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How Textbooks Influence Students' Algebra Learning: A Comparative Study on the initial treatment of the concept of function

Qintong Hu
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To the Graduate Council:

I am submitting herewith a thesis written by Qintong Hu entitled "How Textbooks Influence Students' Algebra Learning: A Comparative Study on the initial treatment of the concept of function." I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Teacher Education.

Ji-Won Son, Major Professor

We have read this thesis and recommend its acceptance:

JoAnn Cady, Vena Long

Accepted for the Council:
Dixie L. Thompson

Vice Provost and Dean of the Graduate School

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How Textbooks Influence Students' Algebra Learning:

A Comparative Study on the Initial Treatment of the Concept of Function

A Thesis

Presented for the

Master of Science

Degree

The University of Tennessee, Knoxville

Qintong Hu

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Abstract

To give insights into cross national differences in schooling this study analyzed the initial treatment of the concept of function in three curricula: a US standards-based text--*Connected Mathematic 2: Variables and Patterns*, a US conventional text--*Glencoe: Mathematics Applications and Concepts: Course 2*, and a Chinese reform text--*Shu Xue: Grade 8, first volume*.

This study examined content organization and problem features in the three textbooks. For content analysis, this study explored how the concept of function was introduced, defined, and developed. The results indicated both of the US textbooks introduce this concept at grade 7 whereas the Chinese text does so at grade 8. *Connected Mathematics* devotes more lessons than the Chinese text and *Glencoe* in the initial treatment of the concept of function. *Connected Mathematics* defines function as rule while *Glencoe* addresses it as relationship; the Chinese text introduces the concept of function as correspondence. *Connected Mathematics* pays equal an amount of attention to the four representations including tables, graphs, verbal descriptions, and equations examined in this study. In contrast, *Glencoe* employs the representations of tables, graphs, and equations and it focuses on the representation of graphs; the Chinese text also employs the representations of tables, graphs, and equations but it focuses on the representation of equation. The Chinese text provides many explanations and illuminations in worked-out examples to tell how the solutions are derived.

Problems were then analyzed extensively with respect to three criteria: (1) contextual feature, (2) response type, and (3) cognitive expectation. Analysis results showed that all the three texts emphasize the cognitive expectation of representation.

Connected Mathematics provides more real-world problems than other texts; and the problems aim at cultivating students' mathematical reasoning. Most of the problems in *Glencoe* are embedded in pure math contexts to help students do procedure practice. The problems in the Chinese text emphasize problem solving. Implications for curriculum developers, teachers, and researchers have been discussed in accordance with the findings.

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Chapter 1

INTRODUCTION

Over the past two decades, increasing the overall level of student understanding and mathematical proficiency has been a major goal in teaching mathematics. The documents from National Council of Teachers of Mathematics [NCTM] (1991, 2000) serve as guidelines for various aspects of classroom mathematics in order to improve students' mathematical understanding and problem reasoning. Along the same line, No Child Left Behind [NCLB] requires high expectations for all students to learn mathematics, in particular, asking students to take algebra no later than the eighth grade.

However, international assessment studies including the *Program for International Student Assessment* (PISA) and the 2003 *Trends in International Math and Science Study* (TIMSS) reveal that American students still trail their international counterparts in math and science when compared to their counterparts from economy competing countries, such as Japan, Korea and China (Gonzales et al., 2004; Lemke, 2004). For example, in the PISA report, the US 15-year-olds were ranked 24th among peers from 29 other nations in math literacy.

What is the best route to improve students' math achievements? According to Schmidt et al. (1997), who compared the mathematics and science curricula of the US to those of 46 nations, "In the most ambitious cross-national study of standards and textbooks ever attempted, the intended curriculum of the US was found not to measure up to the most common expectations for student learning found in other nations" (Valverde & Schmidt, 2000). This indicated that the texts in the US lack the coherence, focus and level of demand that are prevalent across the world. In addition, Schmidt et al. reported that American textbooks are viewed as “a mile long and an inch deep”, which indicates that there are many topics covered by textbooks but none of them is elaborated explicitly to facilitate students’ understanding. Although various factors influence the final math outcomes, textbooks still "dominate instruction in elementary and secondary schools" (Farr, Tulley, & Powell, 1987, p. 59). On average, 75% to 90% of classroom instruction is organized around textbooks (Tyson & Woodward, 1989; Woodward & Elliott, 1990). As Osborn, Jones, and Stein (1985) argue, improving textbooks used in American schools is an essential step toward improving American schooling.

Statement of the Problem

In response to the criticism on the quality of the US textbooks, many organizations and publishing companies began to create standards-based curriculum with the National Science Foundation (NSF) funding in ways that improve the quality of textbooks to meet standard curriculum document. In the late 1980s and early 1990s, the NCTM published the first round of its standards documents, which provided recommendations for improving and reforming K-12 mathematics. In accordance with the NCTM *Standards* documents, several curriculum materials have been developed aiming at engaging students in doing mathematics by understanding the why as well as the how of the mathematics they study.

Accordingly, numerous studies have been conducted to compare traditional curricula with standards-based curricula and examine whether standards-based curriculum is efficient enough for students to learn mathematics (Fuson et al., 2000;

Huntley et al., 2000; Reys, 2003; Riordan & Noyce, 2001). It has been reported that standards-based curricula are more efficient in helping students improve their mathematical abilities and achievements. However, comparison between conventional curriculum and standards-based curriculum is not enough in terms of catching up with the counterparts from other countries in international examinations. Kilpatrick, Swafford, and Findell (2001) argue that research that looks across countries can provide a better picture of what matters in instruction aimed at developing proficiency. Thus, comparing curriculum with other countries is necessary.

This study intended to compare a US standards-based textbook, a US conventional textbook with a reformed Chinese textbook. Unlike US, China is much more competitive in international math examinations and used to have a national curriculum. All regions around the nation have to use the same math textbook which is entirely conducted by the central government. However, with the rapid development of economy in China, different areas of the nation with different needs and resources asked for developing differentiated textbooks. Unlike the previous curriculum reforms, the latest reform initiating from 2005 placed students' development at the center of the curriculum. In addition, this reform intended to implement three critical transformations: the transformation from "centralization" to "decentralization" in curriculum policy, the transformation from "scientific discipline-centered curriculum" to "society construction-centered curriculum" in curriculum paradigm, the transformation from "transmission-centered teaching" to "inquiry-centered teaching" in teaching paradigm (Zhong, 2006). Similar to the reform movement in the US, various kinds of textbooks have been developed, published, and applied to various school districts. Additionally, both reformed curricula in the US and China are viewed to implement inquiry-based teaching. Comparing the US reform-oriented curricula with those in China will provide an insight about whether and how reformed curricula in the US and China give different learning opportunities to the students and whether these differences are the reasons why the students from these two countries have performance gap in international math competitions.

Algebra is one of the five content standards in NCTM's *Principles and Standards*. It is well known that the concept of function is critical for students' algebra learning. NCTM (2000), for example, stressed that the concept of function should be placed as one of the cornerstones of mathematics curricula: algebra. Additionally, this concept is viewed as the underlying theme when developing algebraic ideas (Laugh Baum, E., 2003). Furthermore, algebra is helpful to daily life, from applying formulas for calculating miles per gallon of gasoline to using functions to determine the profit of a business venture.

However, as discussed above, the US students showed the poor performance on the topic of algebra, in particular, on the topic of function on international assessments of mathematics ability (Stedman, 1997). For example, on the algebra subtest of the 2003 Trends in International Mathematics and Science Survey (TIMSS), the US 8th graders scored below many economic competitors, such as Japan, Russia, Korea, Singapore and China. In addition, the US students were 16% behind international average in algebra and 18% behind the international average in functions, but only 14% behind the international average in geometry in Second International Mathematics Study. Thus, much more attention should be paid to algebra and, especially function. However, there are a few studies on how textbooks provide learning opportunities for students to learn the topic of functions.

Purpose of the Study

The purpose of this study is to illuminate the national and the cross-national similarities and differences in ways of conceptualizing and presenting the concept of function in the US textbooks and the Chinese textbook. Additionally, this study examines how the problems are presented in the US and Chinese textbooks on the concept of function. This study has five research questions:

(a) When is the concept of function introduced in the US and Chinese textbooks?
(b) What are the learning goals when the concept of function is initially introduced in each curriculum?
(c) How is the concept of function introduced and developed in each curriculum?
(d) How many and what types of problems in the concept of function are presented in each curriculum?
(e) What kinds of learning opportunities are provided with respect to cognitive expectation in each curriculum?

Significance of the Study

Answers to the research questions will provide various implications to classroom instruction, curriculum development, and mathematics education and so on. First, based on the analysis of learning goals, teachers may enhance the requirements for the students to encourage their algebraic thinking as Cai (2000) suggested that teachers' expectations and encouragements directly influence ways of students' mathematics learning.

Second, the teachers could observe the difference between standards-based curricula and conventional curricula. Since there are still a large number of conventional textbooks dominating the classrooms, now might be a good time to persuade school boards and teachers to rethink their choice. Differences from the comparison of the Chinese text and the US texts will provide teachers some information on how to help students to improve their academic achievements. Additionally, the gap between learning goals, content presentation, and the problems presentation could draw teachers' attention to find other resources to fulfill instructional goals.

