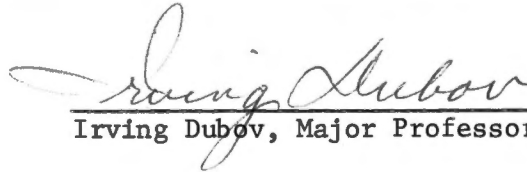
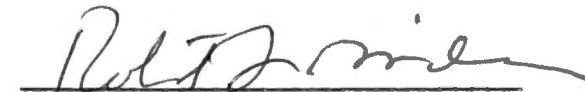


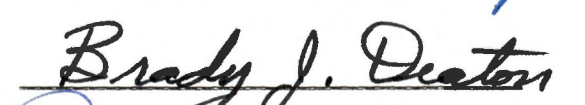
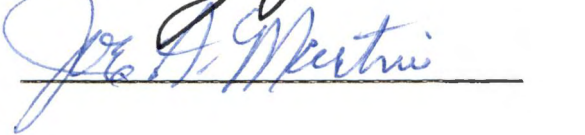
To the Graduate Council:

I am submitting herewith a dissertation written by Ronald D. Weddel entitled "Residential Location Model to Minimize Service Delivery Costs on the Rural-Urban Fringe." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Agricultural Economics.


Irving Dubov, Major Professor

We have read this dissertation
and recommend its acceptance:



Accepted for the Council:


Vice Chancellor
Graduate Studies and Research

1

RESIDENTIAL LOCATION MODEL TO MINIMIZE SERVICE DELIVERY COSTS
ON THE RURAL-URBAN FRINGE

A Dissertation
Presented for the
Doctor of Philosophy
Degree
The University of Tennessee

Ronald D. Weddel

August 1974

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Last, but not least, thanks to my wife, Brenda, whose love, understanding, and mere presence made it all worthwhile.

ABSTRACT

The purpose of this study was to determine a least cost pattern of residential development in terms of the delivery of three services--sewers, solid waste pickup and primary and secondary education. West Knox County, Tennessee, was chosen as the general study area because of its rapid suburban growth and accessibility of data. Five tracts of land were selected as potential service demand sites. Fifteen service origins, including sewage treatment plants, primary/secondary schools, and a solid waste transfer station were designated.

The primary means and procedures used in data assembly for this research involved economic engineering analysis. Data from the Knoxville-Knox County Metropolitan Planning Commission were heavily relied upon, as well as data from the East Tennessee Development District and other local government agencies. Personal interviews also provided a major portion of needed information. A cost minimizing linear programming model was used to allocate population in a minimum cost residential pattern.

The major findings of the research were that medium and high density residential development resulted in lowest cost of providing services. The difference in cost between medium and high density development was, in some instances, slight. Differences in family size between these two densities account for this situation. Average family size for medium density was three persons per dwelling which meant an average of 24 people per acre. Average family size for high density was 1.6 persons per dwelling, indicating an average of 25.6 persons per acre. Average

cost would not differ a great deal under these circumstances if total costs are similar.

The linear programming model has limited use for analyzing overall land-use problems. It can deal only with linear or straight line relationships and with the quantitative aspects of a development problem. It worked in this research because the number of alternatives were small and the variables considered were limited to costs associated with three services--sewers, solid waste collection, and education. Policymakers at the local level might be able to use such research, however, to guide them in evaluating alternatives for residential development with respect to providing certain services.

Policymakers need reliable data upon which to base land-use decisions. Budget information is necessary to determine the costs of various aspects of residential development on the rural-urban fringe. Partial budgets were developed in this research to use in a residential location model. Much more detailed budgets are needed, as well as information on the variables of density and family size. Future research should concentrate efforts in data collection and development of budgets. Once in possession of reliable information, researchers can develop mathematical models to analyze the information.

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CHAPTER I

RESIDENTIAL DEVELOPMENT

I. INTRODUCTION

Both the direction and extent of growth in the suburbs have taxed resources of local governments. The move to metropolitan area fringes was prompted, in part, by desire to escape the crowded conditions of the intercity and to gain access to open space.

Knox County Topography

Open country is still available for residential development in Knox County. The topography of this area is characterized by parallel ridges and valleys in a Northeast-Southwest direction. The differences in elevation between the ridges and valleys range from 180 to 400 feet. Steep narrow slopes have limited the spread of urban development, and the hilly terrain has restricted development to relatively low density uses. Lakes and rivers further imposed barriers to growth. The soils in the county are primarily clay produced from a weathering of underlying formations of limestone, dolomite, and shale. Such soil is very poor for septic tanks. Approximately 65% of the total land area is rated unsuitable for septic tank drain fields (26). This has meant that only very low density dwellings could be supported in the absence of public sewer systems.

Residential Sprawl

Growth in the western portion of Knox County from Knoxville to the county line has proceeded rapidly in the past few years. Most of the

residential construction was low density single family dwellings until recent times. This type of building uses significant amounts of land so that open space and farm lands have been converted to other uses. Even though the terrain exhibited by Knox County limits somewhat the spread of urban growth, sprawl is nevertheless obvious in the western part of the county.

Two kinds of sprawl

Sprawl has taken on two forms. The first form is that represented by low density, large lot residential areas. Especially where septic tanks have to be used, lots of one acre or more are often required for adequate drainage fields. Tremendous amounts of land are taken up by this form of sprawl. Much of the land in West Knox County will not support septic tank systems in anything except extremely low density areas.

A second form of sprawl and more prevalent is that represented by skip development. This happens when land is skipped over for some reason; usually the landowner speculates that the price of land will go up. The developer, rather than pay what the landowner wants, goes farther out to find cheaper land. Such leapfrog development renders useless (e.g., for farming) much of the intervening vacant land. Services are more expensive, however, because sewers, and other utilities and roads have to cross or go by the vacant land to the development farther out. Both forms of sprawl mean inefficient use of the land.

Sprawl in West Knox County

Residential sprawl of both kinds has taken place in West Knoxville-Knox County. Since public sewers were not available when some subdivi-

sions were built, larger lots were needed in order that septic tanks and drain fields be provided. More obvious is skip development. It can be assumed that much of the land that has been passed up was too expensive at the time. In other words, the developer had alternative tracts of land to consider, and the more distant land was chosen for development.

Service costs with sprawl

Provision of services is made more difficult and much more expensive when residential areas are spread out. Since the cost of sewerage a subdivision is a function of the length of sewer lines needed, among other factors, the greater distance between houses in a large lot subdivision means sewers are provided at greater expense. The suburban homeowner rarely pays this full cost. Neutze, writing about land-use problems, said that the major defect in utilities policies was that users did not pay the full capital cost of services. "Instead of charging the costs of mains and arterial streets to actual beneficiaries, either through capital charges or special taxes on the properties actually served, the charges are too frequently averaged out over a very large number of properties," (40, p. 138). Neutze contended that the yearly averaging and front-foot kind of charging systems encourage urban sprawl (39, p. 140).

Noncontiguous subdivisions, those that have vacant land between them, extend the area over which public services are supplied. Costs of busing children to school are increased by residential sprawl. Solid waste collection is made more costly by sprawl. Hirsch has found that the difficulty of collecting garbage depends upon the location of pickup stations--the closer pickup locations are to one another, the less time collection crews spend moving from one location to the next (20, p. 486).

Service costs with density

A study of the economics of open space preservation, mentioned by Clawson, provided insight regarding economics of more closely spaced settlement (9, p. 157). Under the open space program, suburban development was to be confined and the urban service area of the region would include 1114 square miles. Without the program, the service area would include 1441 square miles. It was estimated that savings per square mile, in investment cost, would total \$1,887,000 with the open space program. Clawson estimated that full subdivision contiguity could reduce public service costs appreciably, and additional savings from higher densities within developments might equal or even exceed cost reductions resulting from contiguity (9, p. 158). Densities within residential subdivisions have the greatest impact on the cost of sewer service, according to Downing's work. In his study, the economics of higher densities within the subdivision were substantial at all distances from the treatment plant (11, p. 103-11).

Higher density development does mean more efficient land use, but increased building of multifamily units can lead to difficulties as evidenced in West Knox County. Apartments and townhouses have been built where single-family residences once dominated the neighborhood. Services and public facilities previously adequate have become overburdened by the advent of multifamily structures. As an illustration, roads which were originally built to accommodate low density development are inadequate to handle the increased traffic generated by multifamily complexes. The same can be said for other services. Services become more expensive or their quality deteriorates. Had there been meaningful overall planning for this area, many of the problems could have been avoided.

Combinations of different densities on a single tract are possible if planned. Mixed density developments have gained acceptance and have become quite popular for recreational or resort communities. These developments could have wide ranging appeal. Single family dwellings would attract larger families, and higher density apartments would attract singles, young married couples, and older couples with no children at home. An added advantage of mixed density residential areas would be the cost savings in providing community services, such as sewer services.

Knox County Development in the 1970s

Residential construction

Following World War II and 20 years or so thereafter, living in the suburbs meant living in a single family home on a large lot. In a good many metropolitan areas in the South, apartment construction has recently assumed the lead in building. Tastes and preferences of the housing consumer have been changing. Helping in this change has been the increasing and enormous rise in the cost of constructing houses. Land and building materials have increased tremendously in price in a very few years. Multifamily dwellings for owners and renters have taken a greater share of residential construction in recent years. For example, in 1970, Knox County (outside the city of Knoxville) had 1140 housing units authorized by building permits. All were single family dwellings. In the first six months of 1973, 86.5% of the building permits issued were for multifamily dwellings. The total number was just under 4200 units. Within the city, the percent of multifamily construction increased from 58.4% in 1970 to 87.3% for the first half of 1973 (41, p. 1).

Age distribution of the population

As the recent trend in residential construction has been changing, other relevant factors have also been changing. The age distribution of the population in Knox County was a factor to consider. Two groups, ages 20-34 and 60 and over, account for most of the demand for multifamily residences, according to the Metropolitan Planning Commission (MPC). These two groups made up less than 32% of the population in 1960. They accounted for over 36% of the population in 1970 (34, p. 2).

II. RESEARCH OBJECTIVES

Overall Objective

The objective for this research was to determine a pattern of development which would locate an expected increase in population in western Knox County while minimizing the cost of delivering sewer service, solid waste collection, and primary and secondary education to this population.

Factors to Consider

Determining demand locations

In order to attain the main objective, certain minor objectives had to be met. Potential sites for residential location had to be determined from the existing supply of vacant land in the study area. Each tract of land selected had to be able to accommodate a residential subdivision. The tracts of land and the selected services with existing service supply sites were to become part of a model to meet the overall objective.

Determining an analytical technique

A model was needed which would fit the circumstance of the problem. Given several tracts of land where subdivisions could be located and

given three services to be delivered, a model was sought to minimize costs. Delivery costs for the various services were computed on a per capita basis in order to compare delivery costs among the different subdivisions and services. The costs as considered are only partial costs and reflect transportation costs only. They are not intended to show incidence of costs. Since service delivery costs were to be minimized, a linear programming format was preferred. The number of restrictions which were imposed added to the complexity of the residential model.

III. GENERAL PROCEDURE

Selecting Service Demand Points

The procedure to be followed involved selecting demand points for subdivisions, selecting services and service origin points, determining residential densities to be considered, and estimating delivery costs for the three services. The number of tracts of land selected were limited to keep the problem from becoming too large. Each tract had to be capable of subdivision with some realistic expectation of being developed in the near future.

Data for the demand locations were obtained from the Knoxville-Knox County Metropolitan Planning Commission. A number of parcels of land were vacant in the study area and five tracts were selected with the help of the MPC. Each tract was suitable for residential location, and each had a high probability of being developed in the next few years.

Selecting Services

Sewer service, solid waste disposal, and education were chosen for three reasons. First, control is exercised over each of these

services to some extent by a public agency.¹ Second, these three services are of basic importance to each resident. Septic tanks are often inadequate for sewage, and private open burning of solid wastes, where legal, can only be a realistic alternative in very sparsely settled areas. Regarding education, Clawson believed school location was a critical factor relative to the home. "The school need not be within the subdivision, but the elementary school should be within easy walking distance or with a reasonably short bus ride" (9, p. 143). Closeness of schools is one of the amenities home buyers expect in a subdivision. The third reason for choosing the three services had to do with data availability. The Knoxville-Knox County Metropolitan Planning Commission was an important source of information for all three services.

Residential Densities and Family Size

Three densities of development were considered: two dwelling units per acre, eight dwelling units per acre and 16 dwelling units per acre. Two dwellings per acre was for low density single family residences. Medium density of eight dwellings per acre was for single family units under cluster development or duplexes. Sixteen dwellings per acre was to include higher density row houses, condominiums, or apartments. Where family size is an important factor to consider, changing densities meant that family size estimates changed. Lower densities have traditionally drawn larger families. The high density apartments, generally, attract singles, couples, and, at most, couples with one pre-school age child. Trends are changing, however, and factors such as

¹In the case of sewer service, delivery costs are a mixture of public and private cost depending upon the rules and regulations set by the local government.

declining birth rates, increased costs, the population age distribution, and increased concern with land use might indicate a general move toward higher density dwellings.

Delivery Costs for Selected Services

Sewer service

Delivery cost for each service had to be determined using an economic engineering approach. The cost of sewer service is affected by several factors. Two of the most important factors are distance from a treatment facility and density. Cost varies directly with distance and inversely with density. Other factors affecting the cost of installing sewers are (1) depth of trench, (2) size of sewer pipe installed, (3) type of ground, and (4) sewer pipe material. The slope and size of sewer, roughness of pipe interior, and amount of groundwater seepage affect sewage flow (12, p. 51-3). A further consideration in determining sewerage costs involves the amount of sewage produced by a person or household. A common average figure, and one used in the water-sewer study conducted for the Knoxville Standard Metropolitan Statistical Area, is 100 gallons of sewage produced by a person in a days time (100 gped). Peak flows of sewage are accounted for by multiplying this average figure by a factor of from two to four depending upon the average flowrate in millions of gallons per day (5, p. 11).

When average and peak flows are known, mathematical formulas are available to calculate the size of sewer pipe needed for any number of people or dwelling units. The slope of the land is a factor which is considered in determining rate of flow and pipe size. For many studies and for the present study, minimum slope was assumed.

Distance from the source of sewage production to a treatment facility is a major factor in determining service costs. Where major trunk lines are already in place and adjacent to a potential location, distance was still a factor in determining sewer costs. A rate was determined in order to amortize the capital cost of the trunk line construction. It was then possible to assign sewer service costs to each dwelling or individual affected.

Primary and secondary education

Factors to consider for education are school enrollment, school capacity, number of school age children in the population, and the costs of transporting children from home to school. Enrollment figures were needed for the present and future in order to determine the increase in the number of school children expected for the study area. Capacity figures were necessary to know what excess there might be in area schools.

Number of school age children was a factor needed to be able to allocate children to different locations based on density and family size. More children would be present in a single family than in a multifamily residential area.

Transportation for these children was assumed to be by bus. Both primary and secondary schools were considered for this second service.

Solid waste service

Solid waste collection, for the third and final service, was assumed not to be greatly affected by residential densities. If centrally located refuse receptacles are situated within a subdivision, collection vehicles would have to make only one stop at each central location.

Elements of solid waste disposal costs, such as the expected per capita solid waste generation, were required for the study. The type of collection system to be used, including the nature and cost of the collection equipment, was needed in order to estimate the cost of transporting solid waste from its source to a compaction station.

Having determined the costs involved for all three services under the conditions assumed, a linear programming model was used and adapted to arrive at a least cost service delivery pattern for residential development.

CHAPTER II

THEORETICAL MODEL

I. STUDY AREA

Planning Units

West Knox County, including a small portion of the city of Knoxville, was chosen as the general study area for this investigation because of the very rapid growth experienced by this section in the last ten years. Continued growth is likely for this area in the near future. A second reason for choosing this area concerns the researcher's accessibility to information. Finally, insights obtained from studying this area will be useful in application to other areas that are similar.

The county has been divided into planning units¹ (1) by the Knoxville-Knox County Metropolitan Planning Commission. Three of these planning units, viz., numbers 16, 17, and 18, were selected. They extend from the Knoxville city limits on the east to the county line on the west (see figure 1). The topography of the area is primarily a rolling valley bounded by a series of parallel ridges on the north and the Tennessee River on the south.

