REVIEW

FOUNDATIONS OF DIATONIC THEORY: A MATHEMATICALLY BASED APPROACH TO MUSIC FUNDAMENTALS, BY TIMOTHY A. JOHNSON. LANHAM, MD: SCARECROW PRESS, 2008. (ORIGINALLY PUBLISHED EMERYVILLE, CA: KEY COLLEGE PUBLISHING, 2003.)

MATTHEW SANTA

In *Foundations of Diatonic Theory: A Mathematically Based Approach to Music Fundamentals*, Timothy Johnson presents the work of those theorists who have focused on aspects of the diatonic collection over the past forty or so years (the book is in fact dedicated to John Clough, one of the most prominent figures in the field of diatonic set theory).[1] It is intended as a supplemental text (not a replacement text) for a music fundamentals course, or as reading for a course on music and mathematics, or on diatonic set theory. It is important to emphasize that, unlike most music theory texts, it is truly a supplement: it does not teach theory fundamentals, and it does not address all of the materials that would be covered in a course on music and mathematics, or even a course on diatonic set theory. This is by no means a criticism, as not only does *Foundations of Diatonic Theory* (hereafter *FDT*) accomplish its stated goals, but it does so in such a masterful way, and with such a refreshing approach, that after reading it, any course on

1 See Eytan Agmon, "A Mathematical Model for the Diatonic System," *Journal of Music Theory* 33/1 (1989): 1–25, and *idem*, "Coherent Tone-Systems: A Study in the Theory of Diatonicism," *Journal of Music Theory* 40/1 (1996): 39–59; Gerald Balzano, "The Group-Theoretic Description of 12-Fold and Microtonal Pitch Systems," *Computer Music Journal* 4/4 (1980): 66–84; Richmond Browne, "Tonal Implications of the Diatonic Set," *In Theory Only* 5/6–7 (1981): 3–21; Norman Carey and David Clampett, "Aspects of Well–Formed Scales," *Music Theory Spectrum* 11/2 (1989): 187–206, and *idem*, "Self-Similar Pitch Structures, Their Duals, and Rhythmic Analogues," *Perspectives of New Music* 34/2 (1996): 62–87; John Clough, "Aspects of Diatonic Sets," *Journal of Music Theory* 23/1 (1979): 45–61, *idem*, "Diatonic Interval Sets and Transformational Structures," *Perspectives of New Music* 18/1–2 (1979–80): 461–482, and *idem*, "Diatonic Interval Cycles and Hierarchical Structure," *Perspectives of New Music* 32/1 (1994): 228–253; John Clough and Jack Douthett, "Maximally Even Sets," *Journal of Music Theory* 35/1–2 (1991): 93–173; John Clough and Gerald Myerson, "Variety and Multiplicity in Diatonic Systems," *Journal of Music Theory* 29/2 (1985): 249–269; and Jay Rahn, "Coordination of Interval Sizes in Seven–Tone Collections," *Journal of Music Theory* 35/1–2 (1991): 33–60.

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music fundamentals, music and mathematics, or diatonic set theory taught without its perspective would feel incomplete.

One of the most innovative aspects of the book is its approach based on self-discovery (very reminiscent of the Montessori method). We are used to exercises in theory texts that give students the opportunity to apply their knowledge of general principles to specific musical contexts, not exercises designed to lead the students to the discovery of general principles. However, in *FDT*, students are given exercises throughout the entire book that lead to discoveries. The solutions to all exercises are given in the text itself, which seems appropriate given that the author considers it a supplement, although some instructors might wish that there were some exercises that could be used to test whether or not the reading was done.

The path of discovery in *FDT* has been carefully crafted so that it can be overlaid onto a music fundamentals course: the first chapter requires only a knowledge of whole steps and half steps, and of major and minor scales; the second chapter also requires a knowledge of intervals, key signatures, and the circle of fifths; and, the third and final chapter is the only one that requires a knowledge of triads and seventh chords. Instructors of a fundamentals course may therefore assign Chapter 1 shortly after the beginning of the course, wait until they've finished work on intervals and key signatures to assign Chapter 2, and assign Chapter 3 at the end of the course. Johnson suggests that readings from his book might be given in place of more tedious review exercises, which would certainly be a good idea in courses for non-majors, where speed with music fundamentals is not such an issue. Even in courses for majors, one could argue that the kind of repetitive review exercises found in many fundamentals approaches (e.g., spell sixty different major triads) do little to insure a successful application of those skills in real musical contexts, and that the time might better be spent with more thought-provoking exercises like the ones found in Johnson's book.

Using *FDT* in a course on music and mathematics, or on diatonic set theory, would be a much more efficient and engaging way to present the extant literature on diatonic theory than just assigning readings upon which the book is based. Those readings are intended for the scholarly community and consequently their use of specialized terminology and their lack of exercises would prove difficult for students. In a course on music and mathematics, the book would need to share time with readings on topics ranging from acoustics to set theory;[2] and in a course on diatonic set theory, the book would need to share time with readings on applications to historical studies, to non-western music, and to analysis.[3] Since *FDT* is only 177 pages, this would not be a problem.