Third, through examining how the concept of function is introduced and developed in each curriculum, it is clear to see the whole process where students develop their algebraic thinking. This might further indicate why American students do not perform as well as their Chinese counterparts on algebra tests. Analyzing the problems in the textbooks will indicate what kind of learning opportunities are provided to students, and this might be one of the critical explanations to the statements that Chinese students outperform their American counterparts and students using standards-based texts do better than those with conventional texts (MacIver, 2009).

Fourthly and finally, based on the findings of this study, curriculum developers might improve the qualities of the problems presented in textbooks to provide more learning opportunities and to bridge the gap between the learning goals and the problems for practice. Comparing the learning goals could help curriculum developers to see the difference between China and America in students' learning expectations. As discussed previously, American textbooks lack coherence, focus, and level demands; they may borrow something to promote the organization of the curricula.

Organization of the Study

After an introduction of the problem investigated in this study, a complete review of related literatures, including the research on standards-based curricula, the research on international curriculum comparison, and the research on teaching and learning of functions, is elaborated in Chapter 2. Chapter 3 contains materials employed in the study, the analytical framework, and how the content analysis and the problem analysis are designed. Chapter 4 reports the results from the content analysis and the problem analysis. Chapter 5 contains the summary of the study, the main findings of the study, conclusions, and the implications for the filed of math education.

Chapter 2

LITERATURE REVIEW

In this chapter, previous curriculum studies on standards-based curricula, on international curriculum comparison, and on the teaching and learning of functions will be discussed first to illuminate what has been done in curriculum analysis. After reviewing the results of prior research, the limitations of the current research literature will be discussed in order to provide the needs for the present study.

Research on Standards-based Curricula

With the implementation of standards-based curricula various research has been done to find students' corresponding achievements, to investigate teachers' potential usage and learning of reformed curricula, and to analyze how standards-based curricula are different from conventional curricula in terms of material organization and problem representation. Since the ultimate purpose of developing standards-based curriculum is to improve students' mathematics achievements, a large body of research has compared achievement improvements between students using standards-based textbooks and those using conventional textbooks. Riordan and Noyce (2001), for example, examined the difference between standardized text scores in grade 4 using standards-based textbooks, *Everyday Mathematics* and *Connected Mathematics,* and those using a mix of traditional textbooks. Results indicated that students using either of the standards-based programs as their primary math curriculum outperformed significantly in statewide mathematics test their counterparts attending matched comparison schools. Similar conclusions have been drawn by other studies. Reys (2003), for instance, assessed the impact of standards-based curricula on students' mathematics achievements in middle grades. Significant achievement differences were indentified in content areas of data analysis and algebra between students using standards-based curricula for at least two years and students from comparison districts using other curriculum materials. Similarly, Fuson et al. (2000) investigated arithmetic achievements among three groups—(1) the US students using *Everyday Mathematics* [EM], (2) the US students reading traditional curriculum, and (3) their Japanese counterparts—in order to illustrate the role standards-based curriculum played in increasing students' academic gains. Consistent with previous studies (eg., Reys, 2003), Fuson et al. reported that standards-based curricula performed excellently in improving students' academic achievements. In particular, Fuson and her colleagues showed that: in the number-sense test, the EM second graders were equivalent to the Japanese group who came from middle-class families. This finding is important since previous studies and international mathematics contests always suggest that American students fall behind their Japanese counterparts. The finding from Fuson et al's study recommends a way for the US students to be competitive with students from high-achieving countries when they use EM.

In addition to improving achievement, standards-based curricula have also been reported to stimulate students' *problem solving abilities* while conventional curricula focus on procedure practice. Huntley et al. (2000) researched the effects of the Core-Plus Mathematics Project [CPMP] curriculum and conventional-oriented curricula on the growth of students' understanding, skill, and problem solving ability in algebra. The results show that CPMP is more effective than conventional curricula in developing students' ability to solve algebraic problems embedded in real life contexts while conventional curricula are more effective in developing students' skills in manipulation of symbolic expressions in algebra when the expressions are presented free of application contexts.

Although standards-based textbooks are proven being effective in improving students' mathematics achievements; other studies also reported the effectiveness of traditional textbooks. Sood and Jitendra (2007), for example, compared number-sense instruction in three first-grade conventional math textbooks and one standards-based textbook, *Everyday Mathematics* [EM]. All instructional parts of the textbooks--big ideas, conspicuous instruction, mediated scaffolding, and judicious review--were coded by the authors and they found that the standards-based textbooks emphasized real world connections and did a better job than conventional textbooks in promoting relational understanding and integrating spatial relationship tasks with other more complex skills.

However, Sood and Jitenra reported that the conventional textbooks provided more opportunities for number relationship tasks, more direct and explicit instruction, more common feedback as well as more practice on number-sense skills. Since real world connection, relational understanding, and spatial relationship are more complicated than number-sense skill practice, standards-based textbooks have a more valid base of improving students' problem solving and conceptual understanding. If number-sense is critical in elementary school math where arithmetic is the main point, the idea of algebra is the counterpart of number-sense for middle graders.

Nie et al. (2009) also analyzed the intended treatments of the ideas of variable in a standards-based text and a traditional text. They reported that the standards-based curriculum includes a strong connection among variables, equation solving, and linear functions. On the other hand, the conventional curriculum does not emphasize either the connection between variables and functions or that between algebraic equations and functions, but it concentrates on the relation between variables and equation-solving.

As stated above, standards-based curricula outperform conventional curricula in improving students' academic achievements and their mathematical abilities. However, conventional textbooks still dominate a large number of math classrooms. To stimulate math teachers to make significant changes of applying appropriate standards-based curricula, much more curriculum analysis between standards-based curricula and conventional curricula on various math topics need to be done in the future.

Research on International Curriculum Comparison

A large number of comparative curriculum analyses have been done since curriculum is always regarded as an important element influencing students' academic performance. Different countries have particular culture values, social systems, and economic needs, so their curricula also vary in order to meet the domestic needs. Although curricula must be different basing on different countries' situations, it is necessary to investigate what has been done in the curricula of high-achieving countries, such as Singapore, Japan, and Korea. Among various comparative studies, some emphasize the content organization while others emphasize problem analysis. Content organization is developed by examining topic placement in textbooks, difficulty levels of the content, treatments of specific concept, and students' achievement expectations. On the other hand, problems appearing in textbooks have been analyzed to see what kind of learning opportunities are provided to students based on different types of problems at different instructional points.

Li and Ginsburg (2006) did a textbook analysis in socio-cultural contexts, in which classification and framing of mathematical knowledge in Hong Kong, mainland China, Singapore, and the US was examined. They found that three Asian systems' mathematics textbooks exhibited a higher degree of classification and framing than that of the US textbooks. Being influenced by Confucian culture, curricula from Asian countries receive more government control as they are regarded as centralized national curricula, while the US curricula are decentralized.

In fact, eastern countries not only did a better job in classification and framing of math knowledge but also outscored the US in the efficiency of the treatment of content. Fuson et al. (1998) investigated grade placement of addition and subtraction topics in Japan, China, the Soviet Union, Taiwan and the US. They discovered that multi-digit addition and subtraction appear from one to three years earlier in other countries than in the US. Moreover, American textbooks are more likely to repeat previous topics than the other countries. All these conclusions are consistent to the former statement that the US curricula lack the coherence, focus and level of demand that are prevalent across the world (Valverde & Schmidt, 2000).

Further studies have been done to show differences and similarities in concept treatments and problem representations of curricula from the US and other countries. Cai et al. (2002) compared intended treatment of arithmetic average in the US textbooks and the Asian school mathematics textbooks. The Asian series teach the arithmetic average as computational algorithm in terms of conceptual and procedural understanding, while the US series teach the concept as a representative of a data set in terms of the statistical aspect. Since the concept of fraction is always an emphasis for textbook examiners, Freiman and Volkov (2004) developed a study of the presentation of common fractions in textbooks from China, the US, Canada and Russia. Results suggested that the Chinese series did not attempt to provide any geometrical interpretation for the notion of fractions nor manipulatives to facilitate students' understanding. Instead, they focused on arithmetical methods. However, present North American textbooks have paid their attention to real life situations and visual models.

From these studies, it seems that Asian textbooks devote their efforts to helping students develop arithmetic skills while western countries' textbooks did better jobs provide real life connections, visual representations, and conceptual understandings. Aligning with the results of these studies, Park and Leung (2006) stated that western countries' textbooks helped students find math in real life while Asian countries succeed in conveying math in an economic way but failed to motivate students to learn.

While problem-solving and conceptual understanding are important mathematical skills reflected by NCTM standards, comparative studies also have compared curricula from eastern and western countries to investigate their efforts devoted to develop these critical mathematical abilities. Mayer and Sims (1995) conducted a study to compare how textbooks teach mathematics problem-solving in Japan and the US because there was a hypothesis that typical Japanese textbooks were more oriented toward teaching problem-solving and conceptual understanding skills whereas the US textbooks were more oriented toward teaching isolated facts and rote computation. The research results to some extent confirmed the assumption. The Japanese textbooks devoted much more pages to instructional lesson than the US textbooks while the exercise sets were about the same length in both countries. Moreover, the Japanese textbooks excelled in providing more worked-out examples, concrete explanations as well as illustrations of problem-solving procedures and meaningful instructional methods emphasizing the coordination of multiple representations. On the other hand, the US textbooks were found to focus on unsolved exercises and interesting-grabbing illustrations. Since relevant explanations and illustrations are much more effective than unsolved exercises in helping students develop problem-solving abilities, the results imply that much more concrete examples and illustrations should be included in American textbooks.

