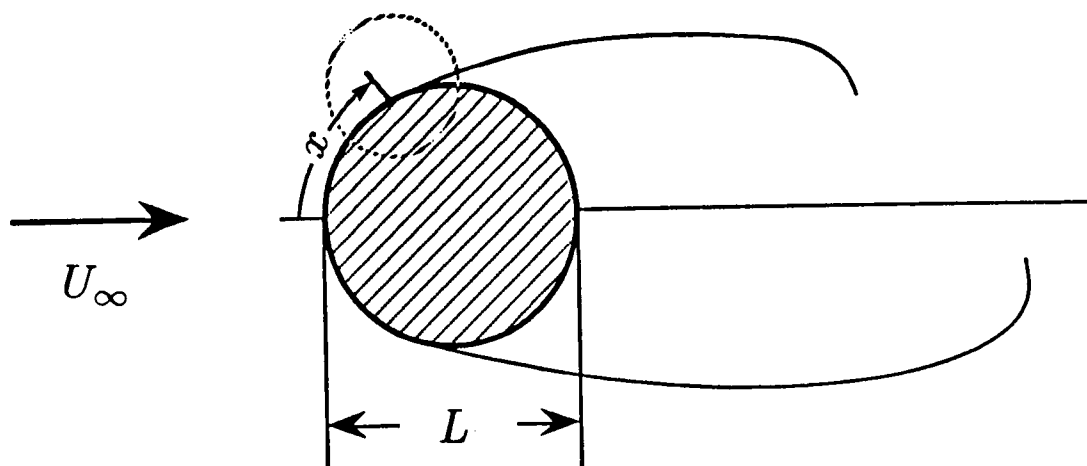
CHAPTER III

MATHEMATICAL FORMULATION AND ANALYTICAL RESULTS

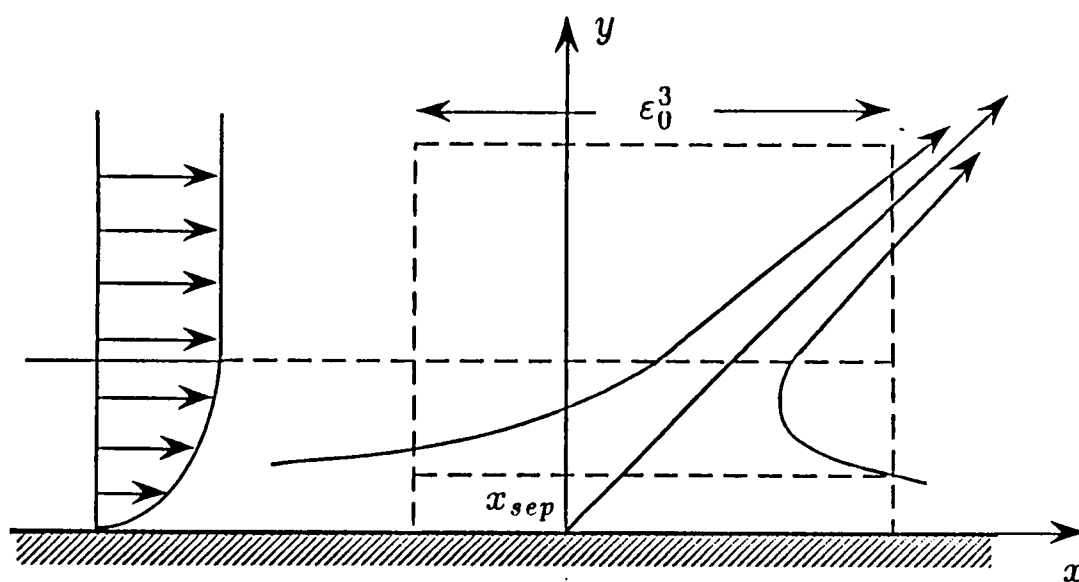
While all previous works on streaming have been more or less confined to attached flows, the occurrence of separation is the basic feature of the present study which causes special difficulties, both analytically and numerically. Nevertheless, we may still appeal to the perturbation technique to obtain approximate estimations and even closed-form solutions. In this chapter we derive the first and second order perturbation equations with proper boundary conditions with an assumption that the basic steady separating flow is given. Then some analytical results are presented, which are the extension and improvement of preliminary results first arrived at by J.Z.Wu in 1989. The general perturbation equations are given in §3.1, which are then simplified near the separation point based on the triple-deck analysis in §3.2. In §3.3 we introduce the boundary vorticity streaming and its flux and discuss the criterion of the shift of separation point. The qualitative results and achievable closed-form quantitative solutions are presented in §3.4 (high frequency), §3.5 (moderate frequency) and §3.6 (Tollmien-Schlichting frequency).

3.1 General Perturbation Equations

Consider a two-dimensional, incompressible, steady separating flow (u_0, v_0, p_0) over a bluff body at $t = t_0$ (see Fig. 7). Assume that an oscillating device located near x_{0s} , as sketched in Fig. 5, is turned on later, such that on a part of the body the flow gains a streamwise periodic velocity with small amplitude in comparison with the oncoming velocity U_∞ . Then we shall study the localized perturbation problem near the separation point (Fig. 7b). Let n be the circular frequency of



(a)



(b)

Figure 7. The steady separating flow over a bluff body. (a) Flow over bluff body. (b) The locally enlarged flow near separation point.

the forcing oscillation, and introduce two independent small parameters

$$\varepsilon_0 = Re^{-1/8} = \left(\frac{\nu}{U_\infty L} \right)^{\frac{1}{8}} \ll 1, \quad (3.1a)$$

$$\varepsilon = St^{-1/2} = \left(\frac{U_\infty}{nL} \right)^{\frac{1}{2}} \ll 1, \quad (3.1b)$$

where ν is the kinematic viscosity and L is the body length. The third important parameter is the ratio of ε_0 and ε :

$$\alpha = \frac{\varepsilon_0}{\varepsilon} = \left(\frac{St}{Re^{\frac{1}{4}}}} \right)^{\frac{1}{2}}. \quad (3.1c)$$

The dimensionless governing equations, scaled by U_∞ , L and n^{-1} are

$$u_x + v_y = 0, \quad (3.2a)$$

$$\varepsilon^{-2} u_t + uu_x + vv_y = -p_x + \varepsilon_0^8 (u_{xx} + u_{yy}), \quad (3.2b)$$

$$\varepsilon^{-2} v_t + uv_x + vv_y = -p_y + \varepsilon_0^8 (v_{xx} + v_{yy}), \quad (3.2c)$$

along with boundary conditions

$$u = 1, \quad v = 0, \quad \text{as } y \rightarrow \infty, \quad (3.3a)$$

$$u = aA(x)e^{it}, \quad v = 0, \quad \text{at } y = 0, \quad (3.3b)$$

where $A(x) \leq 1$, and the constant $a \ll 1$ is the maximum forcing amplitude. Only the real part of a complex variable will be taken in the final analysis.

The solution of (3.2) and (3.3) can be developed as

$$(u, v, p) = (u_0, v_0, p_0) + a(u_1, v_1, p_1) + a^2(u_2, v_2, p_2) + \dots, \quad (3.4a)$$

where

$$(u_1, v_1, p_1) = (\hat{u}, \hat{v}, \hat{p})e^{it} \quad (3.4b)$$

is the first-order perturbed oscillating flow; and

$$(u_2, v_2, p_2) = (u_s, v_s, p_s) + (u_{22}, v_{22}, p_{22})e^{2it} \quad (3.4c)$$

is the second-order flow, with (u_s, v_s, p_s) being the time-independent streaming components. The boundary conditions for $(\hat{u}, \hat{v})$ and (u_s, v_s) are

$$\left. \begin{aligned} \hat{u} = \hat{v} = 0 & \quad \text{as } y \rightarrow \infty, \\ \hat{u} = A(x), \quad \hat{v} = 0 & \quad \text{at } y = 0, \end{aligned} \right\} \quad (3.5a)$$

and

$$\left. \begin{aligned} u_s = v_s = 0 & \quad \text{as } y \rightarrow \infty, \\ u_s = v_s = 0 & \quad \text{at } y = 0. \end{aligned} \right\} \quad (3.5b)$$

Substituting (3.4) into (3.2), for the amplitudes $(\hat{u}, \hat{v}, \hat{p})$ we have the linearized equations

$$\hat{u}_x + \hat{v}_y = 0, \quad (3.6a)$$

$$(i\varepsilon^{-2} + u_{0x})\hat{u} + u_0\hat{u}_x + v_0\hat{u}_y + u_{0y}\hat{v} = -\hat{p}_x + \varepsilon_0^8(\hat{u}_{xx} + \hat{u}_{yy}), \quad (3.6b)$$

$$(i\varepsilon^{-2} + v_{0y})\hat{v} + u_0\hat{v}_x + v_0\hat{v}_y + v_{0x}\hat{u} = -\hat{p}_y + \varepsilon_0^8(\hat{v}_{xx} + \hat{v}_{yy}), \quad (3.6c)$$

and for the streaming components (u_s, v_s, p_s) ,

$$u_{sx} + v_{sy} = 0, \quad (3.7a)$$

$$\begin{aligned} u_{0x}u_s + u_0u_{sx} + v_0u_{sy} + u_{0y}v_s = & -p_{sx} + \varepsilon_0^8(u_{sxx} + u_{syy}) \\ & - \overline{(u_1u_{1x} + v_1u_{1y})}, \end{aligned} \quad (3.7b)$$

$$\begin{aligned} v_{0y}v_s + u_0v_{sx} + v_0v_{sy} + v_{0x}u_s = & -p_{sy} + \varepsilon_0^8(v_{sxx} + v_{syy}) \\ & - \overline{(u_1v_{1x} + v_1v_{1y})}, \end{aligned} \quad (3.7c)$$

an overbar implying time-averaging over a period.

We now change the scales for the basic steady flow and the perturbed time-dependent flow based on their respective physics. At this stage, it is convenient to rescale (u_0, v_0, p_0) according to boundary layer scales. Let

$$Y = \varepsilon_0^{-4} y, \quad V_0 = \varepsilon_0^{-4} v_0, \quad (3.8)$$

such that in an attached boundary layer, $Y, V_0 = O(1)$, and the continuity equation gives

$$u_{0x} + V_{0Y} = 0. \quad (3.9)$$

Some coefficients in (3.6) and (3.7) then become

$$v_0 = \varepsilon_0^4 V_0, \quad u_{0y} = \varepsilon_0^{-4} u_{0Y}, \quad v_{0x} = \varepsilon_0^4 V_{0x}, \quad v_{0y} = V_{0Y}. \quad (3.10)$$

When $x \rightarrow x_{0s}$, V_0, u_{0Y}, V_{0x} and V_{0Y} are no longer of $O(1)$ and should be re-estimated by using the triple-deck theory, as will be done below. On the other hand, for $\varepsilon \ll 1$ it is expected that, at least near the wall, the oscillating motion of fluid is balanced by the viscous diffusion. This leads to a diffusive thickness of oscillating layer (Fig. 8) δ_f , where

$$\delta_f/L \equiv \frac{1}{L} \sqrt{\frac{\nu}{n}} = \varepsilon \varepsilon_0^4 \ll 1. \quad (3.11)$$

Thus in (3.6) and (3.7), for the unknown variables $(\hat{u}, \hat{v}), (u_s, v_s)$ and their normal derivatives, we introduce a rescaling factor $L\sqrt{n/2\nu}$, such that

$$y = \sqrt{2\varepsilon\varepsilon_0^4} \eta, \quad \hat{v} = \sqrt{2\varepsilon\varepsilon_0^4} \hat{V}, \quad v_s = \sqrt{2\varepsilon\varepsilon_0^4} V_s, \quad (3.12)$$

where η is the scaled normal coordinate in oscillating layer, which is depicted with other normal scales in Fig. 9. For attached flows $(\eta, \hat{V}, V_s) = O(1)$; but once more, as $x \rightarrow x_{0s}$, the order of magnitude of these quantities may change, because

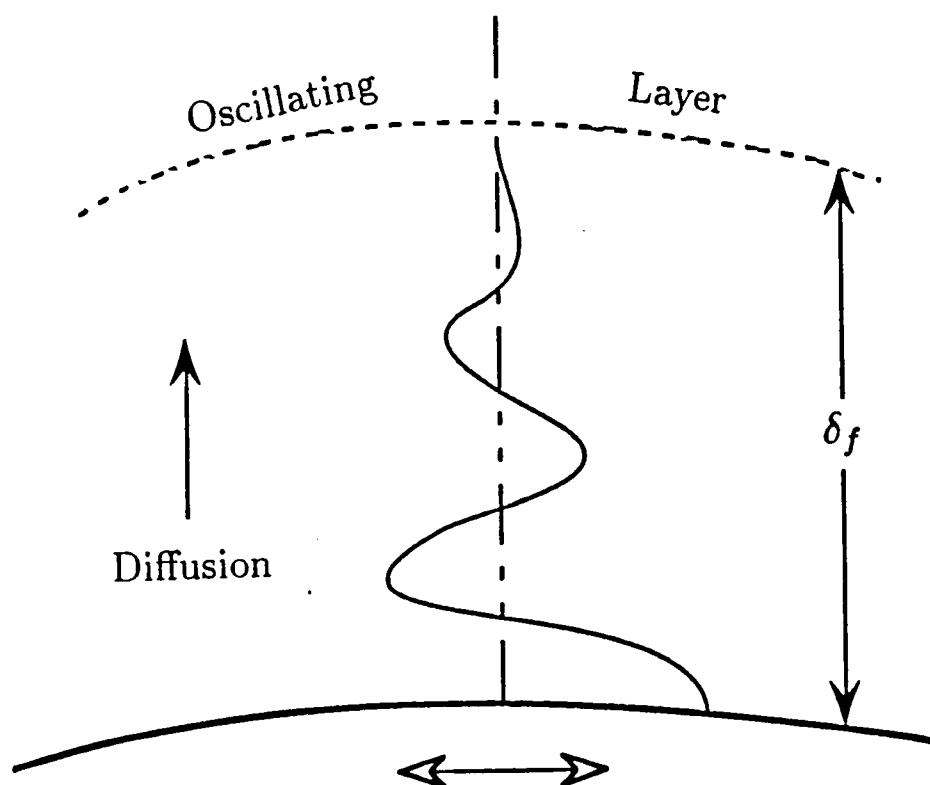


Figure 8. The oscillating layer.

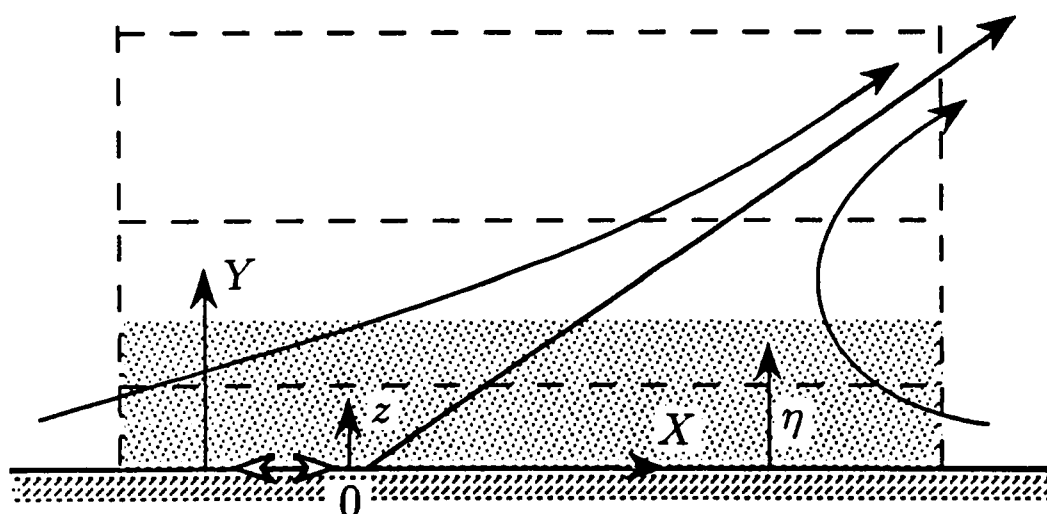


Figure 9. The normal scales in the triple-deck region. The shaded region represents the oscillating layer.

the strong normal convection drastically alters the normal extension of oscillating flow.