Service Demand Points

Potential service demand points were selected and located with the assistance of the Knoxville-Knox County Metropolitan Planning Commission

¹A planning unit includes a relatively homogeneous area of land-use and its boundary lines were drawn to respect natural and man-made barriers while not cutting across other agency divisions, e.g. census tracts.

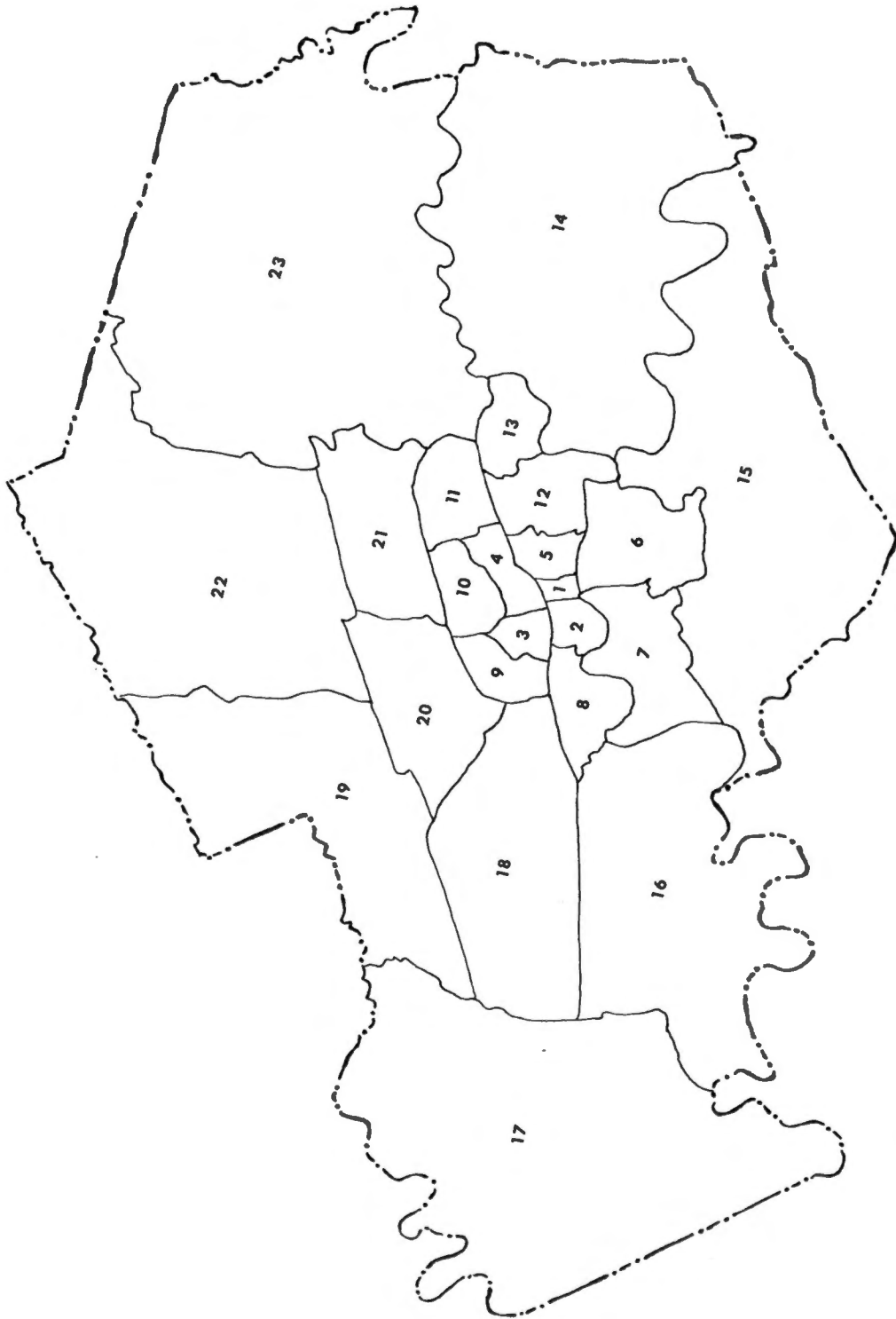


Figure 1. Knox County, Tennessee, with Planning Unit Boundaries, delineated by natural and man-made barriers.

and the Knox County Mapping Service. A service demand point represents a tract of land which has the potential to be subdivided. Five potential residential sites were delineated within Planning Units 16, 17, and 18. Tract 1 was located just north of Interstate 40-75 and west of Weisgarber Road. It was within the Knoxville city limits and contained 18 acres of land. Tract 2 was situated at Northshore and Lyons Bend Road, contained 34 acres, and was also within the Knoxville city limits. Tracts 3, 4, and 5 were outside the city limits. Tract 3, at Middlebrook Pike and Ball Camp Pike, contained 217 acres. Two separate, but contiguous, pieces of land totaling 316 acres formed Tract 4 which was in an area bounded by Northshore and Ebenezer Roads. Available land in Tract 5 totaled 191 acres and was in the Farragut community at Kingston Pike and Concord Road.

The area under consideration has been developing since 1940. Tracts 1 and 2 were in a section of West Knoxville that had developed between 1940 and 1960. The other three sites, in the county, were in an area that has been developing since 1960 (14, p. 161). Practically all this development has been in low density residential units. Only in the 1970s has the majority of residential growth in the county been multifamily building (41, p. 1).

Service Origin Points

The five potential service demand points were served by multiple service origin points (see figure 2). Three treatment plants were involved in sewer service. The Fourth Creek Treatment Plant, completed in 1968, had a designed treatment capacity of approximately eight million gallons per day (5, p. 65). The plant was located on the Tennessee River

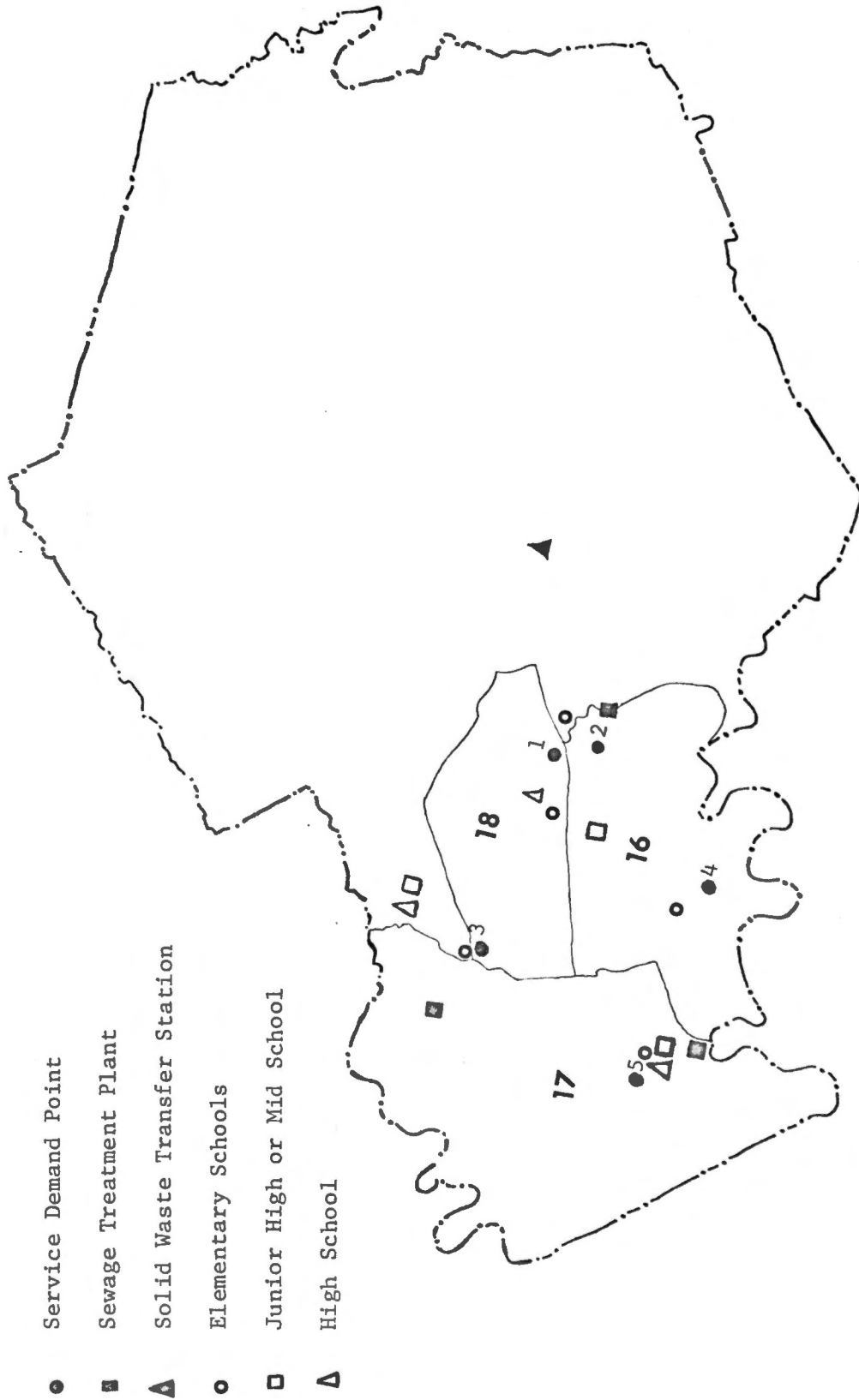


Figure 2. Knox County, showing the location of Planning Units 16, 17, and 18 in Knox County, and the location of five service demand points.

and provided service for Tracts 1 and 2 (within Knoxville) as well as for Tract 4 (in the county). Tract 2 was adjacent to the treatment plant and Tract 1 was on a major trunk line emanating from this treatment facility.

Only one of the five potential service demand points was served by the Byington Treatment Plant. This plant was situated in Beaver Valley and served the communities of Byington and Beaver Ridge and the area along Tennessee Highway 62. Capacity at the Byington facility was 300,000 gallons per day. It provided service for Tract 3 (49).

The third treatment plant also served only one of the five service demand points. The Turkey Creek facility, located near the mouth of Turkey Creek, provided treatment for liquid wastes from Tract 5. This plant had a design capacity of 1.0 million gallons per day (45). There were a total of five potential demand points, each demanding a certain amount of service from one of three treatment plants. Total capacity for all three plants was 9.3 million gallons per day.

Service origin points for education included 11 schools--five elementary, three junior high, and three high. West Hills Elementary served Tract 1. Tract 2 was served by Bearden Elementary School. In the county, Ball Camp, Blue Grass, and Farragut Elementary Schools provided the primary educational services for Tracts 3, 4, and 5 respectively.

One junior high and one senior high school were the service origin points for the two potential service demand points inside the city limits. Bearden Junior High, to be built on a new location at Middlebrook Pike and Francis Road, was referred to as New Bearden Junior High (32, p. 7). Bearden High was a relatively new high school located on Kingston Pike on the western edge of the city.

The new Karns Middle School (planned for 1974) and Karns High School provided secondary education service for Tract 3, situated in the Ball Camp Community. Farragut Middle School and the new Farragut High School served as service origin points for secondary education for service demand points 4 and 5 (29, p. 3).

Solid waste disposal service had only one service origin point to serve the five potential service demand points. All solid waste collected in Knoxville-Knox County must go to a transfer station for compaction before being delivered to a landfill. This transfer station was located in Knoxville and served as the service origin point for solid waste service for all five tracts.

Population Estimates

Knox County's population has increased in the last decade, and a large part of this growth occurred in the western part of the county. Increases in population are expected to continue, and the study area is projected to have an additional 21,000 people by 1980 over 1970. This is almost a 50% increase over 1970 population for the area (31, pp. 1, 2). It was necessary to estimate both the expected population increase for the three planning unit areas during 1973-1978 and the average family size. Depending upon the residential densities used, family size was changed. For low density dwellings, larger family sizes were indicated, and consequently more school age children were indicated.

Density and Family Size

Average family size in Knox County was about three persons per family, according to the 1970 census (7, p. 136). The Knoxville-Knox County Metropolitan Planning Commission (MPC) provided breakdown on

family size. For single family dwellings (low density), average family size was estimated at 3.6 by the MPC (46). Multifamily dwellings, from duplexes to high density apartments were estimated to have an average family size of 1.6 persons (46). For the present research, a distinction had to be made between medium density residential units and high density units. The researcher compared census block data with a map of low and medium density subdivisions in the study area to estimate family size. The results of this analysis are detailed in Chapter III.

II. THE RESIDENTIAL LOCATION MODEL

General Linear Programming Model

Linear programming generally refers to a computational procedure used in prescribing production patterns to maximize profits of firms or to minimize costs of producing or delivering a product (18, p. 4). Straight line relationships are used in linear programming, and the technique involves the maximization or minimization of a linear function subject to linear inequalities.

The mathematical statement of the general linear model for the minimization case was adapted from Hillier and Lieberman (17, pp. 127-28).

$$\begin{array}{ll}
 \text{Minimize:} & Z = c_1x_1 + c_2x_2 + \dots + c_nx_n \\
 \text{Subject to:} & \begin{array}{l}
 a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \geq b_1 \\
 a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \geq b_2 \\
 \cdot \qquad \qquad \cdot \qquad \qquad \cdot \qquad \qquad \cdot \\
 \cdot \qquad \qquad \cdot \qquad \qquad \cdot \qquad \qquad \cdot \\
 \cdot \qquad \qquad \cdot \qquad \qquad \cdot \qquad \qquad \cdot \\
 a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \geq b_m
 \end{array} \\
 \text{and} & x_j \geq 0
 \end{array}$$

where a_{ij} , b_i , and c_j are given constants

The function to be optimized is the objective function. This function is constrained or limited by a series of restricting equations. "The

variables being solved for are called decision variables. Given n competing activities, the decision variables, $X_1, X_2, \dots, X_n$, represent the levels of these activities." For example, if each activity is a set of inputs, then X_j would be the amount of the j^{th} input to be used during a given time period. Z is total cost which is to be minimized. C_j is the increase in cost that would result from each unit increase in X_j . The number of minimum requirements to be met is m ; b_i is the minimum i^{th} requirement for the n activities; a_{ij} is the amount of the i^{th} requirement produced by one unit of the j^{th} activity. The non-negativity restrictions ($X_j > 0$) rule out the possibility of negative activity levels.

Duality

For every solution to a linear programming problem (called a "primal") there exists a "dual" solution. If the primal is a minimizing problem, e.g. cost, the dual solution would be one where, for example, profits were maximized.

Assume a Minimization Primal:

$$\begin{aligned} \phi_{\min} &= \sum_{j=1}^n c_j x_j \\ \text{subject to: } &\sum_{j=1}^n a_{ij} x_j \geq d_i && \text{for } i = 1, 2, \dots, m \\ \text{and} &&& x_j \geq 0 \end{aligned}$$

where c_j is the known cost per unit of input j

x_j is the unknown amount of input j that should be used

a_{ij} is the known number of units of product i that can be obtained from one unit of input j

d_i is the known amount of product i that must be produced

Interpretation: A set of input rates need to be determined "that will minimize the total cost of production for all outputs required, subject to the constraints that the amount of each product produced will be at least as great as the minimum amount required of that product, and the constraints that there can be no 'negative input rates'" (13, p. 3).

The dual of this primal:

$$\begin{aligned} \theta_{\max} &= \sum_{i=1}^m d_i p_i \\ \text{subject to: } \sum_{i=1}^m a_{ij} p_i &\leq k_j \quad \text{for } j = 1, 2, \dots, n \\ \text{and } p_i &\geq 0 \end{aligned}$$

where d_i is the known amount of product i that must be produced

p_i is the unknown "price" of product i

a_{ij} is the known number of units of product i that can be produced from one unit of input j

k_j is the known cost per unit of input j

Interpretation: A set of product "prices" or values need to be determined "that will maximize total returns for all products produced, subject to the constraints that the values of the products that can be made from one unit of each input will be no greater than the known cost per unit of that input, and the constraints that there can be no 'negative price' for any product" (13, p. 4).