To further elaborate the differences between textbooks from eastern and western countries in developing students' problem solving skills, Fan and Zhu (2007) coded problems from Chinese, Singapore, and American textbooks to analyze representations of problem-solving procedures there. The Singapore texts presented specific heuristics in a much more explicit way than the Chinese and American texts. The Chinese texts outperformed the others in presenting various heuristics. Compared to the Chinese and Singapore textbooks, the problems in the US texts were much more like traditional routine work emphasizing visual representations.

Many more problem analyses have been done in comparative studies to find different learning opportunities embedded in textbooks. Li (2000) analyzed problems following selected content representations in the Chinese and American textbooks in terms of what he called the mathematical feature, the contextual feature, and the performance requirements. The results showed that the difference in mathematical feature and contextual feature was smaller than the difference in performance requirements. The American texts presented various kinds of performance requirements while the Chinese texts focused on numerical answers resulting from procedure practice. In addition, Zhu and Fan (2006) conducted a study of comparison of the representation of problem types in the Chinese and American textbooks. Employing another framework, the authors suggested that the US textbook developers should consider including more multiple-step problems as well as much more challenging problems. However, the Chinese textbooks should provide more real-world problems within application contexts.

Since previous research applied either content analysis or problem analysis to illuminate the learning opportunities provided by the US textbooks and the Chinese textbooks, in this study both content analysis and problem analysis will be employed to illustrate a full picture of the difference of the US textbooks and the Chinese textbooks in presenting content knowledge and providing exercise opportunities.

Research on Teaching and Learning of Functions

Function is one of the underlying themes when developing algebra ideas (Laughbaum, 2003). There is a motivation to investigate how school mathematics curriculum treats this critical topic. The function concept is a complex idea whose power and richness permeate almost all areas of mathematics. Previously, this topic did not come to school mathematics until the secondary level. However, NCTM *Standards* documents (1989, 2000) called for a functional emphasis to be integrated throughout the school curriculum from the elementary level. They state that students should be exposed to algebraic ideas from elementary level to be well prepared for formal algebra learning in middle and high school (Cai, 2008).

Because of the complex features and various applications of the concept of function, various kinds of representations, including equations, tables, graphs, and verbal descriptions, can be applied depending on different functional situations. Since understanding of functions in one representation will not necessarily correspond to the understanding in another representation, translating among different representations is important to problem solving. Even (1990) found that when these representations were combined, information from the combination facilitates a more deep and comprehensive understanding of the underlying functional situation. Moreover, NCTM's *Principles and Standards for School Mathematics* (2000) states that students should be able to understand functions, use various representations for them, and convert among these various representations. Thus, curriculum materials should include problems and activities that would help students be efficient in grasping different function representations and translating among these representations. Hartter (2009) also stresses that it is critical to provide experiences that enable students to make connections between multiple representations of the concept of function.

In addition, Lloyd and Wilson (1998) who examined the impact of a teacher's conceptions of functions on his implementation of a reformed curriculum suggested that comprehensive and well-organized conceptions are the necessary precondition fundament for teaching that makes conceptual connections, provides various representations and generate meaningful discussions. In particular, pointing out that textbook materials contribute to teachers' implementation, Lloyd and Wilson suggest that if different representations and the conceptual connections are emphasized by textbook content, teachers might devote more effort to helping students understand the meanings of various representations and capturing the underlying relationships among the representations.

Given that algebra is an abstract concept for students when they first encounter it, to understand the concept of function, instruction should be embedded in contexts familiar to students. NCTM (2000) advocates an increase in real-world problems in the instruction of algebra. In fact, the contextualized settings also reinforce students' grasp of multiple representations of functions. Keller and Hirsch (1996) investigated students' preference regarding representations of functions. They recommended the availability of multiple representations within the classroom to allow students to tie higher order thinking skills to contextualized settings rather than pure mathematical settings. However, it seems that students do not connect the word 'function' with everyday life situations. In particular, they usually have poor understanding about the concept of function: f is viewed as a label while $f(x)$ is referred to as a formula with a graph (Sajka, 2003). Such misunderstanding demonstrates how important it is for students to be introduced to multiple representations. Obviously, in this case, students apply the representations of graphs and equations to describe the concept of function; since there are other representations available and none of a particular type of representation could provide a global understanding, students will overcome this kind of limited perspective if they are able to grasp a wide range of representations of the concept. Although students are expected to build connections among various kinds of representations to better understand the concept of function, they do have their representation preferences. For example, students might prefer to apply the kind of representation which they are frequently exposed to during classroom instruction. Oppositely, representations seldom mentioned in class are probably regarded as inappropriate understandings of the concept. Moreover, several researchers (Elia, 2007; Gerson, 2008) have suggested that for most students, representations of function remain compartmentalized and mathematical thinking fragmentary.

The results of previous studies suggest that textbooks should provide opportunities for students to learn different representations of the concept of function and understand the relationships among them to further help students to grasp how to translate among different kinds of representations. Since it is recommended to integrate the concept of function into contextual situations to facilitate students' understanding, whether problems and examples presented in textbooks are aligned with this advice will directly influence students learning and understanding. From the misunderstanding of students' conceptions of functions, it is found that different meanings and representations of functions should be given equal amount of attention, otherwise students are likely to employ the mostly-mentioned meanings and representations and dismiss the others.

Based on all of the above suggestions, this study will examine what types of problems in the concept of function are presented in each curriculum in order to understand different learning opportunities provided in each curriculum.

Limitations of the Current Research Literature

First, despite the important role of textbooks on students' learning, little research has been done to investigate to what extent textbook materials could provide learning opportunities to help students build the concept of function by applying different representations and applications. As described earlier, the previous studies examined the concept of function in light of three aspects: students' ability to define the concept of function; students' ability to handle different representations of function; and students' function problem-solving ability (Elia, 2007). However, 75% to 90% of classroom instruction is organized around textbooks (Woodward & Elliott, 1990). It is important to examine what learning opportunities are presented in textbooks, in particular, in light of three important aspects mentioned above.

Second, little research particularly emphasized the treatment of the concept of function between standards-based curricula and conventional curricula. Research has reported that standards-based curricula outscore conventional curricula in developing conceptual understanding and problem solving abilities, while traditional textbooks provide more common feedbacks as well as more practice on number-sense skills. Comparing the treatments of the concept in standards-based curricula and conventional curricula could further illustrate the function of curricula in helping students construct a better conception of the concept. More research need to be done in various mathematical topics on curriculum studies involving comparing standards-based curricula and traditional curricula.

Furthermore, comparative studies including curricula from relatively high-achieving countries is an remaining area to be studied in order to find a better way to help the US students to eliminate the performance gap with their counterparts from high-achieving countries. Previous comparative study including curricula from relatively high-achieving countries is unavoidable. Previous comparative studies of curriculum analysis have discussed various math topics in terms of different emphasizes. Some focused on the content organization while the others did problem analysis. In fact, since almost all the materials on textbooks would be applied by students, both content analysis and problem analysis should be employed to determine how textbook materials affect students' conceptions of math topics and their abilities of applying math knowledge to solve problems with real-world contexts.

In this study, both content analysis and problem analysis will be conducted to answer the five research questions aforementioned in Chapter 1. Content analysis could investigate how the concept of function is introduced and whether the instruction materials in textbooks facilitate students' application of different representations in different contextual applications. On the other hand, problem analysis could illustrate what opportunities are provided by textbooks to help students strengthen their conceptual understanding of the concept and their abilities of applying specific representation of function to resolve practical problems. In addition, curriculum analysis involving a traditional textbook, a standards-based textbook, and a Chinese reformed textbook will further illuminate how different instructional approaches and learning opportunities are embedded in standards-based curriculum and curriculum from a high-achieving country.

Chapter 3

METHOD

This chapter begins by introducing textbooks employed in this study. Specifically, particular sections and parts which were analyzed in this study are explicitly illustrated. Then the analytical framework of the study is presented to show the overall research plan. Following that, content analysis and problem analysis frameworks are discussed to illuminate the methods applied to do content and problem analysis.

Materials

Table 3.1 shows three textbooks were analyzed in this study: (1) a standards-based textbook and (2) a conventional textbook in the US, and (3) a reformed Chinese textbook.

Table 3.1 *Materials used in this study*

United States		China
Standards-based text	*Conventional test*	
CMP2 *Variables and Patterns* (Introducing Algebra)	*Glencoe: Mathematics Applications and Concepts:* (Grade 7: *Course 2*).	*Shu Xue* (Grade 8, volume 1).

The data used for this study come from the relevant lessons on the concept of function from the textbooks and the accompanying teachers' manuals from the three mathematics programs.