Nevertheless, by (3.10) and (3.12), (3.6a, b, c) are recasted to

$$\hat{u}_x + \hat{V}_\eta = 0, \quad (3.13a)$$

$$\begin{aligned} (i + \varepsilon^2 u_{0x})\hat{u} + \frac{\varepsilon V_0}{\sqrt{2}}\hat{u}_\eta + \varepsilon^2 u_0 \hat{u}_x + \sqrt{2}\varepsilon^3 u_{0Y} \hat{V} \\ = \frac{1}{2}\hat{u}_{\eta\eta} + \varepsilon^2(-\hat{p}_x + \varepsilon_0^8 \hat{u}_{xx}), \end{aligned} \quad (3.13b)$$

$$\begin{aligned} \varepsilon_0^8 \left\{ 2(i + \varepsilon^2 V_{0Y})\hat{V} + \varepsilon^2 u_0 \hat{V}_x + \sqrt{2}\varepsilon(V_0 \hat{V}_\eta + V_{0x} \hat{u}) \right\} \\ = -\hat{p}_\eta + \varepsilon_0^8(\hat{V}_{\eta\eta} + 2\varepsilon^2 \varepsilon_0^8 \hat{V}_{xx}). \end{aligned} \quad (3.13c)$$

These rescalings are of course applicable to (3.7) as well, yielding that

$$u_{sx} + V_{s\eta} = 0, \quad (3.14a)$$

$$\begin{aligned} \varepsilon^2(u_{0x}u_s + u_0u_{sx}) + \frac{\varepsilon}{\sqrt{2}}V_0u_{s\eta} + \sqrt{2}\varepsilon^3u_{0Y}V_s \\ = \frac{1}{2}u_{s\eta\eta} + \varepsilon^2\varepsilon_0^8u_{sxx} - \varepsilon^2(p_{sx} + \overline{u_1u_{1x}} + \overline{V_1u_{1\eta}}), \end{aligned} \quad (3.14b)$$

$$\begin{aligned} \varepsilon_0^8 \left\{ 2\varepsilon^2(V_{0Y}V_s + u_0V_{sx}) + \sqrt{2}\varepsilon(V_0V_{s\eta} + V_{0x}u_s) \right\} \\ = -p_{s\eta} + \varepsilon_0^8(V_{s\eta\eta} + 2\varepsilon^2\varepsilon_0^8V_{sxx}) - 2\varepsilon^2\varepsilon_0^8(\overline{u_1V_{1x}} + \overline{V_1V_{1\eta}}). \end{aligned} \quad (3.14c)$$

Recall that if the basic steady flow remains attached, the lowest order approximation of (3.13b, c) is

$$\hat{u}_{\eta\eta} - 2i\hat{u} = 0, \quad \hat{p}_\eta = 0, \quad (3.15)$$

which, under condition (3.5a), yields the well-known Stokes sublayer solution

$$\hat{u}(x, \eta) = A(x)e^{-(1+i)\eta}. \quad (3.16)$$

Correspondingly, (3.14c) reduces to

$$p_{s\eta} = 0,$$

showing that p_s equals its value at the outer edge of viscous oscillating layer; hence

$$-a^2 p_{sx} = a^4 (u_s u_{sx} + v_s v_{sx})_{\eta \rightarrow \infty} = O(a^4)$$

which is negligible. Then (3.14b) simply gives

$$u_{s\eta\eta} = 2\varepsilon^2 F(x, \eta), \quad (3.17)$$

where

$$\begin{aligned} F(x, \eta) &\equiv \overline{u_1 u_{1x} + V_1 u_{1\eta}} \\ &= \frac{1}{2} (\hat{u}_r \hat{u}_{rx} + \hat{u}_i \hat{u}_{ix} + \hat{V}_r \hat{u}_{r\eta} + \hat{V}_i \hat{u}_{i\eta}) \end{aligned} \quad (3.18)$$

is the Reynolds stress due to nonlinear convection, with r and i indicating real and imaginary parts respectively. For the present case, by (3.16) we obtain

$$F(x, \eta) = \frac{1}{4} (A^2)' (e^{-2\eta} + e^{-\eta} \sin \eta) \quad (3.19)$$

and hence the solution of (3.17) with (3.19) and (3.5b) is

$$u_s(x, \eta) = -\frac{\varepsilon^2}{8} (A^2)' (3 - e^{-2\eta} - 2e^{-\eta} \cos \eta), \quad (3.20)$$

which is the classical result of Rayleigh's streaming law. In deriving (3.20) we have added a boundary condition

$$u_s \text{ is finite as } \eta \rightarrow \infty. \quad (3.21)$$

3.2 Perturbation Equations near Separation

As $x \rightarrow x_{0s}$, the order of magnitude of those coefficients in (3.13) and (3.14) need to be re-estimated according to the triple-deck theory [49]. Table 1 lists

relevant results based on boundary layer scales (cf. Fig. 9) with triple-deck and oscillating layer parameters ε_0 and ε , as well as their ratio α , defined by (3.1a, b, c).

Table 1 Orders of Magnitude in Triple-Deck Flow

	Δx	ΔY	u_{0x}, V_{0Y}	V_0	u_{0Y}	V_{0x}
Upper deck	ε_0^3	ε_0^{-1}	ε_0^{-1}	ε_0^{-2}	ε_0^3	ε_0^{-5}
Main deck	ε_0^3	1	ε_0^{-2}	ε_0^{-2}	ε_0	ε_0^{-5}
Lower deck	1	ε_0^3	ε_0^{-2}	ε_0^{-1}	1	ε_0^{-4}
Osc. Layer	ε_0^3	ε	ε_0^{-2}	$\varepsilon_0^{-2}\varepsilon$	α	$\varepsilon_0^{-5}\varepsilon$

From Table 1 we notice that, as long as $\varepsilon \ll 1$, (3.13b, c) can be simplified as

$$\hat{u}_{\eta\eta} - \sqrt{2}\varepsilon V_0 \hat{u}_\eta - 2(i + \varepsilon^2 u_{0x}) \hat{u} = 0, \quad (3.22a)$$

$$\hat{p}_\eta = 0. \quad (3.22b)$$

Multiplying both side of (3.22a) by α^2 and using (3.9), we then have

$$\alpha^2(\hat{u}_{\eta\eta} - 2i\hat{u}) - \sqrt{2}(\tilde{V}_0 \hat{u}_\eta - \tilde{V}_{0\eta} \hat{u}) = 0, \quad (3.23)$$

where $\tilde{V}_0 = \alpha^2 \varepsilon V_0$. We see that the first two terms of order $O(\alpha^2)$ represent the viscous and unsteady effect respectively. The last two terms account for convection effect and their order of magnitude should be estimated by the oscillating layer scale, which are $O(1)$ from Table 1 and (3.1c); but in the following study, they will be estimated according to the scales to be used, such as the main deck and lower deck scales. Similarly, (3.14b, c) reduce to

$$u_{s\eta\eta} - \sqrt{2}\varepsilon V_0 u_{s\eta} - 2\varepsilon^2 u_{0x} u_s = 2\varepsilon^2 F(x, \eta), \quad (3.24a)$$

$$p_{s\eta} = 0, \quad (3.24b)$$

where (3.24a) can be further recasted to

$$\alpha^2 u_{s\eta\eta} - \sqrt{2}(\tilde{V}_0 u_{s\eta} - \tilde{V}_{0\eta} u_s) = 2\varepsilon_0^2 F(x, \eta). \quad (3.25)$$

Hence, only streamwise components $\hat{u}$ and u_s need to be studied, showing the vortical wave property of the oscillating flow.

Eqs. (3.23) and (3.25) will be used as basic equations for further analytical study. We see that the Strouhal and Reynolds number ratio α , is indeed an important parameter determining the nature of solutions. In fact by (3.11), we find that the thickness ratio of the oscillating layer to the lower deck,

$$\frac{\delta_f}{h_l} = \alpha^{-1}, \quad (3.26)$$

also depends on α . Thus the essential role of α is to determine whether the oscillating layer is confined in the lower deck or enters the main deck. Therefore, analysis starting from (3.23) and (3.25) has to be further divided into three cases with $\alpha \gg 1$, $\alpha \ll 1$ and $\alpha \sim 1$, respectively (see §3.4 ~ §3.6).

3.3 Boundary Streaming Vorticity and Its Flux

To study the time-averaged shift of separation point due to streaming, two quantities are of crucial importance. one is the boundary streaming vorticity

$$\omega_s(x, 0) = -u_{s,y}(x, 0), \quad (3.27)$$

which directly relates to the streaming wall friction $\tau_s(x)$ by

$$\omega_s(x, 0) = -\varepsilon_0^{-8} \tau_s(x),$$

hence the shifted separation point x_s is determined by the condition

$$\omega_0(x_s, 0) + \omega_s(x_s, 0) = 0. \quad (3.28)$$

Another important quantity is the streaming boundary vorticity flux, defined as

$$\sigma_s(x) = -\varepsilon_0^8 \omega_{sy}(x, 0) = \varepsilon_0^8 u_{syy}(x, 0), \quad (3.29)$$

of which an observation may provide some insight to the physics involved in the shifting of x_s , since this flux measures the vorticity production from the wall. By the well-known relation [35]

$$\sigma(x) = \frac{\partial p}{\partial x} \bigg|_{y=0}, \quad (3.29)$$

it is clear that the separation will be delayed if the streaming causes an additional time-averaged favorable pressure gradient.

It should be emphasized that Eq. (3.28) is applicable only to the time-averaged shift of separation point. For an unsteady separating boundary layer, it is known that the instantaneous separation point does not coincide with the point on the wall with vanishing skin friction (or boundary vorticity); in fact the separation point may be located inside the fluid. However, this complexity can be avoided here, since our concern is only the time-averaged steady effect rather than the transient movement of separation point. Then, owing to the periodicity, the first order effect of the wall oscillation on the instantaneous movement of separation point is canceled by taking average, and to the second order only the streaming effect is left. Therefore, the resulted flow is still a steady one and the use of Eq. (3.28) is justified. Meanwhile, the time-averaged separation point, if any, should be on the wall instead of inside the fluid.

3.4 Very High Frequency: $St \gg Re^{\frac{1}{4}}$

We now consider the extreme case with $\alpha = (St/Re^{1/4})^{1/2} \gg 1$. In this case the oscillating layer is, by (3.26), much thinner than the lower deck (Fig. 10).

The lower-deck equations are of the same form as the classical boundary layer equations, except that the pressure is unknown. Thus the behavior of oscillating layer within the lower deck is similar to that of an attached boundary layer.

To use perturbation method (which is regular), Eqs. (3.23) and (3.25) can be recasted to

$$\hat{u}_{\eta\eta} - 2i\hat{u} - \sqrt{2}\delta^2(\tilde{V}_0\hat{u}_\eta - \tilde{V}_{0\eta}\hat{u}) = 0, \quad (3.30)$$

$$u_{s\eta\eta} - \sqrt{2}\delta^2(\tilde{V}_0u_{s\eta} - \tilde{V}_{0\eta}u_s) = 2\varepsilon^2F(x, \eta), \quad (3.31)$$

where $\delta = 1/\alpha$ ($\ll 1$) is a small parameter. Assume

$$\hat{u} = \hat{u}_0 + \frac{\delta^2}{\sqrt{2}}\hat{u}_1 + \dots, \quad (3.32)$$

$$u_s = u_{s0} + \frac{\delta^2}{\sqrt{2}}u_{s1} + \dots, \quad (3.33)$$

we find that the zeroth order equations of $\hat{u}$ and u_s , are the same as (3.15) and (3.17), and the solutions are therefore given by Eqs. (3.16) and (3.20) respectively.

In terms of η , by (3.12) the boundary vorticity is

$$\omega_{s0}(x, 0) = -\frac{a^2}{\sqrt{2}}\varepsilon^{-1}\varepsilon_0^{-4}u_{s\eta}(x, 0), \quad (3.34)$$

thus from (3.20),

$$\omega_{s0}(x, 0) = \frac{1}{2\sqrt{2}}a^2\varepsilon\varepsilon_0^{-4}(A^2)'. \quad (3.35)$$

Moreover, by (3.28) and (3.17),

$$\sigma_{s0}(x) = \frac{1}{2}\varepsilon^{-2}u_{s\eta\eta}(x, 0) = F(x, 0) = \frac{1}{4}(A^2)', \quad (3.36)$$

which is indeed proportional to the wall Reynolds stress.

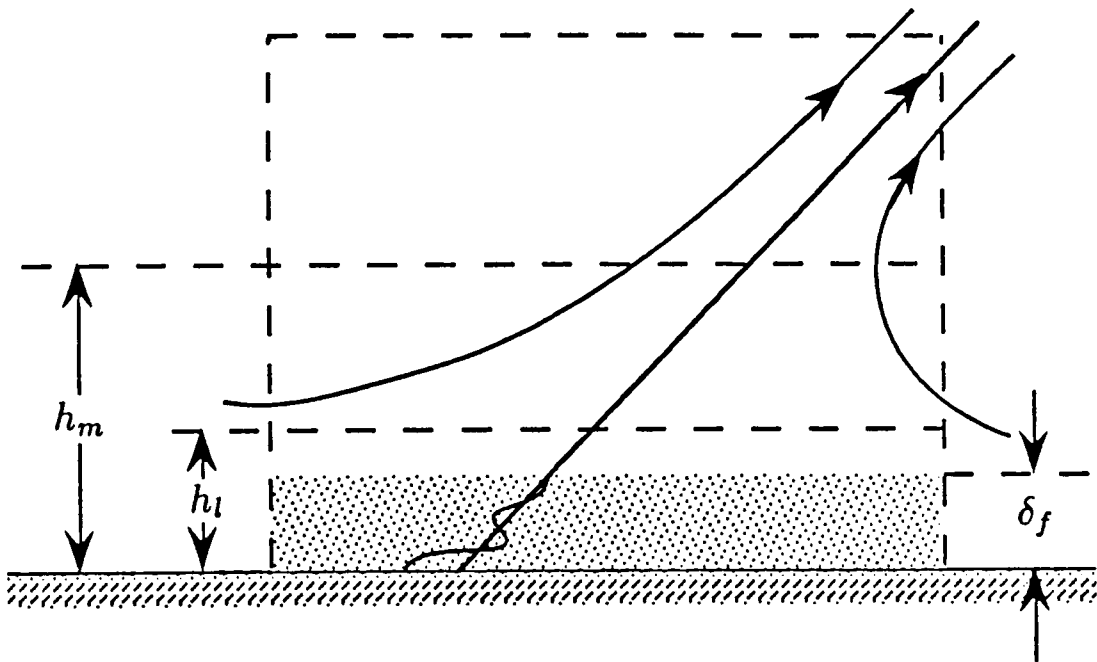


Figure 10. Flow pattern with $\alpha \gg 1$. The oscillating layer remains in the lower deck.

Therefore, for very rapid forcing oscillation with $St \gg Re^{\frac{1}{4}}$, to the leading order, the streaming is independent of basic separating flow structure, and the sign of $\omega_s(x, 0)$ and $\sigma_s(x)$ is the same as that of $(A^2)'$. Since $\omega_0 < 0$ before separation, (3.34) and (3.35) clearly show that in the local region near x_0 , a decreasing $A(x)$ may delay the separation. Note that the device shown in Fig. 5 does provide a possibility of such type of $A(x)$. These results have been confirmed by numerical computations (Chapter 4).

In order to examine the influence of basic separating flow to the streaming, the above analysis has to be extended to $O(\alpha^{-2})$ order. From (3.30) and (3.32), we have

$$\hat{u}_{1\eta\eta} - 2i\hat{u}_1 = 2(\tilde{V}_0\hat{u}_{0\eta} - \tilde{V}_{0\eta}\hat{u}_0). \quad (3.37)$$

Then, letting

$$\hat{u}_1 = A(x)f(x, \eta)e^{-(1+i)\eta},$$

and using (3.16), we get

$$f_{\eta\eta} - 2(1+i)f_{\eta} = -2(1+i)H(x, \eta),$$

subject to the boundary condition

$$f \quad \text{is finite as } \eta \rightarrow \infty, \quad \text{and} \quad f = 0 \quad \text{at } \eta = 0,$$

where $H(x, \eta) = \tilde{V}_0 - \tilde{V}_{0\eta}/(1+i)$. Hence we obtain

$$f(x, \eta) = 2(1+i) \int_0^{\eta} d\tau \int_{\tau}^{\infty} H(x, \varrho) \exp[-2(1+i)(\varrho - \tau)] d\varrho,$$

where τ, ϱ are integration variables. Interchange the order of integration, it follows that

$$\begin{aligned} f(x, \eta) &= \int_0^{\eta} H(x, \varrho) d\varrho - \int_0^{\infty} H(x, \varrho) e^{-2(1+i)\varrho} d\varrho \\ &\quad + \int_{\eta}^{\infty} H(x, \varrho) e^{-2(1+i)(\varrho - \eta)} d\varrho. \end{aligned} \quad (3.38)$$

Therefore the full first-order oscillating solution is

$$u_1(x, \eta) = \hat{u}(x, \eta)e^{it} = A(x) \left[1 + \frac{\delta^2}{\sqrt{2}} f(x, \eta) \right] e^{-\eta} e^{i(t-\eta)}. \quad (3.39)$$

Note that $\hat{u}$ decays exponentially.

As to the full second order streaming effect, denote

$$F(x, \eta) = F_0(x, \eta) + \frac{\delta^2}{\sqrt{2}} F_1(x, \eta) + \dots, \quad (3.40)$$

then from (3.31) and (3.33) we have $O(\delta^2)$ equation

$$u_{s1\eta\eta} = 2(\varepsilon^2 F_1 + \tilde{V}_0 u_{s0\eta} - \tilde{V}_{0\eta} u_{s0}) \quad (3.41a)$$

with the boundary conditions

$$u_{s1} \text{ is finite as } \eta \rightarrow \infty \text{ and } u_{s1} = 0 \text{ at } \eta = 0. \quad (3.41b)$$

Eq. (3.41) can readily be integrated.