Residential Location Model

In the transportation model, movement of goods or services may take place between any origin and any destination. However, the restrictions of the present location problem prevents complete freedom in moving services from all origins to all destinations. The reason being that

bounded drainage areas are defined by topography; sewage treatment plants, located in particular drainage areas serve only subdivisions within these areas. Likewise, school district boundaries determine where children attend school; crossover between districts was not considered.

For the residential model, five tracts of land served as potential demand sites for the services of sewage removal, solid waste removal, and primary and secondary education. Providing these services were three sewage treatment plants, 11 schools (five elementary, three junior high, and three high schools), and one solid waste compaction station. The model was to arrange a projected population increase on five potential sites (each site with three possible density levels) so that the cost of delivering all three services would be minimized.

Subdivision 1 was served by Sewage Treatment Plant 1, West Hills Elementary School, Bearden Junior High School, Bearden High School, and the Knoxville Solid Waste Transfer Station. Subdivision 2 was served by Sewage Treatment Plant 1, Bearden Elementary School, Bearden Junior High School, Bearden High School, and the Knoxville Transfer Station. The third potential subdivision would obtain its services from Sewage Treatment Plant 2, Ball Camp Elementary School, Karns Middle School, Karns High School, and the Solid Waste Transfer Station. Subdivision 4 service origin points were Sewage Treatment Plant 1, Blue Grass Elementary School, Farragut Middle School, Farragut High School, and the Transfer Station. Finally, service origin points for Subdivision 5 were Sewage Treatment Plant 3, Farragut Elementary School, Farragut Middle School, Farragut High School, and the Transfer Station. There was evidently some overlapping, i.e., some service origin points serving more

than one service delivery point. This was limited, however, for the reasons cited above--topography and political boundaries.

The residential model was a cost minimization problem in which distance and the cost of service delivery determined the total cost of providing a package of services to residential subdivisions. Consideration was given to applying the so called "transportation model" but its specifications were not applicable to the problem under study. For example, the transportation model assumes that the supply of products or service from any one origin can move to any destination. Such is not the case for the residential model. Political boundaries and topography prevent the free movement or delivery of services among all possible routings between origins and destinations.

The algebraic form of the residential model used is as follows:²

$$\text{Minimize: } Z = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^t c_{ijk} x_{ijk}$$

$$\text{subject to: } \sum_{j=1}^n x_{.jk} \geq \sum_{j=1}^n d_{.jk}$$

$$\sum_{j=1}^n a_{ijk} x_{ijk} \leq s_i$$

$$\text{and } x_{.jk} \leq d_{.jk}$$

$$x_{ijk} \geq 0$$

$$i = 1, 2, \dots, 15$$

$$j = 1, 2, \dots, 5$$

$$k = 1, 2, \text{ or } 3$$

where m is the number of service origin points

n is the number of service delivery points

²The following model corresponds to the matrix of Table 4-2.

t is the number of density levels

s_i is the available capacity at origin i

c_{ijk} is the cost per year to provide service from origin i to one person located at point j , density k

x_{ijk} is the number of people located and served at point j , density level k , a service delivered from origin i

a_{ijk} is the amount of service capacity at origin point i needed per unit of service delivered to demand point j

$x_{.jk}$ is the number of people located at point j , density k , under the condition that service origins are predetermined

$d_{.jk}$ is the availability of service for demand point j , density k , under the condition that service origins are predetermined

This model was to allocate a population increase to five subdivisions in such a manner that sewers, solid waste pickup, and primary and secondary education could be delivered at the least possible cost. Application of the Parametric Programming routine meant that elements of the basic programming model could be changed to suit the specific requirements of this study.

Parametric Programming--Post Optimal Analysis

It is sometimes of interest to determine the results of simultaneous changes in the objective function or other parameters. A systematic study of these changes in various parameters of a linear programming model is the object of parametric linear programming. This procedure is well suited to sensitivity analysis, and it is also designed for the situation where some flexibility exists in the parameter values, and one wishes to investigate trade-offs in these values.

It may be possible, for example, to increase the unit profit from one activity at the expense of another by shifting personnel and equipment (19, pp. 499-500).

In the present model the interest was in changing the s_i or the right hand side (RHS). The procedure used to change C_j in the objective function is related to changing the S_i . The reason is that changing the S_i is equivalent to changing the coefficients in the objective function of the dual model. The i th constraint then becomes

$$\sum_{j=1}^n a_{ij}x_j \leq s_i + \alpha_i\theta \quad i = 1, 2, \dots, m$$

where α_i are given input constants. The object is to determine the optimal solution as a function of θ . The solution procedure starts with the optimal solution and corresponding set of equations for $\theta = 0$. It then finds the rate at which the RHS of these equations (the value of the basic variables) changes as θ increases by successive steps. If all basic variables remain non-negative for all non-negative values of θ , the corresponding solution will be optimal for $\theta \geq 0$. Otherwise, it remains optimal until some basic variable decreases to zero. At this value of θ , the set of equations is obtained for the next optimal basic solution by using the dual simplex method, choosing the variable that had reached zero as the leaving basic variable (19, pp. 502-3).

The computer program PARARHS is used postoptimally to perform parametric programming on the resource availabilities, the RHS. An explanation of this programming procedure follows in Chapter IV.

CHAPTER III

DATA

I. SERVICE DEMAND POINTS

In order to hypothesize an optimal pattern of residential development, a number of vacant tracts of land were needed. Criteria for selecting land were based on the judgment of the researcher using the knowledge of members of the Knoxville-Knox County Metropolitan Planning Commission. Each piece of land had to have the potential for residential development. What this meant was that each tract had to be: zoned for residential use, reasonably accessible, served by public sewers, and served by primary and secondary schools. In addition, each tract needed to have a relatively high probability of development in the next few years. Five land parcels met these criteria and were therefore selected for this research. The total number of tracts selected was kept relatively small to keep the mathematical model more manageable.

Each piece of land varied in size, location, topography, and shape. It was therefore necessary to make certain assumptions about each piece of land: (1) each tract was assumed relatively flat, (2) each lot (acre) in the tract was assumed to slope from back to the front, and (3) each potential site was assumed composed of square acres, an acre being 208.7 feet on a side. These assumptions were made to facilitate determination of sewer service costs using economic engineering analysis.

II. COST COEFFICIENTS FOR SERVICES

Sewer Service

Past research

In a 1957 study, Isard and Coughlin estimated costs of various public services, including sewerage (22). They assumed three development densities--1, 4, and 16 dwelling units per acre. Estimated total cost of sanitary sewers for a medium density community of 2480 units was \$482,410; for the high density community with the same number of dwelling units, the cost was only \$182,520. Increasing development density reduced sewage costs. (Individual septic tanks were assumed to serve the low density community) (22, pp. 16-17). ✓

In a 1969 study, Downing estimated the cost of sanitary sewer service for an area of 160 acres considering various residential densities. The cost of installed sewer depended on several factors: (1) depth of trench, (2) size of sewer pipe installed, (3) type of ground, and (4) sewer pipe material. Other factors affected sewage flow: (1) slope of sewer, (2) size of sewer, (3) roughness of sewer pipe interior, and (4) amount of ground water seepage into sewer (12, pp. 51-53). Downing's work was similar to earlier studies which indicated that tremendous cost savings in sewer service (in dollars per capita per year) resulted from higher density development. Cost savings from density were also apparent in the present study dealing with West Knoxville-Knox County.

Knoxville's sewer system

The wastewater collection system serving Knoxville is owned and operated by the City. Within the system there are more than 950 miles

of sewer lines varying in size from 8 inches to 72 inches in diameter (most are 8 inches). The collection system was constructed of concrete sewer pipe. The number of connections to the sewer system increased from 28,697 in 1960 to 44,515 in February, 1971. It has been estimated that about 70% of the Cit's population was served by public sewers in 1971 (5, pp. 63-64).

Knoxville operated three wastewater treatment plants. The Third Creek Plant built in 1955 had a capacity for primary sewage treatment of 31.1 MGD (million gallons per day). The present Loves Creek Plant, completed in 1966, had a design capacity of 3.8 MGD. The Fourth Creek Plant, completed in 1968, had a capacity of 8.0 MGD. This plant served portions of the study area (5, pp. 64-65).

The Byington facility, operated outside the city was in the West Knox Utility District system. In a telephone conversation with the utility district engineer, it was found that the Byington Plant will have a designed capacity of 300,000 gallons per day (49). The Turkey Creek Plant, within the First Utility District, was to be completed sometime in 1975. This facility will have a designed capacity of 1.0 MGD (45). The latter three facilities, Fourth Creek, Byington, and Turkey Creek, were the treatment plants considered in this study.

Assumptions for sewer service

The assumptions, previously detailed in the above section on land selection, were made, primarily, for sewer service data building. Land for development was considered relatively flat and sewer pipe slope estimated to be 5 feet for every 1,000 feet. (The State requires a slope of at least 4 feet per thousand). Collection sewers were assumed

to be constructed down the center of streets within the hypothesized subdivisions. Depth of trench, type of ground, and groundwater seepage as they affect cost were reflected in the cost figures from the study conducted by Allen and Hoshall (5, pp. 107-118). The length and diameter of sewer pipe needed for each hypothesized subdivision at each density was determined by using an economic engineering analysis. An average figure of 100 gallons per person per day was used to estimate the amount of sewage produced per day in a subdivision. To account for peak loads, the following table was consulted (5, p. 11).

Ratio of Average Flow to Peak Flow

<u>Average Flowrate (MGD)</u>	<u>Ratio to Convert Average to Peak</u>
0 - 0.99	4.0
1.00 - 1.99	3.0
2.00 - 9.99	2.5
10.00 and greater	2.0

A total figure in gallons of sewage was determined for each subdivision at each density level. This figure was then converted to cubic feet per second for the purpose of determining pipe sizes. Using figures from an engineering manual (44, pp. 1-3), cubic feet per second was converted to gallons per minute or gallons per day. Information was supplied by the Knoxville Wastewater Control Board (37) to determine pipe size and length of pipe needed (Table 3-1).

Pipe size and length

In the 18 acre subdivision there was 1878 feet total sewer pipe needed. A figure of anything less than .65 cubic feet per second (cfs) for sewage flow can be handled by 8-inch pipe. As Table 3-1 indicates, 8-inch pipe was sufficient for all density levels of Subdivision 1. Subdivision 2 also used only 8-inch sewer pipe. The 34 acre tract needed a total of 3548 feet of 8-inch pipe.

Table 3-1. Length and Diameter Sewer Pipe for Each Tract at Three Densities

Subdivision	Density	Footage of					Total Footage Pipe Needed
		8-inch	10-inch	12-inch	15-inch	18-inch	
		Feet					
1	1	1,878	--	--	--	--	1,878
	2	1,878	--	--	--	--	1,878
	3	1,878	--	--	--	--	1,878
2	1	3,548	--	--	--	--	3,548
	2	3,548	--	--	--	--	3,548
	3	3,548	--	--	--	--	3,548
3	1	14,518	8,230	--	--	--	22,748
	2	4,673	3,864	5,620	8,591	--	22,748
	3	4,387	3,622	5,269	9,470	--	22,748
4	1	14,429	12,201	6,345	--	--	32,975
	2	4,566	3,864	5,620	12,645	6,280	32,975
	3	4,281	3,622	5,269	11,854	7,949	32,975
5	1	14,533	5,502	--	--	--	20,035
	2	4,671	3,864	5,620	5,880	--	20,035
	3	4,387	3,622	5,269	6,757	--	20,035

Subdivision 3 with 217 acres of land produced 164,920 gallons of sewage per day at low density development. Allowing for peak flows, this translated to 1.021 cubic feet per second. Above .65 cfs and up to and including 1.2 cfs, 10-inch sewer pipe was called for. It then became necessary to ascertain the amount of 8-inch and 10-inch pipe to employ. The difference between 1.021 and .65 cfs is equal to .371 cfs. Multiplying by a conversion factor¹ changed the cfs figure to gallons. This figure was then divided by the amount of sewage produced by two acres of dwelling units on either side of a common sewer line (called a "run"). The resulting number was multiplied by 208.7 (the length of a "run"). This gave 8230 feet--the number of feet of 10-inch pipe needed. There remained 14,518 feet of sewer pipe, all 8-inch in diameter.

At medium density for Subdivision 3, 8591 feet of 15-inch pipe was required, 5620 feet of 12-inch pipe, 3864 feet of 10-inch pipe, and 4673 feet of 8-inch pipe. When 16 dwelling units per acre were considered, 9470 feet of 15-inch sewer pipe was needed; 5269 feet of 12-inch pipe was required. For 10-inch and 8-inch sewer pipe, 3622 feet and 4387 feet, respectively, were necessary.

Subdivision 4, 316 acres, was the largest piece of land among the five subdivisions. Pipe sizes needed for the low density subdivision were 12-inch, 10-inch, and 8-inch. Lengths were 6345 feet, 12,201 feet, and 14,429 feet. For both medium and high density development on Subdivision 4, the largest pipe size was 18 inches--6280 feet for medium density and 7949 feet for high density. For medium density, 12,645 feet

¹The conversion factor is 646,272, a number to transform cubic feet per second to gallons per day.

of 15-inch pipe were required. Only 11,854 feet of 15-inch pipe were used for the higher density development. For 12-inch pipe, 5620 feet were needed in the medium density subdivision; this amounted to 5269 feet for the high density community. The medium and high densities required 3864 feet and 3622 feet of 10-inch and 4566 feet and 4281 feet of 8-inch sewer pipe.

The last of the five subdivision produced 145,160 gallons of sewage per day at low density. Multiplying by a factor of four to account for peak flows, 5502 feet of 10-inch pipe and 14,533 feet of 8-inch pipe were needed to carry this amount of effluent. If medium density is considered, the peak load rises to over 1.8 million gallons per day requiring footages of 5880 feet of 15-inch, 5620 feet of 12-inch, 3864 feet of 10-inch, and 4671 feet of 8-inch sewer pipe. For high density, footage increased to 6757 feet of 15-inch sewer pipe; amounts of other pipe sizes declined: 5269 feet of 12-inch, 3622 feet of 10-inch, and 4387 feet of 8-inch.

Sewer service costs

When sizes and length of sewer pipe were estimated for each subdivision, costs were determined and allocated on a per capita basis. Distances from each subdivision to each treatment plant were estimated using maps supplied by the Metropolitan Planning Commission of Knoxville.

Costs had to be estimated for construction of collector sewers, including manholes, within each potential location. Transmission costs were estimated considering the distance to the treatment plant and using the per foot cost of the size sewer pipe exiting the subdivision. Cost of manholes were figured into transmission cost. (Manholes were spaced 300 feet apart).

Subdivision 1 included 18 acres and sewer costs were estimated at a little over \$20,000 (see Table 3-2). Transmission costs for two miles of 8-inch pipe and 35 manholes amounted to over \$113,500. Spread out over 50 years, the total cost of \$133,665 was translated (at 6 1/2%) to \$66.26 per person per year in the low density development. (Local municipal bond dealer provided the interest rates to use--6 1/2% for developments within the city and 8 1/2% for developments in the county) (48). The same size and amount of sewer pipe was required for each density, so total cost was the same for medium and high density as for low density, but per capita costs were naturally less. For medium density per capita, cost was \$21.01 per capita per year; for high density, estimated cost was \$19.69 per person per year.

Subdivision 2 sewer cost was estimated at about \$38,300. Transmission cost, \$5,012, was very low because this piece of land was adjacent to the treatment plant. Total cost was over \$43,300 for each density level; per capita costs, reflecting amortization over 50 years at 6 1/2%, were \$11.40 for low density of two dwellings per acre, \$3.60 for medium density (eight dwelling units per acre), and \$3.38 for high density (16 dwelling units per acre).

The third potential site, located in the county, had an estimated total project cost of \$455,385 for low density development. For medium density, total cost increased to over \$628,000, and high density had estimated costs of \$676,577. Amortized at 8 1/2% for 50 years, these costs translated to per capita cost of \$23.88, \$10.43, and \$10.53 respectively for low, medium, and high densities. In this instance, as in Subdivision 5, the medium density developments cost least per capita to sewer.