Among various standards-based curricula, Connected Mathematics Program [CMP] was chosen for this study since it has been often reported that CMP is widely used in the US and CMP provides numerous learning opportunities for students to experience mathematical thinking and conceptual understanding. *Connected Mathematics* dedicates two chapters (which they call units) out of eight to introducing and developing the concept of function in Grade 7: *Variables and Patterns* and *Moving Straight Ahead*. Since the concept of function is initially introduced in *Variables and Patterns* and then developed through *Moving Straight Ahead* focusing on presenting linear relationships, all the lessons from *Variables and Patterns* are counted here as lessons used to initially introduce the concept of function.

The *Glencoe* Mathematics curriculum representing the conventional US textbooks was chosen for the study due to its wide use in middle school classrooms. Since both conventional textbooks and standards-based textbooks are employed in American classrooms; this study applied both kinds of textbooks to further explain the differences between these two kinds of textbooks and to eliminate the coverage limitation. The Glencoe curriculum includes three separate textbooks: *Glencoe: Mathematics Applications and Concepts: Course 1, Course 2 and Course 3*. The concept of function is first introduced in Grade 7 in Course 2, in particular through two lessons: lesson 1.4 *Algebra: Variables and Expressions* and lesson *4.6 functions and linear equations*. This study therefore analyzed these two lessons in order to explore the initial treatment of the concept of function.

To represent a reform-based Chinese textbook, *Shu Xue* (grade 8, first volume) published by the People's Education Press (PEP) was selected for this study since it is the most widely used version of reformed curriculum in China. The concept of function is addressed in Grade 8. Only one section *11.1 variables and function* was devoted for the study topic. Therefore only this section was analyzed for the study. Appendix A shows all the titles of all the lessons devoted to initial treatment of concept of function in each textbook.

Although students' versions of the textbooks were mainly used for analysis, in order to learn more information such as learning goals, instructional suggestions, and other related materials that might be helpful for answering the research questions, the teachers' manuals accompanying these textbooks were also employed. Specific sections and parts of teachers' manuals were examined according to the corresponding parts analyzed in the students' versions.

Analytical Framework

Table 3.2 presents research questions addressed by content analysis and problem analysis and aspects of the textbooks investigated to answer the research questions.

Table 3.2 *Framework used to analyze the content and problems in textbooks*

	Research question	Aspects investigated
Content Analysis	• When is the concept of function introduced in the US and Chinese textbooks?	• Grade level
	• What are the learning goals when the concept of function is initially introduced in each curriculum?	• NCTM process standards: problem solving, reasoning and proof, communication, connections, and representation
	• How is the concept of function introduced and developed in each curriculum?	• Content organization • Content presentation • Definition • Worked-out examples

Problem Analysis	• How many and what types of problems in the concept of function are presented in each curriculum?
	• What kinds of learning opportunities are provided with respect to cognitive expectation in each curriculum?
		• The number of problems
		• Characteristics of problems
		  • Contextual feature
		  • Response type
		  • Cognitive expectation

This study analyzed content and problems presented in all the lessons whose titles are listed in the Appendix A and the corresponding pages in the supplementary materials, including the teacher's manuals. In the content analysis, content organization, learning goals, content presentation, and the definitions of the concept of function were examined. In the problem analysis, problems designed for students to do relevant exercise were counted and coded to investigate what kinds of learning opportunities are provided by each text.

Content Analysis

First, specific learning goals from both students' textbooks and the teacher's guides of the lessons identified for this study were recorded to investigate and compare learning goals of concept of function in the three texts. *Glencoe* presents "what you'll learn" where learning goals of each lesson were identified for this study. These learning goals are embedded in both students' edition and the teacher wraparound edition. However, *Connected Mathematics* and the Chinese text only present learning goals in the teacher's guide. *Connected Mathematics* presents all learning goals at the beginning of each investigation which are aligned with those from each lesson of the investigation. The Chinese text does not have specific learning goals for each lesson but instead has chapter learning goals, learning goals for lesson *11.1* were identified as the first two learning goals from the chapter learning goals.

Second, content presentations of the identified lessons were examined in order to investigate how the concept of function was introduced and developed in each text.

Next, the definitions of the concept of function being used were explored in each text. The topic of this study is to compare initial treatments of the concept of function in the textbooks; how textbooks initially define this concept is important because students' initial conception of the concept of function come from the textbooks. Table 3.3 describes three different perspectives about the definition of the concept of function: relationship, correspondence, and rule. This study examined what kinds of definitions are used in addressing the concept of function in each text.

Table 3.3 *Definitions of the concept of function*

Description	Definition
Relationship	A relationship between two variables, typically x and y is called a function if there is a rule that assigns to each value of x one and only one value of y.
Correspondence	A function is a correspondence between two sets which assigns every one of the elements in the first set to an element in the second set.
Rule	A function is an abstract entity that associates an input to a corresponding output according to some rule.

Problem Analysis

A problem in this study means any task or activity for which the students have no prescribed or memorized rules or methods, nor is there a perception by students that there is a specific "correct" solution method (Hiebert et al., 1997). Only problems related to the concept of function were considered; problems for other unrelated topics and related problems presented in other places such as unit review other than the identified lessons were ignored for the analysis. Each problem was analyzed for three features listed in Table 3.4. The framework was developed by modifying previous researchers' works (e.g., Stigler et al., 1986; Tabachneck, Koedinger, & Nathan, 1995; Li, 2000; Son & Senk, 2010).

Table 3.4 *Problem analysis framework*

Features	Categories (and Codes)
Contextual Feature	Purely mathematical context in numerical or word form (PM)
	Illustrative context using words, diagrams, or combined (IC)
Performance Requirement	
(1) Response Type	Numerical answer (N)
	Equation (E)
	Table (T)
	Graph (G)
	Verbal description (V)
	Explanation or solution required (ES)
(2) Cognitive Expectation	Recall of definition (D)
	Procedure practice (PP)
	Problem solving (PS)
	Representation (R)
	Mathematical reasoning (MR)

All three features in Table 3.4 have been considered critical dimensions for problem analysis and have been used by previous researchers, including Stigler et al. (1986), Tabachneck, Koedinger, & Nathan (1995), Senk, Beckmann, and Thompson (1997), Li (2000), and Son and Senk (2010).

For example, Li (2000) analyzed problems of integral addition and subtraction and employed a three-dimensional framework: Mathematical feature, Contextual feature, and Performance requirements. However, in this study I only applied two dimensions: contextual features and performance requirements. Because this study focused on the initial treatment of the concept of function, the meaningful understandings of the concept and grasping various types of representations of the concept should be emphasized by the texts. The computation issue (i.e., mathematical feature) is not the main concern of this study. Additionally, I identified response types as one feature for the problem analysis, since there are four different types of representation: equations, tables, graphs, and verbal descriptions. Verbal description is from CMP teacher's guide (Lappan et al., 2006, p4), which views it as descriptions of relationships in students' everyday language. Li's cognitive requirements are similar to the category of cognitive expectation as shown in Table 3.4. I borrowed this component from Son & Senk (2010).

In this study, problems are categorized as requiring *recall of definition* when they require students to recall the definition of the concept of function to solve problems. Problems are coded as *procedure practice* if they are aiming at exercising students' abilities to follow given-out rules and algorithms to find correct answers. Problems are coded as *problem solving* engage students to figure out methods to solve real-world applications. If problems require students to translate among four representations of the concept of function, they are coded as *representation*. For example, the function is represented by a table in a problem, and the problem asks the students to describe the patterns reflected in the table; then the problems should be coded as *verbal description* and *representation*. Problems are coded as *mathematical reasoning* if they require students to explain solution, estimate possible answers, or evaluate strategies.

Each problem identified in all three textbooks was coded in terms of the two dimensions described above. A doctoral student who is affluent in both English and Chinese worked as the second coder with me when coding problems in the Chinese text. Across two features, the percentage agreement ranged between 90% and 99%. My advisor coded problems from both American textbooks, our percentage agreement also ranged between 90% and 99%. Table 3.5 shows examples of problems and their codes for the three features of the problems listed in table 3.4.

Table 3.5 *Sample textbook problems and coding*

<table>
<tr><th rowspan="2">Example</th><th colspan="3">Coding</th></tr>
<tr><th>Context</th><th>Response Type</th><th>Cognitive Expectation</th></tr>
<tr><td>1. Write an equation that has (1, 2) as a solution.</td><td>PM</td><td>E</td><td>PS</td></tr>
<tr><td>2. Explain the relationship among input, output, and function rule.</td><td>PM</td><td>ES</td><td>D</td></tr>
<tr><td>3. Use the equation to complete the table.
y=4x+3<table><tr><td>x</td><td>1</td><td>2</td><td>5</td><td>10</td><td>20</td></tr><tr><td>y</td><td></td><td></td><td></td><td></td><td></td></tr></table></td><td>PM</td><td>N</td><td>PP</td></tr>
<tr><td>4. Sean is buying a new DVD player and speakers for $315. The store offers him an interest-free payment plan that allows him to pay in monthly installments of $25.
a. How much will Sean still owe after one payment? After two payments? After three payments?</td><td>IC</td><td>N</td><td>PS</td></tr>
<tr><td>b. Use n to stand for the number of payments and a for the amount still owe. Write an equation for calculating a for any value of n.</td><td>IC</td><td>E</td><td>R</td></tr>
<tr><td>c. Use your equation to make a table and a graph showing the relationship between n and a.</td><td>IC</td><td>T/G</td><td>R</td></tr>
<tr><td>5. Is y the function of x in the following equations? Why? Please give out other function examples.
(1) $y=3x-5$; (2) $y=\frac{x-2}{x-1}$; (3) $y=\sqrt{x-1}$.</td><td>3PM</td><td>3ES</td><td>3D</td></tr>
<tr><td>6. Each side of a square is 3, the area increases by y if the side increases by x, find out the algebraic expression between x and y, to name the variable and the function, and use a table to express the values of y when x equals to 1, 2, 3, 4.</td><td>IC</td><td>E/ES/T</td><td>R/D</td></tr>
</table>

Note. Problems 1-2 are from Bailey, et al., 2006b, p. 179. Problems 3-4 are from Lap pan, 2006, p. 56, 57. Problems 5-6 are from Research Center for Middle School Mathematics Curriculum, 2005, p. 18, 20.