It remains to calculate $F(x, \eta)$. Splitting $f(x, \eta)$ into real and imaginary parts:

$$f = f_r + i f_i,$$

then by (3.39) we have

$$\overline{u_1 u_{1x}} = \frac{1}{2} e^{-2\eta} \left\{ \frac{1}{2} (A^2)' + \frac{\delta^2}{\sqrt{2}} (A^2 f_r)_x \right\} + O(\delta^2).$$

Moreover, from (3.39) and the continuity, and note that $f(x, 0) = f_x(x, 0) = 0$, we get

$$\hat{V} = \frac{e^{-(1+i)\eta}}{1+i} \left\{ A'[1 - e^{(1+i)\eta}] + \frac{\delta^2}{\sqrt{2}} [A'(f - h) + A(f_x - h_x)] \right\},$$

where, by (3.38),

$$\begin{aligned} h(x, \eta) &\equiv h_r + i h_i = \int_0^\eta f_\tau e^{(1+i)(\eta-\tau)} d\tau \\ &= 2 \left\{ \int_0^\eta \tilde{V}_0 e^{(1+i)(\eta-\tau)} d\tau - \int_0^\infty \tilde{V}_0 e^{(1+i)(\eta-2\tau)} d\tau \right. \\ &\quad \left. + \int_\eta^\infty \tilde{V}_0 e^{-2(1+i)(\tau-\eta)} d\tau \right\}. \end{aligned}$$

On the other hand, it follows from (3.39) that

$$\hat{u}_\eta = -A(1+i)e^{-(1+i)\eta}\left(1 + \frac{\delta^2}{\sqrt{2}}g\right),$$

where

$$\begin{aligned} g(x, \eta) &= g_r + ig_i \equiv f - \frac{1}{1+i}f_\eta \\ &= \int_0^\eta \tilde{V}_0 d\tau - \int_0^\infty \tilde{V}_0 e^{-2(1+i)\tau} d\tau - \int_\eta^\infty \tilde{V}_0 e^{-2(1+i)(\tau-\eta)} d\tau. \end{aligned}$$

Therefore, a straightforward calculation yields the Reynolds stress $F(x, \eta)$ as in (3.40), where $F_0(x, \eta)$ is given by (3.19), while

$$\begin{aligned} F_1(x, \eta) &= \frac{(A^2)'}{4} [e^{-2\eta}(2f_r - f_i + h_i + g_i) \\ &\quad + e^{-\eta}(g_r \sin \eta - g_i \cos \eta)] + \frac{A^2}{2} e^{-2\eta}(f_{rx} - f_{ix} + h_{ix}) \end{aligned} \quad (3.42)$$

gives the $O(\delta^2)$ order correction.

Note that since f, g and h all vanish at $\eta = 0$, we have $F^{(1)}(x, 0) = 0$ and

$$F(x, 0) = F_0(x, 0) = \frac{1}{4}(A^2)' + O(\delta^4). \quad (3.43)$$

It then immediately follows from (3.36) and (3.31) that

$$\sigma_s(x) = \frac{1}{2}\varepsilon^{-2}u_{s\eta\eta}(x, 0) = F_0(x, 0) = \frac{1}{4}(A^2)', \quad (3.44)$$

which is exactly the same as (3.36). Thus if $\alpha \gg 1$ the effect of basic steady flow to streaming vorticity flux is at most $O(\delta^4)$ order. However, this is not the case for the streaming boundary vorticity. Rather, by integrating (3.41), we generally obtain a nonvanishing $\omega_{s1} = u_{s1\eta}$. This $O(\delta^2)$ correction to streaming boundary vorticity comes from redistribution of streaming vorticity inside the fluid due to the basic steady convection rather than being newly created from the wall. Its quantitative effect will be discussed in Chapter 4.

From the above analysis, we see that the oscillating flow is only influenced a little (as small as an $O(\delta^4)$ effect) by the steady separating flow, so we expect that it is confined in the oscillating layer (Fig. 10). This can be made clearer by observing the (3.30) in terms of the lower deck scale $z = \sqrt{2}\alpha^{-1}\eta$ which is much thicker than oscillating layer. Now we have $\hat{u}_\eta = \sqrt{2}\alpha^{-1}\hat{u}_z$ and $\hat{u}_{\eta\eta} = 2\alpha^{-2}\hat{u}_{zz}$, so that (3.23) becomes

$$\hat{u}_{zz} - i\alpha^2\hat{u} - (\bar{V}_0\hat{u}_z - \bar{V}_{0z}\hat{u}) = 0, \quad (3.45)$$

where $\bar{V}_0 = \varepsilon_0 V_0 = O(1)$ order in lower deck and thus $\bar{V}_{0z} = O(1)$. Therefore, the leading order in (3.46) is $O(\alpha^2)(\gg 1)$ and which leads to a solution

$$\hat{u} = 0, \quad (3.46)$$

This result does confirm the above assessment. Moreover, by some straightforward derivations, it can be shown that the solution of (3.30) is in fact the inner solution of Eq. (3.45) with the outer solution being given by (3.46). Similar discussion in main deck leads to (3.46) as well.

§3.5 Moderate Frequency: $1 \ll St \ll Re^{\frac{1}{4}}$

The case of external excitation with $\alpha \ll 1$ has been analyzed by Goldstein [26] where the streaming effect was found to be small. Let us re-examine the flow structure in the triple-deck region with our internal excitation (i.e. the tangential wall oscillation). In this case the oscillating (Stokes) layer occurs in part of the main deck as well as the whole lower deck.

We seek the solution of Eq. (3.23) by using the perturbation method with $\alpha(\ll 1)$ as the small parameter. The perturbation must be singular and an inner layer should be introduced. Suppose

$$\hat{u} = \hat{u}_0 + \alpha^2\hat{u}_1 + \alpha^4\hat{u}_2 + \dots,$$

and substitute it into (3.23), we have

$$-\tilde{V}_0 \hat{u}_{0\eta} + V_{0\eta} \hat{u}_0 = 0, \quad (3.47a)$$

$$-\tilde{V}_0 \hat{u}_{1\eta} + V_{0\eta} \hat{u}_1 = \frac{2i\hat{u}_0 - \hat{u}_{0\eta\eta}}{\sqrt{2}} \quad (3.47b)$$

and so on. The solution of (3.47a) is obvious:

$$\hat{u}_0 = C\tilde{V}_0, \quad (3.48)$$

where C is a constant to be determined by the boundary condition (B.C.). Because (3.48) will be used to derive the upper B.C. of the inner equation by matching with the inner solution at $\eta = 0$, the constant C in (3.48) should be determined by the B.C. at $\eta \rightarrow \infty$. Interestingly, two possibilities arise when considering the boundary condition at $\eta \rightarrow \infty$. First, if the conventional concept of Stokes layer is still valid, we would have $\hat{u}_0 \rightarrow 0$ as $\eta \rightarrow \infty$, but since $\tilde{V}_0 = O(1)$, we obtain a trivial solution with $C = 0$. However, this implies that the Stokes layer does not exist at all, leading to a contradiction (for attached boundary layer, $V_0 = O(\varepsilon_0^4)$, such problem does not exist). The other possibility, which is more self-consistent and physically acceptable, is that the Stokes layer concept coming from the attached flow must be altered near the separation point: the oscillating flow is convected upwards by V_0 into the main deck and further into the separating shear layer. The constant C should then be determined by matching (3.48) with the perturbed outer shear flow. This constant might coincide with the coupling coefficient obtained by Goldstein [30] which controls the receptivity mechanism generating Kelvin-Helmholtz instability waves. If this is so, similar receptivity behavior can then be obtained for internal excitation as well. However, we shall not go into such stability analysis; what we need to draw from the above discussion is only the conclusion that the physically acceptable value of constant C must be non-zero.

Then it is not difficult to obtain the closed form solution of the first order equation (3.47b). We have

$$\tilde{V}_0^2 \left(\frac{\hat{u}_1}{\tilde{V}_0} \right)_\eta = \frac{C}{\sqrt{2}} (\tilde{V}_{0\eta\eta} - 2i\tilde{V}_0)$$

and hence

$$\hat{u}_1 = \tilde{V}_0 \left[C_1 - C \int_\eta^\infty \frac{\tilde{V}_{0\eta\eta} - 2i\tilde{V}_0}{\tilde{V}_0^2} d\eta \right] \quad (3.49)$$

where $C_1 = (\hat{u}_1/\tilde{V}_0)_\infty$ needs to be determined by matching with the outer shear flow.

It is obvious that (3.48) can not satisfy the boundary condition at $\eta = 0$ which is $\hat{u} = A(x)$, since $C\tilde{V}_0(x, 0) = 0$. Thus we have to introduce an inner layer. Setting $z = \sqrt{2}\alpha^{-1}\eta$, which yields

$$\hat{u}_{zz} - i\alpha^2 - (\bar{V}_0\hat{u}_z - \bar{V}_{0z}\hat{u}) = 0 \quad (3.50)$$

in the lower deck, where $\bar{V}_0 = \varepsilon_0 V_0 = O(1)$. The boundary conditions are

$$\hat{u} \text{ is finite as } z \rightarrow \infty, \quad (3.51a)$$

$$\hat{u} = A(x) \quad \text{at } z = 0. \quad (3.51b)$$

Note that a direct matching with the outer solution by using the limiting matching principle (van Dyke 1975) would give $\hat{u} \rightarrow 0$ as $z \rightarrow \infty$ which is doubtful from physical point of view. Hence there seems a need of an intermediate expansion for matching the inner and outer solutions. By (1.2) and (3.12), Eq. (3.1) gives $y = \varepsilon_0^5 z$, which is precisely the scale of the lower deck and independent of the frequency. Now, no matter how small α^2 is, since as $z \rightarrow 0$ we have $V_0 \sim z^2$ and $u_{0x} \sim z$, which can be made even smaller than α^2 ; thus there always exists a thin Stokes layer near the wall, where the flow is still governed by (3.15) with $\hat{u} = A(x)$

at $z = 0$, and is therefore inherently unsteady. Then, as z increases, the $O(\alpha^2)$ effect can be neglected and (3.50) gives a quasi-steady flow

$$\hat{u}_{zz} - \bar{V}_0 \hat{u}_z + \bar{V}_{0z} \hat{u} = 0, \quad z \gg \alpha^2,$$

which is in agreement with the quasi-steady nature of the trailing edge analysis by Brown and Daniels [50].

The qualitative behavior of $\hat{u}(x, z)$ can be estimated by rewriting (3.50) with (3.51) as an integral equation

$$\hat{u} = A + \int_0^z d\tau \int_\tau^\infty \bar{V}_{0\sigma} e^{-\int_\tau^\sigma \bar{V}_{0d\rho}} \hat{u} d\sigma. \quad (3.52)$$

Here, for $x < x_0$, we may roughly take $\bar{V}_0 \geq 0$, $\bar{V}_{0z} \geq 0$. Because $\hat{u}(x, 0) = A > 0$, $\hat{u}$ must be positive for $z \ll 1$. Then the integrand of (3.52) is positive, hence $\hat{u}$ increases as z . It finally approaches constant as $z \rightarrow \infty$, because by (3.50) and (3.51a) we have

$$\hat{u}_z(x, z) = \int_z^\infty \bar{V}_{0\tau} e^{-\int_z^\tau \bar{V}_{0d\sigma}} \hat{u} d\tau \quad (3.53)$$

which decreases to zero at infinity.

The second order streaming equation (3.25) can be solved in the way similar to solving (3.23), and similar problem of boundary conditioning could be expected. Substituting

$$u_s = u_{s0} + \alpha^2 u_{s1} + \dots,$$

$$F(x, \eta) = F_0(x, \eta) + \alpha^2 F_1(x, \eta) + \dots$$

into (3.25), we have

$$\tilde{V}_0 u_{s0\eta} - \tilde{V}_{0\eta} u_{s0} = -\sqrt{2}\varepsilon_0^2 F_0(x, \eta), \quad (3.54)$$

where $F_0(x, \eta)$ is given by (3.18) in terms of V_0 . Integrating (3.54) gives

$$u_{s0} = \tilde{V}_0 \left[\sqrt{2}\varepsilon_0^2 \int_{\eta}^{\infty} \frac{F(x, \eta)}{\tilde{V}_0^2} d\eta + C_2 \right], \quad (3.55)$$

where $C_2 = (u_{s0}/\tilde{V}_0)_{\infty}$ is to be determined by the matching mentioned above. Generally, $u_{s0}(x, 0) \neq 0$, indicating that an inner layer should be introduced again, and the governing equation needs to be solved numerically.

We conclude that when $St \ll Re^{\frac{1}{4}}$ the wall oscillation excites an unsteady layer which is first damped in the viscous (inner) layer and then convected away with the separated shear layer and at meantime amplified (Fig. 11). The instability problem has to be studied to determine the streaming effect which is our main concern. Later on, we shall see from numerical experiments that the streaming is stronger for lower frequency excitation and the movement of separation point is not negligible. This is different from Goldstein's conclusion and may imply a different control mechanism of the internal excitation.

It should be kept in mind that the smallness of ε is assumed in our entire study, although $\alpha \ll 1$ in the above discussion. This is necessary to obtain the ordinary differential equations (ODEs), such as (3.23) and (3.25), which are more tractable analytically. However, these ODEs imply that the downstream effect of the wall oscillation is always lost, hence our simplification is too localized to deal with large scale streaming phenomena. This limitation will be seen from the numerical results in Chapter 4.

3.6 Tollmien-Schlichting Frequency: $St \sim Re^{\frac{1}{4}}$

We now consider the case with $St \sim Re^{\frac{1}{4}}$ in which the oscillating layer has the same scale as the lower deck. It is seen from (3.23) and (3.25) that all the terms are of $O(1)$ and mutually balanced, and the equations have to be solved numerically.

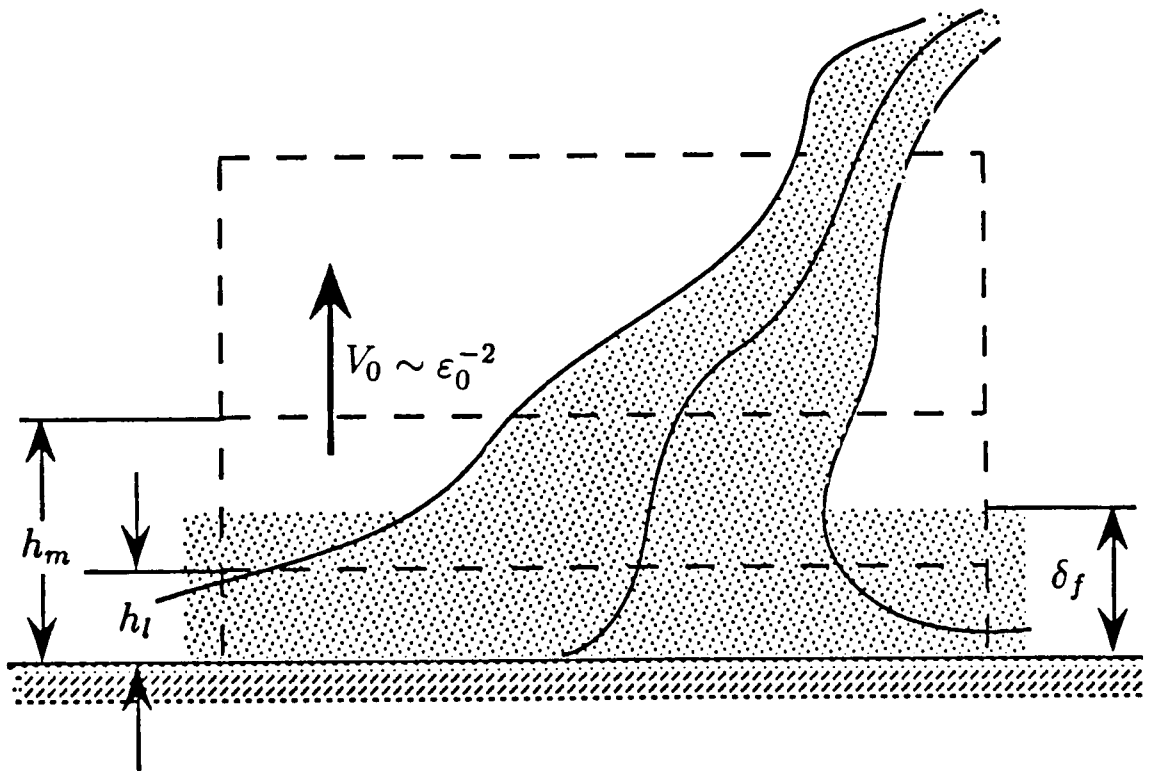


Figure 11. Flow pattern with $\alpha \ll 1$. The oscillating flow is convected upward into the outer shear layer by the strong normal velocity near the separation point.

Nevertheless, we know from the last subsection that when the oscillating layer is much thicker than the lower deck ($\alpha \ll 1$), the oscillating flow is convected upward with the shear layer in the main deck. Therefore, it would be of interest to examine the equations in the main deck for the present critical situation.

For this purpose, we rewrite (3.23) and (3.25) in terms of main deck scale as

$$\varepsilon_0^2 \hat{u}_{YY} - V_0^* \hat{u}_Y - (i\alpha^2 - V_{0Y}^*) \hat{u} = 0, \quad (3.56)$$

$$\varepsilon_0^2 u_{sYY} - V_0^* u_{sY} + V_{0Y}^* u_s = \varepsilon_0^2 F(x, Y), \quad (3.57)$$

where $V_0^* = \varepsilon_0^2 V_0$. Estimated in main deck, both V_0^* and V_{0Y}^* are found to be of $O(1)$ from Table 1. To solve the equations, we need to carry out singular perturbation with respect to the small parameter ε_0 . It is easily seen that the inner layer is just the lower deck and the recasted equations in the lower deck are equivalent to the equations in the oscillating layer, Eqs. (3.23) and (3.25). Hence the inner solutions of (3.56) and (3.57) are nothing but the solutions of (3.23) and (3.25) respectively.