Table 3-2. Total Capital Costs of Sewage Collection and Transmission for Each Tract at Three Densities

Subdivision	Density	Sewer Cost ^a	Transmission Cost ^b	Project Cost
1	1	20,076	113,589	133,665
	2	20,076	113,589	133,665
	3	20,076	113,589	133,665
2	1	38,298	5,012	43,310
	2	38,298	5,012	43,310
	3	38,298	5,012	43,310
3	1	263,380	192,005	455,385
	2	322,047	306,220	628,268
	3	370,356	306,220	676,577
4	1	410,028	556,011	966,040
	2	598,050	886,761	1,484,811
	3	610,421	886,761	1,497,182
5	1	228,110	57,095	285,204
	2	196,626	90,870	287,495
	3	314,175	90,870	405,045

^aSewer cost is the cost to provide collector sewers of 8 inches or greater within the subdivision and includes the cost of installation of pipe and manholes.

^bTransmission cost is the cost to get sewage from the subdivision to the treatment plant and includes the cost of installation of pipe and manholes.

Subdivision 4 had estimated costs of \$966,040 for low density, \$1,484,811 for medium density, and \$1,497,182 for high density. At 6 1/2% and paid off over a 50 year period, this amounted to \$27.31 per person per year at low density, \$13.30 per person per year at medium density, and \$12.57 per person per year at high density.

The fifth and final location had total costs (collection and transmission) of over \$285,000 for the two dwelling units per acre density level; medium density was above \$287,000; and high density was just over \$405,000. Using 8 1/2% as a payoff rate, the per capita per year costs were \$16.98 for low, \$5.42 for medium and \$7.16 for high density development. High density was more expensive than medium density in this instance because more of the larger diameter, and hence more expensive, sewer pipe was required for the high density development than for the medium density development. Difference in family size between these two densities account for this situation. Average family size for medium density was three persons per dwelling, which meant an average of 24 people per acre. Average family size for high density was 1.6 persons per dwelling, indicating an average of 25.6 persons per acre. Average cost would not differ a great deal under these circumstances if total costs are similar.

If only one service, sewerage, were considered in the model, the solution would have been obvious (see Table 3-3). Note the subdivision with least per capita costs is location 2, density level 3. Being first to enter, 870 people would have been brought in at \$3.38 per capita per year. Next to enter would have been Subdivision 5, Density 2. This would have entered 4584 at \$5.42 per person into the solution. Location 3 at medium density would then have entered with 5208 people at a per

Table 3-3. Sewer Service Cost Per Person Per Year for Each of Five Subdivisions at Three Density Levels

Subdivision	Density	Payoff Rate ^a	Amortization Factor for 50 Years	Project Cost (dollars)	Amortized Cost Per Year (dollars)	Population	Cost Per Person Per Year
1	1	6 1/2%	.0679139	133,665	9,078	137	66.26
	2	6 1/2%	.0679139	133,665	9,078	432	21.01
	3	6 1/2%	.0679139	133,665	9,078	461	19.69
2	1	6 1/2%	.0679139	43,310	2,941	258	11.40
	2	6 1/2%	.0679139	43,310	2,941	816	3.60
	3	6 1/2%	.0679139	43,310	2,941	870	3.38
3	1	8 1/2%	.0864633	455,385	39,374	1,649	23.88
	2	8 1/2%	.0864633	628,268	54,322	5,208	10.43
	3	8 1/2%	.0864633	676,577	58,499	5,555	10.53 ^b
4	1	6 1/2%	.0679139	966,039	65,608	2,402	27.31
	2	6 1/2%	.0679139	1,484,811	100,839	7,584	13.50
	3	6 1/2%	.0679139	1,497,181	101,679	8,090	12.57
5	1	8 1/2%	.0864633	285,204	24,660	1,452	16.98
	2	8 1/2%	.0864633	287,495	24,858	4,584	5.42
	3	8 1/2%	.0864633	405,044	35,021	4,890	7.16 ^b

^aPrivate unility districts finance construction of sewers outside the city and must pay a higher rate for borrowed funds.

^bHigh density was more expensive than medium density in these instances because more of the larger diameter, and hence more expensive, sewer pipe was required for the high density development than for the medium density development.

capita cost of \$10.43. Constraint placed on total population to be located (10,000 to 11,000) would have meant the solution had been reached with a population of 10,662.

Solid Waste Pickup

Knoxville-Knox County solid waste system

When this research was begun, three legal dumping sites, landfills, were located in Knox County. Shortly thereafter, all three of these were closed, and presently one landfill site serves all of Knox County.

The City of Knoxville operated the only municipally-owned refuse collection and disposal system in Knox County. Refuse was collected once per week at no direct charge to the customer. Additional pickups were available for a nominal charge. As of July 1, 1971, the municipal collection service included approximately 104,000 customers. The annual budget for this operation was about \$1,110,000; 90% for collection and 10% for disposal (30, p. 16).

Several private disposal companies operated in the county. In most of these operations, customers pay a monthly service charge for weekly collections. The type of equipment most often used in collection of solid waste was the packer truck. These trucks ranged in capacity of from 16 to 30 cubic yards.

After closing landfills in the county, all solid waste collected within the city or county was delivered to one landfill site via a transfer station. The transfer station provided compaction facilities for solid waste bound for the landfill. Other means of disposing of solid wastes, such as recycling, incineration, and composting, were discussed in the Allen and Hoshall study (3, pp. 91-103), but these

methods are in the future. A system to reuse solid waste seems to be in the future for the Knoxville SMSA, but alternatives to landfills, at present, do not exist. There are, of course, advantages to the present practice regarding sanitary landfills.

Sanitary landfills

Sanitary landfills involve the deposition of refuse in natural depressions or man-made trenches where compacting and covering of refuse occurs. If well planned and operated, sanitary landfills are economical and flexible, require relatively small capital investment, may provide reclamation of land, do not require segregation of solid wastes, cause little or no air pollution, and provide for final and complete disposal. One of the major problems associated with landfills is in obtaining necessary land for continuing landfill requirements. Often, the term "sanitary landfill" is synonymous with "dump" in the minds of the public (3, pp. 58-65).

Another problem with sanitary landfills is the long haul cost involved in solid waste collection and disposal. Landfills are generally located some distance away from the collection area. Centrally locating transfer stations where collected garbage is brought and compacted reduces haul distances significantly. For this research, a transfer station was the service origin point for solid waste collection service.

Solid waste generation in Knoxville-Knox County

A necessary determinant of costs of solid waste removal was the per capita amount of solid waste generated and projected to be generated by the study area populace. It was estimated, using figures from the solid waste study conducted by Allen and Hoshall, that approximately 3.5

pounds of solid waste per person per day would be generated in the study area by 1978 (3, pp. 41-51). Using 3.5 pounds per capita per day for solid waste production, figures were obtained for each potential subdivision location (Table 3-4).

A method for collecting and disposing of waste had to be devised or assumed. One plan, known as "Operation Facelift," put into operation by Knox County, provided the basis for the scheme used in this research. Under "Operation Facelift," trash receptacles were placed about the county so that rural or county residents could legally dispose of their refuse. In the present study, containers were assumed located at each potential demand site and were picked up twice per week. Most containers were 8 cubic yards in capacity (with some smaller) and were picked up and emptied by a front loading packer truck. A figure of 250 pounds per cubic yard of uncompacted waste meant that each 8 cubic yard receptacle would contain 2000 pounds of refuse² (36 and 3, p. 124). These receptacles were to be picked up twice per week, and the contents compacted on a packer truck and delivered to a transfer station located in Knoxville.

According to L. H. Kidd, Chief Clerk Deputy Director of Knoxville, the capacity of the transfer station was approximately 600 tons per day (25). However, this capacity could be substantially increased very easily. Located north of downtown Knoxville off I-75, the transfer station was well managed, sanitary, and represented a tremendous improvement over the old method. Prior to its operation, each individual

²Averaging two sources of information resulted in a figure of 250 pounds per cubic yard for the average weight of solid waste in the Knoxville SMSA.

Table 3-4. Solid Waste Container Cost for Five Subdivisions at Three Density Levels

Subdivision	Density	Population	Solid Waste Generated per One-Half Week (lbs.)	Number and Size (cubic yards) Containers Needed for Twice Per Week Pickup	Cost in July 1975 Prices (\$)	Cost	
						Per Subdivision Per Year	Per Capita Per Year (\$)
1	1	137	1678.25	1-8CY	487.30	85.69	.6255
	2	432	5292	2-8CY, 1-6CY	974.60 + 389.84	171.38 + 68.55	.5554
	3	461	5647.25	3-8CY	1461.90	257.07	.5576
2	1	258	3160.5	1-8CY, 1-6CY	487.30 + 389.84	85.69 + 68.55	.5978
	2	816	9996	5-8CY	2436.50	428.45	.5251
	3	870	10657.5	5-8CY, 1-3CY	2436.50 + 237.56	428.45 + 41.78	.5405
3	1	1649	20200.25	10-8CY	4873	856.90	.5196
	2	5208	63798	32-8CY	15593.60	2742.08	.5265
	3	5555	68048.75	34-8CY	16568.20	2913.46	.5245
4	1	2402	29424.5	15-8CY	7309.50	1285.35	.5351
	2	7584	92904	46-8CY, 1-4CY	22415.80 + 316.74	3941.74 + 55.70	.5271
	3	8090	99102.5	50-8CY	24365	4284.50	.5296
5	1	1452	17787	9-8CY	4385.70	771.21	.5311
	2	4584	56154	28-8CY	13644.40	2399.32	.5234
	3	4890	59902.5	30-8CY	14619	2570.70	.5257

collection truck had to haul each load to a landfill. With the transfer station, one 75 cubic yard trailer can be used to transport the contents of at least three 25 cubic yard packer trucks from the transfer station to the landfill. This saved both time and transportation costs.

Solid waste costs

Each hypothetical subdivision was assumed served by central pickup of solid waste. Although this is not entirely realistic (since the city provides house by house service), the assumption was made to compare costs among different possible subdivisions both in the city and in the county.

Equipment costs were supplied by manufacturers for collection equipment, as well as for refuse receptacles. The number of containers needed for each subdivision was determined and costs were adjusted to reflect prices in 1975 (Table 3-4). The Wholesale Price Index for all commodities for 1973 on a monthly basis was used to predict prices for mid year 1975 (56, pp. s-8, s-9). A simple regression line was fitted to the data using "least squares" techniques. Projecting this line to July 1975 provided an index for that month. Cost figures were then inflated using the formula: July 1975 Price = present or base year price times the estimated 1975 index divided by the base year index.

Representative costs for transporting solid wastes are outlined in Table 3-5. Forty cents (\$.40) per mile was the cost obtained for operation of a front loading packer truck. This figure was multiplied by the round trip distance, and the number of truck loads, to get cost per week. Multiplying by 52 and dividing by the population of the sub-

Table 3-5. Solid Waste Transportation Cost

Item	Cost
Front Loading Packer Truck, 38 Cubic Yard ^a	
Depreciation: ^b $24,179 \div 7$	\$ 3454.14
Interest on Investment: ^c $\frac{24179 + 10\%}{2} (.06)$	797.91
License: ^d Zone Tax	210.75
Use Tax	120.00
Insurance: ^d	734.50
Fuel @ 10.475/day	2995.85
Maintenance: ^c 150% of Initial Cost $\div 7$	5181.21
Labor @ 22.00/day	6292.00
Fringes @ 20%	1258.40
	<u>\$21144.76</u>
Administration, Supervision, and Profit: ^e 10%	2114.48
	<u>\$23259.24</u>
Adjusted to July 1975: $\frac{23259.24 (177.01)}{145.3}$	= \$28335.29
$28335.29 \div 286$	= \$ 99.07/day
$99.07 \div 250$	= .40/mile

^a7 year life, 10% salvage value, 286 days per year.

^bThese data obtained from (21).

^cThese data obtained from (50, p. 79).

^dThese data obtained from (57).

^eThese data obtained from (25).

division at each density gave transportation cost per capita per year. On site container costs per capita per year are found in Table 3-4.

Since door to door collection was not considered, density of development made little difference in solid waste costs for any one subdivision. Distance from the subdivision to the transfer station was the primary determinant of cost (Table 3-6).

Primary and Secondary Education

Family size estimates

While considering different levels of density for development, it occurred that family size was at issue. Traditionally, in this area, low density development meant larger families, while apartments and other high density dwellings drew singles and young couples. It was therefore assumed that low density housing would generate more children of school age than the higher densities. Getting at family size, however, was difficult. Almost no detailed information was available connecting density and family size nor on family size and number of school age children. The Knoxville-Knox County MPC provided estimates of family size in the county of 3.6 for single family dwellings and 1.6 for multifamily dwellings. These figures were not wholly satisfactory because they lumped all multifamily, from duplexes to high rise apartments, in one family size category. However, information to the contrary was not readily available.

Block statistics for the 1970 Knox County Census of Housing seemed the only way to check for family size. Using a Homesekers Guide (52, pp. 13-15) with a map of subdivisions in Knox County, it was possible to match block statistics with 11 single family subdivisions

Table 3-6. Solid Waste Transmission Cost Per Capita Per Year for Five Subdivisions at Each of Three Density Levels

Subdivision	Density	Population	Round-trip (miles)	Solid Waste Generated Per Week (lbs.)	Truck Loads Per Week	Cost				Dollars			
						Per Week	Per Subdivision		Per Capita Per Year	Transportation		Per Capita Per Year	Solid Waste Cost Per Year
							Cost of \$.40 Per Mile	Per Year with Transportation		Cost Per Year	Equipment Cost Per Year		
1	1	137	12.32	3356.5	.20		51.25			.3741	.6255	.9996	
	2	432	12.32	10584	.62		158.88			.3678	.5554	.9232	
	3	461	12.32	11294.5	.66		169.13			.3669	.5576	.9245	
2	1	258	14.58	6321	.37		112.21			.4349	.5978	1.0327	
	2	816	14.58	19992	1.17		354.82			.4348	.5251	.9599	
	3	870	14.58	21315	1.25		379.08			.4357	.5405	.9762	
3	1	1649	28.60	40400.5	2.36		1403.92			.8514	.5196	1.3710	
	2	5208	28.60	127596	7.46		4437.80			.8521	.5265	1.3786	
	3	5555	28.60	136097.5	7.96		4735.24			.9524	.5245	1.3769	
4	1	2402	28.0	58849	3.44		2003.46			.8341	.5351	1.3692	
	2	7584	28.0	185803	10.87		6130.69			.8347	.5271	1.3618	
	3	8090	28.0	198205	11.59		6750.02			.8344	.5296	1.3640	
5	1	1452	29.92	35574	2.08		1294.46			.8915	.5311	1.4226	
	2	4584	29.92	112308	6.57		4088.75			.8920	.5234	1.4154	
	3	4890	29.92	119805	7.01		4362.58			.8921	.5357	1.4178	

in the study area. This sample yielded an average figure of 3.8 persons per dwelling unit. For high density apartments and other multifamily dwellings, the average figure of 1.6 persons per dwelling was provided by the Knoxville-Knox County MPC.⁴

Data were obtained for family size relative to medium density of eight dwellings per acre in one instance. Single family dwellings in so-called "cluster developments" would approach this medium density but such developments are not present in Knox County. One census block in the study area was known to contain mostly duplex dwellings. This served as a proxy for medium density development. Matching this block to the information in the Homesekers Guide, family size was estimated to be 3.0 persons per unit. Therefore, family size for low, medium, and high densities were estimated to be 3.8, 3.0, and 1.6 persons per dwelling.

School age children

It was then necessary to arrive at some estimate for the number of school age children per family within the three family sizes. Bits of data were pieced together to come up with estimates of school age children per family. The Knoxville-Knox County MPC did a study for the location of a new junior high school in the study area. Their estimates were that 6.6% of the population in the study area were 12 through 14 years old. This percentage held for several years into the future. Further, 28% of the population in planning units 16, 17, and 18 were of school age, according to the Knoxville-Knox County MPC. From

⁴Several managers and agents of multifamily developments were contacted for information on family size and number of school age children. The researcher was told in every case that this information was confidential and not available.

this information, the number of school age children in each school division was estimated. Projected enrollment figures also were consulted to help with student estimates.