As illustrated in the above table, if a problem has more than one separate question, the questions were coded as individual problems. Problem 5 in Table 3.5 is an example. Within this problem, there are actually three problems; thus, I counted this problem three times. In addition, if within one question, more than one response type and cognitive expectation are required, it was given several codes under specific categories in terms of the characteristics of the problem. For instance, the last problem requires more than one response types and cognitive expectations, I thus gave it more than one codes in terms of the categories of response type and cognitive expectation.

In the next chapter, the results on the content analysis and the problems analysis are discussed.

Chapter 4

RESULTS

Results from the content analysis are discussed first and those from the problem analysis are discussed in the following sections.

Results of the Content Analysis

Content Organization

Appendix A presents all the chapters and lessons related to the concept of function from the eighth grade Chinese textbook and the two seventh grade US textbooks. As Appendix A shows, the US textbooks initially introduce the concept of function in Grade 7 while the Chinese text does this in Grade 8. Thus, the Chinese text falls one year behind its American counterparts in introducing the concept of function. Appendix B shows what algebraic knowledge is included in the US textbooks through kindergarten to Grade 8 and what algebraic knowledge is included in the Chinese text through Grade 1 to Grade 9. The comparison indicates how students from these two countries are prepared to grasp the concept of function.

There are twelve chapters in *Glencoe* for seventh graders, however, one lesson from two different chapters each are devoted to introducing the concept of function. In other words, only two lessons are devoted to the topic studied. All other lessons in these two chapters and the other chapters focus on other mathematical topics. In *Connected Mathematics*, all fourteen lessons from *Variables and Patterns* are dedicated to introducing the concept of function. In contrast, only three lessons were devoted to initially introduce the concept of function in the Chinese text. The first lesson in chapter 1 of the Chinese text employed in this study is dedicated to initially introducing the concept of function. However, this lesson is such a long section, including three topics: variables, functions, and the graphs of functions, which correspondence to three lessons either in *Connected Mathematics* or in *Glencoe*.

Compared to *Glencoe* and the Chinese text, *Connected Mathematics* devotes many more lessons to developing the concept of function. *Connected Mathematics* and the Chinese text present the topics much more systematically than *Glencoe*: lessons in each chapter of *Connected Mathematics* and the Chinese text are designed to introduce one big idea while lessons within the same chapter of *Glencoe* are dedicated to different mathematical ideas.

Learning Goals

Table 4.1 includes all learning goals of the three textbooks. Differences among learning goals in the three different texts are explored with respect to the five process standards articulated in the *Principles and Standards for School Mathematics* (NCTM, 2001)—problem solving, representation, connections, communication, and reasoning and proof.

Table 4.1 *Learning goals from the three texts*

Text	Lesson	Learning goal
Glencoe	1.4 Variables and Expressions	• Evaluate simple algebraic expressions.
	4.6 Functions and Linear Equations	• Graph linear equations.

Table 4.1 Continued

Text	Lesson	Learning goal
Connected Mathematics	Investigation 1: Variables, Tables, and Coordinate Graphs	• Collect experimental data and organize it in a table • Identify patterns and relationships between variables using information in a table • Create a coordinate graph of data in a table • Indentify patterns and relationships between variables using information in a graph • Compare table and graph representations of same data • Consider data values between plotted points • Create a table from data in a coordinate graph • Compare patterns of change in a table and graph • Interpret narrative notes to make a table and a graph
	Investigation 2: Analyzing Graphs and Tables	• Compare data sets given in tables and graphs • Use patterns in data to make predictions about values between and beyond given data values • Make a graph from a table, choosing the variable and scale for each axis • Use tables and graphs to analyze data and make decisions • Predict the pattern in the graph of a relationship between variables • Tell the “story” shown in a graph
	Investigation 3: Rules and Equations	• Write equations to represent relationships between variables and describe how the pattern of change shows up in a table, a graph, or an equation • Use tables, graphs, and equations to answer questions
	Investigation 4: Calculator Tables and Graphs	• Make and use graphing calculator tables • Make and use graphing calculator graphs • Use a graphing calculator to support problem solving
Chinese	11.1 Variables and Functions	• In the background of investigating quantitative relationships and changing rules in real-world problems, students are expected to experience the process of finding out variables and constants, building up and representing function models, discussing function models, and solving real-world problems and to understand that function is a critical mathematical model to describe changing rules of the real-world • Know the concepts of variables constants, and functions by applying real-world problems • Understand the concept of ‘changing and corresponding’ • Know three representations of functions—tables, equations, and graphs, and analyze simple functional relationships by utilizing graphs.

Connected Mathematics requires students to develop the representations of functions--words, tables, graphs, and equation—gradually, to translate among representations, thus it reflects the standards of *representation* and *connections*. It also requires students to identify relationships from these representations, and to make predictions and decisions by using patterns in data. Therefore, the process standard-*reasoning and proof*-is also reflected in the learning goals of *Connected Mathematics*. In addition, it also includes the requirement of using graphing calculators to make graphs or tables of relationships.

Compared to *Connected Mathematics*, *Glencoe* only requires students to represent functions in graphs, which reflects the standard of *representation*. The Chinese text also asks students to know the representations of functions in terms of tables, equations, and graphs, but it does not require students to do translations among different representations; thus the standard of *representation* is embedded in the learning goals. Moreover, unlike the US texts, the Chinese textbook asks students to understand that function is an important mathematical model which can be applied to solve real-world problems, and this requirement reflects *problem solving* standard. *Reasoning and proof* standard is also embedded in the learning goals of the Chinese text, which requires students to analyze simple functional relationships by utilizing graphs.

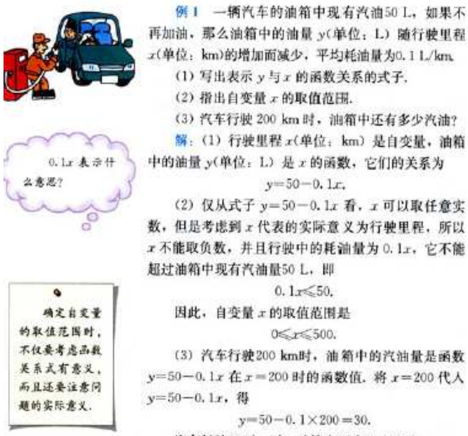
In short, while *Connected Mathematics* emphasizes *reasoning and proof*, *connections*, and *representation* as learning goals for the concept of function, *Glencoe* only includes the *representation* standard; in contrast, the Chinese text stresses *representation*, *reasoning and proof*, and *problem solving*. None of the texts seems to include *communication* process standard as learning goals for learning the concept of function.

Content Presentation

Lessons from *Glencoe* and the Chinese text are organized into teacher-centered approaches while those from *Connected Mathematics* are more inquiry-based and student-centered. Each lesson from *Glencoe* typically is organized into three parts: (1) 'when am I ever going to use this?', (2)worked-out examples, and (3)practice problems. In the first part, a real-world problem with several questions is employed to indicate the topic of the lesson. Following that, the definition of the concept of function is presented in the descriptions of the solutions of this real-world problem. Then in the second part, three to four worked-out examples show how to solve specific problems required in the lesson by providing particular steps. Finally practice problems without solutions at the end of each lesson are provided for students to practice particular kinds of problems introduced in the lesson.

Lessons in the Chinese text are similar to those in *Glencoe*, though the three parts identified in *Glencoe* are not that clear in the Chinese text. However, the Chinese text provides more specific explanations and illuminations than *Glencoe* in the solutions of worked-out examples. Figure 4.1 and Figure 4.2 illustrate this difference.

Unlike the traditional content presentation from *Glencoe*, investigations form the core of *Connected Mathematics*. Each investigation is designed under specific theme, within it there are two to five carefully sequenced problems, which are identified as lessons. These problems typically are embedded in real-world context and include several questions with different response and cognitive expectations aiming at leading students to discover the target mathematical knowledge and develop problem-solving strategies and skills. Although no clues in the textbook indicates specific instruction period, the teacher's guide suggests organizing the lessons in three parts: (1) launch, (2) explore, and (3) summarize.



例 1 一辆汽车的油箱中现有汽油50 L，如果不再加油，那么油箱中的油量 y(单位：L) 随行驶里程 x(单位：km)的增加而减少，平均耗油量为0.1 L/km.

(1) 写出表示 y 与 x 的函数关系的式子.

(2) 指出自变量 x 的取值范围.

(3) 汽车行驶 200 km 时，油箱中还有多少汽油?