To get the outer solutions in main deck, expand $\hat{u}$ as

$$\hat{u} = \hat{u}_0 + \varepsilon_0^2 \hat{u}_1 + \dots,$$

we have

$$-V_0^* \hat{u}_{0Y} - (i\alpha^2 - V_{0Y}^*) \hat{u}_0 = 0, \quad (3.58)$$

$$-V_0^* \hat{u}_{1Y} - (i\alpha^2 - V_{0Y}^*) \hat{u}_1 = -\hat{u}_{0YY}. \quad (3.59)$$

They can be solved as following. Eq. (3.58) can be written as

$$\left(\frac{\hat{u}_0}{V_0^*} \right)_Y / \left(\frac{\hat{u}_0}{V_0^*} \right) = \frac{-i\alpha^2}{V_0^*},$$

which yields

$$\hat{u}_0 = C_3 V_0^* \exp\left(i\alpha^2 \int_Y^\infty \frac{1}{V_0^*} dY\right) \quad (3.60)$$

at once. Then Eq. (3.59) can be written as

$$\left(\frac{\hat{u}_1}{V_0^*}\right)_Y + \frac{i\alpha^2}{V_0^*} \left(\frac{\hat{u}_1}{V_0^*}\right) = \frac{\hat{u}_{0YY}}{V_0^{*2}},$$

and the solution is

$$\hat{u}_1 = V_0^* \left[\int_\infty^Y \frac{\hat{u}_{0\eta\eta}}{V_0^{*2}} \exp\left(i\alpha^2 \int_Y^\eta \frac{d\tau}{V_0^*}\right) d\eta + C_4 \exp\left(i\alpha^2 \int_Y^\infty \frac{d\tau}{V_0^*}\right) \right] \quad (3.61)$$

The constants C_3 and C_4 which have the same form as C_1 are to be determined by the B.C. at the upper boundary of the main deck. The same problem arises for the boundary conditions there as in the case with $\alpha \ll 1$. In fact, Eq. (3.60) is quite similar to Eq. (3.48) except that now $\hat{u}_0$ is unsteady and also oscillatory in Y direction. The similar amplifying mechanism is expected; however, it is not clear whether this solution matches the Tollmien-Schlichting or Kelvin-Helmholtz wave. Nevertheless, since the oscillation term in (3.61) is bounded by 1, the amplification rates in (3.61) and (3.48) are the same.

Finally the outer solution of (3.57) gives the streaming effect in the main deck. It has the similar form as (3.55) and will not be repeated here. We conclude that once the oscillating flow extends into the main deck, it is amplified and convected upward into the outer shear layer. Unlike the $\alpha \ll 1$ case, the flow is unsteady to the leading order in main deck. Again, without matching with the shear flow in the upper deck and beyond, the problem can not be fully solved.

CHAPTER IV

NUMERICAL COMPUTATION AND RESULTS

In the previous chapter, we studied the behavior of local oscillating flow and the induced streaming effect near the separation point analytically and obtained some quantitative and qualitative results. In the analysis we assumed the basic steady separating flow was given, so in this chapter, we shall calculate two types of steady separating flow first, a breakaway separation and a separation bubble. Then the perturbed flow will be solved numerically by using (3.22) and (3.24) (or (3.23) and (3.25)). The numerical results will be checked against the analytical ones and provide clearer physical pictures of the problem. Note that due to the limitation of our simplification and since we leave the instability problem aside, these results are only preliminary and by no means reflecting the complete picture of the reality; however they are very helpful for further studies.

In §4.1, we establish an approximate model for the basic velocity profiles of the breakaway separation. The separation bubble modeling is presented in §4.2, along with a brief discussion of the numerical procedure for solving the bubble flow based on boundary layer equations. In addition to the analysis in Chapter 3, a supplementary study of Eqs. (3.22) and (3.24), including boundary conditioning problem and the numerical procedure to be used, are given in §4.3. Finally, the numerical results and discussion are presented in §4.4.

4.1 Breakaway Separation Modeling

To perform calculations, it is necessary to know the basic steady velocity profiles u_0 and V_0 (assumed given in the perturbation analysis) near the separation

point. These data may be obtained from the proper triple-deck solution of the breakaway separation from smooth surface [51]. Since our major concern here is the shift of separation point represented by the relative amount of the boundary vorticity of the basic steady flow, $\omega_0(x, 0)$, and that of the streaming flow, $\omega_s(x, 0)$, only the solutions in lower deck (or a sublayer of lower deck) are involved if $\alpha \gtrsim 1$, in which the normal distance Y (in boundary layer scale) is always small. This allows us to approximate (u_0, V_0) by normal Taylor expansions and avoid the necessity of re-calculating the whole triple-deck solution.

In fact, we have

$$\begin{aligned} u_0(x, Y) &\simeq Y u_{0Y} + \frac{Y^2}{2} u_{0YY}, \\ V_0(x, Y) &\simeq \frac{Y^2}{2} V_{0YY} + \frac{Y^3}{3!} V_{0YYY}, \end{aligned}$$

where the derivatives are estimated at $Y = 0$, thus by using (2.9) and the relations

$$\omega_0 = -\varepsilon_0^{-4} u_{0Y}, \quad \sigma_0 = p_{0x} = u_{0YY} \Big|_{Y=0},$$

we obtain the approximate expressions of coefficients in (3.22a) or (3.23) and (3.24a) or (3.25) in terms of boundary vorticity ω_0 and its flux σ_0 (or tangential pressure gradient p_{0x}):

$$\begin{aligned} u_{0x} &\simeq -\sqrt{2}\varepsilon\varepsilon_0^4\eta\omega_{0x} + \varepsilon^2\eta^2\sigma_{0x} \\ &= -\sqrt{2}\varepsilon\varepsilon_0\eta\omega_{0\tilde{X}} + \varepsilon^2\varepsilon_0^{-3}\eta^2\sigma_{0\tilde{X}} \\ &= -\varepsilon_0^2z\omega_{0\tilde{X}} + \frac{1}{2}\varepsilon_0^{-1}z^2\sigma_{0\tilde{X}}, \end{aligned} \tag{4.1a}$$

$$\begin{aligned} V_0 &\simeq \frac{Y^2}{2}(\varepsilon_0^4\omega_{0x} - \frac{Y}{3}\sigma_{0x}) \\ &= \varepsilon^2\varepsilon_0\eta^2(\omega_{0\tilde{X}} - \frac{\sqrt{2}}{3}\varepsilon\varepsilon_0^{-4}\eta\sigma_{0\tilde{X}}) \\ &= \frac{1}{2}\varepsilon_0^3z^2(\omega_{0\tilde{X}} - \frac{1}{3}\varepsilon_0^{-3}z\sigma_{0\tilde{X}}), \end{aligned} \tag{4.1b}$$

where $\tilde{X} = \varepsilon_0^{-3}(x - x_{0s})$ is the streamwise coordinate of triple-deck. In (4.1), ω_0 and σ_0 are easily inferred from the numerical result of Smith [51]. Expansion to higher orders are possible but seems unnecessary.

Strictly speaking, Eq. (4.1) is not uniformly valid in the sense of matching asymptotic expansion, since the error grows unboundedly as η or z approaches infinity. But interestingly, a comparison of the present result with the recent study by Shen and Ma [52] with the full triple-deck numerical solution instead of the Taylor expansions showed that, using (4.1) does not cause large error in the resulted streaming as long as α is large enough. But for smaller α , such as $\alpha < 1$ typically, the oscillating layer is thicker and the computation based on the Taylor expansion becomes sensitive to the choice of the upper boundary of the computational domain where the B.C. of infinity is imposed (this problem will be addressed in more detail in §4.4.). However, no such problem occurs when we carried out the calculation with the closed separation bubble which is solved numerically.

4.2 Separation Bubble Modeling

In this subsection, we briefly discuss the numerical method of modeling separation bubble. This can be done based on the boundary layer equations, which simplify the calculation drastically since the integration by forward marching can be used and there is no need of downstream boundary conditions. There are several methods for calculating separation bubble by assuming a given distribution of either skin friction or displacement thickness. Here we shall adopt the method developed by Carter [53] which is based on given displacement thickness. In fact, Carter found that the computation fails to converge for relatively large bubble if the skin friction is given.

The dimensionless boundary layer equations for two-dimensional, laminar, incompressible flow are

$$u_x + V_Y = 0 \quad (4.4a)$$

$$uu_x + Vu_Y = UU_x + u_{YY}, \quad (4.4b)$$

with boundary conditions

$$u(x, 0) = V(x, 0) = 0 \quad (4.4c)$$

and

$$u(x, Y) \rightarrow U(x) \quad \text{as } Y \rightarrow \infty, \quad (4.4d)$$

where Y is the boundary layer scale, V is the scaled normal velocity, U is the velocity at the edge of the boundary layer. For present application, we introduce the vorticity $\omega = -\partial u / \partial Y$ and the stream function Ψ , so that (4.4a-d) are recasted to

$$u = \Psi_Y, \quad V = -\Psi_x, \quad \omega = -\Psi_{YY}, \quad (4.5a)$$

$$u\omega_x + V\omega_Y = \omega_{YY}, \quad (4.5b)$$

with

$$u(x, 0) = \Psi(x, 0) = 0 \quad (4.5c)$$

and

$$\Psi \rightarrow U(Y - \delta^*), \quad (4.5d)$$

where

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dY$$

is the displacement thickness. Then introducing

$$\psi = \Psi - u(Y - \delta^*), \quad (4.6)$$

we obtain a set of equations to be solved numerically:

$$\psi_Y = (Y - \delta^*)\omega, \quad (4.7a)$$

$$u\omega_x - (\psi + (Y - \delta^*)u)_x\omega_Y = \omega_{YY}, \quad (4.7b)$$

subject to boundary conditions

$$u(x, 0) = \psi(x, 0) = 0, \quad (4.7c)$$

$$\omega(x, Y), \psi(x, Y) \rightarrow 0 \quad \text{as } Y \rightarrow \infty. \quad (4.7d)$$

In our calculation, we do not follow Carter [48] to transform the normal coordinate Y , since the same grid will be used to compute Eqs. (3.22) and (3.24) for which the uniform rectangular grid is more convenient.

In the calculation, δ^* is assumed to be given. To be flexible in computing different bubbles, δ^* is prescribed artificially according to Carter. We use polynomials to construct $\delta^*(x)$ which are smooth and has a maximum in the interval $[x_0, x_2]$, while matching the Blasius flat plate solution at the two ends of the interval. Specifically, we have, for $x_0 \leq x \leq x_1$

$$\delta^*(x) = a_1 + a_2(x - x_0) + a_3(x - x_0)^2 + a_4(x - x_0)^3, \quad (4.7a)$$

and for $x_1 \leq x \leq x_2$

$$\delta^*(x) = b_1 + b_2(x - x_1) + b_3(x - x_1)^2 + b_4(x - x_1)^3. \quad (4.7a)$$

The constants a_1 to a_4 and b_1 to b_4 are so determined that, at $x = x_0$, the prescribed δ^* matches with the value and slope of the Blasius flat plate distribution, then at $x = x_1$, δ^* reaches its maximum value $\delta_{\max}^*$, and at $x = x_2$, δ^* has a

and at $x = x_2$, δ^* has a minimum which is generally chosen about the same as the Blasius flat-plate value at that particular x location. Following Carter [53], we may have

$$x_0 = 1.0, \quad x_1 = 1.35, \quad x_2 = 1.884,$$

$$a_1 = 1.7208\sqrt{x_1}, \quad a_2 = 0.5 \times 1.7208/\sqrt{x_1}, \quad a_3 = -a_1/(3d_1^2) - 2a_2/(3d_1),$$

$$b_1 = \delta_{\max}^*, \quad b_2 = -3(b_1 - 2.25)/d_2^2, \quad b_3 = 2(b_1 - 2.25)/d_1^3,$$

where $d_1 = x_1 - x_0$ and $d_2 = x_2 - x_1$, and $\delta_{\max}^*$ can be chosen to give bubbles with different size. A curve of $\delta^*(x)$ is given in Fig. 12 for $\delta_{\max}^* = 4.5$, and the corresponding separation bubble flow is in Fig. 13.

The solution procedure of (4.6) have been given in detail in Ref. [53]. Both forward-marching and global iteration procedures [53] are used in our calculation. It is straightforward to work out the algebraic changes in the scheme because of not using the transformation of normal coordinate. These minor changes does not affect the performance of the algorithm and the scheme without transformation works as well as the original one.

4.3 Numerical Procedure for Solving Streaming Effect

Having had the basic steady separating flow, we can obtain the streaming effect by solving (3.22a) and (3.24a). The equations are repeated here for convenience of the following discussion. We have, for oscillating flow, the homogeneous equation

$$\hat{u}_{\eta\eta} - \sqrt{2}\varepsilon V_0 \hat{u}_\eta - 2(i + \varepsilon^2 u_{0x}) \hat{u} = 0, \quad (4.8a)$$

with non-homogeneous boundary conditions

$$\hat{u}\Big|_{\eta=0} = A(x) \quad \text{and} \quad \hat{u}\Big|_{\eta \rightarrow \infty} \text{ is finite}, \quad (4.8b)$$

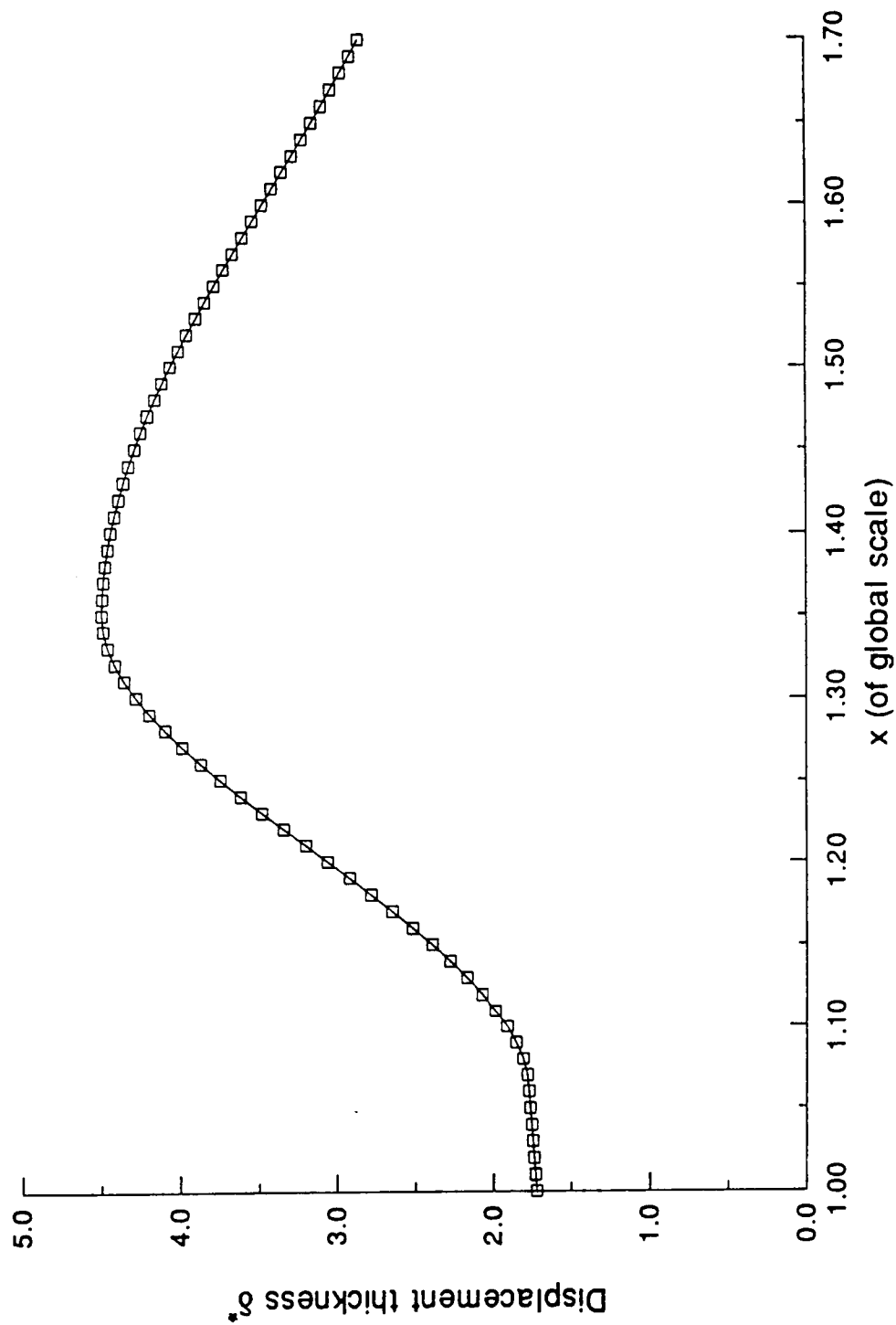


Figure 12. Prescribed distribution of displacement thickness ($\delta_{\max}^* = 4.5$).