Given that 10,000 to 11,000 people were projected to come into the study area between 1973 to 1978, 2800 to 3000 should be of school age. If 6.6% or approximately 700 of these children will be junior high school students, extrapolating from census data indicated that about 6.6% would also be high school students. This meant that about 14%, or approximately 1500, would be of elementary school age.

County school authorities provided planning estimates of the number of school age children per family, and with little change these figures were used (42). For the type development considered for low density housing, an average of two school age children per family was used. For high density development, one school age child for every ten families, or .1 per family, was used. One school age child per family was estimated for medium density developments.

Education service costs

Cost information was obtained from Knox County School officials and from manufacturers and operators of transportation equipment (58 and 6). Information was also obtained from research on transportation costs in a Delaware school system (47, pp. 94-95). Adjusting the costs by the straight line projected Wholesale Price Index to 1975 resulted in a per student per mile cost of \$.0158 (see Table 3-7). Depreciation was based on a 175 day school year, and a salvage value of 10% was used. Salvage value was added to original equipment cost, and interest costs were calculated for half the estimated useful life (50, p. 40). Esti-

Table 3-7. Transportation Cost for Education Service

Item	Cost
School bus ^a	\$13,000.00
Depreciation: ^a $13,000 \div 175$	\$ 10.61
Interest on Investment: ^b $\frac{13000 + 10\%}{2} (.06) = 429 \div 175$	2.45
License: ^a $18.50 \div 175$	.11
Insurance: ^a $320.00 \div 175$	1.83
Fuel: ^a .50/gal., 50 mi./day @ 5 mpg	5.00
Maintenance: ^a $1261.75 \div 175$	7.21
Labor: ^a	10.00
Social Security, Unemployment Insurance: ^a $300 \div 175$	1.71
	<u>\$ 38.92</u>
Administration, Supervision, and Profit: ^c 10%	3.89
Daily Rate X	\$ 42.81
175 Days	<u>\$7491.75</u>
Adjusted to July 1975: $\frac{7491.75 (177.01)}{145.3} = \text{Annual Rate}$	\$9216.76
$9126.76 \div 175 = \text{Daily Rate}$	52.15
$52.15 \div 50 = \text{Per Mile Rate}$	1.04
$1.04 \div 66 = \text{Per Student Per Mile}$	.0158

^a66 passenger--7 year life, 10% salvage value (6).

^bThese data were obtained from (50, p. 79).

^cThese data were obtained from (47, pp. 94-95).

mated annual cost (175 day school year), adjusted for inflation to July 1975, was \$9126.76 per year to operate a school bus. Allowing for the maximum of 66 children on a school bus meant a cost per student per mile of a little over .015 cents.

A map showing school locations and school district lines was provided by the Knoxville-Knox County MPC. From this map distances between potential subdivisions and primary and secondary schools were determined. Distance was the primary cost determining factor for education as it was for solid waste collection. Density had little influence on cost in this instance since school buses were assumed to pick up from a central point at each subdivision.

Students from Subdivision 1 were in an area served by West Hills Elementary School, 1.85 miles away; Bearden Junior High School (new), 2.04 miles away; and Bearden High School, 2.46 miles from Subdivision 1. West Hills Elementary School had sufficient capacity to absorb the additional students, as did the new Bearden Junior High School and Bearden High School. Cost figures per student are shown in Tables 3-8 and 3-9. Buses were assumed to start from the schools in the morning for two round trips per day.

Subdivision 2 was served by Bearden Elementary School, Bearden Junior High School, and Bearden High School. Distances in this case were 1.66 miles to Bearden Elementary, 4.17 miles to the new Bearden Junior High, and 3.69 miles to Bearden High School. All three schools were estimated to have room for these students.

The preceding subdivisions, as well as the schools that serve them, are within the city limits of Knoxville. However, for purposes of the analysis of synthetic cost data, these subdivisions were treated the same

Table 3-8. Primary-Secondary Education Transportation Costs, Per Student Per Day

Tract	Elementary School	Cost Per		Junior High School	Cost Per		High School	Cost Per	
		2 Round Trips Distance (miles)	Student Per Day @ .0158 Per Mile (dollars)		2 Round Trips Distance (miles)	Student Per Day @ .0158 Per Mile (dollars)		2 Round Trips Distance (miles)	Student Per Day @ .0158 Per Mile (dollars)
1	West Hills	7.4	.1169	Bearden	8.16	.1289	Bearden	9.84	.1553
2	Bearden	6.64	.1049	Bearden	16.68	.2635	Bearden	14.76	.2332
3	Ball Camp	0.00	.0000	Karns	10.24	.1618	Karns	10.24	.1618
4	Bluegrass	3.6	.0569	Farragut	31.20	.4930	Farragut	31.20	.4930
5	Farragut	0.00	.0000	Farragut	0.00	.0000	Farragut	0.00	.0000

Table 3-9. Primary-Secondary Education, Numbers of Students, and Cost Per Student Per Year
(Costs Adjusted to Reflect July 1975 Prices)

Subdivision	Density	Population	School Age Population	Elementary School Population	Junior High School Population	High School Population	Per Child			Per Child		
							Per Year	Elementary Cost (\$)	Junior High Cost (\$)	Per Year	Junior High Cost (\$)	Per Year High School Cost (\$)
---(.0158 Per Student Per Mile)---												
1	1	137	72	38	17	17	20.46	20.46	22.56	27.21	27.21	27.21
	2	432	144	76	34	34	20.46	20.46	22.56	27.21	27.21	27.21
	3	461	29	15	7	7	20.46	20.46	22.56	27.21	27.21	27.21
2	1	258	136	72	32	32	18.36	18.36	46.11	40.81	40.81	40.81
	2	816	272	144	64	64	18.36	18.36	46.11	40.81	40.81	40.81
	3	870	55	29	13	13	18.36	18.36	46.11	40.81	40.81	40.81
3	1	1649	868	460	204	204	0	0	28.32	28.32	28.32	28.32
	2	5208	1736	920	408	408	0	0	28.32	28.32	28.32	28.32
	3	5555	347	183	82	82	0	0	28.32	28.32	28.32	28.32
4	1	2402	1264	670	297	297	9.96	9.96	86.28	86.28	86.28	86.28
	2	7584	2528	1340	594	594	9.98	9.98	86.28	86.28	86.28	86.28
	3	8090	506	268	119	119	9.96	9.96	86.28	86.28	86.28	86.28
5	1	1452	764	404	180	180	0	0	0	0	0	0
	2	4584	1528	810	359	359	0	0	0	0	0	0
	3	4890	306	162	72	72	0	0	0	0	0	0

way regarding education as were subdivisions in the county. Too, if the city and county school systems were ever consolidated, this analysis would then be more realistic.

Elementary school transportation costs were assumed to be zero for Subdivision 3. Ball Camp Elementary School is located adjacent to Subdivision 3. Karns Middle School (to be constructed) served the 12 through 14 age group and 2.56 miles distant. Karns High School was also 2.56 miles from Subdivision 3. Ball Camp Elementary School was assumed to have room for a maximum of 120 additional students. This was far short of the demand which would come from Subdivision 3 at any of the three density levels. The new Karns Middle School was assumed to handle most, if not all, new students generated by Subdivision 3; the same was true for Karns High.

Blue Grass Elementary, Farragut Middle School, and Farragut High were service origins for Subdivision 4. They were .9, 7.8, and 7.8 miles respectively, from this subdivision. Blue Grass Elementary did not have sufficient capacity to handle all the potential school children which could be generated by Subdivision 4 except at high density. Farragut Middle School could absorb the amount of 12 through 14 year olds only from certain density levels. This school served two potential subdivisions, and certain density combinations of these two would have more children than could be taken care of. Farragut High School had sufficient excess capacity. With a capacity to take 1232 additional students (by assumption), the combination of Subdivisions 4 and 5 at density level 2 would generate 953 high school students. At low density, the number of high school children generated by Subdivisions 4 and 5 would be 477; at high density, the number decreases to 191.

Other density combinations of these two subdivisions would generate different numbers of children. The particular densities and combinations of subdivisions were left for the linear programming model to resolve.

Subdivision 5 was served by Farragut Elementary School, Farragut Middle School, and Farragut High School. Since this subdivision was adjacent to all three of these schools, transportation costs for education were assumed to be zero. Farragut Elementary was assumed to have excess capacity for 331 students. With 404, 810, and 162 children generated, respectively, by densities 1, 2, and 3, Farragut Elementary had less than adequate space for children who would be in Subdivision 5 at low and medium densities.

Total costs for sewer, solid waste, and education services are shown in Table 3-10. Costs for sewer service and for solid waste service are on a per person basis. In order to get education costs from a per student to a per person basis, costs were weighted and averaged among the various levels of education and densities of development. For example, family size for low density was 3.8 persons with 2.0 school age children per family. At low density, 52.63% of the family was school age. Further, 53% of the school age children were of elementary school age with 47% of the school age children divided evenly between junior and senior high schools. Multiplying .5263 times .53 gave .2789 persons per family of elementary school age. Multiplying .5263 times .235 gave .1237 persons per family of junior high school age; this same figure applied to children of senior high school age. Multiplying each of these figures by the per student costs (see Table 3-9) and adding the results gave the per person per year costs for education at density level

Table 3-10: Combined Cost for All Three Services Per Capita Per Year, Sewage, Primary-Secondary Education, Solid Waste at Five Subdivisions at Three Density Levels

Subdivision	Density	Sewer Service	Solid Waste Service	Education	Total Cost
-----Dollars-----					
1	1	66.26	1.00	11.86	79.12
	2	21.01	.92	7.51	29.44
	3	19.69	.92	1.41	22.02
2	1	11.40	1.03	15.87	28.30
	2	3.60	.96	10.05	14.61
	3	3.38	.98	1.89	6.25
3	1	23.88	1.37	7.00	32.25
	2	10.43	1.38	4.44	16.25
	3	10.53	1.38	.84	12.75
4	1	27.31	1.37	24.12	52.80
	2	13.30	1.36	15.27	29.93
	3	12.57	1.36	2.87	16.80
5	1	16.98	1.42	0	18.40
	2	5.42	1.42	0	6.84
	3	7.16	1.42	0	8.58

1 of Subdivision 1 (Table 3-10). The same process was employed to obtain figures for densities 2 and 3; however, family size was different in each of the last two densities.

After all service costs were put on the same basis (per capita per year), they were added together to obtain total per capita service cost per subdivision for each density (Table 3-10).

The programming model used to resolve the problem of which subdivisions to develop at what densities is fully discussed in the following chapter. Summary and conclusions will succeed in Chapter V.

CHAPTER IV

ANALYSIS OF THE RESIDENTIAL LOCATION MODEL

I. INTRODUCTION

Chapter II dealt with the theoretical model of residential location. Among a number of alternative demand and supply sites, one pattern of residential development was sought which would minimize costs of providing potential residents with three services--sewers, solid waste pick-up, and primary and secondary education. A minimum cost linear programming model was the technique employed to identify an optimal solution.

II. THE MODEL

Cost Coefficients

The table of cost coefficients (Table 4-1) describes the cost associated with flows of services between a set of service supply points and a set of demand points. The service demand points are subdivisions that can be developed at one of three density levels. The service origin points are three sewage treatment plants, one solid waste transfer station, and 11 schools (five elementary, three junior high, and three high schools). Costs shown in Table 4-1 are on a per capita basis. Sewer and solid waste costs are on a per person basis; school costs in this instance are on a per student basis. Later, all costs will be combined and put on the same per capita basis. An explanation of the coefficients of Column 1 of Table 4-1 follows: The first number, in the Sewage Plant 1 row, indicates that the average cost of providing sewer

Table 4.1. A Matrix of Service Origin Points with Available Capacities, Service Demand Points with Potential Demand for Services, and Per Capita Per Year Costs for Sewer and Solid Waste Service and Per Student Cost for Primary and Secondary Education

Service Origin Points	Service Demand Points												Available Capacities at Service Origins ^a
	1			2			3			4			
	1b	2	3	1	2	3	1	2	3	1	2	3	
Dollars													
Sevage Plant 1	66.26	21.01	19.69	11.40	3.60	3.38	23.88	10.43	10.53	27.31	13.30	12.57	9421
Sevage Plant 2													1650
Sevage Plant 3													4890
West Hills Elementary	20.46	20.46	20.46	18.36	18.36	18.36	0	0	0	0	9.96	9.96	510
Bearden Elementary													120
Ball Camp Elementary													150
Bluegrass Elementary													331
Farragut Elementary													406
Bearden Jr. High	22.56	22.56	22.56	46.11	46.11	46.11	28.32	28.32	28.32	86.28	86.28	86.28	549
Karns Mid School													468
Farragut Mid School	27.21	27.21	27.21	40.81	40.81	40.81	28.32	28.32	28.32	86.28	86.28	86.28	300
Bearden High													213
Karns High													1232
Farragut High													
Solid Waste Transfer Station	1.00	.92	.92	1.03	.96	.98	1.37	1.38	1.38	1.37	1.36	1.36	10500
Potential no. of people to be served at each subdivision at 3 density levels	137	432	461	258	816	870	1649	5208	5555	2402	7584	8090	1452
													4890

^aCapacity figures represent the number of people (or students, in the case of schools) who could be provided service from service origins.

^bSubdivisions densities.

service for one year for one person was \$66.26. For a family of four, say, this would be about \$265.00 for one year. In the row for West Hills Elementary School, \$20.46 indicates the cost to deliver, by bus, one student to school from Subdivision 1 for the school year. For Bearden Junior High, the cost to transport one student to school from Subdivision 1 for the school year is \$22.56. The cost to transport one student to Bearden High School from Subdivision 1 is \$27.21 per school year. For solid waste removal the cost indicated is \$1.00 per person per year. These costs are based on the assumptions and techniques described in Chapter III. Only a portion of the total costs of all aspects of these services was considered, that of transportation costs.

The last column of Table 4-1 indicates the amount of capacity available at each service site. Sewage Plant 1 has sufficient capacity to serve an additional 9421 people; Plant 2 available capacity will service 1650 people; and Plant 3 capacity would enable 4890 additional people to be served. Capacities to serve additional students are listed for each school in the final column. The solid waste transfer station had sufficient capacity to serve the total increase in the population for the study area.

Computational Model

Costs for all three services were combined as shown in Row 1 of Table 4-2. There are 15 cost figures in this table, one for each of five subdivisions at three density levels. There are 31 rows, the first 15 of which are (1) the cost row and (2) 14 rows set up to show the relationship between the various service origins and the potential subdivision locations. These 14 rows are constraining conditions which

Table 4-2. Residential Location Model. Origins, Destinations, Cost Coefficients, Demands for and Supplies of Services, and Change Columns for Parametric Programming

Row ^a	Service Demand Points															RHS1 ^b	RHS2	RHS3	RHS4	RHS5	RHS6
	S1D1	S1D2	S1D3	S2D1	S2D2	S2D3	S3D1	S3D2	S3D3	S4D1	S4D2	S4D3	S5D1	S5D2	S5D3						
1 Cost (\$ per capita per year)	79.12	29.44	22.02	28.30	14.61	6.25	32.25	16.25	12.75	52.80	29.93	16.80	18.40	6.84	8.58						
2 Plant 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	9421					
3 Plant 2																1650					
4 Plant 3																4890					-1650
5 WHE	.2789	.1766	.0331													250					-4890
6 BES				.2789	.1766	.0331										150					
7 BCE							.2789	.1766	.0331							120					
8 BGE										.2789	.1766	.0331				510					
9 FES													.2789	.1766	.0331	331			74		
10 BJH	.1237	.0783	.0147	.1237	.0783	.0147	.1237	.0783	.0147							406					
11 KHS							.1237	.0783	.0147							549					
12 FMS							.1237	.0783	.0147	.1237	.0783	.0147	.1237	.0783	.0147	468					
13 BHS	.1237	.0783	.0147	.1237	.0783	.0147										300					
14 KHS							.1237	.0783	.0147							213					
15 FHS										.1237	.0783	.0147	.1237	.0783	.0147	1232					
16 S1D1S	1															137					
17 S1D2S		1														432					
18 S1D3S			1													461					
19 S2D1S																258					
20 S2D2S																816					
21 S2D3S																870					
22 S3D1S							1									1649					
23 S3D2S								1								5208					
24 S3D3S									1							5555					
25 S4D1S										1						2402					
26 S4D2S											1					7584					
27 S4D3S												1				8090					
28 S5D1S													1			1452					
29 S5D2S														1		4584					
30 S5D3S															1	4890					
31 Total	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	10500 ^c			-4584	-4890	-4890

^aValues in Rows 5 through 15 are coefficients to represent the percentage of school age children in each subdivision at each density level.