2 For a course on music and mathematics, one should consider including some of the following readings in addition to those commonly found in courses on acoustics or on set theory: Courtney Adams, "Erik Satie and Golden Section Analysis," *Music and Letters* 77/2 (1996): 242–252; Gerard Assayag, Hans Georg Feichtinger, and Jose Francisco Rodrigues (eds.), *Mathematics and Music: A Diderot Mathematical Forum* (New York: Springer-Verlag, 1997); David J. Benson, *Music: A Mathematical Offering* (Cambridge: Cambridge Univ. Press, 2007); Trudi Hammel Garland and Charity Vaughn Kahn, *Math and Music: Harmonious Connections* (Palo Alto, CA: Dale Seymour Publications, 1995); Leon Harkleroad, *The Math Behind the Music* (Cambridge: Cambridge Univ. Press, 2007); Roy Howat, *Debussy in Proportion: A Musical Analysis* (Cambridge: Cambridge Univ. Press, 1983), and *idem*, "Architecture as Drama in Late Schubert" in *Schubert Studies*, ed. Brian Newbould (London: Ashgate Press, 1998): 168–192; James Jeans, *Science and Music* (Cambridge: Cambridge Univ. Press, 1937; reprinted by Dover, 1968); Ian Johnston, *Measured Tones: The Interplay of Physics and Music* (New York: Adam Hilger, 1989); Jonathan Kramer, "The Fibonacci Series in Twentieth Century Music" *Journal of Music Theory* 17/1 (1973): 111–148; Charles Madden, *Fractals in Music: Introductory Mathematics for Musical Analysis* (Salt Lake City: High Art Press, 1999); Hugo Norden, "Proportions in Music," *Fibonacci Quarterly* 2 (1964): 219–222; Thomas D. Rossing, *The Science of Sound*, 2nd ed. (Reading, MA: Addison-Wesley, 1990); Martin Scherzinger, "The Changing Roles of Acoustics and Mathematics in Nineteenth-Century Music Theory and Their Relation to the Aesthetics of Autonomy," *South African Journal of Musicology* 18 (1998): 17–33; William Sethares, *Tuning, Timbre, Spectrum, Scale* (New York: Springer, 1998); Johann Sundberg, *The Science of Musical Sounds* (San Diego: Academic Press, 1991); Rex Wexler and Bill Gannon, *The Story of Harmony* (Vancouver: Justonic Tuning Inc., 1997); and David Wright, *Music and Mathematics* (online publication, <http://www.math.wustl.edu/~wright/Math109/00Book.pdf>, 2009).

3 For a course on diatonic set theory and its applications, one should consider some of the following readings: Norman Carey and David Clampitt, "Regions: A Theory of Tonal Spaces in Early Medieval Treatises," *Journal of Music Theory* 40/1 (1996): 131–147; John Clough, Jack Douthett, N. Ramanathan, and Lewis Rowell, "Early Indian Heptatonic Scales and Recent Diatonic Theory," *Music Theory Spectrum* 15/1 (1993): 36–58; Robert Gauldin, "The Cycle–7 Complex: Relations of Diatonic Set Theory to the Evolution of Ancient Tonal Systems," *Music Theory Spectrum* 5 (1983): 39–55; Jay Rahn, "Constructs for Modality, ca. 1300–1550," *Canadian Association of University Schools of Music Journal* 8/2 (1989): 5–39; and Matthew Santa, "Analysing Post-Tonal Diatonic Music: A Modulo 7 Perspective," *Music Analysis* 19/2 (2000): 167–201.

FDT is organized into three chapters, with a short introduction and a short conclusion framing them, and with an annotated bibliography, a separate list of sources cited, and index completing the text. Chapter 1 explores the property of maximal evenness (a collection divides the octave as evenly as possible), first informally and then formally, and concludes with the deep-scale property (each interval class appears a unique number of times). Chapter 2 explores the properties of "cardinality equals variety" (the size of each subset equals the number of distinct qualities within that subset) and "structure implies multiplicity" (the distances between elements in a given subset equals the breakdown of how many times each quality appears in the collection); it also defines Myhill's property (having exactly two qualities for each size of interval), well-formed collections (collections that can be generated from a single interval class), generators, and bisectors. Chapter 3 explores how the properties presented in the first two chapters apply (or don't apply) to triads and seventh chords.

One of the most consistent pedagogical elements of the book is having students answer questions about various collections by using circle diagrams. The circle diagrams are sometimes mod-12 clocks (adjacent hash marks represent semitonal distances), sometimes the circle of fifths, and sometimes a diatonic collection (adjacent hash marks on the circle represent fifths within a single diatonic collection). Students plot various collections and subsets onto these diagrams and relate distances measured against a background collection to distances measured by the steps within the given collection.