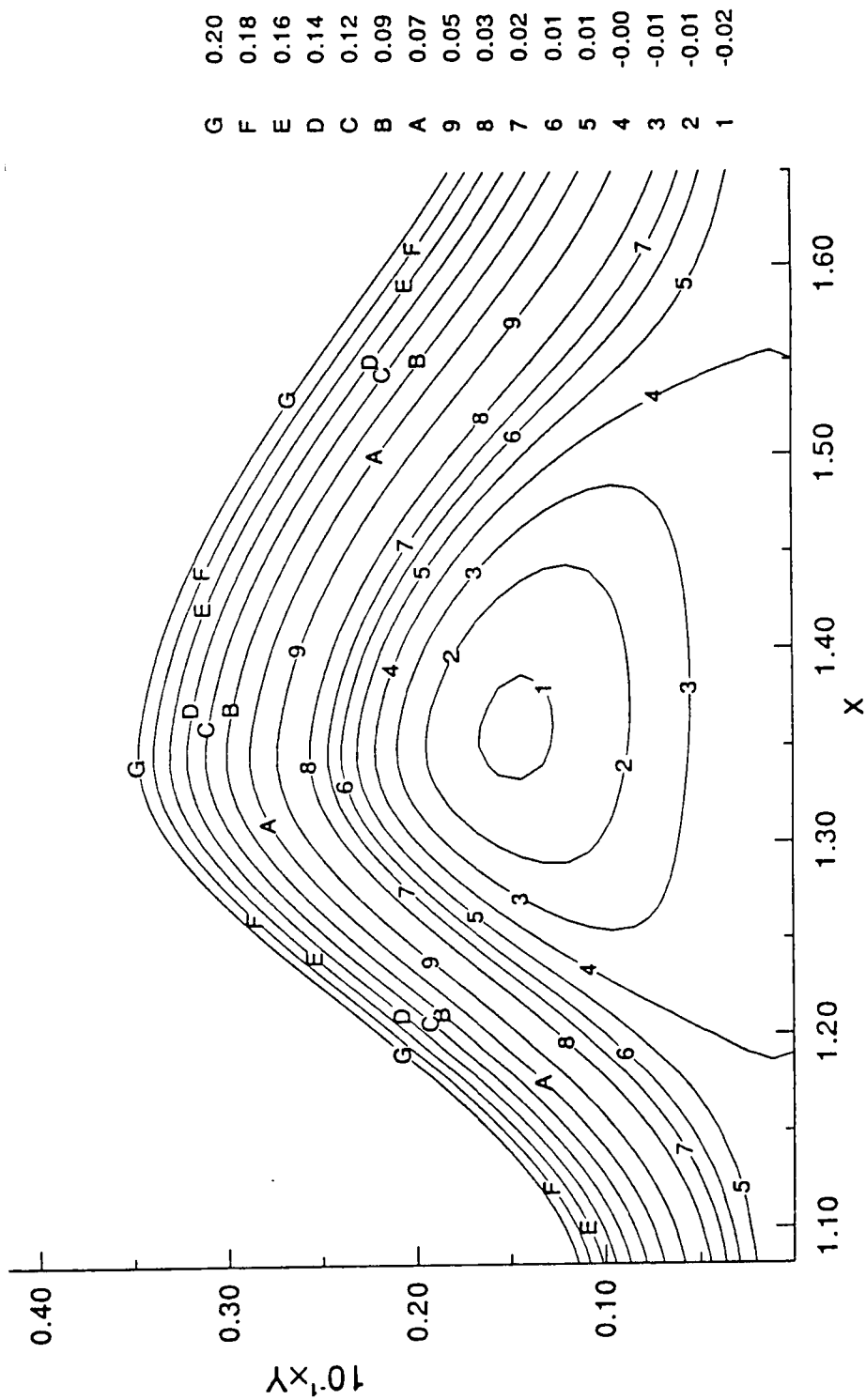


Figure 13. The streamline distribution of a separation bubble calculated by using the modified Carter's scheme.
(with prescribed displacement thickness in figure 12)

and, for second order streaming, the non-homogeneous equation

$$u_{s\eta\eta} - \sqrt{2}\varepsilon V_0 u_{s\eta} - 2\varepsilon^2 u_{0x} u_s = 2\varepsilon^2 F(x, \eta) \quad (4.9a)$$

with

$$u_s \Big|_{\eta=0} = 0 \quad \text{and} \quad u_s \Big|_{\eta \rightarrow \infty} \text{ is finite.} \quad (4.9b)$$

A simple finite difference scheme is used in solving Eqs. (4.8) and (4.9). The derivative terms are discretized on uniform meshes by using central difference so that the scheme is of second-order accuracy, and the resulted linear systems can be solved easily by the usual tridiagonal solver (the complex linear system obtained by discretizing (4.8a) can be solved virtually by the same tridiagonal solver with some minor changes). It should be noted that u_{0x} becomes negative near the separation point which may cause the tridiagonal solver to be unstable, because no pivoting was used. In the computations, we used a solver with double precision and iterative improvement, and obtained accurate results without numerical instability. In fact, same results were obtained when the Gauss-Jordan elimination with full pivoting, which is much slower, was used to solve the linear systems. The fact that the scheme is stable may be due to the smallness of u_{0x} though it is negative. The solution procedure starts with solving (4.8) which yield the forcing terms $F(x, \eta)$ in (4.9a), and then solve (4.9) to obtain the streaming velocity. The shift of separation point can be seen from the calculated skin friction distributions.

Although the solution procedure seems easy, however, we need to look at the equations and the boundary conditions more carefully. The boundary condition at $\eta = 0$ is clear, but we have seen in Chapter 3 that the boundary condition at the upper edge of the oscillating layer ($\eta \rightarrow \infty$) needs to be specified by matching with the outer solutions in the main deck or lower deck, depending on the order of α . Unfortunately, this matching can not be determined for $\alpha \gg 1$ since the

outer solutions are not available. Therefore, we assume that $\hat{u}$ and u_s approach constants as $\eta \rightarrow \infty$ instead; or equivalently,

$$\hat{u}_\eta \Big|_{\eta \rightarrow \infty} \rightarrow 0 \quad \text{and} \quad u_{s\eta} \Big|_{\eta \rightarrow \infty} \rightarrow 0. \quad (4.10)$$

This is more restrictive than (4.8b) and (4.9b) mathematically, but it may give reasonable results and is possible to be implemented. The validity of (4.10) can be justified only by looking at the computational results.

Another problem about the boundary value problems (4.8a) and (4.9a) with (4.10) is that we do not know if they have non-trivial solutions and if their solutions are unique, for which no analytical proof is available. The problem is less serious for (4.8a) which has complex coefficients. For (4.9a), the trouble comes from the u_{0x} term which may be negative near separation. Nevertheless, the numerical results seems to suggest that nontrivial solutions do exist.

4.4 Numerical Results and Discussions

As discussed in Chapter 2, we perform the forcing excitation by a local wall oscillation device as sketched in Fig. 5a. The oscillation generates a step like discontinuous forcing amplitude distribution $A(x)$ (see Eq. (3.3b)) which is difficult to deal with analytically. To avoid this, we consider the excited fluid motion very close to the wall, then the amplitude of the fluid becomes smooth due to the viscous diffusion as shown in Fig. 5b. However, this distribution is not available analytically and has to be obtained by full numerical computation which is not the concern of the present study. Thus, we need to make assumptions for $A(x)$. Introducing the normalized triple-deck coordinates $X = \lambda^{-\frac{1}{3}} \tilde{X}$ with $\lambda = 0.332$ [51], the forcing amplitude function $A(X)$ is chosen as follows: either

$$A_1(X) = \frac{1}{2} \left[\tanh \left(\frac{-X + L_s}{B} \right) + 1 \right] \quad (6.3a)$$

or

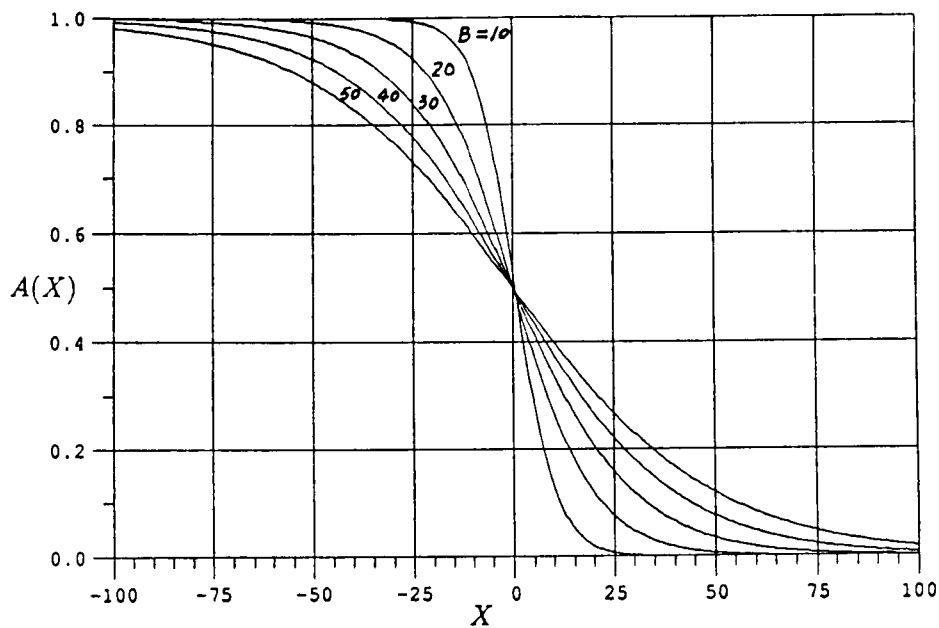
$$A_2(X) = \exp \left[- \left(\frac{X - L_s}{B} \right)^2 \right], \quad (6.3b)$$

see Fig. 14. The parameter L_s determines the central location of the oscillating piece of wall, and B determines the sharpness of the amplitude slope. Obviously, $A_2(X)$ is more realistic, but the part with $A_2'(X) > 0$ may promote separation. So we tested $A_1(X)$ as well, which may be difficult to realize physically and represents a more idealized case where the upstream end of the oscillating piece of wall is sufficiently far from the separation point such that its unfavorable effect is not harmful. Throughout numerical calculations, the amplitude parameter a (see Eq. (3.3b)) is fixed as 0.5, which is about the upper bound of the validity of the present perturbation procedure. Note that for the device shown in Fig. 5, the amplitude is proportional to the frequency, and it is very easy to produce a much larger amplitude than 0.5.

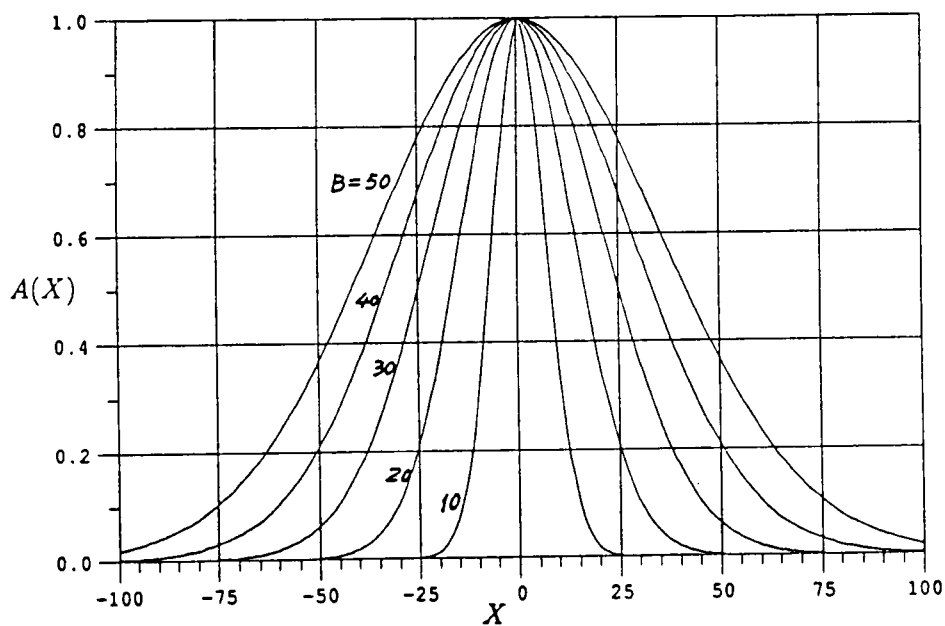
1. Breakaway Separation

By using (4.1a,b) and the scheme described in §4.3, the streaming effect is easily solved. Although (4.1) is valid for $\alpha \gtrsim 1$ as we have seen in §4.1, the $\alpha < 1$ case was also tested because for α smaller than but close to 1, δ_f is still small compared to the main deck thickness. The streaming effect is mainly presented as the skin friction distribution curves (Figs. 15), from which we can see clearly the movement of separation points by comparing the resulted distribution with the basic steady one (indicated as "original" in the figures) from Ref. [51].

Figure 15 shows the numerical results as well as the leading order analytical solution given by (3.35) and (3.20) with $\alpha \gg 1$ for comparison. In Figs. 15a,b the amplitude function is $A_2(X)$, while in Figs. 15c,d $A_1(X)$ was used. In numerical calculation the outer boundary of oscillating layer can not be set at $\eta = \infty$. Instead, $\eta = 5$ and $\eta = 10$ were used and the results are given in Figs. 15a,c and

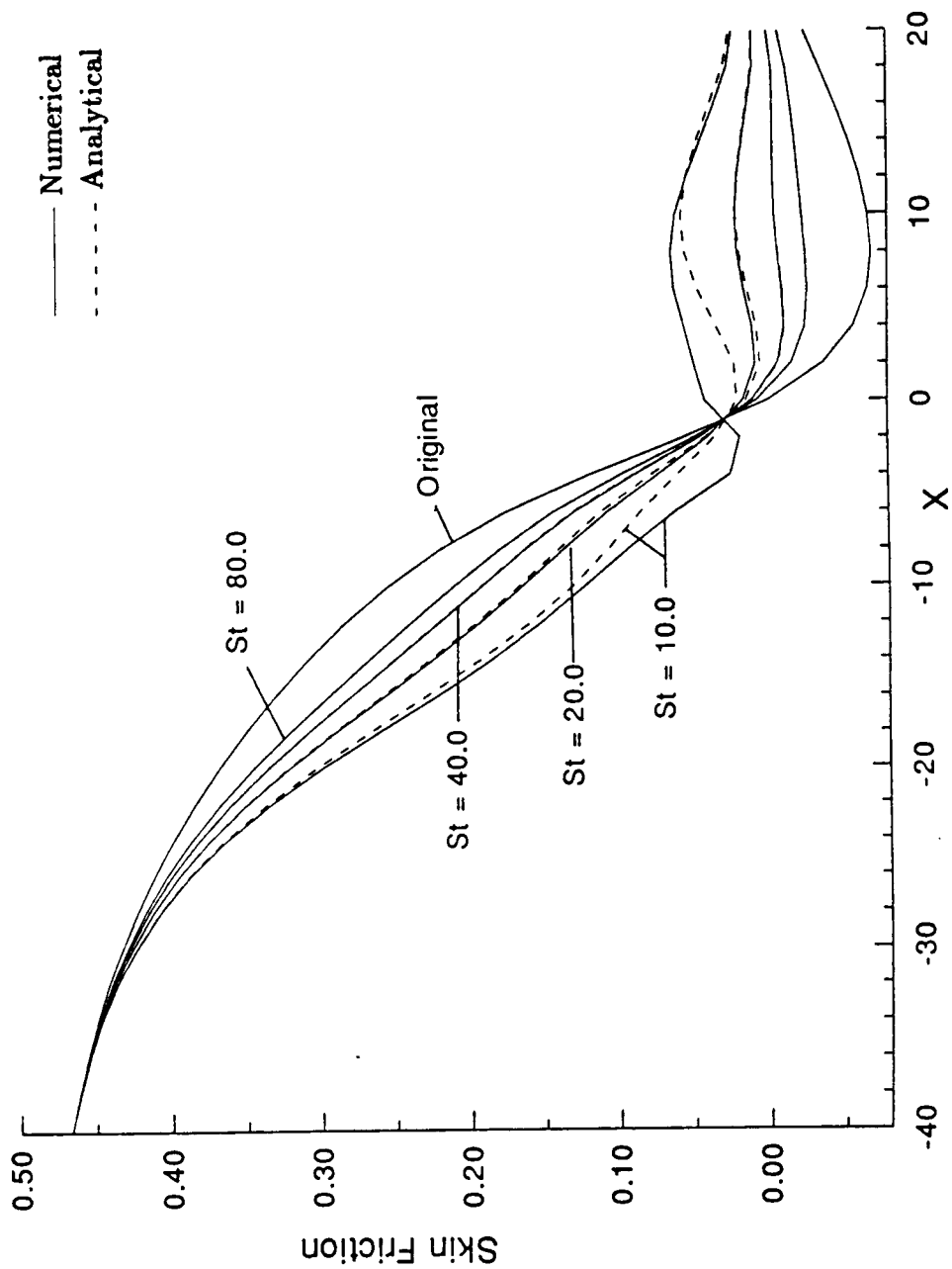


(a)