^bValues in Rows 2 through 15 are the number of people (or students) who could be served by service origins. Values in Rows 16 through 30 are the maximum number of people who could be located at each subdivision at each density level.

^cRow 31 indicates the total number of people to be allocated by the model.

restrict the amount of service which can be supplied by a sewage plant, for example, or a school. Row 16 to Row 30 are also constraining conditions, but on the demand locations. These rows indicate the maximum number of service units to be supplied to a potential subdivision at one of three density levels. An example might be helpful in explaining the roles of the figures, or coefficients, within the body of Table 4-2. Column 1, S1D1, indicates the relationship between Subdivision 1, developed at low density, and the services to be delivered to it. Cost is \$79.12 per person per year as the combined cost of providing sewage and solid waste disposal, and primary and secondary education. Plant 1 is the treatment plant which serves Subdivision 1. West Hills Elementary School (WHE), Bearden Junior High (BJH), and Bearden High School (BHS) provide primary and secondary education service for Subdivision 1.¹ If one person were to be located in Subdivision 1, Density 1, as is shown in Row 16, S1D1S, one unit of sewer service would be required from Plant 1, Average family size at this density is 3.8 persons, two of which are school age. This means that for each person located in S1D1, a fraction of an elementary student is represented (.2789) in Row 5 and a fraction of a junior high and high school student is represented by the figures in Rows 10 and 13 (.1237). Row 16 under the column labeled RHS1 indicates the maximum number of combined service units (in other words, the number of people) that could go to Subdivision 1, Density 1. If 137 people were located on S1D1, then there would be demanded 137 units of sewer service, 137 units of solid waste service, 38 units of elementary school service (137 times .2789), 17 units of junior high school service

¹The cost of solid waste is included in the combined cost of services, but this service was not constraining and is not in the table. In other words, any number of people up to the 10,500 total would be implicitly provided solid waste service at any location.

(137 times .1237), and 17 units of high school service (137 times .1237). Coefficients for education service for Densities 2 and 3 are smaller fractional units because family size and number of school children per family declines as density increases. The column labeled RHS1 (Right Hand Side 1) indicates the capacity available at service origins (Row 2 through 15) in terms of potential people to be served. Row 16 through 30 under RHS1 indicate the maximum number of people that could be located on a particular subdivision at one of three density levels, and also the maximum number of service units that would be required. Row 31, Total, shows that the total number of people to be located (an estimated population increase) is limited to 10,500. Each person to be located in a particular subdivision of density 1, 2, or 3 also adds one person to the total.

III. PARAMETRIC PROGRAMMING

Linear programming is a computational technique for optimizing (maximizing profit or minimizing costs) a certain activity subject to certain restrictions. It can be useful and informative to view the changes in a solution when some of the restrictions are changed. In the present research, for example, what would happen to the solution if Treatment Plant 2 had no available capacity, or if one of the schools had more or less capacity than in the initial solution? Such questions can be easily dealt with using the parametric programming technique.

Parametric programming allows changes to be made in the original conditions of a problem, such as stated above, to examine the effects of these changes upon an initial optimal solution. After an initial solution has been found, the parametric programming routine changes one

or more elements of the problem (i.e. increases or decreases costs or capacities) and then recomputes to find a new optimal solution. Parametric programming in this research involved only changes either in the capacity for one of the service origin points, or the maximum demand for services at one of the potential subdivisions. One change, a -816, (see Table 4-2, RHS2) reduced demand for services to zero at Subdivision 2, Density 2. This meant that S2D2 no longer had potential for development. Various solutions reached using parametric programming will be discussed in detail in Section V.

IV. COMPUTER ROUTINE: PARARHS

This routine informs the computer that parametric programming is to be performed on the right-hand-side of the mathematical array making up the LP matrix (see Table 4-2). The following explanation of this procedure is based on a technical report on the IBM MPS-360 Computer Routine (14, pp. 19-25).

PARARHS (parametric programming on the right-hand-side) is used postoptimally to perform changes in the capacities and/or demands for various services. PARARHS replaces the original RHS (service capacities and demand for services) in Table 4-2 with the original RHS plus a multiple of a "change column." This multiple, XPARAM, is the parameter. The beginning value of XPARAM equals zero; and its value increases to a user designated maximum, XPARAMAX (in this case $XPARAMAX = 1$). If a change column is defined, then each value of XPARAM defines a different related LP program. PARARHS solves this series of problems while varying XPARAM from zero to a predetermined maximum.

For the initial solution, the available capacities at service

origin points and the potential demand at various subdivisions were as shown in Column RHS1 of Table 4-2. The first change made was in the potential of Subdivision 2, Density 2. Under the so-called "change column," RHS2, the negative number instructs the computer to redo the initial solution without capacity at S2D2 and with all other capacities unchanged. XPARAM was set equal to 1 so that PARARHS would increase the value of the capacities in RHS1 according to the instructions in RHS2. In other words, demand at S2D2 would be reduced from a potential of 816 to zero, and a new solution would be found. Each additional change column,--RHS3, RHS4, RHS5, RHS6--instructs the computer to solve the linear programming problem using the capacities of RHS1 plus any changes called for in a change column.

V. RESULTS OF SIX COMPUTER RUNS

Solution Two

Pattern of development

Solution One served as the starting point for other solutions but had absurd results. Solution Two resulted from the parametric programming routine with XPARAM=1. S2D2S (Subdivision 2, Density 2) demand was reduced to zero and the solution was a realistic one. The pattern of development which emerged was one where all development took place at high density. Subdivision 5 was developed both with high and medium densities (Table 4-3).

Subdivision 1 was left undeveloped. Subdivision 2 was developed at high density with 870 people. These 870 people obtained 870 units of sewer service from Sewage Treatment Plant 1, and 870 units of solid waste service from the Transfer Station. With 870 people, about 3%, or 29, were

Table 4-3. Solution 2. Results of the Residential Model: Subdivisions and Densities Developed, Showing the Pattern of Allocating 10,500 People, Including Students. Cost: \$118,292

Service Origin and Service Delivery Points	Capacities at SOP's and SDP's (people) ^a	Pattern of Allocation (people) ^a	Unused Capacities (people) ^a	Dual Solution ^b (dollars)	Opportunity Cost Differential ^c (dollars)
1. Plant 1	9421	3960	5461	--	62.32
2. Plant 2	1650	1650	0	4.05	12.64
3. Plant 3	4890	4890	0	7.82	5.22
4. WHE	250	0	250	--	11.50
5. BES	150	29	121	--	--
6. BCE	120	55	65	--	--
7. BGE	510	102	408	--	--
8. FES	331	331	0	12.12	--
9. BJH	406	13	393	--	--
10. KMS	549	24	525	--	--
11. FMS	468	192	276	--	--
12. BHS	300	13	287	--	--
13. KHS	213	24	189	--	--
14. FHS	1232	192	1040	--	--
15. S1D1	137	0	137	--	62.32
16. S1D2	432	0	432	--	12.64
17. S1D3	461	0	461	--	5.22
18. S2D1	258	0	258	--	11.50
19. S2D2	816	0	0	--	--
20. S2D3	870	870	0	10.55	--
21. S3D1	1649	0	1649	--	19.50
22. S3D2	5208	0	5208	--	3.50
23. S3D3	5555	1650	3905	--	--
24. S4D1	2402	0	2402	--	36.00
25. S4D2	7584	0	7584	--	13.13

Table 4-3 (continued)

	Service Origin and Service Delivery Points	Capacities at SOP's and SDP's (people) ^a	Pattern of Allocation (people) ^a	Unused Capacities (people) ^a	Dual Solution ^b (dollars)	Opportunity Cost Differential ^c (dollars)
26.	S4D3	8090	3090	5000	--	--
27.	S5D1	1452	0	1452	--	12.80
28.	S5D2	4584	1179	3405	--	--
29.	S5D3	4890	3711	1179	--	--
30.	Total	10500	10500	0	-16.80	--

^aFigures for schools, Rows 4 through 14, are in terms of students.

^bRate at which cost would decrease if capacity of some limiting resource were increased by one unit.

^cAmount that per unit service cost would have to be reduced before a particular subdivision, at a particular density, could be developed.

students claiming 29 units of elementary education service from Bearden Elementary School. A little over 1%, or 13 of this 870, were students demanding 13 units of junior high education service from Bearden Junior High School. The same number of students were claiming 13 units of high school service from Bearden High School.

Subdivision 3 was developed at Density 3 for 1650 people. Sewage Treatment Plant 2 had available capacity to serve only 1650 people, however. All 1650 units of available sewer service were demanded from Treatment Plant 2. Fifty-five students were served by Ball Camp Elementary. Karns Middle School served only 24 students. Karns High School at 213 service units available; only 24 students from Subdivision 3 claimed education service from this high school. All 1650 people in Subdivision 3 were provided with solid waste service.

Subdivision 4 was developed at Density 3 with 3090 people. The people in Subdivision 4 obtained 3090 units of sewer service from Treatment Plant 1. A little over 3%, 102, of these people were of elementary school age and demanded 102 units of education service from Blue Grass Elementary School. There was demand from 192 units of education service from Farragut Middle School and demand for 192 units from Farragut High. Subdivision 4 needed 45 units each and Subdivision 5 needed 147 units each for junior and senior high school.

Subdivision 5 was developed partially at medium density and partially at high density. At Density 2, Subdivision 5 was developed for 1179 people--this was a maximum number that could be located on this tract at this density because of limitations in elementary school capacity at Farragut. At Density 3, Subdivision 5 was developed for 3711 people. There were 4890 units of sewer service available, i.e. 4890

people could be served by Treatment Plant 3; Subdivision 5 used up this availability. The available capacity of Farragut Elementary School was claimed by Subdivision 5 students. Claims were made on Farragut Middle School and Farragut High by both Subdivision 4 and 5. Subdivision 4 needed 45 units of both junior high and high school service. Subdivision 5, Density 2, had 92 junior high and 92 high school students needing education service; at Density 3, students from Subdivision 5 made claims for 55 units of education service from both Farragut Middle School and Farragut High School.

The 10,500 people located according to the pattern just explained were provided services at a total cost of \$118,292.

The dual of solution two

The dual solution figures appear in the next to the last column of Table 4-3. The values in this column represent the rate at which cost would decrease if the capacity of some limiting resource were increased by one unit. For example, if capacity at Sewage Treatment Plant 2 were increased from 1650 to 1651, the dual value of 4.05 indicates that overall minimum cost would be reduced by \$4.05. Increasing Plant 2 capacity by one unit would increase the number of people in Subdivision 3 to 1651; S4D3 would be reduced by one to meet the condition that no more than 10,500 people were to be located; one unit of service at S3D3 cost \$12.75; at S4D3 the cost for one unit was \$16.80. Hence, a saving or reduction in cost of \$4.05.

If one unit of education service capacity were added to Farragut Elementary School, then more people could be added to Subdivision 5, and fewer people would be located on the higher cost alternative Subdivision

4. The reduction in cost would be \$12.12 if available capacity at Farragut Elementary School were increased to 332 from 331.

Finally, if the total number of people to be located in subdivisions were increased to 10,501, cost would be increased \$16.80 since each extra unit at S4D3 cost \$16.80. (The -16.80 in Table 4-3 indicates a negative contribution to the optimal solution; hence an addition to total cost).

Reduced cost for solution two

In the final column, Opportunity Cost Differential, the values there indicate the amount that the cost per unit of combined service would have to be reduced in order for a particular subdivision at a given density to be developed. It does not matter which individual service cost is reduced, only that the cost per unit for the package of three services is reduced. For example, cost per unit of combined service at Subdivision 1, Density 1, was \$79.12. If cost would be reduced by \$62.32 per unit, then S1D1 could be developed just as cheaply as could S4D3, the last subdivision to be developed. S1D2 might be developed if its combined unit cost could be reduced by \$12.64. If unit service cost declined \$5.22, S1D3 could possibly be developed. Subdivision 2 was fully developed at high density so it could not be developed further at low density; reduced cost of \$11.50 does not make any sense in this instance. Although Subdivision 3 was developed at Density 3, it was only partially developed, limited by the excess capacity of Treatment Plant 2. However, if it were possible to add one person to S3D2, combined unit service cost would have to be reduced \$3.50 before this would become a feasible alternative to S3D3. To make S3D1 a reason-

able alternative to S3D3, unit service cost at S3D1 would have to be reduced by \$19.50 (\$32.25 - \$12.75). The \$36.00 for S4D1 indicates that cost would have to be reduced this much per unit of service in order for S4D1 to be a feasible alternative to S4D3. S4D2 unit service cost would have to decline by \$13.13 to be able to compete for the resources going to S4D3. For S5D1, the 12.80 in that row, indicates the amount that unit service costs would have to be reduced before this subdivision could be developed.

Solution Three

Pattern of Development

In Solution Three, S2D2 (Subdivision 2, Density 2) and S5D3 (Subdivision 5, Density 3) were blocked out using the parametric programming method. In other words, these two possibilities were eliminated from the solution. S2D2 was blocked because Subdivision 2 was fully developed at Density 3. S5D3 was blocked because S5D2 was developed first. This technique enabled development of subdivisions at only one of three densities. Minimum total cost of all services for this solution was \$141,871.

As in Solution Two, Subdivision 1 was left undeveloped in Solution Three (Table 4-4). Subdivision 2 was again fully developed at high density with 870 people in multifamily dwellings. This indicated a need for 870 units of sewer service from Plant 1, and 870 units of solid waste service. Of these 870 people, 55 would be of school age. There would be demand for 29 units of education service from Bearden Elementary, 13 units of service from Bearden Junior High, and 13 units of service from Bearden High.

Table 4-4. Solution 3. Results of the Residential Model: Subdivisions and Densities Developed, Showing the Pattern of Allocating 10,500 People, Including Students. Cost: \$141,871

Service Origin and Service Delivery Points	Capacities at SOP's and SDP's (people) ^a	Pattern of Allocation (people) ^a	Unused Capacities (people) ^a	Dual Solution ^b (dollars)	Opportunity Cost Differential ^c (dollars)
1. Plant 1	9421	6976	2445	--	
2. Plant 2	1650	1650	0	4.05	
3. Plant 3	4890	1874	3016	--	
4. WHE	250	0	250	--	
5. BES	150	29	121	--	
6. BCE	120	55	65	--	
7. BGE	510	202	308	--	
8. FES	331	331	0	56.40	
9. BJH	406	13	393	--	
10. KMS	549	24	525	--	
11. FMS	468	237	231	--	
12. BHS	300	13	287	--	
13. KHS	213	24	189	--	
14. FHS	1232	237	995	--	62.32
15. S1D1	137	0	137	--	12.64
16. S1D2	432	0	432	--	5.22
17. S1D3	461	0	461	--	11.50
18. S2D1	258	0	258	--	--
19. S2D2	816	0	0	--	--
20. S2D3	870	870	0	10.55	--
21. S3D1	1649	0	1649		19.50
22. S3D2	5208	0	5208		3.50
23. S3D3	5555	1650	3905		--
24. S4D1	2402	0	2402		36.00
25. S4D2	7584	0	7584		13.13

Table 4-4 (continued)

Service Origin and Service Delivery Points	Capacities at SOP's and SDP's (people) ^a	Pattern of Allocation (people) ^a	Unused Capacities (people) ^a	Dual Solution ^b (dollars)	Opportunity Cost Differential ^c (dollars)
26. S4D3	8090	6106	1984		--
27. S5D1	1452	0	1452		11.56
28. S5D2	4584	1874	2710		--
29. S5D3	4890	0	0		--
30. Total	10500	10500	0	-16.80	

^aFigures for schools, Rows 4 through 14, are in terms of students.

^bRate at which cost would decrease if capacity of some limiting resource were increased by one unit.