Figure 1 reproduces the diagram accompanying the first exercise given to the student. The instructions for the exercise are as follows: "Place three, four, and five dots on the crossing lines around the circles so that the dots are spread out as much as possible. The first one is done for you" (5). Students are first allowed to have the realization that there are multiple correct

FIGURE 1. Exercise 1.1 from Johnson, *Foundations of Diatonic Theory* (5)

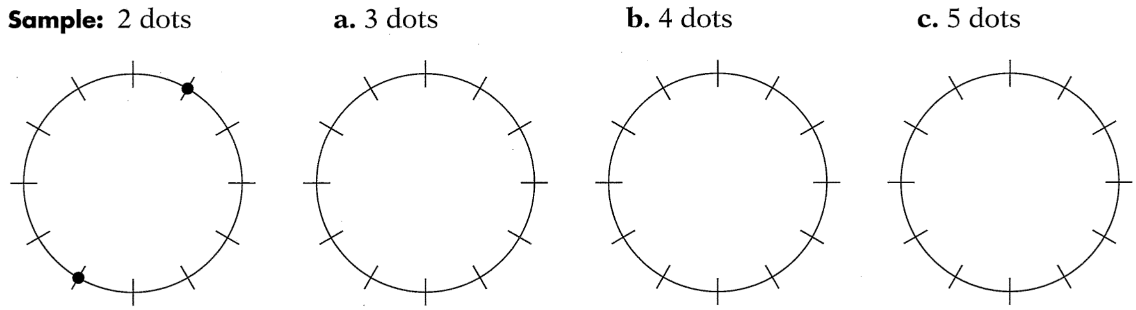
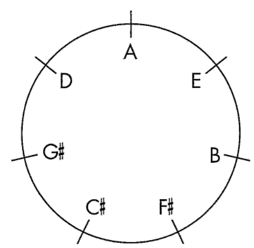


FIGURE 2. Exercise 3.5a from Johnson, *Foundations of Diatonic Theory* (132)



solutions to Exercise 1.1 before they are then taught to calculate the number of correct solutions using the greatest common divisor. Students are also allowed to grapple with the problem of collections that do not divide twelve evenly, before that problem is addressed by the text. Exercise 1.2 is like Exercise 1.1, but uses six, seven, and eight dots. The concept of maximal evenness is introduced in purely abstract terms, as well as the concept of complementary collections, before the musical ramifications of these mathematical properties are discussed. It is not until p. 15 that the circles in Exercises 1.1 and 1.2 are identified as mod-12 clocks and students are encouraged to play their solutions on the piano, revealing how the augmented triad, the diminished seventh chord, the pentatonic scale, the whole-tone scale, the diatonic scale, and the octatonic scale are all maximally even collections (although the term itself is not defined and used formally until p. 26).

Another example from the end of the last chapter confirms how Johnson consistently applies this approach. Figure 2 reproduces the diagram accompanying Exercise 3.5a. The instructions for the exercise are as follows: "First, plot any triad of your choice using the given notes on the circle diagram (arranged in the circle-of-fifths pattern). Then answer the questions that follow" (132). The first question is: "Based on cardinality equals variety for triads, how many different triad qualities can be formed in the diatonic collection?" (132). The next question is a follow-up: "Explain how you determined the answer . . . without writing out all of the triads" (132). The third question is: "Based on structure implies multiplicity (as illustrated by your plotted triad above), how many individual triads will be associated with each quality?" (132). And the last question is another follow-up: "Explain how you determined the answer . . . without writing out all of the triads" (132). The exercise deftly reviews the concepts of cardinality equals variety and structure implies multiplicity introduced in Chapter 2, while at the same time showing how those concepts apply to the most important three-note collection in tonal music: the triad. Exercise 3.5b is like Exercise 3.5a, but with the questions focused on diatonic seventh chords instead of triads.

The conclusion that follows the last chapter summarizes each of the three chapters in turn, and also provides answers to some of the questions initially posed in the introduction: "Why does the major scale seem to work so well? Why has diatonicism formed the backbone of Western music for so long—permeating both classical music of the past (and now the present) and much popular music? And, perhaps most naïve and yet apt, why are the black and white keys of the piano arranged as they are?" (145). Johnson suggests that the answer is at least in part because the diatonic collection is maximally even, that cardinality equals variety and structure implies multiplicity for both itself and for all of its subsets, and that it is a deep scale that is well-formed and has Myhill's property. Johnson has found a brilliant way of leading students to these discoveries, and as a result, students using this text will most likely come to a deeper understanding of the diatonic scale and its properties than is usually found in any traditionally taught undergraduate music theory course. They will also most likely come to a deeper understanding of other scales, both in terms of their properties and in how they compare to the diatonic scale.

Because of its innovative approach, *FDT* will come as a breath of fresh air for those who decide to incorporate it into fundamentals courses. It would also provide courses on music and mathematics, and on diatonic set theory, with a way of getting into the material that encourages students to think critically—a most desirable quality in a text, as critical thinking should be demanded of the student by any graduate or upper-division undergraduate course. If Johnson's Montessori-style approach is contagious among the next generation of textbook authors, the benefits to theory students and instructors alike could be enormous.

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ABOUT THE AUTHOR

Matthew Santa is Associate Professor of Music Theory and Chair of the Theory/Composition Area at Texas Tech University.

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