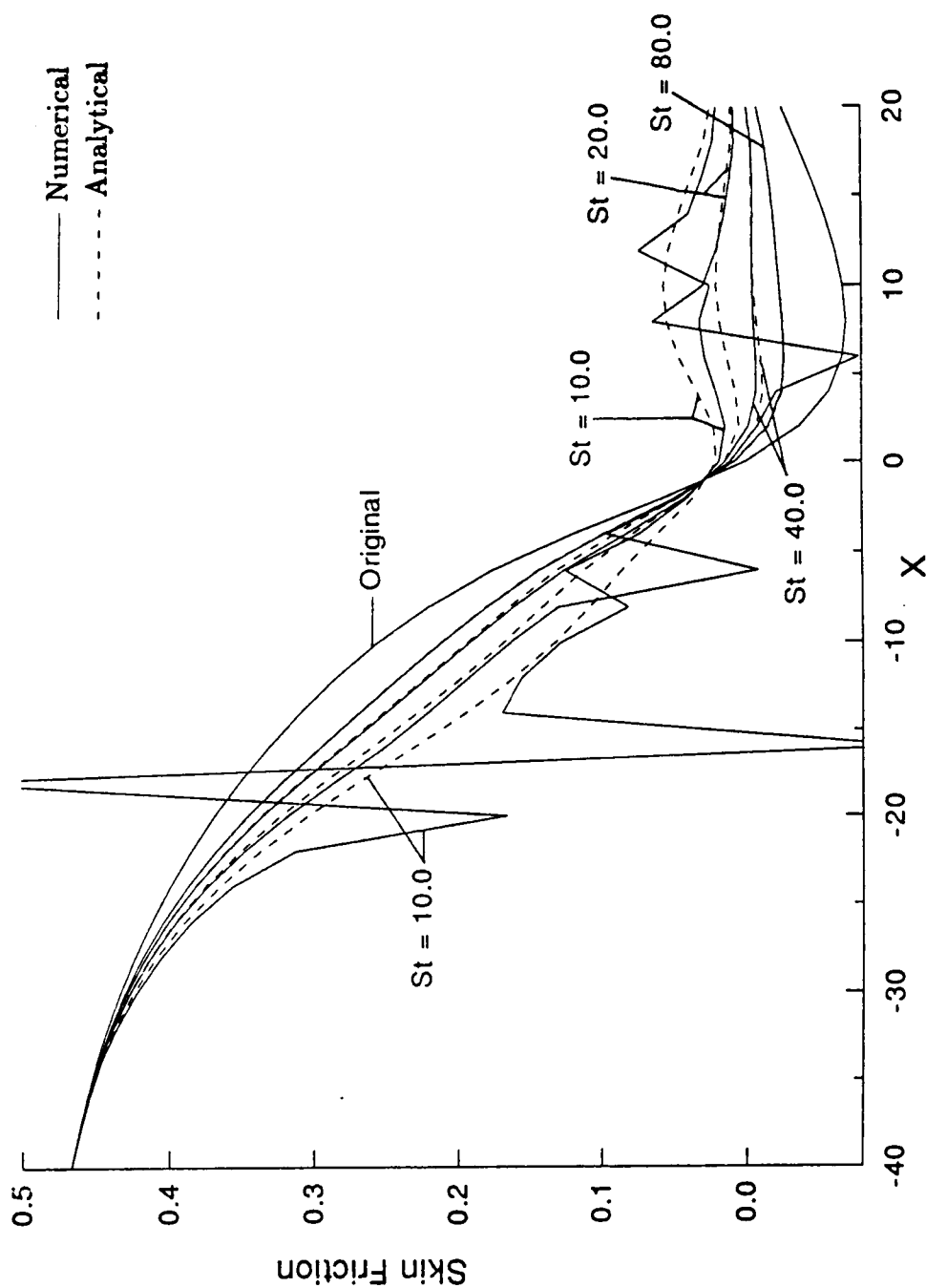
(b)

Figure 14. Assumed amplitude functions. (a) $A_1(X)$, ideal for large favorable streaming effect. (b) $A_2(X)$, more local and realistic. Only cases with $L_s = 0$ are shown in the figure.



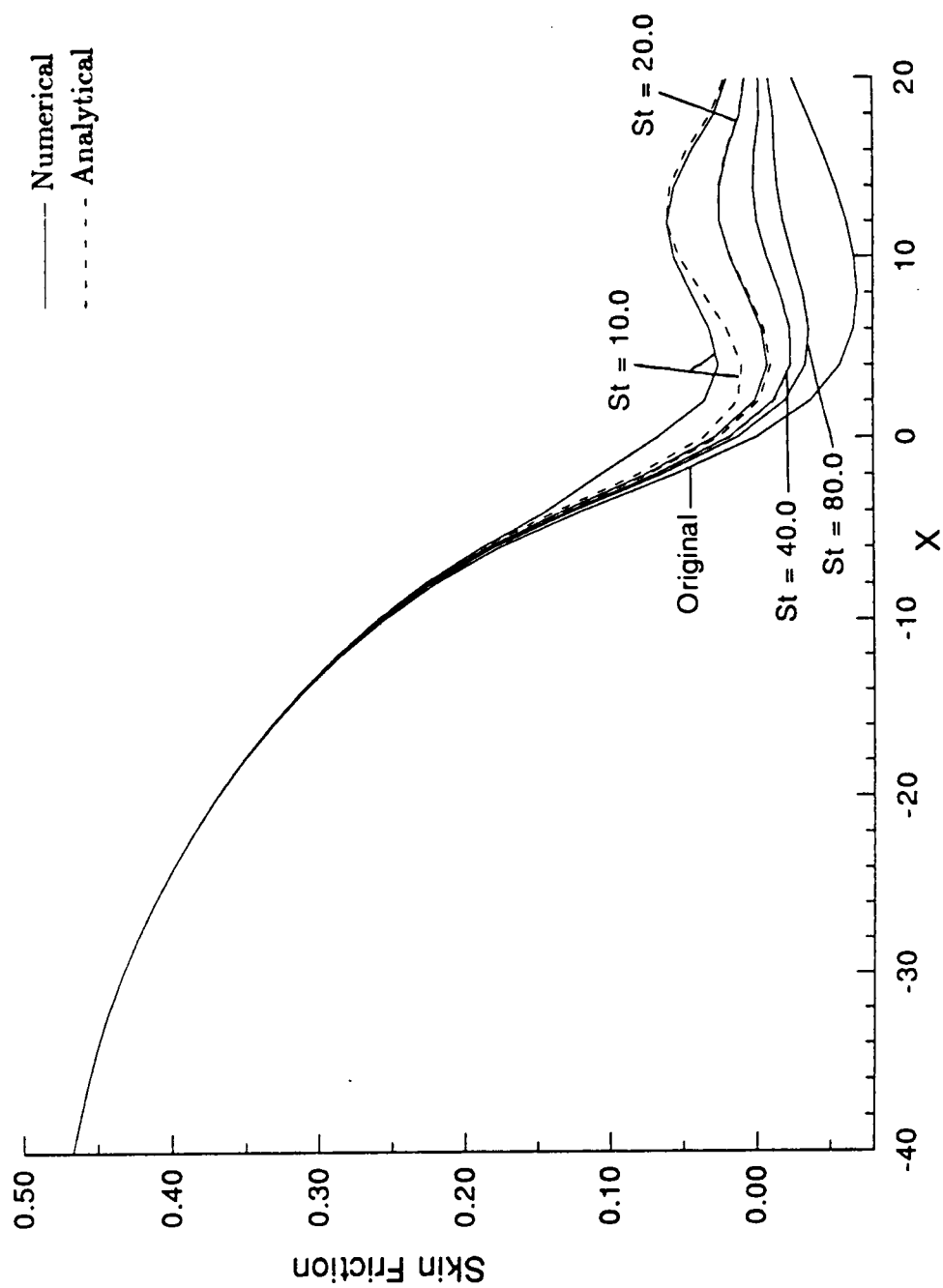
(a) $A_2(X)$ with $B = 20.0$, $L_s = -1.0$; $\eta_\infty = 5.0$.

Figure 15. Streaming effect on the skin friction of breakaway separation ($Re = 2.0 \times 10^5$).



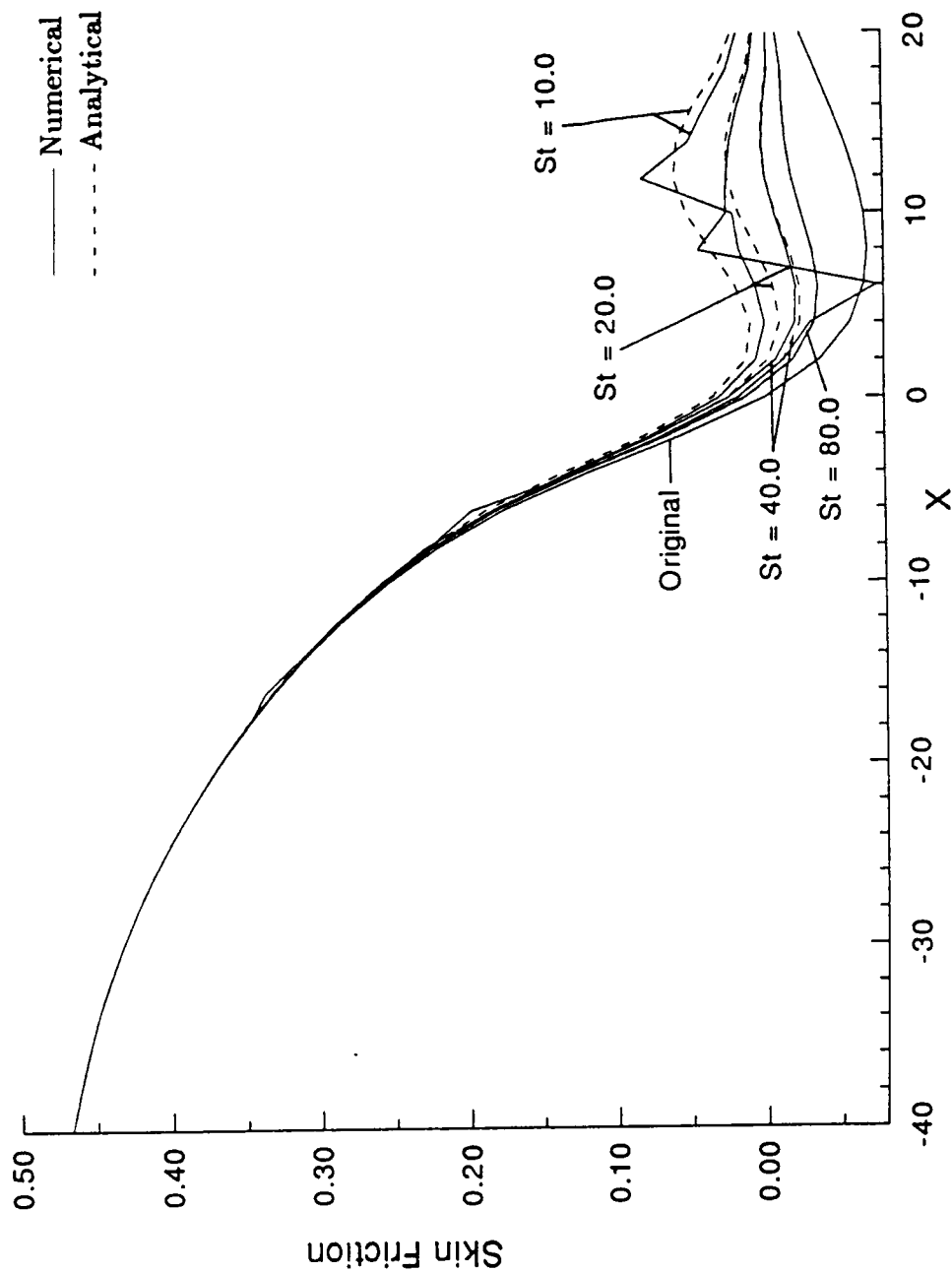
(b) $A_2(X)$ with $B = 20.0$, $L_s = -1.0$; $\eta_\infty = 10.0$.

Figure 15 (continued).



(c) $A_1(X)$ with $B = 10.0$, $L_s = 15.0$; $\eta_\infty = 5.0$.

Figure 15 (continued).



(d) $A_1(X)$ with $B = 10.0$, $L_s = 15.0$; $\eta_\infty = 10.0$.

Figure 15 (continued).

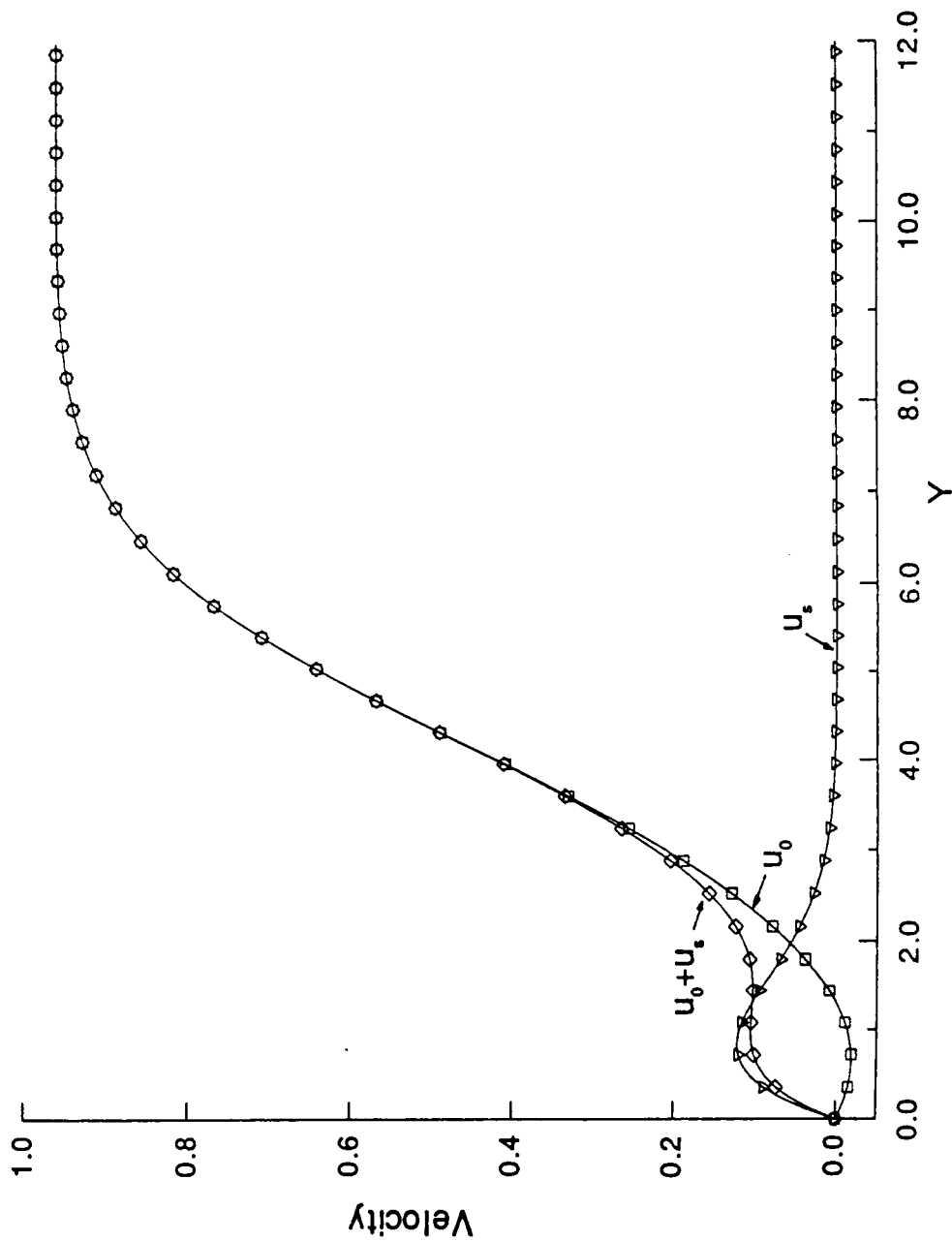
15b,d respectively. It turns out that this selection of numerical outer boundary has quite strong effect to the calculated results. Figure 15a shows very good agreement between the numerical and analytical solution exists even for $\alpha = 0.97$ ($St = 20$). By contrast, in Fig. 15b an x-wise oscillation of the time-averaged skin friction curves for low α (0.69, 0.97) appears although for higher α (1.38, 1.95) the analytical solution still agrees well with the numerical one. Through a series of numerical experiments, we found that the above x-wise oscillation for smaller α is only due to the negative u_{0x} term in the second order (streaming) equation (3.24) (interestingly, the same term in the first order equation (3.22) has no "bad" effect), and the problem can not be attributed to physical or numerical reason but most likely comes from the invalidity of the Taylor expansion for large η when α is small (0.69, 0.97). For larger α (1.38, 1.95), the oscillating layer is thinner so that the Taylor expansion is valid for larger η (> 10), and hence, the oscillation did not occur. Although, it can not be determined whether $\eta = 5$ is large enough, however, this finite boundary was found at least to be good enough for computing the analytical solution by using the same numerical scheme. Similar results for $A_1(X)$ are shown in Fig. 15c and 15d. As we expected, only favorable streaming effect appears in Fig. 15c in contrast to both unfavorable and favorable effects shown in Fig. 15a.