解：(1) 行驶里程 x(单位：km) 是自变量，油箱中的油量 y(单位：L) 是 x 的函数，它们的关系为

$$y=50-0.1x.$$

(2) 仅从式子 $y=50-0.1x$ 看，x 可以取任意实数，但是考虑到 x 代表的实际意义为行驶里程，所以 x 不能取负数，并且行驶中的耗油量为 $0.1x$，它不能超过油箱中现有汽油量50 L，即

$$0.1x \leqslant 50.$$

因此，自变量 x 的取值范围是

$$0 \leqslant x \leqslant 500.$$

(3) 汽车行驶200 km时，油箱中的汽油量是函数 $y=50-0.1x$ 在 $x=200$ 时的函数值. 将 $x=200$ 代入 $y=50-0.1x$，得

$$y=50-0.1\times200=30.$$

汽车行驶200 km时，油箱中还有30 L汽油.

Example 1: There is 50L gasoline in a car's tank. If do not adding gasoline any more, the gasoline in the tank y (unit: L) will decrease with the increase of the distance x (unit: km) the car traveled, the average cost of the gasoline is 0.1L/km.

(1) Write an equation to represent the functional relationship between y and x.

(2) Point out the range of the variable x.

(3) When the car has traveled 200km, how much gasoline is left in the tank?

Answer: (1) the distance of the car traveled x is the variable, the gasoline in the tank y is the function of x, their relationship is: y=50-0.1x.

(2) Depending only on the equation y=50-0.1x, x could be any real number. However, x can not be a negative number considering the real-world meaning that x is presenting is the distance the car traveled. In addition, the cost of gasoline is 0.1x, which can not be bigger than the amount of gasoline in the tank, 50L. Therefore:

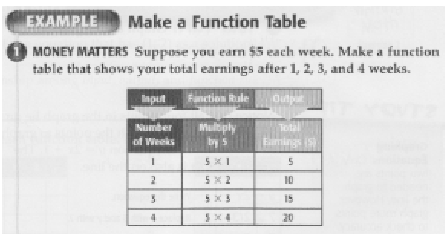
0.1x <=50, thus, the range of the variable x is 0<=x<=500

(3) When the car has traveled 200 km, the gasoline in the tank is the value of the function y=50-0.1x at x=200. Put x=200 into the equation y=50-0.1x, to get:

y=50-0.1*200=30.

When the car has traveled 200km, there is 30L gasoline in the tank.

Figure 4.1 Example from the Chinese text



EXAMPLE Make a Function Table

1 MONEY MATTERS Suppose you earn $5 each week. Make a function table that shows your total earnings after 1, 2, 3, and 4 weeks.

Input: Number of Weeks	Function Rule: Multiply by 5	Output: Total Earnings ($)
1	5 × 1	5
2	5 × 2	10
3	5 × 3	15
4	5 × 4	20

Figure 4.2 Example from *Glencoe*

Definition of the Concept of Function

Among the three concepts of function—(1) function as rule, (2) function as correspondence, and (3) function as relationship, *Connected Mathematics* defines the concept of function as rule at a high level of understanding, the Chinese text defines the concept as correspondence at a middle level of understanding, and *Glencoe* presents the concept as relationship at a low understanding level.

Glencoe uses the following sentence to define this concept: "A relationship where one thing depends on another is called a function." (p.176). This definition indicates the concept of function is a relationship between two variables. Unlike *Glencoe* which gives an explicit definition, there is no formal definition of the concept of function in *Connected Mathematics*, instead, it simply employs the words 'relationship' and later 'rule' in sequence to describe how two variables from real-world problems are related. However, 'rule' still implies a high understanding level. The Chinese text defines the concept in the following sentence: "Generally, in a changing process, if there are two variables x and y, and according to each particular value of x there is only a corresponding value of y, then we call x is an independent variable, y is the function of x." (p.7). Since the Chinese text treats the concept of function as a correspondence between two variables, it defines the concept at a middle level of understanding.

Content analysis and problem analysis are not entirely independent. For example, the learning goal of problem solving indicates the problems presented in the corresponding lessons should be designed as real-world problems which require students to develop their problem-solving strategies. Accordingly, to fulfill the other learning goals, corresponding problems should be developed. Since the learning goals are identified as the intended curriculum, and all materials presented in textbooks, including the problems for students to do for practice, are identified as the potential implemented curriculum, the gap between the intended curriculum and the potential implemented curriculum could be illustrated by examining to what extent the learning goals align with the problems for students to practice. The results from the problem analysis will be presented in the following section.

Results of the Problem Analysis

Contextual Feature

Table 4.2 presents the frequencies of contextual feature used in the problems from the three textbooks.

Table 4.2 *Distribution of Contextual feature of the problems among three textbooks*

	Pure Math context	Illustrative context	Total
Glencoe	90 (88.2%)	12 (11.8%)	102
Connected Mathematics	26 (6.7%)	361 (93.2%)	387
Chinese	17 (51.5%)	16 (48.5%)	33

Connected Mathematics includes the largest number of problems to help students grasp the concept of function (n=387); and the Chinese text provides the smallest number of problems (n=33). In addition, *Connected Mathematics* provides illustrative contexts to most of the problems, *Glencoe* devotes the majority of the problems to pure math contexts, and the Chinese textbook equally contributes the problems to illustrative contexts and pure math contexts. Considering that illustrative contexts are usually identified to provide opportunities to students to do real-world applications, this result indicates that *Connected Mathematics* outscores the Chinese textbook, and the Chinese textbook outscores *Glencoe* in terms of helping students practice how to do real-world applications.

Response Type

Mathematics problems presented in three textbooks are analyzed based on seven response types as addressed in Table 4.3. As noted in Method, some problems require students to use more than one response type; and in this case double codes were used. Therefore the number of the responses counted here is lager than the total number of problems coded in Table 4.2. Table 4.3 illustrates the distribution of response type required in the problems presented in the three texts.

Table 4.3 *Distribution of Response type of the problems*

	Numerical answer	Table	Graph	Verbal	Equation	Explanation	Total
Glencoe	50 (48.0%)	8 (7.6%)	27 (26.0%)	1 (1.0%)	14 (13.4%)	5 (4.0%)	104
Connected Mathematics	105 (24.4%)	49 (11.4%)	47 (10.9%)	52 (12.1%)	40 (9.3%)	136 (31.9%)	429
Chinese	8 (20.5%)	3 (7.6%)	7 (17.9%)	0 (0%)	9 (23.0%)	12 (31.0%)	39

Glencoe emphasizes numerical answers and the representation of graphs. *Connected Mathematics* highlights numerical answers and explanations, and pays equal amount of attention to the representations of tables, graphs, verbal descriptions, and equations. The Chinese text concentrates on numerical answers, explanations, and the representation of equations.

All three texts emphasize numerical answers, which implies that for all the textbook developers correct answers are still an import criteria reflecting to what degree students have understood and grasped mathematical knowledge. Both the Chinese text and *Connected Mathematics* devote efforts to developing students' mathematical reasoning abilities because they employ a relatively high percentage of problems requiring explanation.

If tables, graphs, equations, and verbal descriptions are regarded as different representations of the concept of function rather than different response types, *Glencoe* emphasizes the representation of graphs and overlooks the representation of verbal description; the Chinese text emphasizes the representations of graphs and equations and ignores to use words to describe the concept of function; however, *Connected Mathematics* seems to devote equal amount of effort to the four types of representations. As noted above, multiple representations of the concept of function are beneficial for students to understand and develop the concept of function; *Connected Mathematics* does better than the others in this case. Since both *Glencoe* and the Chinese text overlook the representation of verbal descriptions, to some extent, this representation should be paid much more attention than it is now. The Chinese text emphasizes the representation of equations which are algebraic expressions. This result aligns with Cai (2008), who found Chinese students are more likely than their American counterparts to think algebraically.

Cognitive Expectation

Table 4.4 shows how the three texts distribute problems in terms of five kinds of cognitive expectations. The number of the cognitive expectations is bigger than that of the problems in some textbooks because, as noted previously, there are problems which are double coded when more than one cognitive expectation is identified here.

Table 4.4 *Cognitive expectation of the problems*

	Recall of definition	Procedure practice	Problem solving	Representation	Mathematical reasoning	Total
Glencoe	2 (1.9%)	46 (45.0%)	5 (4.9%)	44 (43.1%)	5 (5.1%)	102
Connected Mathematics	0 (0%)	38 (8.9%)	75 (17.6%)	171 (40.1%)	142 (33.4%)	426
Chinese	6 (16.6%)	2 (5.5%)	8 (22.2%)	15 (41.6%)	5 (14.1%)	36

The three textbooks almost devote equal percentage (around 40%) of the problems to cognitive expectation of *representation.* In addition, similar percent of the problems in *Glencoe* require *procedure practice*. *Connected mathematics* devotes 33.4% and 17.6% of the problems to *mathematical reasoning* and *problem solving*. The Chinese text devotes almost similar percent (around 17%) of the problems to *recall of definition*, *problem solving*, and *mathematical reasoning*. All the three textbooks devote a relatively high percentage of the problems to *representation*, thus *representation* is the common highlight of the cognitive expectations in three texts. Similar percent of the problems in *Connected Mathematics* and in the Chinese text are distributed to practice *problem solving*; therefore *problem solving* is emphasized equally in *Connected Mathematics* and the Chinese text.