^cAmount that per unit service cost would have to be reduced before a particular subdivision, at a particular density, could be developed.

Subdivision 3 was developed at high density but was limited to 1650 people by the capacity of Sewage Plant 2. Solid waste service was provided for 1650 people. The number of school age children in this development was 103--55 units of service were demanded from Ball Camp Elementary; 24 units of service were demanded from Karns Middle School; and 24 units of service were demanded from Karns High School.

Subdivision 4 was subdivided and 6106 people were located there at Density 3. Sewer service for these people was provided by Treatment Plant 1; solid waste service was also provided. Education service was provided by Blue Grass Elementary for 202 students, by Farragut Middle School for 90 students, and by Farragut High School for 90 students.

Subdivision 5 was developed for 1874 people at medium density. Sewer service was provided from Plant 3. Solid waste service was provided from the Transfer Station. Only 1874 people could be located in Subdivision 5 because of the limited capacity assumed for Farragut Elementary School. Service capacity of 331 units at this elementary school was used up by students in Subdivision 5. For junior high and high school, 147 students would go to Farragut Middle School and 147 students would go to Farragut High.

The dual of solution three

An added unit of capacity to Sewage Plant 2 would have brought about a reduction in total cost for all services of \$4.05. Reduction in cost of about \$56.40 would result from a one unit increase in capacity at Farragut Elementary School. Since 17.66% of any population located at S5D2 was assumed to be of elementary school age, raising the available capacity of Farragut Elementary to 332 would enable 1880 people to be

located at S5D2. Cost reduction of \$10.55 per unit would come about if S2D3 could add one unit to the 870 people located there. By increasing total population, cost would be increased by \$16.80 for every unit added above 10,500.

Reduced cost for solution three

To give S1D1 the possibility of being developed, \$62.32 would have to be taken off its combined unit service cost. It is highly unlikely that cost per unit could be reduced by anything approaching \$62.32. S1D2 could possibly be developed if its combined unit service cost were reduced by \$12.64 to \$16.80. This is also unlikely to occur. To make it possible for S1D3 to be developed, its unit cost would have to come down to \$16.80 from \$22.02. For an alternative to S2D3, unit service cost of S2D1 would have to be reduced by \$22.05. Reducing S3D1 by \$19.50 would give this subdivision an opportunity for development equal to S3D3. A small reduction of \$3.50 per unit of combined service and S3D2 would become a feasible alternative to S3D3. For S4D1, unit cost would have to come down by \$36.00, S4D2 would need to drop \$13.13 per unit of combined service, and S5D1 would need an \$11.56 drop in unit service cost to have the opportunity of being developed.

Solution Four

Pattern of development

In the last several years, the majority of developments have been relatively high density. As a matter of fact, about 85% to 90% of the building permits issued in Knoxville have been for high density development (41, p. 1). In prior solutions, all development has taken place at either medium or high density. Realistically, however, there should

probably be some development at low density. If 12% to 15% of the total of 10,500 people were to be located in a low density development, total cost would naturally be increased. In order to see what total cost would be with some low density residential development, S5D1 was forced into the solution with 1452 people. Unit service cost for S5D1 was \$18.40 and would not have come into the solution (would not have been developed) ordinarily. By blocking S5D2 and S5D3 and by forcing S5D1 to equal 1452, S5D1 was developed (Table 4-5). With 1452 people in low density development, about 28%, or 405, were assumed to be of elementary school age. Farragut Elementary School capacity was assumed increased just enough to provide service for the 405 elementary students from S5D1. Education service for 180 students was needed from Farragut Middle School and space for 180 students was also needed at Farragut High School. Solid waste service was provided for 1452 people and 1452 units of sewer service were provided from Treatment Plant 3.

Subdivision 1 was left undeveloped in Solution Four. Subdivision 2 was developed at high density demanding 870 units of service from Sewage plant 1, and 870 units of solid waste service. There were 29 students to be provided transportation to Bearden Elementary School from Subdivision 2. Transportation was provided for 13 students from Subdivision 2 to Bearden Junior High and for 13 students from Subdivision 2 to Bearden High School.

Capacity at Sewage Plant 2 limited development on S3D3 to 1650 people. Of these, 55 were students for Ball Camp Elementary School. Demand for educational service resulted from the 24 students of junior high school age. Educational service for these two groups were provided by Karns Middle School and by Karns High School. Solid waste service was provided S5D3 with 1650 service units from the Transfer Station.

Table 4-5. Solution 4. Results of the Residential Model: Subdivisions and Densities Developed, Showing the Pattern of Allocating 10,500 People, Including Students. Cost: \$162,862

Service Origin and Service Delivery Points	Capacities at SOP's and SDP's (people) ^a	Pattern of Allocation (people) ^a	Unused Capacities (people) ^a	Dual Solution ^b (dollars)	Opportunity Cost Differential ^c (dollars)
1. Plant 1	9421	7398	2023	--	62.32
2. Plant 2	1650	1650	0	4.05	12.64
3. Plant 3	4890	1452	3438	--	5.22
4. WHE	250	0	250	--	11.50
5. BES	150	29	121	--	--
6. BCE	120	55	65	--	--
7. BGE	510	216	294	--	--
8. FES	405	405	0	--	--
9. BJH	406	13	393	--	--
10. KMS	549	24	525	--	--
11. FMS	468	276	192	--	--
12. BHS	300	13	287	--	--
13. KHS	213	24	189	--	--
14. FHS	1232	276	956	--	--
15. S1D1	137	0	137	--	62.32
16. S1D2	432	0	432	--	12.64
17. S1D3	461	0	461	--	5.22
18. S2D1	258	0	258	--	11.50
19. S2D2	816	0	0	--	--
20. S2D3	870	870	0	10.55	--
21. S3D1	1649	0	1649	--	19.50
22. S3D2	5208	0	5208	--	3.50
23. S3D3	5555	1650	3905	--	--
24. S4D1	2402	0	2402	--	36.00
25. S4D2	7584	0	7584	--	13.13

Table 4-5 (continued)

Service Origin and Service Delivery Points	Capacities at SOP's and SDP's (people) ^a	Pattern of Allocation (people) ^a	Unused Capacities (people) ^a	Dual Solution ^b (dollars)	Opportunity Cost Differential ^c (dollars)
26. S4D3	8090	6528	1562	--	--
27. S5D1	1452	1452	0	-1.60	--
28. S5D2	4584	0	0		--
29. S5D3	4890	0	0		--
30. Total	10500	10500	0	-16.80	--

^aFigures for schools, Rows 4 through 14, are in terms of students.

^bRate at which cost would decrease if capacity of some limiting resource were increased by one unit.

^cAmount that per unit service cost would have to be reduced before a particular subdivision, at a particular density, could be developed.

Subdivision 4 was developed for 6528 people in multifamily dwellings at relatively high density. A portion of this number of people were of school age. Blue Grass Elementary School was to provide the capacity for 216 students. Farragut Middle School had to serve both Subdivision 4 and Subdivision 5. The same was true of Farragut High School. Subdivision 4 needed education service for 96 students at Farragut Middle School and for 96 students at Farragut High School. Subdivision 5 has been discussed previously.

Total cost for this pattern of development, i.e., forcing some development to be low density, was significantly higher at \$162,862. Low density development could have occurred elsewhere, but the cost on all except Subdivision 2 would have been even higher.

The dual of solution four

Here again, if capacity were increased at Sewage Treatment Plant 2, S3D3 could have been developed further, and total cost would decrease by \$4.05 per unit. Total cost could be further reduced by \$10.55 per unit if Subdivision 2 were able to add to its 870 people. This cost reduction comes about as long as a higher cost development is reduced by one unit. The negative sign before the 1.60 in column five indicates that additions to this development would have a negative impact on the optimal least cost solution by adding \$1.60 to total cost for each unit over 1452. If extra people were added to the total population, they would be located at S4D3 at a cost per unit of \$16.80. Hence, the addition to total cost is indicated by the negative 16.80, the last entry in column five of Table 4-5.

Reduced cost for solution four

In column six of Table 4-5 are located the Reduced Cost figures. If cost per unit of service could be reduced by \$62.32 in the case of S1D1, then it would have the same opportunity to be developed as would S4D3. A reduction of \$12.64 per combined service unit would make S1D2 a viable alternative to S4D3. If unit service cost were reduced for S1D3 by \$5.22 per unit, it too would be an alternative to development of S4D3. If unit service cost could be reduced by \$19.50 per unit, then S3D1 might become developed instead of S3D3; and if cost per unit of combined service dropped \$3.50, S3D3 might profitably be developed. Unit service cost for S4D1 would have to be reduced by \$36.00 before it would have the opportunity to compete with S4D3 for the limited resource capacities. S4D2 would need a reduction of \$13.13 in its per unit service cost before it could profitably become a part of the solution. S5D2 and S5D3 have been blocked from this solution.

Solution Five

Pattern of development

There existed the possibility that capacities in the amounts assumed for some service origins were set too high. After all, Treatment Plant 3 had not been constructed yet, and the facility at Byington had not as yet been expanded. If for any reason these facilities were not developed or were constructed with less than the assumed service capacities, subdivision development may be severely restricted. To see what changes would result in the optimal least cost solution, first, additional capacity was assumed zero at Sewage Treatment Plant 3. This would mean that development on Subdivision 5 would be without sewer

service. Therefore, residential development could not take place on Subdivision 5. The following solution occurred when Treatment Plant 3 capacity was reduced to zero (Table 4-6).

Subdivision 1 remained undeveloped in this solution. Subdivision 2, the same in all solutions, was developed for 870 people at Density 3. A certain percentage of this number were assumed to be of school age: 29 students to attend Bearden Elementary, 13 for Bearden Junior High, and 13 for Bearden High. S3D3 was next to be developed with 1650 people at high density. Out of the 1650 people, 103 would have been school aged. Only 55 students required service from Ball Camp Elementary. Education service (school capacity and transportation) was required by 24 students of Karns Middle School and the same number of students required service of Karns High School.

Subdivision 4 was developed at high density for 7980 people. This population was served by Treatment Plant 1 for sewer service and by the Transfer Station for solid waste service. Demands for 264 education service units were made upon Blue Grass Elementary by the students of elementary school age assumed for S4D3. Space and transportation for 117 students was required for Farragut Middle School. There were also 117 students assumed for Farragut High School. Since Treatment Plant 3 was assumed to have zero capacity, Subdivision 5 was left undeveloped. Cost increased considerably as development occurred in the absence of excess capacity at Sewage Plant 3. The minimum cost for residential development in this case was \$160,539.

The dual of solution five

Lack of capacity at Treatment Plant 3 was responsible for the

Table 4-6. Solution 5. Results of the Residential Model: Subdivisions and Densities Developed, Showing the Pattern of Allocating 10,500 People, Including Students. Cost: \$160,539

Service Origin and Service Delivery Points	Capacities at SOP's and SDP's (people) ^a	Pattern of Allocation (people) ^a	Unused Capacities (people) ^a	Dual Solution ^b (dollars)	Opportunity Cost Differential ^c (dollars)
1. Plant 1	9421	8850	571	--	
2. Plant 2	1650	1650	0	4.05	
3. Plant 3	0	0	0	9.96	
4. WHE	250	0	250	--	
5. BES	150	29	121	--	
6. BCE	120	55	65	--	
7. BGE	510	264	246	--	
8. FES	331	0	331	--	
9. BJH	406	13	393	--	
10. KMS	549	24	525	--	
11. FMS	468	117	351	--	
12. BHS	300	13	287	--	
13. KHS	213	24	189	--	
14. FHS	1232	117	1115	--	
15. S1D1	137	0	137	--	62.32
16. S1D2	432	0	432	--	12.64
17. S1D3	461	0	461	--	5.22
18. S2D1	258	0	258	--	11.50
19. S2D2	816	0	0	--	--
20. S2D3	870	870	0	10.55	--
21. S3D1	1649	0	1649	--	19.50
22. S3D2	5208	0	5208	--	3.50
23. S3D3	5555	1650	3905	--	--
24. S4D1	2402	0	2402	--	36.00
25. S4D2	7584	0	7584	--	13.13

Table 4-6 (continued)

Service Origin and Service Delivery Points	Capacities at SOP's and SDP's (people) ^a	Pattern of Allocation (people) ^a	Unused Capacities (people) ^a	Dual Solution ^b (dollars)	Opportunity Cost Differential ^c (dollars)
26. S4D3	8090	7980	110	--	--
27. S5D1	1452	0	1452	--	--
28. S5D2	4584	0	4584	--	--
29. S5D3	4890	0	0	--	--
30. Total	10500	10500	0	-16.80	--

^aFigures for schools, Rows 4 through 14, are in terms of students.

^bRate at which cost would decrease if capacity of some limiting resource were increased by one unit.

^cAmount that per unit service cost would have to be reduced before a particular subdivision, at a particular density, could be developed.

significant increase in costs for the development pattern of solution five. It follows, therefore, that cost reduction per unit of added capacity at Plant 3 would be as much as \$9.96. Cost reduction per unit of added capacity at Plant 2 was the same as in past solutions, \$4.05. If more people could possibly have been settled on Subdivision 2, each unit would have brought about a \$10.55 reduction in total cost. Since Subdivision 4 was the last subdivision to be developed at a cost of \$16.80 per service unit, any additional development over the 10,500 maximum population would cost \$16.80 per person. This explains the negative figure in the Total Row.

Reduced cost for solution five

Not much is changed in the Opportunity Cost Differential column from past solutions. S4D3 had a unit service cost of \$16.80 and was the highest cost development. Before those undeveloped tracts could be developed, their unit service costs would have to be reduced to no more than \$16.80. If S1D1 were to become a development alternative to S4D3, its service cost would have to be reduced by \$62.32 per unit (Column 6, Table 4-6). Before it could become an alternative to S4D3, S1D2 unit cost would have to drop \$12.64. Reduced cost for S1D3 was \$5.22. Cost per unit of combined service for S2D1 would need to drop \$22.05 in order to compete with S2D3 for development. Reduced cost for S3D1 was \$19.50, meaning that S3D1 would be a development alternative for S3D3 if unit service costs were reduced to \$12.75. S3D1 could be an alternative to S4D3 if unit cost were reduced by \$15.45 to \$16.80. Of course, S3D1 could not become an alternative unless capacity were increased at Plant 2. Reducing unit service cost of S3D2 by \$3.50 would

mean that S3D2 could compete for the same resources (population) going to S3D3. If service cost could be reduced by \$36.00 per unit, then S4D1 would be an alternative to S4D3. S4D2 unit cost would have to be reduced to \$13.13 to make it a development alternative to S4D3.

Solution Six

Pattern of development

The final pattern of development was obtained without the availability of sewage treatment capacity at Plant 2 (Table 4-7). This meant that Subdivision 3 could have no possibility for development. Overall costs would, naturally, be expected to increase.

As in the five previous development patterns, Subdivision 1 was not developed; its unit combined service costs were too high. Subdivision 2, having the lowest unit service costs at Density 3, was the first subdivision to be developed. The maximum number of people who could be housed at S2D3 was 870. This number would require collection of and treatment capacity for 870 units of sewage. Treatment capacity was provided by Plant 1. Solid waste service for 870 people was implicitly provided by the Transfer Station. With 870 people slated for S2D3, a little over 6% of this population were assumed to be of school age. There were 29 students of elementary school age who would need capacity at Bearden Elementary School. Bearden Junior High had more than enough space for 13 students from Subdivision 2. There were also 13 students of high school age from S2D3 who would need capacity at Bearden High School.