It is clear from (3.35) that the streaming effect increases with Re while decreases with St , and from Fig. 15a to 15d, this trend seems to hold for a wide range of α rather than limited to $\alpha \gg 1$. In order to confirm this and to extend the solution to smaller α range without the above oscillating trouble, therefore, the streaming effect based on the separation bubble model was computed, which is presented below.

2. Bubble Separation

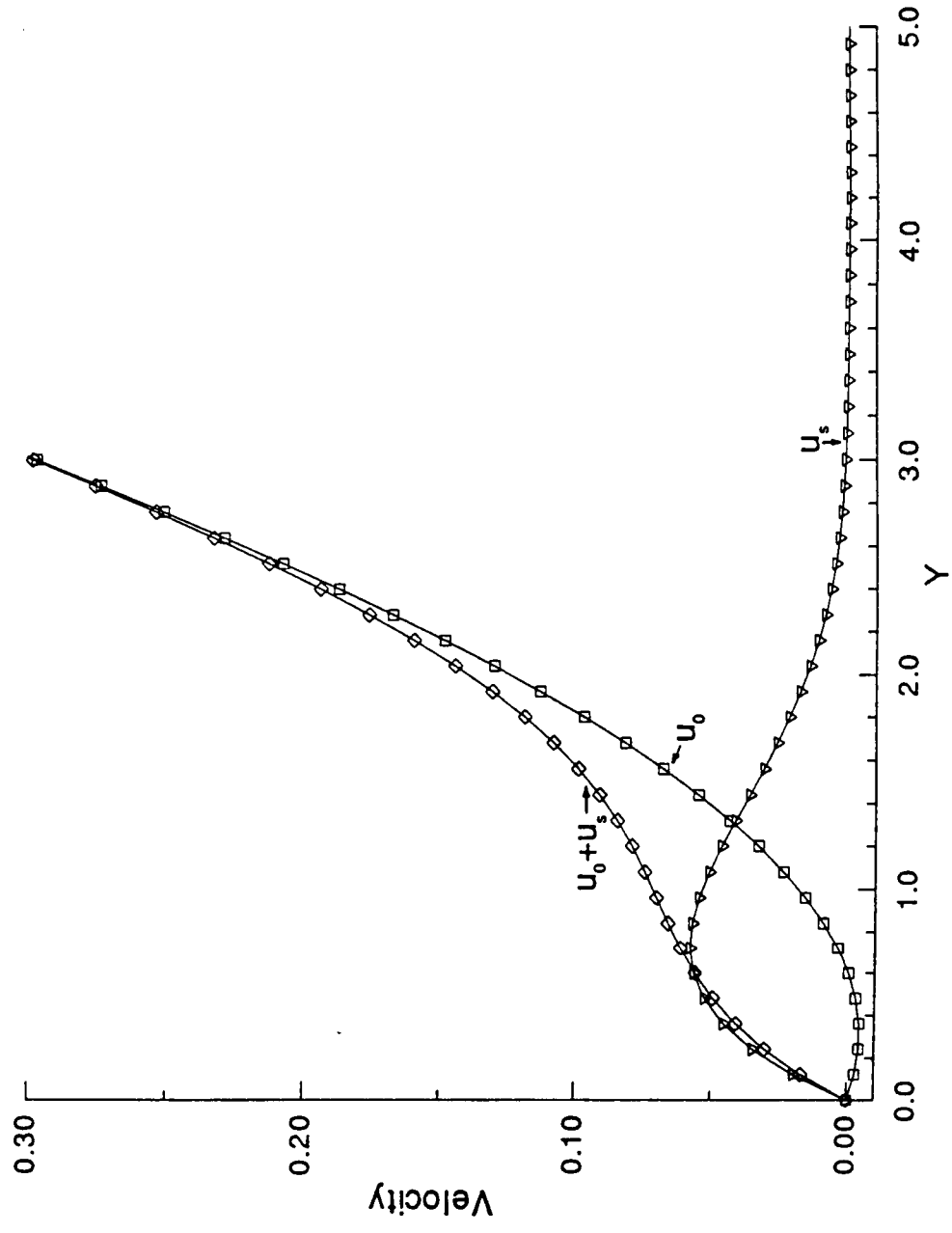
For the bubble separation we present the streaming effect by showing the change of velocity profiles (Fig. 16) as well as the skin-friction curves (Fig. 17). Figure 16a gives the numerically solved streaming velocity for $X = 10$ at $Re = 2.0 \times 10^5$ and $St = 10$ ($\alpha = 0.69$), and $A_2(X)$ with $L_s = 0$, $B = 10$ is used. We see that the streaming changes basic steady separating velocity profile to a non-separating one. Similar result for $Re = 2.0 \times 10^4$ and $St = 5$ ($\alpha = 0.648$) is shown in Fig. 16b. The maximum displacement thickness $\delta_{\max}^*$ used for determining the bubble is 4.5 in both Fig. 16a and 16b. Based on the same bubble, we calculated the skin friction curves with different Reynolds and Strouhal numbers, which are shown in Figs. 17a and 17b. Note that good agreement between analytical and numerical results for higher α similar to that in Fig. 15a can be observed, and there is no x-wise oscillation in the curves.

From these figures we may conclude that the streaming effect may indeed cause fundamental change in the separating flow. The breakaway separation can be delayed and the small separation bubble can be totally eliminated. The streaming effect is stronger as α decreases. To see this more clearly, in Figs. 18a and 18b, the dependence of the skin friction on α is plotted for four different X locations inside the bubble. Note that because when $\alpha = 0$ there is no streaming at all, there must be a maximum streaming between $\alpha = 0$ and $\alpha = 1$. Such a maximum represents the optimum oscillating effect, and was indeed captured for one X location ($X = 1.96$). From Figs. 17a and 17b we see that this location is quite close to the separation point. Unfortunately, the maximum point of $\tau_s \sim \alpha$ curves for other X locations seems to appear at too low α , for which the Strouhal number St is no longer much greater than one (if Re is not assumed unreasonably high) and our model equations are not applicable.



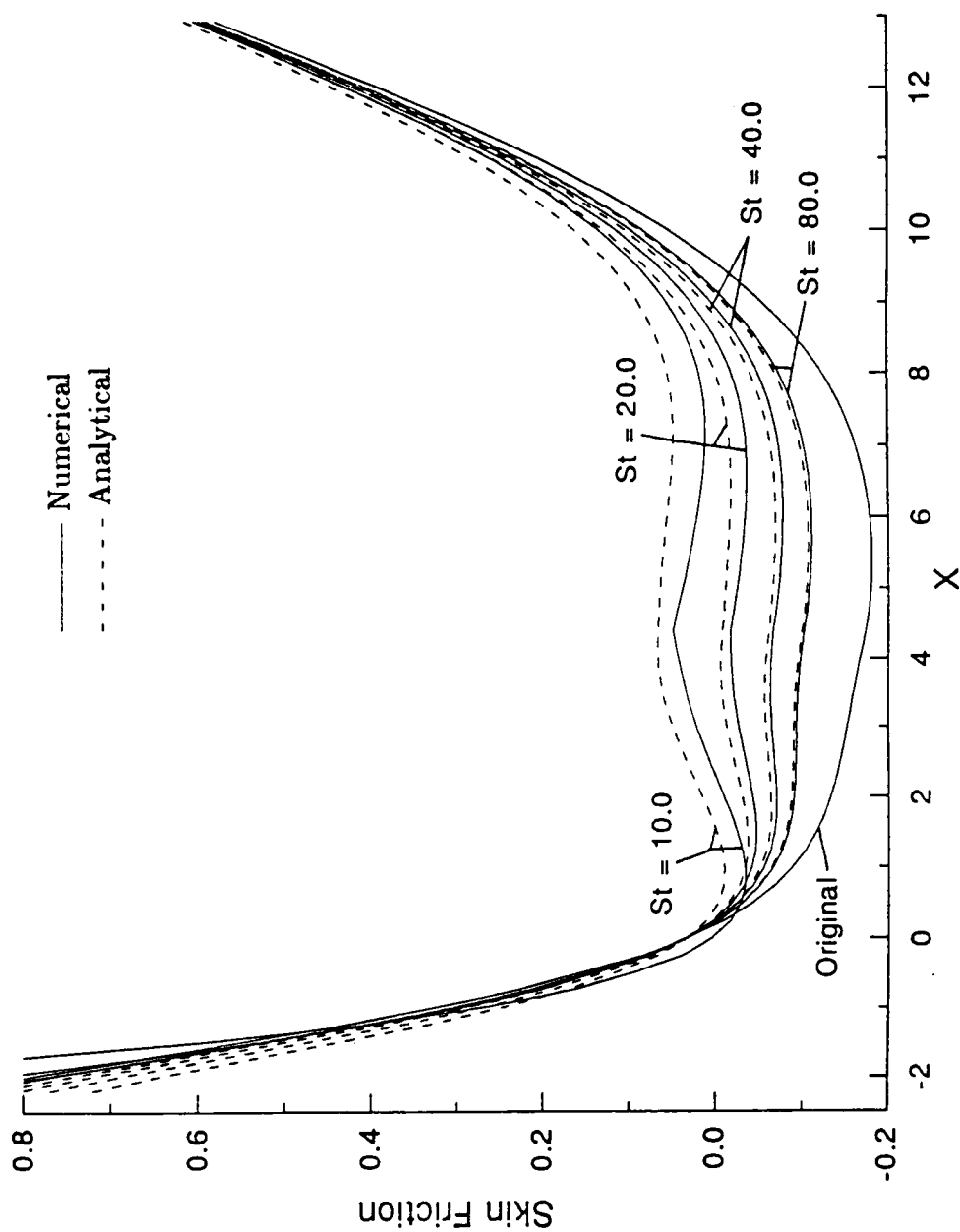
(a) $Re = 2.0 \times 10^5$, $X = 6.13$, $B = 10.0$, $L_s = 0.0$.

Figure 16. Streaming effect on the basic steady separating velocity profile (with $A_2(X)$).



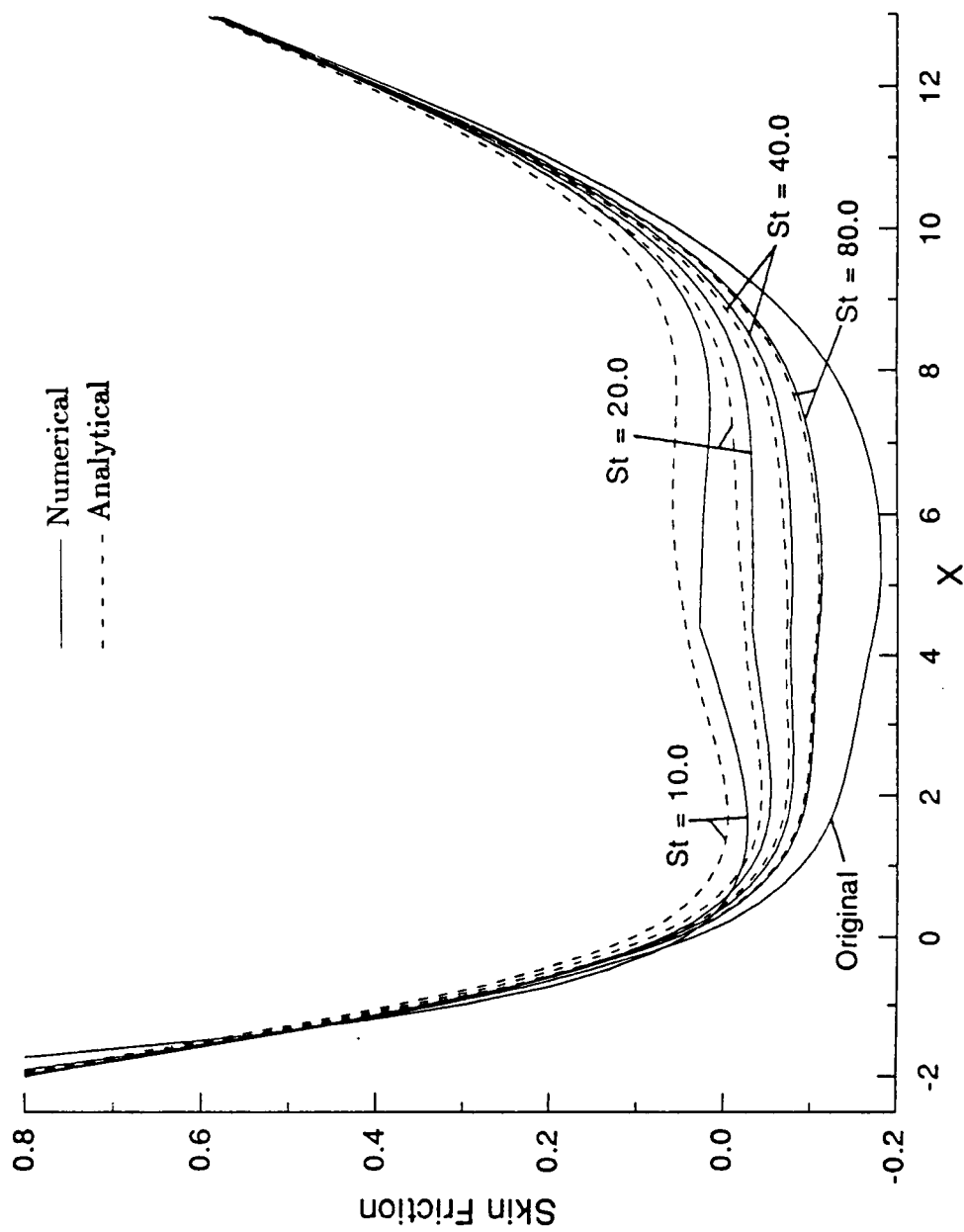
(b) $Re = 4.0 \times 10^4$, $X = 4.69$, $B = 10.0$, $L_s = -1.0$ (enlarged).

Figure 16 (continued).



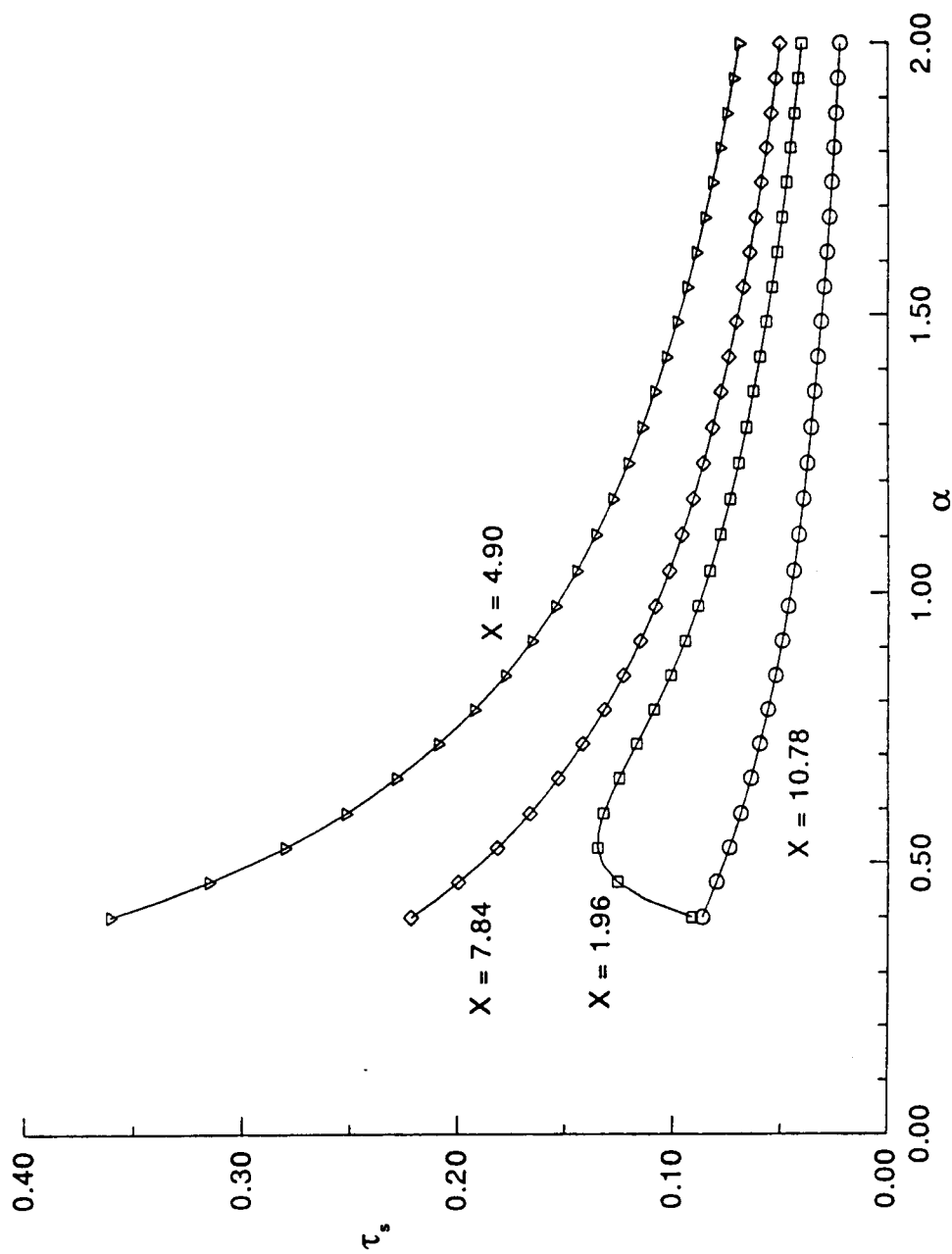
(a) $A_2(X)$ with $B = 10.0$, $L_s = 0.0$.