However, different textbooks still have specific highlights in terms of cognitive expectation. *Glencoe* also aims to help students to do *procedure practice*; *Connected Mathematics* promotes *mathematical reasoning*; while the Chinese text contributes more efforts than the others to develop *problem solving* abilities.

Chapter 5

CONCLUSIONS AND DISCUSSION

This study examined how three textbooks--a conventional US textbook, a standards-based US textbook, and a Chinese reformed textbook--initially treat the concept of function. The purpose of this study was not only to illuminate the quality differences between conventional textbooks and standards-based textbooks but also to investigate mathematical and pedagogical differences between the US textbooks and the Chinese text in hopes of raising some hypotheses about differences in achievement between the US and Chinese students.

Content analysis was conducted among these three textbooks aforementioned by comparing content organization, learning goals, content presentation, and the definition of the concept of function. Problem analysis was executed by coding the problems presented in the textbooks for practice in terms of contextual feature, response type, and cognitive expectation. In the following section, I summarize the findings of the study and discuss conclusions and implications for curriculum developers, teachers, and future study.

Summary of the Findings

Content Analysis

Both of the US textbooks initially introduce the concept of function in Grade 7 while the Chinese text does so in Grade 8. *Connected Mathematics* contains 14 lessons to introduce and develop the concept of function while *Glencoe* and the Chinese text respectively devote two and three lessons to do this. In each lesson, *Glencoe* and the

Chinese text presents the content in a traditional way: (1) present real-world problems to introduce the target topic, (2) formally give the definition of the target topic and the examples of how to apply the knowledge to solve problems, and (3) provide the related practice problems for students to work on. However, I could not identify these three steps of the lessons in *Connected Mathematics*. All the contents in *Connected Mathematics* are real-world problems without solutions. These problems encourage students to discover mathematical knowledge by themselves and to apply what they discovered to solve problems.

All the textbooks require students to know how to represent the concept of function. *Connected Mathematics* asks students to develop multiple representations of function including verbal description, tables, graphs, and equations one at a time. Moreover, it requires students to translate among these representations. *Glencoe* requires students to use graphs only to represent the concept of function. The Chinese text develops multiple representations (especially tables, equations, and graphs) one at a time; however, it asks students to choose and to apply representations that are appropriate for particular contexts instead to do translations among different representations.

Unlike *Glencoe* and the Chinese text, *Connected Mathematics* introduces how to use graphing calculators to represent functions in terms of tables and graphs. In addition to the requirements of the representations, the Chinese textbook asks students to understand function is an important mathematical model which can be applied to solve real-world problems. Although the US textbooks present real-world problems, they do not articulate problem solving as a specific learning goal.

Problem Analysis

Connected Mathematics provides the greatest number of problems for students to solve while the Chinese text provides the smallest number of problems. Most of the problems in *Connected Mathematics* are presented in illustrative contexts while most of the problems in *Glencoe* have pure math contexts. The Chinese text distributes the problems between illustrative contexts and pure math contexts almost equally. Most of the problems presented in the three textbooks focus on the representation. Every textbook makes different choices regarding which of the representations to use and to emphasize. As recommended Keller and Hirsch (1998), multiple representations of the concept of function should be available; thus equally developing all four representations might be a useful way to help students to construct conceptual understanding of the concept of function. *Connected Mathematics* pays an equal amount of attention to four representations examined in this study when designing problems for practice. *Glencoe* only focuses on the representation of graphs while the Chinese text emphasizes the representations of graphs and equations as well. In addition to these representations, *Connected Mathematics* also aims to develop students' ability of mathematical reasoning by providing problems that require such cognitive expectation. The other emphasis of the problems is procedure practice in *Glencoe* and is problem solving in the Chinese text.

Conclusions and Discussion

The standards-based US textbook is designed for student-centered instruction while the conventional US textbook seems to facilitate teacher-centered instruction. The standards-based US textbook puts problems in illustrative contexts and emphasizes representation, connections, and reasoning and proof. This practice provides students various learning opportunities to develop different mathematical abilities. In contrast, the conventional US text emphasizes procedure practice, which can not help students to understand the concept of function and to develop various mathematical abilities.

Similar to the conventional US textbook, many illustrations and explanations in the Chinese text make it seems to be designed for teacher-centered instruction. However, in terms of providing problems for students to work on, the Chinese text is much more similar to the standards-based US text. For example, with respect to cognitive expectation, the Chinese text put more emphasis on problem solving rather than on procedure practice. This trend suggests that the Chinese text applies a balanced approach: presenting content in a traditional way but providing problems to improve critical mathematical abilities.

Problems presented in a textbook are only one part of the potential implemented curriculum. Indeed, how the learning goals presented in textbook and\or teacher's manuals align with the identified problems could indicate whether there is a gap between the intended curriculum (i.e., learning goals) and the potential implemented curriculum (e.g., content and problem presentation in textbooks). The learning goals from *Glencoe* reflect the standard of *representation*. These learning goals align with the performance requirements of the problems because, as noted above, *Glencoe* emphasizes the representation of *graphs* in terms of response-type and *representation* in terms of cognitive expectation. The standards of *representation* and *reasoning and proof* reflected by the learning goals of *Connected Mathematics* are realized since a large proportion of problems in it require students to use representation and mathematical reasoning.

In addition, *Connected Mathematics* sets expectation related to the standard of *connections*: it is difficult to examine whether this standard is fulfilled by focusing on the cognitive expectations of the problems. But, the Connections section of the homework from *Connected Mathematics* does provide the opportunity to connect new knowledge with prior knowledge. The Chinese text focuses on *problem solving* and *representation* in terms of cognitive expectation and this aligns with the standards of *representation* and *problem solving*. However, there is no clue to conclude that the standard of *reasoning and proof* is also reflected by the problems from the Chinese text since the response type of *explanation* is highlighted in it but it does not emphasize the cognitive expectation of *mathematical reasoning*.

The reformed Chinese text provides analysis, illustrations and explanations in the solutions of worked-out examples. In addition to merely presenting the correct solutions, the Chinese text provides illustrations and explanations to tell how the correct answers were derived as shown the example below. When giving solutions to the examples that employ the representations of tables and graphs in the contexts, the Chinese textbook prefaces the solutions with analyses where some information that can not easily be found are discussed.

Implications

This study employed only one textbook each to represent the US standards-based curricula, the US conventional curricula, and the reform-oriented Chinese curricula. This limitation in the number of the textbooks might lead to errors in the generalization of the findings. Nevertheless, this study still has implications for curriculum developers, teachers, and researchers.

First, the comparative analysis of the US textbooks and the Chinese text illustrates how curriculum developers in different countries value the concept of function. Based on the *Common Core States Standards for Mathematics*, the concept of function is required for students in Grade 8 and algebraic expressions are required for students in Grade 7. In this case, there is no difference between the US and the Chinese curriculum standards. However, the American textbooks introduce the concept of function earlier than the Chinese text. In addition, instead of systematically introducing the concept of function, the US conventional textbook seems to introduce this concept in Grade 7 by applying one kind of representation in one lesson. Although the US standards-based textbook also initially treat the topic of the concept of function in Grade 7, it develops the concept in a systematical way. The Chinese textbook reserves the concept of function for Grade 8 as it is systematically introduced. From the findings of study, curriculum developers should compare nationally and internationally to find the most beneficial way for students to acquire knowledge presented in curriculum standards.

Second, this study showed a gap between intended curriculum and potential implemented curriculum. Curriculum developers should try to minimize this gap by providing problems aimed to help students to develop certain mathematical abilities outlined in the learning goals. In addition, teachers could use their content knowledge and understandings of the curriculum to bridge this gap. For instance, if teachers know how important it is to translate the four representations of the concept of function, they can find extra materials from other resources to do classroom instruction when their textbooks do not provide these materials. This study reaffirms the important role of teachers in implementing curriculum materials.

Third, this study further confirms the previous statement that *Connected Mathematics* is a standards-based curriculum and *Glencoe* is a conventional textbook. Previous researchers have suggested the American textbooks outscored Chinese textbooks in developing students' conceptual understanding and mathematical reasoning (Li, 2000). Cai (2001) indicated that Chinese students outperformed their American counterparts when doing computations and routine problems but were less effective than their American counterparts when developing nonroutine problem-solving skills and creative thinking. Consistent with previous studies, this study showed that *Connected Mathematics* provides more problems that intend to develop the critical mathematical ability (i.e., mathematical reasoning) and a relatively high percent of the problems requiring explanations. However, this study also reported that the Chinese text emphasizes the process standards of *representation, reasoning and proof,* and *problem solving*. In particular, this study showed that the Chinese textbook uses the balanced approach by presenting not only how to use a procedure but also why the procedure works. Although *Connected Mathematics* is proven to be effective in helping students to improve their critical mathematical abilities, curriculum developers may consider the Chinese approach in presenting mathematical content in order for students to develop the mathematical proficiency on the concept of function.

Fourthly and finally, this study has implication for future research. This study focused only on the initial treatment of the concept of function. Future studies on other algebra topics to compare reformed-based textbooks from different countries might indicate changes in students' performance.