Subdivision 5 was next to develop. S5D2 could have accommodated 4584 people; only 1874 people were located there. The reason for this has been pointed out earlier. Available capacity at Farragut Elementary

Table 4-7. Solution 6. Results of the Residential Model: Subdivisions and Densities Developed, Showing the Pattern of Allocating 10,500 People, Including Students. Cost: \$148,553

Service Origin and Service Delivery Points	Capacities at SOP's and SDP's (people) ^a	Pattern of Allocation (people) ^a	Unused Capacities (people) ^a	Dual Solution ^b (dollars)	Opportunity Cost Differential ^c (dollars)
1. Plant 1	9421	8626	795	--	62.32
2. Plant 2	0	0	0	4.05	12.64
3. Plant 3	4890	1874	3016	--	5.22
4. WHE	250	0	250	--	--
5. BES	150	29	121	--	--
6. BCE	120	0	120	--	--
7. BGE	510	257	253	--	--
8. FES	331	331	0	56.40	--
9. BJH	406	13	393	--	--
10. KMS	549	0	549	--	--
11. FMS	468	261	207	--	--
12. BHS	300	13	287	--	--
13. KHS	213	0	213	--	--
14. FHS	1232	261	971	--	--
15. S1D1	137	0	137	--	--
16. S1D2	432	0	432	--	--
17. S1D3	461	0	461	--	--
18. S2D1	258	0	258	--	--
19. S2D2	816	0	0	--	--
20. S2D3	870	870	0	10.55	--
21. S3D1	1649	0	1649	--	19.50
22. S3D2	5208	0	5208	--	3.50
23. S3D3	5555	0	5555	--	--
24. S4D1	2402	0	2402	--	36.00
25. S4D2	7584	0	7584	--	13.13

Table 4-7 (continued)

Service Origin and Service Delivery Points	Capacities at SOP's and SDP's (people) ^a	Pattern of Allocation (people) ^a	Unused Capacities (people) ^a	Dual Solution ^b (dollars)	Opportunity Cost Differential ^c (dollars)
26. S4D3	8090	7756	334	--	--
27. S5D1	1452	0	1452	--	11.56
28. S5D2	4584	1874	2710	--	--
29. S5D3	4890	0	0	--	--
30. Total	10500	10500	0	-16.80	--

^aFigures for schools, Rows 4 through 14, are in terms of students.

^bRate at which cost would decrease if capacity of some limiting resource were increased by one unit.

^cAmount that per unit service cost would have to be reduced before a particular subdivision, at a particular density, could be developed.

School was 331, and 331 students were assumed of elementary school age from a total population of 1874 at S5D2. Students of junior high school age number 147 at S5D2 (by assumption). Sufficient capacity existed at Farragut Middle School for these students. There were also 147 students of high school age requiring service capacity at Farragut High School.

Subdivision 4 was next and last to develop in Solution Six. S4D3 developed for 7756 people. Treatment Plant capacity was available from Plant 1 and solid waste service was implicitly provided from the Transfer Station. There were 257 students assumed who would need service from Blue Grass Elementary. Farragut Middle School would need to provide capacity for 114 junior high school students from S4D3; and Farragut High would need to provide capacity for 114 high school students from S4D3.

Total minimum cost of providing services for 10,500 people located according to the pattern established by Solution Six was \$148,553.

The dual of solution six

The cost reduction for a unit of capacity added to Treatment Plant 2 was \$4.05 as indicated in column five of Table 4-7. The most significant reduction in cost could be brought about by adding capacity to Farragut Elementary School. One unit of service capacity added to this school would have meant a reduction of about \$56.40 in total cost. Adding capacity to Farragut Elementary would have enabled more people to be located on Subdivision 5 and less people to be located on Subdivision 4. Significant cost savings would have resulted. If S2D3 could possibly take more than 870 people, the cost reduction per unit would be \$10.55. This is not possible because S4D3 is fully developed with 870

people. Adding more units to the total allocation of 10,500 would mean adding \$16.80 per unit to total cost. This is indicated by the negative number in column five in the last row of Table 4-7.

Reduced cost for solution six

Figures in the Opportunity Cost Differential Column (Table 4-7) appear basically the same as they did in all previous solutions. The primary reason for this stems from the fact that input costs (cost per unit of combined service) have not changed in any solution. Unit service cost for S1D1 was still \$62.32 higher than the cost per unit for the last subdivision to be developed, S4D3. S1D2 was \$12.64 above the unit service cost of S4D3, \$16.80. Service cost would need to be reduced by \$5.22, from \$22.02 per combined service unit to \$16.80 per combined service unit, before S1D3 could profitably be developed. To enable S3D1 to profitably enter the solution, its unit service cost would need to be reduced by \$19.50; it would then be an alternative to S3D3. To make S3D2 an alternative for S3D3, unit service cost of S3D2 would need to be reduced only \$3.50. But, since Plant 2 capacity was assumed zero in this pattern, the reduced cost figures for Subdivision 3 are meaningless. Unit service cost would need to come down \$36.00 to make S4D1 a viable alternative to S4D3. If cost per combined service unit dropped \$13.13, S4D2 could be developed profitably. The last figure in column six indicates that unit service cost would have to be reduced by \$11.56 before S5D1 could profitably be developed.

CHAPTER V

SUMMARY

I. SUMMARY

The approach of this study was to develop a model of residential location such that the delivery cost of three services--sewers, solid waste disposal, and primary and secondary education--could be minimized. An economic engineering analysis was used to "build" representative cost data for a set of five subdivisions. These subdivisions differed in size, location, and distances from service origin points. They ranged in size from 18 acres to 316 acres and were located in a corridor bounded by a ridge on the north and by a river on the south. The study area was about 13 miles long by about 6 miles wide and went from the city limits of a municipality to the county line. The greatest distance between any two subdivisions was about nine miles; the shortest distance between any two subdivisions was just under two miles. Distance from subdivisions to service origin points varied from zero to about 15 miles. Cost information was needed for each service, and data were obtained that gave the combined cost per capita for the three services. Delivery costs considered were only partial costs--transportation costs only. The linear programming model used to analyze the data has limited applicability to overall land-use planning problems. However, the results of such an analysis could be used to indicate a less costly direction of residential growth in terms of the services supplied to a relatively small area.

II. MAJOR FINDINGS

As expected, medium and high density developments (eight dwellings per acre and 16 dwellings per acre) were able to provide services at lower costs than were the low density developments (two dwellings per acre). Cost savings were generally greater between low density and medium density development than between medium density and high density development. Difference in family size between these two densities account for this situation. Average family size for medium density was three persons per dwelling which meant an average of 24 people per acre. Average family size for high density was 1.6 persons per dwelling, indicating an average of 25.6 persons per acre. Average cost would not differ a great deal under these circumstances if total costs were similar.

Subdivision 1, at low density, had a combined cost of \$79.12 per capita per year for the three services. This was over 2 1/2 times the cost for medium density development, and just over 3 1/2 times the cost of providing services to the high density development.

Subdivision 2 had a cost of \$28.30 per capita per year for services at low density. This was almost twice as much as at medium density and 4 1/2 times as much as at high density development.

Subdivision 3 could be delivered the package of services for \$32.25 per capita per year at low density. This was about twice as much as at medium density and just over 2 1/2 times the cost per capita per year for delivering the three services to the high density development.

Subdivision 4 per capita cost per year was \$52.80 for low

density. This was about 1 3/4 times the same cost for medium density and just over 3 times as much as the per capita cost at high density.

Finally, per capita service delivery cost was \$18.40 per year for the low density development on Subdivision 5. This was less than 3 times the cost for medium density, but it was just over 2 times the cost of three services for the high density development. For Subdivision 5, medium density development was lower in service delivery costs than was the higher density development. The reason for this was that sewer collection and transmission cost for Subdivision 5 was only \$5.42 per capita per year for medium density, but \$7.16 per capita per year for high density. More of the larger diameter, and more expensive, sewer pipe was required for high density development than for medium density development.

The subdivision that had the lowest service delivery cost per capita per year was Subdivision 2, Density 3. The highest cost subdivision, Subdivision 1, Density 1, was over 12 1/2 times more costly than the lowest cost subdivision. Per capita service delivery costs for high density development ranged from a low of \$6.25 per year for Subdivision 2 to \$22.02 per year for Subdivision 1. For medium density development, per capita service delivery cost ranged between \$6.84 per year for Subdivision 5 to \$29.93 per year for Subdivision 4. For low density, service delivery cost for the three services ranged from \$18.40 per capita per year for Subdivision 5 to \$79.12 per capita per year for Subdivision 1.

The least expensive subdivision to the most expensive subdivision, in terms of service delivery cost, were arranged as follows:

- (1) Subdivision 2, Density 3
- (2) Subdivision 5, Density 2
- (3) Subdivision 5, Density 3
- (4) Subdivision 3, Density 3
- (5) Subdivision 2, Density 2
- (6) Subdivision 3, Density 2
- (7) Subdivision 4, Density 3
- (8) Subdivision 5, Density 1
- (9) Subdivision 1, Density 3
- (10) Subdivision 2, Density 1
- (11) Subdivision 1, Density 2
- (12) Subdivision 4, Density 2
- (13) Subdivision 3, Density 1
- (14) Subdivision 4, Density 1
- (15) Subdivision 1, Density 1

This arrangement is the order in which subdivisions would be developed if there were no restrictions placed upon the linear programming model.

Sewer service costs exercised the greatest influence on the cost per capita of the package of three services. Education service costs had somewhat less influence on combined costs, and solid waste service costs had very little influence on the relative costs of the combined package of services under the assumptions made in the analysis. Sewer service delivery costs ranged from \$3.38 per capita per year for Subdivision 2 Density 3, to \$66.26 per capita per year for Subdivision 1, Density 1. Education service delivery costs ranged from zero (because this subdivision was located adjacent to schools and transportation cost was zero) for all three densities at Subdivision 5 to a high of \$24.12 per capita per year at Subdivision 4, Density 1. Solid waste service delivery cost ranged between \$.92 per capita per year for Subdivision 1, Densities 2 and 3, to \$1.42 per capita per year for all three densities of Subdivision 5. Density within subdivisions had little influence on the delivery cost of solid waste service. Since Subdivision 3, 4, and 5 were all approximately the same distance from the solid waste transfer

station, there was not a very great difference in costs--about \$1.36 per capita per year from Subdivision 4 to \$1.42 per capita per year for Subdivision 5.

Service cost data were analyzed using a linear programming framework with the post optimal technique of "parametric programming." There were six solutions using this analytical technique. The first solution gave absurd results. Solution Two, obtained by using parametric programming on the variables of solution one, allocated 10,500 people in the following manner:

Subdivision 2, Density 3 was allocated its maximum of 870 people, 55 of whom were school age children. Next to be developed was Subdivision 5. There were 1179 people at Density 2 and 3711 people at Density 3. This was the only solution where a subdivision was developed at more than one density. The number of school age children at Subdivision 5 was 625. Following Subdivision 5, Subdivision 3 was developed with 1650 people at high density. Of this number, 103 were school age children. Subdivision 4 was then developed with 3090 people, 192 of whom were school age children. This concluded the total allocation of 10,500 people for Solution Two at a total cost of \$118,292. Allocation to Subdivision 2 was limited by the maximum number of people (870) that could be housed on it at high density. Subdivision 5 was constrained by the available capacity of its elementary school and its treatment plant capacity. Subdivision 3 was constrained by available capacity at its treatment plant. The allocation to Subdivision 4 was limited by the total number of people to be located by the model--10,500 people. There were already 7410 people in Subdivision 2, 3, and 5, leaving 3090 to be located in Subdivision 4.

The pattern of development for Solution Three was as follows:

Subdivision 2 and 3 were the same as in Solution Two. Subdivision 5 was developed at Density 2 for 1874 people, 625 of whom were students. It was constrained to 1874 by the available capacity at the elementary school of 331 students. Subdivision 4, Density 3, was developed for 6106 people, of whom 382 were school age children. Subdivision 4 was constrained to 6106 by the total number, 10,500, of people to be allocated in a solution. Total cost for Solution Three was \$141,871.

Solution Four allocation was the same as for previous solutions with respect to Subdivision 2 and 3. Subdivision 5 was forced into the solution at low density with 1452 people. This was done to see how much costs would rise over other solutions where no low density development occurred. There were 765 students in Subdivision 5, Density 1. (This solution required that capacity at the elementary school be increased to 405 students). Subdivision 4, Density 3, was developed for 6528 people, the remainder of the total allocation. There were 408 students in Subdivision 4 in this solution. Total cost for this residential development pattern was \$162,862.

Solution Five reflected the effect upon the pattern of development of eliminating available capacity at Treatment Plant 3. Here again, Subdivisions 2 and 3 were developed as before. Subdivision 5 was not developed because there was no treatment plant capacity. Subdivision 4, Density 3, was developed with 7980 people, 498 of whom were students. Total cost of this pattern of development was \$160,539.

Solution Six reflected the effect of eliminating available capacity at Treatment Plant 2 upon the pattern of development and total costs. Subdivision 2 was developed as before but Subdivision 3 was left

undeveloped since there was no available capacity at Treatment Plant 2. Subdivision 5, Density 2, was developed for 1874 people, 625 of whom were students. The limited capacity of the elementary school kept Subdivision 5 from developing further. Subdivision 4, Density 3, was allocated 7756 people, of whom 485 were students. Total cost for Solution Six was \$148,553. The least cost solution was Solution Two at \$118,292.

In conclusion, this research has indicated a pressing need for reliable data for land-use planning in the study area. The area has grown rapidly, both commercially and residentially. There has been opposition from homeowners to the high density developments that have been forced upon them. Neither the opposition nor those who favor more dense residential development have been backed up in their positions with the research to indicate the consequences of such development. This research has shown, for example, that high density residential developments place very little additional burden on existing school facilities in the area. High density developments usually have smaller families per dwelling (estimated at 1.6 persons per dwelling by the Knoxville-Knox County Metropolitan Planning Commission) and only an average of one school age child for every ten families, according to estimates of county school officials.

This research determined which subdivision density combinations were cheapest to provide sewerage for the alternatives considered. Armed with this information, planners and city officials could influence the direction of residential growth by providing sewerage to those areas which they wanted to develop first. The model used in this research pointed to one subdivision-density combination where all three services could be provided at minimum cost. Municipal leaders might possibly

provide incentives in some form, to influence private developers to build on this piece of land.

The analytical model was limited to the consideration of only three quantitative variables in this research. The costs of providing sewers, solid waste disposal, and primary and secondard education are important considerations for any development. There are other variables that are also important--shopping facilities, location of employment, recreational areas, police and fire protection, and environmental amenities. These additional variables need to be looked at with respect to residential location. A minimum cost linear programming model such as the one used in this research is too limited to deal, practically, with all these variables. Linear programming is further limited as implied by the name, "linear." This means that relationships are always straight line relationships when using this technique. For example, if one unit of service cost one dollar, ten units of this service cost ten dollars. But linear programming is a useful tool that can be used on a small scale while recognizing its limitations. A related technique, transportation programming, could be used to help locate shopping centers, parks, libraries, cultural facilities, and other services to serve the greatest number of people by minimizing the cost of transportation for residents to these facilities.

This research has provided only a small start in researching the problems associated with land use. More information is needed for the study area as concerns the relationship between residential densities and family size. It would also be helpful to know how services are paid for and whether or not the ones who benefit from services pay the full cost of these services. West Knox County is projected to grow rapidly in

the future. What will be the composition of this growth? Will there be more demand for high density residential development, or will there be continued demand for relatively low density single family development? Budgets need to be developed for providing residents with various services, and the costs associated with expanding existing services facilities as well as the cost of new facilities needs to be known with certainty.

The technical expertise is available at institutions such as The University of Tennessee for providing policymakers with research analysis of land-use problems. Planners and government officials could help in this endeavor by developing continuing, reliable data series.

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VITA

Ronald Doyle Weddel was born in Fort Worth, Texas, on June 28, 1940. He attended public schools in that city and was graduated cum laude from Polytechnic High School in 1958. The following September he entered Texas Christian University and attended one year. He worked as a surveyor's assistant for a year and a half, then entered and spent four years in the U.S. Air Force. Upon separation from the military, he entered The University of Texas at Arlington, and in August 1966 graduated with honors with a Bachelor of Arts degree in Economics. In September 1966, he accepted a research assistantship at The University of Texas at Arlington and began study toward a Master's degree. He received this degree in January 1968 and shortly thereafter went to work as a financial economist in the Research Department of the Federal Reserve Bank of Dallas, Texas.

He was awarded an assistantship and entered the Graduate School in the Department of Agricultural Economics and Rural Sociology at The University of Tennessee in July 1969. He received the Doctor of Philosophy degree with a major in Agricultural Economics in August 1974. He is a member of the American Agricultural Economics Association and the Southern Agricultural Economics Association.

He is married to the former Brenda Carol Owen of Fort Worth, Texas.