Figure 17. Streaming effect on the skin friction of separation bubble ($Re = 2.0 \times 10^5$).



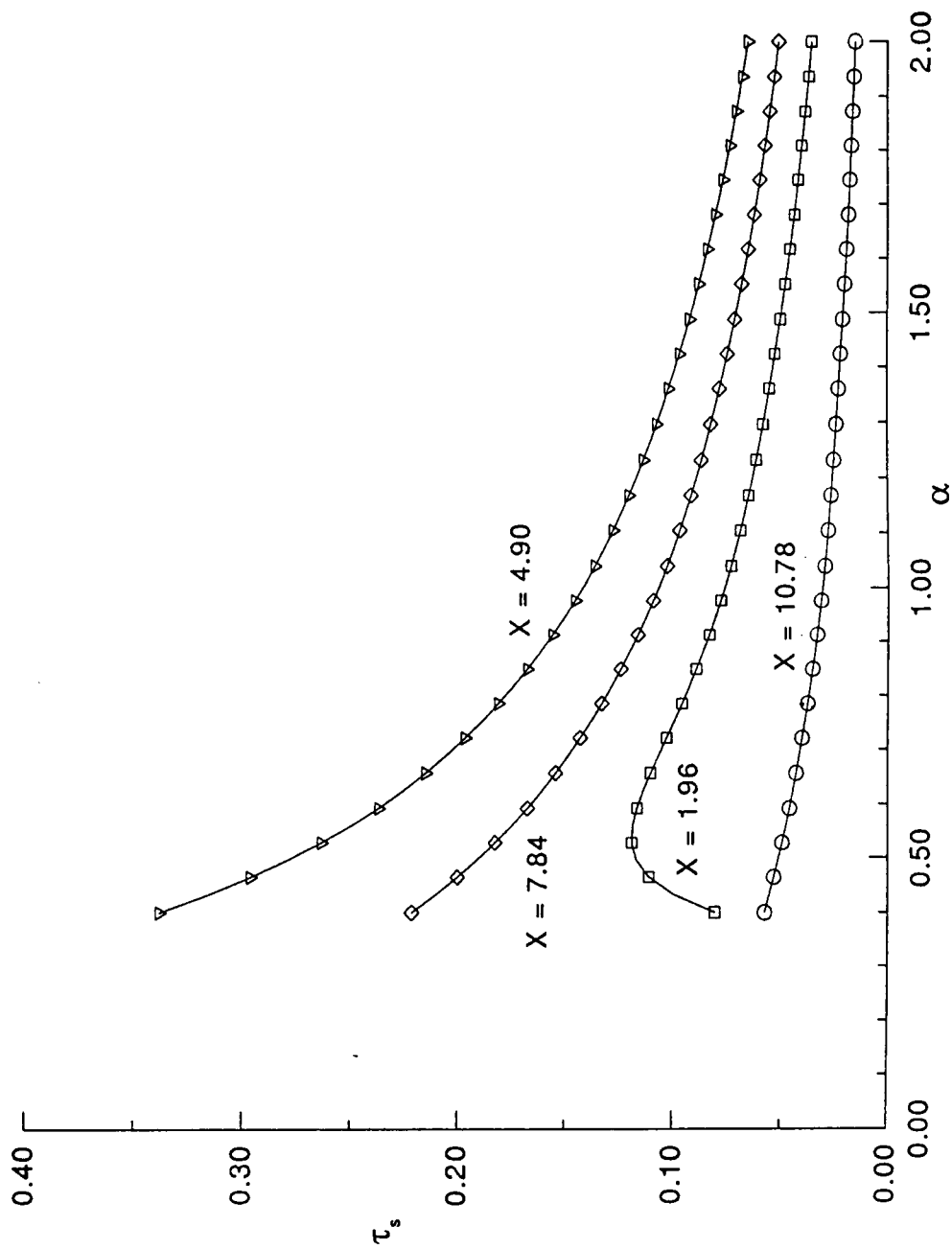
(b) $A_1(X)$ with $B = 5.0$, $L_s = 7.5$.

Figure 17 (continued).



(a) $A_2(X)$ with $B = 10.0$, $L_s = 0.0$.

Figure 18. Effect of α on $\tau_s = \partial u_s / \partial Y$ ($Re = 2.0 \times 10^5$).



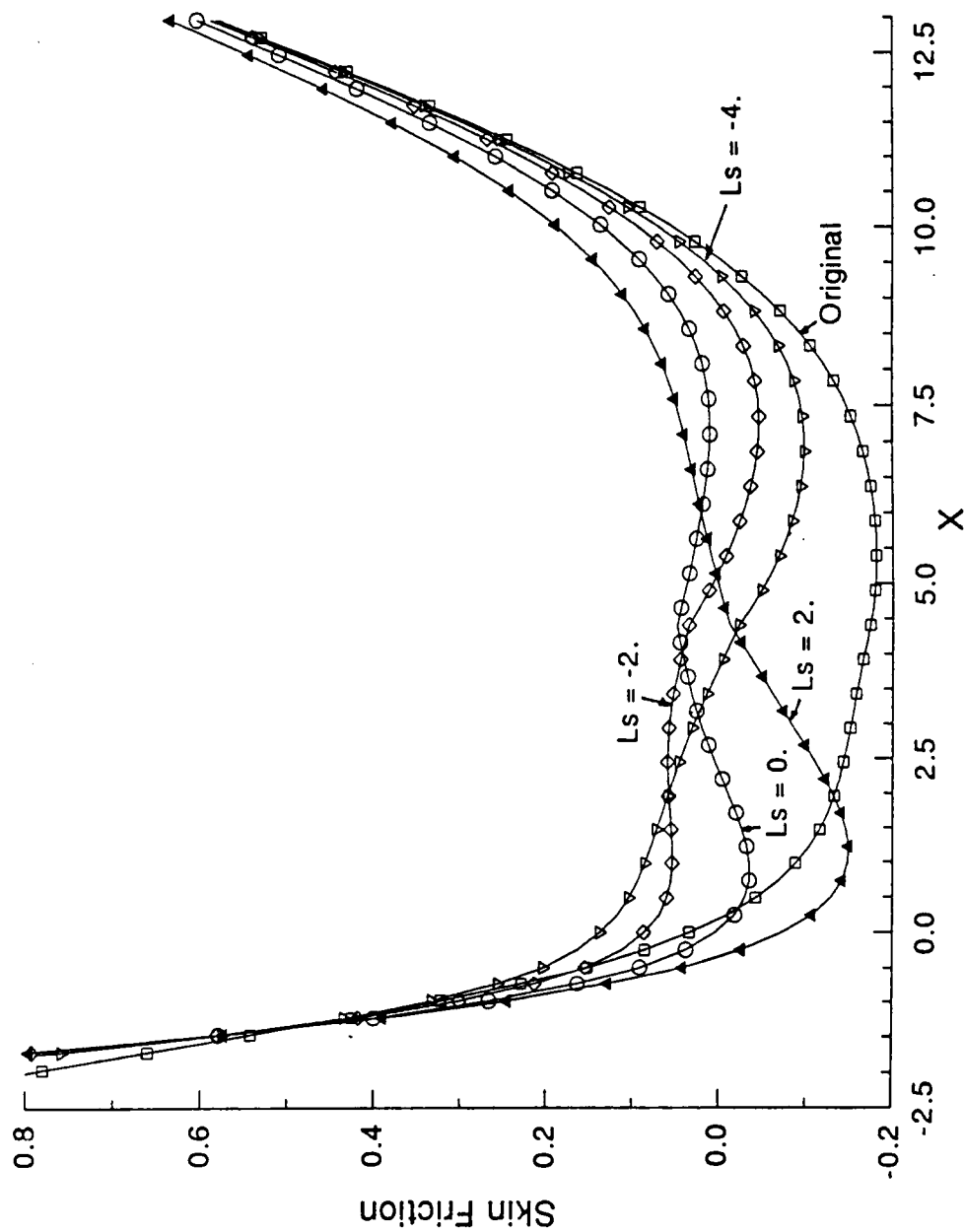
(b) $A_1(X)$ with $B = 5.0$, $L_s = 7.5$.

Figure 18 (continued).

It needs to be pointed out that the peaks in the $\tau \sim \alpha$ curves may occur with different α values at different X locations. These peaks represent only the local maximum streaming effect, but the overall maximum streaming effect on the whole body and the corresponding optimum α should be determined by other criteria, such as the changes in the lift or drag coefficient due to the streaming. As mentioned in Chapter 2, the maximum streaming may be achieved under the resonance state [54]. This is a very important and interesting problem, however, it is difficult to analyze in the present theoretical model because the long wave limit is used.

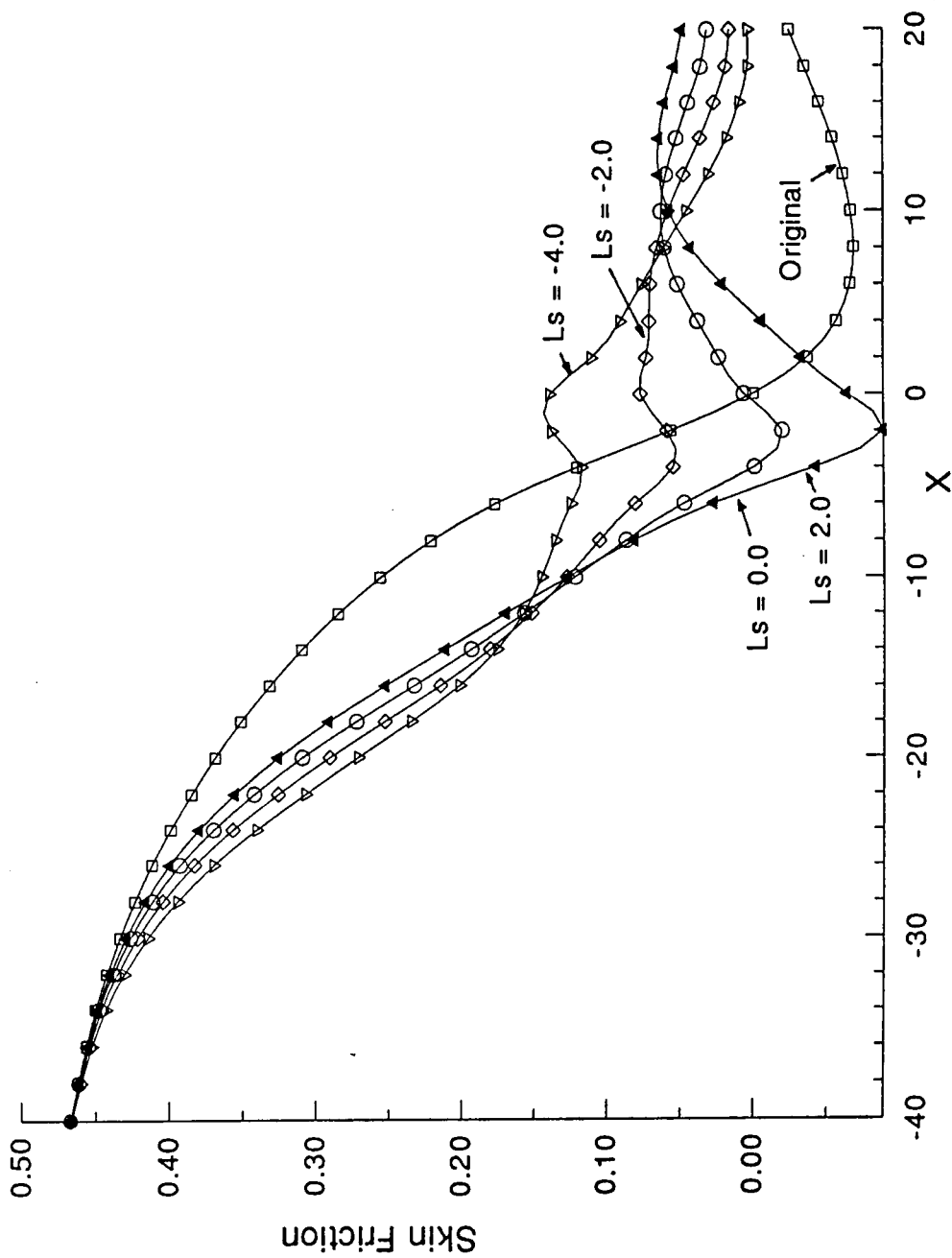
The influence of exiting location L_s is shown in Figs. 19a and 19b for the bubble and breakaway separations respectively. At certain locations, excitation may delay separation while at some others it may eliminate a given steady separation bubble or change the breakaway separation to a local separation bubble. It is seen that the results are strongly dependent on L_s .

The calculated results are also highly dependent on B which determines the slope of $A(X)$ and the range of the excitation. We have seen from the above figures that the better separation control is associated with larger values of B (but not too large to give necessary sharpness of $A(X)$'s slope). In fact, for bubble separation, if B is of order 10, the forcing excitation range becomes about the same as the bubble length, and thus no longer be of local type. However, we have to remember that in our theory the downstream effect of the local excitation is neglected; it is this neglect that reduces the original governing equations to ODEs in which the streamwise coordinate X appears only as a parameter as we mentioned before. Therefore, it is reasonable to expect that the streaming effect is over-localized in the present study, and in reality the required range of forcing excitation for achieving effective separation control can be further reduced.



(a) $A_2(X)$ with $B = 10.0$ (bubble separation).

Figure 19. Effect of the excitation location L_s ($Re = 2.0 \times 10^5$, $St = 10.0$).



(b) $A_2(X)$ with $B = 20.0$ (breakaway separation).

Figure 19 (continued).

CHAPTER V

CONCLUDING REMARKS AND RECOMMENDATIONS

On the physical mechanism underlying the separation control by forcing waves, the streaming effect on the breakaway and bubble separation is studied. Under the assumption of $St \gg 1$, the governing equations can be simplified to ODEs, from which the analytical results are obtained. Then the problem is closed by adding simplified boundary conditions at the upper boundary of the oscillating layer and solved numerically. The acquired analytical and numerical results have provided a useful understanding of the streaming effect on the separating flow. These are summarized below:

1. It has been confirmed that, under the proper forcing frequency and the distribution of forcing amplitude, the streaming effect can suppress or delay the separation for both bubble and breakaway separation cases.
2. The important parameter $\alpha = (St/Re^{1/4})^{1/2}$, the ratio of the lower deck thickness to that of the oscillating layer, plays crucial roles in separation controls. In fact, under fixed Re , α determines the nature of the excited flow. For $\alpha \gg 1$, the oscillating flow is confined in the lower deck and the streaming effect is independent of the basic steady flow to the leading order. While for $\alpha \lesssim 1$, once the oscillating flow gets into the main deck, it will be further convected upward by the strong upwelling velocity near the separation point into the outer shear flow, where the amplified vortical waves couple with the instability wave. Thus, the interaction between the streaming and instabilities makes the separation control problem even more complicated than that generally understood in the literature. And for the same reason, the

boundary conditions of the oscillation and the streaming equations can not be determined completely without the knowledge of the outer shear flow. Here we introduced the simplified local boundary condition such that the feedback effect of outer instability waves was ignored.

3. The importance of α can also be seen from the fact that the streaming effect increases as α decreases for $\alpha \gtrsim O(1)$, and that in the range of $0 < \alpha < 1$, there exists a maximum streaming effect for a certain optimum α . The frequencies corresponding to the optimum α are in qualitative agreement with relevant experiments.
4. It has been shown that the streaming effect is strongly dependent on the location of forcing and the local variation of forcing amplitude, which can be attributed to the over localization of the theoretical model used in this study. The negative slope of the forcing amplitude generates the favorable streaming effect, like the Rayleigh streaming law, and the strength of the effect is proportional to the magnitude of the slope.

The results obtained so far are still preliminary, however, they give physical insights to the streaming phenomenon. In fact, the present work is the first attempt of solving the streaming effect on the separating flow. Further works should be done to complete the study and extend it to more general and practical situations, including the low frequency excitation cases. The recommended study includes the following topics:

1. The present theoretical work can be further extended to include the study of instabilities of the outer shear flow and its interaction with the streaming effect. This is important not only for completing the present theoretical study, but also for practical purposes, because in practice it is hoped that the streaming is coupled with the instabilities of the flow so that it can be

amplified. By the same reason, the coupling between the streaming and the resonance effect should also be studied.

2. It is highly desirable to use the full triple-deck numerical solution of the basic steady separation to replace the Taylor expansion in the calculation of breakaway separation, though, the matching of solutions in the three decks may be difficult.
3. The global changes in the separating flow structure, including the downstream effect, could probably be solved more efficiently by direct appealing to the numerical methods for the Navier-Stokes equations. The numerical methods can also be used to study the strong nonlinear streaming under larger amplitude forcing which is beyond the ability of perturbation method, but is often encountered for separation control of high Re and turbulent flow with large angle of attack.
4. No experimental work specifically addressing the streaming effect on the separating flow has been done. It is highly desirable and important for practice, and may provide further directions in the study of the streaming phenomenon.

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