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Appendices

Appendix A

The Development of the Concept of Function in the Three Textbooks

Textbook	Glencoe (Grade 7)		Connected Mathematics (Grade 7)		Chinese (grade 8)
Lessons	Chapter 1 Decimal Patterns and Algebra	Chapter 4 Algebra: Linear Equations and Functions	Variables and Patterns	Moving Straight Ahead	Shu Xue
	1.1 A Plan for Problem Solving	4.1 Writing Expressions and Equations	**Investigation 1 Variables, Tables, and Coordinate Graphs**	Investigation 1 Predicting from Patterns	**11.1 Variables and functions**
	1.2 Powers and Exponents	4.2a Hands-On Lab: Solving Equations Using Models	**1.1 Preparing for a Bicycle Tour: Interpreting Tables**	1.1 Conducting an Experiment	**11.1.1 Variables**
	1.3 Order of Operations	4.2 Solving Addition and Subtraction Equations	**1.2 Making Graphs**	Investigation 2 Walking Rates	**11.1.2 Functions**
	1.4 Algebra: Variables and Expressions	4.3 Solving Multiplication Equations	**1.3 Day1: Atlantic City to Lewes: Interpreting Graphs**	2.1 Walking to the Yogurt Shop	**11.1.3 Graphs of functions**
	1.5a Problem-Solving Strategy: Guess and Check	4.4a Problem-Solving Strategy: Work Backward	**1.4 Day2: Lewes to Chincoteague Island: Reading Data from Graphs**	2.2 Changing the Walking Rate	11.2 Linear Function
	1.5 Algebra: Equations	4.4 Solving Two-Step Equations	**1.5 Day3: Chincoteague Island to Norfolk: Finding Average Speed**	2.3 Walking for Charity	11.2.1 Proportional Function
	1.6 Algebra: Properties	4.5 Inequalities	**Investigation 2 Analyzing Graphs and Tables**	2.4 Walking to Win	11.2.2 Linear Function
	1.7 Sequences	**4.6a Hands-On Lab: Functions and Graphs**	**2.1 Renting Bicycles: Analyzing a Table and a Graph**	2.5 Crossing the Line	11.3 Look at Equations and Inequalities from the Perspective of Function
	1.7b Hands-On Lab: Exploring Sequences	**4.6 Functions and Linear Equations**	**2.2 Finding Customers: Making and Analyzing a Graph**	Investigation 3 Exploring Lines with Graphing Calculator	11.3.1 Linear Function and Linear Equation with one unknown
	1.8 Measurement: The Metric System	4.7 Lines and Slope	**2.3 What's the Story? Interpreting Graphs**	3.1 Getting to the point	11.3.2 Linear Function and Linear inequality with one unknown

The development of the concept of function in the three textbooks (Continued)

Textbook	Glencoe (grade seven)		Connected Mathematics (Grade seven)		Chinese (grade 8)
	Chapter 1 Decimal Patterns and Algeba	Chapter 4 Algebra: Linear Equations and Functions	Variables and Patterns	Moving Straight Ahead	Shu Xue
	1.9 Scientific Notation		**Investigation 3 Rules and Equations**	3.2 Graphing Lines	11.3.3 Linear Function and System of linear equations with two unknowns
			3.1 Writing Equations: Equations With One Operations	3.3 Finding Solutions	
			3.2 Writing More Equations: Equations With Two Operations	3.4 Planning a Skating Party	
			3.3 Paying Bills and Counting Profits: Equations for Revenue, Expenses, and Profit	Investigation 4 Solving Equations	
			Investigation4 Calculator Tables and Graphs	4.1 Paying in Installments	
			4.1 Making and Using Calculator Tables	4.2 Using the Symbolic Method	
			4.2 Making and Using Calculator Graphs	4.3 Analyzing Bones	
			4.3 Extending the Tour: Comparing Relationships	Investigation 5 Exploring Slope	
				5.1 Climbing Stairs	
				5.2 Finding the Slope of a Line	
				5.3 Connecting Points	
				Investigation 6 Writing an Equation for a Line	
				6.1 Solving Alphonso's Puzzle	
				6.2 Converting Temperatures	
				6.3 Solving the Mystery of the Irish Elk	

Appendix B

The algebra development in curriculum standards from the US and China

<table>
<tr><th colspan="2">US</th><th colspan="2">China</th></tr>
<tr><th>Grade level</th><th>Algebra development</th><th>Grade level</th><th>Algebra development</th></tr>
<tr><td>Kindergarten</td><td>Children identify, duplicate, and extend simple number patterns and sequential and growing patterns (e.g., patterns made with shapes) as preparation for creating rules that describe relationships.</td><td rowspan="3">Grade 1-3</td><td rowspan="3">In this period of time, students will learn numbers within ten thousand, simple fractions and decimals, common quantities; understand the meanings of numbers and operations; grasp the basic computations; explore and understand simple quantitative relationships.</td></tr>
<tr><td>Grade 1</td><td>Through identifying, describing, and applying number patterns and properties in developing strategies for basic facts, children learn about other properties of numbers and operations, such as odd and even (e.g., “Even numbers of objects can be paired, with none left over”), and 0 as the identity element for addition.</td></tr>
<tr><td>Grade 2</td><td>Children use number patterns to extend their knowledge of properties of numbers and operations. For example, when skip counting, they build foundations for understanding multiples and factors.</td></tr>
<tr><td>Grade 3</td><td>Understanding properties of multiplication and relationship between multiplication and division is a part of algebra readiness that develops at grade 3. The creation and analysis of patterns and relationships involving multiplication and division should occur at this grade level. Students build a foundation for later understanding of functional relationships by describing relationships in context with such statements as, “The number of legs is 4 times the number of chairs.”</td><td>Grade 4-6</td><td>In this period of time, students will learn integers, fractions, decimals, percents and the related operations; begin to know negative numbers and equations; begin to use calculators to do complicated operations and to explore mathematical problems; obtain the abilities to solve simple problems in real-world life.</td></tr>
<tr><td>Grade 4</td><td>Students continue identifying, describing, and extending numeric patterns involving all operations and nonnumeric growing or repeating patterns. Through these experiences, they develop an understanding of the use of a rule to describe a sequence of numbers or objects.</td><td rowspan="2">Grade 7-9</td><td rowspan="2">In this period of time, students will learn real numbers, integral expressions, fractional expressions, equations, system of equations, inequalities, system of inequalities, and functions; explore numbers, expressions, and relationships and rules from real-world problems; begin to grasp some effective tools to express, handle, and communicate quantitative relationships and changing rules; develop symbolic senses; understand the close relationship between math and real life, increase application senses to use algebraic knowledge and method to solve problems.</td></tr>
<tr><td>Grade 5</td><td>Students use patterns, models, and relationships as contexts for writing and solving simple equations and inequalities. They create graphs of simple equations. They explore prime and composite numbers and discover concepts related to the addition and subtraction of fractions as they use factors and multiples, including applications of common factors and common multiples. They develop an understanding of the order of operations and use it for all operations.</td></tr>
</table>

The algebra development in curriculum standards from the US and China (Continued)

US		China	
Grade level	Algebra development	Grade level	Algebra development
Grade 6	Students use the commutative, associative, and distributive properties to show that two expressions are equivalent. They also illustrate properties of operations by showing that two expressions are equivalent in a given context (e.g., determining the idea in two different ways for a rectangle whose dimensions are x+3 by 5). Sequences, including those that arise in the context of finding possible rules for patterns of figures or stacks of objects, provide opportunities for students to develop formulas.		
Grade 7	Students use the arithmetic of rational numbers as they formulate and solve linear equations in one variable and use these equations to solve problems. They make strategic choices of procedures to solve linear equations in one variable and implement them efficiently, understanding that when they use the properties of equality to express an equation in a new way, solutions that they obtain for the new equation also solve the original equation.		
Grade 8	Students encounter some nonlinear functions (such as the inverse proportions that they studied in grade 7 as well as basic quadratic and exponential functions) whose rates of change contrast with the constant rate of change of linear functions. They view arithmetic sequences, including those arising from patterns or problems, as linear functions whose inputs are counting numbers. They apply ideas about linear functions to solve problems involving rates such as motion at a constant speed.		

Note. I developed the table basing on *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* and *Mathematics Curriculum Standards.*

Vita

Qintong Hu was born in Qingdao, China. Since elementary school, she has been a top student with huge interest in mathematics. After twelve years in school, her hard work and excellent grades allowed her to enroll in the Department of Mathematics, Education College, Qingdao University, China. During the college time, she learned advanced mathematics, pedagogical knowledge, and shadowed mentor teachers in a local middle school for three months as a teaching intern.

To learn more about becoming a master teacher, who not only has exceptional teaching skills and content knowledge but also can do research, she decided to pursue her Master's degree in math education in the University of Tennessee at Knoxville [UTK]. During two years' study of math education at UTK, she worked as a part-time research assistant for Dr. Son in order to assist her in researching various math education topics. With Dr. Son's encouragement and guidance, in 2010, she presented "How standards-based curricula and conventional curricula influence students' learning opportunities" at PME-NA, a prestigious national conference on math education.

She also worked as a Chinese teaching assistant in the Department of Modern Foreign Languages and Literatures where she became familiar with American students. Not only her language expression but also her teaching strategies and classroom organization skills were reinforced during the year-long teaching experience.

Upon graduation, Qintong plans to travel with her parents in America, and then continue her Ph.D study in mathematics education.